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Modeling Long Memory in REITs

Abstract

One stylized feature of financial volatility impacting the modeling process is long memory. This paper examines long memory for alternative risk measures, observed absolute and squared returns for Daily REITs and compares the findings for a market equity index. The paper utilizes a variety of tests for long memory finding evidence that REIT volatility does display persistence. Trading volume is found to be strongly associated with long memory. The results do however suggest differences in the findings with regard to REITs in comparison to the broader equity sector.

Keywords: Long Memory, FGARCH, REITs

Modeling Long Memory in REITs

1. Introduction

The continued development and increased investor awareness of the Real Estate Investment Trusts (REIT) sector has led to a dramatic increase in daily trading in the sector in recent years. SNL Financial estimate that average daily volume has increase from just over 7m shares in 1996 to over 40m shares in 2005. In addition, as the sector continues to mature and develop there will be is increased interest in derivative products based on the sector. At present a number of OTC (over-the-counter) products are available, while the Chicago Board Options Exchange (CBOE) provides traded options on the Dow Jones Equity REIT Index. The growth in both traded and OTC derivative products based on REITs furthers the interest in the dynamics of the sector at higher frequencies such as daily intervals. Furthermore, an examination of the volatility of the sector becomes more important as not only daily trading increases, but also as a result of its role in derivative pricing.

In contrast to the large literature that has examined the return behavior of REITs, very few examine volatility in the sector, and even fewer use high frequency data. Two early papers on REIT volatility (Devaney, 2001 and Stevenson, 2002a) both analyzed monthly data. The analysis conducted by Devaney (2001) was primarily concerned with the sensitivity of REIT returns and volatility to interest rates and was undertaken using a GARCH-M framework. The Stevenson (2002a) paper was, in contrast, concerned with volatility spillovers across both different REIT sectors, and between REITs and the equity and fixed-income markets. Four recent papers have examined various aspects of daily REIT volatility. Winniford (2003) concentrates on seasonality in REIT volatility. The author finds strong evidence that volatility in Equity REITs varies on a seasonal basis, with observed increased volatility in April, June, September, October and November. Cotter & Stevenson (2006) utilize a multivariate GARCH model to analyze dynamics in REIT volatility. Using a relatively short and quite distinct period of study (1999-2003) they find an increasing relationship between Equity REITs and mainstream equities in terms of both returns and volatility. Bredin et al. (2008), as with the Devaney (2001) paper, concentrate on the specific issue of interest rate sensitivity, examine the impact of unanticipated changes in the

Fed Funds Rate on REIT volatility. The results show a significant response in REIT volatility to unanticipated rate changes, however in contrast to much of the broader equity market evidence no evidence of asymmetry in the response in found. The final paper to have examined daily REIT volatility is the one most similar to the current paper. Najand & Lin (2004) utilize both GARCH and GARCH-M models in their analysis, reporting that volatility shocks are persistent.

This persistence in volatility is a common empirical finding in financial economics and is studied extensively in Taylor (1986). Whereas asset returns have largely been found to contain very little autocorrelation, it has been noted in a large number of papers across different asset classes that autocorrelation in various measures of volatility does exist at significant levels and remains over a large number of lags[1]. This effect, referred to as long memory, has been documented across a large sphere of the finance literature from macroeconomic series such as GNP (Diebold & Rudebusch, 1989) to exchange rate series (Baillie et al., 1996; Andersen & Bollerslev, 1997a, 1997b) at low and relatively high frequencies. Moreover, it is documented for equity index series at daily intervals (Ding et al., 1993, Ding & Granger, 1996).

This paper examines the long memory properties of alternative risk measures, observed absolute and squared returns for REITs and compares these to the S&P500 composite index. Analysis of long memory has been overlooked and we benchmark our REIT findings against the broader equity market. Specifically, the long memory property and its characteristics are explored. The long memory property occurs where volatility persistence remains at large lags and the series are fractionally integrated. Fractionally integrated series are integrated to order d where $0 < d < 1$ unlike integrated series of order 1, $d = 1$, and non-stationary series of order 0, $d = 0$. Fractionally integrated series have observations far apart in time that may exhibit weak but non-zero correlation. Much focus has been on the absolute returns series, $|R_t|^k$, or a squared returns series, $[R_t^2]^k$, for different power transformations, $k>0$. This property adds to the general clustering condition usually referred to in the context of squared returns persistence originally modeled in Engle's (1982) ARCH paper. There are daily cycles to the dependence structure giving rise to daily seasonality that exhibits a slow decay of the autocorrelation structure but also involves a *u*-shaped cyclical pattern (Andersen & Bollerslev, 1997a, 1997b). In addition, Ding et al. (1993) indicate that this non-linear dependence is strongest for absolute volatility with a power transformation of k=1 and as a consequence they suggest that parametric modeling of volatility should focus on absolute returns rather than the commonly used squared returns.

This paper begins by examining the autocorrelation structure of the REIT returns and volatility series. It then formally tests for the long memory property and measures the magnitude of the fractional integration parameter. In terms of model building, there are several approaches from linear and non-linear perspectives that could be applied. This paper fits two long memory volatility models, Fractionally Integrated GARCH (FIGARCH) and Fractionally Integrated Exponential GARCH (FIEGARCH) that allow for asymmetry. Baillie et al. (1996) find that these models have considerable success in modeling daily equity returns and we will investigate whether these GARCH models can capture the long memory properties of daily REITS. The paper examines the association between volume and volatility in the long memory volatility models. The paper proceeds as follows. In Section 2, long memory is discussed. The section incorporates a presentation and discussion of our GARCH models that are fitted to the daily series'. Details of the series and data capture follow in Section 3. Section 4 presents the empirical findings. It begins by briefly describing the indicative statistics of the volatility series', followed by a thorough analysis of their long memory characteristics. In addition, the ability of the GARCH processes to model volatility persistence is presented. Finally, a summary of the paper and some conclusions are given in section 5.

2. Long Memory

Baillie (1996) shows that long memory processes have the attribute of having very strong autocorrelation persistence before differencing, and thereby being non-stationary, whereas the first differenced series does not demonstrate persistence and is stationary. However, the long memory property of these price series is not evident from just first differencing alone, but has resulted from analysis of the associated risk measures. In fact financial returns themselves have only been found to exhibit short memory, with significant first order dependence that dissipates rapidly over subsequent lags. Thus the finance literature has concentrated its analysis of long memory on the volatility series and we follow this convention.

Long memory properties may be investigated by focusing on the absolute returns series denoted absolute volatility, $|R_t|^k$, or the squared returns series denoted squared volatility, $[R_t^2]^k$, and on their power transformations, where k>0.[2] Absolute volatility is examined as Davidian & Carroll (1987) find that absolute realizations are more robust in the presence of fat-tailed observations found in financial series than their squared counterparts. Moreover, empirical analysis of financial time series suggests that the long memory feature dominate for absolute over squared realizations (see Ding & Granger, 1996). Whereas squared volatility is utilized given that it underpins the commonly used risk measures such as standard deviation and variance.

Models with a long memory property have dependency between observations of a variable for a large number of lags so that Cov[R_{t+h}, R_{t-j}, $j \geq 0$] tends to zero as the number of lags h gets large.[3] In particular, long memory in financial time series has concentrated on volatility realizations where unexpected shocks affect the series for a large time frame. Thus confirmation of long memory properties for REITS would have major implications for the associated investments strategies that need to take account of the persistence and characteristics of the dependence structure in REIT volatility. However, if the dependency between observations of a variable disappears for a small number of lags, h, such as for a stationary ARMA process, then the data is described as having a short memory property and Cov[R_{t+h}, R_{t-j}, $j \geq 0$] $\rightarrow 0$. Formally, long memory is defined for a weakly stationary process if its autocorrelation function $\rho(\cdot)$ has a hyperbolic decay structure:

$$\rho(j) \sim C_j^{2d-1} \text{ as } j \rightarrow \infty, C \neq 0, 0 < d < \frac{1}{2} \tag{1}$$

where d represents the long memory parameter, or degree of fractional integration.

In contrast, short memory, or anti-persistence is evident if $-1/2 < d < 0$.

The corresponding shape of the autocorrelation function for a long memory process is hyperbolic if there is a relatively high degree of persistence in the first lag(s) that declines rapidly initially and is followed by a slower decline over subsequent lags. Thus the decay structure remains strong for a very large number of time periods. Previous analysis of equity returns suggest that the long memory parameter, *d*, is generally found to be between 0.3 and 0.4 (e.g. Andersen and Bollerslev, 1997a; and Taylor, 2000).

