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1 August 2011

Online at https://mpra.ub.uni-muenchen.de/34878/
MPRA Paper No. 34878, posted 08 Dec 2011 18:37 UTC
Unemployment Insurance and Home Production *

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November, 2011

Abstract

In this paper, we incorporate home production into a quantitative model of unemployment and show that realistic levels of home production have a significant impact on the optimal unemployment insurance rate. Motivated by recently documented empirical facts, we augment an incomplete markets model of unemployment with a home production technology, which allows unemployed workers to use their extra non-market time as partial insurance against the drop in income due to unemployment. In the benchmark model, we find that the optimal replacement rate in the presence of home production is roughly 40% of wages, which is 40% lower than the no home production model’s optimal replacement rate of 65%. The 40% optimal rate is also close to the estimated rate in practice. The fact that home production makes a significant difference in the optimal unemployment insurance rate is robust to a variety of parameterizations and alternative model environments.

J.E.L. Classification: D13, E21, J65.

Keywords: Unemployment insurance, home production, incomplete markets, self-insurance.

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*This paper is a part of my Ph.D. thesis at the University of Rochester. I am grateful to Mark Aguiar for his support and guidance. I would also like to thank Árpád Ábrahám, Mark Bils, Yongsung Chang, Gregorio Caetano, Yavuz Arslan, and the seminar and conference participants at the University of Rochester, European University Institute, Central European University, Izmir Economy University, Middle East Technical University, Central Bank of Turkey, Society for Economic Dynamics Meeting in Montreal, Midwest Macro Meeting at Michigan State University, and Canadian Economic Association Meeting in Quebec City for helpful discussions. All errors are mine and comments are very welcome. The views expressed here are those of the author and do not reflect those of the Central Bank of Turkey.

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1 Introduction

In this paper, we study the optimal rate of unemployment insurance in an economy where individuals do home production as well as market production. If there were complete private insurance against unemployment shocks, then government-provided unemployment insurance would be unnecessary. Therefore, it is important to account for the amount of self-insurance of unemployed workers when designing unemployment insurance programs. In this paper, we consider home production as a self-insurance mechanism and study the role of home production in determining the optimal unemployment insurance rate. The results suggest that the optimal replacement rate in the presence of home production is roughly 40% of wages, which is significantly lower than the no home production model’s optimal replacement rate of 65%. The 40% optimal rate is also close to the estimated rate in practice for the United States.

To formalize the idea of self-insurance through home production, we augment an incomplete markets model of unemployment with a home production technology, which allows unemployed workers to use their extra non-market time as partial insurance against the drop in income due to unemployment. The model features a heterogeneous agent framework due to idiosyncratic employment shocks. Government provides unemployment insurance as a constant fraction of lost earnings during the unemployment spell, which is the current design of policy implemented in the United States. It is financed through proportional income taxation. Along with government-provided unemployment insurance, individuals can partially self-insure by increasing home production during unemployment spells and/or accumulating savings.

The role of home production in determining the optimal rate of unemployment insurance is quantified by solving the model twice: once with home production and once without home production. By introducing home production into the model, we dif-
ferentiate consumption from expenditure, which is not an accurate measure of actual consumption.\footnote{Aguiar and Hurst (2005, 2007)} Due to the nature of the unemployment shocks, the budget and time constraints of individuals change during unemployment spells. They have looser time constraints and tighter budget constraints, and their optimal behavior adjusts accordingly. In equilibrium, individuals do more home production during unemployment spells, which provides smoother consumption compared to the case in which there is no home production. Eventually, the optimal rate of unemployment insurance turns out to be significantly smaller due to the additional self-insurance through home production. The result is robust to a variety of parameterizations and alternative model environments.

The quantitative implications of the model rely on two crucial points: first, the response of individual time allocation to unemployment shock and second, the elasticity of substitution between time and goods in the home production function. Therefore, it is important to calibrate the model to be consistent with empirical facts at these two points. We do so by comparing the model’s predictions with the recent empirical literature.\footnote{Aguiar and Hurst (2007), and Burda and Hamermesh (2010)}

The paper contributes to the quantitative unemployment insurance literature by computing the optimal rate of unemployment insurance in an economy where individuals do home production as well as market production. In general, incomplete markets models - including the quantitative models of unemployment insurance\footnote{See for instance: Hansen and Imrohoroglu (1992), Hopenhayn and Nicolini (1997), Acemoglu and Shimer (2000).} - ignore the partial insurance role\footnote{See Heathcote et al. (2009) for a survey on the partial insurance mechanisms in incomplete markets.} of home production and how it varies with employment status. However, the recent empirical literature provides evidence that home production is quantitatively important in time-use surveys.\footnote{Aguiar and Hurst (2007)} In particular, the unemployed allocate their non-market

\begin{thebibliography}{9}
\footnotesize
\item Aguiar and Hurst (2005, 2007)
\item Aguiar and Hurst (2007), and Burda and Hamermesh (2010)
\item See Heathcote et al. (2009) for a survey on the partial insurance mechanisms in incomplete markets.
\item Aguiar and Hurst (2007)
\end{thebibliography}
time differently,\textsuperscript{6} and this is important for policy analysis. Therefore, our paper closes
the gap between the home production literature and unemployment insurance literature.

