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Abstract: This paper demonstrates that a pollution tax with a fixed cost component may lead, by itself, to segregation between clean and dirty firms without heterogeneous preferences or increasing returns. We construct a simple model with two locations and two industries (clean and dirty) where pollution is a by-product of dirty good manufacturing. Under proper assumptions, a completely stratified configuration with all dirty firms clustering in one city emerges as the only equilibrium outcome when there is a fixed cost component of the pollution tax. Moreover, a stratified Pareto optimum can never be supported by a competitive spatial equilibrium with a linear pollution tax. To support such a stratified Pareto optimum, however, an effective but unconventional policy prescription is to redistribute the pollution tax revenue from the dirty to the clean city residents.

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"If you visit American city,
You will find it very pretty.
Just two things of which you must beware:
Don't drink the water and don't breathe the air."
(Tom Lehrer, *Pollution*)

1 Introduction

It is evident that the development of many local economies has featured adjacent but segregated clean and dirty cities. Examples of such pairs include Seattle/Tacoma and San Francisco/Oakland with larger clean cities, Ann Arbor/Detroit and Aurora/Denver with smaller clean cities, as well as Washington, D.C./Baltimore and Champaign/Urbana with comparably sized clean and dirty cities. A natural question arises: Why are dirty firms clustered in one location and why is such an outcome sustainable over time? Certainly one might address the question with heterogeneity in preferences or increasing returns in production.[1] Our paper proposes an alternative: a pollution tax with a fixed cost tax component alone may lead to segregation between clean and dirty firms without heterogeneous preferences or increasing returns.

Since 1972, the OECD has adopted the polluter pays principle, trying to internalize environmental costs based on the idea first advanced by Pigou (1920). More recently, the OECD (1994) categorized three types of pollution taxes: (i) a proportional tax on the actual pollution output, for example according to the amount of emission; (ii) a proportional tax on a proxy for pollution output, for example according to water consumption, electricity usage or each unit of product when the production process harms the environment; and (iii) a fixed cost tax levied on each company or each household. In this paper, we consider both the second and the third types. Whereas a fixed cost tax levied on each firm is considered, the proportional Pigouvian tax is generalized to a linear tax that includes a fixed cost tax component as proposed by Carlton and Loury (1980).[2]

The key difference between this work and the classical literature on Pigouvian taxation is: We assume that there is an independent local government with taxing power in each region that must

[1]See Porter (1990) for a comprehensive discussion of industrial clustering from a business strategy viewpoint. Our paper is also related to the locational stratification literature, where stratification is caused by human capital (cf. Benabou 1996a,b, Chen, Peng and Wang 2009), local public goods (cf. Nachyba 1997 and Peng and Wang 2005), and the environment (cf. Chen, Huang and Wang 2011).

[2]See also Baumol (1972) and Buchanan and Tullock (1975) on direct control versus taxation. While there is an existing literature on the welfare consequences of pollution taxation (see citations in Section 7 below), none explores the implication of pollution taxation for production agglomeration.

balance its own budget. We take the tax system of each local government to be exogenous, with no tax competition.[3] There are 3 related potential distortions in our framework: a negative pollution externality from dirty firm production imposed on consumers, a positive local agglomeration externality for polluting firms, and a migration incentive for consumers induced by the different, independent tax and redistribution schedules in the two regions. Regarding the last distortion, local tax revenue and local profits are distributed back to the residents of that location only. The setting would be classical if there were only one national government with the power to tax differentially and redistribute to consumers independent of region of residence. In that case, the standard welfare theorems would go through under Pigouvian taxes, since correction for the pollution externality and migration incentives (the first and third distortions) can be made in the usual way, whereas the fixed cost component of the firm tax/transfer system can account for the agglomeration externality. However, with independent regional government taxation as in our setting, equilibrium allocations might not be Pareto optimal unless transfers between the regional governments are made so that the regional governments can mimic a national government.

To illustrate the possibility that a pollution tax causes agglomeration of dirty firms, we construct a simple model featuring two locations and two industries, clean and dirty. Both industries use homogeneous labor as inputs. Whereas the clean service production is Ricardian (constant returns), dirty manufactured good production is socially constant-returns-to-scale and privately diminishing returns with positive spillovers of the Romer type. Pollution is a by-product of dirty good manufacturing. To eliminate unnecessary complications associated with a wealth effect, utility is assumed to be quasi-linear, linear in clean good consumption and pollution but strictly increasing and strictly concave in dirty good consumption. The pollution tax schedule features a fixed cost tax component that is independent of pollution (or dirty good output) and may also contain a marginal tax component that is proportional to dirty good output.

We establish that under proper assumptions, a completely stratified equilibrium with all dirty firms clustering in one city is supported and such a stratified equilibrium cannot emerge in the absence of the fixed payment pollution tax. In some circumstances, an integrated equilibrium is impossible, but a stratified equilibrium exists. Under suitable conditions, we show that the presence of pollution and a pollution tax with a fixed cost tax component, rather than the Romer-type positive spillovers, are necessary for agglomeration of dirty firms.

Next we turn to the examination of Pareto optima. Depending on exogenous parameter values,

[3] The reader is referred to Markusen, Morey, and Oleviler (1995) for modeling fiscal competition in pollution taxes with firms choosing the location of plants.

both integrated and stratified configurations can arise as optima. Whereas an integrated Pareto optimum can be supported by a competitive spatial equilibrium with a linear pollution tax, a stratified Pareto optimum cannot. Specifically, regardless of the linear pollution tax schedule, a stratified equilibrium is always over-polluted compared to the optimum. To support the stratified Pareto optimum, one must redistribute pollution tax revenues from the dirty to the clean city residents. This suggests a new instrument to rectify competitive equilibrium inefficiency when there is pollution generated by dirty good production.

We wish to emphasize that in this paper, we consider only equilibrium or optimal configurations that are completely stratified in terms of production or that are completely integrated in that production is symmetric across locations. We relegate the discussion of other possible configurations to the concluding section.

The remainder of the paper is organized as follows. Section 2 contains the notation and basic model. Section 3 provides first order necessary conditions for equilibrium. Section 4 analyzes the two types of equilibria we consider here, namely integrated and stratified. Section 5 analyzes the conditions on parameters that generate each of these types of equilibria. Section 6 gives further results, particularly about stability of equilibrium, that can be derived with specific functional forms, namely an example. Section 7 discusses Pareto optima and the welfare theorems, whereas section 8 concludes.

2 The Model

Consider a local economy consisting of two regions/cities ($i = A, B$) and two sectors (a clean/service good X and a dirty/manufactured good Y). Each region has an abundant supply of land of density one in a featureless landscape. This local economy is populated with three groups of active agents: (i) a continuum of households of a fixed mass one, who are all both consumers and workers; (ii) a continuum of clean (non-polluting) firms of mass one, and (iii) a continuum of dirty (polluting) firms of mass $M > 0$. Whereas households require land for residential purposes, firms can undertake production without land input. Goods are freely mobile, but land is immobile and workers are not allowed to commute between the two regions – that is, a worker's residential location is identical to her work location. Let the clean good be the numéraire. The within-city commuting cost is zero. Given the abundant supply of land, we shall set land rent to zero. In addition to the three groups of active agents, there is a local government ruling each region, whose only activity is to collect pollution taxes/fees for redistribution to consumers. To close the economy, we shall assume that dirty firms in a particular region are owned by consumers in the same region. There is no cost to transport any commodity between regions.

2.1 Firms

The clean good is produced with labor input under a Ricardian technology,

$$x^i(j) = \psi \cdot n_x^i(j), i = A, B, j \in [0, k^i] \tag{1}$$

where $x^i(j)$ denotes the output of clean firm j in location i, $\psi > 0$ is the inverse of the unit labor requirement for clean good production, $n_x^i(j)$ represents clean firm j's demand for labor, and $k^i \in [0,1]$ denotes the mass of clean firms in region i. The total local supply of the clean good in region i is given by $X^i = \int_0^{k^i} x^i(j)dj$ and the total local clean industry employees in the region i can be specified as:

$$N_x^i = \int_0^{k^i} n_x^i(j)dj \quad i \in A, B \tag{2}$$

Under *ex post* symmetry of firms in a region, imposed throughout, we have $N_x^i = k^i n_x^i$.

Denote by m^i the mass of dirty firms in region i, by $n_y^i(j)$ the labor demand by a dirty firm j in region i, and by N_y^i the total local dirty industry employees in region i, where:

$$N_y^i = \int_0^{m^i} n_y^i(j)dj \quad i \in A, B \tag{3}$$

Each dirty good firm employs labor as the sole private input under a privately decreasing-returns-to-scale and socially constant-returns-to-scale production technology $\widetilde{f}$:

$$y^i(j) = \widetilde{f}\left(n_y^i(j), N_y^i\right) = N_y^i f\left(\frac{n_y^i(j)}{N_y^i}\right), i \in A, B, j \in [0, m^i] \tag{4}$$

where $y^i(j)$ is the output of dirty firm j in region i. We assume that $\widetilde{f}$ is strictly increasing and strictly concave in each argument, satisfying the boundary condition $\widetilde{f}\left(0, N_y^i\right) = 0$ and the Inada conditions $\lim_{n_y^i(j)\to 0} \frac{\partial \widetilde{f}\left(n_y^i(j), N_y^i\right)}{\partial n_y^i(j)} \to \infty$ and $\lim_{n_y^i(j)\to \infty} \frac{\partial \widetilde{f}\left(n_y^i(j), N_y^i\right)}{\partial n_y^i(j)} \to 0$. Under social constant returns, we can divide firm output by the total number of local dirty industry employees to obtain f, where the properties of $\widetilde{f}$ imply that f is strictly increasing and strictly concave in the fraction of firm employees in the local dirty industry. The incorporation of N_y^i into a dirty firm's production function captures positive spillovers of the Romer (1986) type, where N_y^i is a positive measure of small firms, and where each firm is of measure zero. Under an *ex post* symmetric equilibrium, $N_y^i = m^i n_y^i$. The presence of uncompensated positive externalities provides an agglomeration force for dirty firms.

Both goods (clean and dirty) are traded and freely mobile. Let p denote the global relative price of the dirty good. Further denote the wage rate prevailing in region i as w^i. Let the region-specific pollution tax in region i be τ^i (to be specified later), where $\frac{\tau^i}{p}$ represents a typical *ad valorem* tax.[4] Each dirty firm in region i chooses labor demand to maximize its profit; its optimization problem is then given by:

$$\pi^i(j) = \max_{n_y^i} p\left[N_y^i f\left(\frac{n_y^i}{N_y^i}\right) - \tau^i\right] - w^i n_y^i(j) \tag{5}$$

The aggregate output of the dirty good in region i is $Y^i = \int_0^{m^i} y^i(j)dj$.

