Auctions with Loss Averse Bidders

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October 16, 2012

Abstract

We theoretically and experimentally study independent private value auctions in the presence of bidders who are loss averse in the sense of Köszegi and Rabin (2007). In one specification, we consider gains and losses in two dimensions separately, about whether they receive the object or not, and how much they pay (narrow bracketing of gains and losses); in the other specification, we consider gains and losses over the entire risk neutral pay off, i.e. the valuation less the bid (wide bracketing of gains and losses). With wide bracketing, we show that the expected revenue for the auctioneer is higher in the first price auction than in the all pay auction, and with narrow bracketing, we show that the opposite is true for the revenue ranking between the first price auction and the all pay auction. In order to test the theoretical predictions, we conduct laboratory experiments, in which money and a real object is auctioned in both a first price auction and an all pay auction. In both settings, the average revenue is significantly higher in the first price auction, suggesting that bidders may behave according to the one dimensional model, although a real object is auctioned. Whereas our findings are inconsistent with narrow bracketing of gains and losses, they are consistent with wide bracketing of gains and losses.

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1 Introduction

Since Kahneman and Tversky (1979), loss aversion and reference dependent preferences have been applied to a variety of empirical and theoretical economic problems. When applying models of loss aversion, the modeller is required to decide over what individuals have feelings of gains and losses, which is the problem of narrow versus wide bracketing. To illustrate the problem, consider the series of experiments conducted by Kahneman, Knetsch, and Thaler (1990), who study the endowment effect in competitive markets. When subjects are given actual goods, the endowment effect has an impact on trading volumes; if, however, subjects are endowed with money rather than a good, they observe no endowment effect. The explanation given is that when trading money for coffee mugs, there is a friction caused by a loss in one and a gain in the other dimension. When money is traded for money, this friction disappears. Köszegi and Rabin (2006) propose a model which rationalizes the experimental findings mentioned, using the concept of consumption dimensions, over which individuals have gain loss utility in an additively separable manner.

Applying the model of Köszegi and Rabin (2006) and Köszegi and Rabin (2007), we derive the equilibrium bidding behavior in the first price auction (FPA) and in the all pay auction (APA) for general environments with independent private values (IPV), study the behavioral implications of loss aversion on bidding strategies, and compare the revenue across auction formats. In one specification, we consider gains and losses in two dimensions separately, about whether they receive the object or not, and how much they pay; in the other specification, we consider gains and losses over the entire risk neutral pay off, i.e. the valuation less the bid. The first specification represents narrow bracketing, while the second one represents wide bracketing. With wide bracketing, we show that the expected revenue for the auctioneer is higher in the FPA than in the APA, and with narrow bracketing, we show that the opposite is true for the revenue ranking between the FPA and the all pay auction.

In order to test the theoretical predictions, we conduct laboratory experiments, in which either money or a real object is auctioned in both a FPA and an APA. In both settings, the average revenue is significantly higher in the FPA, suggesting that bidders may behave according to the one dimensional model, although a real object is auctioned. Whereas our findings are inconsistent with the two dimensional model, they are consistent with the one dimensional one.

The paper contributes to the literature on loss aversion and reference dependent preferences in several ways. Comparing our results to the ones in Kahneman, Knetsch, and Thaler (1990), we conclude that whether individuals apply a narrow or a wide form of bracketing gains and losses depends on the environment under consideration. While competitive markets and auctions are similar in may ways, there is a lot more uncertainty in auctions. Additionally, we provide an estimate for the ratio of marginal disutility of losses to marginal utility of gains of 1.42, using the generalized method of moments for the data obtained in the induced value experiments. Furthermore, we show that when applying the Köszegi and Rabin (2007) model, the theoretical predictions crucially depend on the modeller's decision about how to define the consumption dimensions over which individuals feel gains and losses. Finally, our experimental data shows that there is no measurable difference between auctioning an actual good or simply money in auctions with induced valuations.

1.1 Related Literature

1.1.1 Auction Theory and Risk Preferences

Riley and Samuelson (1981), Maskin and Riley (1984), Matthews (1987), and Fibich, Gavious, and Sela (2006) study the implications of risk averse bidders in auction settings. Lange and Ratan (2010) consider the case of loss averse bidders for the FPA and the Vickrey auction and show that the FPA yields higher expected revenue than the Vickrey auction, independent of whether bidders consider gambles in one or two dimensions. Shunda (2009) shows that under a different notion of reference dependence, the auctioneer can increase his expected revenue by introducing a buy now price. In the present paper, we focus on a specific class of hybrid auctions, incorporating both the FPA and the APA, and study the bidders' behavior and revenue (non) equivalence across different auction formats. Furthermore, while the revenue ranking of the FPA and the Vickrey auction in both models is the same (Lange and Ratan (2010)), our analysis provides another testable implication of reference dependence with revenue data alone. Using a general mechanism design approach in the spirit of Myerson (1981), Eisenhuth (2012) shows that in the one dimensional model, the FPA maximizes the auctioneer's expected revenue, and that in the two dimensional model, any optimal auction is fully all pay.

1.1.2 Experimental Economics

To the best of our knowledge, Noussair and Silver (2006) provide the only empirical analysis comparing the APA and the FPA in a laboratory setting with independent private values. They replicate the environment in Cox, Smith, and Walker (1982) and Cox, Roberson, and Smith (1988), who study the FPA, and compare the revenue data from these studies to their revenue data on the APA. Their finding is that the APA yields significantly higher revenue than the FPA. One confounding effect which might be driving their results is that they provide subjects with an initial endowment of nearly seven times as much as Cox, Smith, and Walker (1982) and Cox, Roberson, and Smith (1988). Thereby, Noussair and Silver (2006) lose some control over their data comparison and higher APA bids might be driven by a nearly seven times higher endowment. Furthermore, they observe bids of 0 for the lowest types in either auction format. Real object auctions are not studied. Lucking-Reiley (1999) studies real object field auctions using the FPA, the Vickrey auction, the English auction, and the Dutch auction with Magic cards and refutes revenue equivalence; an analysis of the APA is missing. Moreover, as the data are collected through online auctions, bidders do not know how many opponents they are facing in the auction. We contribute to the experimental literature by studying revenue equivalence between the APA and the FPA, explicitly differentiating between auctioning money and an actual object.

2 Model

2.1 Preferences

We consider two different specifications of reference dependent preferences. The first one is a specification according to which the bidders consider gambles for the object and money separately; the second specification treats the difference between the valuation for the object and the amount paid as one dimension, and gambles are evaluated over this difference only. As proposed in Köszegi and Rabin (2006), the first specification of bidders' preferences is given by

$$u^2(c^g, c^m | r^g, r^m, \theta) := \underbrace{\theta c^g + c^m}_{\text{intrinsic utility}} + \underbrace{\eta^g \mu^g(\theta(c^g - r^g)) + \eta^m \mu^m(c^m - r^m)}_{\text{gain loss utility}},$$

where $c^g, r^g \in \{0, 1\}$ captures the good dimension, $c^m, r^m \in \mathbb{R}$ captures the money dimension. For $l \in \{g, m\}$, c^l is true consumption, r^l is the reference level of consumption, $\eta^l > 0$, measures the weight attached to gain loss utility in dimension l, and $\theta \geq 0$ is the bidder's intrinsic valuation for the good. The second specification is given by

$$u^1(c^g, c^m | r^g, r^m, \theta) := \underbrace{\theta c^g + c^m}_{\text{intrinsic utility}} + \underbrace{\eta \mu(\theta c^g + c^m - (\theta r^g + r^m))}_{\text{gain loss utility}}.$$

Moreover,

$$\mu^l(x) := \begin{cases} x, & \text{if } x \geq 0 \\ \lambda^l x, & \text{if } x < 0, \end{cases}$$

where $\lambda^l > 1$, $l \in \{g, m\}$, and the second specification with only one dimension is implied when the index on the parameters is suppressed. These preferences capture loss aversion through the Kahneman and Tversky (1979) value function, μ^l, $l \in \{g, m\}$. A deviation from the reference point is disliked more if it is a loss than it is liked if it is a gain.