The rest of the paper is organized as follows. Section 2 summarizes the related liter-
ature. Section 3 presents some interesting facts about employment status and home
production. Section 4 describes the model in which we analyze the optimal unemploy-
ment insurance. Section 5 explains the calibration of the model parameters. Section 6
discusses the results. Finally, Section 7 concludes.

\section{Related Literature}

The paper is related to the quantitative unemployment insurance literature and recent
empirical findings on home production. Many papers, including Hansen and Imro-
horoglu (1992), Davidson and Woodbury (1997), Acemoglu and Shimer (2000), and
Chetty (2008), look for the optimal rate of unemployment insurance conditioning on a
certain type of policy as we do in this paper. Our paper is different from those papers
since we consider home production as well as market production in the model economy.

There is a large literature on the optimal profile of unemployment insurance pay-
ments over the unemployment spell. The results vary. Some of the studies (Shavell
and Weiss 1979, Hopenhayn and Nicolini 1997, Wang and Williamson 2002, Alvarex and
Sanchez 2010) claim that the optimal profile of payments should be decreasing over the
unemployment spell. Other papers (Kocherlakota 2004, Hagedorn et al. 2005, Shimer
and Werning 2008) support flat or increasing payments over the unemployment spell. In
this paper, we abstract from the problem of optimal design. Instead, we focus on the
current design in practice in the United States and look for the optimal rate of insurance
conditional on this design.

\textsuperscript{6}Burda and Hamermesh (2010)
On the other hand, because of the recent availability of time-use surveys, the number of studies emphasizing the role of home production has increased substantially. In particular, using time-use data from some developed countries, including the U.S., Burda and Hamermesh (2010) document that time spent on home production increases significantly due to unemployment. Moreover, Aguiar and Hurst (2007) estimate parameters of a home production function for the U.S. by using two micro data sets. In that sense, we benefit from these two papers in determining parameters of our quantitative model.

Moreover, the home production approach has been employed to explain various puzzles in macroeconomics. Aguiar and Hurst (2005) explain the retirement puzzle using a home production approach, where home production has a consumption-smoothing role. Chang and Hornstein (2007) employ home production in a business cycle model to better understand aggregate fluctuations in labor supply and the small correlation between employment and wages. Benhabib et al. (1991), Greenwood and Hercowitz (1991), Canova and Ubide (1998), and Chang (2000) are other examples. In contrast, motivated by recent empirical facts, we employ home production as a self-insurance mechanism in a quantitative unemployment insurance model.

3 Data

In this section, we provide complementary results to the recent empirical literature on home production. We document the relationship between employment status and home production using the American Time Use Survey (ATUS). ATUS is a repeated cross-sectional data set that is a supplement to the Current Population Survey (CPS). It has been conducted since 2003, and it is nationally representative. We use 2003-2008 samples, which were obtained from the Bureau of Labor Statistics web page.\footnote{http://www.bls.gov/tus/} The survey collects
information through time-use diaries from individuals. It measures time spent on a rich set of activities, including personal care, household activities, work-related activities, education, socializing, leisure, traveling and volunteer activities.\(^8\) The unit of time is minutes per day, and we convert it to hours per week by multiplying by 7/60. Also, the survey provides detailed information about the employment status and demographic characteristics of individuals.

The type of activities considered in home production are time spent on housework, home and vehicle maintenance, caring for household members (child care and adult care), consumer purchases, gardening, pet care, and travel related to these activities.\(^9\)

Since the paper focuses on the use of extra time when individuals move from employment to unemployment, we restrict the sample to individuals in the labor force between the ages of 20-65. The number of observations is 52,652, the rate of unemployment is 5.3%, and the average time spent on home production is 14.28 hours per week in the selected sample. Table 1 presents the demographics and summary statistics of the sample.

We use the following equation to estimate the effect of employment status on home production:

\[
HP_i = \beta_0 + X_i \beta + U_i \phi + D_i \gamma + \epsilon_i
\] (1)

where \(HP_i\) is the weekly hours spent on home production, \(U_i\) is employment status (1 if individual is unemployed, 0 otherwise), \(D_i\) is a set of dummy variables for each state and year, \(X_i\) is a set of explanatory variables including age, education, their interaction, race, gender, family size, and spouse employment status for individual \(i\).

\(^8\)For a detailed description and a guide to the potential uses of the data, see Hamermesh et al. (2005).
Table 2 compares the decline in working hours and the increase in home production and housework hours. There is an increase of 12 hours/week in time spent for home production as opposed to a 37 hours/week decrease in time spent for market production. The increase in time spent on housework at the transition from employment to unemployment is only 5.4 hours/week. We also decompose the increase in home production hours in Table 3.

The empirical results suggest that individuals do not completely substitute leisure for the decline in working hours during unemployment spells. Instead, some part of the decline in working hours is substituted with an increase in home production. We interpret this as a consumption insurance during unemployment spells against the loss of earnings and compute the optimal rate of unemployment insurance in this environment.

4 Model

We augment an incomplete markets model of unemployment with a home production technology, which allows unemployed workers to use their extra non-market time as partial insurance against the drop in income due to unemployment. We use a dynamic general equilibrium environment with home production to understand the role of home production in determining optimal unemployment insurance policy. The markets are incomplete because there is only storage technology for individuals. The unemployment insurance is financed by a proportional income tax. There is a continuum of ex ante identical individuals and heterogeneity arises due to idiosyncratic employment opportunities. We explain each component of the model in detail in the following subsections.
4.1 Employment Process

Individuals receive shocks to employment opportunities every period. It follows a two-state Markov chain. The transition probabilities are defined as \( \chi(i, j) = P(e' = j|e = i) \), where \( i, j \in \{0, 1\} \). For example, given that the individual did not get an offer in the last period, the probability of getting an offer in the current period is equal to \( P(e' = 1|e = 0) = \chi(0, 1) \). Each employed individual earns the same wage rate denoted with \( y \).