2.2 Households

Each household values the consumption of the clean good and the dirty good as well as immobile land, and suffers disutility from pollution. Each household is endowed with one unit of labor. Since a household does not value leisure, the entire one unit of labor is supplied inelastically. Let Q^i measure the level of pollution in region i. Following conventional wisdom, we assume that pollution is a by-product of the production of dirty goods, taking a simple linear form:

$$Q^i = \theta Y^i = \theta \int_0^{m^i} y^i(j)dj \tag{6}$$

where $\theta > 0$. Let h denote land consumption. The utility of a household residing in $\left[-D^i, D^i\right]$ (where D^i is the endogenously determined city boundary) of region i takes a quasi-linear form:

$$U^i = c_x^i - \gamma \cdot Q^i + \begin{cases} 0, & \text{if } h < 1 \\ u(c_y^i), & \text{if } h \geq 1 \end{cases} \tag{7}$$

This utility function is quasi-linear in the spirit of Bergstrom and Cornes (1983): linear in clean good consumption c_x and total pollution Q, but nonlinear in c_y, as $u(c_y)$ is the utility obtained from consuming the dirty good. It is strictly increasing and strictly concave, satisfying the boundary condition $u(0) = 0$ and the Inada conditions $\lim_{c_y^i \to 0} u'(c_y^i) \to \infty$ and $\lim_{c_y^i \to \infty} u'(c_y^i) \to 0$. Thus, the manufactured good is required to be consumed with land (think of watching TV in a house). This ensures that each household will consume exactly one unit of land ($h = 1$) and (given the supply of land and no commuting cost) that households must reside with uniform density in each of the two regions, which simplifies the analysis greatly.

In the absence of commuting cost and land rent, the household's budget constraint in region i is simply specified as follows:

$$c_x^i + pc_y^i = w^i + z^i \tag{8}$$

[4] The pollution tax schedule is written in this form for analytical convenience.

where z^i represents the sum of government rebates (of pollution tax collection) and firm profit redistribution in region i:

$$z^i = \frac{1}{N^i}\int_0^{m^i} \left[\pi^i(j) + p\tau^i\right] dj, \quad i \in A, B \tag{9}$$

2.3 The Local Government

The pollution tax levies on the dirty firm are given as follows:

$$\tau^i = \begin{cases} 0, & \text{if } y^i(j) = 0, \forall \;\; j \\ g^i(y^i(j), Y^i), & \text{otherwise} \end{cases}$$

When pollution is nondegenerate, we shall consider two specific regimes of interest, namely, a fixed pollution tax regime and a linear pollution tax regime:

$$g^i = \begin{cases} F/m^i, \text{ under fixed pollution tax regime} \\ L + ty^i(j), \text{ under linear pollution tax regime} \end{cases}$$

Under the fixed pollution tax regime, a fixed levy $F > 0$ is imposed on region i so that each firm pays an equal share $\frac{F}{m^i}$; under the linear pollution tax regime, in addition to a lump-sum tax $L > 0$, a marginal tax $t > 0$ is imposed on firm output y^i. While the former can best illustrate the role of pollution taxation played in firm agglomeration, the latter is important because it encompasses Pigouvian taxation as a special case and allows practical welfare analysis. For notational convenience, we shall denote generally the marginal tax rate as:

$$\zeta \equiv \frac{\partial \tau^i}{\partial y^i(j)} = \begin{cases} 0, \text{ under fixed pollution tax regime} \\ t, \text{ under linear pollution tax regime} \end{cases}$$

3 Optimization and Equilibrium

We are now prepared to derive individual optimizing conditions and to specify market clearing conditions.

3.1 Optimization

The first-order condition for profit maximization of each clean and dirty firm is, respectively, given by:

$$\psi = w^i \tag{10}$$

$$VMP_L \equiv p(1-\zeta)MPL_y^i = p(1-\zeta)f'\left(\frac{n_y^i}{N_y^i}\right) = w^i \tag{11}$$

where VMP_L denotes the value of the marginal product of labor (or marginal revenue product) and MPL denotes the marginal product of labor. Denote the dirty firm's surplus accrued from uncompensated spillovers as:

$$\sigma(e) \equiv f(e) - (1-\zeta)ef'(e)$$

where $e \equiv \frac{n_y^i}{N_y^i}$. It is convenient to denote the dirty firm's surplus excluding pollution tax as $\widetilde{\sigma}(e) = f(e) - ef'(e)$. Given our assumptions on the production function for dirty firms, both $\sigma(e)$ and $\widetilde{\sigma}(e)$ are strictly increasing in e. Substituting the *ex post* symmetry condition, $N_y^i = m^i n_y^i$ as well as (11) and (3) into (5) yields the profit for every firm j in region i:

$$\pi^i(j) = \pi^i = p\left[n_y^i m^i \sigma\left(1/m^i\right) - \tau^i\right] \tag{12}$$

The lump-sum distribution to each household follows immediately:

$$z^i = \frac{(m^i)^2}{N^i} \cdot \sigma\left(1/m^i\right) \cdot p n_y^i, \; \forall \; \tau^i \tag{13}$$

The household's optimization problem can be written more simply in two steps, solving backward. In the second step, households choose their best consumption bundle subject to their budget in each region. In the first step, they choose their region of residence.

Beginning with the second step, each household residing in region i maximizes their utility subject to the budget constraint by choosing c_y^i:

$$\max_{c_y^i} \; w^i + z^i - pc_y^i - \gamma Q^i + u\left(c_y^i\right) \tag{14}$$

The first-order condition of (14) with respect to c_y^i is given by:

$$u'\left(c_y^i\right) = p \tag{15}$$

It is immediate that, since the relative price of the dirty good across the two regions is one, the consumption of the dirty good in the two regions must be identical too. From the budget constraint (8) and (15), we then solve the clean good consumption as:

$$c_x^i = w^i + z^i - pc_y^i = w^i + z^i - c_y^i u'\left(c_y^i\right) \tag{16}$$

Substituting (11), (13), and (15) into (16), we have the consumption of the clean good in region i as:

$$c_x^i = u'\left(c_y^i\right)\left[(1-\zeta)f'\left(1/m^i\right) + \frac{(m^i)^2}{N^i}\sigma\left(1/m^i\right)n_y^i - c_y^i\right] \tag{17}$$

In the first step, the household's residential location can be determined by:

$$i = \arg\max_i U^i \tag{18}$$

3.2 Market Clearance

Denote region i's labor supply as N^i and recall that total labor supply is normalized to one (the total measure of consumers). The regional and overall labor market clearing conditions are thus:

$$\begin{aligned} N_x^i + N_y^i &= N^i && (19) \\ N^A + N^B &= 1 && (20) \end{aligned}$$

Since each household consumes exactly one unit of land and region i's land supply is given by $2D^i$, the land market clearing condition is simply:

$$N^i = 2D^i \tag{21}$$

which pins down the boundary of each city. Moreover, goods market clearing conditions are:

$$\sum_{i=A,B} N^i c_x^i = \sum_{i=A,B} \int_0^{k^i} x^i(j)dj = X^A + X^B \tag{22}$$

$$\sum_{i=A,B} N^i c_y^i = \sum_{i=A,B} \int_0^{m^i} y^i(j)dj = Y^A + Y^B \tag{23}$$

By symmetry, we have:

$$\begin{aligned} \sum_{i=A,B} N^i c_x^i &= \sum_{i=A,B} k^i x^i = X^A + X^B && (24) \\ \sum_{i=A,B} N^i c_y^i &= \sum_{i=A,B} m^i y^i = Y^A + Y^B && (25) \end{aligned}$$

where $m^A + m^B = M$.

Finally, if both locations are occupied, locational equilibrium requires:

$$U^A = U^B \tag{26}$$

4 Equilibrium Configuration

A *competitive spatial equilibrium* is a tuple of quantities, $\{n_x^i(j), n_y^i(j), N_x^i, N_y^i, N^i, k^i, m^i, c_x^i, c_y^i, x^i(j), y^i(j), Q^i\}$, and prices, $\{w^i, p\}$, such that: (i) all households and firms optimize; (ii) labor markets clear; (iii) goods markets clear; (iv) the population identity holds; and (v) the locational equilibrium condition is met.[5] Among all possible equilibrium configurations, we are particularly interested in

[5]The equilibrium concept is based on the multi-class equilibrium concept constructed by Hartwick, Schweizer and Varaiya (1976).

two equilibria: The first type is an *integrated equilibrium* where all clean and dirty firms are spread symmetrically over the two regions so that both types of firms are completely integrated locationally. The second type is a *stratified equilibrium* where all dirty manufacturing firms agglomerate in one region (without loss of generality, let it be region A) and all clean service firms are located in region B (where workers face better environmental conditions). In order to compare the endogenous variables obtained under the two types of equilibria, we shall use arguments I and S to denote integrated and stratified patterns, respectively.

4.1 Case I: Integrated Equilibrium

In an integrated equilibrium, both firms and households are symmetrically distributed across the two regions. Thus, we have:

$$N_x^A = N_x^B, N_y^A = N_y^B, N^A = N^B = \frac{1}{2}$$
$$k^i = k = \frac{1}{2}, m^i = m = \frac{M}{2},$$
$$n_x^i(j) = n_x = \frac{N_x}{k}, n_y^i(j) = n_y = \frac{N_y}{m}$$
$$n_x + Mn_y = 1$$

Moreover, wages must be equalized between the clean and the dirty sectors in each region. From (10) and (11), we can thus depict in Figure 1 the labor allocation between clean and dirty sectors under the integrated equilibrium.

Insert Figure 1 here

Figure 1 illustrates that dirty firm's labor demand, which is a downward-sloping function of n_y^i/N_y^i, is determined by wage equalization between the clean and the dirty sectors (see point E^{I}), namely where:

$$p(1-\zeta)MPL_y = p(1-\zeta)f'\left(2/M\right) = w = \psi \tag{27}$$

which determines the relative price of the dirty good as a decreasing function of the mass of dirty firms. The Inada conditions assumed are sufficient for the existence of an interior level of dirty industry employment and production.

Under symmetry, a dirty firm's output is now given by, $y = f\left(1/m^i\right) m^i n_y^i = \frac{M}{2} f\left(2/M\right) n_y$. From the dirty good market clearing condition, $c_y^i = c_y = My$, so we have:

$$c_y = M \cdot y = \frac{M^2}{2} f\left(2/M\right) n_y \tag{28}$$

This dirty good market clearing condition enables us to express the dirty good demand as a linear, upward-sloping function of the induced demand for labor starting from the origin, which is referred to as the *dirty good market-clearing* (DM) locus (see Figure 2). Moreover, we can combine (27) and (15), yielding the *dirty good optimization* (DO) locus:

$$u'(c_y) = \frac{\psi}{(1-\zeta)f'(2/M)} \tag{29}$$

Thus, the demand for the dirty good is independent of the induced demand for labor.