The explanations for long memory are varied. One economic rationale results from the aggregation of a cross-section of time series with different persistence levels (Andersen & Bollerslev, 1997a; Lobato & Savin, 1998). Alternatively, regime switching may induce long memory into the autocorrelation function through the impact of different news arrivals (Breidt et al., 1998). The corresponding shape of the autocorrelation function is hyperbolic, beginning with a high degree of persistence that reduces rapidly over a few lags, but that slows down considerably for subsequent lags to such an extent that the length of decay remains strong for a large number of time periods. Also, with a slight variation, it may follow a slowly declining shape incorporating cycles that correspond to, for example, daily seasonality (Andersen et al, 1997a).

We test for the existence of long memory in REITs by using an informal analysis of autocorrelation dependence of our volatility series augmented by two formal tests for the existence of the property. We are interested in two issues: whether REITs exhibit long memory properties and how the characteristics of the dependence structure of REITs compares to the broader equity market. The first test statistic is the parametric Modified Rescale Range (*R/S*) statistic developed by Lo (1991):

$$Q_n = \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\le k\le n}\sum_{j=1}^{k}\left(z_j-\bar{z}\right)-\min_{1\le k\le n}\sum_{j=1}^{k}\left(z_j-\bar{z}\right)\right] \tag{2}$$

Where $\hat{\sigma}_n$ is the estimate of the long run variance for sample size *n*, and for any series *z*, we compare the realized value, z_j, to its mean, $\bar{z}$, and examine the range of the variation. The Modified *R/S* allows for short memory in the time series but can distinguish if long memory exists separately, whereas in contrast, the original *R/S* statistic (Hurst, 1951) is not able to distinguish between long and short memory. Given, that microstructure issues such as bid-ask bounce induces first order correlation and short memory in returns series (Andersen et al., 2001) we may have both long and short memory characteristics in the series analyzed[4]. As a by product, we can also obtain an estimate of the degree of fractional integration, *d*, from applying this test denoted *R/S d*. This describes the degree of fractional integration and allows us to compare to different benchmarks, for example whether it is in the domain $0 < d < \frac{1}{2}$, and whether its magnitude differs across REIT and broad market series.

In addition, long memory is investigated by using the semi-nonparametric Geweke & Porter-Hudak (1983) log-periodogram regression approach (*GPH*) updated for non-Gaussian volatility estimates by Deo & Hurvich (2000). This adjustment is required given the fat-tailed and skewed behavior of financial time series. We also obtain semi-nonparametric estimates of the long memory parameter denoted *GPHd*. Assuming, $I(\omega_j)$ stands for the sample periodogram at the j^{th} fourier frequency, $\omega_j = 2\pi j/T$, j=1, 2, …, [T/2), the log-periodogram estimator of *GPHd* is based on regressing the logarithm of the periodogram estimate of the spectral density against the logarithm of ω over a range of frequencies ω:

$$\log[I(\omega_j)] = \beta_0 + \beta_1 \log(\varpi_j) + U_j \tag{3}$$

where j=1, 2,…, m, and $d = -1/2\beta_1$. This approach allows us to determine if the long memory property is evident in the series analyzed and also gives estimates of the long memory parameter. Again like the *R/S* approach, estimates of *d* are dependent on the choice of *m*. We estimate the test statistic by using $m = T^{4/5}$ as suggested by Andersen et al. (2001). This implies that for our sample size, a sample of 788 periodogram estimates is employed in our analysis.

Given, that long memory is not evident in financial returns series, but is strongly found in their volatility counterparts we need to examine volatility models and their suitability in describing the persistence patterns of the REIT and broad market series. Whilst second order dependence is a characteristic of financial returns, usually modeled by a stationary GARCH process, these specifications have been questioned as to their ability to model the long memory property adequately in contrast to their Fractionally Integrated GARCH counterparts (Baillie, 1996). For instance, while stationary GARCH models show the long memory property of financial returns volatility series occurs by having $[R_t^2]$ and $|R_t|$ with strong persistence, they assume that the autocorrelation function follows an exponential pattern not corresponding to a long memory process. In particular, the correlation between $[R_t^2]$ and $|R_t|$ from stationary GARCH models and their power transformations remain strong for a large number of lags, with the rate of decline following a constant pattern (Ding et al., 1993), or an exponential shape (Ding & Granger, 1996). In contrast, a number of returns series, both $[R_t^2]$ and $|R_t|$, have been found to decay in a hyperbolic manner, namely, they decline rapidly initially, and this is followed by a very slow decline (Ding & Granger, 1996).[5]

Turning to the set of conditional volatility models applied in this study, we first use the Fractionally Integrated GARCH (FIGARCH) model introduced by Baillie et al. (1996). These incorporate the standard time-varying volatility models and estimate the short run dynamics of a GARCH process. More importantly, they also measure the long memory characteristic of the data by estimating the degree of fractional integration *d*. First, taking a GARCH (*p*,*q*) process time varying volatility σ_t^2 is given as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \tag{4}$$

With $\alpha(L)$ and $\beta(L)$ being polynomials of order q and p in the lag operator. The process can be written as an ARMA *(m, p)* process in ε_t^2 where $m = \max(p,q)$:

$$\{1-\alpha(L)-\beta(L)\}\varepsilon_t^2 = \omega + \{1-\beta(L)\}v_t \tag{5}$$

For $v_t = \varepsilon_t^2 - \sigma_t^2$ are the innovations in the conditional variance process.

Converting it back into a GARCH type process gives the FIGARCH (*p,d,q*) model:

$$\phi(L)(1-L)^d\,\varepsilon_t^2 = \omega + \{1-\beta(L)\}v_t \tag{6}$$

Where $\phi(L)=\{1-\alpha(L)-\beta(L)\}(1-L)^{-1}$ is of order *m - 1*, and all the roots of $\phi(L)$ and $\{1-\beta(L)\}$ lie outside the unit circle.

This model can be expanded to deal with further stylized features of financial data. For instance, Black (1976) empirically notes a leverage effect where bad news tends to drive the price of an equity down thus increasing the debt-equity ratio (its leverage) and causing the equity to be more volatile. The leverage effect has an asymmetric impact on volatility with bad news having a greater impact than positive news. This leverage effect led Nelson (1991) to introduce the (Exponential) EGARCH process with a specific variable that distinguished between good news volatility and bad news volatility. If this variable's coefficient was negative bad news shocks have a greater impact on volatility than good news shocks. Engle and Ng (1993) provide further support for the existence of leverage effects in equity data following their introduction of a news impact curve that graphically separates the impact of good new and bad news shocks on volatility. If the effects of news are long lasting as suggested by the fractionally integrated process we should also determine if the long memory exhibits asymmetric effects. In order to allow for asymmetric effects we also apply the Exponential version of the FIGARCH model, the FIEGARCH developed by Bollerslev & Mikkelsson (1996):

$$\log(\sigma_t^2)=\omega+\phi(L)^{-1}(1-L)^{-d}\left[1-\lambda(L)\right]g(\xi_{t-1}) \tag{8}$$

Where

$$g(\xi_t)=\theta\xi_t+\gamma\left[|\xi_t|-E|_t|\right]$$

With the volatility shocks following an asymmetric function, and all the roots of $\phi(L)$ and $\lambda(L)$ lie outside the unit circle. The function has a slope of $\theta-\gamma$ when ξ is negative (market falls) and when ξ is positive (market rises) the slope is $\theta+\gamma$.

The residuals from both FIGARCH and FIEGARCH processes were initially assumed to be from a conditionally fat-tailed process in line with the commonly found characteristics in financial returns. We assume that the underlying data conditionally followed a student-*t* distribution as in Baillie & DeGennaro (1990).

3. Data

The data used in this paper consists of daily logarithmic returns for the period January 1 1990 through December 30 2005 totaling 4175 observations. During this time the popularity of REITS has expanded dramatically with massive growth in investor awareness and interest that focused in on the return and volatility characteristics of the sector. As we are interested in the long memory of the REIT sector we compare the findings to the broad equity market, as represented by the S&P 500 Composite.

Some descriptive statistics of the respective series are outlined in Table 1 detailing the first four moments of each series and a test for normality. Separate analysis is completed for the returns series and the two proxies of volatility, absolute and squared volatility. Starting with returns we find that the average daily returns of both series are near zero but positive for the time frame analyzed suggesting that for the mainstream equity market the 1990s boom has slightly outweighed the downturn at the start of this decade. Accordingly, the reverse is true for the REIT sector, with their strong recent performance outweighing the underperformance of the sector observed during the late nineties. Overall however, the average risk of REITs approaches 1% and is almost identical to the S&P. The time series behaviour of both series is given in Figure 1. Here we can see the increase in volatility at the turn of the decade associated with amongst other events, the fall out of the Asian crises and September 11, and the technology bubble where equity markets in general exhibited greater turbulence and very poor return performance. In the last couple of years the markets have settled down to some degree.

