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The Role of Agency Costs in Financial Conglomeration*

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Abstract

This paper focuses on the role of managerial agency costs in financial conglomerations. We model conglomerations as the integration of commercial and investment banking in one organizational unit where bank managers accomplish both activities. We assume that managers differ in their abilities to undertake the individual tasks. The higher is a manager's ability in undertaking one task, the lower is her disutility of effort for that activity and the higher is her disutility of effort for the other task. When there is no managerial moral hazard, it is not optimal for the bank to form a conglomerate. We show that under managerial moral hazard, forming a conglomerate may be in the bank's interest because it may entail lower agency costs and a larger group of borrowers to fund.

Keywords: Financial Conglomerates, Commercial Banking, Investment Banking, Banking Organization, Multi-task, Moral Hazard.

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1 Introduction

What is the optimal scope of a bank’s activities? A few would deny the benefits of financial conglomeration, but the determinants of the scope and size of a bank’s activities are not well-defined. This paper focuses on the role of agency costs in determining bank size and whether banks should be organized as financial conglomerates or specialized intermediaries.

The idea developed in the paper is that under financial conglomeration bank managers’ efforts may be a source of economies of scope. We show that managerial agency costs may be lower when banks engage in multiple activities, e.g. lending and non-lending financial services, rather than specialize in individual activities. We argue that financial conglomeration occurs when the agency cost of providing a set of services by generalist bank managers is lower than the agency cost of providing the same set of services by specialist bank managers. In fact, conglomeration may entail a reduction in agency costs because generalist bank managers accomplish a multiplicity of tasks: their compensation can be conditioned on the success of more than one task.

In recent years, we could observe the appearance of financial conglomerates engaging in traditional banking as well as other, non-interest income generating business such as insurance or investment banking. The process has been supported by regulatory changes that abolished the limits to the formation of financial conglomerates both in Europe and the US. Nevertheless, in most countries conglomerates and specialized banks coexist and financial institutions differ in the extent they diversify their activities. The results of our analysis are consistent with this observation. We suggest that managerial agency costs affect whether a profit-maximizing bank adopts a conglomerate structure or breaks up its organization into specialized institutions, as well as the chosen bank size.

In the model, bank managers are agents of a profit-maximizing financier (bank) and may perform one or two tasks: commercial banking and investment banking. We define commercial banking as a combination of lending to and monitoring of an endogenously chosen set of borrowers under moral hazard. In turn, investment banking may be any non-lending activity that brings a return on capital. A financially unconstrained bank maximizes profits by lending to all borrowers for whom the moral hazard problem can be overcome through the means of monitoring and by choosing an

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1 A specialist banker is skilled at one type of activity whereas a generalist banker has an intermediate ability to perform more than one banking services. We clarify below the distinction between generalist and specialist bankers.

2 We analyze managerial moral hazard in financial conglomerates without focusing on the effect of conglomeration on managerial risk-taking behavior. For an analysis of the trade-off between the co-insurance benefits of conglomeration and managerial risk taking incentives, see Boot and Schmeits (2000) or Freixas et al. (2007).

3 In the European Union, conglomeration and universal banking has been supported by the implementation of the Second Banking Directive in 1989. In the US, the formation of affiliations between commercial banks, securities firms, and other financial companies has been allowed by the Financial Services Modernization (or Gramm-Leach-Bliley) Act that was enacted in 1999.
optimal form of banking organization. An optimal organization is defined by i) a set of borrowers
to lend capital and thereby the size of the bank, ii) manager(s) to accomplish the lending and non-
lending tasks, iii) an organizational form based on the separation or integration of the lending and
non-lending tasks. We define an ‘integrated’ organizational structure encompassing both lending
and non-lending activities a financial conglomerate. In our context, therefore, a conglomerate is
characterized as a unified institution that assimilates the bank’s multiple activities.4

In the model, bank managers differ in their relative abilities to undertake the lending and non-
lending activities. Managers having a comparative advantage in one task are specialists in the
task whereas managers with an intermediate ability in both tasks are considered as generalists.
We model the difference in managers’ relative abilities by their respective disutility of effort to
undertake the two activities. The higher is a manager’s ability in undertaking one activity, the
lower is her disutility of effort for the given task but the higher is her disutility of effort for the other
task. Consequently, managers cannot be specialists in both the lending and non-lending activities.
Managers are therefore heterogeneous in their ability, which increases the benefits of specialization
for a financier creating an organization as a group of specialized banking units.

Our key insights are as follows. When there is no managerial moral hazard, a profit-maximizing
financier hires bank managers based on their comparative advantages in the individual tasks. Con-
sequently, the lending and non-lending tasks will be accomplished by two specialist managers. The
 corresponding organization of activities is such that commercial and investment banking services
are provided by specialized banks. In contrast, under managerial moral hazard, the size of the
agency costs determines the optimal organizational form. We show that, under managerial moral
hazard, agency costs may be lower when the financier hires a generalist manager who undertakes
both the lending and non-lending tasks. Therefore, the financier may maximize profits through
the integration of the two tasks within a single bank. The corresponding organizational form is a
financial conglomerate where generalist bank managers perform both services for the bank’s clients.

At first, it may seem unusual to assume that a typical bank manager accomplishes both lend-
ing and non-lending activities within the realm of one banking organization. In many financial
conglomerates however, bank employees are relationship bankers that engage in a range of ser-
 vices required by firms belonging to the bank’s clientele. Relationship bankers allocate loans to
corporate clients, acquire information to monitor and renegotiate existing loan agreements, act as
lead underwriter at corporate security issues, and advise their clients regarding decisions about
capital market investments. Even if in some banks, separate teams specialize in commercial and
investment banking activities, the same employees may become engaged in the provision of different

4This definition of a financial conglomerate may differ from the definition applied in the earlier literature. We
provide arguments in support of our definition of a conglomerate as a unified banking institution that performs both
lending and non-lending activities below.
types of services for a particular corporate client. For example, Citigroup is organized into two major segments, Citicorp and Citi Holdings. Nevertheless, substantial efforts have been aimed at the integration of the two units by stimulating commercial and investment bankers to put product lines together and make joint calls on corporate clients.\textsuperscript{5,6}

In the model, optimal hiring decisions and the choice of organizational form have important implications for credit allocation and bank size. A profit-maximizing financier chooses the size of the borrower group such that funding is provided to all borrowers transparent enough to be eligible for funding under monitoring. In particular, we assume that the effectiveness of monitoring and thereby funding possibilities depend on borrowers’ opacity (transparency). Less opaque (more transparent) borrowers, can be easily monitored and thus funded. Bank size is therefore determined by the opacity of the marginal borrower that is fundable under moral hazard and under the chosen equilibrium organizational structure.

Our results suggest that a profit-maximizing financier chooses to integrate rather than separate the lending and non-lending tasks if and only if the equilibrium size of the bank for an integrated organization is larger. In equilibrium, the profit-maximizing organizational form is the one that entails the financing of the larger borrower group. In other words, it is in the interest of the profit-maximizing financier to choose the organization such that the number of borrowers funded in maximized. Consequently, financial conglomerates would naturally arise when, as a consequence of high agency costs, specialized banks have little capacity to fund financially constrained borrowers.

Our model provides insights for the literature on the valuation effects of conglomeration in the financial intermediation industry. Laeven and Levine (2007) find evidence of a valuation discount associated with financial firms that engage in multiple activities. They argue that the discount is due to agency problems inherent in the conglomorate structure and that economies of scope generated by conglomeration would be eliminated by the discount. Schmid and Walter (2009) and Stiroh and Rumble (2006) also provide evidence of a valuation discount. In contrast, Baele, De Jonghe and Vander Vennet (2007), Elsas, Hackethal and Holzhäuser (2010) and Van Lelyveld and Knot (2009) characterize a valuation premium that may be attributed to economies of scope. Our model shows that, under managerial moral hazard, conglomeration may bring about economies of scope and thus a valuation premium. At the same time, when agency costs associated with conglomeration are high, conglomerates will be characterized by a valuation discount.

The paper contributes to the literature on financial conglomeration. Focusing on managerial

\textsuperscript{5}“According to media reports, Citi is creating a new unit that would officially combine the two disciplines. For many clients, the bank has already identified a single relationship manager to handle both needs, reports CNBC.”, FierceFinance, December 17, 2008, http://www.fiercefinance.com/story/citi-combine-commercial-and-investment-banking/

risk-taking incentives, Boot and Schmeits (2000) argue that market discipline, i.e. investors’ understanding of the risk choice of an institution, mitigates the coinsurance benefits of diversification associated with financial conglomerates. In their model, conglomeration decreases the sensitivity of a firm’s funding cost to managerial risk-taking. Consequently, under perfect market discipline, risk-taking incentives are lower in stand-alone institutions. Under imperfect market discipline, conglomeration may be optimal: the diversification benefits may dominate the negative incentive effect on managerial risk-taking. Freixas et al. (2007) focus on the optimal organization of divisions in conglomerates composed of a bank and a non-bank (insurance) units. They show that under an integrated organizational structure the diversification benefits of conglomeration may be diminished by the increased risk-taking induced by the extension of the deposit insurance safety net to the firm’s non-bank division. Rather than focusing on the trade-off between the benefits of diversification and divisional risk-taking incentives, we investigate whether the integration of individual activities an institution is engaged in into one organization may generate lower agency rents than breaking up the institution into specialized organizations.

Complementing our paper, Ross (2007) compares universal and specialized banks from an agency cost perspective. In his model, when the lending and non-lending tasks are mutually independent the integration of tasks (universal banking) is optimal because it entails lower agency costs. When tasks are complementary, however, agency costs are higher under universal banking. If both a lending and a non-lending task have to be accomplished, a risk-averse banker incurs a larger loss when the borrower turns out to be of low credit quality. This may distort incentives for information acquisition and lending. In contrast to Ross (2007), we assume that without managerial moral hazard, the integration of the lending and non-lending tasks is suboptimal for the bank. We show that, under moral hazard, conglomeration may result in lower agency costs than the allocation of tasks to financial intermediaries that specialize in individual activities.

The empirical literature analyzing the different services banks perform focuses on the conflict of interest arising from the participation of commercial banks in the underwriting of corporate security issues. Because of their involvement in lending, commercial banks have an informational advantage relative to investment banks in the underwriting business. The evidence suggests that commercial banks do not exploit their informational advantage by selling low quality securities to the uninformed public (Ang and Richardson (1994), Kroszner and Rajan (1994), Puri (1994), Hebb and Fraser (2002), Konishi (2002)). Rather than investigating the specific conflict of interests generated by commercial banks’ participation in the underwriting business, we focus on the effect of heterogenous managerial ability on the optimal organization of lending and non-lending banking activities in a multi-task setting.

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5 Banks’ involvement in activities other than collecting deposits and lending has also been considered by Berlin, John, and Saunders (1994) and Puri (1996, 1997).

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A number of papers investigated empirically whether conglomerate in banking leads to higher cost, revenue, and operational efficiency. The literature has not come to an unambiguous conclusion in favor of either of the two organizational structures (see Allen and Rai (1996) and Berger, Hunter and Timme (1993)). Benston (1989) and Saunders and Walter (1994) suggest that the combination of different financial intermediary services is revenue efficient and does not increase overall risk. Vander Vennet (2002) argues that financial conglomerates are more cost and revenue efficient than their specialized competitors. Furthermore, in line with our results, Berger, Hancock, and Humprey (1993) find evidence suggesting that larger banks are more efficient.

The paper is also related to the literature on multi-task moral hazard analysis. In the general analysis of Holmström and Milgrom (1991), the effort cost of performing one task may increase or decrease in the effort exerted on the other task. In our task allocation problem, we assume that managers highly skilled in one task have a high disutility of effort when undertaking the other task. We show that even under this assumption the efficient organization of tasks may demand an integrated organizational structure (i.e. financial conglomerate). In a related vein, Laux (2001) provides a rationale for the allocation of multiple projects to a single agent by showing that the multiplicity of tasks may improve on the limited liability-incentive provision trade-off under moral hazard. We show that heterogeneity in the managers’ abilities limits the extent to which this result holds. Moreover, our framework allows us to address the problem of bank size. Dewatripont and Tirole (1999) analyze the integration versus separation of substitute managerial tasks. They show that, when allocating tasks to two competing agents, each collecting one signal rather than one gathering two, the principal enhances incentives for information collection and thereby improves the quality of decision-making. In contrast, in our model selecting one agent to undertake the two tasks may allow the financier to reduce agency costs and thus increase profits.