Insert Figure 2 here

As depicted in Figure 2, one can see that the integrated equilibrium quantity of the dirty good and employment are jointly determined at point E^{I}.

Clean good market clearance implies:

$$c_x = x = \psi n_x(I) = \psi[1 - M \cdot n_y(I)]$$

One may easily check that one of (8), (28) and the above equation are redundant, i.e., Walras' law is verified. Substituting the equilibrium $n_y(I)$ and (29) into (12), we have:

$$\pi(I) = \frac{M}{2}\frac{\psi}{(1-\zeta)f'(2/M)}\left[\sigma\,(2/M)\,n_y(I) - (2/M)\,\tau^i\right] \tag{30}$$

Finally, locational equilibrium (26) in this case is trivial. See Table 1 for a summary of the values of the endogenous variables at equilibrium.

4.2 Case II: Stratified Equilibrium

Now, we move to examine stratified equilibrium. At a stratified equilibrium, assume that the dirty firms agglomerate in region A, and the clean firms agglomerate in region B. Then stratified equilibrium is as shown in Table 1, and we have:

$$k^A = 0, k^B = 1, m^A = M, m^B = 0, \pi^B = z^B = 0$$

$$N^A = Mn_y, N^B = n_x, n_x + Mn_y = 1$$

Thus, we obtain the dirty good production for each dirty firm in region A as: $y = f\left(\frac{1}{m^A}\right) m^A n_y^A = Mf\,(1/M)\,n_y$. In this case, wages need not be equalized between the two regions: those residing in the dirty region receive a higher wage but suffer from pollution. The utility level of workers in the two regions is equal. The wages in the regions A and B are $w^A = p(1-\zeta)f'(1/M)$ and $w^B = \psi$, respectively. The dirty good market clearing condition implies:

$$c_y = My = M^2 f(1/M)\, n_y \tag{31}$$

which can be combined with (15) to yield:

$$p = u'\left[M^2 f(1/M)\, n_y\right] \tag{32}$$

By diminishing marginal utility, the above expression entails a negative relationship between dirty good price and employment (see the bottom panel of Figure 3). From the clean good market clearing condition, one obtains: $N^A c_x^A + N^B c_x^B = x = \psi n_x$, or, using (8) and Table 1,

$$c_x(S) = x = \psi n_x = \psi(1 - M n_y)$$

which can again be used with (8) and (31) to verify Walras' law.

Next, we can rewrite (12) under stratified equilibrium as:

$$\pi(S) = Mu'\left[f(1/M)\, M^2 n_y(S)\right]\left[\sigma(1/M)\, n_y(S) - (1/M)\,\tau^i\right] \tag{33}$$

The equilibrium level of pollution in region A is given by: $Q^A = \theta Y^A = \theta My = \theta M^2 f(1/M)\, n_y(S)$. We can derive the utility level attained by households residing in region A as:

$$U^A = Mf(1/M)\left\{u'\left[f(1/M)\, M^2 n_y(S)\right]\left[1 - Mn_y(S)\right] - \gamma\theta M n_y(S)\right\} + u\left[f(1/M)\, M^2 n_y(S)\right]$$

Since there are no dirty firms and thus no pollution in region B, in equilibrium there is no pollution tax revenue nor redistribution of dirty firm profits in region B. The utility level attained by a household residing in region B is:

$$U^B = \psi - f(1/M)\, M^2 n_y(S) u'\left[f(1/M)\, M^2 n_y(S)\right] + u\left[f(1/M)\, M^2 n_y(S)\right]$$

We can then compute the utility difference between regions A and B as:

$$\Delta U \equiv U^A - U^B = Mf(1/M)\left\{u'\left[f(1/M)\, M^2 n_y(S)\right] - \gamma\theta M n_y(S)\right\} - \psi \tag{34}$$

By employing (32) and (34), we determine the stratified equilibrium relative price p and dirty firm labor demand $n_y(S)$ as shown in Figure 3. Specifically, from the top panel of Figure 3, utility equalization pins down the equilibrium level of dirty industry employment under stratification, which can be plugged into the bottom panel to obtain the relative price of the dirty good.

Insert Figure 3 here

5 Characterization of Equilibrium

Before turning to each of the two specific pollution tax regimes, one may compare dirty sector employment, $n_y(I)$ and $n_y(S)$, under integrated and stratified equilibrium, respectively.

In an integrated equilibrium, we can use the dirty good market clearing condition and the dirty good demand, (28) and (29), to derive:

$$u'\left[\frac{M^2}{2}f\left(2/M\right)n_y\right] = \frac{\psi}{(1-\zeta)f'(2/M)} \tag{35}$$

In a stratified equilibrium, we can apply the location equilibrium condition in (34) to obtain:

$$\delta(n_y) \equiv u'\left[M^2 f\left(1/M\right)n_y\right] - \gamma\theta M n_y = \frac{\psi}{Mf\left(1/M\right)} \tag{36}$$

where $\delta(n_y)$ measures the household's net surplus from consuming the dirty good.

These equilibrium relationships can be referred to as the *dirty good market equilibrium* loci, DE(I) and DE(S), respectively, under integrated and stratified configurations (see Figure 4). Whereas the DE(I) locus yields the equilibrium $n_y(I)$ as shown in the top panel of Figure 4, the DE(S) locus pins down the equilibrium $n_y(S)$ as depicted in the bottom panel of Figure 4. In the top panel of Figure 4, the LHS of DE(I) yields a downward sloping locus as a result of diminishing marginal utility, whereas the RHS is simply a constant that is decreasing in the mass of dirty firms. Thus, the integrated equilibrium is pinned down at point E^I. In the bottom panel, the LHS of DE(S), $\delta(n_y)$, is also a downward sloping locus and the RHS a constant depending negatively on the mass of dirty firms. These loci determine the stratified equilibrium at point E^S. To establish nice sufficient conditions for stratification in the next two subsections, we shall restrict our attention to a plausible scenario with $n_y(I) < n_y(S)$, i.e., dirty industry employment under integration is lower than that under stratification. It is clear from the definition of $\delta(n_y)$ that the above scenario is more likely to arise the smaller $\gamma\theta$ is.

Insert Figure 4 here

5.1 Fixed Pollution Tax Regime

We examine under what conditions the stratified equilibrium emerges under the fixed pollution tax regime but the integrated equilibrium does not, where the pollution tax levied by the local government under the two different configurations is given by:

$$\tau^i = \begin{cases} 2F/M, & \text{for Case } I \\ F/M, & \text{for Case } S \end{cases}$$

For purposes of comparison, in the stratified case only one local government raises pollution tax revenue, whereas in the integrated case each local government raises the same revenue as the dirty city in the stratified case. One interpretation of this assumption is that the simple presence of pollution in a city is enough to trigger a tax.

We impose a regularity condition on the dirty firm's surplus from uncompensated spillovers:

Condition R-1: (Regularity Condition on a Dirty Firm's Surplus)

$$\frac{1}{4}\widetilde{\sigma}\left(2/M\right) < \widetilde{\sigma}\left(1/M\right)$$

Under Condition R-1, we then consider the following:

Condition S-1: (Sufficient Condition for Stratification Under a Fixed Tax)

$$\frac{1}{4}\widetilde{\sigma}\left(2/M\right) < \frac{F}{M^2 n_y(S)} < \widetilde{\sigma}\left(1/M\right)$$

We can then establish:

Theorem 1: (Stratified Equilibrium) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small. Under Condition R-1, we suppose that the fixed pollution tax is moderate so that the inequalities in Condition S-1 are met. Then the stratified configuration arises as an equilibrium outcome, but the integrated configuration does not.*

Proof. The proofs of all the theorems and propositions are relegated to the Appendix. ■

Thus, under Condition R-1, Condition S-1 is sufficient to ensure that the stratified configuration is an equilibrium outcome, but the integrated configuration is not. Intuitively, the first inequality of Condition S-1 implies negative profit received by dirty firms under integration, whereas the second inequality guarantees positive profit obtained by dirty firms under stratification. The main tipping point here is the gains from clustering under the fixed pollution tax regime.

Remark 1: (Impossibility of Integrated Equilibrium) It is not difficult to show that when F is large enough to satisfy $F > \frac{M}{4}\sigma\left(2/M\right)$, then dirty firms always incur negative profit, implying that an integrated configuration can never arise in equilibrium.

Remark 2: (On the Role of Agglomerative Externalities) It is important to note that despite the agglomeration force from uncompensated spillovers, the key driving force for all dirty firms to cluster in one region (A) is the presence of a fixed pollution tax that is independent of an individual firm's output. Specifically, with $F = 0$, it is clear that $\pi(I) > 0$, implying that the integrated configuration always arises in equilibrium. Moreover, we can compute the profits under the two configurations as follows:

$$\begin{aligned}\pi(I) &= \frac{M}{2}\frac{\psi}{f'(2/M)}\widetilde{\sigma}\,(2/M)\,n_y(I)\\ \pi(S) &= Mu'\left[M^2 f\,(1/M)\,n_y(S)\right]\widetilde{\sigma}\,(1/M)\,n_y(S)\end{aligned}$$

Further, assume that $\frac{1}{2}\widetilde{\sigma}\,(2/M) > \widetilde{\sigma}\,(1/M)$. Then, dirty firms will incur higher profit under integrated equilibrium compared to stratified equilibrium when the following inequality is met:

$$\frac{\frac{1}{2}\widetilde{\sigma}\,(2/M)}{\widetilde{\sigma}\,(1/M)} > \frac{u'\left[M^2 f\,(1/M)\,n_y(S)\right]n_y(S)}{\frac{\psi}{f'(2/M)}n_y(I)}$$

Refer to the top panel of Figure 4. The ratio on the right-hand side of the above inequality is measured by the ratio of the lightly shaded area covering E^O to the shaded area covering E^I. As long as this ratio is less than $\frac{\frac{1}{2}\widetilde{\sigma}(2/M)}{\widetilde{\sigma}(1/M)}$ (which is greater than one under the additional condition stated above), it is sufficient to ensure that dirty firms will fully integrate with clean ones.

5.2 Linear Pollution Tax Regime

Under the linear pollution tax regime, $\zeta = t$ and

$$g = \begin{cases} L + \frac{tM}{2}f(\frac{2}{M})n_y(I), \text{ in integrated equilibrium} \\ L + tMf(\frac{1}{M})n_y(S), \text{ for stratified equilibrium}\end{cases}$$

We impose a stronger regularity condition on the dirty firm's surplus from uncompensated spillovers:

Condition R-2: (Regularity Condition on a Dirty Firm's Surplus)

$$\frac{1}{2}\widetilde{\sigma}\,(2/M) < \widetilde{\sigma}\,(1/M)$$

Under Condition R-2, we further consider the following condition:

Condition S-2: (Sufficient Condition for Stratification Under Linear Tax)

$$\frac{1}{2}\widetilde{\sigma}\,(2/M) < \frac{L}{(1-t)Mn_y(S)} < \widetilde{\sigma}\,(1/M)$$

This ensures:

Theorem 2: (Stratified Equilibrium) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small. Under*

Condition R-2, we suppose that the lump-sum component of the linear pollution tax is moderate and the marginal tax rate is not too high so that the inequalities in Condition S-2 are met. Then the stratified configuration arises as an equilibrium outcome but the integrated configuration does not.