In Table 1 evidence on higher moments of returns suggests negative skewness recorded by both series suggesting that the weights of the large negative returns are dominating their positive counterparts. Consistent with the literature, we also find excess kurtosis suggesting that the series exhibit a fat-tailed property. Combining these findings for skewness and kurtosis, we find that all series are non-normal using the Jarque-Bera test statistic and therefore need to incorporate this property later in our modeling approach.

Turning to the proxies of the volatility series, we first reiterate the findings for the returns series, namely, that the REIT index exhibit similar volatility to the S&P and behaviour over the sample period. Average volatility (regardless of proxy) in Table 1 are similar for both indexes. Looking at the plots in Figure 1 we see the behaviour of the volatility associated with the series' since 1990. We clearly see the volatility clustering property where periods of high volatility or low volatility can remain persistent for some time before switching. This property suggests that volatility on any day is dependent on the previous day's values and we will model this phenomenon using a GARCH process that specifically incorporates long memory. The lack of independence of either absolute or squared volatility is clearly seen by the lack of normality and excess kurtosis reported in Table 1 for both series. We also get strong positive skewness for all series that is reasonably similar across the series. Comparing the two measures of volatility, we see that the magnitude of the squared realizations dominate their absolute counterparts but that the squared values are more prone to extreme outliers regardless of which series you examine.

4. Empirical Analysis

Our main focus in this paper is to examine the long memory properties of REITs and it is to this issue that we now turn. We begin by discussing the autocorrelation plots; followed by formal testing for long memory and determining the magnitude of the long memory parameter, and finally we outline our findings from applying two time-varying long memory volatility models. First, looking at dependence using the autocorrelation function (ACF), we provide plots over 100 lags for the volatility series and these are given in Figure 2 for absolute volatility and Figure 3 for squared volatility.[6] Ding et al. (1993) suggest that, as volatility is unobservable, the long memory in equity data should be examined for different power transformations of the volatility proxy series. We follow this suggestion by examining the volatility series for 5 different power transformations [k=0.25, 0.5, 1, 1.5, 2]. This supports the analysis of Beran (1994) in his seminal work in the area. In its strictest sense, the ACF plots in Figures 2 and 3 do not offer conclusive evidence that REITs exhibit long memory in volatility but are much more striking in their support for the property in the broad market index. Moreover there is strong variation in the strength of the long memory feature for the different power transformations and it tends to be stronger for lower *k*. These findings are consistent for squared and absolute volatility. It is noticeable that REITs appear to display less persistence in volatility than the general market. The ACF plots for the S&P indexes report enhanced long memory. It can be seen that in general the first lag for the REIT volatility ACF's tends to be of a greater magnitude but that the persistence reduces at a faster rate than for mainstream equities.

Table 2 reports details of the initial tests for long memory using the approaches described in Section 2. There is extensive evidence of long memory in both the absolute and squared volatility series'. This is consistent across all of the different power transformations, although the effect is generally enhanced as *k* reduces, particularly in the case of REITs. Furthermore, the magnitude of the test statistics is generally lower for the REIT sector than for the S&P. The findings from fitting the long memory volatility models are given in Table 3. The results generally show that both the FIGARCH and FIEGARCH models provide good fits for the data, and are broadly in line with expectations and the previously reported findings. The degree of fractional integration, as measured by the d-values, is in the range of 0.3-0.4 for the FIGARCH model for both series and is consistent with the previous empirical evidence. In relation to the FIEGARCH model the significant negative leverage coefficients also implies asymmetry in the long memory process with the greater impact of negative shocks over positive shocks affecting not only immediate volatility, but also on a persistent basis.

In the literature volume is seen as an important explanatory variable for time varying volatility[7]. To investigate whether trading volume is important for the long memory inherent in the volatility series we analyse the role of trading volume. In Figure 4 we see the large increase in trading activity in equities and this is particularly pronounced for REITs that had very low volume at the start of the sample. In Figure 5 we see that the change in trading volume shows similar patterns to that of the price series, namely, there is clustering of inactive (active) trading periods followed by active (inactive) trading periods.

Taking the volume data we fit a FIEGARCH model and results are reported in Table 4 with the associated time series plots given in Figure 6.[8] We are trying to determine whether volume is an important mixing variable for long memory in volatility. Trading volume is clearly an important explanatory variable for our conditional volatility with a strong statistical significance. Also, economically a 1% change in REIT volume is associated with a 0.01% change in its volatility and this effect is approximately doubled for the S&P series. Interestingly, by including the change in volume variable we see a major revision in the volatility specification with GARCH and ARCH coefficients being considerably amended in comparison to the FIEGARCH model results excluding volume. The main coefficients of the GARCH process, whilst remaining significant, reduce in magnitude considerably, and provide support for the hypothesis that volume and volatility are strongly related. The impact of volume, however, is even more pronounced on long memory with the long memory parameter, d, increasing to approximately 0.8 for both REITs and S&P series. Thus the long memory characteristic is no longer present in the volatility series if we include trading volume as an explanatory variable. Overall, changes (increases) in volume are strongly associated with the long memory in property found in REIT (and market) data.

5. Conclusion

This paper has examined the long memory properties in the volatility of the REIT sector at daily frequencies. As the sector develops and daily trading volume increases not only will interest in the daily dynamics in REITs increase but it will also in all likelihood increase interest in derivative instruments based on the sector. The paper illustrates that as with the general equity market volatility persistence occurs. However, there is evidence that long memory in REIT volatility is not of the same magnitude as that observed in the S&P 500 index. Moreover, changes in volume is an important explanatory variable in modeling long memory of REIT volatility.

References

Andersen, T.G. & Bollerslev, T. (1997a). Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns, *Journal of Finance*, **52**, 975-1005.

Andersen, T.G. & Bollerslev, T. (1997b). Intraday Periodicity and Volatility Persistence in Financial Markets, *Journal of Empirical Finance*, **4**, 115-158.

Andersen, T.G., Bollerslev, T., Diebold, F.X. & Labys, P. (2001). The Distribution of Realized Stock Return Volatility, *Journal of Financial Economics*, **61**, 43-76.

Baillie, R.T. & DeGennaro, R.P. (1990). Stock Returns and Volatility, *Journal of Financial and Quantitative Analysis*, **25**, 203-214.

Baillie, R.T. (1996). Long Memory Processes and Fractional Integration in Econometrics, *Journal of Econometrics*, **73**, 5-59.

Baillie, R.T., Bollerslev, T. & Mikkelsen, H.O. (1996). Fractionally Integrated Generalized Conditional Heteroskedasticity, *Journal of Econometrics*, **74**, 3-30.

Beran, J. (1994). *Statistics for Long-Memory Processes*, Chapman & Hall: New York.

Black, F. (1976). *Studies in Stock Price Volatility Changes*, Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Statistical Association, 177-181.

Bollerslev, T., & Mikkelsen, H.O. (1996). Modeling and Pricing Long Memory in Stock Market Volatility. *Journal of Econometrics*, **73**, 151-184.

Bredin, D., O'Reilly, G. & Stevenson, S. (2008). Monetary Shocks and REIT Returns, *Journal of Real Estate Finance & Economics*, forthcoming.

Breidt, F.J., Crato, N. & deLima, P. (1998). On the Detection and Estimation of Long-Memory in Stochastic Volatility, *Journal of Econometrics*, **83**, 325-348.

Cotter, J. (2005). Uncovering Long Memory in High Frequency UK Futures. *European Journal of Finance*, **11**, 325-337.

Cotter, J. & Stevenson, S. (2006). Multivariate Modeling of Daily REIT Volatility. *Journal of Real Estate Finance and Economics.* **32**, 305-325.

Davidian, M., & Carroll, R. J. (1987). Variance Function Estimation, *Journal of the American Statistical Association*, **82**, 1079-1091.

Deo, R. S. & Hurvich, C. M. (2000). *On the Periodogram Regression Estimator of the Memory Parameter in Long Memory Stochastic Volatility Models,* New York University, Mimeo.

Devaney, M. (2001). Time-Varying Risk Premia for Real Estate Investment Trusts: A GARCH-M Model, *Quarterly Review of Economics & Finance*, **41**, 335-346.

Diebold, F. X. & Rudebusch, G.D. (1989). Long Memory and Persistence in Aggregate Output, *Journal of Monetary Economics*, **24**, 189-209.

Ding, Z., Granger, C.W.J., & Engle, R.F. (1993). A Long Memory Property of Stock Returns, *Journal of Empirical Finance*, **1**, 83-106.