The rest of the paper is as follows. Section 2 describes the model. Section 3 provides a benchmark solution assuming efforts on the two managerial tasks are observable. Section 4 provides our solution under the assumption of managerial moral hazard and derive the condition under which the financier’s profits are higher when choosing a conglomerate structure. Section 5 considers the robustness of the model. Section 6 concludes.

2 The Model

Consider the problem of a financier engaged in lending as well as a non-lending activity. The lending and non-lending activities are carried out in two subsequent periods. In the first period,
the financier lends capital to a group of borrowers. In the second period, the financier invests the amount of capital available at the end of the first period, from each borrower’s project, in the capital market. The gross expected rate of return demanded by the financier is \((1 + i)\).

The model has three types of agents: besides the financier, borrowers, and managers. Each borrower may invest in a project that requires investment \(I\) and may yield a positive outcome \(R > I\) in case of success and 0 in case of failure. Borrowers may work or shirk on their projects. If the borrower works, the probability of obtaining a positive outcome is \(p_H\). If the borrower shirks, the probability of obtaining a positive outcome is \(p_L < p_H\) and the borrower derives private benefit of size \(B\). Each borrower can be characterized with a level of transparency \((1 - s)\), where \(s\) is uniformly distributed on the interval \([0, 1]\). Finally, each borrower has a specific amount of financial capital \(A\), where \(A\) is uniformly distributed on the interval \([0, A]\).

In both periods, the financier may hire an agent (manager). In the first period, to support the lending activity, the manager may monitor the borrowers and thereby reduce private benefits from \(B\) to \(sB\). A borrower with a high \(s\) is opaque and is thus more difficult to be monitored. In what follows, we will refer to \(s\) as the ‘opacity’ of the borrower’s project. In the second period, the manager may carry out the non-lending task utilizing his skills to increase the return on capital available from loans repaid at the end of the first period. If the manager exerts effort, the probability of earning a gross return \(r\) on invested capital is \(\lambda_H\). If the manager shirks on this second task, the probability of earning a gross return \(r\) on invested capital is \(\lambda_L < \lambda_H\).

Managers differ in their abilities across tasks. In particular, the effort cost of carrying out the two activities depends on the manager’s ability, which we denote by \(\theta \in [0, 1]\). The effort cost of monitoring a borrower and thereby reducing his private benefit is \(c_1(\theta)\) where \(\frac{dc_1(\theta)}{d\theta} \geq 0\) and \(\frac{d^2c_1(\theta)}{d\theta^2} \geq 0\). The effort cost of earning a gross return \(r\) on invested capital is \(c_2(\theta)\) where \(\frac{dc_2(\theta)}{d\theta} \leq 0\) and \(\frac{d^2c_2(\theta)}{d\theta^2} \geq 0\). Essentially, the setup captures the idea that, depending on their abilities, managers may be generalists or specialists in a particular task. Managers with a low disutility of effort in either task can be thought of as specialists. In turn, managers with an intermediate ability to accomplish both tasks are considered as generalists. In the remaining of the paper, we will refer to managers with \(\theta = 0\) and \(\theta = 1\) by the term ‘specialist bank managers’.9

Finally, the financier may integrate or separate the two tasks by hiring one or two managers and choose among managers with different abilities.

The timing of events is as follows. First, the financier decides whether to hire one or two managers for the lending and non-lending tasks. In the beginning of the first period, the financier chooses the group of borrowers to finance and thereby the total amount of capital to lend. Then borrowers exert effort and the manager with the lending task monitors the borrowers. At the end of

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9 Notice that our specification is equivalent to assuming that a manager has ability \(\theta\) for one task and \(1 - \theta\) for the other, both effort costs being decreasing in the respective managerial ability.
the period, borrowers’ projects yield a positive outcome or zero. In the second period, the manager with the non-lending task invests the capital available at the end of the first period. The financier’s final profit is realized at the end of the second period.

We make the following assumptions. Every project has a positive expected value when the borrower works, even if the project is monitored by a specialist bank manager:

\[ p_{H}R(\lambda_{H}(r-1)+1) - I - c_{1}(1) \geq 0 \]

Furthermore, the borrower’s work is essential for the project to have a positive value ex-ante:

\[ p_{L}R(\lambda_{H}(r-1)+1) - I + B \leq 0 \]

3 Benchmark: Observable Managerial Effort

In this section, we provide a benchmark solution for the financier’s problem of organizing the bank as a financial conglomerate or creating two separate banking organizations. Our benchmark model assumes that the managers’ efforts are observable.\(^{10}\)

3.1 Separation

We assume here that the lending and non-lending activities are separated. We refer to the manager with the monitoring task by the term ‘first manager’ and to the manager with the non-lending task by the term ‘second manager’. We denote the two managers’ types by \(\theta_{1}\) and \(\theta_{2}\) and their respective shares in the return on a borrower’s project by \(R_{m1}\) and \(R_{m2}\). Furthermore, we denote the borrower’s share in final project returns by \(R_{b}\). In what follows, first we solve the financier’s credit allocation problem for given levels of \(s, \theta_{1}, \theta_{2}\). Then we solve the financier’s profit-maximization problem to find the equilibrium bank size and managerial types to be hired by the financier.

Since there is no managerial moral hazard, the financier will compensate the managers only for the cost of exerting effort:

\[ p_{H}R_{m1} \geq c_{1}(\theta_{1}) \]
\[ p_{H}\lambda_{H}R_{m2} \geq c_{2}(\theta_{2}) \]

The borrower’s effort is not observable. His incentive compatibility constraint is:

\[ p_{H}R_{b} \geq p_{L}R_{b} + sB \]

\(^{10}\)To preserve the role of monitoring in this benchmark model, we assume that the entrepreneur’s effort is unobservable. Therefore, our benchmark model does not provide a first-best solution. It assumes that the financier is informed about the efforts exerted by the bank managers but remains uninformed about the borrower’s effort choice.
The financier’s participation constraint (for every borrower) can thus be written as:

\[ p_H (\lambda_H R r + (1 - \lambda_H) R) - p_H R_b - p_H R_{m_1} - p_H \lambda_H R_{m_2} \geq (1 + i) (I - A) \]

Rearranging the constraint, we obtain the financing condition:

\[ A(s, \theta_1, \theta_2) \geq I + \frac{p_H s B - p_H (\lambda_H R r + (1 - \lambda_H) R) + c_1(\theta_1) + c_2(\theta_2)}{(1 + i)} = A^B(s, \theta_1, \theta_2) \]

The financier maximizes profits by providing funding to every borrower with a profitable investment project. Consequently, for given \((\theta_1, \theta_2)\), the optimal amount to lend will be determined by the level of opacity of the marginal borrower that is eligible for financing when monitored by the first manager, \(s^B(\theta_1, \theta_2)\). Essentially, the level of transparency (opacity) of the marginal borrower determines the equilibrium size of the group of borrowers funded by the financier. We will therefore refer to \(s^B(\theta_1, \theta_2)\) as the benchmark equilibrium bank size. As \(A\) is uniformly distributed on \([0, A]\), the financier’s profits can be expressed as:

\[
\Pi^*_B(s^B, \theta_1, \theta_2) = \int_0^{s^B} \left[I - A^B(s, \theta_1, \theta_2)\right] \Pr \left[A^B(s, \theta_1, \theta_2) \leq A \leq A^B(1, \theta_1, \theta_2)\right] ds \\
= \frac{i}{(1 + i)^2 s^B} \left(p_H (\lambda_H R r + (1 - \lambda_H) R) - \frac{p_H s^B}{2} - c_1(\theta_1) - c_2(\theta_2)\right) \left(1 - s^B\right) \frac{p_H s^B}{A}
\]

Solving the model, we obtain the following intuitive result.

**Lemma 1** When the lending and non-lending tasks are separated and the managers’ efforts are observable, the financier’s optimal choice of the managers’ type is \(\theta_1^B = 0, \theta_2^B = 1\). Moreover, the benchmark equilibrium bank size \(s^B(0, 1)\) is:

\[
s^B(0, 1) = \frac{\left[\alpha_B(0, 1) + \frac{p_H B}{\Delta p}\right] - \sqrt{\left(\alpha_B(0, 1)\right)^2 + \left(\frac{p_H B}{\Delta p}\right)^2 - \frac{p_H B}{\Delta p} \alpha_B(0, 1)}}{\frac{3p_H B}{\Delta p}}
\]

where \(\alpha_B(0, 1) = 2 \left[p_H R [\lambda_H (r - 1) + 1] - C_B(0, 1)\right]\),

and \(C_B(0, 1) = c_1(0) + c_2(1)\).

**Proof.** The Proof is in the Appendix. □

When the managers’ efforts are observable, the financier maximizes the amount of capital to lend to borrowers and thereby its profits by hiring two specialist managers for the lending and non-lending tasks. Since there are no agency problems on the managers’ side, the financier selects the managers with the highest ability in both tasks. The financier does not pay agency rents, but has to make the two managers participate by compensating them for their costs of effort.
3.2 Integration

When the lending and non-lending activities can not be separated, the financier chooses a manager with ability $\theta^B$ so that profits are maximized. The financier compensates the manager only for the cost of exerting effort:

$$p_H \lambda_H R_m \geq c_1(\theta) + c_2(\theta)$$

The borrower’s incentive constraint is:

$$p_H R_b \geq p_L R_b + s^B B$$

The financier’s participation constraint (for every borrower) can thus be written as follows.

$$p_H (\lambda_H R r + (1 - \lambda_H) R) - p_H R_b - p_H \lambda_H R_m \geq (1 + i) (I - A)$$

Rearranging the constraint, we obtain the financing condition:

$$A(s, \theta) \geq I + \frac{s^B \frac{p_H B}{2 \delta} - p_H R (1 + \lambda_H (r - 1)) + c_1(\theta) + c_2(\theta)}{(1 + i)} = A^B(s, \theta)$$

Similar to the case of task separation, the threshold level of capital $A^B(s(\theta), \theta)$ that is required for the borrower to get funding decreases in the level of transparency (increases in the parameter $s$). The financier maximizes profits by lending to all borrowers that are eligible for funding. Therefore, for given $\theta$, the optimal amount to invest will be determined by the opacity of the marginal borrower. We denote this level of opacity by $s^B(\theta)$. The financier’s choice of $s^B(\theta)$ determines the size of the borrower group to be funded and thereby the equilibrium bank size. The financier’s profits can be written as:

$$\Pi_B^I(s^B(\theta), \theta) = \int_0^{s^B} [I - A(s(\theta), \theta)] \Pr [A(s^B(\theta), \theta) \leq A \leq A^B(1, \theta)] \, ds$$

Solving for the equilibrium $s^B(\theta)$, we obtain the following result.

**Lemma 2** When the lending and non-lending tasks can not be separated and the manager’s efforts are observable,

i) if $c_1(\theta)$ and $c_2(\theta)$ are such that for all $\theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) > 0$, then the financier maximizes profits by choosing a manager specialized in lending $\theta^B = 0$.

ii) if $c_1(\theta)$ and $c_2(\theta)$ are such that for all $\theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) < 0$, then the financier maximizes profits by choosing a manager specialized in the non-lending activity $\theta^B = 1$.
iii) if there exists \( \hat{\theta} \) such that \( \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) = 0 \), then there exists an interior solution \( \theta^B = \hat{\theta}^B \) and the financier chooses a generalist manager to accomplish both tasks.

Moreover, the benchmark equilibrium bank size \( s^B (\theta^B) \in (0, \frac{1}{2}) \) and:

\[
s^B (\theta^B) = \frac{\alpha_B (\theta^B) + \frac{p_H B}{\Delta P}}{\sqrt{\left( \alpha_B (\theta^B) + \frac{p_H B}{\Delta P} \right)^2 - \frac{3p_H B}{2p} \alpha_B (\theta^B)}}
\]

where \( \alpha_B (\theta^B) = 2 \left[ p_H R \left( \lambda_H (r - 1) + 1 \right) - C_B (\theta^B) \right] \),
and \( C_B (\theta^B) = c_1 (\theta^B) + c_2 (\theta^B) \).

Proof. The Proof is in the Appendix.

When the cost of the lending activity is more sensitive to the financier’s choice between a specialist and a generalist bank manager than the cost of the non-lending task, in order to minimize effort costs, the financier will hire a manager specialized in lending. On the other hand, when the effort cost of the non-lending activity is more sensitive to the manager’s ability than the effort cost of the lending task, an investment banking specialist will be hired. Finally, when the efforts costs of the two tasks are equally sensitive to \( \theta \), a generalist manager will be hired to perform both tasks.