In Section 5 below, we shall verify that both the presence of pollution and the presence of a fixed tax are crucial for a stratified equilibrium with all dirty firms clustering in one location to arise.

6 The Case with Specific Functional Forms

Under the fixed pollution tax regime, we are left to check whether the stratified equilibrium is stable. Due to the difficulty of examining stability in the general setting, we shall conduct our analysis under specific functional forms for the dirty good production technology and the subutility for the dirty good. Specifically, we assume that $\widetilde{f}$ and u both take simple Cobb-Douglas forms:

$$\begin{aligned}\widetilde{f}\left(n_y^i(j), N_y^i\right) &= \phi[n_y^i(j)]^{\beta}[N_y^i]^{1-\beta}, \phi > 0 \text{ and } \beta \in (0,1) \\ u\left(c_y\right) &= \eta\left(c_y\right)^{\alpha}, \eta > 0 \text{ and } \alpha \in (0,1)\end{aligned}$$

Before deriving the stability condition, it is useful to provide explicit conditions in this special case under which the stratified configuration is an equilibrium outcome but the integrated configuration is not.

6.1 Fixed Pollution Tax Regime

Under the fixed pollution tax regime with the specific functional forms, we can derive a sufficient condition to ensure existence of a stratified equilibrium as follows:

Condition S-1′: (Stratified Equilibrium)

$$\beta\left[1 + \frac{\gamma\theta F}{(1-\beta)\phi\psi M^{1-\beta}}\right] < \frac{\alpha\beta\eta\phi}{\psi} M^{1-\beta}\left(\frac{1-\beta}{F}\right)^{1-\alpha} < 2^{2-\alpha-\beta}$$

We can establish:

Proposition 1: (Stratified Equilibrium under Fixed Pollution Tax) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small, and Condition R-1 is met. Then, under a fixed pollution tax regime with Condition S-1′, a stratified competitive spatial equilibrium emerges.*

We are now ready to check whether the stratified equilibrium is stable. Informally, stability is defined using small perturbations of firms from one region to the other, checking to see whether or not they would return to their equilibrium region.

Consider,

Condition I: (Instability without Pollution Tax)

$$\psi + \gamma\theta \left[\phi(\frac{\alpha\eta\beta}{\psi})^{\beta} M^{1-\beta}\right]^{\frac{1}{1-\alpha\beta}} > \left\{\psi^{\beta(1-\alpha)}(\alpha\eta\beta\phi^{\alpha})^{1-\beta} M^{\alpha[2-\beta(2-\beta)]}\right\}^{\frac{1}{1-\alpha\beta}}$$

We can then obtain:

Proposition 2: (Instability of Stratified Equilibrium) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small, and Condition R-1 is met. Then, under Condition I, a stratified competitive spatial equilibrium is unstable in the absence of the pollution tax.*

Remark 3: (On Pollution vs. Corporate Tax) One may inquire whether our analysis applies to general corporate taxation. First, thinking of τ as a corporate tax in an economy without pollution concerns is not economically sensible, since it is not a tax on profits. Second, even if we ignore economic considerations, should $\gamma\theta = 0$, Conditions S and I would contradict each other if

$$M^{\frac{2\alpha-(1-\alpha)\beta^2}{1-\beta}} > \left[\frac{(1-\beta)\phi}{F}\right]^{1-\alpha}$$

That is, should the above inequality be met, pollution concerns are crucial for supporting the stratified equilibrium as a stable equilibrium configuration. Third, Condition S-1′ (particularly the second inequality) cannot hold when there is no fixed pollution tax ($F = 0$). In summary, we have shown that pollution and a fixed tax are crucial for a stable stratified equilibrium to arise with all dirty firms clustered at one location.[6]

Remark 4: (Equilibrium Classification and Bifurcation Diagram) It is possible to delineate numerically a diagram in (M, F), namely the exogenous measure of dirty firms and the exogenous fixed cost tax revenue, that shows how changes in the values of (M, F) result in different types of equilibria, i.e. integrated versus stratified. Specifically, we set $\alpha = \beta = \gamma = 0.5$, $\phi = \eta = 1$, $\psi = 0.1$ and $\theta = 0.02$. We can then vary the values of each of M and F from 0 to 30. As shown

[6] If there is no tax and both industries have (different) CRS production functions (with no Romer externality) but there is still pollution, then an integrated equilibrium will arise with both wage and utility equalized but with a corner solution in consumption (only the dirty good is consumed).

in Figure 5(a), stratification is more profitable than integration for lower values of M and higher values of F: the indifference boundary between the two configurations is given by $\widetilde{BEC}$. Of course, a configuration can be supported only under positive profit, which is met for the area under $\widetilde{AES}$ in the case of stratification and for the area under $\widetilde{OEI}$ in the case of integration. Thus, a stratified equilibrium arises in the area of $OAEB$ (shaded with horizontal lines) whereas an integrated equilibrium emerges in the area of $BEID$ (shaded with vertical lines). We now set $F = 5$ and vary the measure of dirty firms, M. As long as $M > 1.60$, an equilibrium exists where firms earn sufficient profits to pay for the pollution tax. Over the range $M \in (1.60, 12.42)$, the equilibrium configuration is stratified and the fraction of dirty firms in city A is one. As firms continues to enter, the equilibrium configuration becomes integrated and the fraction of dirty firms in city A drops to $\frac{1}{2}$. This is depicted in the bifurcation diagram, Figure 5(b).[7]

Insert Figure 5(a,b) here

6.2 Linear Pollution Tax Regime

We turn next to examining the case of a linear pollution tax. Consider,

Condition S-2′: (Stratified Equilibrium)

$$\beta(1-t) + \frac{\beta\gamma\theta ML}{\phi\psi(1-\beta)} < \frac{\alpha\beta\eta\phi M^{\alpha-\beta}(1-t)^{2-\alpha}}{\psi}(\frac{1-\beta}{L})^{1-\alpha} < 2^{1-\beta}$$

We now have:

Proposition 3: (Stratified Equilibrium under Linear Pollution Tax) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small, and Condition R-2 is met. Then, under a linear pollution tax regime with Condition S-2′, a stratified competitive spatial equilibrium emerges.*

7 Can Pareto Optimum Be Price Supported?

Since individuals are *ex ante* identical, we restrict our attention to within-region-equal-treatment Pareto optimum in the sense that all households within a given region reach an identical indirect utility level and consume the same bundle. Such a Pareto optimum must satisfy the following

[7] If one allows the location of dirty city to be in either A or B, then a standard fork bifurcation diagram is obtained.

constraints:

$$\begin{aligned} x^i(j) &= \psi n_x^i(j), \quad i = A, B, j \in [0, k^i] \\ y^i(j) &= N_y^i f\left(\frac{n_y^i(j)}{N_y^i}\right), \quad i \in A, B, j \in [0, m^i] \end{aligned}$$

$$N_x^i = \int_0^{k^i} n_x^i(j)dj, \quad N_y^i = \int_0^{m^i} n_y^i(j)dj, \quad N_x^A + N_x^B + N_y^A + N_y^B = 1$$

$$\sum_{i=A,B} N^i c_x^i = \sum_{i=A,B} \int_0^{k^i} x^i(j)dj, \quad \sum_{i=A,B} N^i c_y^i = \sum_{i=A,B} \int_0^{m^i} y^i(j)dj$$

where the first two specify production technologies, the third gives labor material balance and the population identity, and the last represents commodity material balance. Such Pareto optima are found by solving the following optimization problem:

$$\begin{aligned} \max U^A &= c_x^A - \gamma\theta \int_0^{m^A} y^A(j)dj + u(c_y^A) \\ \text{s.t. } U^B &= c_x^B - \gamma\theta \int_0^{m^B} y^B(j)dj + u(c_y^B) = \overline{U} \\ &\quad \text{and the above technology and material balance constraints.} \end{aligned}$$

We consider equilibria with linear taxes in the next two subsections.

7.1 Case I: Integrated Optimum

In an integrated optimum, we have all interior allocations. We can establish:

Theorem 3: (Equilibrium Support of Integrated Configuration) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently small, and Condition R-2 is met. Then, the Pareto optimum with an integrated configuration can be supported by a competitive spatial equilibrium under the following marginal tax rate:* $\zeta = \{1 + 2\psi / [\gamma\theta f'(2/M)]\}^{-1}$.

Intuitively, the higher the pollution damage (captured by larger $\gamma\theta$) is, the greater the marginal pollution tax will be.

7.2 Case II: Stratified Optimum

In stratified optimum, we have: $k^A = m^B = 0$. We can establish:

Theorem 4: (Suboptimality of Stratified Equilibrium) *Consider a local economy in which pollution production is not too severe and pollution disutility is not too high, in other words $\gamma\theta$ is sufficiently* *small, and Condition R-2 is met. Then, a stratified competitive spatial equilibrium is suboptimal with over-employment and over-production in the dirty goods sector relative to the stratified Pareto optimum.*

Thus, a stratified equilibrium can never reach Pareto optimality by means of a linear pollution tax (which encompasses Pigouvian taxation). In fact, the equilibrium employment in the dirty sector under the stratified configuration is always too large, implying that dirty goods and pollution are both over-produced. Such an over-polluting equilibrium outcome can never be corrected by a linear pollution tax.

To understand the result, it is best to refer to Figure 6, where we plot the downward-sloping after-tax MPL locus in the top panel and repeat the locational equilibrium diagram (the top panel of Figure 3) in the bottom panel of Figure 6. A high marginal tax will shift down the after-tax MPL locus without altering any other curves. Thus, the only change is the corresponding reduction in the dirty industry wage, w^A. As long as $w^A > \psi$ still holds after the tax increase, the lower wage will be fully offset by the tax and profit redistribution, keeping consumers in region A as well off as before the tax increase. This is equivalent to saying that although dirty good demand is elastic, dirty good supply is perfectly inelastic. As a result, dirty good employment and production in stratified equilibrium remain at levels higher than the respective optimum quantities, regardless of the linear pollution tax levied.

Insert Figure 6 here

In the conventional literature, Pigouvian taxes (a special form of a linear tax without the lump-sum component) need not work in practice due to the difficulty of computing marginal damages at the optimum (Baumol 1972), or when firms have monopoly power so that they can transfer the tax burden (Buchanan and Tullock, 1975), or when oligopolistic firms have dynamic strategic interactions (Benchekroun and Van Long 1998), or when lobbying groups care about the distribution of income in political games (Aidt 1998)). In our paper, assuming away all of these issues, we show that even a generalized Pigouvian tax as proposed by Carlton and Loury (1980) cannot restore first best under a static, competitive environment, when we allow locational choice with endogenous stratification.

