Asset price, asset securitization and financial stability

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Abstract: Prior to the Global Financial Crisis in 2008, securitization has been widely perceived as a way to disperse credit risks, and to enhance financial system's capacity in dealing with defaults. This paper develops a model of securitization and financial stability in the form of amplification effects. This model has illustrated three different scenarios: A negative shock in the economy will lead to downturn of the economy and falling of the asset prices, deteriorating balance sheets and tightening financing conditions. However, if there is no shock or a positive shock, banks can improve its profitability significantly through securitization. While securitization decreases the probability of systemic crisis, banks tend to suffer more when the crisis happens as a result of over-borrowing and over-investing. This paper uses a three-period theoretical model to demonstrate the impact of securitization on the financial stability, and provides clear analytical guidelines for a new regulatory framework of securitization that account for systemic risk and systemic externalities.

Key words: Asset Price; Asset Securitization; Systemic Risk; Financial Stability

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1 Introduction

The financial crisis started in the US in 2007 quickly spread out to all developed and emerging economy, and is considered as the most serious crises since the Great Depression. Its global effects include the failure and bailout of key financial institutions, the decline in private wealth, substantial financial commitments incurred by many governments, and a significant decline in economic activities. This crisis raised the need for the researchers and policymakers around the globe to take unprecedented policy measures to deal with systemic risks (Gorton 2008, Brunnermeier 2009, Diamond and Rajan 2009, Blanchard 2009 and Krishnamurthy 2009). There is no coherent definition for systemic risk, but most existing research considers it as the danger or probability that financial institutions become insolvent in a large scale. This concept is different from so-called "systematic risk' which sometimes called market risk, aggregate risk, or undiversifiable risk, is the risk associated with aggregate market returns.
Comparing to the crisis in the 20^{th} century, the recent crisis has three main features. Firstly, the immediate cause or trigger of the crisis is neither the bankruptcy of traditional banks nor currency crisis in the traditional sense, but the burst of the United States housing bubble which peaked in approximately 2005–2006. Secondly, the asset lead to crisis is not the long term investment project as before, but the mortgage-backed security (MBS). Thirdly, in order to keep liquidity, the financial institutions made panic selling of securities, which sharpened the severity of the crisis.

The fore-mentioned features of the crisis are associated with the operating model of the modern banking. Traditionally, the main source of bank's revenue is interest on the capital it lends out to customers. And the bank profits from the difference between lending and deposit interest, as shown in figure 1.

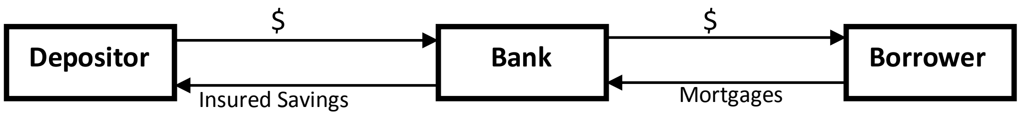


Figure 1: Traditional Banking

Under the traditional banking model, the arrival of some negative information on banks' investment returns may lead to severe bank runs. There is a large range of research related to this issue. For example, Corrado(2005) analyse how a supranational institution which acts as an international lender of last resort can cope with banking crises by guaranteeing run-proof bank deposit contracts in the traditional banking crisis.
In the past decades, banks, especially American banks, have adopted different measures to remain profitable in a rapidly changing financial system. One popular measure adopted by large modern banks is to participate in financial markets by originating and distributing securities as well as operating in the monetary market. Originally, the securitization market served as a source of financing and many assumed that securitization would provide a more resilient source of financing compared with the conventional market that depends on

borrowing from other financial intermediaries. Figure 2 shows the structure of the securitized banking system.

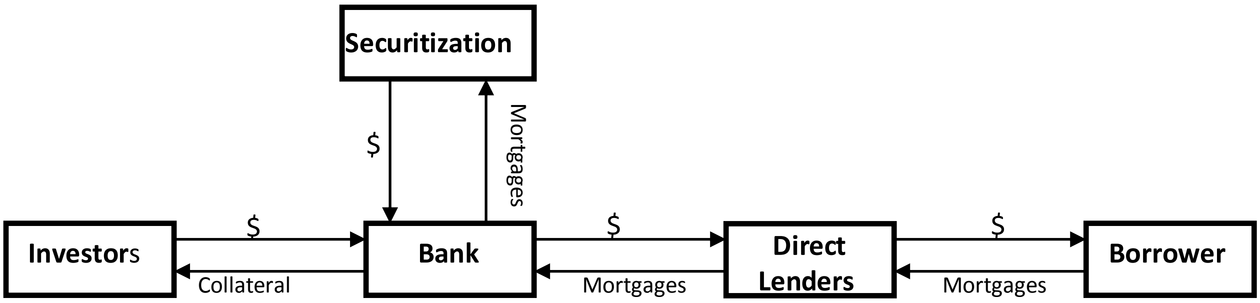


Figure 2: Securitized Banking

Through securitization, banks can increase their leverages. Securitization can offer perfect matched funding by eliminating funding exposure in terms of both duration and pricing basis. Apart from these advantages, the securitization can also help banks to reduce capital requirements, lock in profits, and transfer risks. Given those benefits, it is not surprising that securitization has grown rapidly in banking industry. As shown in Figure 3, private bond issuance of residential and commercial mortgage-backed securities (RMBS and CMBS), asset-backed securities (ABS), and collateralized debt obligations (CDOs) peaked in 2006 to nearly $2 trillion. In 2009, private issuance dropped to less than $150 billion, and almost all of it was asset-backed issuance supported by the Federal Reserve's TALF (Term Asset-Backed Securities Loan Facility) program to aid credit card and small-business lenders.

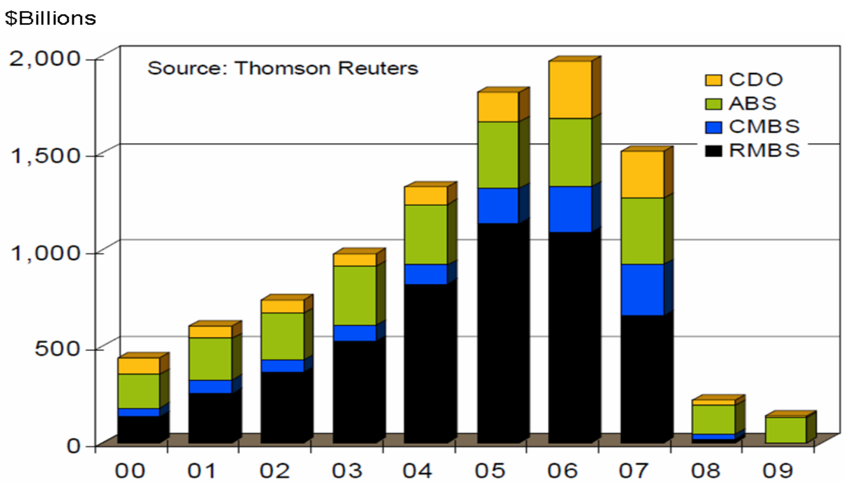
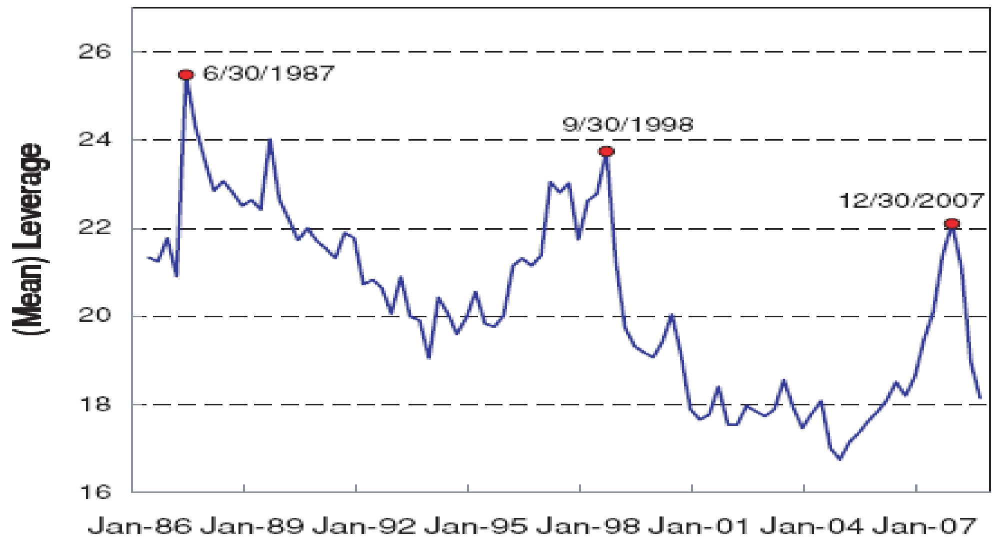


Figure 3: Securitization market in U.S ($ billions)

Does securitization lead to the financial stability? It is difficult to answer this question without formally modelling the underlying externalities associated with systemic financial crises. This paper attempts to model the impact of asset securitization on financial systemic risk. We measure the likelihood of a crisis by the probability that a bank liquids all its asset, and its scale (impact) in terms of the asset price. Lower asset prices correspond to more serious crises.

Leverage is another important factor to consider on the financial stability and securitization is

crucial in understanding the leverage of the financial system as a whole. Securitization enables credit expansion through higher leverage of the financial system as a whole. If the expansion of assets entailed by the growth in financial system leverage drives down lending standards, securitization may decrease financial stability rather than promoting it.
Figure 4 plots the leverage US primary dealers: First, leverage tends to decrease overall since 1986. This decline in leverage is due to the bank holding companies in the sample—a sample consisting only of investment banks shows no such trend in leverage (see Adrian and Shin, 2007). Secondly, each of the peaks in leverage was immediately followed by a financial crisis (the peaks are 1987Q2, 1998Q3, 2008Q3). Financial crisis tend to follow marked increases of leverage.