4.2 Household Decisions

Individuals enjoy utility from consumption and leisure. They have two continuous decisions at every period: one is the saving/spending decision, and the other is the time allocation decision. Individuals can choose the amount of time spent on home production and leisure given their employment status. The time constraint is looser and the budget constraint is tighter when the individual is unemployed. Given that the individual has a job offer, he or she also makes an accept/reject decision.

They maximize their lifetime utility:

\[
E \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)
\]

where \( u(\cdot) \) is a period utility function, \( \beta \) is a time discount factor, \( c_t \) is consumption and \( l_t \) is leisure.

Individuals have a time constraint in each period, which depends on the employment shock in the current period:

\[
h_t + l_t + n(e) = 1
\]

where \( h_t \) is time spent on home production, \( l_t \) is leisure and \( n(e) \) is labor supply con-
ditional on employment status. Individuals have an employment decision only at the extensive margin. If an individual is unemployed, then \( n(0) = 0 \), if he or she is employed, then \( n(1) = \bar{n} \). Therefore, unemployed individuals have looser time constraints compared to employed individuals. In particular, employed individuals face the following time constraint:

\[
h_t + l_t + \bar{n} = 1 \tag{3}
\]

and unemployed individuals face the following time constraint:

\[
h_t + l_t = 1 \tag{4}
\]

### 4.3 Storage Technology

Individuals can accumulate wealth through a non-interest-bearing asset and the assets evolve according to the following equation:\(^{10}\)

\[
x_t + a_{t+1} = a_t + y_t^d(e)
\]

\[
0 \leq a_t \tag{6}
\]

where \( x_t \) is expenditure on market goods and services, and \( a_{t+1} \) is the amount of wealth carried to the next period. Disposable income \( (y_t^d) \) depends on employment opportunity and qualification for unemployment insurance, we will return to this later on. If individuals are not working, they can consume their stock of wealth and they can also possibly consume unemployment benefits.

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\(^{10}\) Although this asset structure is pretty standard in the UI literature, we would like to indicate that the optimal replacement rates would tend to be smaller with a richer asset market structure or a borrowing option. However, the wealth of unemployed workers is close to zero in the U.S. data, and our asset structure matches this fact.
4.4 Home Production

Individuals produce consumption goods and services using market goods and services and time according to a production function:

$$c_t = f(h_t, x_t)$$ (7)

where $c_t$ is the amount of consumption goods and services, $h_t$ is time spent on home production and $x_t$ is expenditure on market goods and services. In this paper, we assume that every individual has the same home production technology with function $f(\cdot)$. We will explain this function in detail later on, when we explain functional specifications.

The individual’s two constraints (2) and (5) are important in our analysis. Employed individuals have less time remaining for leisure and home production, so the time resource is more scarce for them. On the other hand, unemployed individuals have fewer resources for market goods and services. Due to this difference, they will allocate their time and goods differently.

4.5 Moral Hazard

In the model economy, we also allow for moral hazard. Moral hazard is caused by imperfect government monitoring of job offers. In this case, individuals can turn down job offers and still collect unemployment benefits with some probability. The probability of obtaining benefits conditional on rejecting an offer is denoted with $\pi(\eta)$, where $\eta$ represents last period employment status. We allow $\pi(\cdot)$ to depend on $\eta$, because the effectiveness of government monitoring might be different for searchers and quitters. If an individual is employed in the previous period and rejects a job offer in the current period, he/she is a quitter and probably better monitored by the government. If he/she is not employed in the previous period and rejects a job offer in the current period, he/she
is a searcher and probably not monitored as well as a quitter. Note that a positive value of either \( \pi(0) \) or \( \pi(1) \) refers to moral hazard in the society.

4.6 Unemployment Insurance System

We define the unemployment benefits system as follows: If an individual has no employment opportunity, then he or she qualifies for unemployment benefits directly; If an individual has an employment opportunity and accepts it, then he or she does not qualify for unemployment benefits; If an individual has an employment opportunity and rejects it, then he or she qualifies for the unemployment benefits with probability \( \pi(\eta) \) depending on his or her employment status in the last period.

In the scheme below, \( e \) is the indicator of employment opportunity with \( e \in \{0, 1\} \). If \( e \) is equal to 1, an individual has an employment opportunity; if \( e \) is equal to 0, an individual has no employment opportunity. We denote the unemployment insurance qualification indicator with \( \mu \in \{0, 1\} \). If it is equal to 1, the individual is qualified for unemployment benefits; otherwise, he or she is not qualified. The last period’s employment status is denoted with \( \eta \in \{0, 1\} \). If \( \eta = 1 \), then the individual was employed in the last period; otherwise, he or she was not employed in the last period. We summarize the unemployment benefit system as follows:
gets no offer \((e = 0) \Rightarrow \mu = 1\)

gets offer \((e = 1), \text{ accepts } (\eta' = 1) \Rightarrow \mu = 0\)

gets offer \((e = 1), (\eta = 0), \text{ rejects } (\eta' = 0) \Rightarrow \mu = 1 \text{ with probability } \pi(0) \text{ and } \mu = 0 \text{ with probability } 1 - \pi(0)\)

gets offer \((e = 1), (\eta = 1), \text{ rejects } (\eta' = 0) \Rightarrow \mu = 1 \text{ with probability } \pi(1) \text{ and } \mu = 0 \text{ with probability } 1 - \pi(1)\)

Note that government monitoring is an exogenous process with \(\pi(.)\). The government does not make a decision on the degree of monitoring job offers. Optimal government monitoring is not analyzed in this paper.