Whereas the linear pollution tax cannot correct equilibrium inefficiency, it should be noted that an appropriate redistribution scheme may do the job. In particular, consider a lump-sum redistribution from polluted region A to clean region B. This induces ΔU to shift down and hence equilibrium employment in the dirty industry to fall. Thus, as long as $\gamma\theta$ is not too large, there exists an appropriate level of such a redistribution to support the Pareto optimal level of dirty industry employment as an equilibrium.

8 Concluding Remarks

This paper has established the interesting proposition that taxing pollution with a fixed component independent of dirty good output can cause firm agglomeration. The key argument is as follows. At a symmetric, integrated equilibrium, wages equalize both across sectors and locations. Then, in the presence of a fixed total pollution damage payment in each polluted region, dirty factories may not have sufficient profitability to pay the tax and thus no integrated equilibrium exists. Now if dirty firms cluster, as they do in a stratified equilibrium, then they share the fixed pollution tax in the one region where they cluster. Moreover, wage equalization between the two locations is no longer required in equilibrium because clean and dirty firms are in two different locations, so there is no wage equalization even across sectors. All we need is utility equalization, which only requires that pollution disutility balance with the wage differential. This is consistent with firm profitability under stratification.

We have also established that whereas an integrated Pareto optimum can be supported by a competitive spatial equilibrium with a linear pollution tax, a stratified Pareto optimum cannot. Regardless of the linear pollution tax schedule, a stratified equilibrium is always over-polluted compared to the optimum. To support the stratified Pareto optimum, however, an effective (but practically not implementable) policy prescription is to redistribute the pollution tax revenue from the dirty to the clean city residents. Such a policy will induce migration to the clean city, thereby reducing production of the dirty good and thus of pollution.

In this paper, we have considered only equilibrium configurations that are completely stratified in terms of production or that are completely integrated in that production is symmetric across locations. One may inquire whether other configurations may emerge in equilibrium. The answer is positive: it is possible that one city is mixed with both clean and dirty industries present, whereas another has only the clean industry. In this configuration, clean industry workers must have equal utility across locations and all workers must have the same wage in the city with mixed industries. Under the Ricardian technology where clean workers are paid an exogenously fixed wage, the two equalization conditions can be met only in knife-edge cases. It is therefore innocuous to ignore this partially integrated configuration.

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Appendix

Proof of Theorem 1: A condition sufficient to show that the stratified configuration is an equilibrium but the integrated configuration is not is:

$$\pi(I) < 0 < \pi(S)$$

That is, the dirty firms only operate under stratification. From (30) and (33), in turn, we need the following inequality condition:

$$\frac{1}{4}\widetilde{\sigma}\,(2/M)\,n_y(I) < \frac{F}{M^2} < \widetilde{\sigma}\,(1/M)\,n_y(S)$$

When $n_y(I) < n_y(S)$, the above inequality holds under Condition S-1, which can be met only under Condition R-1. Since $n_y(I)$ and $n_y(S)$ are endogenous, we must further investigate their magnitudes in order to establish precise sufficient conditions on primitives. This can be accomplished utilizing Figure 4, by comparing the positions of point E^I and point E^S. We can see from the top panel of Figure 4 that, as long as $\gamma\theta$ is not too large, we can have $n_y(I) < n_y(S)$ (as shown in the bottom panel of Figure 4). Given this and Condition R-1, we can always choose F to satisfy Condition S-1, which subsequently ensures the existence of a stratified configuration but not the integrated configuration as an equilibrium outcome, as illustrated diagrammatically where $n_y(S)$ is pinned down by Figure 4 with Conditions R-1 and S-1 being met as in Figure 7a. ■

Proof of Theorem 2: In this case, the profits generated by each dirty firm under integrated and stratified configurations become:

$$\begin{aligned}
\pi(I) &= \frac{M}{2}\frac{\psi}{(1-t)f'(\frac{2}{M})}[(1-t)\widetilde{\sigma}(\frac{2}{M})n_y(I) - \frac{2}{M}L] \\
\pi(S) &= Mu'[f(\frac{1}{M})M^2 n_y(S]][(1-t)\widetilde{\sigma}(\frac{1}{M})n_y(S) - \frac{1}{M}L]
\end{aligned}$$

Similar to the fixed tax case, here is a sufficient condition to ensure that the stratified configuration is an equilibrium but the integrated configuration is not:

$$\pi(I) < 0 < \pi(S)$$

which can be rewritten as the following inequalities:

$$\frac{1}{2}\widetilde{\sigma}\,(2/M)\,n_y(I) < \frac{L}{(1-t)M} < \widetilde{\sigma}\,(1/M)\,n_y(S)$$

Under Condition R-2, as shown in Figure 7b and the circumstances delineated by Figure 4, $n_y(I) < n_y(S)$ and thus Condition S-2 is sufficient to ensure the inequalities above. ■

Proof of Proposition 1: With specific functional forms, dirty firm employment in integrated equilibrium can be solved explicitly:

$$n_y(I) = \frac{1}{2}\left[\frac{\alpha\eta\beta\phi^{\alpha}}{\psi(M/2)^{1-\alpha(2-\beta)}}\right]^{\frac{1}{1-\alpha}}$$

whereas dirty firm employment in stratified equilibrium must satisfy:

$$\frac{\alpha\eta\phi^{\alpha}}{M^{1-\alpha(2-\beta)}[n_y(S)]^{1-\alpha}} = \psi + \gamma\theta\phi M^{2-\beta} n_y(S)$$

Substituting these into Condition S-1 gives the result. ■

Proof of Proposition 2: Suppose the stratified equilibrium is not stable. We have deviation of dirty firms of positive measure ε moving from A to B, receiving joint profit given by:

$$\pi_{\varepsilon} = p\phi(\tilde{n}_{y_{\varepsilon}})^{\beta}\varepsilon^{1-\beta} - \psi\tilde{n}_{y_{\varepsilon}} - p\tilde{\tau}$$

From the clean and dirty firms' first-order conditions for profit optimization, we have:

$$\tilde{n}_{y_{\varepsilon}} = \varepsilon(\frac{\beta\phi p}{\psi})^{\frac{1}{1-\beta}}$$

Combining these expressions, we obtain:

$$\pi_{\varepsilon} = \psi(\frac{1-\beta}{\beta})\varepsilon(\frac{\beta\phi p}{\psi})^{\frac{1}{1-\beta}} - p\tilde{\tau}$$

Thus, the per deviating firm profit can be computed as follows:

$$\tilde{\pi} = \frac{\pi_{\varepsilon}}{\varepsilon} = \psi(\frac{1-\beta}{\beta})(\frac{\beta\phi p}{\psi})^{\frac{1}{1-\beta}} - p\frac{F}{\varepsilon}$$

Recall that the profit of a firm that doesn't deviate is:

$$\pi^{A} = p\left[(1-\beta)\phi M^{1-\beta} n_y - \frac{F}{M}\right]$$

To ensure stability, we therefore need: $\lim_{\varepsilon\to 0}\tilde{\pi} < \pi^{A}$, which holds trivially as $\lim_{\varepsilon\to 0}\tilde{\pi} = -\infty$. It remains to check that the Romer positive externality alone cannot lead to stable dirty firm agglomeration. This is equivalent to showing that, with $F = 0$,

$$\tilde{\pi} > \pi^{A}$$

which requires:

$$n_y < \left[\frac{\alpha\eta\beta\phi^{\alpha}}{\psi M^{\frac{1-\alpha\beta(2-\beta)}{\beta}}}\right]^{\frac{\beta}{1-\alpha\beta}}$$

Using (36), we can rewrite the inequality above in primitives, yielding Condition I. ■

Proof of Proposition 3: Under a linear pollution tax, it is easily verified that the sufficient condition S-2 becomes Condition S-2$'$. ■

Proof of Theorem 3: Upon substituting out the production technologies, this problem can be solved by setting up the Langrangian as follows:

$$\begin{aligned}
\mathcal{L} &= c_x^A - \gamma \cdot \theta \int_0^{m^A} N_y^A f\left(\frac{n_y^A(j)}{N_y^A}\right) dj + u(c_y^A) \\
&+ \lambda_U [c_x^B - \gamma\theta \int_0^{m^B} N_y^B f\left(\frac{n_y^B(j)}{N_y^B}\right) dj + u(c_y^B) - \overline{U}] \\
&+ \lambda_N \left[1 - \int_0^{k^A} n_x^A(j)dj - \int_0^{k^B} n_x^B(j)dj - \int_0^{m^A} n_y^A(j)dj - \int_0^{m^B} n_y^B(j)dj\right] \\
&+ \lambda_X \sum_{i=A,B} \left[\int_0^{k^i} \psi n_x^i(j)dj - \left(\int_0^{k^i} n_x^i(j)dj + \int_0^{m^i} n_y^i(j)dj\right) c_x^i\right] \\
&+ \lambda_Y \sum_{i=A,B} \left[\int_0^{m^i} N_y^i f\left(\frac{n_y^i(j)}{N_y^i}\right) dj - \left(\int_0^{k^i} n_x^i(j)dj + \int_0^{m^i} n_y^i(j)dj\right) c_y^i\right]
\end{aligned}$$

where λ_U, λ_N, λ_X and λ_Y are Lagrange multipliers associated with the utility constraint and labor and goods market clearing constraints, respectively. Since the measures k^A and m^B are zero under a stratified configuration, we must derive Pareto optimum under each configuration separately.