Source: SEC 10-K and 10-Q filings, sited in "Leverage, Securitization and Global Imbalances"
Note: the set of 18 banks that has a daily trading relationship with the Fed. They consist of US investment banks and US bank holding companies with large broker subsidiaries (such as Citigroup and JP Morgan Chase)

Figure 4: Mean Leverage of US Primary Dealers

More research is needed on the links between bank leverage and the securitization. In our model, banks make, securitize, distribute, and trade asset good, or they hold consume good. They also borrow capital good, using their security holdings as collateral, which will form different leverages. And we then examine the relationship between security, leverage and financial stability.

Our model predicts that asset securitization makes crises less likely since banks have more consume good to keep liquid by asset securitization. As a result, direct lenders are more willing to lend, allowing banks to increase their borrowing and initial investment. But, if a crisis does occur, losses will be greater. Overall, asset securitization may serve as a measure to reduce the likelihood of crises but to magnify their potential impact.
The paper proceeds as follows. Section 2 reviews the related literature. Section 3 sets up the benchmark model. Section 4 shows the model with securitization. Section 5 concludes the paper. The Appendix contains all the proofs.

2 Related Literature

Our paper is related to three broad strands of research: asset prices and financial stability, asset securitization and financial stability, financial leverage and financial stability. We attempt to combine these three strands by describing banks' choices of securitization and financial leverage as well as the impact on financial stability.

2.1 Asset prices and financial stability

The idea that the asset price influences on financial constraint and financial stability can be traced back at least as far as Veblen (1904, chap. 5), who described the positive interactions between asset prices and collateralized borrowing. Later, many researchers followed his idea and built up a large body of literature in this area. Bernanke and Gertler (1989), for example, construct an overlapping generation model in which financial market imperfections cause temporary shocks in net worth, and such shocks are to be amplified and to persist. In their model, a positive technology shock increases the labour demanded by the entrepreneurs who have been funded, and allows for more projects to be undertaken. Moreover, the accompanying rise in wage improves the financial position of the next generation of entrepreneurs, so more of their projects will be funded. They will subsequently demand more labour, and the cycle goes on. Kiyotaki and Moore (1997) extend the Bernanke and Gertler's story by illustrating the positive feedback through asset prices and the associated intertemporal multiplier process. During a business cycle, a major channel for shocks to net worth is through changes in the values of firms' assets or liabilities. Asset prices reflect future market conditions. When the effects of a shock persist (as in Bernanke and Gertler 1989), the cumulative impact on asset prices, and hence on net worth at the time of the shock, can be significant. They show that small, temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices.

Similar to Kiyotaki and Moore (1997), Lorenzoni (2008) studies the welfare properties of competitive equilibria in an economy with financial frictions hit by aggregate shocks. In particular, it shows that competitive financial contracts can result in excessive borrowing ex ante and excessive volatility ex post. The model provides a framework to evaluate preventive policies, which can be used during a credit boom to reduce the expected costs of a financial crisis. Existing studies allow for state-contingent financial contracts. However it is unclear to what extent the underlying externality drives their results. In contrast, our model compares the difference of the over borrowing rate with or without securitization between the actual financial contract and the state-contingent financial contracts which is clearer. Qiu(2011) build a general equilibrium model to analyze how the ability of banks to create money can affect asset prices and financial stability. His research focuses on the monetary policy while our research puts more attention to the relationship between asset price and asset securitization.

2.2 Asset securitization and financial stability

Traditional theories of financial intermediation describe banks as accepting deposits, primarily from households, and issuing loans, primarily to firms (Diamond and Dybvig, 1983; Diamond, 1984). The involvement of banks in security markets requires a revision of this theory. Gorton and Pennacchi (1995) and Allen and Gale (1997) are pioneers in modeling bank's operation in financial markets. There is now substantial evidence which suggests that securitization, the act of converting illiquid loans into liquid securities, contributed to bad loans. (Dell'Ariccia et al., 2008; Mian and Sufi,2008; Purnanandam,2008; Keys et al.,2009). By creating distance between the originators of loans and the investors who bear the final risk of default, securitization weakened lenders' incentives to screen borrowers, exacerbating the potential information asymmetries which lead to problems of moral hazard.

Using Bank Lending Survey, Maddaloni and Peydró (2009) study the determinants of bank lending standards in the Euro Zone, and find that high securitization activity amplifies the positive impact of low short-term interest rates on bank risk-taking. Using a large data set of securitized subprime loans in the U.S., Keys et al.(2009) find that loans originated by banks tend to default more relative to independent lenders. Their conclusions are well supported by Purnanandam(2008) and Loutskina and Strahan (2008). A central question surrounding the current subprime crisis is whether the securitization process reduced the incentives of financial intermediaries to carefully screen borrowers. Keys et al. (2010) examine this issue using data on securitized subprime mortgage loan contracts in the United States. Their findings suggest that existing securitization practices did adversely affect the screening incentives of subprime lenders. Although lots of empirical research shows the relationship between securitization and financial stability, there is little theoretical research in this area. We compare the securitization model to the benchmark model and test the validity findings of the empirical research.

2.3 Financial leverage and financial stability

Finally, this study is related to the extensive literature on leverage and financial systemic risk. Geanakoplos (1997, 2003) introduced the idea of endogenous margins or equilibrium leverage. Geanakoplos (2003) especially identified increasing volatility and increasing disagreement as causes of increased margins, and hence of the leverage cycle. Adrian and Shin (2008) put forward a theory of pro-cyclical leverage and credit availability based on the optimizing behaviour of financial intermediaries. In their model, pro-cyclical leverage comes from investment bank's focus on value at risk. Adrian and Shin argued that volatility is countercyclical, allowing banks to take more leveraged bets when asset prices are high.

Our approach is to build on Kiyotaki and Moore (1997) with three key differences.
First, we focus on asset securitization and its effect on financial leverage and systemic risk. In contrast, Kiyotaki and Moore (1997) focus on the dynamic interaction between credit limits and asset prices. Focusing on securitization allows us to closely explore its impact on financial leverage and systemic risk. Second, Kiyotaki and Moore (1997) analyse the economic shock on net value and asset price through linearization. In this study, we calculate the equilibrium and then analyse the impact on the two factors. This allows us to illustrate the results in a more intuitive way. Third, Kiyotaki and Moore (1997) construct an economy with indefinite periods while we consider a model with just three periods, so as to make the model simpler as well as illustrating the impact of shock.

Another closely related paper is Shleifer and Vishny (2010). They also look at banks' endogenous choice of liquidity which involves fire sales of illiquid assets. However, there are two major differences between our model and theirs. First, while their research only concentrates on at the choice of the securitization, our paper combines both the choice of securitization and its impact on asset price. Second, Shleifer andVishny (2010) doesn't consider any economic shock in the model while we introduce shock in our model to show the relationship between securitization, asset price and financial stability.

3 The Benchmark Model

We consider a model with three periods: 0(initial period), 1(intermediate period), and 2(final period). The economy produces two goods, consumption good and capital good. Consumption goods can be turned into capital goods on one for one basis at any point of time by the bankers, but the opposite is not feasible. There are two important assumptions. Firstly, we assume there are two types of players in the economy: bankers and direct lenders. They are risk neutral and identical within their group. Secondly, we assume the complete competition in the banking industry following similar practice in Allen and Gale (1998), Rochet andVives (2004), and Korinek (2008). Free entry into the banking industry forces banks to compete by offering deposit contracts that maximize the expected utility of the firms. Thus, the bankers in our model can also be interpreted as entrepreneurs -in a sense that they make financing decisions and are subject to business risk.

Both the banker and direct lender can produce consume good and are indifferent to consume in period 0, 1, 2. At period 0, 1, 2, the respective expected utilities of a banker and a direct lender are $\pi_0^b + \pi_1^b + \pi_2^b$ and $\pi_0^c + \pi_1^c + \pi_2^c$. There is no discounting and here is no interest conflict between the bank owners and the bankers. The objective functions of a banker and a direct lender are:

$$\text{Max } E_0(\pi_0^b + \pi_1^b + \pi_2^b) \text{ and Max } E_0(\pi_0^c + \pi_1^c + \pi_2^c) \qquad (1)$$

3.2 Endowments and Projects

Each banker has an endowment *n* of consumption goods at the beginning of his life and receives no further endowment in the following periods. Each direct lender receives a constant endowment *e* of consumption goods in each period.
Each direct lender owns a firm in the "traditional sector". Firms in the traditional sector invest capital k_{1s}^T in period 1 to produce consumption goods in period 2. The technology of the traditional sector is represented by the production function $G(k_{1s}^T)$. The function is increasing, strictly concave, twice differentiable and satisfies the following properties: $G(0) = 0$, $G^{'}(0) = 1$.

At the period t=1, there is a competitive spot market in which the consume good is exchanged for capital good at a price of q_{1s} .The other market is a one-period credit market in which one unit of consume good at period t=1 and t=2, is exchanged for a claim to R(R is a positive constant with the value no less than 1) units of consume good at period t=1 and t=2. For simplicity, we assume that the economy begins with no capital, so the price of capital is one at period 0, as long as some investment takes place.

Figure 5 depicts the timeline of events. The banker has access to the following technology. In period 0, they choose the level of investment k_0. In period 1, this investment yields $a_{1s}k_0$ units of consumption good, with $a_{1s} > 0$. Without economic shock, a_{1s} is a positive constant with a value larger than R. The economic shock, a_{1s} is random and follows the normal distribution depends on the aggregate state s. At the end of period 1, the banker chooses the capital stock for next period, k_{1s} , by making the net investment $k_{1s} - k_0$ ($k_{1s} - k_0 < 0$ means the banker sell part of his capital good). The capital stock k_{1s} produces A k_{1s} units of consumption goods in period 2, with A >R> 1. Capital fully depreciates at the end of period 2. To maximize his consumption, the banker can lend from the direct lenders b_t and repay R b_t in the period t+1(t=0,1). Both the banker and the direct lender can consume all the capital or consume good left in period 2. Since the economy ends at period 2, so there is no debt after period 2.

Let E(a_{1s}) >R, so that early investment in period 0 is expected to be profitable. If a_{1s} turns out to be less than 1, the bank has two options: it can either sell a portion of its capital to direct lenders and continue with the investment project; or it can go into liquidation, abandoning the project and selling all of its capital to direct lenders.