The financier’s choice of the manager therefore depends on the relative sensitivity of the disutilities of efforts to managerial ability. This is due to the fact that the financier’s expected revenue from the lending and non-lending activities (the pledgeable income) decreases in the total disutility of efforts on both tasks. Consequently, it is optimal for the financier to select the manager’s ability so that total effort costs are minimized.

We characterized the financier’s optimal choice of managers and bank size assuming the separation and the integration of tasks, under the assumption that managerial effort is observable. The optimal organizational form is determined by the size of the financier’s profits.

**Proposition 1** When managerial effort is observable, the financier chooses an organization based on the separation of lending and non-lending activities.

Proof. The Proof is in the Appendix.

The result is not surprising. Indeed, the separation of lending and non-lending activities allows the financier to minimize effort costs by hiring specialist managers based on their comparative advantages in the two tasks. Given managerial effort is observable, the financier optimally chooses to break the institution into two specialized financial intermediaries. The total expected wage the financier pays as a compensation for managerial effort equals the total cost of effort.

In this benchmark case, it would never be in the financier’s interest to choose the integration of tasks: when there are no managerial agency problems, conglomeration should not occur.
4 Task Allocation Under Moral Hazard

In this section, we consider the financier’s choice of optimal task allocation under the assumption of unobservable managerial effort. First, we solve the model for the financier’s choice of equilibrium bank size and managerial types under the assumption that the lending and non-lending tasks are separated. Then, we consider the optimal bank size and hiring choice assuming that the financier may hire only one manager to execute the two tasks. Finally, we compare the financier’s profits under the two banking organizational structures: the separation and integration of managerial tasks.

4.1 Two Managers (Separation of Tasks)

Assume the lending and non-lending activities are separated. The financier chooses managers with optimal abilities $\theta_1^*, \theta_2^*$ so that the amount of capital to lend $\Pi^S(s, \theta_1^*, \theta_2^*, \theta_1, \theta_2)$ is maximized. The financier is not capital constrained: funding is provided for every project transparent enough so that the moral hazard problem can be overcome through the means of monitoring. The first manager exerts monitoring effort if the following incentive compatibility constraint holds.

$$p_H R_{m_1} \geq p_L R_{m_1} + c_1(\theta_1)$$

The incentive constraint for the manager with the non-lending task is as follows.

$$p_H \lambda_H R_{m_2} \geq p_H \lambda_L R_{m_2} + c_2(\theta_2)$$

Given that the first manager monitors, the borrower’s incentive compatibility constraint is:

$$p_H R_b \geq p_L R_b + sB$$

For each borrower’s project, the financier’s participation constraint can be written as:

$$p_H (\lambda_H R + (1 - \lambda_H) B) - p_H R_b - p_H R_{m_1} - p_H \lambda_H R_{m_2} \geq (1 + i)(I - A)$$

Rearranging the constraint, we obtain the per project financing condition:

$$A(s, \theta_1, \theta_2) \geq I + \frac{p_H \left( \frac{sH}{\Delta p} \right) - p_H R (1 + \lambda_H (r - 1)) + p_H \frac{c_1(\theta_1)}{\Delta p} + p_H \lambda_H \frac{c_2(\theta_2)}{\Delta \lambda}}{(1 + i)} = A^*(s, \theta_1, \theta_2)$$

The condition shows that the threshold level of capital $A^*(s, \theta_1, \theta_2)$ required for the borrower to get funding decreases in the level of the borrower’s transparency (increases in the parameter $s$): monitoring transparent borrowers reduces moral hazard and increases pledgeable income to a larger extent than monitoring opaque borrowers. Furthermore, given the effort cost functions $c_1(\theta_1)$ and $c_2(\theta_2)$, the threshold level of capital required for funding decreases in the abilities of specialist
managers skilled in their respective tasks. When hired to monitor, a manager skilled in the lending
task may benefit from local information and thereby reduce borrower side moral hazard at a low
cost. In contrast, a manager skilled in the non-lending task needs substantial rents to have the
incentive to monitor. Hiring the latter for the non-lending task will, however, increase pledgeable
income and thereby eliminate funding constraints. Agency rents thus depend on the managers’
disutilities of efforts for the two tasks. As the pledgeable income decreases in the agency rent,
credit rationing is less severe when total effort costs are lower. In fact, the lower $\theta_1$ and the higher
$\theta_2$, the lower the agency costs are.

Under the assumption that efforts are unobservable, the financier has to pay agency rents to
induce the two managers to exert effort on their respective tasks. Consequently, for every borrower
with a specific level of opacity $s$, the threshold level of own capital required to obtain funding is
higher than in the benchmark case $A^*(s, \theta_1, \theta_2) > A^B(s, \theta_1, \theta_2)$.

The financier maximizes profits by lending to all borrowers eligible for funding. Therefore, for
given $(\theta_1, \theta_2)$, the optimal amount to lend will be determined by the level of opacity of the marginal
borrower that is eligible for financing. We denote this level of opacity by $s^*(\theta_1, \theta_2)$ and refer to it
as the optimal bank size under the separation of tasks. As $A$ is uniformly distributed on $[0, \overline{A}]$,
the financier’s profits can be expressed as:

$$\Pi^S(s(\theta_1, \theta_2), \theta_1, \theta_2) = \int_0^{s^*} [I - A(s(\theta_1, \theta_2), \theta_1, \theta_2)] \Pr[A(s(\theta_1, \theta_2), \theta_1, \theta_2) \leq A \leq A(1, \theta_1, \theta_2)] ds$$

$$= \frac{i}{(1 + \frac{r}{\lambda_H})} s^* \left( p_H R (1 + \lambda_H (r - 1)) - \frac{p_H B (s^*)}{\Delta p} \right) \frac{1}{\overline{A}} (1 - s^*) \frac{p_H B}{\Delta p}$$

Solving for the optimal $s^*(\theta_1, \theta_2)$ gives the following result.

**Lemma 3** When the lending and non-lending tasks are separated, the equilibrium bank size is given
by $s^*(\theta_1, \theta_2)$ where

$$s^*(\theta_1, \theta_2) = \left[ \alpha_S (\theta_1, \theta_2) + \frac{p_H B}{\Delta p} \right] - \sqrt{\left( \alpha_S (\theta_1, \theta_2) \right)^2 + \left( \frac{p_H B}{\Delta p} \right)^2 - \frac{p_H B}{\Delta p} \alpha_S (\theta_1, \theta_2)}$$

where $\alpha_S (\theta_1, \theta_2) = 2 [p_H R [\lambda_H (r - 1) + 1] - C_S (\theta_1, \theta_2)]$,
and $C_S (\theta_1, \theta_2) = \frac{p_H}{\Delta p} c_1 (\theta_1) + \frac{\lambda_H}{\Delta \lambda} c_2 (\theta_2)$.

and $s^*(\theta_1, \theta_2) \in (0, \frac{1}{2})$.

**Proof.** The Proof is in the Appendix. ■

The financier chooses managers with abilities $\theta_1^*, \theta_2^*$ such that the amount of capital to lend
and thus profits are maximized. The equilibrium choice of manager types can easily be derived by
expressing the derivative of the function \( \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2) \) with respect to \( \theta_1 \) and \( \theta_2 \). By the Envelope Theorem:

\[
\frac{d\Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} = \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*}{\partial \theta_1} + \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1}
\]

\[
= - \frac{i}{(1+i)^2} \frac{1}{A} \Delta p (1-s^*) s^* \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1} + \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1}
\]

\[
\frac{d\Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} = \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*}{\partial \theta_2} + \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2}
\]

\[
= - \frac{i}{(1+i)^2} \frac{1}{A} \Delta p (1-s^*) s^* \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2} + \frac{\partial \Pi^S (s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2}
\]

The following proposition summarizes the result.

**Proposition 2** When the lending and non-lending tasks are separated, the financier’s optimal choice of managers is such that \( \theta^*_1 = 0, \theta^*_2 = 1 \). Moreover, in equilibrium \( \frac{ds^*_1}{d\theta_1} < 0 \) and \( \frac{ds^*_2}{d\theta_2} > 0 \), therefore the size of the bank \( s^*(\theta^*_1, \theta^*_2) \) is the highest possible.

**Proof.** The Proof is in the Appendix. ■

Similar to the benchmark case of observable managerial effort, when tasks are separated, the financier hires two specialist managers: one with a low disutility of effort for the lending task and another with a low disutility of effort for the non-lending task. Since the agency rent to be paid by the financier to compensate the managers for their efforts decreases in the monitoring ability of the first manager (increases in \( \theta_1 \)) while decreases in the ability for the non-lending task, of the second manager, (increases in \( \theta_2 \)) the financier maximizes profits when hiring specialist managers for both tasks. The financier’s choice of the managers’ types minimizes agency rents, maximizes pledgeable income, and, consequently, the size of the borrower group to fund.

### 4.2 The One-Manager Case (Integration)

We assume now that effort is unobservable and that the lending and non-lending tasks can not be separated. The financier chooses a manager with optimal ability \( \theta^* \) so that profits are maximized.

The manager monitors the project and subsequently exerts effort on the non-lending task if the following incentive condition holds:

\[
p_H \lambda_H R_m \geq p_L \lambda_L R_m + c_1(\theta) + c_2(\theta)
\]

Given the manager monitors, the borrower’s incentive constraint is:

\[
p_H R_b \geq p_L R_b + sB
\]
For every borrower’s project, the financier’s participation constraint can be written as follows.

\[ p_H (\lambda_H Rr + (1 - \lambda_H) R) - p_H R_b - p_H \lambda_H R_m \geq (1 + i) (I - A) \]

Rearranging the constraint, we obtain the financing condition:

\[ A(s, \theta) \geq I + \frac{p_H \left( s^B \frac{p_H}{A} \right) - p_H R (1 + \lambda_H (r - 1)) + p_H \lambda_H c_{1}(\theta) + c_{2}(\theta)}{(1 + i)} = A^*(s, \theta) \]

Similar to the case of task separation, the threshold level of capital \( A^*(s(\theta), \theta) \) required for the borrower to get funding decreases in the transparency of the borrower (increases in the parameter \( s \)). Furthermore, the lower the manager’s disutility of effort in the lending task, the lower is the effort cost of monitoring, and, at the same time, the higher is the effort cost of the non-lending activity. Specialist managers will have a low cost of effort only in the task they are skilled at. Generalist managers will have an intermediate cost of effort in both tasks. The overall impact of the manager’s type on credit allocation will depend on the specific form of the effort cost functions \( c_{1}(\theta) \) and \( c_{2}(\theta) \).

The financier maximizes profits by lending to all borrowers that are eligible for funding. Therefore, for given \( \theta \), the optimal amount to invest and the size of the bank will be determined by the level of opacity of the marginal borrower. We denote this level of opacity by \( s^*(\theta) \). The financier’s profits can be written as:

\[ \Pi^I(s(\theta), \theta) = \int_0^{s^*} \left[ I - A(s(\theta), \theta) \right] \Pr [A(s(\theta), \theta) \leq A \leq A(1, \theta)] ds \]

Solving for \( s^*(\theta) \), we obtain the following result.

**Lemma 4** When the lending and non-lending tasks can not be separated, the equilibrium bank size is given by the level of opacity \( s^*(\theta) \) such that

\[ s^*(\theta) = \frac{\alpha_1(\theta) + \frac{p_H B}{A}}{\frac{3p_H B}{A}} - \sqrt{\left(\frac{\alpha_1(\theta)}{\alpha_1(\theta)}\right)^2 + \left(\frac{p_H B}{A}\right)^2 - \frac{p_H B}{A} \alpha_1(\theta)} \]

where \( \alpha_1(\theta) = 2 [p_H R [\lambda_H (r - 1) + 1] - C_1(\theta)] \),

and \( C_1(\theta) = \frac{\lambda_H p_H}{\lambda_H p_H - \lambda_H \lambda_{PL}} [c_1(\theta) + c_2(\theta)] \).

where \( s^*(\theta) \in (0, \frac{1}{2}) \).
The financier chooses the manager’s type to maximize the amount of capital to lend and thus profits. His equilibrium choice will depend on the form of the effort cost functions, \( c_1(\theta_1) \) and \( c_2(\theta_2) \). To show this, we express \( \frac{d\Pi^I(s^*(\theta), \theta)}{d\theta} \) by the Envelope Theorem.

\[
\frac{d\Pi^I(s^*(\theta), \theta)}{d\theta} = \frac{\partial \Pi^I(s^*(\theta), \theta)}{\partial \theta} \frac{ds^*(\theta)}{d\theta} + \frac{\partial \Pi^I(s^*(\theta), \theta)}{\partial \theta} = -\frac{i}{(1+i)^2} \frac{1}{A} \Delta \beta \left(1 - s^*\right) \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} s^* \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right)
\]

The following proposition summarizes the result concerning the financier’s equilibrium choice of the manager’s type.