Ding, Z. & Granger, C.W.J. (1996). Modeling Volatility Persistence of Speculative Returns, *Journal of Econometrics*, **73**, 185-215.

Engle, R.F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation, *Econometrica*, **50**, 987-1008.

Engle, R. F. and Ng, V. (1993). Measuring and testing the impact of news on volatility, *Journal of Finance*, **48**, 1749-1778.

Geweke, J. & Porter-Hudak, S. (1983). The Estimation and Application of Long Memory Time Series Models, *Journal of Time Series Analysis,* **4**, 221-238.

Hurst, H. E. (1951). Long Term Storage Capacity of Reservoirs, *Transactions of the American Society of Civil Engineers*, **116**, 770-799.

Kleiman, R.T., Payne, J.E. & Sahu, A.P. (2002). Random Walks and Market Efficiency: Evidence from International Real Estate Markets, *Journal of Real Estate Research*, **24**, 279-297.

Lamoureux, C. G. & Lastrapes, W. D. (1990). Heteroskedasticity in stock return data: volume versus GARCH effects, *Journal of Finance*, **45**, 221-229.

Lo, A. (1991). Long Memory in Stock Market Prices, *Econometrica*, **59**, 1279-1313.

Lobato, I. N. & Savin, N.E. (1998). Real and Spurious Long-Memory Properties of Stock-Market Data, *Journal of Business and Economic Statistics*, **16**, 261-268.

Najand, M. & Lin, C. (2004). *Time Varying Risk Premium for Equity REITs: Evidence from Daily Data*, Working Paper, Old Dominion University.

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, **59**, 347-370.

Scholes, M. and Williams, J. (1977). Estimating Beta From Non-Synchronous Data, *Journal of Financial Economics,* **5**, 309-327.

Stevenson, S. (2002a). An Examination of Volatility Spillovers in REIT Returns, *Journal of Real Estate Portfolio Management*, **8**, 229-238.

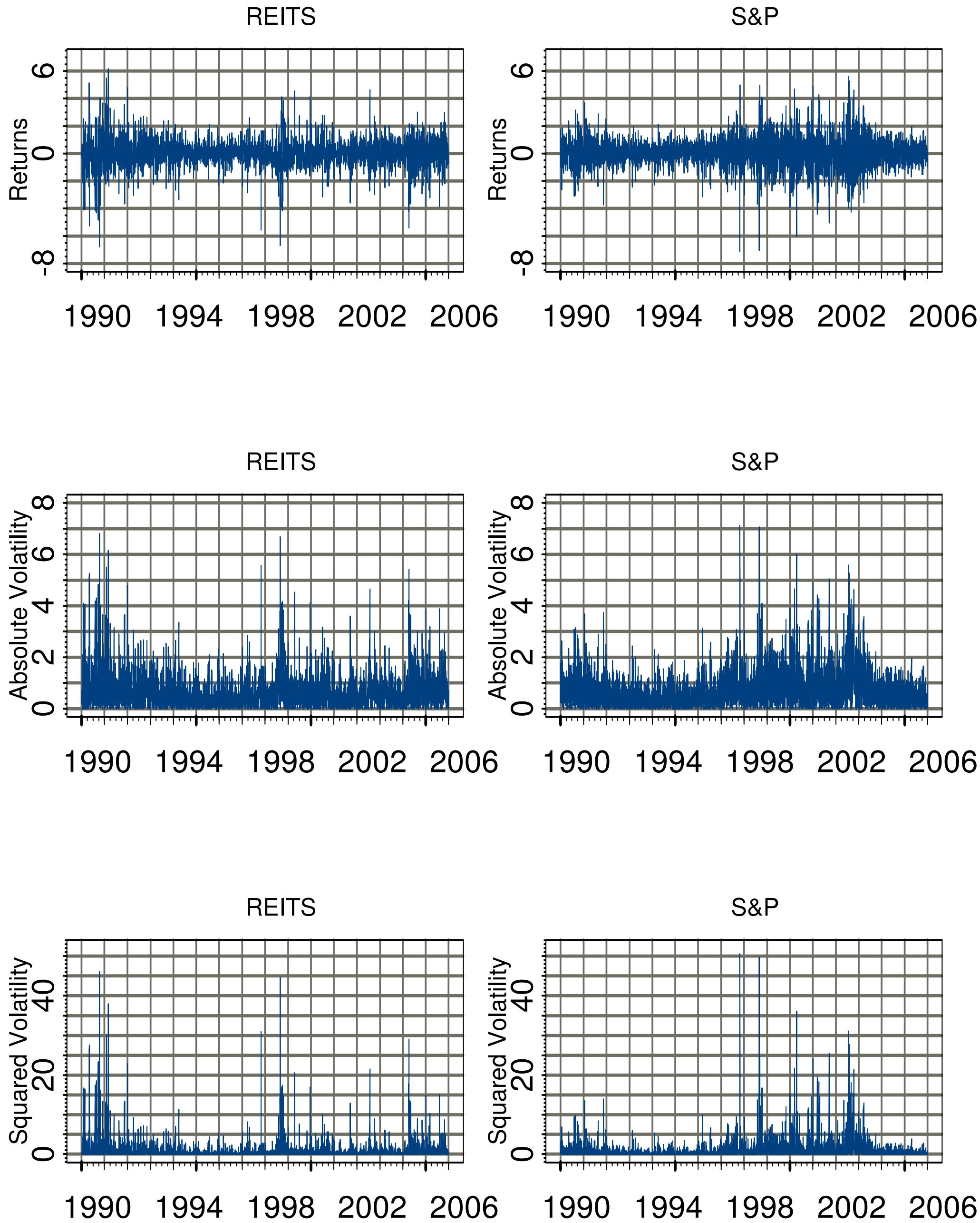
Stevenson, S. (2002b). Momentum Effects and Mean Reversion in Real Estate Securities, *Journal of Real Estate Research*, **23**, 48-65.

Taylor, S.J. (1986). *Modeling Financial Time Series*, Wiley: London.

Taylor, S.J. (2000) *Consequences for Option Pricing of a Long Memory in Volatility*, Working Paper, University of Lancaster.

Winniford, M. (2003). Real Estate Investment Trusts and Seasonal Volatility: A Periodic GARCH Model, Working Paper, Duke University.

Figure 1: Time Series Plots of Daily Series



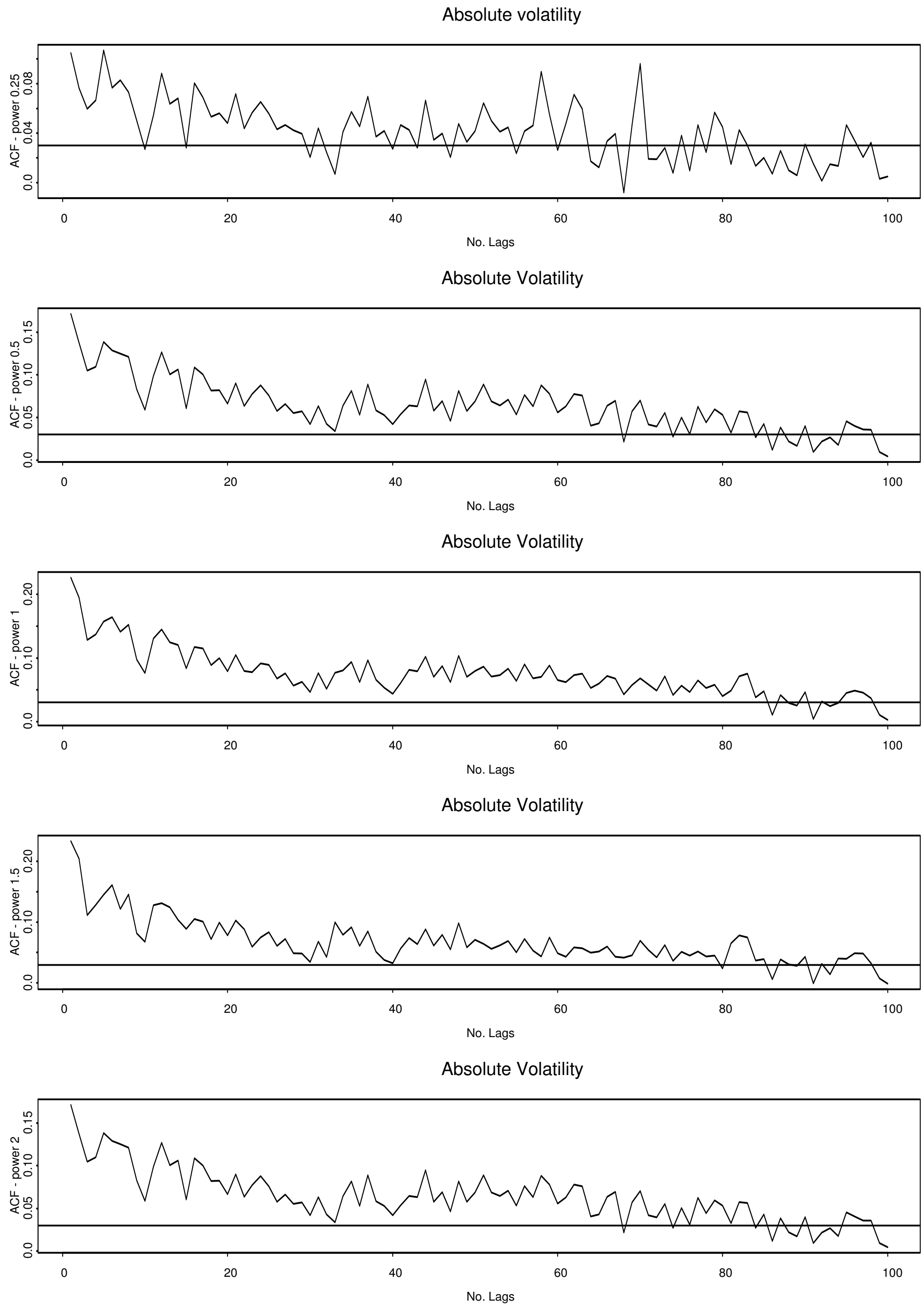
Notes: The plots show the time series behaviour of daily percentage values for the returns, absolute returns (absolute volatility) and squared returns (squared volatility) series' between 1990 and 2005 inclusive.

