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Pre-Entry Advertising, Entry Deterrence and Multi-Informational Signaling

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Abstract

Advertising is commonly regarded as a strategic tool to increase demand and steal business from competitors. The present work studies the competitive effects of advertising in a two-period game with incomplete information about the opponent's cost structure. Bagwell and Ramey (1988) showed that deterring entry is possible by signaling lower costs even if the post-entry game is independent of the pre-entry advertising decision. Assuming that pre-entry advertising by an entrant affects the post-entry game, then the incumbent is forced to do more than in the Bagwell/Ramey case to deter entry; he needs to distort costs downwards more extensively. On the other hand, introductory advertising does not facilitate entry if the entrant learns from the signal that competition with the incumbent is unpro&table. In this case, the entrant abstains from entry after performing introductory advertisng. Furthermore, if the incumbent has private information on cost and advertising effectiveness, then he can deter entry by acting as if he had lower production costs and a better advertising effectiveness. In this scenario, entry deterrence is associated with overinvestment in advertising but not with limit pricing, which is a new prediction. The use of multi-informational signals, i.e. pooled information on more than one type of private information transferred by one signal, is methodologically a new development of the classical signaling game.

1 Introduction

Advertising is a tool for incumbent firms to defend or extend their market position against newcomers. On the other hand, newcomers can apply advertising as a tool to introduce a new product. Bain (1949) developed the idea that brand loyalty is a critical aspect in market entry. According to Bain, the incumbent can establish a credible threat of aggressive competition by overinvesting in advertising, if he engages in high budgets before entry to strengthen brand loyalty. However, as Schmalensee (1974) pointed out, this threat is not credible. If a newcomer enters the market, it is not beneficial for the incumbent to maintain a high advertising budget, even if he has engaged in overinvestment in advertising prior to entry. Instead, it is better to accommodate with the entrant, and consider overinvestment as sunk. As Needham (1974) clearly pointed out, advertising by the incumbent before market entry only affects the new firm's entry decision if there is a definite relation between this advertising and the entrant's expected profit after entry. Because such a relation does not exist, the incumbent's effort to deter entry by overinvestment in advertising is regarded as ineffective.

In 1983, Schmalensee presented a new model showing that underinvestment in informative advertising can be a credible reputation of market power. However, this finding was based on complete information on part of the consumers and potential entrants. They need to be fully aware of the incumbent's advertising efforts. This prediction has two consequences, neither of which is ultimately true: 1. The entrant knows the incumbent's cost and demand parameters. 2. Market participants observing the incumbent's advertising can categorize advertising perfectly into informative and persuasive classes. However, advertising cannot be perfectly classified in this regard, as it always contains features of both and because consumers have different perceptions. Therefore, it is not reasonable to derive results based on classifications of consumer perceptions.

The impossibility of classifying advertising is obviously due to the fact that consumers have different awareness of a firm's product and brand, and because of different perceptions of qualitative product features. In an attempt to measure the level of information consumers have about a large number of brands and their quality perceptions, Clark *et al.* (2007) found that advertising significantly affects brand awareness, but not perceived quality. At first glance, this result seems strange. If advertising outlays are not correlated to perceived quality, then firms with lower advertising budgets may be able to achieve a higher perceived quality for their brands than more intensively advertising firms. This would be quite in accordance with Hertzendorf's (1993) finding that quality perception is not necessarily a result of the amount of advertising spend. On the other hand, it is questionable why advertising significantly increases brand awareness in the study by Clark *et al.* (2007), but not perceived quality. According to Hertzendorf, more awareness resulting from high advertising levels should antagonize biased quality perception. In this regard, the study by Clark *et al.* (2007) is obviously in conflict with Hertzendorf. Another study by Moorthy and Zhao (2000) made the contrary finding that advertising outlays are significantly associated with perceived quality. These findings show that economic statements based on advertising classifications are unreliable. It is therefore wise to abstain from classifications of this kind.