### 4.7 Taxation and Disposable Income

The government collects a proportional income tax \((\tau)\) to finance the unemployment benefits. The rate of tax is adjusted to balance the government budget. Individuals who qualify for unemployment benefits receive a certain fraction \((\theta)\) of their lost income. As noted before, an individual’s disposable income \((y_t)\) depends on the current and previous employment status. When individuals qualify for unemployment insurance, they receive a benefit equal to \(\theta(1 - \tau)y\), and we denote it with \(b\). We summarize disposable income at different states as follows:
gets no offer \((e = 0)\) ⇒ \(y^d_i = b\) \hspace{1cm} (8)

gets offer \((e = 1)\), accepts \((\eta' = 1)\) ⇒ \(y^d_i = (1 - \tau)y\) \hspace{1cm} (9)

gets offer \((e = 1)\), rejects \((\eta' = 0)\), gets benefit \((\mu = 1)\) ⇒ \(y^d_i = b\) \hspace{1cm} (10)

gets offer \((e = 1)\), rejects \((\eta' = 0)\), no benefit \((\mu = 0)\) ⇒ \(y^d_i = 0\) \hspace{1cm} (11)

In the above scheme, \(y^d_t\) represents disposable income in period \(t\). We denote the wage of an employed worker with \(y\), which is normalized to 1. If an individual has no employment opportunity, he or she enjoys the after-tax unemployment insurance benefits. If an individual has an employment opportunity and accepts it, then he or she enjoys the after-tax wage. If an individual has an employment opportunity and rejects it and qualifies for benefits, then he or she enjoys after tax unemployment benefits. If an individual does not qualify for benefits when he or she rejects the employment opportunity, then he or she has 0 disposable income in that period.

4.8 Timing of Events

At each period an individual’s state is determined by current employment opportunity \((e)\), previous employment status \((\eta)\), and current asset level \((a)\). Given the current state, the individual makes an employment decision, a saving/spending decision, and a time allocation decision. First, the employment opportunity shock is received. If there is an employment opportunity, then the individual makes an employment decision by accepting or rejecting the opportunity. If the individual rejects the opportunity, then he or she finds out if he or she qualifies for unemployment benefits. Then the individual makes a saving/spending decision and a time allocation decision depending on whether he or she receives unemployment benefits. On the other hand, if there is no employment opportunity, he or she enjoys after-tax unemployment benefits.
opportunity, then the individual makes saving/spending and time allocation decisions directly. We summarize the timing of events in Figure ??.

4.9 Recursive Formulations

Individuals make their decisions depending on three state variables: current asset levels \(a\), existence of a job offer in the current period \(e\), and the previous period’s employment status \(\eta\). We denote the value function of an individual with state variables \(a, e, \eta\) with \(V(a, e, \eta)\).

The problem of an individual with no employment opportunity can be summarized with the following recursive formulation:

\[
V(a, 0, \eta) = \max_{a', x, h, l} \left\{ u(c, l) + \delta \sum_{e'} \chi(0, e') V(a', e', 0) \right\}
\]

s.t.

constraints (4), (5), (7), (8).

On the left-hand side of equation (12), \(a\) represents the current asset level and \(\eta\) represents last period employment status. Note that \(\eta\) has no role in the value function of individuals with no employment opportunity\(^{11}\) because they directly qualify for unemployment insurance. Also, note that the employment opportunity indicator \(e\) is 0, in this case. On the right-hand side, the inputs of period utility are consumption and leisure. Consumption is represented as a function \(f(.)\) of time spent on home production and expenditures on market goods. Time left for leisure equals \(1 - h\), because the labor supply \(n\) is zero. In the next period’s value function, \(\eta\) is equal to zero, because the individual is not working in the current period. Since the individual qualifies for unemployment insurance, disposable income equals a certain fraction \((\theta)\) of lost after-tax

\(^{11}\)We write it as an input in order to be consistent with the general notation of the model.
earnings \(((1 - \tau)y)\).

The problem of an individual with an employment opportunity can be summarized with the following recursive formulation:

\[
V(a, 1, \eta) = \max \{V_A(a, 1, \eta), V_R(a, 1, \eta)\}
\]

(13)

where, \(V_R(a, 1, \eta) = \pi(\eta)V_B(a, 1, \eta) + (1 - \pi(\eta))V_N(a, 1, \eta)\)

(14)

where \(V_f\) is the value of an offer, \(V_A\) is the value of accepting the offer, \(V_R\) is the value of rejecting the offer, \(V_B\) is the value of qualifying for unemployment insurance after rejecting the offer, and \(V_N\) is the value of not qualifying for unemployment insurance after rejecting the offer. Recall that \(\pi(\eta)\) represents the probability of obtaining unemployment benefits for those who reject employment opportunities. Therefore, the value of rejecting an offer is equal to \(\pi(\eta)\) times the value of obtaining unemployment benefits, and \((1 - \pi(\eta))\) times the value of not obtaining benefits upon rejecting job offers.