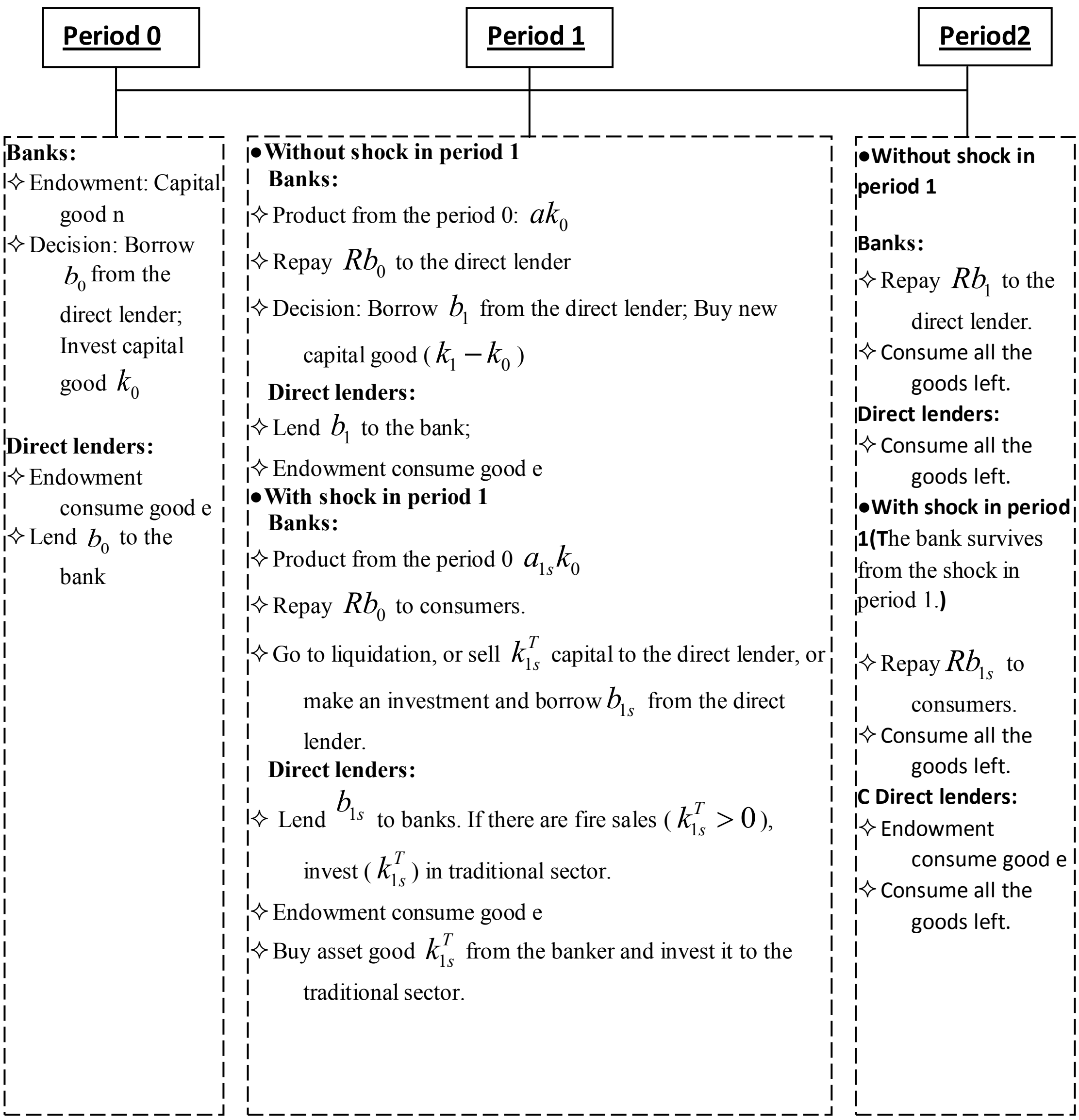


Figure 5: Timeline of the model

3.4 Equilibrium with no shock

We now solve for equilibrium with no shock. Since direct lenders expect investment in the productive sector of the economy to be profitable and, since they have very large endowments relative to bankers, they always meet the borrowing demands of the banker. If the banker has capital good k_t at the beginning of the period, then he can borrow b_t in total, as long as the repayment does not exceed the market value of his capital goods at period t+1(t=0,1), i.e.

$0 \le b_0 \le \theta q_0 n$ and $0 \le b_1 \le \theta q_1 k_0$ (2)

The two constraints specify that the banker can only borrow up to a fraction θ of the value of their assets in each period, where we define θ to be the maximum loan-to-value ratio as in Jermann and Quadrini (2006) and Gai et al. (2006). Since the economy ends at period 2, the banker has no incentive to borrow at period 2.

The banker can expand his scale of production by investing in more capital goods. We consider a banker who holds endowment capital good n and has incurred a total debt b_0. At the beginning of the period, he can invest k_0 which should satisfies

$k_0 \le n + b_0$ (3)

At period 1 the banker harvests ak_0 capital good, which, together with a new loan b_0, is available to cover the cost of buying new capital good, and to repay the accumulated debt Rb_0 (which includes interest). The banker's flow-of-funds constraint at the period 1 is thus

$q_1(k_1 - k_0) + Rb_0 \le ak_0 + b_1$ (4)

For the direct lender, without buying asset good from the fire sale of the banker, the direct lender would choose the debt $b_0^{'}$ and $b_1^{'}$ lends to the banker to maximize the utility.

$$Max(e - b_0^{'} + e + Rb_0^{'} - b_1^{'} + e + Rb_1^{'}) \text{ i.e. } Max[3e + (R-1)b_0^{'} + (R-1)b_1^{'}] \quad (5)$$

Market equilibrium is defined as a sequence of capital good prices and allocations of capital good, debt, and consumption, such that the banker maximizes the expected discounted utility (1) subject to the borrowing constraint (2) and the flow-of-funds constraint in period 0 and period 1; Each direct lender maximizes the expected discounted utility (1); and the markets for capital good and debt clear.

Before we identify the equilibrium, it's useful to begin with a preliminary result, as shown in Lemma 1.

Lemma 1.
In equilibrium, asset prices are characterized by the conditions $q_s = G^{'}(k_s^T)$, $k_{1s}^T = (k_0 - k_{1s})_+$ *for any state s.*

From this lemma, it follows that two cases are possible in the capital market. In the first case, the price of capital is one, the traditional sector chooses no investment, and the entrepreneurial sector makes positive investment $k_{1s} - k_0 > 0$. While in the second case, the price of capital $q_s = G^{'}(k_s^T)$ and the traditional sector chooses investment $k_0 - k_{1s} > 0$.

With these conditions, we can obtain proposition 1 which gives a characterization of optimal financial contracts between the banker and direct lender without economic shock.

Proposition 1.

In equilibrium without economic shock:

$q_0 = q_1 = 1$,

$b_0 = \theta n \qquad k_0 = (\theta+1)n \qquad b_1 = \theta(\theta+1)n \qquad k_1 = (\theta+1)(a+\theta+1)n - \theta nR$

$\pi_0^b = \pi_1^b = 0$

$\pi_2^b = Ak_1 - b_1 R = A[(\theta+1)(a+\theta+1)n - \theta nR] - \theta(\theta+1)nR$

The total consume of the direct lender would be $\pi_T^c = 3e + (R-1)\theta n + (R-1)\theta(\theta+1)n$

Proof can be found in the Appendix. The intuition behind Proposition 1 is simple. Without shock in period 1, the output of period 0 is ak_0 with certain. So the banker will borrow as much as possible. The banker and the direct lender reach the goal to maximize their utility. And we can see that, caeteris paribus, the consumption of the banker increases with a and θ ($\frac{\partial \pi_2^b}{\partial a} > 0$ and $\frac{\partial \pi_2^b}{\partial \theta} > 0$), and the consume of the direct lender increases with θ.

3.5 Equilibrium with shock

In this section, we solve for the competitive equilibrium by considering the optimization problem of the representative banker. Under the economic shock, the representative banker's optimization problem is given by:

$$\max E_0 \sum_{t=0,1,2} \pi_t^b$$

Subject to:

$k_0 \le n + b_0 \qquad (6)$

$q_{1s}(k_{1s} - k_0) + Rb_0 \le a_{1s}k_0 + b_{1s}$ Vs: partial or no liquidation (7)

$\pi_{1s}^{b} = q_{1s}k_{1s} - Rb_0$ Vs: total liquidation in period 1 (8)

$\pi_{2s}^{b} = Ak_{1s} - Rb_{1s}$ Vs: partial or no liquidation (9)

$\pi_{2s}^{b} = 0$ Vs: total liquidation in period 1 (10)

$0 \le b_0 \le \theta q_0 n$ (11)

$0 \le b_{1s} \le \theta q_{1s} k_0$ (12)

Equation (6) represents the banker's budget constraint in period 0: investment costs and any profits taken by the banker in period 0 must be financed by its endowment (initial net worth) and borrowing from direct lenders. In period 1, provided that the investment project continues (i.e. provided that the bank does not go into total liquidation), the bank's budget constraint is given by (7): Financing is provided by the start of the period assets at their market value and net period 1 borrowing, adjusted for the revenue surplus or shortfall. In period 2, profits are then given by (9).

By contrast, if the bank goes into total liquidation in period 1, it sells all of its capital at the market price, yielding $q_{1s}k_1$ in revenue. Therefore, its period 1 profits are given by (8), while period 2 profits are zero. Finally, (11) and (12) simply represent combined and simplified versions of the borrowing constraints.

Since the expected returns on investment are always high, it is clear that the bank will never take any profit until period 2 unless it goes into total liquidation. Therefore, $\pi_0 = \pi_{1s} = 0$ for all state s. Moreover, given that the high return between periods 1 and 2 is certain, banks wish to borrow as much as possible at period 1. So (12) binds at its upper bound and $b_0 = \theta q_0 n$. Finally, the asset price is only endogenous in period 1: $q_0 = 1$ because of the large supply of consumption goods in period 0 and no capital good sale.