Bagwell and Ramey (1988) presented a new idea related to advertising as a tool for entry deterrence. They did not specify information technology or demand effects of advertising. This is important as the relation of advertising effects and consumer preferences is in conflict with generalized model assumptions. Bagwell/Ramey's model focused on the asymmetry of a specific information between entrant and incumbent, namely the incumbent's production cost. This idea resulted in axiomatic conclusions about the application of advertising in market entry scenarios. Accordingly, advertising can be a signal for true information about the incumbent's production cost and demand situation (Bagwell and Ramey, 1988 and 1994). In this scenario, the incumbent can signal truthfully by overinvesting in advertising. The overinvestment distorts the incumbent's true production cost downwards, and allows him to present himself as a low-cost firm, not contestable by an entrant. Overinvestment in advertising occurs to signal true information, and does not depend on a threat. However, some questions remain unanswered, namely *i*) can the entrant benefit from carrying out advertising prior to entry? and *ii*) can the incumbent more efficiently deter entry by signaling more than one type of private information? It may be profitable to signal more than one type of information. It is therefore the aim of the present work to provide a model of multi-informational signaling for the examination of a broader class of entry scenarios.

The remaining part of the article is organized as follows. Section 2 introduces the case of pre-entry advertising on part of the entrant. In section 3, we provide the characterization of separating equilibria for the case of multi-informational signals. That is, the incumbent sends signals on two kinds of private information, cost level and advertising effectiveness. Section 4 summarizes the results.

2 Pre-entry advertising and entry deterrence

Empirical evidence supports both views of advertising. Firstly, advertising for new products facilitates entry and erode the incumbent's market position. Secondly, advertising is used as an entry deterrence strategy by established firms through overinvestment. Studies showed that *i*) market admission by new firms or products caused a response by incumbent firms, mostly by increasing advertising efforts (Telser, 1962; Alemson, 1970; Cubbin and Domberger, 1988, Thomas, 1999), *ii*) advertising facilitated market entry (Telser, 1962; Ferguson, 1967; Alemson, 1970), *iii*) new entry contributed to decreasing market prices (Alemson, 1970), but also that *iv*) introductory advertising led to unsuccessful entry (Telser, 1962; Alemson, 1970). In some industries, it was also observed that incumbents' response to entry failed to appear, if entry is small-scale (Thomas, 1999), and that new entry was not accompanied by intensive advertising (Brown, 1978). The fourth (*iv*) observation limits the potential of introductory advertising, and suggests that newcomers may consider to abstain from market admission even after carrying out advertising, if competition with the incumbent firms in later stages would be unprofitable. This is an important aspect as advertising has the characteristics of an irreversible investment. In a world of uncertainty, expected profitability can be positive for a newcomer, although negative thereafter. This could induce a newcomer to test a market by applying marketing, but abstain from entry after receiving additional information and sink advertising.

The empirical studies have shown that the level of advertising outlays increases in the light of imminent entry. Theoretical analysis has confirmed that increasing advertising is a rational response to the threat of entry (Bagwell/Ramey, 1988). It is interesting to analyze the question of how introductory advertising by newcomers affects the incumbent's entry deterrence behavior in a theoretical model like that of Bagwell and Ramey. To investigate this question, we extended the signaling model by Bagwell and Ramey (1988) by introducing entrant advertising. The model used to answer the question is no longer a standard signaling game, since the uninformed party makes one of its key decisions simultaneously with the informed party. The actual entry decision is the second key decision of the uninformed party. Thus, the advertising choice of the uninformed party is based on its prior beliefs and its expectations as to the kind of equilibrium being played.

2.1 The model

Assume the model by Bagwell and Ramey (1988) with an incumbent (firm I) operating solely on a market where consumers can be influenced by the provision of advertising. The incumbent is privately informed about his marginal costs, indicated by c_I^i with $i \in \{L, H\}$. A potential competitor (firm E) does not know the incumbent's true costs, but knows that it is one of the two possible states c_I^H and c_I^L. Also, firm E has a belief of the distribution of states given by the common knowledge prior probability distribution þ$(c_I^i) \in [0, 1]$. The entrant's posterior belief that the incumbent operates with high cost is indicated by $\rho(A_I) \in [0, 1]$, which is calculated by Bayes' rule. The entrant can advertise prior to entry simultaneously to the incumbent's first period action, and first-period advertising affects second-period sales. After observing the incumbent's advertising decision, the challenger may choose either to enter the market, or not to enter. In the case of entry, the firms set to duopoly behavior in the second stage of the game, and they compete for market shares based on their pre-entry advertising spend.