**Proposition 3** When the lending and non-lending tasks can not be separated, the financier’s choice of the manager’s type depends on the form of the functions \( c_1(\theta) \) and \( c_2(\theta) \).

i) If \( c_1(\theta) \) and \( c_2(\theta) \) are such that for all \( \theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) > 0 \), then the financier maximizes profits by choosing a specialist manager skilled in the lending task \( \theta^* = 0 \). Moreover, in equilibrium \( \frac{ds^*}{d\theta} < 0 \), therefore the size of the bank \( s^*(0) \) is the highest possible.

ii) If \( c_1(\theta) \) and \( c_2(\theta) \) are such that for all \( \theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) < 0 \), then the financier maximizes profits by choosing a specialist manager skilled in the non-lending task \( \theta^* = 1 \). Moreover, in equilibrium \( \frac{ds^*}{d\theta} > 0 \), therefore the size of the bank \( s^*(1) \) is the highest possible.

iii) If there exists \( \tilde{\theta} \) such that \( \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) = 0 \), then there exists an interior solution \( \theta^* = \tilde{\theta} \) and the financier chooses a generalist manager. In equilibrium \( \frac{ds^*}{d\theta} = 0 \) and again, the size of the bank \( s^*(\tilde{\theta}) \) is the highest possible.

**Proof.** The Proof is in the Appendix.

Again, the financier maximizes profits by choosing a manager depending on the relative sensitivity of the costs of exerting effort to her ability. Moreover, the sensitivity of effort costs to managerial ability affects the agency rent and pledgeable income and consequently the size of the borrower group the financier lends to. The financier chooses the size of the bank such that all borrowers for whom the moral hazard problem can be overcome receive funding. The financier’s optimal choice of the manager’s type therefore always entails the largest bank size.

4.3 Integration vs Separation of Tasks

In this section we compare the financier’s profits under the separation and integration of the two managerial tasks, in order to understand the motive to choose one or the other organizational
structure. Since profits depend on the size of the borrower group to fund, we first compare the equilibrium bank size given the financier’s profit-maximizing choices of managerial types under the integration and separation of the two managerial tasks.

Given the result in Lemma 3 and Proposition 2, when tasks are separated, the equilibrium bank size \( s^*(0,1) \) can be expressed as follows.

\[
s^*(0,1) = \frac{\left[ \alpha_S(0,1) + \frac{p_H B}{\Delta p} \right] - \sqrt{\left( \alpha_S(0,1) + \frac{p_H B}{\Delta p} \right)^2 + \left( \frac{p_H B}{\Delta p} \alpha_S(0,1) \right)}}{\frac{3p_H B}{\Delta p}}
\]

where \( \alpha_S(0,1) = 2 [p_H R [\lambda_h (r - 1) + 1] - C_S(0,1)] \),
and \( C_S(0,1) = \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \).

Let us denote the financier’s profit-maximizing choice of managerial type under task integration by \( \theta^* \). According to Proposition 3, \( \theta^* \in \{0, \tilde{\theta}\} \). Given this result and the result in Lemma 4, the equilibrium bank size assuming the integration of tasks \( s^*(\theta^*) \) is given by the following expression.

\[
s^*(\theta^*) = \frac{\left[ \alpha_I(\theta^*) + \frac{p_H B}{\Delta p} \right] - \sqrt{\left( \alpha_I(\theta^*) + \frac{p_H B}{\Delta p} \right)^2 + \left( \frac{p_H B}{\Delta p} \alpha_I(\theta^*) \right)}}{\frac{3p_H B}{\Delta p}}
\]

where \( \alpha_I(\theta^*) = 2 [p_H R [\lambda_h (r - 1) + 1] - C_I(\theta^*)] \),
and \( C_I(\theta^*) = \frac{\lambda_H p_H}{\lambda_H p_H - \lambda_L p_L} [c_1(\theta^*) + c_2(\theta^*)] \).

Using the above expressions, we define the function \( s(C) \) as:

\[
s(C) = \left( \frac{2 [p_H R [\lambda_h (r - 1) + 1] - C] + \frac{p_H B}{\Delta p}}{4 [p_H R [\lambda_h (r - 1) + 1] - C]^2 + \left( \frac{p_H B}{\Delta p} \right)^2 - 2 \frac{p_H R}{\Delta p} \left[ p_H R [\lambda_h (r - 1) + 1] - C \right]} \right)
\]

It follows that the equilibrium bank size under task separation \( s^*(0,1) \) and task integration \( s^*(\theta^*) \) can be expressed as the same function of \( C_S(0,1) \) and \( C_I(\theta^*) \), respectively. Indeed:

\[
s^*(0,1) = s(C_S(0,1)),
\]
\[
s^*(\theta^*) = s(C_I(\theta^*)).
\]

The following result obtains as the function \( s(C) \) is non-increasing in \( C \).

**Lemma 5** The equilibrium bank size under the integration of tasks \( s^*(\theta^*) \) is larger than the equilibrium bank size under the separation of tasks \( s^*(0,1) \) if and only if

\[
\frac{\lambda_H p_H}{\lambda_H p_H - \lambda_L p_L} [c_1(\theta^*) + c_2(\theta^*)] - \left[ \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \right] \leq 0.
\]

where \( \theta^* \in \{0, \tilde{\theta}\} \) is the financier’s choice of the manager’s type defined in Proposition 3.
Proof. The Proof is in the Appendix. ■

The result in Lemma 5 is intuitive. Given the financier’s profit-maximizing choices of managerial types in the two organizational structures, the equilibrium bank size under the separation of tasks is larger than under the integration of the two activities when the total agency rent paid to the managers is lower. A reduction in managerial agency costs increases the financier’s pledgeable income and thereby allows for the funding of a larger borrower group. Whenever conglomeration entails lower agency costs, the size of the borrower group the conglomerate lends to will be larger than the aggregate size of the group of borrowers funded by two specialized institutions. Given the result in Lemma 5, we are able to compare the financier’s profits under the two organizational structures. Under the integration of tasks, the profits are:

\[ 
\Pi^I (s^* (\theta^*), \theta^*) = \frac{i}{(1+i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} \left( \frac{p_H R (1 + \lambda_H (r - 1))}{p_H \lambda_H - p_L \lambda_L} \left( c_1 (\theta^*) + c_2 (\theta^*) \right) - \frac{p_H B (s^* (\theta^*))}{\Delta p} \right) (1 - s^* (\theta^*)) s^* (\theta^*) 
\]

Under task separation, the financier’s profits are:

\[ 
\Pi^S (s^* (0, 1), 0, 1) = \frac{i}{(1+i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} \left( \frac{p_H R (1 + \lambda_H (r - 1))}{p_H \lambda_H - p_L \lambda_L} \left( c_1 (0) + \lambda_H c_2 (1) \right) - \frac{p_H B (s^* (0, 1))}{\Delta p} \right) (1 - s^* (0, 1)) s^* (0, 1) 
\]

The result concerning the financier’s choice of optimal banking organization follows.

**Proposition 4** There exists parameters values \( p_H, p_L, \lambda_H, \lambda_L \) and cost functions \( c_1 (\cdot), c_2 (\cdot) \) such that the financier chooses an organization integrating the lending and non-lending tasks. In particular, the financier’s profits are higher under the integration than under the separation of the two tasks if and only if:

\[
\frac{\lambda_H p_H}{\lambda_H p_H - \lambda_L p_L} \left[ c_1 (\theta^*) + c_2 (\theta^*) \right] - \left[ \frac{p_H}{\Delta H} c_1 (0) + \frac{\lambda_H}{\Delta H} c_2 (1) \right] \leq 0.
\]

where \( \theta^* \in \left\{ 0, 1, \hat{\theta} \right\} \) and \( \hat{\theta} \) is defined in Proposition 3.

Proof. The Proof is in the Appendix. ■

Even though, under the assumption that effort is observable, total effort costs are minimized when the financier hires two specialist managers for the lending and non-lending tasks, the above proposition shows that under moral hazard, the financier may optimally choose to hire one manager to accomplish the two tasks. Notice that Proposition 3 states that this manager may be either a specialist or a generalist depending on the relative sensitivity of the effort costs to managerial ability. In what follows, to focus on the most interesting case we assume that the effort costs of the two tasks are equally sensitive to managerial ability. Under this assumption \( \theta = \hat{\theta} \) and the
equilibrium organization is a financial conglomerate where generalist bank managers perform both the lending and non-lending tasks.

The intuition for the above result is that conglomeration allows the financier to condition the manager’s compensation on the success of multiple tasks and thereby increases the pledgeable income. Due to this effect, when the tasks can be integrated within one organization, there exists an optimal level of managerial ability such that the expected agency cost the financier pays for the accomplishment of the two tasks is lower than the expected agency cost assuming the organization is broken up into two specialized institutions. In equilibrium, the remuneration the financier pays to a generalist bank manager is always higher than the total remuneration paid to two specialist bank managers (i.e. \( c_1(\theta^*) + c_2(\theta^*) > c_1(0) + c_2(1) \)). Nevertheless, under conglomeration, expected agency costs may be lower, since the financier conditions the remuneration of the generalist bank manager on the success of both tasks. Intuitively, under conglomeration the financier pays a higher agency rent but pays less often than when the bank is broken up into two specialized institutions.

To interpret the result in Proposition 4 further, we characterize the circumstances that ensure the optimality of the integration of the lending and non-lending tasks (conglomeration). The result suggests that conglomeration is more likely if the values of \( \Delta p \) and \( \Delta \lambda \) are low relative to the value of \( \lambda_{HPH} - \lambda_{LP_L} \). In fact, \( \Delta p \) and \( \Delta \lambda \) express the marginal productivity of managerial effort for the lending and non-lending tasks, respectively, while \( \lambda_{HPH} - \lambda_{LP_L} \) expresses the marginal productivity of a generalist manager exerting effort on both tasks. Our result therefore states that conglomeration is more likely when the marginal productivity of exerting effort on the two tasks is high relative to the marginal productivities of effort exertion on the individual tasks. This may occur, for instance, when \( \Delta p \) and \( \Delta \lambda \) take intermediate values. Indeed, when \( \Delta p \) and \( \Delta \lambda \) are high, the agency rents to be paid to specialist bank managers to induce them to exert effort are low. To maximize profits, the financier therefore chooses to separate the two tasks. In contrast, when \( \Delta p \) and \( \Delta \lambda \) are low, the agency problems are severe for specialist as well as generalist managers. Hiring a specialist for each task will thus be in the bank’s interest. Consequently, conglomeration may only be optimal when the severity of agency problems is intermediate for the individual tasks. Even in this case, however, the equilibrium organizational form will depend on the financier’s trade-off between paying managers a low compensation more often or a high compensation but less often.

Finally, Lemma 5 and Proposition 4 imply that the financier hires a single manager to carry out the lending and non-lending tasks when the equilibrium size of the bank is larger with an integrated organizational structure than with an organization where the two tasks are separated. The profit-maximizing organizational form is therefore the one that entails the financing of the larger borrower group. The result suggests that it is in the financier’s interest to choose the organization of the bank so that the number of borrowers funded is maximized. An important policy implication of this insight is that, when the purpose is to alleviate credit rationing, policy makers should not
necessarily aim at the regulation of a bank’s organizational form.

5 Robustness

In the previous sections, we have assumed that the amount of financial capital $A$ held by each borrower was uniformly distributed on the interval $[0, \bar{A}]$. In order to check the robustness of our results, assume instead that each borrower has a specific amount of financial capital $A$, where $A$ is distributed on $[0, A(1)]$, with a cumulative distribution function $F$ and a density function $f(.)$. We only make a usual monotone hazard rate assumption on this distribution: $\frac{1-F(.)}{f(.)}$ is non increasing.

The following Proposition proves that our main result is robust to the introduction of this general distribution function.

**Proposition 5** Assuming a general distribution function for the financial capital $A$ held by borrowers, the financier’s profits are higher under the integration than under the separation of the lending and non-lending tasks if and only if agency costs under the integration of tasks are lower than under the separation of tasks.