The value of accepting an offer can be summarized as follows:

\[
V_A(a, 1, \eta) = \max_{a', x, h, l} \{u(c, l) + \delta \sum_{e'} \chi(1, e')V(a', e', 1)\}
\]

s.t.

constraints (3), (5), (7), (9).

(15)

The value of qualifying for unemployment insurance upon rejecting an offer is equal to:

\[
V_B(a, 1, \eta) = \max_{a', x, h, l} \{u(c, l) + \delta \sum_{e'} \chi(1, e')V(a', e', 0)\}
\]

s.t.

constraints (4), (5), (7), (10).

(16)
The value of not qualifying for benefits upon rejecting an offer is defined as:

\[
V_N(a, 1, \eta) = \max_{a', x, h, l} \{u(c, l) + \delta \sum_{e'} \chi(1, e') V(a', e', 0)\}
\]  
(17)

s.t.

constraints (4), (5), (7), (11).

4.10 Equilibrium

In this economy, a \textit{stationary competitive equilibrium} is defined as:

- a set of decision rules of expenditure \(x(\omega)\), stock of wealth \(a'(\omega)\), home production \(h(\omega)\), leisure \(l(\omega)\), and employment \(\eta'(a, e, \eta)\), where \(\omega = (a, e, \eta, \mu)\),

- a tax rate \(\tau\),

- an invariant measure \(\lambda(\omega)\),

- such that:

- the decision rules solve the individuals’ problem defined in equations (12), (14), (15), (16), (17),

- the goods market clears:

\[
\sum_{\omega} \lambda(\omega)x(\omega) = \sum_{\omega} \lambda(\omega)\eta'(\omega)y
\]

- the government budget is balanced:

\[
\sum_{a} \{[\lambda(a, 1, \eta, 1) + \lambda(a, 0, \eta, 1)](1 - \tau)\theta y - \\
\lambda(a, 1, \eta, 0)\eta'(a, 1, \eta, 0)y\tau\} = 0
\]
and the invariant measure $\lambda(\omega)$ solves:

$$
\lambda(\omega') = \begin{cases}
0; & \text{if } e' = 0, \mu' = 0 \\
\sum_\mu \sum_\eta \sum_e \sum_{a \in \Omega} \chi(e, e') \lambda(\omega); & \text{if } e' = 0, \mu' = 1 \\
\sum_\mu \sum_\eta \sum_e \sum_{a \in \Omega} \chi(e, e') \lambda(\omega) \times \\
\{\eta'(\omega') + [1 - \pi(\eta'(\omega'))][1 - \eta'(\omega')]\}; & \text{if } e' = 1, \mu' = 0 \\
\sum_\mu \sum_\eta \sum_e \sum_{a \in \Omega} \chi(e, e') \lambda(\omega) \times \\
\pi(\eta'(\omega'))[1 - \eta'(\omega')]; & \text{if } e' = 1, \mu' = 1
\end{cases}
$$

where $\Omega = \{a : a' = a'(a, e, \eta, \mu)\}$.

Note that, in equilibrium, decision rules solve the individuals’ optimization problems. Total unemployment insurance payments for unemployed workers must be equal to the taxes paid by employed workers. The invariant distribution ensures that the distribution of agents doesn’t change across time. In the invariant distribution, the first line means that the fraction of the population that gets offers and also obtains unemployment benefits is zero. The second line means that all agents who do not get an offer qualify for unemployment insurance benefits for sure. The third line represents the fraction of the population that gets an offer and does not qualify for benefits. The last line is the fraction of the population that gets an offer and also qualifies for benefits.

## 5 Calibration

In the unemployment insurance literature, most of the studies are quarterly or six-week periods.\textsuperscript{12} We define each period as six weeks to be in line with the existing literature. The employment opportunities follow a two state Markov process. We follow Hansen and Werning (2008).\textsuperscript{17}

\textsuperscript{12}There are exceptions that use weekly periods, for example: Acemoglu and Shimer (2000), Shimer and Werning (2008).
Imrohoroglu (1992) in using a transition matrix of employment opportunities with the following probabilities, which matches the average rate and duration of unemployment in the United States:

\[
\begin{bmatrix}
0.9681 & 0.0319 \\
0.5 & 0.5
\end{bmatrix}
\]

With the above transition matrix, agents receive employment opportunities 94% of the time, and the average duration of time without employment opportunities is 12 weeks.

We have a constant labor supply of employed workers denoted with \( \bar{n} \), which equals 0.45. We take this constant labor supply to be the same as that in the closely related studies, such as Hansen and Imrohoroglu (1992), Abdulkadiroglu et al. (2002), and Pallage and Zimmermann (2005), which matches the average working hours in the United States.\(^{13}\)

We choose a value of 0.995 for parameter \( \beta \). This is standard for monthly or quarterly periods. Therefore, this is a reasonable value for our model as well. This value returns a plausible value of the average wealth/income ratio for the unemployed workers in the model. Empirical studies\(^ {14} \) report this ratio to be near 0. The model returns a value around 0.15.