In the first-stage, the entrant calculates profitability of pre-entry advertising based on prior beliefs. This calculation affects the equilibrium path, which in turn determines the incumbent's considerations as to the kind of equilibrium being played. Let type i's first-period equilibrium signal be depicted by $(\hat{P}_I^i, \hat{A}_I^i)$, and let the entrant's strategy be a tuple of actions (A_E^1, α), where A_E^1 is the entrant's first-period advertising and α the entry decision, with $\alpha = 1$ for entry. First-period advertising enables the entrant to attract sales in the second period by stealing some business from the incumbent ($\partial\pi_E^i/\partial A_E^1 > 0$ and $\partial\pi_D^i/\partial A_E^1 < 0$). For the sake of simplicity, there is no dilution of advertising effects over the two periods.

Let the incumbent's demand be a continuous function $X_I(P_I, A_I)$ depending on price and advertising. According to Bagwell and Ramey, the incumbent's first-period profit function is defined by:

$$\pi_I^i(P_I, A_I) = (P_I - c_I^i)X_I(P_I, A_I) - A_I \tag{1}$$

with (P_I^i, A_I^i) as the unique first-period maximizer. The profit function has a unique maximum. Cross-effects are so small that optimal price is positively (and optimal advertising negatively) associated with changes in marginal costs c. Demand is assumed to be decreasing in price and increasing in advertising. The entrant chooses to join the market only if the incumbent operates with high cost, and we assume the following order of second-period entrant profits: $0 < \pi_E^L < F < \pi_E^H$. The type-dependent incumbent's second-period duopoly profits may have the following relation: $\pi_D^L > \pi_D^H > 0$.

2.2 Optimality conditions for signaling equilibria

In our entry game, the incumbent has to choose a price-advertising combination in the first period as the best response to the entrant's entry and first-period advertising decision. The first condition for a signaling equilibrium is thus as follows.

Condition 1 (Optimality for the incumbent) *The incumbent with cost level $i \in \{L, H\}$*

chooses a price-advertising combination $\left(\tilde{P}_I^i, \tilde{A}_I^i\right)$ *in the first period such that for all* $A_E^1 \in \Re_+$ *:*

$$\left(\tilde{P}_I^i, \tilde{A}_I^i\right) \in \arg\max_{(P_I, A_I)} \left\{\pi_I^i (P_I, A_I) + \delta [E_I]\right\}$$

with $E_I = \alpha (P_I, A_I, A_E^1)\, \pi_D^i + (1 - \alpha (P_I, A_I, A_E^1)) [\pi_I^i (P_I^i, A_I^i)]$.

The entrant may choose an advertising level according to the prior probability distribution simultaneous with the incumbent's first-period action. Since competition with a low-cost incumbent is unprofitable, the challenger enters only when he observes the low-cost signal. Thus we get the following condition.

Condition 2 (Optimal advertising for the entrant) *The entrant chooses an advertising level* $\tilde{A}_E^1 \in \Re_+$ *in the first period to maximize the expected profit from period two under prior beliefs:*

$$\tilde{A}_E^1 \in \arg\max_{A_E^1} \left\{\delta \hat{E}_E - A_E^1\right\}$$

with $\hat{E}_E = \hat{\rho}\pi_E^H (A_E^1) + (1 - \hat{\rho})\, \pi_E^L (A_E^1) - F$
and with $\pi_E^L = 0$ *if* $\hat{\rho}\pi_E^H (A_E^1) + (1 - \hat{\rho})\, \pi_E^L (A_E^1) - A_E^1 > F$.