2.2 Auction Rules

A single, indivisible object is sold among $N \geq 2$ loss averse bidders who share the same η^l and λ^l, $l \in \{g, m\}$, and whose valuations, $\{\theta_i\}_{i=1}^N$, are the realizations of N independent draws from the continuous distribution function, $F : \Theta \to [0, 1]$, where $\Theta := [\theta_{\min}, \theta_{\max}] \subset \mathbb{R}_+$, with strictly positive density everywhere. The valuation of bidder i, θ_i, is bidder i's private information. The bidders and the auctioneer share the same prior beliefs. We consider the following class of auctions with all pay component, $\alpha \in [0,1]$. Bidders simultaneously submit their bid, and the bidder with the highest bid wins the object and pays his entire bid. All other bidders walk away without the object but have to pay α of their bid. In case of a winning tie, the winner is selected among the highest bidders with equal probability. For $\alpha = 0$, we have the FPA and for $\alpha = 1$, the APA. This formulation, incorporating both the APA and the FPA, appears first in Siegel (2010). Other common auction formats, as, for instance, the Vickrey auction are excluded from our analysis, partially because Lange and Ratan (2010) study the Vickrey auction in the same setting and partially because Eisenhuth (2012) shows that this is without loss of generality, using a general mechanism design approach in the spirit of Myerson (1981). In our theoretical analysis, we focus on symmetric equilibrium bidding functions.

2.3 Solution Concept

In the above described auction setting, each bidder learns his valuation before submitting his bid and therefore, maximizes his interim expected utility. Using Köszegi and Rabin (2006)'s notation, if the distribution of reference points is G, and the distribution of actual consumption outcomes is H, the decision maker's interim expected utility is given by

$$U(H|G,\theta) := \int_{\{(c^g,c^m)\}} \int_{\{(r^g,r^m)\}} u(c^g,c^m|r^g,r^m,\theta)dG(r^g,r^m|\theta)dH(c^g,c^m|\theta).$$

Definition 1 *(Köszegi and Rabin (2007)) Conditional on the realization of the type, θ, for any choice set, D, $H \in D$ is an interim CPE if $U(H|H,\theta) \geq U(H'|H',\theta)$, for all $H' \in D$.*

Fixing all other bidders' behavior, each bidder's bid, b_i, induces a distribution, $H_i(\mathcal{A}|b_i,b_{-i})$, over the set of alternatives, $\mathcal{A} := \{0,1\}^N \times \mathbb{R}^N$. Therefore, the definition can be modified in the following way to match the auction setting under consideration.

Definition 2 *Conditional on the realization of the type, θ_i, $b : \Theta \to \mathbb{R}_+$ is a symmetric interim CPE bidding function if for all $i, \theta_i, \theta_{-i}, b' \geq 0$,*

$$\begin{aligned} &U\left(H_i(\mathcal{A}|b(\theta_i), b_{-i} = b(\theta_{-i}))|H_i(\mathcal{A}|b(\theta_i), b_{-i} = b(\theta_{-i})), \theta_i\right) \\ &\quad \geq U\left(H_i(\mathcal{A}|b', b_{-i} = b(\theta_{-i}))|H_i(\mathcal{A}|b', b_{-i} = b(\theta_{-i})), \theta_i\right). \end{aligned}$$

The interpretation of CPE is that each bidder understands that once consumption occurs, i.e. once the auction is over, he evaluates this consumption outcome against the actual lottery as his reference lottery. As laid out in Köszegi and Rabin (2007), CPE is the most appropriate solution concept for decisions under risk, whose uncertainty is resolved long after the decision is made. An alternative solution concept, *choice unacclimating personal equilibrium* (UPE), requires the decision to be optimal, given the expectations at the time the decision is made. Below we discuss the relationship between UPE and CPE in the auction setting under consideration. For the following analysis, it is convenient to define $\Lambda^l := \eta^l(\lambda^l - 1) > 0$, $l \in \{g, m\}$, which can be viewed as an overall measure of the degree of loss aversion in the respective dimension. The following assumption, as proven in the appendix, guarantees that all bidders participate in the auction for any realization of their own type, and that their equilibrium bidding functions derived below are strictly increasing.

Assumption 1 ***(No Dominance of Gain Loss Utility)*** $\Lambda^g \leq 1$.

This assumption places, for a given η (λ), an upper bound on λ (η). In Herweg, Müller, and Weinschenk (2010), this assumption is referred to as *no dominance of gain loss utility*. In the following, we consider each specification of reference dependent preferences, one and two dimensional, at a time.

3 Analysis

3.1 Two Dimensions — Narrow Bracketing

Consider the ex post utility of bidder i when his bid is x, and x_{-i} is the vector of all other bidders' bids. Let $q_i(x) = P(i \text{ wins } |x, x_{-i}) = P(x > \max_{j \neq i}\{x_j\})$ be the probability that bidder i wins the auction, conditional on his own and all other bidders' bids. When he ends up with the object and pays x, his utility is

$$\underbrace{\theta_i - x}_{\text{intrinsic utility}} + \underbrace{\eta^g (1 - q_i(x))\, \theta_i - \eta^m \lambda^m (1 - q_i(x))\, (1 - \alpha) x}_{\text{gain loss utility}}.$$

The first term represents intrinsic utility, and the second term captures gain loss utility. Compared to the situation in which the bidder does not win the auction, which happens with probability $(1 - q_i(x))$, he experiences a gain in the good dimension and a loss in the money dimension. In case bidder i ends up without the object and his bid is x, his utility is

$$\underbrace{-\alpha x}_{\text{intrinsic utility}} + \underbrace{\eta^g \lambda^g q_i(x)(-\theta_i) + \eta^m q_i(x)(1 - \alpha) x}_{\text{gain loss utility}},$$

since, compared to the situation in which he wins the auction, which happens with probability, $q_i(x)$, this is considered a loss in the good dimension and a gain in the money dimension. Therefore, bidder i's interim expected utility is

$$\begin{aligned} &q_i(x) \left(\theta_i - x + \eta^g (1 - q_i(x))\, \theta_i - \eta^m \lambda^m (1 - q_i(x))\, (1 - \alpha) x\right) \\ &\quad + (1 - q_i(x)) \left(-\alpha x - \eta^g \lambda^g q_i(x) \theta_i + \eta^m q_i(x)(1 - \alpha) x\right). \end{aligned}$$

By varying their bid, x, each bidder changes the probability of winning, and therefore his reference lottery. We look for strictly increasing, symmetric equilibrium bidding functions. Hence, dropping the i subscript, the bidder's program is

$$\begin{aligned} V(\theta) &:= \max_{x\in\mathbb{R}_+} \{q(x)\left(\theta - x + \eta^g\left(1-q(x)\right)\theta - \eta^m\lambda^m\left(1-q(x)\right)(1-\alpha)x\right) \\ &\qquad + \left(1-q(x)\right)\left(-\alpha x - \eta^g\lambda^g q(x)\theta + \eta^m q(x)(1-\alpha)x\right)\} \\ &= F^{N-1}(\theta)\left(1-\Lambda^g\left(1-F^{N-1}(\theta)\right)\right)\theta \\ &\quad -F^{N-1}(\theta)\left(1+\Lambda^m\left(1-F^{N-1}(\theta)\right)\right)(1-\alpha)b_\alpha(\theta) - \alpha b_\alpha(\theta), \end{aligned} \tag{1}$$

where the ultimate equality follows from independence of the types, b_α being strictly increasing (and hence, invertible), and the definition of Λ^l, $l \in \{g, m\}$. Since, $V(\theta_{\min}) = 0$, applying the envelope theorem yields the following expression for the symmetric CPE bidding function:

$$b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\theta_{\min}}^{\theta}\beta(s)ds}{(1-\alpha)F^{N-1}(\theta)\Delta(\theta)+\alpha},$$

where $\Delta(\theta) := (1+\Lambda^m(1-F^{N-1}(\theta))) \geq 1$ and $\beta(\theta) := F^{N-1}(\theta)\left(1-\Lambda^g\left(1-F^{N-1}(\theta)\right)\right)$. Furthermore, by the envelope theorem, the bidder's pay off is independent of the auction format, α, and depends, as in the risk neutral case, only on the probability of winning the auction.

Proposition 1 *Suppose assumption 1 holds. Then, $b_\alpha(\theta)$ is strictly increasing, for almost all θ and constitutes the unique symmetric pure strategy CPE bidding function.*

Proposition 1 and all formal results which follow are proven in the appendix. In order to study the equilibrium bidding behavior of loss averse bidders, it is instructive to first consider the case in which bidders are only loss averse in the money dimension ($\Lambda^m > 0$) and risk neutral in the good dimension ($\Lambda^g = 0$). Letting b_α^{RN} denote the equilibrium bidding function with risk neutral bidders in the same environment, the CPE bidding function then reads

$$b_\alpha(\theta) = \frac{1}{\psi_\alpha(\theta)} b_\alpha^{RN}(\theta),$$

where

$$\psi_\alpha(\theta) := \frac{(1-\alpha)F^{N-1}(\theta)\Delta(\theta)+\alpha}{(1-\alpha)F^{N-1}(\theta)+\alpha} \geq 1.$$

If bidders only have gain loss considerations in the money dimension, then the equilibrium bid is the distorted risk neutral bid. Regarding the comparative statics results with respect to the parameter, Λ^m, the following result holds.