Table 1: Summary Statistics for Daily Series

	REITs	S&P 500
Panel A: Returns		
Mean	0.029	0.030
Std Dev	0.944	0.997
Skewness	-0.256*	-0.100*
Kurtosis	8.297*	7.011*
Normality	4925.84*	2805.12*
Panel B: Absolute Volatility		
Mean	0.659	0.701
Std Dev	0.677	0.709
Skewness	2.617*	2.244*
Kurtosis	14.55*	11.66*
Normality	27963*	16541.8*
Panel C: Squared Volatility		
Mean	0.893	0.994
Std Dev	2.405	2.432
Skewness	8.535*	8.396*
Kurtosis	109*	119*
Normality	2003006*	2387327*

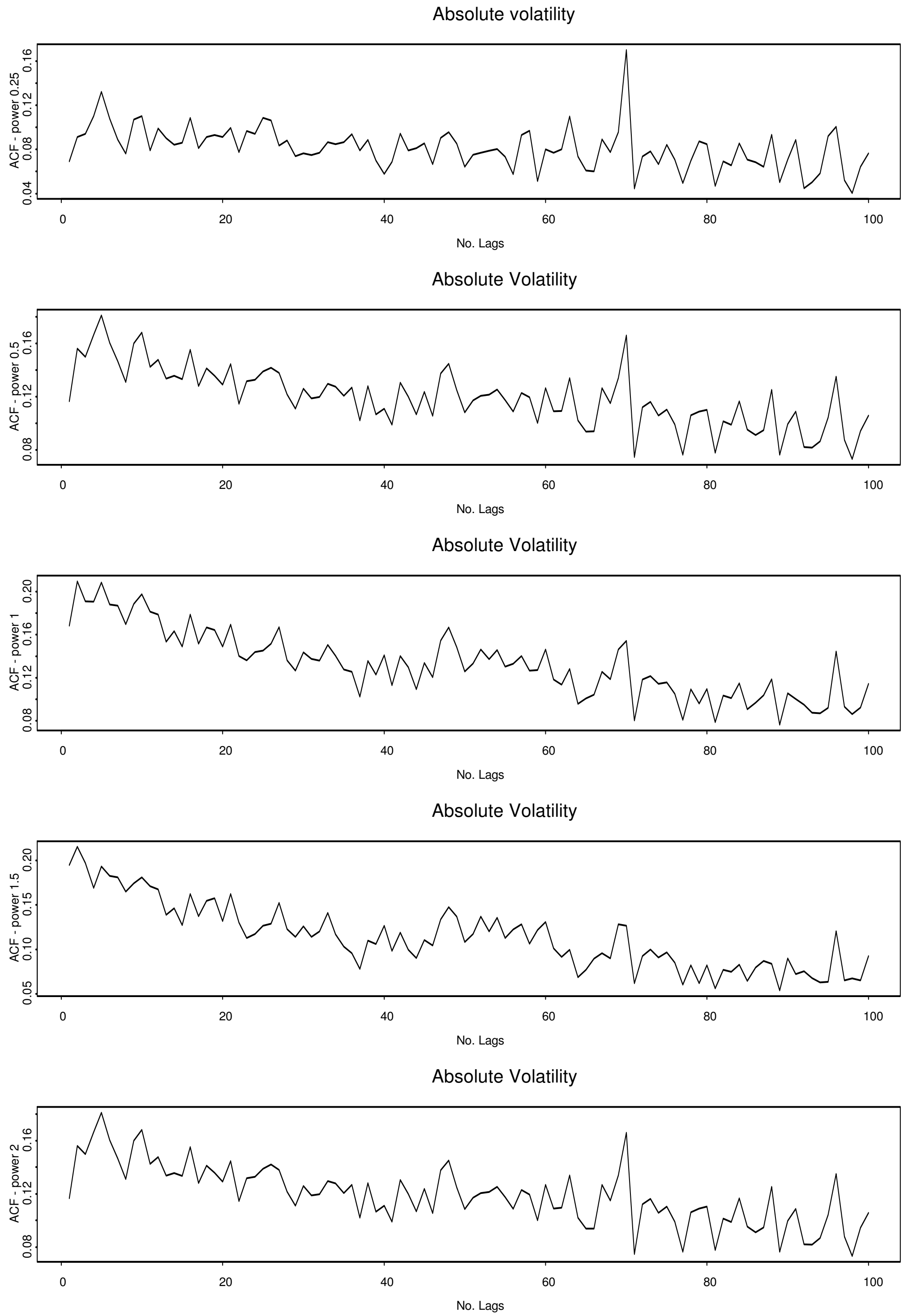
Notes: Estimates are given for returns (Panel A), for Absolute Volatility (Panel B) and Squared Volatility (Panel C). Mean and standard deviations are expressed in percentage form. Skewness and kurtosis are tested using Fisher's G and Fisher's G2 statistics respectively. Normality is tested for using the Jarque-Bera test statistic. The skewness, kurtosis and normality statistics have a value of 0 for a normal distribution. All skewness, kurtosis and normality statistics are significant at 5% significance levels indicated by *.