$\tilde{A}_E^1$ is positive only if $\hat{\rho}\pi_E^H (A_E^1) + (1 - \hat{\rho})\, \pi_E^L (A_E^1) - A_E^1 > F$. However, in this case, the equilibrium is separating, and the entrant enters only if the incumbent reveals to be high-cost. Thus, we need to consider $\pi_E^L = 0$ in this case. Consistency of beliefs is given by Bayes' rule. If $\left(\tilde{P}_I^L, \tilde{A}_I^L\right) \neq \left(\tilde{P}_I^H, \tilde{A}_I^H\right)$, then $\rho\left(\tilde{P}_I^L, \tilde{A}_I^L\right) = 0$ and $\rho\left(\tilde{P}_I^H, \tilde{A}_I^H\right) = 1$ and if $\left(\tilde{P}_I^L, \tilde{A}_I^L\right) = \left(\tilde{P}_I^H, \tilde{A}_I^H\right)$ then $\rho\left(\tilde{P}_I^L, \tilde{A}_I^L\right) = \hat{\rho}$. The first case is referred to as separating equilibrium.

The entry decision at the beginning of the second period is independent of the cost of pre-entry advertising as it is irreversible in the second period. The entrant decides to enter if the expected profit under posterior beliefs covers the investment F, as expressed in Condition 3.

Condition 3 (Optimal entry) $\hat{e}\left(P_I, A_I, A_E^1\right) = 1$, *for all* (P_I, A_I), *if* $E_E > F$, *with* $E_E = \rho \pi_E^H + (1-\rho)\, \pi_E^L - A_E^1$.

If entry is unprofitable under prior beliefs ($\hat{\rho}$) the equilibrium can be separating or pooling. If Condition 3 is fulfilled under prior beliefs, and if the incumbent is a low-cost type, the entrant does not enter. The entrant has to consider this in his optimality condition such that $\pi_E^L = 0$ under our requirements in this specific case. Consequently, the game always results in a separating equilibrium if the entrant's advertising spend is positive.

2.3 Separating equilibria

Assume Condition (3) is fulfilled under prior beliefs. Then, entry is profitable to the entrant before observing the incumbent's signal, i.e.:

$$\hat{\rho}\pi_E^H\left(A_E^1\right) + (1-\hat{\rho})\, \pi_E^L\left(A_E^1\right) - A_E^1 > F$$

In this case, the game's outcome must be a separating equilibrium and the challenger enters only if he observes the high-cost signal. In the case of entry, the firms set to duopoly behavior. In the first period, the entrant does not know the incumbent's true costs, but chooses an advertising level according to the objective:

$$\max_{A_E^1} G = \hat{\rho}\pi_E^H\left(A_E^1\right) - A_E^1$$

We assume that competition with a low-cost incumbent remains unprofitable to the entrant, even after advertising (i.e. $\pi_E^L\left(A_E^1\right) < F \ \forall \ A_E^1$), to look at the interesting case where signaling by the incumbent determines the entrant's entry decision. We can then find the incumbent's best response to the entrant's first-period advertising decision by setting $\rho\left(P_I, A_I\right) = 1$ for $(P_I, A_I) \neq \left(\hat{P}_I^L, \hat{A}_I^L\right)$, and Condition 1 is satisfied for $c_I^i = c_I^H$ in any separating equilibrium if:

$$\pi_I^H\left(\hat{P}_I^L, \hat{A}_I^L\right) \leq (1-\delta)\, \pi_I^H\left(P_I^H, A_I^H\right) + \delta \pi_D^H\left(A_I^H, A_E^1\right) \equiv \hat{e}_I^H \qquad (2)$$

where δ is the discount factor. Let $\hat{\mathcal{A}}$ be the set of $\left(\hat{P}_I^L, \hat{A}_I^L\right)$ satisfying (2). Furthermore, Condition 1 is satisfied for $c_I^i = c_I^L$ in any separating equilibrium if:

$$\pi_I^L \left(\hat{P}_I^L, \hat{A}_I^L\right) \geq (1-\delta)\, \pi_I^L \left(P_I^L, A_I^L\right) + \delta \pi_D^L \left(A_I^L, A_E^1\right) \equiv \hat{e}_I^L \tag{3}$$

Let the set of $\left(\hat{P}_I^L, \hat{A}_I^L\right)$ satisfying (3) be indicated as $\hat{\mathcal{L}}$. We look for pairs $\left(\hat{P}_I^L, \hat{A}_I^L\right) \in \hat{\mathcal{A}} \cap \hat{\mathcal{L}}$ that constitute separating equilibria. Also, we look at the interesting case $\left(P_I^L, A_I^L\right) \notin \hat{\mathcal{A}}$.