Proposition 2 *$b_\alpha(\theta)$ is strictly decreasing in Λ^m, for almost all θ, for all $\alpha \in [0, 1)$. For $\alpha = 1$, Λ^m has no effect on $b_\alpha(\theta)$, for all θ.*

In order to see the intuition behind of this result, consider the FPA. As above, let $q(x)$ be the probability of winning the object when submitting a bid of x. In case he wins, a bidder pays x which is a loss of x relative to paying nothing (in case he loses). This loss sensation is weighted with the probability of not having to pay, which is $(1 - q(x))$ since the reference point is formed at the interim stage. Since a bidder wins with probability $q(x)$, from an interim perspective, this feeling of loss occurs with probability $q(x)$, and thus there is an interim expected loss of $\eta^m \lambda^m q(x)(1 - q(x))x$. Likewise, in the event of losing, a bidder considers the bid saved a gain of x in the money dimension, and the outcome of winning is weighted with $q(x)$ in the reference lottery. Furthermore, from an interim perspective, a bidder expects to lose with probability $(1 - q(x))$ so his interim expected gain is $\eta^m q(x)(1 - q(x))x$. Consequently, the overall expected gain loss sensation is $-\eta^m(\lambda^m - 1)q(x)(1 - q(x))x$, and since losses loom larger than gains ($\lambda^m > 1$), this is always negative when there are multiple monetary outcomes. Hence, a bidder's interim benefit of winning is less and hence, in equilibrium, this lowers the bidders' bids relative to the APA, in which payments are certain from an interim perspective. More specifically, the reduction in interim expected pay off consists of three parts, the overall degree of loss aversion, $\eta^m(\lambda^m - 1) = \Lambda^m$, the variance of the Bernoulli distributed outcome of winning or losing the auction, $q(x)(1 - q(x))$, and the wedge between the amount paid when winning and losing. Since the last part is identical to 0 in the APA, gain loss considerations about whether to pay or or not have no effect on the interim expected pay off, whereas for all $\alpha \in [0, 1)$, the pay off is reduced. In a symmetric equilibrium, the interim probability of winning and receiving the good is the same across auction formats, in particular, $q(x) = F^{N-1}(\theta)$, and therefore, does not affect the comparison across auction formats. In order to examine how loss aversion in the good dimension affects the bidding behavior, consider the case in which $1 \geq \Lambda^g > 0$ and $\Lambda^m = 0$. Then, the equilibrium bidding function is given by $b_\alpha(\theta) = (1 - \Lambda^g)b_\alpha^{RN}(\theta) + \Lambda^g \kappa_\alpha(\theta)$, where

$$\kappa_\alpha(\theta) := \frac{F^{2(N-1)}(\theta)\theta - \int_{\theta_{\min}}^{\theta} F^{N-1}(s)ds}{(1 - \alpha)F^{N-1}(\theta) + \alpha}.$$

The following proposition summarizes the impact of loss aversion in the good dimension on the equilibrium bid.

Proposition 3 *If $0 < \Lambda^g \leq 1$ and $\Lambda^m = 0$, there is a unique interior threshold, $\bar{\theta}$, such that $b_\alpha(\theta)$ is strictly decreasing in Λ^g, for almost all $\theta < \bar{\theta}$, and strictly increasing in Λ^g, for all $\theta > \bar{\theta}$ $\alpha \in [0, 1]$.*

To see the intuition behind this result, again, let $q(x)$ be the interim probability of winning. Then, from an interim perspective, expected gain loss utility in the good dimension is $-\eta^g(\lambda^g - 1)q(x)(1 - q(x))\theta$. As above, the variance of the Bernoulli distributed outcome of winning or losing the auction reduces the interim expected pay off. This implies that the loss is maximized at $q(x) = 1/2$, and minimized at $q(x) = 0$ and $q(x) = 1$. Hence, whenever $q(x)$ is less than $1/2$ a bidder has an incentive to lower $q(x)$ in order to reduce this feeling of loss while whenever $q(x)$ is greater than $1/2$ he has an incentive to increase $q(x)$ in order to lower this feeling of loss. Of course, in equilibrium, the probability of winning for a bidder is the probability that he is the highest type, which is unaffected by loss aversion. Therefore, loss aversion in the good dimension increases the bid of the highest types and reduces the bid of the lowest types through an indirect effect caused by this preference for certain outcomes. Figure 1 depicts the equilibrium bidding functions for $N = 2$ and $\theta \sim U[0, 1]$, compared to the same situation with risk neutral bidders ($\Lambda^g = \Lambda^m = 0$). In the APA, the bidding functions with risk neutrality and loss aversion in the money dimension coincide (the dashed line in the right panel of Figure 1).

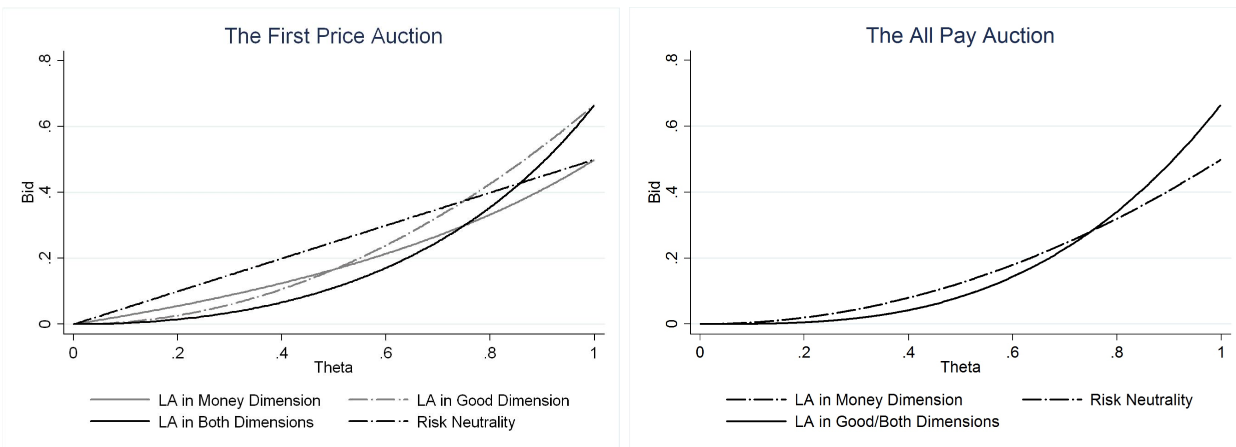

Figure 1: FPA and APA with $\Lambda^g = 1$, $\Lambda^m = 1$, $N = 2$, and $\theta \sim U[0, 1]$.

So far, it has been assumed that assumption 1 is satisfied. As Lange and Ratan (2010) show, if assumption 1 is not met, there are some bidders who choose to not participate in the auction and submit a bid of 0. The argument is that by choosing a bid of 0, a bidder can secure himself a pay off of 0. However, if a set of types of strictly positive measure bid 0, then these types tie at 0 and win with positive probability. Taking into account the ties at 0, the implications of a violation of assumption 1 are the following.

Proposition 4 *Suppose assumption 1 does not hold, i.e.* $\Lambda^g > 1$. *Then, in the unique symmetric pure strategy CPE, there is a unique interior threshold,* $\hat{\theta} \in (\theta_{\min}, \theta_{\max})$, *given by* $F^{N-1}(\hat{\theta}) = (\Lambda^g - 1)/\Lambda^g$, *such that for all* $\theta \geq \hat{\theta}$,

$$b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta} \beta(s)ds}{(1-\alpha)F^{N-1}(\theta)\Delta(\theta) + \alpha},$$

and for all for all $\theta < \hat{\theta}$, $b_\alpha(\theta) = 0$, *for all* $\alpha \in [0, 1]$. *Additionally,* $\hat{\theta}$ *is strictly increasing in* Λ^g *and the number of bidders,* N.