Proposition 2 shows the optimal financial contracts between the banker and the direct lender with economic shock in period 1.

Proposition 2. *There exists a unique symmetric competitive equilibrium. In equilibrium, asset prices satisfy* $0 \le q_{1s} \le 1$. *Depending on the shock* a_{1s} *, the optimal debt is one of the following types:*

Type1: $a_{1s} + q_{1s} < R$

$$Z_0 - Z_{1s}R = Z_{1s}(a_{1s} + q_{1s}) - Z_{1s}R = Z_{1s}(a_{1s} + q_{1s} - R) < 0$$

Then optimum b_0^* *=0, in this case the bank will go to bankrupt or keep insolvent by liquidating part of capital good.* The over borrowing rate is θ.

Type2: $a_{1s} + q_{1s} = R$

$$Z_0 - Z_{1s}R = 0$$

Then optimum $b_0^* \in (0, \theta q_0 n)$ *, in this case the bank will liquidate part of his capital good to keep insolvent.* The over *borrowing rate lies within the pale of (0,* θ*).*

Type3: $a_{1s} + q_{1s} > R$

$$Z_0 - Z_{1s} R > 0$$

Then optimum debt $b_0^* = \theta q_0 n$ *, which is similar with the equilibrium without shock*

Where $Z_0 = \frac{A}{q_{1s}}(a_{1s} + q_{1s})$ *and* $Z_{1s} = \frac{A}{q_{1s}}$. *The over borrowing rate is 0.*

The variables Z_0 and Z_{1s} defined above, respectively, are the Lagrange multipliers on the budget constraints at periods 0 and 1; they represent the rates of return on bank wealth in periods 0 and 1.The banker must exhaust his borrowing capacity in the high state ($a_{1s} + q_{1s} > R$) while must borrow nothing in the low state($a_{1s} + q_{1s} < R$). In the media state ($a_{1s} + q_{1s} = R$), the banker is indifferent between borrow any from the direct lender. Obviously, in the low and media state, the banker is over borrowing ($b_0 = \theta q_0 n > b_0^*$). As $E(a_{1s}) > R$, the banker in the market aiming at maximum profit will always exhaust his borrowing capacity. The over borrowing rate in the low state is higher than in the media and high state.

Then we focus on the critical point that the bank liquidates all the capital good and the asset price. Using the result that the financial constraint is always binding in period 2 (from Appendix C), we can get

$$(k_{1s} - k_0) q_{1s} + b_0 R = a_{1s} k_0 + b_{1s} \quad (13)$$

Let $k_0 - k_{1s} = k_{1s}^T$, we can get

$$q_{1s} = \frac{b_0 R - a_{1s} k_0 - b_{1s}}{k_{1s}^T} \quad (14)$$

For $b_{1s} = \theta q_{1s} k_0$, we can rewrite it as

$$q_{1s} = \frac{\theta n R - a_{1s}(\theta + 1) n}{k_{1s}^T + \theta(1+\theta) n} \quad (15)$$

Equation (15) shows that the more assets the banker sells, the more deeply decreasing the asset price will be. Figure 6 gives a graphical illustration of the equilibrium in the state of low and media states for given values of k_0, b_0 and b_{1s}. Curve S plots the banker's supply of capital as a function of q_{1s}. The more the capital is sold, the further the price will decline.

An increase in b_{1s} increases the capital stock at period 0. The choice of b_0 affects the equilibrium price in another way. Figure 6 shows the effect of an increase in borrowing in period 0, leading to a rightward shift of the banker's supply and to a lower equilibrium price. For completeness, the figure includes the regions where $q_{1s} \geq A$ arise in equilibrium although such prices never arise in equilibrium as when q_{1s} goes above A, the banker's investment becomes unprofitable and will sell all the capital good k_0.

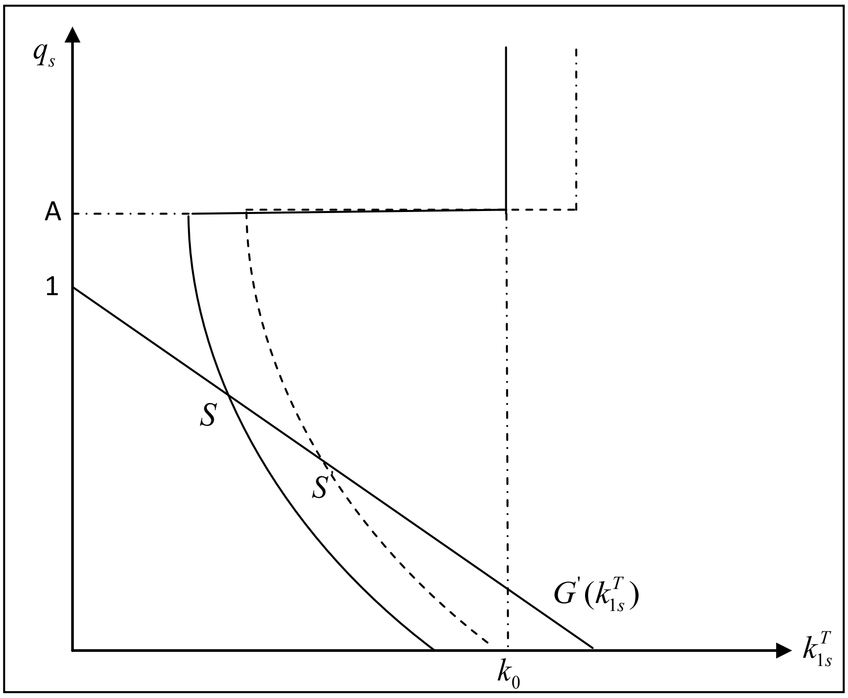


Figure 6 Asset market equilibrium

If the banker can't make the repayment in period 1, then the bank will go bankrupt. There exist two extreme cases:

Case 1: No capital good sale. In this case, there is a positive shock that allows the banker to make the repayment in period 1 without fire sale.

$k_{1s} - k_0 \geq 0$

From lemma 1, we can clearly find that $G'(0) = q_{1s} = 1$. Let $k_{1s} = k_0$ and we can get the threshold for a_{1s} from (13), which can be rewritten as

$$b_0 R = a_{1s} k_0 + b_{1s} \quad (16)$$

Let $b_0 = \theta n$ and $b_{1s} = \theta q_{1s} k_0$ into (16). We can get $a_{1s}^* = \frac{\theta}{\theta+1} R - \theta$. So we can see for any $a_{1s} \geq a_{1s}^*$, there will be no fire sale.

Case 2: Financial crisis. In this case, there is a negative shock that leads the representative banker to sell all the capital good and still can't keep solvent. Let $k_{1s}=0$ *and we can get the threshold for* a_{1s} *from (13).*

$$b_0 R = a_{1s}k_0 + b_{1s} + k_0 q_{1s} \qquad (17)$$

Let $b_0=\theta n$ and $b_{1s}=\theta q_{1s}k_0$ into (17). We can get $a_{1s}^{**}=\frac{\theta}{\theta+1}R-\theta q_{1s}-q_{1s}$. For $q_{1s}=G'(k_0)$, we can get $a_{1s}^{**}=\frac{\theta}{\theta+1}R-\theta G'(k_0)-G'(k_0)$. So we can see for any $a_{1s}\le\frac{\theta}{\theta+1}R-\theta G'(k_0)-G'(k_0)$, the bank will surely go bankrupt.

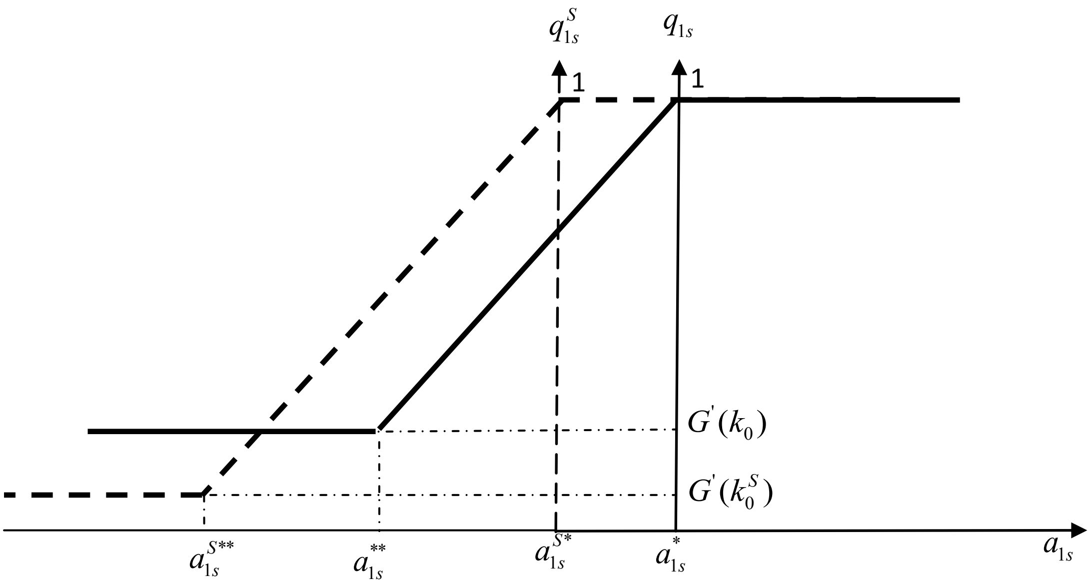


Figure7 The Asset price as a function of the shock

From figure 7, we can see clearly that the shock can directly have effect on asset price. When $a_{1s}>a_{1s}^{*}$, there will be no asset sale and $q_{1s}=1$. When $a_{1s}\le a_{1s}^{**}$, the bank will sell all the capital good and $q_{1s}=G'(k_0)$. With securitization, the threshold will be smaller. But when a crisis happens, the loss will be larger than the situation without securitization which we will show in next section.

4 The Model with Asset Securitization

We model securitization as the sale of cash flow claims that would otherwise be held by banks. We do not model packaging and tranching of loans, which are based on the securitization and essentially a process of re-securitization. It has similar influence on financial systemic risk like securitization. As in Gorton and Pennacchi (1995) and shleifer and Vishny(2010), we assume that the bank must initially keep a fraction *d* of the loan on its own books when it sells a loan in the market. If N projects are financed and the corresponding loans are securitized, the bank must hold dN of these securities on its balance sheet at the time of the underwriting. We assume that the bank does not need to hold on to these securities for more than one period.