Corresponding to Bagwell and Ramey (1988), the low-cost incumbent may choose any $\left(\hat{P}_I^L, \hat{A}_I^L\right)$ in $\hat{\mathcal{A}} \cap \hat{\mathcal{L}}$ by the threat of certain entry following a deviation, and we look for elements not dominated by others. An element that fulfills this requirement is located at the intersection point of $\hat{e}_I^H$ and $\psi_I(c_I) = (P_I(c_I), A_I(c_I))$, where the latter determines the location of complete-information price-advertising combinations that maximize the incumbent's profit at different cost levels. The following proposition tells us the equilibrium strategies.

Proposition 1 *Let $\left(P_I^L, A_I^L\right) \notin \hat{\mathcal{A}}$, if $\delta \hat{b}_E > F + A_E^1$ then the unique undominated separating equilibrium consists of*

$$\left\{ \left(\hat{P}_I^i, \hat{A}_I^i\right)_{i=L,H} \mid \left(\hat{P}_I^L, \hat{A}_I^L\right) = \left(P_I(\underline{c}_I), A_I(\underline{c}_I)\right),\ \left(\hat{P}_I^H, \hat{A}_I^H\right) = \left(P_I^H, A_I^H\right) \right\}$$

such that $\underline{c}_I < c_I^L$.

The proof is available in the Appendix. The difference between Theorem 2 of Bagwell and Ramey and Proposition 1 is that $\underline{c}_I < c_I^0$, i.e. the cost distortion necessary to deter entry is larger in the context of pre-entry advertising. $(P_I(c_I^0), A_I(c_I^0)) = (P_I^0, A_I^0)$ can be chosen profitably by the high-cost incumbent. Therefore, in the light of challenging entrant advertising, (P_I^0, A_I^0) cannot be a separating equilibrium anymore.

Proposition 2 *The undominated separating equilibrium stated in Proposition 1 contains a cost distortion of $\underline{c}_I < c_I^0 < c_I^L$.*

Figure 1 shows two isoprofit curves for the high-cost incumbent. The exterior curve is $\tilde{\pi}_I^H = \pi_I^H(\underline{P}_I, \underline{A}_I)$, which determines the the case of $A_E^1 > 0$. The interior curve maps the case $A_E^1 = 0$, depicted as $\overline{\pi}_I^H = \pi_I^H(P_I^0, A_I^0)$.

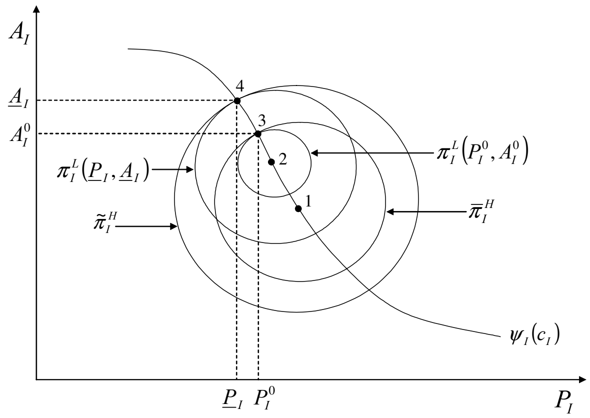

Figure 1: The undominated seperating equilibrium if $A_E > 0$. Legend: (1) (P_I^H, A_I^H), (2) (P_I^L, A_I^L), (3) (P_I^0, A_I^0), (4) $(\underline{P}_I, \underline{A}_I) = (\tilde{P}_I^L, \tilde{A}_I^L)$.