**Proof.** The Proof is in the Appendix. ■

6 Conclusion

Our paper analyzes the role of agency costs in determining whether it is in a bank’s interest to organize itself as a financial conglomerate. We show that financial conglomeration creates economies of scope through the reduction of managerial agency costs to be paid to induce bank managers to exert effort for the banking task they have been assigned to. We set-up a model where conglomereration, i.e. the integration of lending and non-lending activities in one organization, would never occur without managerial agency problems. In this benchmark case, a profit-maximizing financier optimally selects specialist bank managers based on their comparative advantages in the individual tasks. However, under managerial moral hazard, the bank’s optimal organizational form is determined by the size of the expected managerial agency costs. We show that agency costs may be lower when a generalist bank manager is hired to perform both the lending and non-lending tasks. This result is due to the fact that the integration of tasks in one organization allows the financier to condition managerial compensation on the success of several tasks. A financial conglomerate structure where generalist bank managers perform both lending and non-lending activities for the bank’s clients may therefore dominate the organization of activities into specialized institutions. We also show that a conglomerate structure is optimal for the bank whenever it ensures a larger group of borrowers to fund and thus a larger bank size.
The results of our model have implications concerning the value creation in financial conglomerates. The insights may reconcile the controversial evidence in relation to the existence and size of a diversification discount for financial conglomerates. We characterize the conditions that ensure that the organization of lending and non-lending activities into a single bank, is beneficial for a profit-maximizing financier. Furthermore, we show that conglomerates create value whenever it allows the bank to finance a larger group of borrowers.

We believe that our paper contributes to the current discussion on the optimal design of banking organizations in the financial intermediation industry. Focusing on the role of agency costs, we suggest that the profit-maximizing organization may be built on a combination of lending and non-lending activities in the same organizational unit. We point out that agency costs may affect the economies of scope generated by financial conglomeration and therefore whether banks should be organized as conglomerates or specialized intermediaries. The main conclusion from our analysis is that financial conglomerates and specialized banks should coexist and that agency costs will affect to what extent financial institutions diversify their activities.

7 Appendix

Proof of Lemma 1. For each borrower, the financing condition provides the threshold level of capital the borrower is required to contribute to the project to be eligible for funding. Assume that managerial effort is observable and denote this threshold level of capital by \( A^B(s, \theta_1, \theta_2) \).

\[
A^B(s, \theta_1, \theta_2) = I + \frac{p_H sB - p_H (\lambda_H Rr + (1 - \lambda_H) R) + c_1(\theta_1) + c_2(\theta_2)}{(1 + i)}
\]

The optimal amount to lend is determined by the level transparency of the marginal borrower that is still eligible for financing \( s^B(\theta_1, \theta_2) \). As \( A \) is uniformly distributed on \([0, A]\), the financier’s profits can be expressed as:

\[
\Pi^S_B(s^B, \theta_1, \theta_2) = \int_0^{s^B} \left[ I - A^B(s, \theta_1, \theta_2) \right] \Pr \left[ A^B(s, \theta_1, \theta_2) \leq A^B(1, \theta_1, \theta_2) \right] ds
\]

\[
= \frac{i}{(1 + i)^2} s^B \left( p_H (\lambda_H Rr + (1 - \lambda_H) R) - \frac{p_H}{\Delta p} s^B B - c_1(\theta_1) - c_2(\theta_2) \right) \frac{(1 - s^B) \frac{\Delta p}{\Delta p} B}{A}
\]

In what follows, we solve for the profit-maximizing level of \( s^B(\theta_1, \theta_2) \) that determines the size of the borrower group funded by the financier.

\[
\frac{d\Pi^S_B}{ds^B} = \begin{pmatrix}
[I - A^B(s^B, \theta_1, \theta_2)] (1 - s^B) \\
-s^B \left[ I - A^B(s^B, \theta_1, \theta_2) + \frac{s^B \Delta p}{2 \Delta p} B \right]
\end{pmatrix} = 0
\]

\[
\iff
\begin{pmatrix}
p_H (\lambda_H Rr + (1 - \lambda_H) R) - c_1(\theta_1) - c_2(\theta_2) \\
-\left( 2p_H (\lambda_H Rr + (1 - \lambda_H) R) - 2(c_1(\theta_1) + c_2(\theta_2)) + \frac{p_H}{\Delta p} B \right) s^B + \frac{3}{2} \left( s^B \right)^2 \frac{p_H}{\Delta p} B
\end{pmatrix} = 0
\]
The expression on the left hand side of the above equation is a second degree convex polynomial $ax^2 + bx + c = 0$, with $a \geq 0, b \leq 0$ and $c \geq 0$. Therefore we have 2 positive roots. The polynomial is positive for $s^B = 0$ and negative for $s^B = 1$. In addition, $s^B (\theta_1, \theta_2)$ is such that $s^B (\theta_1, \theta_2) < \frac{1}{2}$.

Indeed:

\[
\frac{d\Pi^S_B}{ds^B} (s^B = 0) = I - A^B (0, \theta_1, \theta_2) \geq 0
\]
\[
\frac{d\Pi^S_B}{ds^B} (s^B = 1) = - \left( I - A^B \left( \frac{1}{2}, \theta_1, \theta_2 \right) \right) \leq 0
\]
\[
\frac{d\Pi^S_B}{ds^B} (s^B = \frac{1}{2}) = - \frac{1}{8} B \leq 0
\]

It follows that the level of borrower’s transparency that determines the equilibrium bank size is such that $s^B \in (0, \frac{1}{2})$.

Moreover, by the Envelope Theorem:

\[
\frac{d\Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} = \frac{\partial \Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^B} \frac{\partial s^B (\theta_1, \theta_2)}{\partial \theta_1} + \frac{\partial \Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1}
\]
\[
= - \frac{i}{(1 + i)^2} \frac{p_H}{A \Delta \rho} B (1 - s^B) \frac{dc_1}{d\theta_1}
\]
\[
\frac{d\Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} = \frac{\partial \Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^B} \frac{\partial s^B (\theta_1, \theta_2)}{\partial \theta_2} + \frac{\partial \Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2}
\]
\[
= - \frac{i}{(1 + i)^2} \frac{p_H}{A \Delta \rho} B (1 - s^B) \frac{dc_2}{d\theta_2}
\]

Since $\frac{dc_1}{d\theta_1} > 0$ and $\frac{dc_2}{d\theta_2} < 0$, in equilibrium $\frac{d\Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} < 0$ and $\frac{d\Pi^S_B (s^B (\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} > 0$.

Consequently, when the two tasks can be separated, to maximize profits the financier will choose managers such that $\theta_1^B = 0, \theta_2^B = 1$.

This implies that the benchmark equilibrium bank size $s^B (0, 1)$ is:

\[
s^B (0, 1) = \frac{\left[ 2p_H R \left[ \lambda_H \left( r - 1 \right) + 1 \right] - 2 \left[ c_1 (\theta_1) + c_2 (\theta_2) \right] + \frac{p_H B}{\Delta \rho} \right] \left( 2p_H R \left[ \lambda_H \left( r - 1 \right) + 1 \right] \right)^2}{3p_H B \Delta \rho}
\]
\[
\left( \left( 2p_H R \left[ \lambda_H \left( r - 1 \right) + 1 \right] \right)^2 - 2 \left[ c_1 (\theta_1) + c_2 (\theta_2) \right] \right)^2 + \frac{p_H B}{\Delta \rho} \right) - 4 \frac{p_H B}{\Delta \rho} \left( p_H R \left[ \lambda_H \left( r - 1 \right) + 1 \right] \right)
\]
\[
\left( \left[ \alpha_B (0, 1) + \frac{p_H B}{\Delta \rho} \right] - \sqrt{\left[ \alpha_B (0, 1) \right]^2 + \left( \frac{p_H B}{\Delta \rho} \right)^2} \right) - \left[ c_1 (0) + c_2 (1) \right] \right]
\]

where $\alpha_B (0, 1) = 2 \left[ p_H \left( \lambda_H R r + (1 - \lambda_H) R \right) - \left[ c_1 (0) + c_2 (1) \right] \right]$. 


Proof of Lemma 2. For each borrower, the financing condition provides the threshold level of capital the borrower is required to contribute to the project to be eligible for funding. Assume that managerial effort is observable and denote this threshold level of capital by $A^B(s, \theta)$. The calculation is similar to the calculation in the proof of Lemma 1. In what follows, we determine the profit-maximizing level of $s^B(\theta)$ that determines the optimal bank size.

$$
\frac{d\Pi_B^I}{ds^B} = \frac{1}{A} \left( [I - A(s^*, \theta)] (1 - s^*) p_H \frac{B}{\Delta p} \right) = 0
$$

$$
\iff \left( -s^* \left[ I - A(s^*, \theta) + \frac{s^* p_H B}{\Delta p} \right] \frac{B}{\Delta p} \right) = 0
$$

The expression on the left hand side of the above equation is a second degree convex polynomial $ax^2 + bx + c = 0$, with $a \geq 0, b \leq 0$ and $c \geq 0$. We therefore have 2 positive roots. As the polynomial is positive for $s^B = 0$, and negative for $s^B = 1$, only one of the two roots is lower than one. This root is lower than $\frac{1}{2}$, since the polynomial is negative for $s^B = \frac{1}{2}$. Indeed:

$$
\frac{d\Pi_B^I}{ds^B}(s^B = 0) = I - A^B(0, \theta) \geq 0
$$

$$
\frac{d\Pi_B^I}{ds^B}(s^B = 1) = - \left( I - A^B \left( \frac{1}{2}, \theta \right) \right) \leq 0
$$

$$
\frac{d\Pi_B^I}{ds^B}(s^B = \frac{1}{2}) = - \frac{1}{8} \frac{p_H B}{\Delta p} \leq 0
$$

It follows that in equilibrium $s^B(\theta) \in (0, \frac{1}{2})$.

By the Envelope Theorem,

$$
\frac{d\Pi_B^I(s^B(\theta), \theta)}{d\theta} = \frac{\partial \Pi_B^I(s^B(\theta), \theta)}{\partial s^B} \frac{d s^B(\theta)}{d \theta} + \frac{\partial \Pi_B^I(s^B(\theta), \theta)}{\partial \theta}
$$

$$
= - \frac{i}{(1 + i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} (1 - s^B) s^B \left( \frac{d c_1}{d \theta} + \frac{d c_2}{d \theta} \right)
$$

It follows that we have three cases for the financier’s choice of the equilibrium $\theta^B$:

- If $c_1(\theta)$ and $c_2(\theta)$ are such that for all $\theta$, $\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) > 0$, then $\frac{d\Pi_B^I}{d\theta} < 0$ and the financier maximizes profits by choosing a specialist manager such that $\theta^B = 0$.

- If $c_1(\theta)$ and $c_2(\theta)$ are such that for all $\theta$, $\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) < 0$, then $\frac{d\Pi_B^I}{d\theta} > 0$ and the financier maximizes profits by choosing a specialist manager such that $\theta^B = 1$.