The utility function is a Constant Relative Risk Aversion (CRRA) function composed of consumption and leisure with a risk aversion parameter of \( \sigma \), and the composition of consumption and leisure is formed as a Cobb-Douglas function:

\[
u(c, l) = \frac{(c^{1-\rho} l^\rho)^{1-\sigma} - 1}{1 - \sigma}
\]

\(^{13}\)In the business cycle literature, Kydland and Prescott (1991) report the same value for \( \bar{n} \).

We choose a benchmark value for \( \sigma \) in order to have results comparable to those in the aforementioned related studies in the unemployment insurance literature. Although the acceptable range for \( \sigma \) is 1.5 to 10 in the business cycle literature, unemployment insurance studies usually set it between 0.5 to 4.\(^{15}\) In the benchmark case, we pick 2.50 for \( \sigma \).

The share of leisure in the utility function is denoted with \( \rho \), and the value for this parameter is 0.67 in the benchmark case. We follow Kydland and Prescott (1991) for the benchmark value of \( \rho \), which is standard in the business cycle literature.\(^{16}\)

The home production function takes a Constant Elasticity of Substitution (CES) form with an elasticity of substitution between time and goods, \( 1/(1 - \nu) \):

\[
f(h, x) = (\psi h^\nu + x^\nu)^{\gamma/\nu}
\]

The parameters of the home production function are estimated by Aguiar and Hurst (2007) using U.S. micro data.\(^{17}\) They have multiple estimations for \( \psi \) and \( 1/(1 - \nu) \). In the benchmark case, we choose \( \psi = 0.31 \) and \( 1/(1 - \nu) = 1.45 \). We solve the model with other values as well. Using these parameter values, the model returns a difference of about 5 hours per week between the average home production time of the unemployed and the employed.

To have a benchmark model, we need to choose a value for the average replacement rate (\( \theta \)). There are empirical studies on the average replacement rate in the United States. Gruber (1997) finds an average replacement rate of about 40%. Clark and Summers (1982) estimate an average replacement rate of around 65%. Keeping in mind

\(^{15}\)For example, the value of \( \sigma \) is 1 in Shavell and Weiss (1979), 0.5 Hopenhayn and Nicolini (1997), 2.5 in Hansen and Imrohoroglu (1992), 2 in Alvarez-Parra and Sanchez (2010), and 4 in Acemoglu and Shimer (2000).

\(^{16}\)Also, Jacobs (2007) estimates a range of 0.63 to 0.68 for the value of \( \rho \) using the PSID data set.

\(^{17}\)For a detailed discussion of the data sets and estimations, see Aguiar and Hurst (2007).
that replacement rates have decreased over time in the United States, and that Gruber’s work is more recent, we pick the current level of the replacement rate as 40% in the benchmark case. The parameters are reported in Table 4.

We choose parameters $\pi(0)$ and $\pi(1)$ to determine the degree of moral hazard in the society. Higher values for these parameters mean higher moral hazard. Pallage and Zimmermann (2005) assume that $\pi(0) > 0$ and $\pi(1) = 0$ and predict a value of about 0.2 for $\pi(0)$ in the United States using a quantitative dynamic general equilibrium model. We do quantitative analysis in several different cases for parameters $\pi(0)$ and $\pi(1)$, and we pick values close to the ones reported in Pallage and Zimmermann (2005). We analyze the role of home production in determining optimal unemployment insurance policy for the different cases.

6 Quantitative Results

We solve the model computationally and simulate 50,000 periods to calculate the moments. In the quantitative exercises, we aim to find the role of home production in determining the optimal rate of unemployment insurance. We divide the $[0,1]$ interval into grids. The replacement rate takes values from these grids. We compute the equilibrium for each possible value of the replacement rate. We pick the one that maximizes average utility in the society as the optimal replacement rate. In order to see the role of home production in determining optimal unemployment insurance, we solve the model twice: once with home production and once with no home production. We also repeat the same exercises allowing for moral hazard in the society. This allows us to understand how moral hazard affects the role of home production in determining unemployment insurance policy.

In general, our results imply that optimal unemployment insurance levels are smaller
when we allow for self-insurance through home production and the stock of wealth. In the following subsections, we quantify the role of home production in determining the optimal unemployment insurance policy using various parameter values.

6.1 Benchmark Model

We are going to use the benchmark economy as an example to illustrate how agents behave. The average wealth to income ratio is 0.03 for unemployed individuals and 0.36 for employed individuals in this economy. We define unemployed individuals as the fraction of the population that does not get offers and would accept one if they did, plus those who qualify for unemployment insurance upon refusing a job offer.\(^ {18} \) The unemployment rate is 5.5\% in the benchmark case. The average unemployment duration is of about 10 weeks. The standard deviation of log consumption is 0.12, and the standard deviation of log expenditure is 0.26. Note that the dispersion in consumption is smaller than the dispersion in expenditure due to the smoothing role of home production in this economy. The agents who receive employment opportunities accept or refuse offers depending on their asset levels. They reject offers if they have asset levels greater than or equal to 0.52. Unemployed individuals spend about 40 minutes per day (4.66 hours per week) more than the employed individuals on home production activities.