Introductory entrant advertising causes the incumbent to react with an expansion of his own advertising effort. The low-cost incumbent's effort to deter entry is larger than that required if there is no entrant advertising. Entry deterrence is thus more costly to the low-cost incumbent in the light of entrant advertising. Accommodation with the entrant is less attractive to the low-cost incumbent as long as $\tilde{A} \cap \tilde{E}$ is not empty. The incumbent's cost distortion necessary to deter entry depends on the level of advertising provided by the challenger. The more the entrant advertises the more he steals business from the incumbent. If advertising by the entrant would not be efficient under prior beliefs, then (P_I^0, A_I^0), the equilibrium price-advertising pair in the non-entrant-advertising case, would be the unique separating equilibrium.

Another interesting feature of the present model is that the entrant does not join the market if he observes the low-cost signal, even though he has advertised in the first period. In fact, this outcome is a consequence of the players' rational behavior. A potential entrant should perform advertising when the expected profit is positive. Advertising prior to entry is always risky, and we may observe in reality that potential entrants enter the previously advertised market not knowing if the attempt will be profitable. However, we learned that it is not always rational to enter the market, even if advertising has already been performed. In the moment true information is revealed, advertising outlays are sunk and cannot be included in the entry decision anymore. In the real world, such introductory advertising may be carried out by newcomers under similar conditions of incomplete information, but entry may occur despite competition is not profitable. It is thus beneficial to abstain from entry even if advertising outlays have been performed.

3 Multi-informational signaling of cost level and advertising effectiveness

The present study sought to take another option for increasing barriers to entry into account, namely the exploitation of private information on more than one aspect. The consideration of multi-informational signaling is methodologically a relatively new and scarcely examined approach. Few papers have been published on this approach, which is also called multi-dimensional signaling. However, this term is used for different scenarios, e.g. for the case of one type of private information and two or more signals (Wilson 1985, McNally 1999), and a continuum of characteristcs of one private information type (Quinzii and Rochet, 1985). As the term "multi" here refers to the number of types of private information transferred by one or more signals, we call the game a multi-informational signaling game. In this category of games, Quinzii and Rochet (1985) revisited the Spence (1973) labor market signaling model to show that a separating equilibrium exist, and that there is a qualitative difference in the results between the signaling of one and multiple types of information. Chen (1997) presented a model in which firms have private information as to cost level and demand. He found that separating equilibria are difficult to derive, and that most equilibria are "hybrid" in nature. That is, an equilibrium can be separating regarding one information type and pooling reagrding the other. Thus, in his model both aspects of private information can give rise to different and separate signals.

In contrast, in our model the incumbent is able to exploit his knowledge over two kinds of private information in a unique separating equilibrium to increase profits compared with the benchmark outcome of one-type information signaling.

For comparison with the benchmark outcome, we modified the model presented above by considering cost and advertising effectiveness as private information. Whereas, pre-entry entrant advertising is not considered. We found that in this scenario, the incumbent plays advertising enhancement, but not limit pricing, which is a new prediction. Simultaneously, the inclumbent can gain profits. The rationale may be that the incumbent suffers a loss from overinvestment in advertising, but regains profits by maintaining a monopoly-like price level. Thus, the application of multi-informational signaling enables the incumbent to enforce price stability that compensates for advertising overinvestment.

An interesting aspect in this model is the aspect of advertising effectiveness. As information is an economic good, and advertising is a source of information, we would expect more intensive advertising for new products as they need greater publicity than established ones. On the other hand, established firms may respond by more intensive advertising to increase sales and steal business, as was observed in the cigarette industry in the first half of the twentieth century (see Telser, 1962). These considerations include aspects of the impact of advertising's persuasive effects. To include these considerations in a model, we need to spell out concrete relations of return to advertising and consumer interaction, which are not considered in the present model.

Assuming that both advertising effectiveness and marginal cost can be either high or low, then there can be four kinds of incumbent types: *i*) high cost and high effectiveness, *ii*) high cost and low effectiveness, *iii*) low cost and high effectiveness and *iv*) low cost and low effectiveness. The natural idea would be to assume that *ii* and *iv* are the most frequently observed constellations in reality. In fact, these are also the most interesting constellations, as we want to examine whether the low-cost incumbent may have another option to deter entry or increase profitability. Therefore, we restrict the analysis to the two incumbent types specified in *ii* and *iv*.