This result indicates that when loss aversion in the good dimension is too pronounced, there is a set of types of strictly positive measure, for which it is not optimal to submit a positive bid. The cut off point, $\hat{\theta}$, is identical across all auction formats, $\alpha \in [0,1]$. If bidders are given the option of not participating in the auction, with a certain pay off of 0, the cut off in the above proposition changes, but the analysis remains the same. In fact, the cut off with the option of non participation is obtained as the cut off in proposition 4, as $N \to \infty$, since the probability of winning with a winning tie at 0 goes to 0 as the number of bidders increases, and when not participating, the probability of winning is 0, as well. As argued above, the variance of the Bernoulli distributed outcome of winning or losing the auction reduces the equilibrium pay off. The above result says that, depending on the value of Λ^g, this reduction can be too pronounced to make bidding a positive amount worth while for the lowest types, since they have the lowest information rents to start with. If loss aversion is very pronounced ($\Lambda^g > 1$), it is not profitable for the bidders at the bottom of the distribution to take the risk of submitting a positive bid. Loss averse bidders prefer certain outcomes. If gain loss utility dominates intrinsic utility ($\Lambda^g > 1$), then the lowest types have to be compensated for taking the risk associated with participating in the auction, which translates into the non negativity constraint on the submitted bid to be binding for these types. Figure 2 depicts the equilibrium bidding function for the APA in the setting of the previous example in figure 1 if assumption 1 is violated ($\Lambda^g = \Lambda^m = 2$).

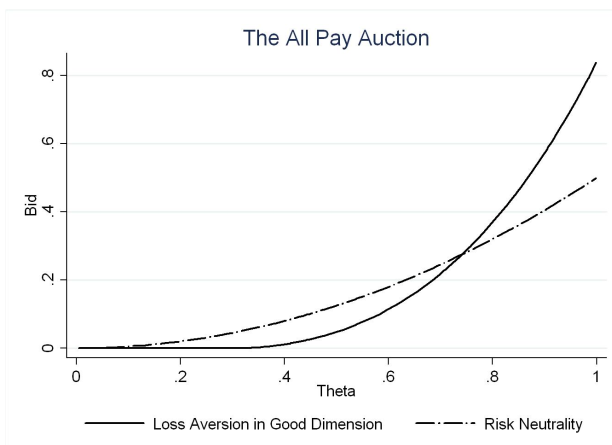

Figure 2: APA with $\Lambda^g = 2$, $N = 2$, and $\theta \sim U[0,1]$.

3.2 One Dimension — Wide Bracketing

Similar to the considerations in the previous subsection, with only one dimension, a bidder of type θ solves

$$V(\theta) \quad := \quad \max_{x \in \mathbb{R}_+} \{q(x)\,(\theta - x + \eta\,(1 - q(x))\,(\theta - x(1-\alpha)))$$

$$
\begin{aligned}
&\quad + (1-q(x))\left(-\alpha x - \eta\lambda q(x)(\theta-(1-\alpha)x)\right)\} \\
&= F^{N-1}(\theta)\left(1-\Lambda\left(1-F^{N-1}(\theta)\right)\right)\theta \\
&\quad -F^{N-1}(\theta)\left(1-\Lambda\left(1-F^{N-1}(\theta)\right)\right)(1-\alpha)b_\alpha(\theta) - \alpha b_\alpha(\theta). \qquad (2)
\end{aligned}
$$

Application of the envelope theorem yields the following expression for the symmetric CPE bidding function.

$$
b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\theta_{\min}}^{\theta} \beta(s)ds}{(1-\alpha)\beta(\theta)+\alpha},
$$

where $\beta(\theta)$ is as defined above. Analogous to the case with two dimensions, the following results hold for one dimensional reference dependence.

Proposition 5 *Suppose assumption 1 holds. Then,* $b_\alpha(\theta)$ *is strictly increasing, for almost all* θ *and constitutes the unique symmetric pure strategy CPE bidding function.*

Proposition 6 *Suppose assumption 1 does not hold, i.e.* $\Lambda > 1$*. Then, in the unique symmetric pure strategy CPE, there is a unique interior threshold,* $\hat{\theta} \in (\theta_{\min}, \theta_{\max})$*, given by* $F^{N-1}(\hat{\theta}) = (\Lambda-1)/\Lambda$*, such that for all* $\theta \geq \hat{\theta}$*,*

$$
b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta} \beta(s)ds - \beta(\hat{\theta})\hat{\theta}}{(1-\alpha)\beta(\theta)+\alpha},
$$

and for all $\theta < \hat{\theta}$*,* $b_\alpha(\theta) = 0$*, for all* $\alpha \in (0,1]$*, and* $b_\alpha(\theta) = \theta$*, for* $\alpha = 0$*. Additionally,* $\hat{\theta}$ *is strictly increasing in* Λ *and the number of bidders,* N*.*

For all auctions which are not an FPA, the above result is essentially identical to the two dimensional model, and the intuition from above applies. For the FPA, the intuition is now different. Whereas in the two dimensional model and for all auctions with $\alpha \in (0,1]$, there is no possibility for the bidders to secure themselves a pay off of 0, there is in the FPA, which can be achieved by always bidding the value, θ, so that the ex post pay off is 0, and therefore, also the expected pay off. Regarding the comparative statics properties with respect to the parameter, Λ, the following result holds, as an analogue to propositions 2 and 3.

Proposition 7 *If* $\alpha = 1$ *and* $0 < \Lambda \leq 1$*, there is a unique interior threshold,* $\bar{\theta}$*, such that* $b_\alpha(\theta)$ *is strictly decreasing in* Λ*, for almost all* $\theta < \bar{\theta}$*, and strictly increasing in* Λ*, for all* $\theta > \bar{\theta}$*. If* $\alpha = 0$*,* $b_\alpha(\theta)$ *is strictly increasing in* Λ*, for almost all* θ*.*

As argued above, loss averse bidders exhibit an aversion to the variance and the wedge between the pay off when winning and losing, $\theta - x$ and $-\alpha x$. In the FPA, the wedge between the pay off when winning and losing decreases if the bid increases. This effect drives bidders to increase their bid when the degree of loss aversion increases. Since the CPE bidding function is continuous in α, the above result implies that for auctions with a low enough all pay component, the higher the degree of loss aversion, the higher the bid. Overbidding behavior is observed in the experimental literature (e.g. Filiz-Ozbay and Ozbay (2007) and Noussair and Silver (2006)). In the APA, the bid is always paid for sure, and due to the linearity of the Kahneman and Tversky (1979) value function, does not enter the bidders' gain loss considerations, so that the intuition from proposition 3 applies. Figure 3 shows the CPE bidding function in the same environment as the previous examples.

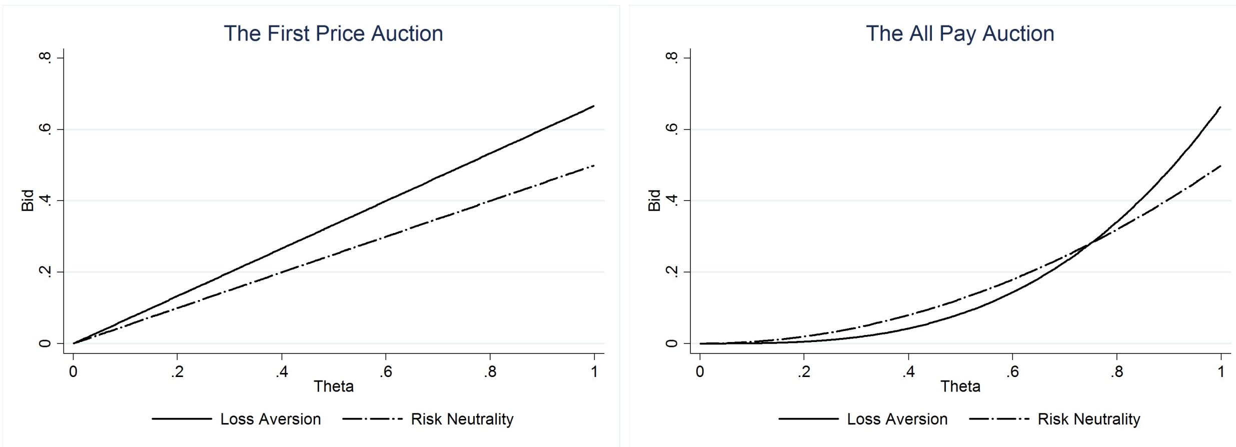

Figure 3: FPA and APA with $\Lambda = 1$, $N = 2$, and $\theta \sim U[0, 1]$.

3.3 Relationship between CPE and UPE

As mentioned above, in the auction setting under consideration, CPE and UPE are equivalent. For convenience, we first state the definition of UPE.

Definition 3 *(Köszegi and Rabin (2006)) Conditional on the realization of the type, θ, for any choice set, D, $H \in D$ is an interim CPE if $U(H|H,\theta) \geq U(H'|H,\theta)$, for all $H' \in D$.*

In CPE, the bidder picks a lottery which maximizes his expected pay off taking into account that his reference lottery adjusts accordingly; in UPE, given a reference lottery, the bidder needs to be willing to pick this very lottery. While the interpretation of CPE and UPE is different, every CPE can be supported as a UPE in the auction setting under consideration. In order to see this, suppose that each bidder has some reference lottery, say H. Given this reference lottery, each bidder maximizes his expected pay off by submitting a bid. The essence of the argument involves noticing that given the reference lottery, maximizing the expected pay off over the submitted bid leads to a probability of winning of $F^{N-1}(\theta)$, just as in CPE. More specifically, for any reference lottery, each bidder's pay off maximizing probability of winning is $F^{N-1}(\theta)$. In UPE, both lotteries have to coincide, so that $H = F^{N-1}(\theta)$.

