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Group Lending with Endogenous Group Size*

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Abstract

This paper focuses on the size of the borrower group in group lending. We show that, when social ties in a community enhance borrowers’ incentives to exert effort, a profit-maximizing financier chooses a group of limited size. Borrowers that would be fundable under moral hazard but have insufficient social ties do not receive funding. The result arises because there is a trade-off between raising profits through increased group size and providing incentives for borrowers with less social ties. The result may explain why many micro-lending institutions and rural credit cooperatives lend to groups of small size.

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1 Introduction

Group lending is an unconventional lending arrangement that has been successfully applied to provide credit to poor people in low income communities. The special feature is that loans are allocated individually to group members, but all members face consequences if one cannot fulfill repayment obligations. Such joint liability arrangements are effective in dealing with asymmetric information problems, enhancing the availability of credit for poor borrowers that traditional commercial banks would not have as customers (Ghatak and Guinnane, 1999). This paper contributes to the literature by developing a simple model of group lending to address the issue of optimal group size.

The idea behind group lending is that people with connections based on geographical proximity or shared norms may be able to meet contractual obligations that would be impossible under conventional banking
agreements. Members of a community may have knowledge of each other’s types, projects, and actions. Furthermore, they may inflict non-financial sanctions on delinquent borrowers. The effects of potential retribution on borrowers’ incentives to repay may depend on the strength of social ties among community members.

Although the majority of today’s joint liability lending institutions are microfinance institutions like the well-known Grameen Bank in Bangladesh or BancoSol in Bolivia, the traditions of group lending date back to the mid-19th century when the German rural credit cooperatives were established. Both types of institutions have become well known of their ability to make small loans to borrowers without collateral valuable for a commercial lender. An important feature for both lending institutions has been that the borrowers lived in small rural communities, interacted frequently, and belonged to groups organized on the basis of different economic and social ties.

Group lending institutions differ in the size of the borrower group. At the turn of the 20th century, most rural credit cooperatives in Germany used to lend to groups of between 75 and 250 members (Ghatak and Guinnane, 1999). The Grameen bank in Bangladesh is known of its preference towards small groups of five members. In their study on group lending programs in Ghana, Owusu and Tetteh (1982) find that the number of members in a group varies between 10 and 100. FINCA (Fundacion Integral Campesina), the international organization, lends to borrower groups of between 10 and 50 members. Devereux and Fishe (1993) argue that, in the Dominican Republic, small group size is an important feature of successful micro-lending programs.

In this paper, we focus on the equilibrium size of the borrower group from the perspective of a profit-maximizing financier. We build a model where social ties affect incentives to exert effort on borrowers’ individual projects under moral hazard. In particular, we assume that every borrower can be characterized by a level of social capital that represents the strength of the borrower’s social attachment to the community she is part of. Borrowers with a higher level of social capital are easier to provide incentives to work. This may be the case, for example, because borrowers with strong social ties are more sensitive to non-financial sanctions than borrowers less attached to their community. Social ties are therefore important but the model does not require that borrowers are jointly liable for the loan. We show the existence of an optimal group size determined by the level of social capital of the marginal borrower that is eligible for funding under moral hazard. Our result suggests that if the group is chosen to maximize the financier’s profits, group size is limited: it depends on the strength of the borrower’s social ties whether the borrower becomes part of the group. We show that the chosen group size increases in the project’s profit potential and decreases in the expected agency cost the financier is required to pay to compensate borrowers for their efforts.

In our model, group size has two countervailing effects on the financier’s profits. An increase in group size

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1 An important difference between the two types of organizations is that while microfinance institutions obtain most of their lending capital from external financial institutions, in credit cooperatives members’ capital contributions represent a major source of funding.
increases the bank’s customer base and thus the amount of capital to lend. On the other hand, increasing group size entails the involvement of borrowers with less social capital. The financier needs to remunerate those borrowers by paying large agency costs, thereby reducing profits. Consequently, agency costs together with borrowers’ heterogeneity in terms of social ties suffice to show that the number of borrowers in the group is limited.