When the bank securitizes a loan, it can sell the securities it does not retain in the market. We denote by p_t (t=0, 1) as the price of the securities at time t, which is an exogenous positive constant (The price can deviate from the rational price of 1 because of investor sentiment, which is not the focus of the paper.). The banker has an incentive to securitize the capital good only if $E(a_{1s}^S)-1>1-p_0$ in period 0 and $A-1>1-p_1$ in period 1.In the case of identical projects, all securities are obviously identical.

For the purpose to maximise his utility, the banker can securitize these loans. A suitably large portfolio of assets is "pooled" and transferred to a "special purpose vehicle" or "SPV" (the issuer), a tax-exempt company or trust formed for the specific purpose of funding the assets. And the SPV get them with price p_t at the beginning of period t and sell them to the buyer in the financial market. At the end of the period t+1, the banker should repay the buyer with the price of 1.

4.1 Securitization without shock

Let k_0^S, k_1^S, b_0^S and b_1^S denote the investment and borrowing in period 0 and 1 respectively. The superscript "S" denotes the state with securitization. As shown in section 3.5, the shock in the economy is $a^S=E(a_{1s})>R$. We assume that $a^S-1>1-p_0$ and $A-1>1-p_1$ so the banker has the incentive to securitize the capital good. Since the banker expects investment in the economy to be profitable and he will invest as much as possible. The direct lender has very large endowments relative to banker and he always meets the borrowing demands of the banker.

Excepting borrowing from the direct lenders, the banker can securitize these loans and sell them in the financial market (distribution). The banker can get $(\frac{1}{d}-1)np_0$ income from the selling in period 0 and $(\frac{1}{d}-1)k_0^S p_1$ in period 1.

If at period 0 and 1 the banker has capital good k_t, then he can borrow b_t in total, as long as the repayment does not exceed the market value of his capital goods and the income from securitization at period t+1(t=0,1), i.e.,

$$0\le b_0^S\le\theta[n+(\frac{1}{d}-1)np_0] \text{ and } 0\le b_1^S\le\theta[q_{1s}k_0^S+(\frac{1}{d}-1)k_0^S p_1] \quad (18)$$

In period 0, the banker can invest his initial wealth plus the amount borrowed from the direct lender and the income from securitization,

$$k_0^S\le n+b_0^S+(\frac{1}{d}-1)np_0 \quad (19)$$

The banker can expand production by investing in more capital goods. Consider a banker who holds investment k_0^S and has incurred a total debt b_0^S. At period 1 the bank harvests $a^S k_0^S$ capital good, which, together with a new loan b_1^S and profit from securitization $(\frac{1}{d}-1)k_0^S p_1+(a^S-1)(\frac{1}{d}-1)n$, is available to cover the cost of buying new capital good, to repay the accumulated debt Rb_0^S (which includes interest) and the face value of the securitization. The banker's flow-of-funds constraint at the period 1 is thus

$$(k_1^S-k_0^S)q_1+b_0^S R+(\frac{1}{d}-1)n \le a^S k_0^S+b_1^S+(\frac{1}{d}-1)k_0^S p_1+(a^S-1)(\frac{1}{d}-1)n \qquad (20)$$

At the end of period 2, the banker can harvests Ak_1^S capital good, get the profit from securitization $(A-1)(\frac{1}{d}-1)k_0^S$, repay the debt $b_1^S R$ and the face value of securitization in period 1. And then he can consume all the good left, that is,

$$Ak_1^S-b_1^S R-(\frac{1}{d}-1)k_0^S+(A-1)(\frac{1}{d}-1)k_0^S \qquad (21)$$

We can get proposition 3 by solving the problem the banker faces.

Proposition 3.

In equilibrium without economic shock:

$q_0=q_1=1$,

$$b_0^S=\theta n[1+(\frac{1}{d}-1)p_0]$$

$$k_0^S=(\theta+1)n[1+(\frac{1}{d}-1)p_0]$$

$$b_1^S=\theta(\theta+1)n[1+(\frac{1}{d}-1)p_1][1+(\frac{1}{d}-1)p_0]$$

$$k_1^S=(\theta+1)[a^S+1+(\frac{1}{d}-1)p_1][n+(\frac{1}{d}-1)np_0]+a^S(\frac{1}{d}-1)n-\theta[n+(\frac{1}{d}-1)np_0]R$$

$$\pi_0^{Sb}=\pi_1^{Sb}=0$$

$$\pi_2^{Sb} = Ak_1^S - b_1^S R = A\{(\theta+1)[a^S + 1 + (\frac{1}{d} - 1)p_1][n + (\frac{1}{d} - 1)np_0] + a^S(\frac{1}{d} - 1)n - \theta[n + (\frac{1}{d} - 1)np_0]R\}$$
$$-\theta[q_1 + (\frac{1}{d} - 1)p_1](\theta+1)[n + (\frac{1}{d} - 1)np_0]$$

The Appendix gives the detail poof of it. Comparing the Proposition 1 with Proposition 3, we can see obviously that:

$$\frac{\partial b_0^S}{\partial d} < 0,\ \frac{\partial b_1^S}{\partial d} < 0,\ \frac{\partial k_0^S}{\partial d} < 0,\ \frac{\partial k_0^S}{\partial d} < 0$$

$$b_0^S - b_0 > 0,\ b_1^S - b_1 > 0,\ k_0^S - k_0 > 0,\ k_1^S - k_1 > 0,\ \pi_2^{Sb} - \pi_2^b > 0$$

Through securitization, the banker improves his borrowing ability and investment ability greatly. So the number of projects financed in the economy becomes larger and the balance sheet expands. Also, profit at the end of period 2 is higher than the case of without securitization under no economic shock. The bank has greatly increased its profit ability through securitization, which will improve the social welfare.

4.2 Securitization with shock

Let k_0^S, k_{1s}^S, b_0^S and b_{1s}^S denote the investment and borrowing in period 0 and 1 respectively. With an economic shock, we need some additional conditions. Similar to the section 4.1, we assume that $E(a_{1s}^S) - 1 > 1 - p_0$ and $A - 1 > 1 - p_1$ so the banker has the incentive to securitize the capital good. As in part 3.5, the shock in the economy is denoted as $E(a_{1s}^S) > R$. Since the banker expects investment in the economy to be profitable and he will invest as much as possible. The direct lender has very large endowments relative to banker and he always meets the borrowing demands of the banker.

Excepting borrowing from the direct lenders, the banker can securitize these loans and sell them in the financial market (distribution). The banker can get $(\frac{1}{d} - 1)np_0$ income from the selling in period 0 and $(\frac{1}{d} - 1)k_0^S p_1$ in period 1.

If the banker has capital good k_t^S, then he can borrow b_t^S in total, as long as the repayment does not exceed the market value of his capital goods and the income from securitization at period t+1(t=0,1), i.e.,

$$0 \le b_0^S \le \theta[n + (\frac{1}{d} - 1)np_0] \text{ and } 0 \le b_{1s}^S \le \theta[q_{1s}k_0^S + (\frac{1}{d} - 1)k_0^S p_1] \quad (22)$$

In period 0, the banker can invest his initial wealth plus the amount borrowed from the direct lender and the income from securitization,

$$k_0^S \le n + b_0^S + (\frac{1}{d} - 1)np_0 \quad (23)$$

The banker can expand his scale of production by investing in more capital goods. Consider a banker who holds k_0^S, and has incurred a total debt of b_0^S. At period 1 the bank harvests $a_0^S k_0^S$ capital good, which, together with a new loan b_{1s}^S and profit from securitization $(\frac{1}{d}-1)k_0^S p_1 + (a_{1s}^S - 1)_+(\frac{1}{d}-1)n$, is available to cover the cost of buying new capital good, to repay the accumulated debt Rb_0^S (which includes interest) and the face value of the securitization. The banker's flow-of-funds constraint at the period 1 is thus

$$(k_{1s}^S - k_0^S)q_{1s} + b_0^S R + (\frac{1}{d}-1)n \le a_{1s}^S k_0^S + b_{1s}^S + (\frac{1}{d}-1)k_0^S p_1 + (a_{1s}^S - 1)_+(\frac{1}{d}-1)n \qquad (24)$$

At the end of period 2, the banker can harvests Ak_{1s}^S capital good get the profit from securitization $(A-1)_+(\frac{1}{d}-1)k_0^S$, repay the debt $b_{1s}^S R$ and the face value of securitization in period 1. And then he can consume all the good left, that is,

$$Ak_{1s}^S - b_{1s}^S R - (\frac{1}{d}-1)k_0^S + (A-1)_+(\frac{1}{d}-1)k_0^S \qquad (25)$$

Proposition 4 shows the optimal financial contracts between the banker and the direct lender with economic shock and securitization.

Proposition 4

Depending the shock of a_{1s}, there exist three cases:

Case1: $(A-1)(\frac{1}{d}-1) + (\frac{A}{q_{1s}}p_1 - 1)(\frac{1}{d}-1) + Z_{1s}^S(a_{1s}^S + q_{1s} - R) < 0$

$$Z_0^S - Z_{1s}^S R < 0$$

Then optimum $b_0^* = 0$, in this case the bank will go bankrupt or liquidate part of the capital good to keep insolvency. The over borrowing rate is $\theta[1+(\frac{1}{d}-1)p_0]$.