6.2 Role of Home Production

In this subsection, our purpose is to quantify the role of home production in determining the optimal unemployment insurance. In order to do that, we perform a series of quantitative exercises. First, we quantify the role of home production in determining the optimal unemployment insurance in a society with no moral hazard (\( \pi(0) = \pi(1) = 0 \)).\(^ {18} \)

\(^ {18} \)In the definition of unemployment, we include the fraction of the population that refuses offers but still qualifies for unemployment insurance, because they report themselves as unemployed.
We solve the model for two cases: (i) individuals are not allowed to do home production; that is $\psi = 0$, and (ii) they are allowed to do home production.

In the first case, we assume that consumption is equal to expenditure on market goods and services. In this case, we compute the optimal replacement rate to be about 65% of earnings. Employed individuals spend all of the time remaining after inelastically supplying labor on leisure, and unemployed individuals spend all of their time enjoying leisure. Note that consumption is assumed to be equal to expenditures on market goods and services in this case. Therefore, the standard deviation of consumption is equal to the standard deviation of expenditure in this case.

Next, keeping all the parameters the same, we solve for the second case, in which individuals are allowed to do home production. In this case, we find that the optimal rate of unemployment insurance is 0.40. In this equilibrium, the unemployed spend 4.6 hours per week more on home production compared to the employed. Individuals reduce the cost of unemployment by changing their time allocation during unemployment spells, and that makes the optimal replacement rate smaller compared to the no home production case. Because of home production, the optimal level of unemployment insurance decreases by 25% which corresponds to about 40% or earnings. Since consumption is a function of time and market goods, it deviates from expenditures in this case. The standard deviation of consumption is about half of the standard deviation of expenditures in this case. Therefore, the home production option decreases consumption inequality in the society.

### 6.3 Role of Elasticity of Substitution Between Time and Goods

In order to see how the role of home production depends on the elasticity of substitution between time and goods, we solve the model with various values for the elasticity of substitution between time and goods. We use the estimated parameters from Aguiar
and Hurst (2007): 1.45, 1.78, and 2.13. The optimal replacement rate is 40%, 45%, and 45%, respectively, for the corresponding values of elasticity.\textsuperscript{19} The optimal replacement rate of the no home production case does not change, since the elasticity parameter has no role in the no home production case. Therefore, the fact that the optimal replacement rate is substantially smaller in the model with home production is robust.

### 6.4 Role of Moral Hazard

Now we quantify the role of home production in a society with moral hazard. We introduce some moral hazard with $\pi(0) = 0.1$. This means that job searchers can qualify for unemployment insurance with probability 0.1, although they receive job offers. At this level of moral hazard, the optimal replacement rate in the economy with home production decreases to 35%; however, it remains 65% in the economy with no home production. When the level of moral hazard goes up to 0.2, the optimal replacement rate goes down to 25% in the economy with home production and to 40% in the no home production economy. Therefore, our result regarding the difference between optimal replacement rates in home production and no home production economies is robust in a society with moral hazard.

### 6.5 Unemployment Insurance and Stock of Wealth

The model implies a negative relationship between the average stock of wealth and the replacement rate. Due to the partial replacement for lost earnings, individuals accumulate less precautionary wealth with increased replacement rates. This is in line with previous empirical research on the substitution of public and self-insurance for individuals. Engen and Gruber (2001) provide empirical evidence on this relationship between

\textsuperscript{19}Note that we change parameter $\psi$ along with elasticity to keep the difference between the home production levels of the unemployed and the employed.
precautionary wealth accumulation and the rate of unemployment insurance.

We summarize the quantitative results in Table 5. In general, the optimal replacement rates are smaller when we allow for home production in the model economy. This is due to the consumption smoothing role of home production during unemployment spells. We perform quantitative exercises in several different environments, and the effect of home production is robust in all of them. The results also imply that the current average replacement rate in the United States (about 40%) is optimal only if there is no moral hazard in the society.

7 Conclusion

In this paper, we make a quantitative analysis of optimal unemployment insurance, where we incorporate self-insurance through home production and the stock of wealth. In the benchmark model, we find that the optimal replacement rate in the presence of home production is roughly 40% of wages, which is 40% lower than the no home production model’s optimal replacement rate of 65%. The presence of home production decreases the optimal replacement rate, and this result is robust under various parameterizations. The reason behind this result is the nature of the unemployment shock. During unemployment spells, individuals have tighter constraints while purchasing market goods and services and looser time constraints, and they respond by increasing their home production against unemployment shocks. Since consumption is a function of time and market goods, in the presence of home production, unemployed individuals enjoy smoother consumption levels compared to the no home production case. Eventually, that lowers the optimal replacement rate significantly.
References


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8 Tables

Table 1: Summary Statistics (ATUS, 2003-2008)