The theoretical literature on group lending has mainly focused on the effect of joint liability on group members’ repayment incentives. Besley and Coate (1995) show that the possibility of imposing social sanctions decreases group members’ incentives to default. Ghata (2000) argues that joint liability lending can be used as a screening device through the instrument of peer selection. Ghatak and Guinnane (1999) provide a comprehensive analysis on how joint liability lending may mitigate information and enforcement problems in communities with strong social ties. We contribute to this literature by showing, in a framework without joint liability but with socially connected borrowers, that moral hazard considerations impose an upper limit on group size.

The next section describes the basic model and our main result. Section 2 and 3 consider robustness issues. Section 4 concludes.

2 Basic Model

Consider the problem of a financier engaged in lending to a group of financially constrained borrowers. Borrowers are subject to moral hazard and differ in terms of the level of social and financial capital they possess. The model has one period. At the beginning of the period, each borrower decides whether to invest in a project that requires investment \( I \). The investment project yields \( R \) in case of success and 0 in case of failure. If the borrower exerts effort on her project, the probability of success is \( p_H \). If the borrower does not exert effort, the probability of success is \( p_L \) and the borrower derives private benefit of size \( s_B \). Hence, the opportunity cost of working depends on the borrower’s social capital \( (1 - s) \). A borrower with a higher level of social capital obtains private benefits of a lower amount when shirking. This assumption captures the idea that borrowers with strong within-group social ties are easily punished by non-financial sanctions. We assume that \( s \) is uniformly distributed on the interval \([0, 1]\). Each borrower has a specific amount of financial capital \( A \), where \( A \) is uniformly distributed on the interval \([0, A]\). Finally, we assume that the financier requires a gross return \((1 + i)\) on the investment.

The borrower exerts effort if the incentive compatibility constraint holds.

\[
p_H R_b \geq p_L R_b + sB \\
\iff R_b \geq \frac{sB}{\Delta p}
\]  

2 Other important papers adressing adverse selection issues in group lending include Armendariz de Aghion and Gollier (2000), Lafront and NGuessan (2000), and Lafront (2003).
The financier’s participation constraint and the condition for financing can be written as:

\[ p_H (R - R_b) \geq (1 + i) (I - A), \]

\[ A \geq I - p_H \frac{R - s \bar{H}}{(1+i)} = A(s) \] (2)

The financier may provide funding for every project that satisfies the financing condition. Notice that \( A(s) \) is increasing in \( s \) and therefore decreasing in borrowers’ social capital.

We denote the financier’s profits by \( \Pi \). Since \( s \) is uniformly distributed on \([0,1]\), we can write \( \Pi \) as a function of the level of social capital of the marginal borrower that can be funded under moral hazard, \( \bar{s} \).

\[ \Pi = (1 + i) \int_0^{\bar{s}} [I - A(s)] \Pr[A(\bar{s}) \leq A \leq A(1)] ds \] (3)

In what follows, we denote the financier’s equilibrium choice of the bank’s customer base by \( s^* \) and refer to \( s^* \) as the optimal group size. The following proposition states that a profit-maximizing financier chooses the bank’s customer base such that \( s^* < 1 \).

**Proposition 1** There exists a level of social capital \( s^* \in (0,1) \) that maximizes the financier’s profits and thereby defines the optimal size of the borrower group. The optimal size of the group increases in the project’s expected profit potential \( p_H R \) and decreases in the expected agency cost to be paid to borrowers \( p_H B_p \).