3.4 Revenue Non Equivalence

In this section, we compare the expected ex ante revenue for the auctioneer across auction formats. Since the results depend on whether the two or one dimensional model is applied, the revenue properties are discussed separately.

3.4.1 Two Dimensions

As seen above, the interim expected pay off of a bidder of type θ is identical across auction formats. Hence, gain loss considerations in the good dimension leave the interim expected pay off unaffected across auction formats. By proposition 2, each bidder's bid is reduced by loss aversion in the money dimension if $\alpha < 1$; if $\alpha = 1$, then loss aversion in the money dimension has no effect on the equilibrium bid. Hence, for $\alpha = 1$, bidders bid as if they are risk neutral in the money dimension. Consequently, the interim expected payment of each bidder is reduced by loss aversion in the money dimension if $\alpha < 1$, so revenue equivalence breaks down, as summarized by the following propositions.

Proposition 8 *If bidders are loss averse in the money dimension ($\Lambda^m > 0$), the expected revenue for the auctioneer is strictly increasing in α.*

Gain loss considerations in the money dimension distort the equilibrium bid downwards. By requiring bidders to pay their bid regardless of whether they win the object or not, gain loss distortions in the money dimension are minimized. If $\alpha < 1$, loss averse bidders realize gains in the money dimension if they lose, and losses if they win. Since, under loss aversion, losses loom larger than gains, bidders bid more hesitantly in any auction with $\alpha < 1$ than in the APA. Therefore, among all auctions with fixed all pay component, α, the APA maximizes the auctioneer's expected revenue. In addition, pay off equivalence implies that loss aversion in the good dimension is irrelevant for the revenue ranking across auction formats, as summarized in the following proposition.

Proposition 9 *If bidders are loss averse in the good dimension and risk neutral in the money dimension ($\Lambda^g > 0, \Lambda^m = 0$), the expected revenue for the auctioneer is identical, for all $\alpha \in [0, 1]$.*

This result confirms that the revenue ranking across auction format is solely driven by loss aversion in the money dimension. An immediate implication is that the revenue ranking is unaffected by whether assumption 1 is met or not. Furthermore, as the bidders' pay off depends solely on the allocation rule, the above results imply the inefficiency of any auction that is not an APA if $\Lambda^m > 0$, since when increasing α bidders remain indifferent, but the auctioneer strictly gains.

3.4.2 One Dimension

As in the two dimensional model, the interim expected pay off for each bidder is identical across auction formats. Consider the pay off from (2), which can be rewritten as

$$V(\theta) = F^{N-1}(\theta)\theta - \left(F^{N-1}(\theta)(1-\alpha) + \alpha\right) - \Lambda F^{N-1}(\theta)\left(1 - F^{N-1}(\theta)\right)(\theta - (1-\alpha)b_\alpha(\theta)).$$

Similar to the case with two dimensions, the bidder's pay off is reduced by the variance of the Bernoulli distributed outcome of winning and losing the auction, the degree of loss aversion, and the wedge between the pay off when winning and losing, $\theta - (1-\alpha)b_\alpha(\theta)$. The bidder's objective function satisfies strictly increasing differences in $(\alpha, -x)$, so that $b_\alpha(\theta)$ is non increasing in α, for all θ. Increasing α lowers the bidder's bid, since an increased fraction is paid for sure. In addition, keeping the (non negative) bid fixed, increasing α leads to an increase in the wedge between the pay off when winning and losing. Both of these effects reduce the bidder's pay off. Since the pay off is determined by the allocation rule alone, this pay off reduction is compensated for by bidding less aggressively in the APA compared to the FPA, so that the following result holds.

Proposition 10 *If $\Lambda > 0$, the expected revenue for the auctioneer is strictly decreasing in α.*

3.5 Risk Aversion or Loss Aversion?

A natural question to ask is whether the results derived above are driven by risk aversion rather than loss aversion. Auctions with risk averse bidders are studied in Riley and Samuelson (1981), Maskin and Riley (1984), and Matthews (1987), where bidders' preferences take the form $u(\theta, -x)$, and u is strictly increasing and strictly concave in both arguments. As a special case of this formulation, which is studied in Fibich, Gavious, and Sela (2006), bidders' preferences take the form, $u(\theta - x)$, where u is strictly increasing and strictly concave. Fibich, Gavious, and Sela (2006) compare the expected revenue in the APA and the FPA. Their finding is that the revenue ranking is ambiguous in the sense that there are utility functions and distributions for which either the APA or the FPA yields higher expected revenue for the auctioneer. Maskin and Riley (1984) study optimal auctions with risk averse bidders. They find that a *perfect insurance auction* is optimal with homogeneously risk averse bidders, who differ only in their type, θ. A perfect insurance auction is an auction with two payment schemes, one for bidders who win the auction, x^W, and one for bidders who lose the auction, x^L, that depend on the reported type, but are deterministic otherwise, and have the property that for highest type, the marginal utility of money is identical in each state. The APA is nested in the class of perfect insurance auctions, for $x^W = x^L$, and the FPA is nested for $x^L = 0$. The results in Maskin and Riley (1984) imply that a necessary condition for the APA ($x^W = x^L$) to yield the highest expected revenue for the auctioneer is that the marginal utility of money is independent of the valuation, θ, e.g. $u(\theta, -x) = \theta - m(x)$. Furthermore, the insights obtained by Maskin and Riley (1984) rationalize the ambiguous revenue ranking between the APA and the FPA reported in Fibich, Gavious, and Sela (2006). Furthermore, every risk averse bidder with the above preferences is locally risk neutral, which implies that every risk averse bidder participates in the auction and submits a positive bid, because he obtains non negative expected pay off from doing so. As seen above, this is not necessarily the case if bidders are loss averse. This raises the question whether the limited participation results derived above for high degrees of loss aversion in the good dimension can be explained by first order risk aversion. If bidders have rank dependent expected utility preferences as in Yaari (1987), which allow for first order risk aversion, the revenue ranking and the participation is as with additively separable risk aversion[1], i.e. the APA yields the highest revenue for the auctioneer.

4 Experiment

As seen above, the revenue ranking between the FPA and the APA is opposite in the one and two dimensional model. In this section, we describe the experiments, which are designed to test the theoretical results derived above and which contribute to a better understanding of methodological robustness concerning laboratory and field experiments. A common method in experimental economics to analyze IPV auctions is the induced value (IV) method, where money is auctioned. Bidders are assigned a randomly drawn valuation, and if they win, they receive a monetary pay off equal to their valuation. This is in contrast to a real object (RO) auction, where actual goods are auctioned. Since money is auctioned in the IV method, only the one dimensional model is applicable in this setting; for RO auctions, the two dimensional model is more plausible. Therefore, we examine revenue equivalence between the APA and the FPA for RO and IV auctions separately and compare the results of the IV and RO method. Based on the theoretical results above, we seek to test the following hypotheses.

Hypothesis 1 *IV Auction: The expected revenue is higher for the FPA than for the APA (Proposition 10).*

Hypothesis 2 *IV Auction: Almost every bidder submits a positive bid in the FPA, but not necessarily in the APA (Proposition 6).*

Hypothesis 3 *RO Auction: The expected revenue is higher for the APA than for the FPA (Proposition 8).*

Hypothesis 4 *RO Auction: The fraction of bidders submitting zero bids is identical in the APA and in the FPA (Proposition 4).*

[1] The results can be obtained from the authors upon request.

Two of the above hypotheses can be tested with revenue data alone; the other two require data on the bidders' behavior. The ability to test using revenue data alone is due to the diverging revenue rankings in the one and two dimensional model of reference dependence for the APA and the FPA. Comparing the FPA to the Vickrey auction, Lange and Ratan (2010) find that the theoretical revenue ranking is identical in the one and two dimensional model, so one can only reject (or not reject) both models together when using revenue data.