3.1 The model

In this section, we want to show that the low-cost incumbent's equilibrium strategy changes if both incumbent types fiffer in production cost *and* advertising effectiveness. To make the strategy change explicit we take the Bagwell/Ramey equilibrium (depicted by point 3 in Figure 2) as benchmark, where incumbent types differ in cost only. According to the Bagwell/Ramey case, first-period advertising has no effect on second-period sales again.

To make explicit that the incumbent's demand also depends on the effectiveness of advertising, let $X^i(P, A)$ depicting the incumbent's demand at cost level $i \in \{L, H\}$, with price P and advertising level A, and $X^L(P, A) = X^H(P, A) + bA$ with $b > 0$, $\forall A > 0$. We save subscript I depicting the incumbent's strategies and profits as there is no second player's first period action. We only keep subscript E to denote second-period entrant profits and D for incumbent's second-period duopoly profits. Let the entrant be fully informed about the effects of type-dependent advertising, such that he is able to assign any profit-maximizing price-advertising pair to a particular profit level. The incumbent's first-period profit function is defined as follows:

$$\pi^i(P, A) = (P - c^i)X^i(P, A) - A \tag{4}$$

with $\partial X/\partial A > 0$ and $\partial X^L/\partial A > \partial X^H/\partial A \; \forall A > 0$. By this specification, the optimal level of advertising shifts upwards with increasing advertising effectiveness. Modification of the optimal low-cost advertising condition $\partial X^H/\partial A + b = 1/(P - c^L)$ yields:

$$\frac{P - c^L}{P}\eta_{X^H,A}\left(1 + b\frac{\partial A}{\partial X}\right) = \frac{A}{PX^H}$$

which is a variant of the Dorfman-Steiner equation showing that the relation of advertising to revenue is scaled up by the term in brackets. Thus, optimal advertising and monopoly profits increase in line with more effective advertising. In turn, firms with less effective advertising have to advertise more than more effectively advertising firms to generate the same amount of sales. Consequently, higher advertising effectiveness increases optimal advertising and price in $\Re_+^2 = P \times A$, and thereby induces $\psi(c)$ to shift upwards, as shown in Figure 2.

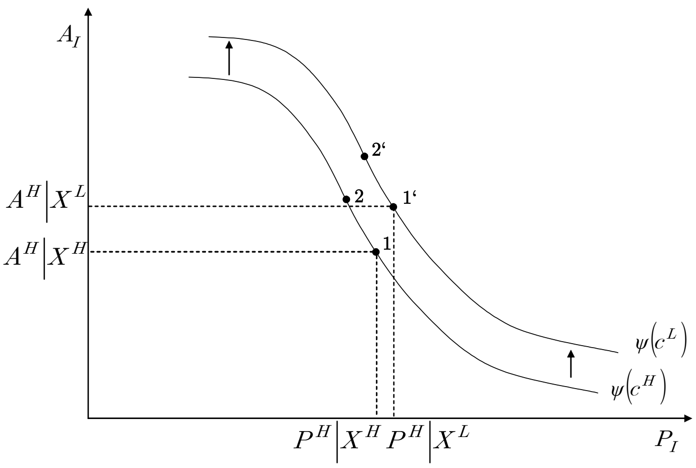

Figure 2: Advertising effectiveness shifts $\psi(c^i)$.

The optimality conditions for sequential Bayesian equilibria are as follows.

Condition 1 (Optimality for the incumbent) *$\forall i \in \{L, H\}$:*

$$\left(\hat{P}^i, \hat{A}^i\right) \in \arg\max_{(P,A)} \left\{\pi^i(P,A) + \delta\left[\alpha\pi_D^i + (1-\alpha)\pi^i\left(P^i, A^i\right)\right]\right\}$$

Condition 2 (Optimality for the entrant) *$\alpha(P,A) = 1$, for all (P,A), if $E_E > F$, with $E_E = \rho\pi_E^H + (1-\rho)\pi_E^L$.*

Third condition requires consistency of beliefs in the way as stated in the previous section.