The proposition suggests that group size is limited: a profit-maximizing financier does not provide financial capital to all borrowers that are fundable under moral hazard. The financier may increase the amount of capital to be lent by including a larger number of borrowers in the group. Providing funding to borrowers with low social capital will however decrease profits because of the inherent moral hazard problem. The financier has a trade-off between raising profits by increasing group size and paying a high agency rent to socially less connected borrowers. The high agency cost to be paid to borrowers with low social capital makes the financier choose the size of the group in a manner that not all borrowers that would otherwise be fundable, even under moral hazard, may realize their investment projects. The result is in line with Devereux and Fishe (1993) suggesting that the borrower group must be composed of fairly homogenous individuals.

Our finding is also consistent with lending practices of the historical German cooperatives. Ghatak and Guinnane (1999) cite examples of German credit cooperatives that denied loans to their members. In 1888, the cooperative in Diestedde (Münsterland) rejected the application of a skilled artisan for a small loan. In 1913, the Limbach cooperative in Saarland did not admit two individuals as members and did not give justification for the decision.\(^3\) Furthermore, cooperatives gave loans to some members while having required additional security. For example, the cooperative in Leer (Münsterland) provided a loan to a borrower in 1909 on the condition that two persons co-sign the agreement.\(^4\)

\(^3\) Ghatak and Guinnane (1999) cite the cooperative records of the ‘Protokollbuch für den Vorstand’ as their resource for such examples.

\(^4\) Co-signing is a form of joint liability between the borrower and the co-signer.
3 Financier’s Information Advantage

In this section, we extend the basic model of section 2 and show that our result holds for a more general model set-up where the financier obtains information about project quality. The borrower has no ability to assess the quality of her project.

We assume that, before the borrower exerts effort, the financier observes information about the projects’ payoff perspectives. Assume that for any project, the ex-ante probability of a good payoff perspective is $\pi$. The financier observes the state of the project with probability $(1 - q)$. When payoff perspectives are good, the borrower’s effort does not matter—the project will succeed. When payoff perspectives are bad, the borrower needs to exert effort to achieve a high success probability.

The borrower’s incentive constraint is thus as follows.

$$(1 - \pi) p_H R_B \geq (1 - \pi) p_L R_B + (1 - \pi) sB$$

$\iff R_B \geq \frac{sB}{\Delta p}$ (4)

The financier’s participation constraint and the financing condition are:

$$(\pi + (1 - \pi) p_H) R - \left[ q (\pi + (1 - \pi) p_H) \frac{sB}{\Delta p} + (1 - q) (1 - \pi) p_H \frac{sB}{\Delta p} \right] \geq (1 + i) (I - A)$$

$\iff A \geq I - \frac{[\pi + (1 - \pi) p_H] R - ((1 - \pi) p_H + q\pi) \frac{sB}{\Delta p}}{1 + i} = A(s)$ (5)

The following proposition states that the result obtained in section 2 is robust to the introduction of asymmetric information into the basic model.

**Proposition 2** Assume the financier has information about project quality unattainable for the borrower. Even under this assumption, there exists a level of social capital $s_e^* \in (0, 1)$ that maximizes the financier’s profits and thereby defines the optimal size of the borrower group. The optimal size of the group $s_e^*$ increases in the expected revenues of the project $[\pi + (1 - \pi) p_H] R$ and decreases in the expected agency cost $\left[ (1 - \pi) p_H + q\pi \right] \frac{B}{\Delta p}$.

4 Robustness

In previous sections, we assumed that borrowers’ financial $A$ capital was uniformly distributed on the interval $[0, \bar{A}]$. We now consider the robustness of our results by assuming a general distribution function for $A$.

We assume that each borrower has a specific amount of financial capital $A$, where $A$ is distributed on $[0, A(1)]$, with a cumulative distribution function $F$ and a density function $f(.)$. For this distribution, we require that the monotone hazard rate assumption holds: $\frac{[1-F(.)]}{F(.)}$ is non-increasing.

The following Proposition states that our main result is robust to the introduction of a general distribution function.