Case2: $(A-1)(\frac{1}{d}-1) + (\frac{A}{q_{1s}}p_1 - 1)(\frac{1}{d}-1) + Z_{1s}^S(a_{1s}^S + q_{1s} - R) = 0$

$$Z_0^S - Z_{1s}^S R = 0$$

Then optimum $b_0^* \in (0, \theta[n+(\frac{1}{d}-1)np_0])$*, in this case the bank will liquidate part of his capital good. The over borrowing rate lies in (0,* $\theta[1+(\frac{1}{d}-1)p_0]$*).*

Case3: $(A-1)(\frac{1}{d}-1)+(\frac{A}{q_{1s}}p_1-1)(\frac{1}{d}-1)+Z_{1s}^S(a_{1s}^S+q_{1s}-R)>0$

$$Z_0^S - Z_{1s}^S R > 0$$

Then optimum $b_{1s}^S = \theta[n+(\frac{1}{d}-1)np_0]$. *The over borrowing rate is 0.*

Where $Z_0^S = (A-2)(\frac{1}{d}-1)+Z_{1s}^S a_{1s}^S + Z_{1s}^S(\frac{1}{d}-1)p_1 + Z_{1s}^S q_{1s}$ *and* $Z_{1s}^S = \frac{A}{q_{1s}}$

We can see clearly that the over borrowing rates in the low and media economic shock states decrease with d. Comparing to Proposition 2, we can obtain that the over borrowing rates with securitization in the low and media economic shock states is larger than the situation without securitization.

Then we focus on the critical point that the banker keeps solvency or liquidates all the capital good and the asset price. Using the result that the financial constraint is always binding in period 2 (from Appendix C), we can get

$$(k_{1s}^S - k_0^S)q_{1s} + b_0^S R + (\frac{1}{d}-1)n = a_{1s}^S k_0^S + b_{1s}^S + (\frac{1}{d}-1)k_0^S p_1 + (a_{1s}^S - 1)_+(\frac{1}{d}-1)n. \quad (26)$$

If the banker can't make the repayment in period 1, then the bank will go bankrupt. There exist two extreme cases:

Case 1: No capital good sale. In this case, there is a positive shock that make the banker can make the repayment in period 1 without fire sale.

$k_{1s} - k_0 \geq 0$

From lemma 1, we can clearly find that $G'(0) = q_{1s} = 1$. Let $k_{1s}^S = k_0^S$ and we can get the threshold for a_{1s}^S from (24), which can be rewritten as

$$a_{1s}^S = \frac{b_0^S R + (\frac{1}{d}-1)n - b_{1s}^S - (\frac{1}{d}-1)k_0^S p_1 - (a_{1s}-1)_+(\frac{1}{d}-1)n}{k_0^S} \quad (27)$$

Let $b_0^S = \theta[n+(\frac{1}{d}-1)np_0]$ and $b_{1s}^S = \theta[q_{1s}k_0^S + (\frac{1}{d}-1)k_0^S p_1]$ into (27). We can get $a_{1s}^{S*} = \frac{\theta}{\theta+1}R - \theta - (\frac{1}{d}-1)[p_1 - \frac{1}{(\theta+1)[1+(\frac{1}{d}-1)p_0]}]$. So we can see for any $a_{1s}^S \geq a_{1s}^{S*}$, there will be no fire sale.

Case 2: The banker liquidates all the good. In this case, there is a negative shock that leads the bank to sell all the capital good and still can't keep solvent. Let $k_{1s}^S = 0$ and we can get the threshold for a_{1s} from (26).

$$a_{1s}^{S**} = \frac{b_0^S R + (\frac{1}{d}-1)n - b_{1s}^S - (\frac{1}{d}-1)k_0^S p_1 - (a_{1s}^S - 1)_+ (\frac{1}{d}-1)n - k_0^S q_{1s}}{k_0^S} \quad (28)$$

Let $b_0^S = \theta[n + (\frac{1}{d}-1)np_0]$ and $b_{1s}^S = \theta q_{1s} k_0^S + (\frac{1}{d}-1)k_0^S p_1$ into (28). We can get $a_{1s}^{S**} = \frac{\theta}{\theta+1}R - \theta q_{1s} - (\frac{1}{d}-1)\frac{1}{(\theta+1)[1+(\frac{1}{d}-1)p_0]} - q_{1s}$. For $q_{1s} = G'(k_0^S)$, we can get $a_{1s}^{S**} = \frac{\theta}{\theta+1}R - \theta G'(k_0^S) - (\frac{1}{d}-1)\frac{1}{(\theta+1)[1+(\frac{1}{d}-1)p_0]} - G'(k_0^S)$ So we can see for any $a_{1s}^S \le a_{1s}^{S**}$, the bank will surely go bankrupt.

Comparing to the situation without securitization under economic shock, obviously, $a_{1s}^{S*} < a_{1s}^*$, $a_{1s}^{S**} < a_{1s}^{**}$. When the banker liquidates all the capital good, the price will decline more than the situation without securitization ($G'(k_0^S) < G'(k_0)$). We can watch directly from figure 7. Since a_{1s} follows normal distribution, we can obtain that $P(a_{1s} \le a_{1s}^{S**}) < P(a_{1s} \le a_{1s}^{**})$ which shows that securitization decrease the probability that banks go to liquidation.

5 Welfare Analysis

Let us next investigate the behaviour of a social planner who optimizes the banker' allocations. The social planner's objective is the same as that of the banker. However, whereas decentralized bankers take asset prices q_{1s} as given, the social planner internalizes that the valuation of assets declines the more she sells.

This changes the social planner's first-order condition on land k_0^{SP} and k_{1s}^{SP} to

$$\text{FOC}(k_0^{SP}):\ (\frac{1}{d}-1)A - Z_0^{SP} + Z_{1s}^{SP} a_{1s} + Z_{1s}^{SP}(\frac{1}{d}-1)p_1 + Z_{1s}^{SP} q_{1s} - Z_{1s}^{SP}(k_{1s}^{SP} - k_0^{SP})\frac{\partial q_{1s}}{\partial k_0^{SP}} \le 0$$

$$\text{FOC}(k_{1s}^{SP}):\ A - Z_{1s}^{SP} q_{1s} - Z_{1s}^{SP}(k_{1s}^{SP} - k_0^{SP})\frac{\partial q_{1s}}{\partial k_{1s}^{SP}} \le 0$$

For $k_0^{SP} > 0$ and $k_{1s}^{SP} > 0$, we can get

$$(\frac{1}{d}-1)A - Z_0^{SP} + Z_{1s}^{SP} a_{1s} + Z_{1s}^{SP}(\frac{1}{d}-1)p_1 + Z_{1s}^{SP} q_{1s} - Z_{1s}^{SP}(k_{1s}^{SP} - k_0^{SP})\frac{\partial q_{1s}}{\partial k_0^{SP}} = 0$$

$$A - Z_{1s}^{SP} q_{1s} - Z_{1s}^{SP} (k_{1s}^{SP} - k_0^{SP}) \frac{\partial q_{1s}}{\partial k_{1s}^{SP}} = 0$$

As long as $k_{1s}^{SP} - k_0^{SP} > 0$, we can see that there is no capital good sale , $q_{1s} = 1$ and $\frac{\partial q_{1s}}{\partial k_0^{SP}} = 0$.

The planner's first-order condition coincides with that of the banker. However, when there is capital good sale in period 1, then the social planner's valuation of liquidity becomes

$$Z_0^{SP} = (\frac{1}{d} - 1)A + Z_{1s}^{SP} a_{1s} + Z_{1s}^{SP} (\frac{1}{d} - 1) p_1 + Z_{1s}^{SP} q_{1s} - Z_{1s}^{SP} (k_{1s}^{SP} - k_0^{SP}) \frac{\partial q_{1s}}{\partial k_0^{SP}}$$

$$Z_{1s}^{SP} = \frac{A}{q_{1s} + (k_{1s}^{SP} - k_0^{SP}) \frac{\partial q_{1s}}{\partial k_{1s}^{SP}}}$$

From (26), $q_{1s} = \frac{(\frac{1}{d} - 1)n + b_0^{SP} R - a_{1s} k_0^{SP} - b_{1s}^{SP} - (\frac{1}{d} - 1) k_0^{SP} p_1 - (a_{1s} - 1)_+ (\frac{1}{d} - 1) n}{(k_0^{SP} - k_{1s}^{SP})}$, we can get that $\frac{\partial q_{1s}}{\partial k_{1s}^{SP}} > 0$ and $\frac{\partial q_{1s}}{\partial k_0^{SP}} < 0$.The asset price q_{1s} declines the more of the asset is sold from bankers to direct lender (i.e. the smaller k_{1s}). And we can get $Z_0^{SP} < Z_0^S$ and $Z_{1s}^{SP} > Z_{1s}^S$. We can summarize the result in the following proposition:

Proposition 5

When facing financing constraints, the social planner values liquidity more since he internalizes that higher liquidity would reduce the quantity of resales required and would therefore mitigate the decline in asset price and the tightness of economy-wide financing constraints.

Proposition 5 shows that: When financing constraints are binding, a decline in asset prices hurts all bankers since it reduces the amount of liquidity that they can raise from the sale of each unit of assets. Bankers take asset prices as given since they realize that their individual behaviour has only an infinitesimal effect on asset prices. However, the behaviour of all bankers together can lead to large fluctuations in asset prices.

The externality in our setup stems from the failure of decentralized bankers to internalize the effects of their risk-taking decisions on asset prices and by implication the effects on the financial constraints faced by other bankers. First-best policy measures against the described systemic externalities would attempt to break the feedback cycle underlying the financial amplification effects. Financial regulators should induce market participants to take precautions against some of the risk they are holding on their balance sheets and limit the quantity holding of off - balance sheet.

6 Conclusions

Prior to the Global Financial Crisis in 2008, the general view regarding securitisation is that it reduces credit risk and enhances the resilience of the financial system in dealing with defaults. Attention has been paid on distorted incentives developed at all stages of the securitisation process which expands the balance sheet and over-borrowing.

In this paper we analyse the effects of securitization on the risk of default of a representative bank. Our model shows that banks can increase its profitability considerably through securitization if there is no shock or a positive shock. At the same time securitization decreases the probability that systemic crisis happen. But by securitization, bankers take on too much risk in both their financing and investment decisions; more generally they over-borrow and over-invest, which will suffer more loss when systemic crisis happen.

This paper demonstrate the effect that securitization has on the financial stability using a relatively simple three period model. Our model lays a theoretical foundation for the empirical research. Moreover, by extending the model to social welfare analysis, we illustrate the implication for the regulatory direction of securitization. Given the connection between securitization and systemic risk and systemic externalities, this model can be utilised by regulators in reducing financial instability and avoiding future systemic financial crises.