The first-order conditions with respect to the 4 consumption and the 4 labor variables are given by:

$$\frac{\partial \mathcal{L}}{\partial c_x^A} = 1 - \lambda_X \left(\int_0^{k^A} n_x^A(j)dj + \int_0^{m^A} n_y^A(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_x^B} = \lambda_U - \lambda_X \left(\int_0^{k^B} n_x^B(j)dj + \int_0^{m^B} n_y^B(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_y^A} = u'(c_y^A) - \lambda_Y \left(\int_0^{k^A} n_x^A(j)dj + \int_0^{m^A} n_y^A(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_y^B} = \lambda_U u'(c_y^B) - \lambda_Y \left(\int_0^{k^B} n_x^B(j)dj + \int_0^{mB} n_y^B(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_x^A(i)} = -\lambda_N + \lambda_X(\psi - c_x^A) - \lambda_Y c_y^A = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_x^B(i)} = -\lambda_N + \lambda_X(\psi - c_x^B) - \lambda_Y c_y^B = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_y^A(i)} = -\gamma\theta f'\left(\frac{n_y^A(j)}{N_y^A}\right) - \lambda_N - \lambda_X c_x^A + \lambda_Y \left[f'\left(\frac{n_y^A(j)}{N_y^A}\right) - c_y^A\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_y^B(i)} = -\lambda_U\gamma\theta f'\left(\frac{n_y^B(j)}{N_y^B}\right) - \lambda_N - \lambda_X c_x^B + \lambda_Y \left[f'\left(\frac{n_y^B(j)}{N_y^B}\right) - c_y^B\right] = 0$$

Straightforward manipulation and simplification yields:

$$u'(c_y^A) = u'(c_y^B) = \frac{\lambda_Y}{\lambda_X} \tag{A1}$$

$$c_x^B - c_x^A = u'(c_y^A)c_y^A - u'(c_y^B)c_y^B \tag{A2}$$

$$\left\{[u'(c_y^A) - \gamma\theta N^A]f'\left(\frac{n_y^A}{N_y^A}\right) - \psi\right\} = n_y^B\left\{[u'(c_y^B) - \gamma\theta N^B]f'\left(\frac{n_y^B}{N_y^B}\right) - \psi\right\} = 0 \tag{A3}$$

$$1 = k^A n_x^A + k^B n_x^B + m^A n_y^A + m^B n_y^B \tag{A4}$$

$$\psi(k^A n_x^A + k^B n_x^B) = (k^A n_x^A + m^A n_y^A)c_x^A + (k^B n_x^B + m^B n_y^B)c_x^B \tag{A5}$$

$$m^A N_y^A f\left(\frac{n_y^A}{N_y^A}\right) + m^B N_y^B f\left(\frac{n_y^B}{N_y^B}\right) = (k^A n_x^A + m^A n_y^A)c_y^A + (k^B n_x^B + m^B n_y^B)c_y^B \tag{A6}$$

Under an integrated configuration, we have: $N^A = N^B = \frac{1}{2}$, $n_y^A = n_y^B = n_y$ and $n_x = 1 - Mn_y$. From (A1) and (A2), we must have the same consumption bundles across the two locations. Using (A4) and (A5) then yields:

$$c_x = \psi(1 - Mn_y) \tag{A7}$$

Combining (A4) and (A6), one obtains:

$$c_y = 2(\frac{M}{2})^2 f(\frac{2}{M})n_y \tag{A8}$$

Both consumptions are identical to the equilibrium ones. Substituting (A8) into (A3) implies:

$$\left[u'\left(2(\frac{M}{2})^2 f(\frac{2}{M})n_y\right) - \frac{1}{2}\gamma\theta\right] = \frac{\psi}{f'(\frac{2}{M})} \tag{A9}$$

By setting n_y in the equilibrium captured by (35) and in the Pareto optimum captured by (A9) equal to one another, one obtains:

$$\frac{\psi}{f'(\frac{2}{M})} + \frac{1}{2}\gamma\theta = \frac{\psi}{(1-\zeta)f'(\frac{2}{M})}$$

which can be manipulated to derive the marginal tax rate given in the statement of the theorem. ■

Proof of Theorem 4: At a stratified optimum, we have: $k^A = m^B = 0$, together with the following 6 first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_x^A} = 1 - \lambda_X\left(\int_0^{k^A} n_x^A(j)dj + \int_0^{m^A} n_y^A(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_x^B} = \lambda_U - \lambda_X\left(\int_0^{k^B} n_x^B(j)dj + \int_0^{m^B} n_y^B(j)dj\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_y^A} = u'(c_y^A) - \lambda_Y \left(\int_0^{k^A} n_x^A(j)dj + \int_0^{m^A} n_y^A(j)dj \right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_y^B} = \lambda_U u'(c_y^B) - \lambda_Y \left(\int_0^{k^B} n_x^B(j)dj + \int_0^{mB} n_y^B(j)dj \right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_x^B(i)} = -\lambda_N + \lambda_X(\psi - c_x^B) - \lambda_Y c_y^B = 0$$

$$\frac{\partial \mathcal{L}}{\partial n_y^A(i)} = -\gamma\theta f'\left(\frac{n_y^A(j)}{N_y^A}\right) - \lambda_N - \lambda_X c_x^A + \lambda_Y \left[f'\left(\frac{n_y^A(j)}{N_y^A}\right) - c_y^A \right] = 0$$

in conjunction with two corner labor allocations: $n_x^A(i) = n_y^B(i) = 0$. Manipulations similar to those in the proof of Theorem 3 above give (A1) and (A2) – so consumption bundles must still be the same across the two locations – together with:

$$[u'(c_y^A) - \gamma\theta N^A] f'\left(\frac{n_y^A}{N_y^A}\right) - \psi = 0 \tag{A10}$$

$$1 = k^B n_x^B + m^A n_y^A \tag{A11}$$

$$\psi k^B n_x^B = m^A n_y^A c_x^A + k^B n_x^B c_x^B \tag{A12}$$

$$m^A N_y^A f\left(\frac{n_y^A}{N_y^A}\right) = m^A n_y^A c_y^A + k^B n_x^B c_y^B \tag{A13}$$

Under a stratified configuration, we have: $n_x^A = n_y^B = 0$, $n_y^A = n_y$, $N^A = Mn_y^A$, $N^B = 1 - Mn_y^A$, and $n_x^B = n_x = 1 - Mn_y$. Whereas clean good consumption still takes the same form as in (A7), (A4) and (A6) together give:

$$c_y = M^2 f(\frac{1}{M}) n_y \tag{A14}$$

implying again that both Pareto optimal consumption bundles are identical to the equilibrium ones. From (A10) and (A14), we have:

$$\left[u'\left(M^2 f(\frac{1}{M}) n_y \right) - \gamma\theta M n_y \right] = \frac{\psi}{f'(\frac{1}{M})} \tag{A15}$$

Since n_y in a stratified equilibrium is determined by (36), and since $\widetilde{\sigma}(\frac{1}{M}) = f(\frac{1}{M}) - \frac{1}{M} f'(\frac{1}{M}) > 0$, we can see that:

$$\frac{\psi}{M f(\frac{1}{M})} < \frac{\psi}{f'(\frac{1}{M})}$$

This implies that n_y in a stratified equilibrium exceeds the Pareto optimal level, which completes the proof. ■

Table 1. Population Accounting

Integrated	City A		City B		Aggregate	
	Firms	Workers	Firms	Workers	Firms	Workers
Clean Sector	$k^A = \frac{1}{2}$	$N_x^A = \frac{1}{2} n_x$	$k^B = \frac{1}{2}$	$N_x^B = \frac{1}{2} n_x$	1	n_x
Dirty Sector	$m^A = \frac{M}{2}$	$N_y^A = \frac{M}{2} n_y$	$m^B = \frac{M}{2}$	$N_y^B = \frac{M}{2} n_y$	M	$N_y = Mn_y$
Total	$\frac{1}{2}(1+M)$	$\frac{1}{2}$	$\frac{1}{2}(1+M)$	$\frac{1}{2}$	$1+M$	1

Stratified	City A (Dirty)		City B (Clean)		Aggregate	
	Firms	Workers	Firms	Workers	Firms	Workers
Clean Sector	0	0	1	$N_x = n_x$	1	$N_x = n_x$
Dirty Sector	M	$N_y = Mn_y$	0	0	M	$N_y = Mn_y$
Total	M	Mn_y	1	$N_x = n_x$	$1+M$	1