4.1 Experimental Design

In order to experimentally test our hypotheses, we employ two different methodological approaches. Both approaches are run in the laboratory instead of in the field in order to contribute to the analysis of auctions with experimental methods. For the IV method, subjects were anonymously matched into groups of three. Each subject was given an endowment of 700 points, where 100 points are worth €1, which was \$1.42 at that time, to submit a bid in the auction. Subjects' valuations in the auction were independently drawn from the uniform distribution on $\{0, 1, 2, \ldots, 299, 300\}$, which was made common knowledge to the subjects. The maximum valuation is therefore 300 points and lower than the endowment. Bids were allowed to have up to two decimal points, e.g. 2.99, as we did not want bidders with low valuations to floor their bids down to zero, which would inhibit the testability of our hypothesis on limited participation. The important part of the design of the induced value method is the following. Subjects were provided with a list of ten different valuations, such that a subject participated in ten auctions. The subjects had to bid for each valuation, however only one of the ten auctions was pay off relevant, and each auction was equally likely to be pay off relevant. We adapted this procedure from Filiz-Ozbay and Ozbay (2007). Figure 1 in the appendix shows an example of such a list. In the FPA, the subject with the highest bid in the group of three won the auction and received the valuation plus the endowment minus the respective bid as her pay off. If more than one subject submitted was the highest bidder, the computer chose the winner with equal probability. The other subjects lost the auction and received only the endowment. The rules of the APA were exactly the same, except for the losers' pay off, which was the endowment minus the bid. Finally, to secure the understanding of the game, we asked several control questions (see Appendix). In the RO auction, we auctioned a real good. The good was chosen, such that subjects valuations do not largely differ from the induced value auction, and are plausibly independent and private. Therefore, we decided to auction a blackboard cup with a piece of chalk. The cup has a blackboard sheathing, on which can be written with chalk (see Appendix). Every subject had the possibility to have a look at the cup before bidding in the auction. The buying price of the cup was €1.75, which was not revealed to the subjects. As in the IV auction, subjects received an initial endowment of 700 points before the auction began. In contrast to the IV auction, each subjects can only take part in one RO auction. The rules of the auction formats are the same as in the IV method. The IV auction and the RO auction were played on different days with different subjects in the BonnEconLab at the University of Bonn with 192 participants from various fields of study, recruited via Greiner (2004), out of which 96 subjects participated in the IV auction. The experiment was programmed and conducted with the software z-Tree Fischbacher (2007).

4.2 Results

Result 1 *The average revenue for the FPA is significantly greater than for the APA, for both the IV and the RO auction.*

A group of three subjects bids in one auction. Since 24 subjects are in one session, we have eight groups per session. In the induced value method, each subject had to submit ten bids for ten valuations. Thus, each group performs ten auctions and we receive ten revenues per group. We take the average of the respective ten revenues of one group as an independent observation and therefore use 8 independent revenue observations per session for the data analysis. The average revenue in the IV method for the FPA is 170 points and 152 points in the APA. Summary statistics of the data collected are shown in Table 1. We will use a t-test to show whether this sizable difference in means is also significant. Performing a one sided t-test, we reject the hypothesis that the average revenues are equal in favor of the alternative hypothesis that the average revenue of the FPA is greater than the average revenue of the APA with $p = 0.04$[2], which confirms our Hypothesis 1, that the revenue of the FPA is larger than of the APA. The results are similar in the RO auction, with an even larger difference in average revenue, which is 263 points for the FPA and 150 points for the APA. Therefore, our data does not confirm Hypothesis 2, that the opposite should occur when bidders think in two dimensions. Using a two sided t-test rejects the hypothesis that average revenues are equal with a $p - value = 0.03$. For the IV auction, the results are consistent with the model of one dimensional reference dependence. In the RO auction, the results reject the model of two dimensional reference dependence, yet are consistent with the one dimensional model. One possible explanation for this is that although a real object is auctioned, subjects behave according to the one dimensional model of reference dependence.

Result 2 *The fraction of bidders submitting zero bids is significantly greater in the APA than in the FPA, for both the IV and the RO auction.*

A strikingly large amount of subjects submits a bid of zero in the APA, 27.7% in the IV auction and 39.6 % in the RO and only 2.5% and 8.3% respectively for the FPA. Using a t-test, we find that zero bids occur significant more often in the APA compared to the FPA with $p < 0.01$. As with the revenue data, these findings are consistent with the one dimensional model fo reference

[2]The p-value of a two sided t-test is 0.08.

	Induced Value Auction				Real Object Auction			
	Mean	Std. Dev.	N	Zero Bids	Mean	Std. Dev.	N	Zero Bids
FPA	170	29.8	16	2.5 %	263	150	16	8.3 %
APA	152	26.5	16	27.7 %	150	129	16	39.6 %

N is the number of independent valuations

Table 1: Summary Statistics

dependence, but not with the two dimensional one.

4.2.1 Structural Estimation For the Induced Value Method

The above results consider the revenue data alone. While we do not have data on the valuations in the real object auctions, we have the induced valuations and the submitted bids in the induced value auctions, which enables to structurally estimate the parameter, Λ. By doing this, we can compare our estimate to the ones reported in the literature and obtain an internal consistency check. We employ the Generalized Method of Moments (GMM) using the moment conditions,

$$E[b_0] = E[b_0(\theta|\Lambda)] \qquad \text{and} \qquad E[b_1] = E[b_1(\theta|\Lambda)].$$

We estimate $\hat{\Lambda} = 0.42$ with a standard error of 0.16, which is statistically different from 0 and 1 at all conventional significance levels using a Wald test. Since we have two moment conditions and only one parameter to estimate, we perform a J-test for over identifying restrictions, which does not reject the null that the model is valid at all conventional significance levels ($\chi^2(1) = 2.31$). The following figure depicts the predicted bids compared to the risk neutral benchmark. Recall that $\Lambda = \eta(\lambda - 1)$, so that η and λ are not identified. Once we normalize $\eta = 1$, we can identify $\hat{\lambda} = 1.42$.

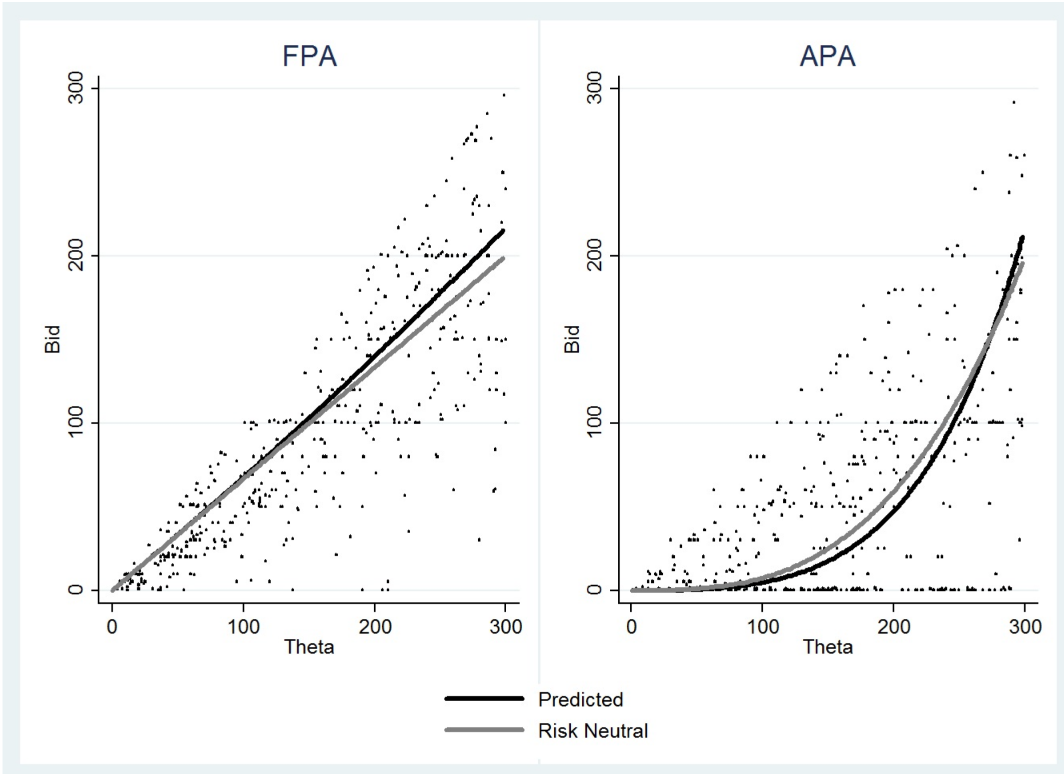

Figure 4: Bids with GMM estimate, $\hat{\Lambda} = 0.42$.