Appendix

A. Proof of lemma 1

The consumer chooses k_s^T to maximize expected utility, that is Max[$G(k_s^T) - q_s k_s^T$]. For G( )is a strictly concave function with $G^{'}(0) = 1$. Therefore, if $q_s \geq 1$, optimal investment is k_s^T =0, while if $q_s < 1$, k_s^T is positive and satisfies the first-order condition $q_s = G^{'}(k_s^T)$.

B. Proof of Proposition 1

For the banker, consider the problem of maximizing consumption subject to (A1.1)–(A1.4) and non-negativity constraints for k_0 and k_1.

$Max(Ak_1 - b_1 R)$

s.t. $k_0 \leq n + b_0$ (A1.1)

$(k_1 - k_0)q_1 + b_0 R \leq ak_0 + b_1$ (A1.2)

And $0 \leq b_0 \leq \theta q_0 n$ (A1.3)

$0 \leq b_1 \leq \theta q_1 k_0$ (A1.4)

Let Z_0 and Z_1 denote the Lagrange multipliers associated, respectively, to (A1.1) and (A1.2). The Lagrange equation would be:

$L = Ak_1 - b_1 R + Z_0(n + b_0 - k_0) + Z_1[ak_0 + b_1 - b_0 R - (k_1 - k_0)q_1] \leq 0$

An optimum is characterized by the following first-order conditions:

$\frac{\partial L}{\partial k_0} = -Z_0 + Z_1 a + Z_1 q_1] \leq 0$ with strictly equation if $k_0 > 0$ .And we can obtain that

$Z_0 = Z_1(a + q_1)$

$\frac{\partial L}{\partial k_1} = A - Z_1 q_1] \leq 0$ with strictly equation if $k_1 > 0$ .And we can obtain that $Z_1 = \frac{A}{q_1} > 1$, so $Z_0 = Z_1(a + q_1) > 1$. For both Z_0 and Z_1 are strictly positive, the constraint (A1.1) and (A1.2)are binding.

$k_0 = n + b_0$ and $(k_1 - k_0)q_1 + b_0 R = ak_0 + b_1$

For $\frac{\partial L}{\partial b_0} = Z_0 - Z_1 R$ and $\frac{\partial L}{\partial b_1} = Z_1 - R$, we can find:

if $Z_0 - Z_1 R < 0$, then b_0=0,

if $Z_0 - Z_1 R = 0$, then $b_0 \in (0, \theta q_0 n)$,

if $Z_0 - Z_1 R > 0$, then $b_0 = \theta q_0 n$,

and

if $Z_1 - R < 0$, then $b_1 = 0$,

if $Z_1 - R = 0$, then $b_1 \in (0, \theta q_1 k_0)$,

if $Z_1 - R > 0$, then $b_1 = \theta q_1 k_0$.

From Assumption A> a >R, we can get that $Z_0 = Z_1(a + q_1) > Z_1 R$, and $Z_0 - Z_1 R > 0$, so $b_0 = \theta q_1 n$.

Since $Z_1 = \dfrac{A}{q_1} \geq A > R$, we can get $b_0 = \theta q_1 k_0$.

For the direct lender, with no buying asset good from the fire sale of the banker, the direct lender would choose the debt $b_0^{'}$ and $b_1^{'}$ lends to the banker to maximize the utility.

$$Max(e - b_0^{'} + e + Rb_0^{'} - b_1^{'} + e + Rb_1^{'}) \text{ i.e. } Max[3e + (R-1)b_0^{'} + (R-1)b_1^{'}]$$

For R>1, we can find that the more the direct lender lend, the more his utility would be, with the debit market clear condition $b_0 = b_0^{'}$ and $b_1 = b_1^{'}$.

For $q_0 = q_1 = 1$, in equilibrium,

$$b_0 = \theta n \qquad k_0 = (\theta+1)n \qquad b_1 = \theta(\theta+1)n \qquad k_1 = (\theta+1)(a+\theta+1)n - \theta nR$$

$$\pi_2^b = Ak_1 - b_1 R = A[(\theta+1)(a+\theta+1)n - \theta nR] - \theta(\theta+1)nR$$

C. Proof of Proposition 2

Similarly with the proof of proposition 2, for the banker, consider the problem of maximizing (A2.1)subject to (A2.2)–(A2.5) and non-negativity constraints for k_0 and k_1.

$Max(Ak_{1s} - b_{1s}R)$ (A2.1)

s.t. $k_0 \leq n + b_0$ (A2.2)

$(k_{1s} - k_0)q_{1s} + b_0 R \leq ak_0 + b_{1s}$ (A2.3)

And $0 \leq b_0 \leq \theta q_0 n$ (A2.4)

$0 \le b_{1s} \le \theta q_{1s} k_0$ (A2.5)

Let Z_0 and Z_{1s} denote the Lagrange multipliers associated, respectively, to (A2.2) and (A2.3). The Lagrange equation would be:

$L = Ak_{1s} - b_{1s}R + Z_0(n + b_0 - k_0) + Z_{1s}[a_{1s}k_0 + b_{1s} - b_0R - (k_{1s} - k_0)q_{1s}] \le 0$

An optimum is characterized by the following first-order conditions:

$\frac{\partial L}{\partial k_0} = -Z_0 + Z_{1s}a_{1s} + Z_{1s}q_{1s}] \le 0$ with strictly equation if $k_0 > 0$.And we can obtain that

$Z_0 = Z_{1s}(a_{1s} + q_{1s})$

$\frac{\partial L}{\partial k_{1s}} = A - Z_{1s}q_{1s}] \le 0$ with strictly equation if $k_{1s} > 0$.And we can obtain that $Z_{1s} = \frac{A}{q_{1s}} > 1$, so $Z_0 = Z_{1s}(a_{1s} + q_{1s}) > 1$. For both Z_0 and Z_{1s} are strictly positive, the constraint (A2.2) and (A2.3) are binding.

$k_0 = n + b_0$ and $(k_{1s} - k_0)q_{1s} + b_0R = a_{1s}k_0 + b_{1s}$

For $\frac{\partial L}{\partial b_0} = Z_0 - Z_{1s}R$ and $\frac{\partial L}{\partial b_{1s}} = Z_{1s} - R$, we can find:

if $Z_0 - Z_{1s}R < 0$, then b_0=0,

if $Z_0 - Z_{1s}R = 0$, then $b_0 \in (0, \theta q_0 n)$,

if $Z_0 - Z_{1s}R > 0$, then $b_0 = \theta q_0 n$,

and

if $Z_{1s} - R < 0$, then $b_{1s} = 0$,

if $Z_{1s} - R = 0$, then $b_{1s} \in (0, \theta q_{1s} k_0)$,

if $Z_{1s} - R > 0$, then $b_{1s} = \theta q_{1s} k_0$.

From Assumption A>R, we can get $Z_{1s} > R$, so $b_{1s} = \theta q_{1s} n$.

For the direct lender, with no buying asset good from the fire sale of the banker, the direct lender would choose the debt $b_0^{'}$ and $b_1^{'}$ lends to the banker to maximize the utility.

$Max(e - b_0^{'} + e + Rb_0^{'} - b_1^{'} + e + Rb_1^{'} + G(k_{1s}^T) - q_{1s}k_{1s}^T)$ i.e.

$Max[3e + (R-1)b_0^{'} + (R-1)b_1^{'} + G(k_{1s}^T) - q_{1s}k_{1s}^T]$

For R>1, we can find that the more the direct lender lend, the more his utility would be, with the debit market clear condition $b_0 = b_0^{'}$ and $b_1 = b_1^{'}$, and the capital good clear condition $q_{1s} = G^{'}(k_{1s}^T)$, $k_{1s}^T = (k_0 - k_{1s})_+$.

Depending the shock of ($a_{1s} + q_{1s}$), there exist three cases:

Case1: $a_{1s} + q_{1s} < R$

$$Z_0 - Z_{1s}R = Z_{1s}(a_{1s} + q_{1s}) - Z_{1s}R = Z_{1s}(a_{1s} + q_{1s} - R) < 0$$

Then optimum b_0=0, in this case the bank will go to bankrupt.

Case2: $a_{1s} + q_{1s} = R$

$$Z_0 - Z_{1s}R = 0$$

Then optimum $b_0 \in (0, \theta q_0 n)$, in this case the bank will liquidate part of his capital good.

Case3: $a_{1s} + q_{1s} > R$

$$Z_0 - Z_{1s}R > 0$$

Then optimum debt $b_0 = \theta q_0 n$, which is simsssilar with the equilibrium without shock in section 3.4.

D. Proof of Proposition 3

$$Max[Ak_1^S - b_1^S R - (\frac{1}{d} - 1)k_0^S + (A-1)(\frac{1}{d} - 1)k_0^S] \quad \text{(A3.1)}$$

s.t. $k_0^S \le n + b_0^S + (\frac{1}{d} - 1)np_0$ (A3.2)

$$(k_1^S - k_0^S)q_1 + b_0^S R + (\frac{1}{d} - 1)n \le a^S k_0^S + b_1^S + (\frac{1}{d} - 1)k_0^S p_1 + (a^S - 1)(\frac{1}{d} - 1)n \quad \text{(A3.3)}$$

And $0 \le b_0^S \le \theta[n + (\frac{1}{d} - 1)np_0]$ (A3.4)

$$0 \le b_1^S \le \theta[q_{1s}k_0^S + (\frac{1}{d} - 1)k_0^S p_1] \quad \text{(A3.5)}$$

Let Z_0^S and Z_1^S denote the Lagrange multipliers associated, respectively, to (A3.2) and (A3.2). The Lagrange equation would be:

$$L = Ak_1^S - b_1^S R - (\frac{1}{d}-1)k_0^S + (A-1)(\frac{1}{d}-1)k_0^S + Z_0^S(n + b_0^S + (\frac{1}{d}-1)np_0 - k_0^S) +$$
$$Z_1^S[a^S k_0^S + b_1^S + (\frac{1}{d}-1)k_0^S p_1 + (a^S-1)(\frac{1}{d}-1)n - (k_1^S - k_0^S)q_1 - b_0^S R - (\frac{1}{d}-1)n] \le 0$$

An optimum is characterized by the following first-order conditions:

$\frac{\partial L}{\partial k_0^S} = (\frac{1}{d}-1)A - 2(\frac{1}{d}-1) - Z_0^S + Z_1^S a^S + Z_1^S(\frac{1}{d}-1)p_1 + Z_1^S q_1 \le 0$ with strictly equation if $k_0^S > 0$.