Figure 2a: Plots of Autocorrelation Values for REIT Daily Absolute Volatility



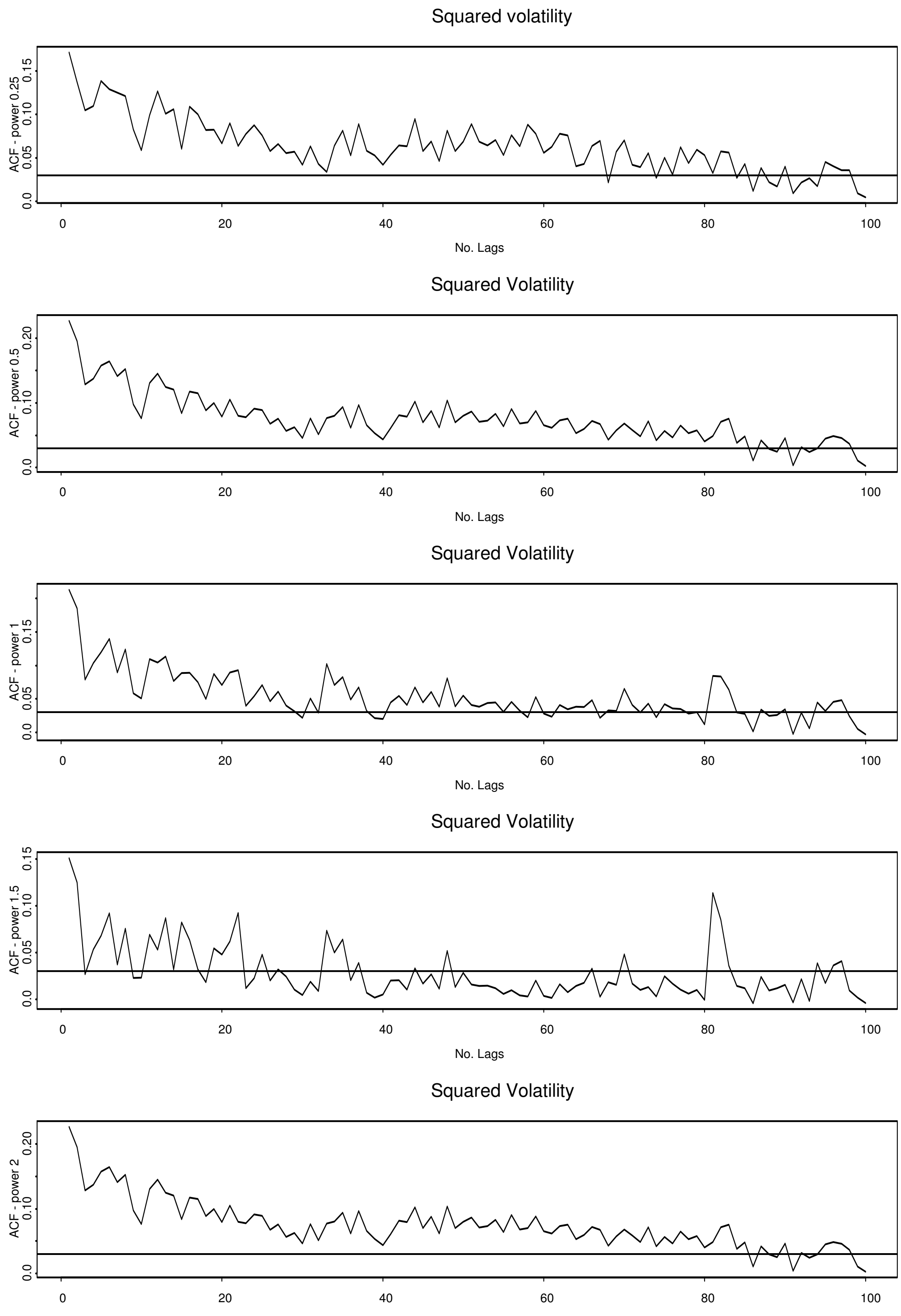
Notes: The plots show the dependence in REIT daily absolute volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm 1.96/\sqrt{n}$) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Figure 2b: Plots of Autocorrelation Values for S&P Daily Absolute Volatility



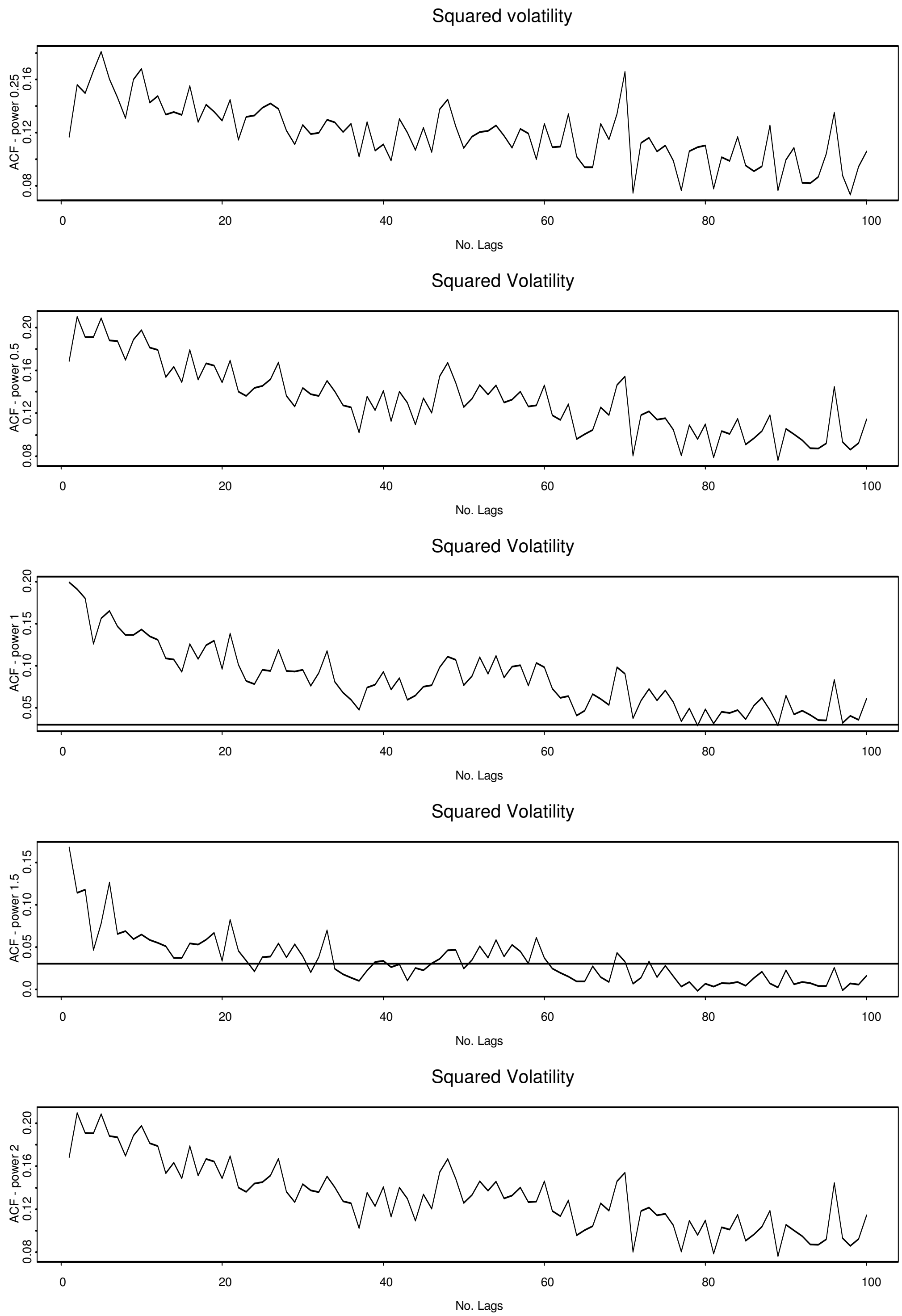
Notes: The plots show the dependence in S&P daily absolute volatility for 5 different power transformations [k=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm$ 1.96/$\sqrt{n}$) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Figure 3a: Plots of Autocorrelation Values for REIT Daily Squared Volatility



Notes: The plots show the dependence in REIT daily squared volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm$ 1.96/√n) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Figure 3b: Plots of Autocorrelation Values for S&P Daily Squared Volatility



Notes: The plots show the dependence in S&P daily squared volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm$ 1.96/$\sqrt{}$n) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Table 2: Long Memory Diagnostics for Daily Series

		R/S	GPH	R/S d	GPHd
Panel A: Absolute Volatility					
REIT	k = 0.25	4.1083**	3.703**	0.139796	0.2791813
	0.5	4.2062**	4.0009**	0.162991	0.3362795
	1	3.8566**	4.5319**	0.174352	0.3619588
	1.5	3.5013**	4.1893**	0.168648	0.3556661
	2	3.1706**	3.7235**	0.15553	0.312547
S&P	k = 0.25	5.7324**	5.5284**	0.12405	0.3157433
	0.5	6.0847**	5.9749**	0.138839	0.385766
	1	5.9084**	5.7912**	0.144529	0.4221734
	1.5	5.4511**	5.4709**	0.142107	0.4150753
	2	4.8572**	4.7648**	0.134813	0.3655939
Panel B: Squared Volatility					
REIT	k = 0.25	4.2062**	4.0009**	0.162991	0.3362795
	0.5	3.8566**	4.5319**	0.174352	0.3619588
	1	3.1706**	3.7235**	0.15553	0.312547
	1.5	2.5481**	2.8687**	0.129716	0.2333424
	2	2.0718*	2.2246*	0.110291	0.1758527
S&P	k = 0.25	6.0847**	5.9749**	0.138839	0.385766
	0.5	5.9084**	5.7912**	0.144529	0.4221734
	1	4.8572**	4.7648**	0.134813	0.3655939
	1.5	3.6671**	3.5083**	0.117991	0.2193724
	2	2.725**	1.7617	0.103408	0.1310806