Figure 1. Labor Allocation Under Integrated Equilibrium

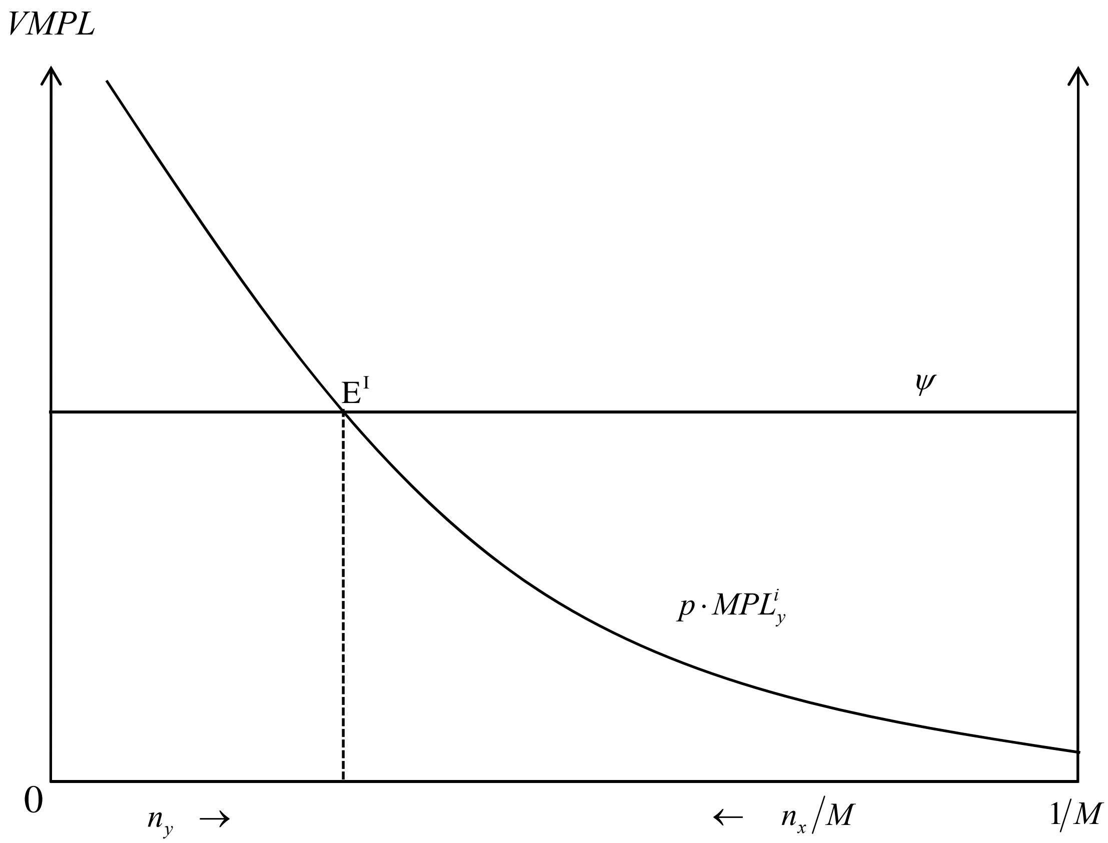


Figure 2. Dirty Good Equilibrium under Integrated Configuration

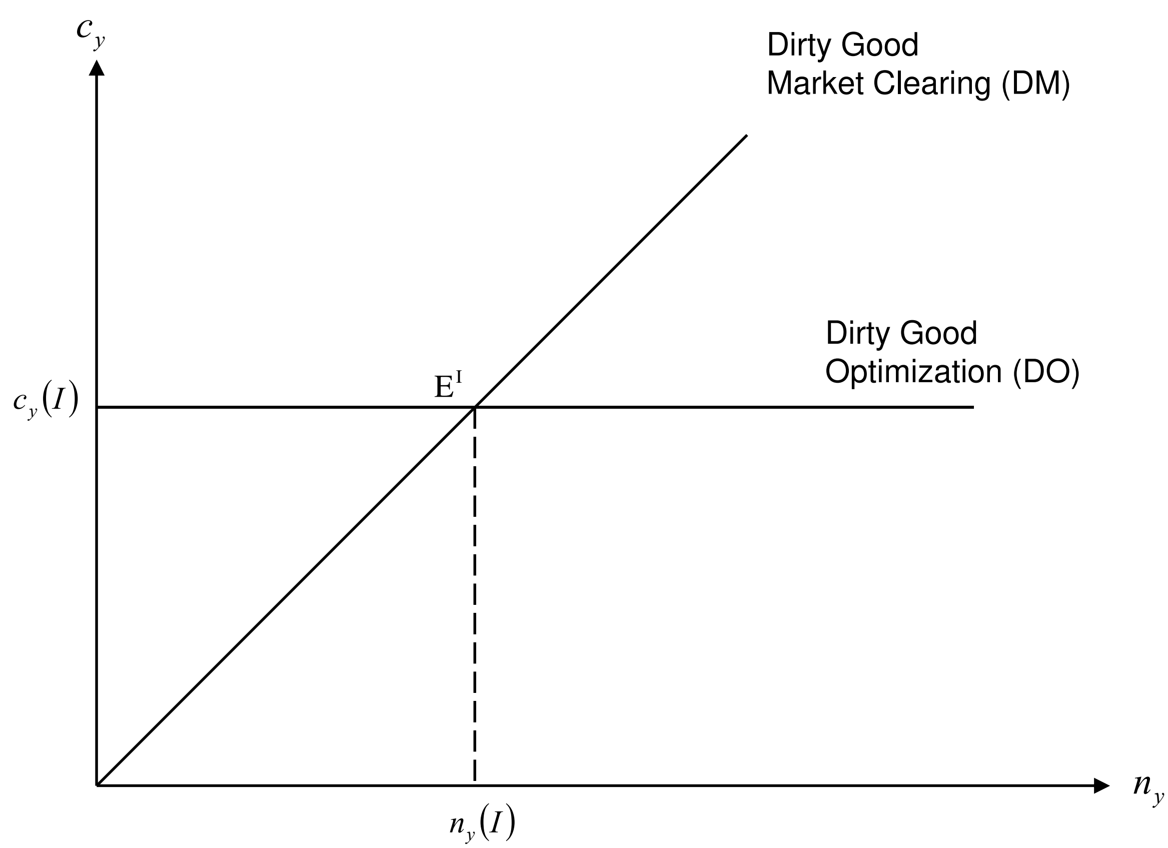


Figure 3. Locational Equilibrium Under Stratified Configuration

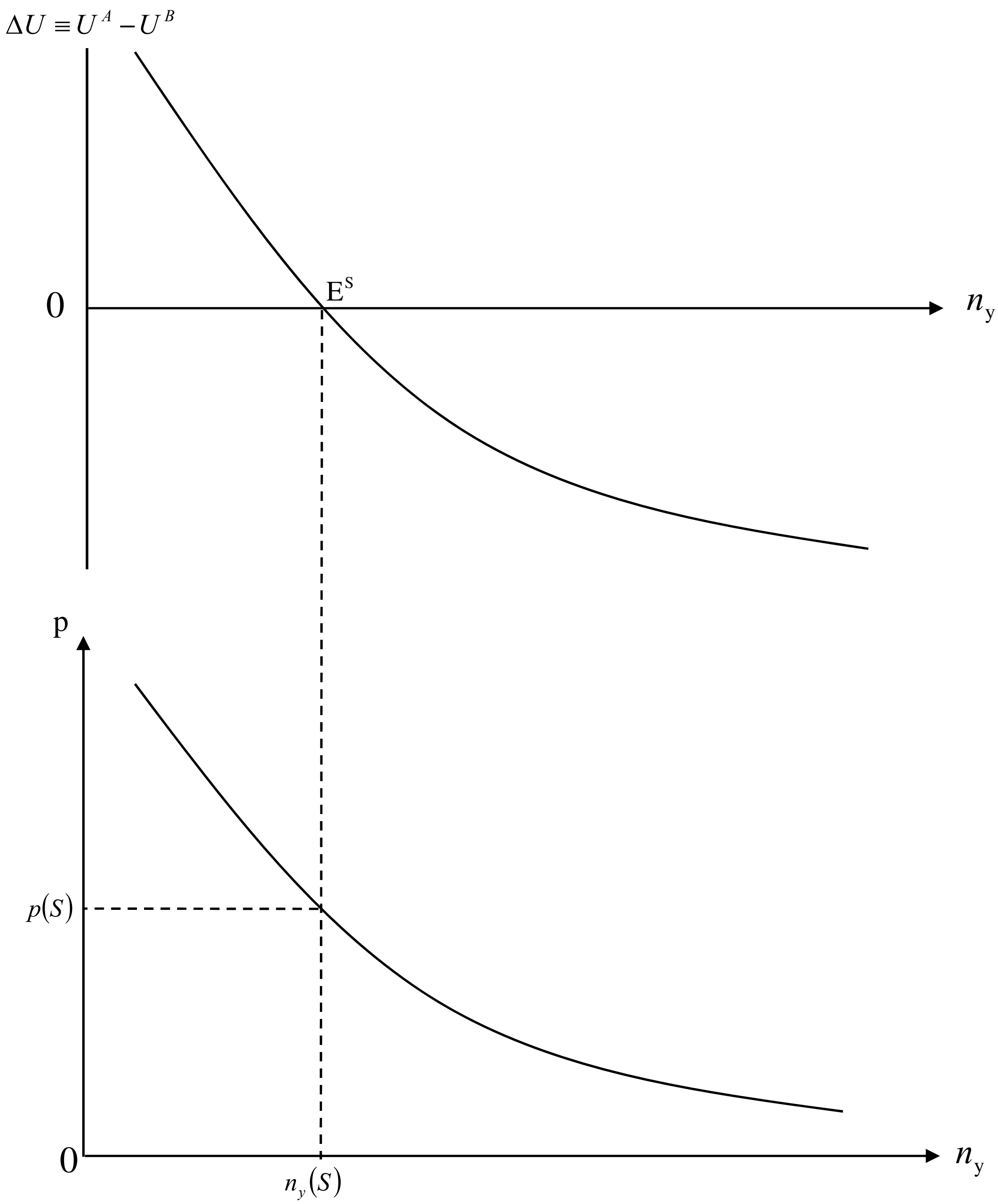


Figure 4. Integrated vs. Stratified Equilibrium

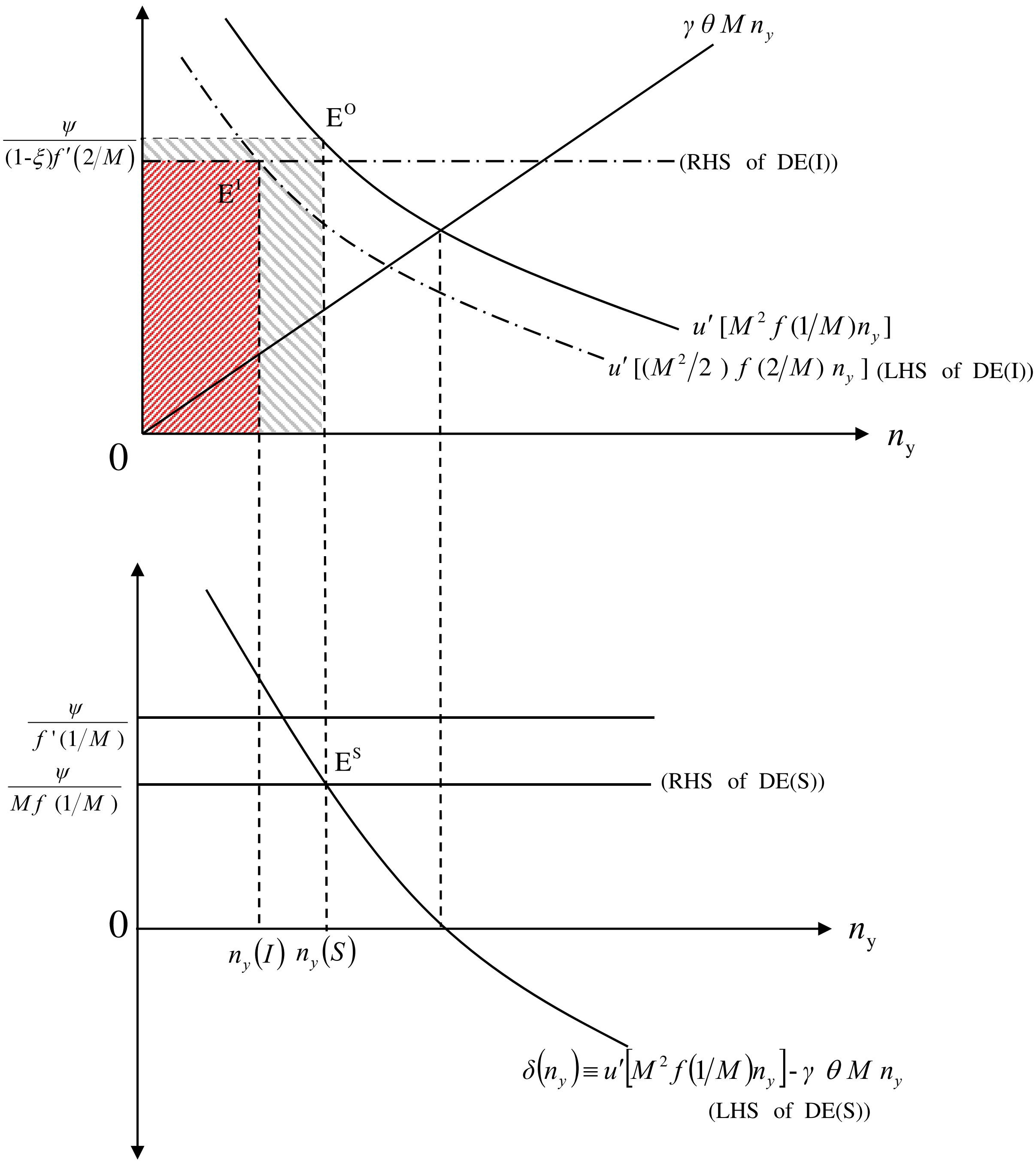
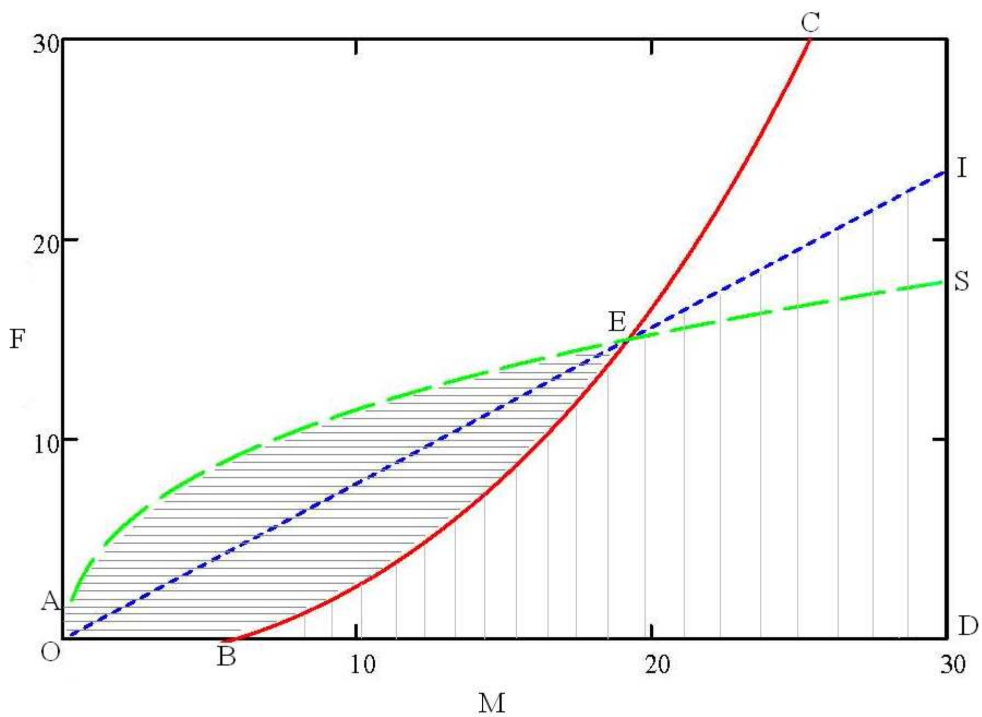