Since $k_0^S > 0$, we can get $(\frac{1}{d}-1)A - 2(\frac{1}{d}-1) - Z_0^S + Z_1^S a_0^S + Z_1^S(\frac{1}{d}-1)p_1 + Z_1^S q_1 = 0$. So

$$Z_0^S = (\frac{1}{d}-1)A - 2(\frac{1}{d}-1) + Z_1^S a_0^S + Z_1^S(\frac{1}{d}-1)p_1 + Z_1^S q_1$$

$\frac{\partial L}{\partial k_1^S} = A - Z_1^S q_1 \le 0$ with strictly equation if $k_1^S > 0$. And we can obtain that $Z_1^S = \frac{A}{q_1} > 1$, so

$$Z_0^S = (\frac{1}{d}-1)A - 2(\frac{1}{d}-1) + Z_1^S a_0^S + Z_1^S(\frac{1}{d}-1)p_1 + Z_1^S q_1 > Z_1^S > 1.$$

For $\frac{\partial L}{\partial b_0^S} = Z_0^S - Z_1^S R$ and $\frac{\partial L}{\partial b_1^S} = Z_1^S - R$, we can find:

if $Z_0^S - Z_1^S R < 0$, then $b_0^S = 0$,

if $Z_0^S - Z_1^S R = 0$, then $b_0^S \in (0, \theta[n + (\frac{1}{d}-1)np_0])$,

if $Z_0^S - Z_1^S R > 0$, then $b_0^S = \theta[n + (\frac{1}{d}-1)np_0]$,

and

if $Z_1^S - R < 0$, then $b_1^S = 0$,

if $Z_1^S - R = 0$, then $b_1^S \in (0, \theta[q_1 k_0^S + (\frac{1}{d}-1)k_0^S p_1])$,

if $Z_1^S - R > 0$, then $b_1^S = \theta[q_1 k_0^S + (\frac{1}{d}-1)k_0^S p_1]$.

Since $Z_1^S = \frac{A}{q_1} > R$ and $Z_0^S > Z_1^S > 0$, we can obtain that $b_1^S = \theta q_1 k_0^S + (\frac{1}{d}-1)k_0^S p_1$ and

$b_0^S = \theta[n + (\frac{1}{d}-1)np_0]$.

For both Z_0^S and Z_1^S are strictly positive, the constraint (A3.2) and (A3.3) are binding. We can get $k_0^S = n + b_0^S + (\frac{1}{d} - 1)np_0 = (\theta+1)[n + (\frac{1}{d}-1)np_0]$ and

$$(k_1^S - k_0^S)q_1 + b_0^S R + (\frac{1}{d}-1)n = a^S k_0^S + b_1^S + (\frac{1}{d}-1)k_0^S p_1 + (a^S - 1)(\frac{1}{d}-1)n \quad \text{(A3.6)}$$

For $b_1^S = \theta(\theta+1)n[1+(\frac{1}{d}-1)p_1][1+(\frac{1}{d}-1)p_0]$ and $b_0^S = \theta n[1+(\frac{1}{d}-1)p_0]$, we can rewrite it as

$$k_1^S = (\theta+1)[a^S + 1 + (\frac{1}{d}-1)p_1][n+(\frac{1}{d}-1)np_0] + a^S(\frac{1}{d}-1)n - \theta[n+(\frac{1}{d}-1)np_0]R$$

Then we can get the utility of the banker in the period 2:

$$\pi_2^b = Ak_1^S - b_1^S R - (\frac{1}{d}-1)k_0^S + (A-1)(\frac{1}{d}-1)k_0^S = A\{(\theta+1)[a^S + 1 + (\frac{1}{d}-1)p_1][n+(\frac{1}{d}-1)np_0] + a^S(\frac{1}{d}-1)n -$$
$$-\theta[q_1 + (\frac{1}{d}-1)p_1](\theta+1)[n+(\frac{1}{d}-1)np_0]$$

E. Proof of Proposition 4

$$Max[Ak_{1s}^S - b_{1s}^S R - (\frac{1}{d}-1)k_0^S + (A-1)(\frac{1}{d}-1)k_0^S] \quad \text{(A4.1)}$$

s.t. $k_0^S \le n + b_0^S + (\frac{1}{d}-1)np_0$ (A4.2)

$$(k_{1s}^S - k_0^S)q_{1s} + b_0^S R + (\frac{1}{d}-1)n \le a_{1s}^S k_0^S + b_{1s}^S + (\frac{1}{d}-1)k_0^S p_1 + (a_{1s}^S - 1)_+(\frac{1}{d}-1)n \quad \text{(A4.3)}$$

And $0 \le b_0^S \le \theta[n+(\frac{1}{d}-1)np_0]$ (A4.4)

$$0 \le b_{1s}^S \le \theta[q_{1s}k_0^S + (\frac{1}{d}-1)k_0^S p_1] \quad \text{(A4.5)}$$

Since A>1, we can get $(A-1)_+ = A-1$.

Let Z_0^S and Z_{1s}^S denote the Lagrange multipliers associated, respectively, to (A4.2) and (A4.3). The Lagrange equation would be:

$$L = Ak_{1s}^S - b_{1s}^S R - (\frac{1}{d}-1)k_0^S + (A-1)(\frac{1}{d}-1)k_0^S + Z_0^S(n + b_0^S + (\frac{1}{d}-1)np_0 - k_0^S) +$$

$$Z_{1s}^S[a_{1s}^S k_0^S + b_{1s}^S + (\frac{1}{d}-1)k_0^S p_1 + (a_{1s}^S - 1)_+(\frac{1}{d}-1)n - (k_{1s}^S - k_0^S)q_{1s} - b_0^S R - (\frac{1}{d}-1)n] \leq 0$$

An optimum is characterized by the following first-order conditions:

$\frac{\partial L}{\partial k_0^S} = (A-2)(\frac{1}{d}-1) - Z_0^S + Z_{1s}^S a_{1s} + Z_{1s}^S(\frac{1}{d}-1)p_1 + Z_{1s}^S q_{1s} \leq 0$ with strictly equation if $k_0^S > 0$.

Since $k_0^S > 0$, we can get $(A-2)(\frac{1}{d}-1) - Z_0^S + Z_{1s}^S a_{1s} + Z_{1s}^S(\frac{1}{d}-1)p_1 + Z_{1s}^S q_{1s} = 0$. So

$$Z_0^S = (A-2)(\frac{1}{d}-1) + Z_{1s}^S a_{1s} + Z_{1s}^S(\frac{1}{d}-1)p_1 + Z_{1s}^S q_{1s}$$

$\frac{\partial L}{\partial k_{1s}^S} = A - Z_{1s}^S q_{1s} \leq 0$ with strictly equation if $k_{1s}^S > 0$. And we can obtain that $Z_{1s}^S = \frac{A}{q_{1s}} > 1$, so $Z_0^S = Z_{1s}^S(a_{1s}^S + q_{1s}) > 1$. For both Z_0^S and Z_{1s}^S are strictly positive, the constraint (A4.2) and (A4.3) are binding.

For $\frac{\partial L}{\partial b_0^S} = Z_0^S - Z_{1s}^S R$ and $\frac{\partial L}{\partial b_{1s}^S} = Z_{1s}^S - R$, we can find:

if $Z_0^S - Z_{1s}^S R < 0$, then b_0^S=0,

if $Z_0^S - Z_{1s}^S R = 0$, then $b_0^S \in (0, \theta[n + (\frac{1}{d}-1)np_0])$,

if $Z_0^S - Z_{1s}^S R > 0$, then $b_0^S = \theta[n + (\frac{1}{d}-1)np_0]$,

and

if $Z_{1s}^S - R < 0$, then $b_{1s}^S = 0$,

if $Z_{1s}^S - R = 0$, then $b_{1s}^S \in (0, \theta[q_{1s}k_0^S + (\frac{1}{d}-1)k_0^S p_1])$,

if $Z_{1s}^S - R > 0$, then $b_{1s}^S = \theta[q_{1s}k_0^S + (\frac{1}{d}-1)k_0^S p_1]$.

Since $Z_{1s}^S = \frac{A}{q_{1s}} > R$, we can obtain that $b_{1s}^S = \theta[q_{1s}k_0^S + (\frac{1}{d}-1)k_0^S p_1]$.

Depending the shock of ($a_{1s}^S + q_{1s}$), there exist three cases:

Case1: $(A-1)(\frac{1}{d}-1)+(\frac{A}{q_{1s}}p_1-1)(\frac{1}{d}-1)+Z_{1s}^S(a_{1s}^S+q_{1s}-R)<0$

$$Z_0^S-Z_{1s}^S R<0$$

Then optimum $b_0^*=0$, in this case the bank will go bankrupt or liquidate part of the capital good to keep insolvency.

Case2: $(A-1)(\frac{1}{d}-1)+(\frac{A}{q_{1s}}p_1-1)(\frac{1}{d}-1)+Z_{1s}^S(a_{1s}^S+q_{1s}-R)=0$

$$Z_0^S-Z_{1s}^S R=0$$

Then optimum $b_0^*\in(0,\theta[n+(\frac{1}{d}-1)np_0])$, in this case the bank will liquidate part of his capital good.

Case3: $(A-1)(\frac{1}{d}-1)+(\frac{A}{q_{1s}}p_1-1)(\frac{1}{d}-1)+Z_{1s}^S(a_{1s}^S+q_{1s}-R)>0$

$$Z_0^S-Z_{1s}^S R>0$$

Then optimum $b_{1s}^S=\theta[q_{1s}k_0^S+(\frac{1}{d}-1)k_0^S p_1]$.

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