<table>
<thead>
<tr>
<th></th>
<th>Housework</th>
<th>Home Production</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>6.69</td>
<td>20.08</td>
<td>100</td>
</tr>
<tr>
<td>Employed</td>
<td>6.40</td>
<td>19.50</td>
<td>95</td>
</tr>
<tr>
<td>Unemployed</td>
<td>11.72</td>
<td>30.29</td>
<td>5</td>
</tr>
<tr>
<td>Female</td>
<td>10.27</td>
<td>24.83</td>
<td>47</td>
</tr>
<tr>
<td>Male</td>
<td>3.53</td>
<td>15.89</td>
<td>53</td>
</tr>
<tr>
<td>Single</td>
<td>5.61</td>
<td>15.75</td>
<td>35</td>
</tr>
<tr>
<td>Married</td>
<td>7.26</td>
<td>22.37</td>
<td>65</td>
</tr>
<tr>
<td>Not-White</td>
<td>6.76</td>
<td>18.33</td>
<td>17</td>
</tr>
<tr>
<td>White</td>
<td>6.67</td>
<td>20.44</td>
<td>83</td>
</tr>
<tr>
<td>High school Grad.</td>
<td>6.57</td>
<td>20.19</td>
<td>91</td>
</tr>
<tr>
<td>Less than High school</td>
<td>7.84</td>
<td>18.98</td>
<td>9</td>
</tr>
<tr>
<td>College Grad.</td>
<td>6.26</td>
<td>20.74</td>
<td>32</td>
</tr>
<tr>
<td>Less than College</td>
<td>6.89</td>
<td>19.77</td>
<td>68</td>
</tr>
</tbody>
</table>

Note: Unmarried couples are included in the “Married” sample.

Table 2: Home Production and Unemployment in ATUS

<table>
<thead>
<tr>
<th></th>
<th>Work</th>
<th>Home Production</th>
<th>Housework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployed</td>
<td>-37.793***</td>
<td>12.025***</td>
<td>5.408***</td>
</tr>
<tr>
<td></td>
<td>(0.365)</td>
<td>(0.610)</td>
<td>(0.347)</td>
</tr>
<tr>
<td>Observations</td>
<td>53024</td>
<td>53024</td>
<td>53024</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.091</td>
<td>0.122</td>
<td>0.141</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Table 3: Home Production and Unemployment in ATUS: Decomposed

<table>
<thead>
<tr>
<th>Unemployed</th>
<th>Housework</th>
<th>House &amp; Vehicle Maintenance</th>
<th>Household Care</th>
<th>Purchasing Goods</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>5.408***</td>
<td>2.493***</td>
<td>2.098***</td>
<td>1.017***</td>
<td>1.335***</td>
</tr>
<tr>
<td>(0.347)</td>
<td>(0.341)</td>
<td>(0.249)</td>
<td>(0.257)</td>
<td>(0.245)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>53024</td>
<td>53024</td>
<td>53024</td>
<td>53024</td>
<td>53024</td>
</tr>
<tr>
<td>R²</td>
<td>0.141</td>
<td>0.041</td>
<td>0.154</td>
<td>0.016</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

* p < 0.05, ** p < 0.01, *** p < 0.001

Table 4: Parameters of the Benchmark Economy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.995</td>
</tr>
<tr>
<td>σ</td>
<td>2.50</td>
</tr>
<tr>
<td>ρ</td>
<td>0.67</td>
</tr>
<tr>
<td>n</td>
<td>0.45</td>
</tr>
<tr>
<td>θ</td>
<td>0.40</td>
</tr>
<tr>
<td>χ(0,0)</td>
<td>0.50</td>
</tr>
<tr>
<td>χ(1,1)</td>
<td>0.9681</td>
</tr>
<tr>
<td>ψ</td>
<td>0.31</td>
</tr>
<tr>
<td>1/(1 − ν)</td>
<td>1.45</td>
</tr>
<tr>
<td>γ</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5: Optimal Replacement Rates (Summary of the Quantitative Results)

<table>
<thead>
<tr>
<th>1/(1 − ν)</th>
<th>ψ</th>
<th>π(0)</th>
<th>π(1)</th>
<th>Optimal θ, HP</th>
<th>Optimal θ, no HP</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.45</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
<td>0.65</td>
<td>40</td>
</tr>
<tr>
<td>1.45</td>
<td>0.31</td>
<td>0.10</td>
<td>0.00</td>
<td>0.25</td>
<td>0.65</td>
<td>60</td>
</tr>
<tr>
<td>1.45</td>
<td>0.31</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.20</td>
<td>50</td>
</tr>
<tr>
<td>1.45</td>
<td>0.31</td>
<td>0.20</td>
<td>0.00</td>
<td>0.05</td>
<td>0.25</td>
<td>80</td>
</tr>
<tr>
<td>1.45</td>
<td>0.31</td>
<td>0.20</td>
<td>0.10</td>
<td>0.05</td>
<td>0.20</td>
<td>75</td>
</tr>
<tr>
<td>1.78</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>0.45</td>
<td>0.65</td>
<td>30</td>
</tr>
<tr>
<td>2.13</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.45</td>
<td>0.65</td>
<td>30</td>
</tr>
</tbody>
</table>

Notes: This table shows the summary of our quantitative results on the optimal unemployment insurance policy. The optimal replacement rates for different set of parameters are reported. In the last column, we report percentage differences between optimal replacement rates with and without home production option.
9 Figures

Figure 1: Timing of the Events

- $e$ is revealed
- Gets offer
  - Accepted
    - $(h, l)$ and $(a', x)$ decisions
    - Gets UI benefits
      - $(h, l)$ and $(a', x)$ decisions
  - Rejected
    - $\mu$ is revealed
    - Gets UI benefits
      - $(h, l)$ and $(a', x)$ decisions
    - No UI benefits
      - $(h, l)$ and $(a', x)$ decisions
- No offer

Notes: In this scheme, $e$ represent employment opportunities. Indicator of unemployment insurance qualification is denoted with $\mu$. We denote home production, leisure, expenditure, and saving decisions with $h$, $l$, $x$, and $a'$, respectively.