- If there exists $\tilde{\theta}^B$ such that $\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) = 0$, then we have an interior solution such that $\theta^B = \tilde{\theta}^B$ since $\frac{d^2 \Pi_B^I(s^B(\theta), \theta)}{d\theta^2} \leq 0$ as both cost functions are convex.
$s^B(\theta^B)$ can therefore be expressed as follows:

$$s^B(\theta^B) = \frac{\alpha_B(\theta^B) + \frac{p_H B}{\Delta p} - \sqrt{(\alpha_B(\theta^B) + \frac{p_H B}{\Delta p})^2 - \frac{3p_H B \alpha_B(\theta^B)}{\Delta p}}}{\frac{3p_H B}{\Delta p}}$$

where $\alpha_B(\theta^B) = 2 [p_H R [\lambda_H (r - 1) + 1] - [c_1(\theta^B) + c_2(\theta^B)]]$. ■

**Proof of Proposition 1.** Given the financier’s profit-maximizing choices of managerial types and group of borrowers to fund, we compare the financier’s profits under the two organizational structures: the integration and the separation of the two tasks.

$$\Pi^S_B(s^B(0,1),0,1) - \Pi^I_B(s^B(\theta^B),\theta^B) = \left[ \frac{\Pi^S_B(s^B(0,1),0,1) - \Pi^S_B(s^B(\theta^B),0,1)}{1 + \frac{1}{(1+i^2)} \frac{p_H B}{\Delta p} (c_1(\theta^*) + c_2(\theta^*)) - [c_1(0) + c_2(1)](1 - s^B(\theta^B))s^B(\theta^B)} \right]$$

As $\Pi^S_B(s^B(0,1),0,1) - \Pi^I_B(s^B(\theta^B),0,1) \geq 0$ because $s^B(0,1) = \text{Arg max} \{\Pi^S_B(.,0,1)\}$, and $\frac{dc_2(\theta)}{d\theta} \leq 0$ and $\frac{dc_2(\theta)}{d\theta} \geq 0$, it is immediate that:

$$\Pi^S_B(s^B(0,1),0,1) - \Pi^I_B(s^B(\theta^B),\theta^B) \geq 0.$$ ■

**Proof of Lemma 3.** As in the benchmark case, the financing condition provides the threshold level of capital the borrower needs to contribute to be eligible for funding $A^*(s, \theta_1, \theta_2)$. Under the assumption that the managerial effort is observable, $A^*(s, \theta_1, \theta_2)$ can be expressed as follows.

$$A^*(s, \theta_1, \theta_2) = I + \frac{p_H \left[ \frac{s^B}{\Delta p} \right]}{(1+i)} - p_H R (1 + \lambda_H (r - 1)) + p_H \frac{c_1(\theta_1) + c_2(\theta_2)}{\Delta p} + p_H \lambda_H \frac{c_2(\theta_2)}{\Delta p}$$

For given $(\theta_1, \theta_2)$, the financier chooses the size of the bank $s^*(\theta_1, \theta_2)$ to maximize profits. As $A$ is uniformly distributed on $[0,A]$, the financier’s profits can be expressed as follows.

$$\Pi^S(s(\theta_1, \theta_2), \theta_1, \theta_2) = \int_{0}^{s^*} \left[ I - A(s(\theta_1, \theta_2), \theta_1, \theta_2) \right] \Pr [A(s(\theta_1, \theta_2), \theta_1, \theta_2) \leq A(1, \theta_1, \theta_2)] ds$$

$$= \int_{0}^{s^*} \left[ \frac{p_H R (1 + \lambda_H (r - 1)) - \frac{p_H B (s^*)}{\Delta p}}{\Delta p} - \frac{\lambda_H c_2(\theta_2)}{\Delta p} \right] \frac{1}{A} (1 - s^*) \frac{p_H B}{\Delta p}$$

In what follows, we solve for $s^*(\theta_1, \theta_2)$:

$$\frac{d\Pi^S}{ds^*} = \left( \begin{array}{c} \left[ I - A(s^*, \theta_1, \theta_2) \right] (1 - s^*) \\ -s^* \left[ I - A(s^*, \theta_1, \theta_2) + s^* \frac{B p_H}{\Delta p} \right] \end{array} \right) = 0$$

$$\iff \left( \begin{array}{c} R (1 + \lambda_H (r - 1)) - \frac{c_1(\theta_1)}{\Delta p} - \lambda_H \frac{c_2(\theta_2)}{\Delta p} \\ - \left( 2R (1 + \lambda_H (r - 1)) - 2 \left( \frac{c_1(\theta_1)}{\Delta p} + \lambda_H \frac{c_2(\theta_2)}{\Delta p} \right) \right) (s^*) + \frac{3}{2} (s^*)^2 \frac{B}{\Delta p} \end{array} \right) = 0$$

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The expression on the left hand side of the above equation is a second degree convex polynomial
\[ ax^2 + bx + c = 0, \] with \( a \geq 0, b \leq 0 \) and \( c \geq 0 \). We therefore have 2 positive roots. As the polynomial
is positive for \( s^* = 0 \), and negative for \( s^* = 1 \), only one of the roots is lower than 1. Moreover,
\( s^*(\theta_1, \theta_2) < \frac{1}{2} \). Indeed:
\[
\begin{align*}
\frac{d\Pi^S}{ds^*}(s^* = 0) &= I - A(0, \theta_1, \theta_2) \geq 0 \\
\frac{d\Pi^S}{ds^*}(s^* = 1) &= - \left( I - A \left( \frac{1}{2}, \theta_1, \theta_2 \right) \right) \leq 0 \\
\frac{d\Pi^S}{ds^*}(s^* = \frac{1}{2}) &= - \frac{1}{8} \frac{i}{(1 + i)^2} \frac{p_H B}{\Delta p} \leq 0
\end{align*}
\]
Notice that \( \Pi^S \) is concave:
\[
\frac{d^2\Pi^S}{ds^*ds^*} = -2 (I - A(s^*, \theta_1, \theta_2)) - (1 - s^*) \frac{i}{(1 + i)^2} \frac{p_H B}{\Delta p} \leq 0
\]
\( s^*(\theta_1, \theta_2) \) can therefore be expressed as follows:
\[
s^*(\theta_1, \theta_2) = \left[ \alpha_S(\theta_1, \theta_2) + \frac{p_H B}{\Delta p} \right] \sqrt{\left( \alpha_S(\theta_1, \theta_2) + \frac{p_H B}{\Delta p} \right)^2 - \frac{3p_H B}{\Delta^p \alpha_S(\theta_1, \theta_2)}}
\]
where \( \alpha_S(\theta_1, \theta_2) = 2 \left[ p_H R [\lambda_H (r - 1) + 1] - \left( \frac{p_H}{\Delta p} c_1(\theta_1) + \frac{\lambda_H}{\Delta c_2}(\theta_2) \right) \right] \).

**Proof of Proposition 2.** The financier chooses the managers' types \( \theta_1^*, \theta_2^* \) such that \( \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2) \)
maximized.
\[
\Pi^S(s(\theta_1, \theta_2), \theta_1, \theta_2) = \frac{i}{(1 + i)^2} s^* \left( \frac{p_H R (1 + \lambda_H (r - 1)) - \frac{p_H B(s^*)^2}{2}}{\Delta p} \right) \frac{1}{A} (1 - s^*) \frac{p_H B}{\Delta p}
\]
By the Envelope Theorem:
\[
\begin{align*}
\frac{d\Pi^S}{d\theta_1}(s^*(\theta_1, \theta_2), \theta_1, \theta_2) &= \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*}{\partial \theta_1} + \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1} \\
&= - \frac{i}{(1 + i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} (1 - s^*) s^* \frac{p_H}{\Delta p} \left( \frac{dc_1}{d\theta_1} \right) \\
\frac{d\Pi^S}{d\theta_2}(s^*(\theta_1, \theta_2), \theta_1, \theta_2) &= \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*}{\partial \theta_2} + \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2} \\
&= - \frac{i}{(1 + i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} (1 - s^*) s^* \frac{\lambda_H}{\Delta \lambda} \left( \frac{dc_2}{d\theta_2} \right)
\end{align*}
\]
Since \( \frac{dc_1}{d\theta_1} > 0 \) and \( \frac{dc_2}{d\theta_2} < 0 \), in equilibrium \( \frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} < 0 \) and \( \frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} > 0 \).
Consequently, when the two tasks are separated, the financier will choose managers such that
\( \theta_1^* = 0, \theta_2^* = 1 \). Moreover,

\[
\begin{align*}
\frac{d^2 \Pi^S}{ds^* d\theta_1} &= \frac{i}{(1+i)^2} \frac{p_H}{\Delta p} \left( \frac{dc_1}{d\theta_1} \right) (2s^* - 1) \\
\frac{d^2 \Pi^S}{ds^* d\theta_2} &= \frac{i}{(1+i)^2} \frac{p_H \lambda_H}{p_H(\lambda_H - \lambda_L)} \left( \frac{dc_2}{d\theta_2} \right) (2s^* - 1) \\
\frac{d^2 \Pi^S}{ds^* ds^*} &= -2 (1 - A^*(s^*, \theta_1, \theta_2)) - (1 - s^*) \frac{i}{(1+i)^2} \frac{p_H B}{\Delta p} \leq 0
\end{align*}
\]

Furthermore,

\[
\begin{align*}
\frac{ds^*}{d\theta_1} &= -\frac{d^2 \Pi^S}{ds^* d\theta_1}, \\
\frac{ds^*}{d\theta_2} &= -\frac{d^2 \Pi^S}{ds^* d\theta_2}
\end{align*}
\]

Since \( s^* (\theta_1, \theta_2) \in (0, \frac{1}{2}) \), \( \frac{ds^*}{d\theta_1} < 0 \) and \( \frac{ds^*}{d\theta_2} > 0 \).

**Proof of Lemma 4.** Under the assumption that effort is unobservable and tasks are integrated, the threshold level of capital the borrower needs to contribute to be eligible for funding \( A^*(s, \theta) \) can be written as follows.

\[
A^*(s, \theta) = I + \frac{p_H \left( \frac{s B}{\Delta p} \right) - p_H R(1 + \lambda_H(r - 1)) + p_H \lambda_H \frac{c_1(\theta) + c_2(\theta)}{p_H \lambda_H - p_L \lambda_L}}{(1+i)}
\]

The financier chooses the size of the bank \( s^*(\theta) \) to maximize the amount of capital to lend and thereby profits. Since \( A \) is uniformly distributed on \( [0, \bar{A}] \), for given \( \theta \), the financier’s profits can be expressed as:

\[
\Pi^F(s(\theta), \theta) = \int_0^{s^*} \left[ I - A(s(\theta), \theta) \right] \Pr \left[ A(s(\theta), \theta) \leq A \leq A(1, \theta) \right] ds
\]

\[
= i \left( F[A(1, \theta)] - F[A(s^*, \theta)] \right) \int_0^{s^*} \left[ I - A(s, \theta) \right] ds
\]

\[
= \frac{i}{(1+i)^2} s^* \left( p_H R(1 + \lambda_H(r - 1)) - p_H \frac{B(s^*)}{\Delta p} - \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} (c_1(\theta) + c_2(\theta)) \right) \frac{1}{\bar{A}} (1 - s^*) \frac{p_H B}{\Delta p}
\]
Therefore, solving for the optimal $s^*$:

\[
\frac{d\Pi^I}{ds^*} (s^* = 0) = I - A(0, \theta) \geq 0 \\
\frac{d\Pi^I}{ds^*} (s^* = 1) = -\left(I - A \left(\frac{1}{2}, \theta\right)\right) \leq 0 \\
\frac{d\Pi^I}{ds^*} (s^* = 1/2) = - \frac{1}{8} p_H B \frac{\Delta}{\Delta p} \leq 0
\]

It follows that in equilibrium $s^*(\theta) \in (0, 1/2)$. Moreover, notice that $\Pi^I (s^*)$ is concave. Indeed,

\[
\frac{d^2\Pi^I}{ds^*ds^*} = -2 (I - A (s^*, \theta)) - \frac{i}{(1 + i)^2} (1 - s^*) \frac{p_H B}{\Delta p} \leq 0
\]

$s^* (\theta)$ can be expressed as follows:

\[
s^*(\theta) = \left[\alpha_1(\theta) + \frac{v_H B}{\Delta p}\right] - \sqrt{\left(\alpha_1(\theta) + \frac{v_H B}{\Delta p}\right)^2 - \frac{3v_H B}{\Delta p} \alpha_1(\theta)}
\]

where $\alpha_1(\theta) = 2 \left[p_H R \left[\lambda_H (r - 1) + 1\right] - \frac{\lambda_H}{\lambda_H - \frac{p_H}{p_H - p_L}} [c_1(\theta) + c_2(\theta)]\right]$. 

**Proof of Proposition 3.** The financier chooses the manager’s type $\theta^*$ such that profits are maximized.

\[
\Pi^I(s(\theta), \theta) = \frac{i}{(1 + i)^2} s^* \left(\frac{p_H R \left(1 + \lambda_H (r - 1)\right) - p_H B \frac{\Delta}{\Delta p} \left(s^*\right)}{2} - \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} (c_1(\theta) + c_2(\theta))\right) \frac{1}{A} (1 - s^*) \frac{p_H B}{\Delta p}
\]

By the Envelope Theorem,

\[
\frac{d\Pi^I (s^*(\theta), \theta)}{d\theta} = \frac{\partial \Pi^I (s^*(\theta), \theta)}{\partial s^*} \frac{ds^*(\theta)}{d\theta} + \frac{\partial \Pi^I (s^*(\theta), \theta)}{\partial \theta}
\]

\[
= - \frac{i}{(1 + i)^2} \frac{1}{A} \frac{p_H B}{\Delta p} (1 - s^*) \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} s^* \left(\frac{dc_1}{d\theta} + \frac{dc_2}{d\theta}\right)
\]
Furthermore,
\[
\frac{d^2 \Pi^I}{ds^* d\theta} = \frac{i}{(1+i)^2} \frac{\mu}{\mu - p_H \lambda_H} \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) (2s^* - 1)
\]
\[
\frac{d^2 \Pi^I}{ds^* ds^*} = -2 (I - A(s^*, \theta)) - \frac{i}{(1+i)^2} (1 - s^*) \frac{\mu}{\mu - p_H \lambda_H} \Delta_p \leq 0
\]
Moreover,
\[
\frac{ds^*}{d\theta} = -\frac{\frac{d^2 \Pi^I}{ds^* d\theta}}{\frac{d^2 \Pi^I}{ds^* ds^*}} = \frac{i}{(1+i)^2} \frac{\mu}{\mu - p_H \lambda_H} \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) (2s^* - 1)
\]
\[
2 (I - A(s^*, \theta)) + \frac{i}{(1+i)^2} (1 - s^*) \frac{\mu}{\mu - p_H \lambda_H} \Delta_p
\]

It follows that we have three cases for the financier’s choice of the equilibrium \(\theta\):

- If \(c_1(\theta)\) and \(c_2(\theta)\) are such that for all \(\theta\) \(\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) > 0\), then \(\frac{d\Pi^I}{d\theta} < 0\) and the financier maximizes profits by choosing a specialist manager such that \(\theta^* = 0\). As in equilibrium \(s^*(\theta) < \frac{1}{2}\), the above implies that \(\frac{ds^*}{d\theta} < 0\), i.e. the size of the bank \(s^*(0)\) is the highest possible.