Figure 5. Equilibrium Configuration

(a) Equilibrium Classification



(b) Burfication Diagram

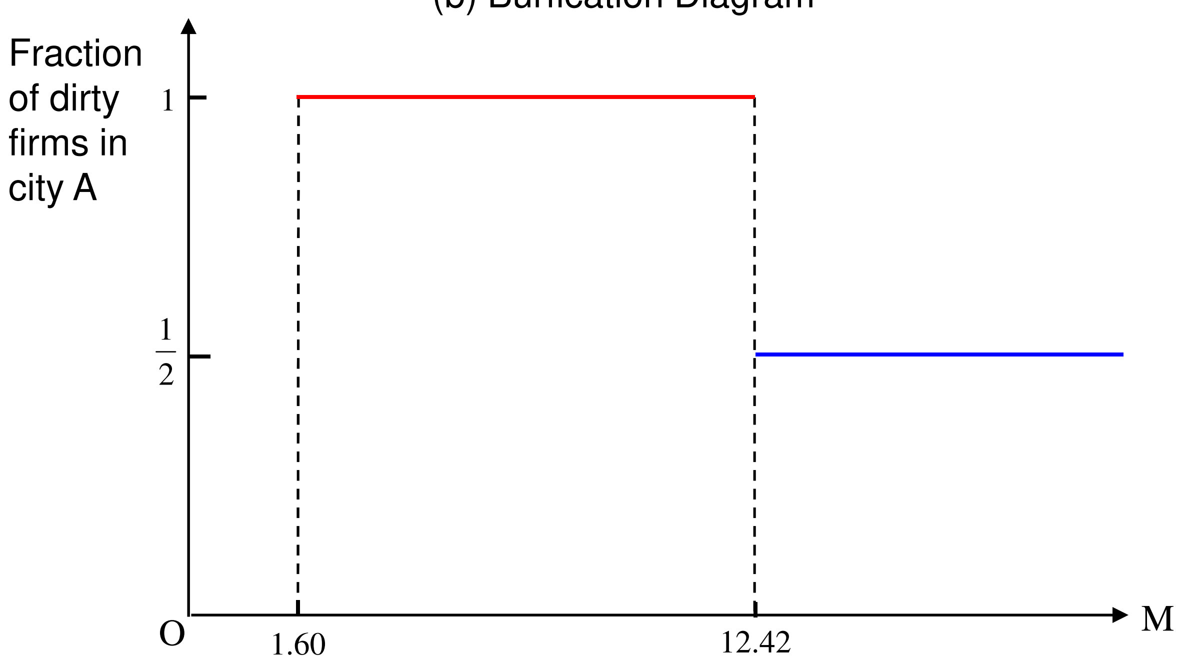


Figure 6. Tax Effects in Stratified Equilibrium

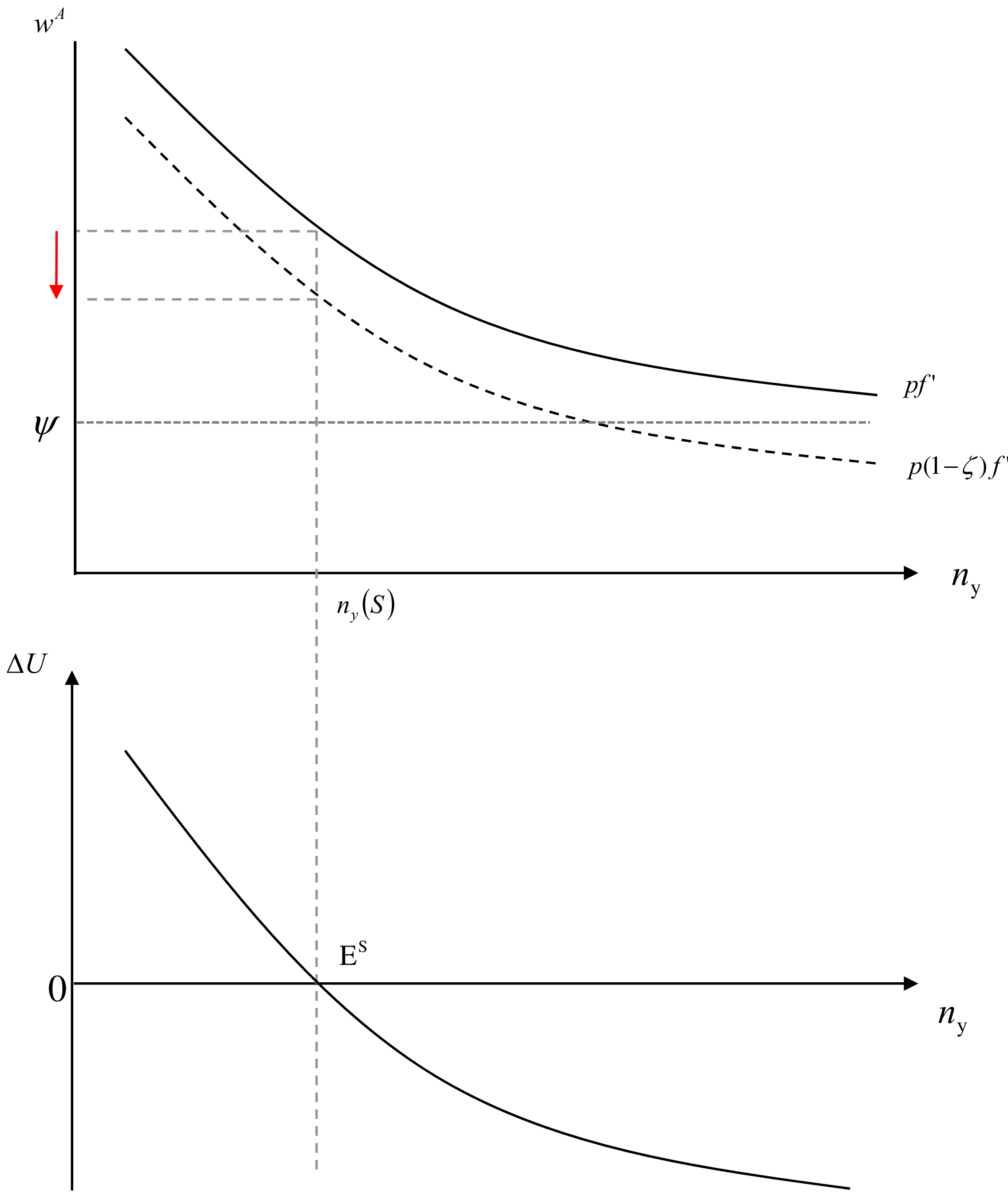
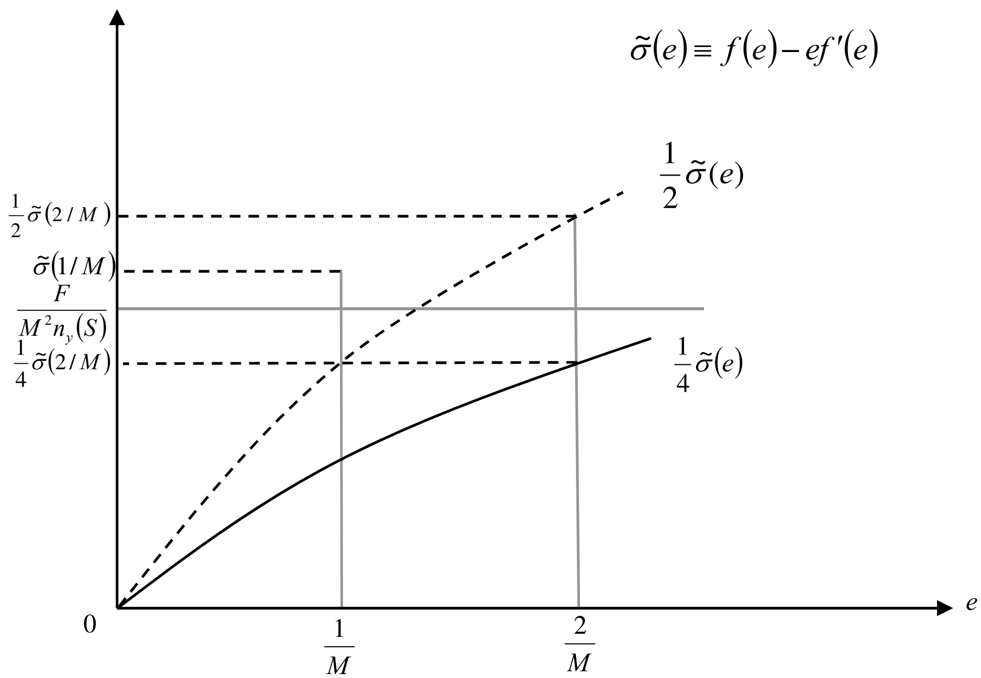


Figure 7. Surplus from Uncompensated Spillovers

a. Fixed Pollution Tax Regime: Conditions S-1 and R-1



b. Linear Pollution Tax Regime: Conditions S-2 and R-2