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5Another interpretation would be that the financier is able to monitor borrowers with an imperfect monitoring technology.
Proposition 3 Assume a general distribution function for borrowers’ financial capital $A$. There exists a level of social capital $\tilde{s}_g \in (0, 1)$ that maximizes the financier’s profits and thereby defines optimal group size. The optimal size of the group increases in expected project revenues and decreases in the expected agency cost to be paid to borrowers.

5 Conclusion

In this paper we focused on group lending from the perspective of a profit-maximizing financier. We showed that when social ties in a community affect borrowers’ incentives to work the optimal size of the group the financier lends to is limited. A profit-maximizing financier chooses the size of the borrowing group in a way that borrowers that would be fundable under moral hazard but have insufficient social ties do not receive funding. Consequently, the size of the borrowing group chosen by the financier is limited. The result arises because both social and financial capital matter for the funding of financially constrained borrowers: the financier faces a trade-off between raising the size of the bank and providing incentives for borrowers with limited social connections. The result may explain why many micro-lending institutions and rural credit cooperatives lend to groups of small size.

6 Appendix

Proof of Proposition 1. Given the financing condition defined in (2), we write the amount of capital to be lent as a function of the level of social capital of the marginal borrower that can be funded under moral hazard, $\tilde{s}$.

\[
\Pi = (1 + i) \int_0^{\tilde{s}} [I - A(s)] \Pr[A(\tilde{s}) \leq A \leq A(1)] ds
\]

\[
= (1 + i) (F[A(1)] - F[A(\tilde{s})]) \int_0^{\tilde{s}} [I - A(s)] ds
\]

\[
= \left( p_H R \tilde{s} - \frac{(\tilde{s})^2}{2} \frac{p_H B}{\Delta p} \right) (F[A(1)] - F[A(\tilde{s})])
\]
The financier will choose the size of the group by maximizing the amount of capital to be lent and thereby profits. Since $A$ is uniformly distributed on $[0, \overline{A}]$, we can write $\frac{d\Pi}{ds}$ as follows:

\[
\frac{d\Pi}{ds} = \left( \frac{1+i}{A} \right) \left( I - A(\hat{s}) \right) \left( F[A(1)] - F[A(\hat{s})] \right) - \hat{s} \left( I - A(\hat{s}) + \frac{1}{1+i} \frac{\hat{s} p_H B}{\Delta p} \right) \frac{p_H B}{\Delta p} f[A(\hat{s})]
\]

\[
= \frac{1}{A} \left( (1+i) \left( I - A(\hat{s}) \right) \left( A(1) - A(\hat{s}) \right) \right) - \hat{s} \left( I - A(\hat{s}) + \frac{1}{1+i} \frac{\hat{s} p_H B}{\Delta p} \right) \frac{p_H B}{\Delta p}
\]

\[
= \frac{1}{A} \left( I - A(\hat{s}) \right) \left( 1 - \hat{s} \right) p_H B \frac{\Delta p}{\Delta p}
\]

Solving for the optimal size of the group $s^*$:

\[
\frac{d\Pi}{ds} = \left( I - A(\hat{s}) \right) (1 - \hat{s}) \left( I - A(\hat{s}) + \frac{1}{1+i} \frac{\hat{s} p_H B}{\Delta p} \right) = 0
\]

\[
\iff \left( I - A(\hat{s}) \right) (1 - 2\hat{s}) - \frac{1}{1+i} \frac{\hat{s} p_H B}{\Delta p} = 0
\]

\[
\iff \frac{1}{1+i} \left[ p_H R - \hat{s} B \frac{\Delta p}{\Delta p} \right] (1 - 2\hat{s}) - \frac{1}{1+i} \frac{\hat{s} p_H B}{\Delta p} = 0
\]

\[
\iff p_H R - \hat{s} \left( 2p_H R + \frac{p_H B}{\Delta p} \right) + \frac{3p_H B}{\Delta p} \hat{s}^2 = 0
\]