- If \(c_1(\theta)\) and \(c_2(\theta)\) are such that for all \(\theta\) \(\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) < 0\), then \(\frac{d\Pi^I}{d\theta} > 0\) and the financier maximizes profits by choosing a specialist manager such that \(\theta^* = 1\). As in equilibrium \(s^*(\theta) < \frac{1}{2}\), the above implies that \(\frac{ds^*}{d\theta} > 0\), i.e. the size of the bank \(s^*(1)\) is the highest possible.

- If there exists \(\hat{\theta}\) such that \(\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) = 0\), then we have an interior solution \(\theta^* = \hat{\theta}\) since \(\frac{d^2 \Pi^I(s^*(\theta), \theta)}{d\theta^2} \leq 0\) as both cost functions are convex. In equilibrium \(s^*(\theta) < \frac{1}{2}\) and \(\frac{ds^*}{d\theta} = 0\).

**Proof of Lemma 5.** We define the expressions \(C_S(0, 1) = \frac{\lambda_H \mu}{\lambda H_H - \lambda L_H} c_1(0) + \frac{\lambda_H \mu}{\lambda L_H} c_2(1)\) and \(C_T(\theta^*) = \frac{\lambda_H \mu}{\lambda H_H - \lambda L_H} [c_1(\theta^*) + c_2(\theta^*)]\), where \(\theta^* = \{0, 1, \hat{\theta}\}\). Furthermore, we define the function \(s(C)\) as follows:

\[
s(C) = \left( \frac{\sqrt{2 \left[ p_H R [\lambda_H (r - 1) + 1] - C \right] + \frac{\mu}{\mu - p_H} \Delta_p}}{\sqrt{\frac{4 \left[ p_H R [\lambda_H (r - 1) + 1] - C \right]^2 + \left( \frac{\mu}{\mu - p_H} \Delta_p \right)^2}{2 \frac{2 \mu}{\Delta_p} [p_H R [\lambda_H (r - 1) + 1] - C]}} - 2 \frac{\mu}{\mu - p_H} \frac{\mu}{\mu - p_H} \right)
\]

Therefore,

\[
s^*(0, 1) = s(C_S(0, 1)),
\]
\[
s^*(\theta^*) = s(C_T(\theta^*)).
\]
This function $s(C)$ is non-increasing in $C$:

$$\frac{ds(C)}{dC} = \frac{1}{3\frac{\Delta H}{\Delta p}} \left( \begin{array}{c} 8 [p_H R \{\lambda_H (r - 1) + 1 - C\} - 2\frac{\Delta H}{\Delta p} - 2] \\ \sqrt{4 [p_H R \{\lambda_H (r - 1) + 1 - C\}^2 + \left(\frac{\Delta H}{\Delta p}\right)^2} \\ -2\frac{\Delta H}{\Delta p} \left[p_H R \{\lambda_H (r - 1) + 1 - C\} - 2\frac{\Delta H}{\Delta p}\right] \right) \leq 0$$

Indeed, the expression in the nominator is non-positive.

$$\left( \begin{array}{c} 4 [p_H R \{\lambda_H (r - 1) + 1 - C\} - \frac{p_H B}{\Delta p}] \\ -2\sqrt{4 [p_H R \{\lambda_H (r - 1) + 1 - C\}^2 + \left(\frac{p_H B}{\Delta p}\right)^2} \\ -2\frac{p_H B}{\Delta p} \left[p_H R \{\lambda_H (r - 1) + 1 - C\} - \frac{p_H B}{\Delta p}\right] \right) \leq 0$$

$$\Leftrightarrow \left(4 [p_H R \{\lambda_H (r - 1) + 1 - C\} - \frac{p_H B}{\Delta p}]^2 \geq 4 \left(4 [p_H R \{\lambda_H (r - 1) + 1 - C\}^2 + \left(\frac{p_H B}{\Delta p}\right)^2} \\ -2\frac{p_H B}{\Delta p} \left[p_H R \{\lambda_H (r - 1) + 1 - C\} - \frac{p_H B}{\Delta p}\right] \right) \right) \Leftrightarrow -3 \left(\frac{p_H B}{\Delta p}\right)^2 \leq 0$$

As we have shown that $s(C)$ is non-increasing in $C$, we can state that:

$$s^*(0, 1) \geq s^*(\theta^*)$$

$$\Leftrightarrow s(C_S(0, 1)) \geq s(C_1(\theta^*))$$

$$\Leftrightarrow C_S(0, 1) \leq C_1(\theta^*)$$

The result follows:

$$\frac{\lambda_{HPH}}{\lambda_{HPH} - \lambda_T p_L} [c_1(\theta^*) + c_2(\theta^*)] - \left[\frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1)\right] \geq 0.$$
Proof of Proposition 4. Given the financier’s equilibrium choices of managerial types, we compare the proﬁts under the separation and the integration of the two managerial tasks:

\[
\Pi^S(s^*, (0, 1) , 0, 1) - \Pi^I(s^*(\theta^*), \theta^*)
\]

\[
= \left[ \Pi^I(s^*(0, 1) , \theta^*) - \Pi^I(s^*(\theta^*), \theta^*) \right] + \frac{1}{\Delta p} \left[ \frac{\lambda_H R}{\lambda_H p - \lambda_L p L} \left[ c_1(\theta^*) + c_2(\theta^*) \right] - \left[ \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \right] \right] (1 - s^*(0, 1)) s^*(0, 1)
\]

Moreover:

\[
\Pi^S(s^*(0, 1) , 0, 1) - \Pi^I(s^*(\theta^*), \theta^*)
\]

\[
= \left[ \Pi^I(s^*(0, 1) , 0, 1) - \Pi^S(s^*(\theta^*), 0, 1) \right] + \frac{1}{\Delta p} \left[ \frac{\lambda_H R}{\lambda_H p - \lambda_L p L} \left[ c_1(\theta^*) + c_2(\theta^*) \right] - \left[ \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \right] \right] (1 - s^*(\theta^*)) s^*(0, 1)
\]

As \( \Pi^I(s^*(0, 1) , \theta^*) - \Pi^I(s^*(\theta^*), \theta^*) \leq 0 \) and \( \Pi^S(s^*(0, 1) , 0, 1) - \Pi^S(s^*(\theta^*), 0, 1) \) because \( s^*(\theta^*) = \text{Arg} \max \{ \Pi^I(\cdot, \theta^*) \} \) and \( s^*(0, 1) = \text{Arg} \max \{ \Pi^S(\cdot, 0, 1) \} \), it is immediate that:

\[
\Pi^S(s^*(0, 1) , 0, 1) - \Pi^I(s^*(\theta^*), \theta^*) \leq 0
\]

\[
\iff \left( \frac{\lambda_H R}{\lambda_H p - \lambda_L p L} \left[ c_1(\theta^*) + c_2(\theta^*) \right] - \left[ \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \right] \right) \leq 0.
\]

■

Proof of Proposition 5. Assume that managerial effort is unobservable.

- Separation. The per project financing condition provides the threshold level of capital the borrower needs to contribute to be eligible for funding \( A^S(s, \theta_1, \theta_2) \). \( A^S(s, \theta_1, \theta_2) \) can be expressed as follows:

\[
A^S(s, \theta_1, \theta_2) = I + \frac{p_H \left( \frac{S^H}{\Delta p} \right) - p_H R (1 + \lambda_H (r - 1)) + p_H c_1(\theta_1) + p_H \lambda_H c_2(\theta_2)}{(1 + i)}
\]

The financier’s proﬁts can be expressed as follows:

\[
\Pi^S(s(\theta_1, \theta_2), \theta_1, \theta_2) = i \int_0^{s^*} [I - A(s(\theta_1, \theta_2), \theta_1, \theta_2)] \Pr[A(s(\theta_1, \theta_2), \theta_1, \theta_2) \leq A \leq A(1, \theta_1, \theta_2)] ds
\]

\[
= \frac{i}{1 + i} s^* \left[ \frac{p_H R (1 + \lambda_H (r - 1)) - \frac{p_H B (s^* - \theta^*)}{\Delta p} - \frac{p_H \lambda_H c_2(\theta_2)}{p_H \Delta \lambda}}{2} \right] [1 - F(A^S(s^*, \theta_1, \theta_2))]
\]

In what follows, we solve for \( s^*(\theta_1, \theta_2) \):

\[
\frac{d\Pi^S}{ds^*} = \left( \begin{array}{c} 0 \\ - s^* \left[ I - A(s^*, \theta_1, \theta_2) \right] \left[ 1 - F(A^S(s^*, \theta_1, \theta_2)) \right] \\ - s^* \left[ I - A(s^*, \theta_1, \theta_2) + \frac{s^* p_H B}{\Delta p} \right] \frac{p_H B}{\Delta p} f(A^S(s^*, \theta_1, \theta_2)) \\
- \left[ 1 - F(A^S(s^*, \theta_1, \theta_2)) \right] \frac{p_H B}{\Delta p} f(A^S(s^*, \theta_1, \theta_2)) \\
\end{array} \right) = 0
\]

\[
\iff \left( \begin{array}{c} \frac{1 - F(A^S(s^*, \theta_1, \theta_2))}{p_H B} \frac{f(A^S(s^*, \theta_1, \theta_2))}{\Delta p} \\
- \frac{(s^*)^2}{2 [I - A^S(s^*, \theta_1, \theta_2)]} \frac{p_H B}{\Delta p} \\
\end{array} \right) = 0
\]
By assumption, we have \( \frac{d}{ds} \left( \frac{1-F(A^S(s^*,\theta_1,\theta_2))}{f(A^S(s^*,\theta_1,\theta_2))} \right) \leq 0 \)

and \( \frac{d}{ds} \left( \frac{(s^*)^2}{2[I-A^S(s^*,\theta_1,\theta_2)]} \right) = \frac{2s^*[I-A^S(s^*,\theta_1,\theta_2)] + \frac{p_H B}{\Delta p}(s^*)^2}{2[I-A^S(s^*,\theta_1,\theta_2)]} \geq 0 \). Consequently, \( \Pi^S \) is concave. Moreover:

\[
\frac{d\Pi^S}{ds^*}(s^* = 0) = \left[ I - A^S(0, \theta_1, \theta_2) \right] \left[ 1 - F \left( A^S(0, \theta_1, \theta_2) \right) \right] \geq 0
\]

\[
\frac{d\Pi^S}{ds^*}(s^* = 1) = - \left[ I - A^S(\frac{1}{2}, \theta_1, \theta_2) \right] \frac{p_H B}{\Delta p} f \left( A^S(1, \theta_1, \theta_2) \right) \leq 0
\]

This implies that the equation \( \frac{d\Pi^S}{ds^*} = 0 \) admits a unique solution \( s^*(\theta_1, \theta_2) \) on \([0, 1]\).