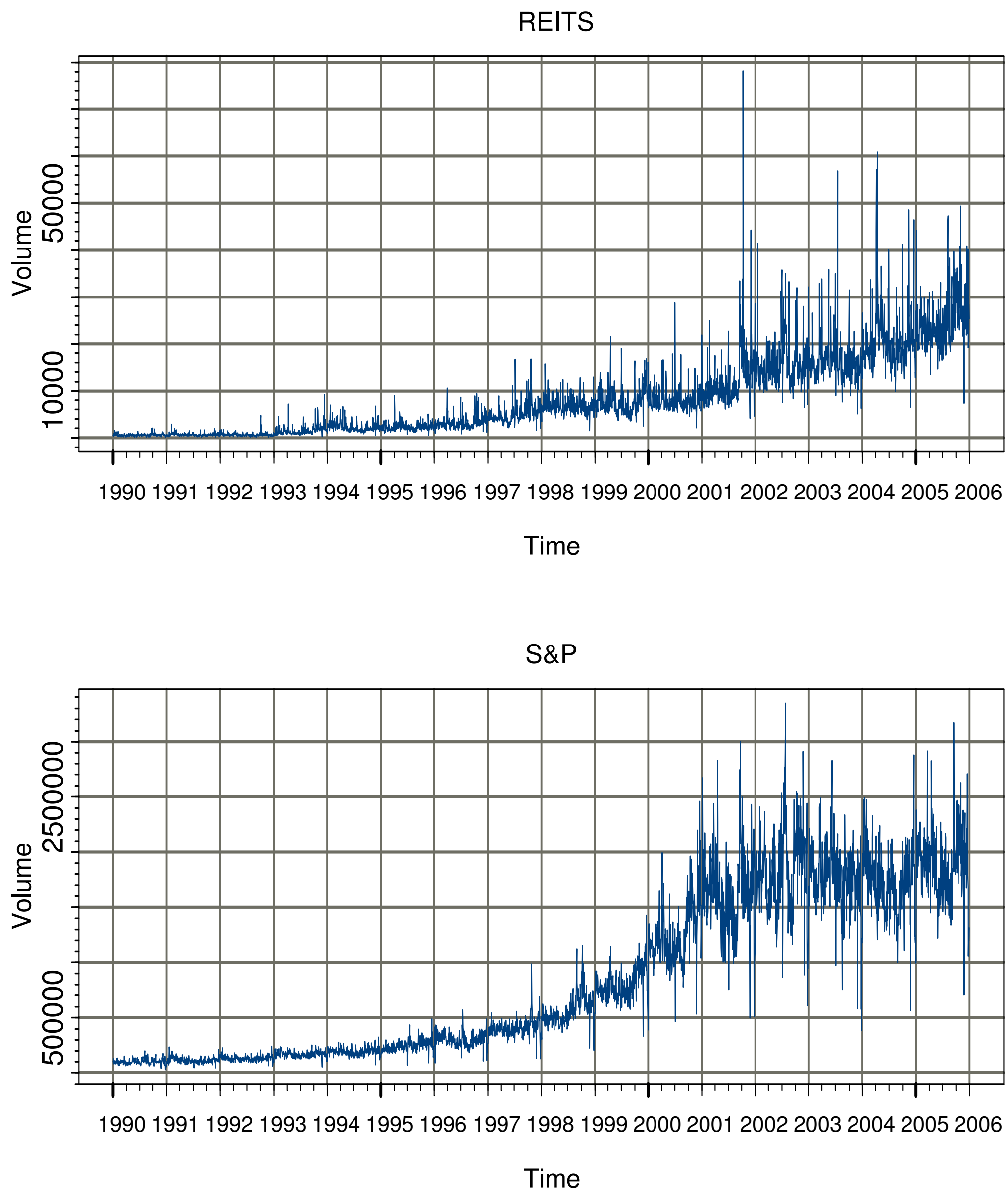
Notes: Further technical details of the long memory tests and parameter estimates are given in the text. The R/S test is the modified R/S statistic (Lo, 1991). The GPH test is the Geweke & Porter-Hudak (1983) semi-nonparametric statistic. The R/S *d* is the R/S long memory parameter. The *GPHd* is the periodogram long memory parameter. Estimates are given for Absolute Volatility (Panel A) and Squared Volatility (Panel B) with different power transformations, [*k*=0.25, 0.5, 1, 1.5, 2]. A single asterisk represents significance at the 5% level whereas two represents significance at the 1% level.

Table 3: Fractionally Integrated GARCH Models for Daily Return Series

	REITs		S&P 500	
	Coefficient	**p-value**	**Coefficient**	**p-value**
Panel A: FIGARCH (1,1)				
A	0.04641***	3.66E-06	0.02803***	2.10E-07
GARCH(1)	0.50543***	1.01E-11	0.54397***	0.00E+00
ARCH(1)	0.37822***	1.69E-08	0.18921***	8.22E-15
d	0.3175***	0.00E+00	0.39632***	0.00E+00
LM (12)	20.9*	0.05189	7.942	0.7896
Q^2 (12)	20.29*	0.06178	20.32*	0.06127
Panel B: FIEGARCH (1,1)				
A	-0.24867***	0.00E+00	-0.10651***	0.00E+00
GARCH(1)	0.13449**	2.72E-02	0.452***	7.38E-08
ARCH(1)	0.33099***	0.00E+00	0.13623***	0.00E+00
Leverage	-0.05662***	1.41E-08	-0.0983***	0.00E+00
d	0.59397***	0.00E+00	0.63067***	0.00E+00
LM (12)	17.94	0.1176	9.205	0.6853
Q^2 (12)	17.61	0.1279	9.118	0.6928

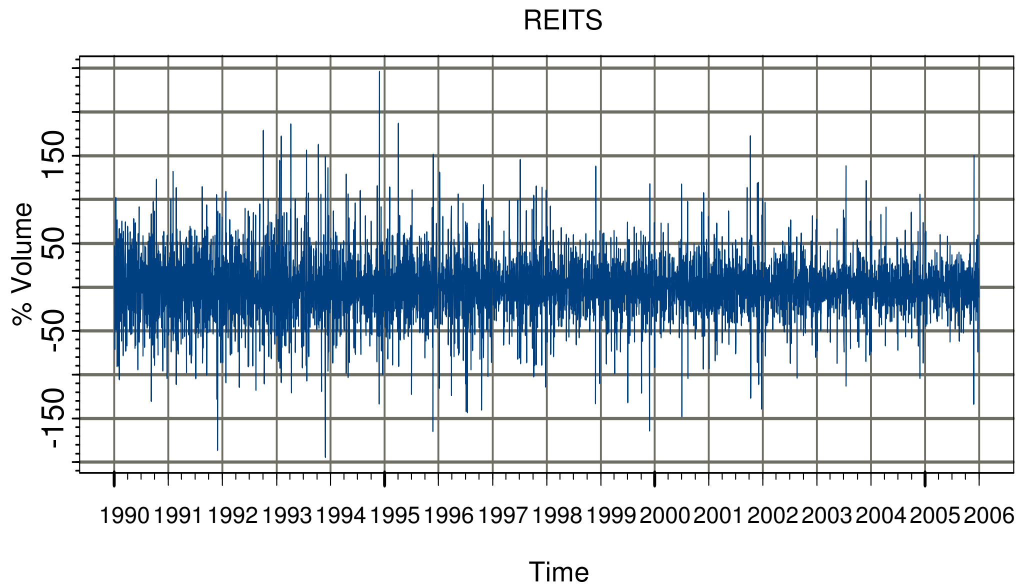
Notes: Coefficients and marginal significance levels for the FI(E)GARCH models are presented with full details of the models given the text. The respective optimal model is chosen based on Akaike's (AIC) and Schwarz's (BIC) selection criteria. A single asterisk denotes statistical significance at the 10%, two denotes statistical significance at the 5% level, while three denotes statistical significance at the 1% level. The FIEGARCH model incorporates a leverage variable that is significant for both indexes. Significant (G)ARCH effects are reported for both indexes. The long memory parameter, *d*, is tested for statistical significance from 0 and occurs in all cases. The diagnostics are supportive of a good fit for both fractionally integrated models. The diagnostics used are the $Q^2(12)$ Ljung-Box test on the squared standardised residual series and Engle's (1982) LM test for up to 12th order ARCH effects on the squared standardised returns series.