The above expression is a second degree polynomial $ax^2 + bx + c = 0$, with $a \geq 0, b \leq 0$ and $c \geq 0$. We therefore have 2 positive roots. Moreover, this polynomial is positive for $\hat{s} = 0$, and negative for $\hat{s} = 1$. Indeed:

\[
\frac{d\Pi}{ds} (\hat{s} = 0) = \frac{1}{A} \frac{1}{1+i} p_H R > 0
\]

\[
\frac{d\Pi}{ds} (\hat{s} = 1) = \frac{1}{A} \frac{1}{1+i} p_H \left( \frac{1}{2} B R - R \right) = \frac{1}{\overline{A}} \left( \frac{B}{2} - 1 \right) < 0.
\]

Let $s^*$ be the lowest root of this polynomial, $\frac{d\Pi}{ds}$ is positive for all $\hat{s} \in [0, s^*)$ and negative for all $\hat{s} \in [s^*, 1]$. Since $\Pi$ is concave in $\hat{s}$, $s^*$ is a maximum of the function $\Pi(\hat{s})$. Indeed,

\[
\frac{d^2\Pi}{ds^2} = -2p_H R - \frac{p_H B}{\Delta p} + \frac{3p_H B}{\Delta p} \hat{s} = -2 \left( p_H R - s \frac{p_H B}{\Delta p} \right) - \frac{p_H B}{\Delta p} (1 - s) \leq 0
\]
Using equation (8) in the proof of Proposition 1 we define the optimal size of the group \( s^* \), as follows:

\[
  s^* = \frac{\left(2p_H R + \frac{p_H B}{\Delta p}\right) - \sqrt{\left(2p_H R + \frac{p_H B}{\Delta p}\right)^2 - 6\frac{p_H B}{\Delta p} p_H R}}{3 \frac{B}{\Delta p}}
\]

\[
  \iff s^* = \frac{1}{3} \left( \left[ \left(2p_H R + \frac{p_H B}{\Delta p}\right) + 1 \right] - \sqrt{\left[ \left(2p_H R + \frac{p_H B}{\Delta p}\right) + 1 \right]^2 - 2\frac{p_H B}{\Delta p} p_H R} \right)
\]

\[
  \iff s^* = \frac{1}{3} \left( \left[ \left(2p_H R + \frac{p_H B}{\Delta p}\right) + 1 \right] - \sqrt{4 \left(\frac{p_H R}{\Delta p}\right)^2 + 1} \right)
\]

\[
  \iff s^* = \frac{1}{3} \left( 2y + 1 - \sqrt{(2y + 1)^2 - 2y} \right) = \frac{1}{3} \left( 2y + 1 - \sqrt{4y^2 + 1} \right) \quad (12)
\]

where \( y = \frac{p_H R}{\Delta p} \).

\[
  \frac{\partial s^*}{\partial y} = \frac{1}{3} \left( 2 - \frac{4y}{\sqrt{4y^2 + 1}} \right) = \frac{2}{3} \left( 1 - \frac{2 \frac{p_H R}{\Delta p}}{\sqrt{4 \left(\frac{p_H R}{\Delta p}\right)^2 + 1}} \right) \geq 0 \quad (13)
\]

Therefore, \( \frac{\partial s^*}{\partial y} \geq 0 \). The result follows. ■

**Proof of Proposition 2.** Given the financing condition defined in (5), we write the amount of capital to be lent as a function of the level of social capital of the marginal borrower that can be funded under moral hazard, \( \hat{s} \). If \( s \) is uniformly distributed on \([0,1]\), we have:

\[
  \Pi = (1 + i) \int_0^{\hat{s}} [I - A(s)] \Pr [A(\hat{s}) \leq A \leq A(1)] \, ds
\]

\[
  = \left( \pi + (1 - \pi) p_H \right) R \hat{s} - \left( (1 - \pi) p_H + q \pi \right) \left( \hat{s} \frac{B}{\Delta p} \right) \left( F[A(1)] - F[A(\hat{s})] \right) \quad (14)
\]

Again, financier chooses the size of the group by maximizing the amount of capital to be lent and thereby profits. The results are therefore the same as in Proposition 1 replacing the expected revenues of the project \( p_H R \) by \( (\pi + (1 - \pi) p_H) R \) and the expected agency cost to be paid to the borrower \( \frac{B}{\Delta p} \) by \( ((1 - \pi) p_H + q \pi) \frac{B}{\Delta p} \).