The financier chooses the managers’ types \( \theta_1^*, \theta_2^* \) such that \( \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2) \) is maximized.

\[
\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2) = \frac{i}{1+i} \left( \frac{p_H R (1 + \lambda_H (r - 1)) - \frac{p_H \lambda H}{2}}{-\frac{\partial c_1(\theta_1)}{\Delta \rho} - \frac{\lambda H c_2(\theta_2)}{\Delta \lambda}} \right) \left[ 1 - F \left( A^S(s^*(\theta_1, \theta_2)) \right) \right]
\]

By the Envelope Theorem:

\[
\frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} = \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*(\theta_1, \theta_2)}{\partial \theta_1} + \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_1}
\]

\[
= - \frac{i s^* \frac{p_H}{\Delta p} \left( \frac{dc_1}{d\theta_1} \right)}{1+i} \left( 1 - A^S(s^*(\theta_1, \theta_2)) \right) \left[ 1 - F \left( A^S(s^*(\theta_1, \theta_2)) \right) \right] \leq 0
\]

\[
\frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} = \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial s^*} \frac{\partial s^*(\theta_1, \theta_2)}{\partial \theta_2} + \frac{\partial \Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{\partial \theta_2}
\]

\[
= - \frac{i s^* \frac{\lambda_H}{\Delta \lambda} \left( \frac{dc_2}{d\theta_2} \right)}{1+i} \left( 1 - A^S(s^*(\theta_1, \theta_2)) \right) \left[ 1 - F \left( A^S(s^*(\theta_1, \theta_2)) \right) \right] \geq 0
\]

Since \( \left( \frac{dc_1}{d\theta_1} \right) > 0 \) and \( \left( \frac{dc_2}{d\theta_2} \right) < 0 \), in equilibrium \( \frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_1} < 0 \) and \( \frac{d\Pi^S(s^*(\theta_1, \theta_2), \theta_1, \theta_2)}{d\theta_2} > 0 \). Consequently, when the two tasks are separated, the financier will choose managers such that \( \theta_1^* = 0, \theta_2^* = 1 \).

**Integration.** Under the assumption that effort is unobservable and tasks are integrated, the threshold level of capital the borrower needs to contribute to be eligible for funding \( A^I(s, \theta) \) can be written as follows.

\[
A^I(s, \theta) = I + \frac{p_H \left( \frac{s_H}{\Delta p} \right) - p_H R (1 + \lambda_H (r - 1)) + p_H \lambda_H c_1(\theta) + c_2(\theta)}{p_H \lambda_H - p_L \lambda_L} \left( 1 + i \right)
\]

The financier chooses the size of the bank \( s^*(\theta) \) to maximize the profits. For given \( \theta \), the
financier’s profits can be expressed as:

$$
\Pi^f(s(\theta),\theta) = i \int_0^{s^*} [I - A(s(\theta),\theta)] \Pr[A(s(\theta),\theta) \leq A \leq A(1,\theta)] ds
$$

$$
\frac{d\Pi^f}{ds^*} = \left( [I - A(s^*,\theta)] (1 - F[A^f(s^*,\theta)]) - s^* \frac{p_H B(s^*)}{2} - \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} (c_1(\theta) + c_2(\theta)) \right) (1 - F[A^f(s^*,\theta)])
$$

Therefore,

$$
\frac{d^2 \Pi^f}{ds^*} = \left( [I - A^f(s^*,\theta)] f(A^f(s^*,\theta)) \right)
$$

By assumption, we have \( \frac{d \Pi^f}{ds^*} \geq 0 \) and

\( \frac{d \Pi^f}{ds^*} \left( \frac{(s^*)^2}{2[I - A^f(s^*,\theta)]} \right) = \frac{2s^* [I - A^f(s^*,\theta)] + \frac{p_H B(s^*)}{A_f} (s^*)^2}{2[I - A^f(s^*,\theta)]} \geq 0 \). \( \Pi^f \) is thus concave. Moreover:

$$
\frac{d \Pi^f}{ds^*} (s^* = 0) = [I - A^f(0,\theta)] \left( 1 - F(A^f(0,\theta)) \right) \geq 0
$$

$$
\frac{d \Pi^f}{ds^*} (s^* = 1) = -\left( I - A^f(\frac{1}{2},\theta) \right) \frac{p_H B}{A_f} f(A^f(1,\theta)) \leq 0
$$

This implies that the equation \( \frac{d \Pi^f}{ds^*} = 0 \) admits a unique solution \( s^*(\theta) \) on \([0,1]\).

The financier chooses the manager’s type \( \theta^* \) such that profits are maximized.

$$
\Pi^f(s(\theta),\theta) = i \int_0^{s^*} \left( p_H R \left( 1 + \lambda_H (r - 1) \right) - \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} (c_1(\theta) + c_2(\theta)) \right) (1 - F[A^f(s^*,\theta)])
$$

By the Envelope Theorem,

$$
\frac{d \Pi^f(s^*(\theta),\theta)}{d\theta} = \frac{\partial \Pi^f(s^*(\theta),\theta)}{\partial s^*} \frac{d s^*}{d\theta} + \frac{\partial \Pi^f(s^*(\theta),\theta)}{\partial \theta}
$$

$$
= -\frac{i}{1 + i} \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} \left( \frac{d^2 s^*}{d\theta^2} \right) s^* \left( \frac{d s^*}{d\theta} \right) \left( 1 - F(A^f(s^*,\theta)) \right)
$$

It follows that we have three cases for the financier’s choice of the equilibrium \( \theta \):

1. If \( c_1(\theta) \) and \( c_2(\theta) \) are such that for all \( \theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) > 0 \), then \( \frac{d\Pi^f}{d\theta} < 0 \) and the financier maximizes profits by choosing a specialist manager \( \theta^* = 0 \).
2. If $c_1(\theta)$ and $c_2(\theta)$ are such that for all $\theta \left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) < 0$, then $\frac{d\Pi}{d\theta} > 0$ and the financier maximizes profits by choosing a specialist manager $\theta^* = 1$.

3. If there exists $\tilde{\theta}$ such that $\left( \frac{dc_1}{d\theta} + \frac{dc_2}{d\theta} \right) = 0$, then we have an interior solution $\theta^* = \tilde{\theta}$ since $\frac{d^2\Pi(s^*(\theta), \theta)}{d\theta^2} \leq 0$ as both cost functions are convex.

- **Shape of $s(.)$.** If we call $C$ the total agency rent paid to the managers, by the implicit function theorem, we have:

\[
\frac{\partial s(\theta_1, \theta_2)}{\partial C} = -\frac{\partial \left( \frac{d\Pi}{d\theta} \right)}{\partial s}.
\]

Moreover, $\frac{\partial \left( \frac{d\Pi}{d\theta} \right)}{\partial s} \leq 0$ by concavity of $\Pi$ and

\[
\frac{\partial \left( \frac{d\Pi}{d\theta} \right)}{\partial C} = \frac{i}{1+i} \left( -\frac{1}{1-F(A(s,\theta_1,\theta_2))} - f(A(s,\theta_1,\theta_2)) \left[ I - A(s,\theta_1,\theta_2) \right] \right) + s \frac{pHB}{\Delta p} \left( f(A(s,\theta_1,\theta_2)) - [I - A(s,\theta_1,\theta_2)] \right) + s \frac{pHB}{\Delta p} \frac{df}{dA} \left( A(s,\theta_1,\theta_2) \right) \right)
\]

\[
= i \left( -\frac{\left( \frac{s pHB}{\Delta p} \right)^2}{s \left[ I - A(s,\theta_1,\theta_2) \right]} - [I - A(s,\theta_1,\theta_2)] \right) + s \frac{pHB}{\Delta p} \left( [I - A(s,\theta_1,\theta_2)] \right) \right) \right)
\]

\[
\leq i \left( -\frac{2 \left( \frac{s pHB}{\Delta p} \right)^2}{s \left[ I - A(s,\theta_1,\theta_2) \right]} - 1 \right) + s \frac{pHB}{\Delta p} \left( \frac{df}{dA} \left( A(s,\theta_1,\theta_2) \right) \right) \right) \right)
\]

The previous inequality comes, first, from the monotone hazard rate condition which gives:

\[
- \frac{df}{dA} (A(s,\theta_1,\theta_2)) \leq \frac{f(A(s,\theta_1,\theta_2))}{1 - F(A(s,\theta_1,\theta_2))}
\]

and, then, from the first order condition:

\[
\frac{[1 - F(A(s,\theta_1,\theta_2))]}{f(A(s,\theta_1,\theta_2))} = \left[ \frac{pHB}{\Delta p} s + \frac{\left( \frac{s pHB}{\Delta p} \right)^2}{2 \left[ I - A(s,\theta_1,\theta_2) \right]} \right]
\]

\[
- \left[ F(A(s,\theta_1,\theta_2)) + s \frac{pHB}{\Delta p} f(A(s,\theta_1,\theta_2)) = -s \frac{\left( \frac{s pHB}{\Delta p} \right)^2}{[I - A(s,\theta_1,\theta_2)] \frac{pHB}{\Delta p} f(A(s,\theta_1,\theta_2))}
\]
Consequently

\[
\frac{\partial s(\theta_1, \theta_2)}{\partial C} = -\frac{\partial (\frac{\partial \Pi^I}{\partial s})}{\partial (\frac{\partial \Pi^I}{\partial s})} \leq 0
\]

- **Optimal Organization.** We have:

\[
\Pi^S (s^*(0, 1), 0, 1) - \Pi^I (s^*(\theta^*), \theta^*)
\]

\[
= \left[ \begin{array}{c}
+ \frac{i}{1^2} s^*(0, 1) \\
- \frac{p_H R (1 + \lambda_H (r - 1))}{p_H R (1 + \lambda_H (r - 1))} \left[ (1 + \frac{\lambda_H}{\Delta \lambda} c_2(1)) - \frac{p_H R (s^*(0, 1))}{\Delta \lambda} \right] \left[ 1 - F \left( A^S (s^*(0, 1), 0, 1) \right) \right]
\end{array} \right]
\]

Hence,

\[
\left( \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} (c_1(\theta^*) + c_2(\theta^*)) - \left( \frac{p_H \lambda_H}{p_H \lambda_H - p_L \lambda_L} \right) (c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1)) \right) \leq 0
\]

\[
\Rightarrow \Pi^S (s^*(0, 1), 0, 1) - \Pi^I (s^*(\theta^*), \theta^*) \leq 0
\]

because in this case, \(A^I [s^*(0, 1)] \leq A^S [s^*(0, 1)]\) and \(\Pi^I (s^*(0, 1), \theta^*) - \Pi^I (s^*(\theta^*), \theta^*) \leq 0\) because \(s^*(\theta^*) = \text{Arg max } \{ \Pi^I (, \theta^*) \} \).
Moreover, we also have:

\[
\Pi^S(s^*(0,1),0,1) - \Pi^I(s^*(\theta^*),\theta^*) = \left[ \left( \Pi^S(s^*(0,1),0,1) - \Pi^S(s^*(\theta^*),0,1) \right) - \left( \Pi^I(s^*(\theta^*),\theta^*) - \Pi^I(s^*(\theta^*),\theta^*) \right) \right] 
\]

\[
= + \frac{i}{1+i} s^*(\theta^*) \left[ \left( \frac{p_H R (1 + \lambda_H (r - 1))}{p_H \lambda_H - \Delta p c_1(0) + \Delta \lambda c_2(1)} - \frac{p_H B (s^*(\theta^*))}{2} \right) \left[ 1 - F(A^S(s^*(\theta^*),0,1)) \right] \right] 
\]

\[
= + \frac{i}{1+i} s^*(\theta^*) \left[ \left( \frac{p_H R (1 + \lambda_H (r - 1))}{p_H \lambda_H - \Delta p c_1(0) + \Delta \lambda c_2(1)} - \frac{p_H B (s^*(\theta^*))}{2} \right) \left[ 1 - F(A^I(s^*(\theta^*),\theta^*)) \right] \right] 
\]

As \( \Pi^S(s^*(0,1),0,1) - \Pi^S(s^*(\theta^*),0,1) \geq 0 \) because \( s^*(0,1) = \text{Arg max} \{ \Pi^S(.,0,1) \} \), it is immediate that:

\[
\left( \frac{p_H \lambda_H}{p_H \lambda_H - \Delta p c_1(0) + \Delta \lambda c_2(1)} \right) \geq 0
\]

\[
\iff \Pi^S(s^*(0,1),0,1) - \Pi^I(s^*(\theta^*),\theta^*) \geq 0
\]

because in this case, \( A^I[s^*(0,1)] \geq A^S[s^*(0,1)] \). And finally:

\[
\Pi^S(s^*(0,1),0,1) - \Pi^I(s^*(\theta^*),\theta^*) \leq 0
\]

\[
\iff \left( \frac{\lambda_H p_H}{\lambda_H p_H - \lambda_L p_L} [c_1(\theta^*) + c_2(\theta^*]) - \left( \frac{p_H}{\Delta p} c_1(0) + \frac{\lambda_H}{\Delta \lambda} c_2(1) \right) \right] \leq 0.
\]

The result therefore follows. □

References