5 Conclusion

While the above analysis is robust to considering more general, non linear specifications of the Kahneman and Tversky (1979) value function in the two dimensional model, the analysis becomes analytically intractable for non linear functions in the one dimensional model. By continuity and the revenue ranking being strict, it follows that introducing small amounts of non linearities will not change any of the above results. However, as in Fibich, Gavious, and Sela (2006), no closed form expression for the equilibrium bidding function can be obtained, and one has to resort to approximate perturbation methods. As the above analysis shows, the implications of bidders with reference dependent preferences in auction environments differ depending on which specification of preferences (one or two dimensional) is assumed. In a working paper preceding their published articles, Köszegi and Rabin (2004) discuss the distinction between consumption and hedonic dimensions in their model of reference dependent preferences. The dimensions across which gambles are considered separately are not necessarily equal to physical consumption dimensions. They give the example of peanut butter, jelly, and bread, where one is plausibly interested only in the total number of sandwiches produced. Which of the two specifications studied above is more appropriate in which context demands further research. The separable model is commonly used in applications, e.g. Köszegi and Rabin (2006), Heidhues and Köszegi (2008), Herweg, Müller, and Weinschenk

(2010). As the above analysis illustrates, the theoretical implications may differ drastically, depending on which specification of reference dependent preferences is employed. Using experiments, we can reject the two dimensional model, but not the one dimensional model.

A Appendix

A.1 Proofs

Proof of Proposition 1: If assumption 1 is satisfied, then

$$F^{N-1}(\theta)\left(1-\Lambda^g(1-F^{N-1}(\theta))\right) = F^{N-1}(\theta)(1-\Lambda^g) + F^{2(N-1)}(\theta)\Lambda^g,$$

is strictly increasing in θ, so the pay off is strictly increasing in the type. Differentiating $b_\alpha(\theta)$ with respect to θ gives

$$\frac{\partial}{\partial\theta}b_\alpha(\theta) > 0 \Longleftrightarrow (1-\Lambda^g+2F^{N-1}(\theta)\Lambda^g)\theta - b_\alpha(\theta)(1-\alpha)(1+\Lambda^m-2\Lambda^m F^{N-1}(\theta)) > 0. \tag{3}$$

(3) is equivalent to

$$\begin{aligned}
&(1-\Lambda^g+2F^{N-1}(\theta)\Lambda^g)F^{N-1}(\theta)\theta - b_\alpha(\theta)(1-\alpha)F^{N-1}(\theta)(1+\Lambda^m-2\Lambda^m F^{N-1}(\theta)) > 0\\
\Longleftrightarrow\quad &\beta(\theta)\theta + \Lambda^g F^{N-1}(\theta) - b_\alpha(\theta)(1-\alpha)F^{N-1}(\theta)(1+\Lambda^m-\Lambda^m F^{N-1}(\theta)) + b_\alpha(\theta)\Lambda^m F^{N-1}(\theta) > 0\\
\Longleftrightarrow\quad &V(\theta) + \Lambda^g F^{N-1}(\theta) + b_\alpha(\theta)\Lambda^m F^{N-1}(\theta) > 0
\end{aligned}$$

which is true since, the expected pay off is positive. Additionally, since b_α is strictly increasing, the envelope representation of the bidder's pay off applies, the type space is convex and has no mass points, uniqueness follows from Myerson (1981). □

Proof of Proposition 2: Immediate by inspection. □

Proof of Proposition 3:

$$\begin{aligned}
&\frac{\partial}{\partial\Lambda^g}b_\alpha(\theta) = -b_\alpha^{RN}(\theta) + \kappa_\alpha(\theta) \le 0 \Longleftrightarrow \kappa_\alpha(\theta) \le b_\alpha^{RN}(\theta)\\
\Longleftrightarrow\quad &F^{2(N-1)}(\theta)\theta - \int_{\theta_{\min}}^{\theta} F^{2(N-1)}(s)ds \le F^{N-1}(\theta)\theta - \int_{\theta_{\min}}^{\theta} F^{N-1}(s)ds.
\end{aligned}$$

For $\theta = \theta_{\min}$, both the LHS and the RHS of the above expression are equal to 0. The derivative of the expression on the RHS is greater than the derivative of the expression on the LHS if and only if $F^{N-1}(\theta) \le 1/2$, which implies that the bid of the lowest types is always reduced by an increase in Λ^g. For $\theta = \theta_{\max}$, the bidding function is increased by an increase in Λ^g. □

Proof of Proposition 4: If $\Lambda^g > 1$, then $\beta(\theta)$ is minimized at $F^{N-1}(\theta) = (\Lambda^g-1)/(2\Lambda^g)$, since $\beta(\theta)$ is quadratic in $F^{N-1}(\theta)$, so always increasing above its minimum and decreasing below it. Therefore, $\beta(\theta) < 0$, for $F^{N-1}(\theta) < (\Lambda^g-1)/\Lambda^g$. These two facts will be used in the remainder of the proof. If all $\theta \le \hat{\theta}$ bid 0, then they face a probability of winning of $F^{N-1}(\hat{\theta})/N$, since $F^{N-1}(\hat{\theta})$ is the probability of winning, in which case the object is allocated among the winning bidders with equal probability. Hence, one candidate symmetric equilibrium bidding function is $b_\alpha(\theta) = 0$, if $\theta < \hat{\theta}$ and

$$b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta}\beta(s)ds - (\hat{\theta}-\theta_{\min})\beta(\hat{\theta}) - V(\theta_{\min})}{\alpha + (1-\alpha)F^{N-1}(\theta)\Delta(\theta)} = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta}\beta(s)ds - \beta(\hat{\theta})\hat{\theta}}{\alpha + (1-\alpha)F^{N-1}(\theta)\Delta(\theta)},$$

for all $\theta \geq \hat{\theta}$, where the last equality follows since $V(\theta_{\min}) = \beta(\hat{\theta})\theta_{\min}$. Following this strategy, all $\theta < \hat{\theta}$ receive an interim expected pay off of $V(\theta) = \beta(\hat{\theta})\theta$. Since $N \geq 2$ and $\beta(\theta)$ is quadratic in $F^{N-1}(\theta)$, so always decreasing below its minimum, $V(\theta) = \beta(\hat{\theta})\theta < 0$, for all $\theta < \hat{\theta}$. Consider a deviation of a type $\theta < \hat{\theta}$ to any bid $b_\alpha(\theta_{\max}) > b > 0$. Then there is a $\theta_{\max} > \theta^* > \hat{\theta}$ with $b_\alpha(\theta^*) = b$. Hence, the pay off from the deviation is

$$\beta(\theta^*)(\theta-\theta^*) + \int_{\theta_{\min}}^{\theta^*}\beta(s)ds + \beta(\hat{\theta})\theta_{\min} = \beta(\theta^*)(\theta-\theta^*) + \int_{\hat{\theta}}^{\theta^*}\beta(s)ds + \beta(\hat{\theta})\hat{\theta} \tag{4}$$

$$< \beta(\theta^*)(\theta-\theta^*) + (\theta^*-\hat{\theta})\beta(\theta^*) + \beta(\hat{\theta})\hat{\theta} = \beta(\theta^*)(\theta-\hat{\theta}) + \beta(\hat{\theta})\hat{\theta} \tag{5}$$

$$= \beta(\theta^*)(\theta-\hat{\theta}) + \beta(\hat{\theta})\hat{\theta} + \beta(\hat{\theta})\theta - \beta(\hat{\theta})\theta = (\beta(\theta^*)-\beta(\hat{\theta}))(\theta-\hat{\theta}) + \beta(\hat{\theta})\theta, \tag{6}$$

where the above inequality follows from $\beta(\theta)$ being quadratic and increasing above its minimum and $N \geq 2$. The pay off from deviating is greater than the pay off from following the strategy only if $(\beta(\theta^*) - \beta(\hat{\theta}))(\theta - \hat{\theta}) \geq 0$, which is never satisfied, since $\theta < \hat{\theta}$ and $\beta(\theta^*) > \beta(\hat{\theta})$. If types above $\tilde{\theta}$ deviate and submit a bid of 0, they earn a pay off of $\beta(\tilde{\theta})\theta$, and if they stick to the above candidate CPE strategy, they earn a pay off of $\beta(\tilde{\theta})\tilde{\theta} + \int_{\tilde{\theta}}^{\theta}\beta(s)ds$. Since $\beta(\theta) > 0$, for all $\theta > \tilde{\theta}$, a deviation is profitable only if $\beta(\tilde{\theta})\tilde{\theta} < \beta(\tilde{\theta})\theta$, which is never true since $\beta(\tilde{\theta}) < 0$, and $\theta > \tilde{\theta}$. Finally, consider the threshold type, $\tilde{\theta}$. This type is indifferent between deviating downwards or upwards, so that there is no profitable deviation. Hence, the second candidate CPE bidding function constitutes a symmetric pure strategy CPE. Since $V(\theta) \geq 0$, for all types who submit a positive bid, the argument in the proof of proposition 2 shows that $b_\alpha(\theta)$ is strictly increasing for all $\theta \geq \hat{\theta}$, so that for all $\theta > \tilde{\theta}$, Myerson (1981)'s condition implies that the symmetric CPE bidding function is unique. Suppose that there is another cut off with the equality defining $\hat{\theta}$ as a strict inequality in either direction. Then, either the types slightly above or slightly below this cut off can earn a higher pay off by deviating to either bidding 0 or a slightly positive amount or the bidding function is not strictly increasing, which cannot be part of a symmetric CPE. □