■

**Proof of Proposition 3.** The optimal group size is defined by the following condition:

\[
  \frac{d\Pi}{ds} = \left( (1 + i) [I - A(\hat{s})] [F[A(1)] - F[A(\hat{s})]] \right. \\
  \left. - \hat{s} [I - A(\hat{s})] + \frac{1}{(1 + i) 2} \hat{s} \frac{B}{\Delta p} \left( (1 - \pi) p_H + q \pi \right) \right) = 0
\]

\[
  \iff \left( \frac{1 + i - 1 - F(A(\hat{s}))}{(1 + i)(1 - F(A(\hat{s})) \frac{B}{\Delta p} f[A(\hat{s})]} \right) \left( \frac{(1 - \pi) p_H + q \pi B}{\Delta p} f[A(\hat{s})] \right) = 0 \quad (15)
\]
By assumption: $\frac{d}{d\hat{s}} \left( \frac{[1-F(A(\hat{s}))]}{f(A(\hat{s}))} \right) \leq 0$ and $\frac{d}{d\hat{s}} \left( \frac{(\hat{s})^2}{f(A(\hat{s}))} \right) = \frac{2[1-A(\hat{s})] + \frac{1}{(1+i)} \frac{1}{A(\hat{s})^2} (\hat{s})^2}{[1-A(\hat{s})]^2} \geq 0$. This implies that $\Pi$ is concave in $\hat{s}$. Moreover:

$$\frac{d\Pi}{d\hat{s}}(\hat{s} = 0) = [1 - A(0)][1 - F(A(0))] > 0$$  \hspace{1cm} (16)

$$\frac{d\Pi}{d\hat{s}}(\hat{s} = 1) = -\left[ 1 - A\left( \frac{1}{2} \right) \right] \frac{((1 - \pi)_p + q\pi)B}{\Delta p} f(A(1)) < 0$$  \hspace{1cm} (17)

This implies that the equation $\frac{d\Pi}{d\hat{s}} = 0$ admits a unique solution $\hat{s}^*_s$ on $[0,1]$. If we denote by $E(R)$, the expected project revenues and by $E(B)$, the expected agency cost to be paid to borrowers, we obtain:

$$\frac{d\Pi}{d\hat{s}} = \left( \frac{[E(R) - \hat{s}E(B)](1 - F[A(\hat{s})])}{-\frac{1}{(1+i)} \hat{s} [E(R) - \hat{s}E(B)] + \frac{1}{2} E(B) f[A(\hat{s})]} \right) = 0$$  \hspace{1cm} (18)

Using the implicit function theorem, we have:

$$\frac{ds^*_s}{dE(R)} = -\frac{\frac{d^2\Pi}{d\hat{s}dE(R)}}{\frac{d^2\Pi}{d\hat{s}^2}} = -\left( 1 - F[A(\hat{s})] \right) - \frac{1}{(1+i)} \left[ E(R) - \hat{s}E(B) \right] E(B) f[A(\hat{s})] \geq 0$$  \hspace{1cm} (19)

$$\frac{ds^*_s}{dE(B)} = -\frac{\frac{d^2\Pi}{d\hat{s}dE(B)}}{\frac{d^2\Pi}{d\hat{s}^2}} = -\hat{s} \left( 1 - F[A(\hat{s})] \right) - \frac{1}{(1+i)} \left[ E(R) - \hat{s}E(B) \right] E(B) f[A(\hat{s})] \leq 0$$  \hspace{1cm} (20)

### References