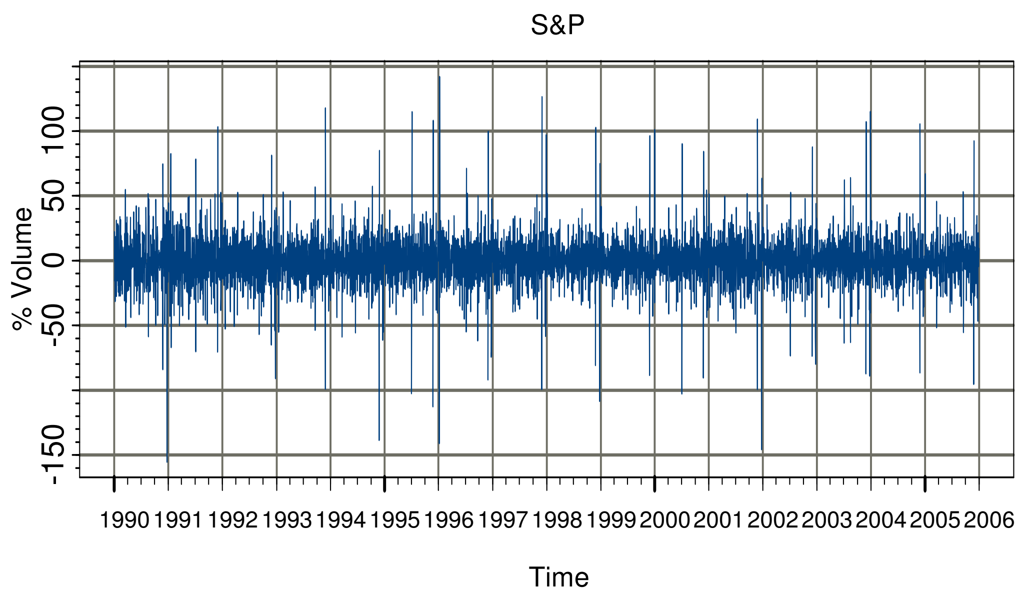
Figure 4: Time Series Plots of Daily Volume Series



Notes: The plots show the time series behaviour of daily trading volume for both indexes between 1990 and 2005 inclusively.

Figure 5: Time Series Plots of Daily Change in Volume Series





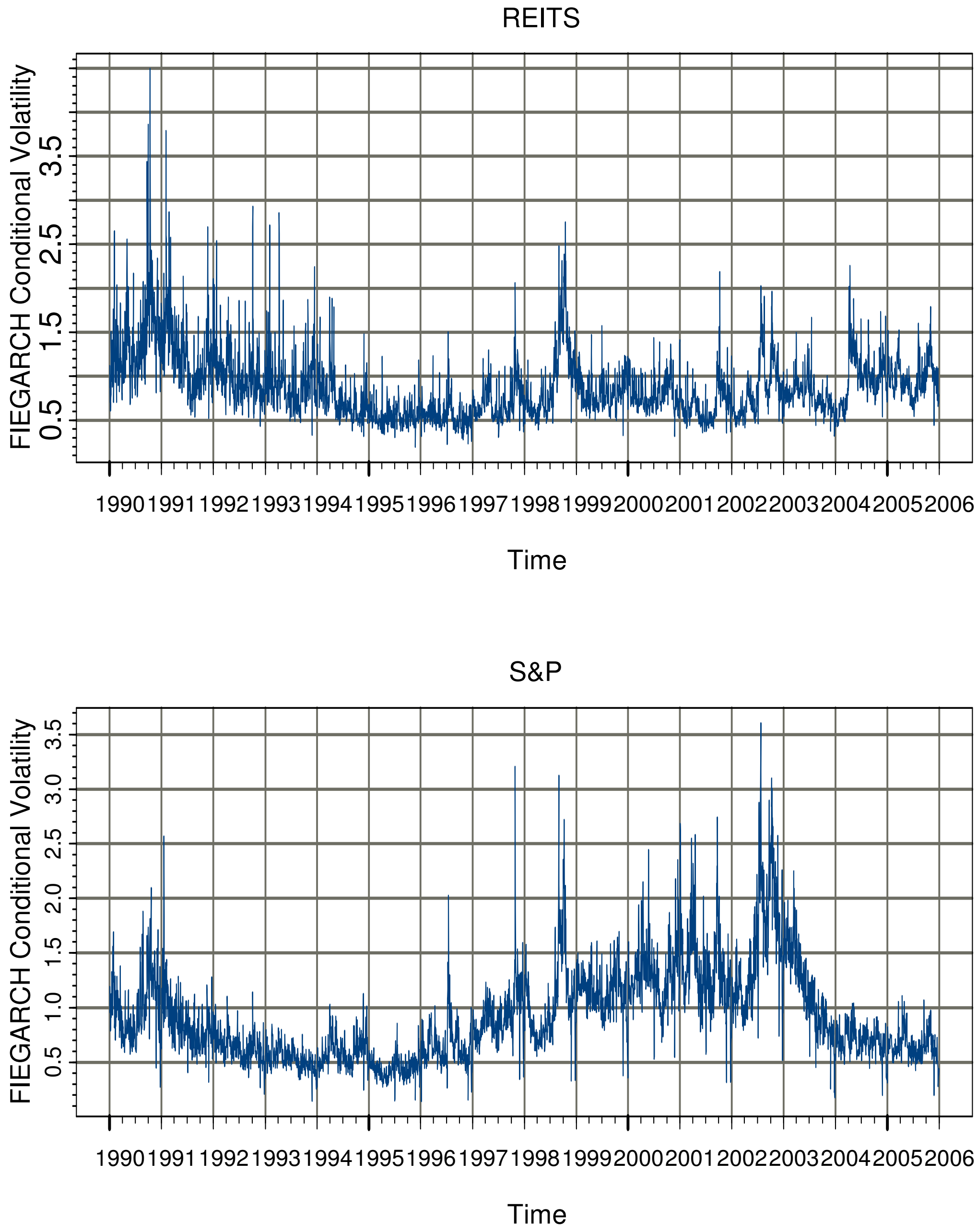
Notes: The plots show the time series behaviour of daily percentage values for the change in volume for both indexes between 1990 and 2005 inclusive.

Table 4: Fractionally Integrated EGARCH Model with Volume

	REITs		S&P 500	
	Coefficient	**p-value**	**Coefficient**	**p-value**
FIEGARCH (1,1)				
A	-0.14787***	0.00E+00	-0.08081***	2.00E-15
GARCH(1)	0.17908***	9.65E-04	0.08229*	8.82E-02
ARCH(1)	0.19329***	0.00E+00	0.1001***	2.07E-14
Leverage	-0.03489***	9.13E-08	-0.06085***	1.45E-13
Volume	0.01184***	0.00E+00	0.02012***	0.00E+00
d	0.77339***	0.00E+00	0.86554***	0.00E+00
LM (12)	20.84*	0.05277	21.21*	0.04734
Q^2 (12)	20.71*	0.05477	21.95*	0.03811

Notes: Coefficients and marginal significance levels for the FIEGARCH model are presented with full details of the model given the text. The respective optimal model is chosen based on Akaike's (AIC) and Schwarz's (BIC) selection criteria. A single asterisk denotes statistical significance at the 10%, two denotes statistical significance at the 5% level, while three denotes statistical significance at the 1% level. The FIEGARCH model incorporates both volume and leverage variables. Significant (G)ARCH effects are reported for both indexes. Both volume and leverage variables are significant for both indexes. The long memory parameter, *d*, is tested for statistical significance from 0 and occurs in all cases. The diagnostics are supportive of a good fit for the fractionally integrated model. The diagnostics used are the $Q^2(12)$ Ljung-Box test on the squared standardised residual series and Engle's (1982) LM test for up to 12^{th} order ARCH effects on the squared standardised returns series.

Figure 6: Time Series Plots of FIEGARCH Daily Conditional Volatility Series



Notes: The plots show the time series behaviour of daily percentage conditional volatility for both indexes between 1990 and 2005 inclusive. Conditional volatility was obtained from fitting the FIEGARCH model with volume included as an explanatory variable.

Endnotes:

[1] Two recent papers to have examined persistence and mean reversion in REIT and international real estate security returns are Kleiman et al. (2002) and Stevenson (2002b).

[2] As volatility is a latent unobservable variable proxies of volatility such as absolute and squared returns are examined in the literature.

[3] For an excellent treatment of long memory processes see Beran (1994).

[4] Extensions of the Hurst (1951) *R/S* statistic involve replacing the sample standard deviation of the series, *Z*, with the square root of the Newey-West estimate of the long run variance.

[5] One such example of a relatively successful application of standard GARCH models is the application of the APARCH model (see Cotter, 2005; for an example). The APARCH specification, developed by Ding et al. (1993) nests seven commonly applied GARCH models. However, the specification has an exponential decline structure that shows strong dependence but is not fully consistent with the long memory decline structure.

[6] We also examine dependence of returns formally through long memory tests and informally through ACF plots. In line with previous studies we find negligible evidence to support the presence of long memory of returns. Results are available on request.

[7] See Lamoureux & Lastrapes (1990). They find that trading volume reflects the dependence in information flows to the market that feeds directly into price volatility.

[8] We avoid fitting the FIGARCH specification as our exogenous variable, change in trading volume, is not always positive as can be seen from the time series plot and would result in negative conditional variance values. Also we have already documented asymmetric effects in the long memory of volatility.