Proof of Proposition 5: Exactly the same as the proof of proposition 1. □

Proof of Proposition 6: In the one dimensional model, the analysis is different for $\alpha \in (0, 1]$ and $\alpha = 0$. Consider first all auction except the FPA and the the following candidate symmetric equilibrium bidding function

$$b_\alpha(\theta) = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta}\beta(s)ds - (\hat{\theta}-\theta_{\min})\beta(\hat{\theta}) - V(\theta_{\min})}{\alpha + (1-\alpha)F^{N-1}(\theta)\Delta(\theta)} = \frac{\beta(\theta)\theta - \int_{\hat{\theta}}^{\theta}\beta(s)ds - \beta(\hat{\theta})\hat{\theta}}{\alpha + (1-\alpha)\beta(\theta)},$$

and $b_\alpha(\theta) = 0$, for all $\theta < \hat{\theta}$. The steps proving that this candidate CPE bidding function constitutes the unique symmetric COPE biding function is exactly the same as in proposition 4. Consider now the FPA. If all types below $\hat{\theta}$ bid 0, they tie and win with strictly positive probability, which yields a negative expected pay off, since $\beta(\theta) < 0$, for all $\theta < \hat{\theta}$. Instead of bidding 0, these types can secure an expected pay off of 0 if they bid their valuation rather than 0. Therefore, we have the following candidate CPE bidding function,

$$b_0(\theta) = \theta - \frac{\int_{\hat{\theta}}^{\theta} \beta(s)ds}{\beta(\theta)},$$

for all $\theta \geq \hat{\theta}$, and $b_0(\theta) = \theta$, for all $\theta < \hat{\theta}$. By L'Hopital's rule, $\lim_{\theta \downarrow \hat{\theta}} = \theta$, so that this candidate CPE bidding function is continuous and strictly increasing. Uniqueness follows from Myerson (1981)'s condition. $\square$

Proof of Proposition 7: The first part of the proof is exactly the same as the proof of proposition 3, the second part follows from differentiation of the bidding function for $\alpha = 0$ with respect to Λ. This expression is equal to 0 for $\theta = \theta_{\min}$, and strictly negative for $\theta = \theta_{\max}$, in addition to being strictly decreasing in θ. $\square$

Proof of Proposition 8: This result follows from the interim expected pay off only depending only on the probability of winning, which is identical for a given type in all auctions with the same allocation rule. $\square$

Proof of Proposition 9: The expected payment, $p_\alpha(\theta)$, of a bidder of type, $\theta \geq \hat{\theta}$, conditional on the other bidders' behavior, is $p_\alpha(\theta) = \alpha b_\alpha(\theta) + F^{N-1}(\theta)(1-\alpha)b_\alpha(\theta)$, i.e. αb_α with certainty, and $(1-\alpha)b_\alpha$ only if he wins, which happens with probability $F^{N-1}(\theta)$. Differentiating the above expression with respect to α yields

$$\frac{\partial}{\partial \alpha} p_\alpha(\theta) = \frac{F^{N-1}(\theta)(\Delta(\theta) - 1)}{(1-\alpha)F^{N-1}(\theta)\Delta(\theta) + \alpha} b_\alpha(\theta),$$

which is strictly positive, for all $\theta > \hat{\theta}$, since $\Delta(\theta) > 1$, for all $\theta > \hat{\theta}$. For $\theta < \hat{\theta}$, the interim expected payment is 0 and remains unchanged in α. Since the interim expected payment is non decreasing for all types and strictly increasing for a set of types of strictly positive measure, this implies that the ex ante expected revenue for the auctioneer, $N \int p_\alpha(\theta)dF(\theta)$, is strictly increasing in α. $\square$

Proof of Proposition 10: Replace Δ by $\beta(\theta)/F^{N-1}(\theta)$ in the proof of proposition 8. $\square$

A.2 Experimental Appendix

Instructions

Instructions, translated into English. General instructions were identical across treatments. Instructions for the real object auction and induced value auction differed.

GENERAL INSTRUCTIONS

You are taking part in a decision-making experiment in which you have the opportunity to earn money. The amount of money you earn is paid to you upon completion of the experiment. Please read the instructions carefully. The instructions are identical for all participants. If you have any questions, please raise your hand. The experimenter will answer your question at your place. During the experiment, you have to remain silent. Violation of this rule leads to immediate exclusion from the experiment and all payments.

All monetary units in the experiment are measured in points, and 100 points = 1 Euro.

INSTRUCTIONS *Real object auction*

You now take part in an auction. For this you get an endowment of 700 points.

Task

At the beginning of the auction you will be divided into groups of three. You will not learn who the other participants in your group are. Your task in the three-group is that of a bidder who bids for an item in an auction. For this you can spend an arbitrary amount of your endowment of 700 points. Your bid must have a maximum of two decimal places.

The item

The auctioned item is a chalk-cup with one piece of chalk. The cup can always be rewritten.

Rules

The auction rules for each three-person group are that the participant with the highest bid wins the auction in their group and thus the cup. If several bidders have the same highest bid, we will then toss a coin to determine the winner.

As the winner you will receive the cup, plus the endowment of 700 points minus your bid:

Payoff = cup + 700 - your bid.

First price auction instructions

If your bid is less than the highest bid, you lose the auction. As a loser, you get the endowment of 700 points:

Payoff = 700.

All pay auction instructions

If your bid is less than the highest bid, you lose the auction. As a loser, you get the endowment of 700 points minus your bid:

Payoff = 700 - bid.

Any questions?

Please enter your cabin number and your bid.

Cabin number:

Bid:

INSTRUCTIONS *Induced value auction*

You now take part in an auction. For this you get an endowment of 700 points.

Task

At the beginning of the auction you will be divided into groups of three. You will not learn who the other participants in your group are. Your task in the three-group is that of a bidder who bids for a fictitious item in an auction. For this you can spend an arbitrary amount of your endowment of 700 points. Your bid must have a maximum of two decimal places.

Your value

Before the auction starts, you will see on your computer screen a list of 10 numbers. Each of these numbers is between 0 and 300 points. The numbers are chosen randomly by the computer, where each number can occur with equal probability. Each number represents a possible value for you for the fictitious item in the auction. The process of generating the values is identical for all participants. This means that every participant in your group of three got a different list of 10 numbers, where each number is chosen randomly and independently from your numbers from the interval of [0,300].

Figure of a sample list:

We ask you to enter a bid for each of your 10 possible values in the column next to the values. For this you can spend an arbitrary amount of your endowment of 700 points.

Thus, you enter bids for ten auctions. However, only one of the ten auctions performed will be payoff relevant. The computer will randomly select one of the ten auctions, where each auction is equally likely. This means that you should enter for each of the ten possible auctions your bid such as if it were the only auction that is conducted. So, for each auction you have an endowment of 700 points and its value on which you can bid is a number between 0 and 300 points.

Rules

The auction rules for each three-person group are that the participant with the highest bid wins the auction in their group and thus the cup. If several bidders have the same highest bid, we will then toss a coin to determine the winner.

As the winner you will receive the cup, plus the endowment of 700 points minus your bid:

Payoff = cup + 700 - your bid.

First price auction instructions

If your bid is less than the highest bid, you lose the auction. As a loser, you get the endowment of 700 points:

Payoff = 700.

All pay auction instructions

If your bid is less than the highest bid, you lose the auction. As a loser, you get the endowment of 700 points minus your bid:

Payoff = 700 - bid.

Do you have any questions on this?

After you have entered all 10 bids on the screen, please press the OK button. You are then asked again to confirm your choices and you can once again decide whether you want to make changes.

The auction begins now with several control questions to ensure that all participants understand the rules.

Any questions?

Control questions on screen

Question 1: What is the smallest value you can get?
Question2: What is the highest value you can get?

Example of an auction: Player 1 has a value of 1 and bids 1, player 2 has a value of 20 and bids 2 and player 3 has a value of 30 and bids 3.
Question 3: Who wins the auction?
Question 4: What is the payoff for player 1?
Question 5: What is the payoff for player 2?
Question 6: What is the payoff for player 3?

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