3.2 Separating equilibria

Taking the case of identical advertising effectiveness as benchmark, we will see that equilibrium strategies change. This is because position and shape of isoprofit curve $\overline{\pi}^H$ change when advertising effectiveness change. Let us depict the new curve by $\widehat{\pi}^H$. Then, we can define the conditions for a separating equilibrium as follows:

$$\pi^H\left(\tilde{P}^L,\tilde{A}^L\right) \le (1-\delta)\,\pi^H\left(P^H,A^H\right) + \delta\pi_D^H \equiv \widehat{\pi}^H \tag{5}$$

and:

$$\pi^L\left(\tilde{P}^L,\tilde{A}^L\right) \ge (1-\delta)\,\pi^L\left(P^L,A^L\right) + \delta\pi_D^L \equiv \widehat{\pi}^L \tag{6}$$

Let $\widehat{L}$ be the set of $\left(\tilde{P}^L,\tilde{A}^L\right)$ satisfying (5) and $\widehat{H}$ be the set of $\left(\tilde{P}^L,\tilde{A}^L\right)$ satisfying (6). Under our extension of the benchmark model, as discussed above, low-cost sales extend high-cost sales by bA. $\widehat{\pi}^H$ is not identical to $\overline{\pi}^H$ in the benchmark case by lowered advertising effectiveness. So as to examine the most interesting scenario again, we assume that $\left(P^L,A^L\right) \notin \widehat{H}$. Moreover, we define $\widehat{\pi}^H$ to "intersect" $\psi\left(c^L\right)$ at $\left(\hat{P},\hat{A}\right)$, the first-period maximizer at cost-level $\hat{c}$ [1]. We know from the proof of Proposition 2 that the equilibrium price-advertising pair is located on $\widehat{\pi}^H$ and thus $\pi^H\left(\hat{P},\hat{A}\right) = \pi^H\left(\tilde{P}^L,\tilde{A}^L\right)$. We are then prepared to find the element that maximizes the low-cost incumbent's profits on $\widehat{\pi}^H$, which is the game's unique undominated separating equilibrium $\left(\tilde{P}^L,\tilde{A}^L\right)$. The following proposition tells us the properties of this equilibrium.

Proposition 3 *Let advertising be more effective to the low-cost than to the high-cost incumbent's sales, with* $X^H(P,A) = X^L(P,A) - bA$*, and let* $\left(P^L,A^L\right) \notin \widehat{H}$*, then the unique undominated separating equilibrium is not located on* $\psi\left(c^L\right)$*, but above of it, and the strategy profile is characterized by:*

$$\left(\tilde{P}^L,\tilde{A}^L\right) \neq \left(\hat{P},\hat{A}\right) \wedge \pi^L\left(\tilde{P}^L,\tilde{A}^L\right) > \pi^L\left(P^0,A^0\right) \wedge \pi^H\left(\tilde{P}^L,\tilde{A}^L\right) = \pi^H\left(\hat{P},\hat{A}\right)$$

$$\left(\tilde{P}^H,\tilde{A}^H\right) = \left(P^H,A^H\right)$$

with $c^0 < \hat{c} < c^L$*, where* $\left(\hat{P},\hat{A}\right) = \left(P(\hat{c}),A(\hat{c})\right)$ *is the first-period maximizers under cost level* $\hat{c}$ *and sales schedule* X^L*.*

Thus, the equilibrium price-advertising pair $\left(\hat{P}^L, \hat{A}^L\right)$ located on $\hat{b}^H$ provides the low-cost incumbent a higher profit than that of the benchmark equilibrium under equally effective advertising. Also, it contains a cost distortion to $c = \hat{b} > c^0$, which is smaller than in the benchmark equilibrium. Put differently, it is profitable for the low-cost incumbent to act as if his advertising is more effective than it really is. If he does so, then the unique separating equilibrium essentially involves a cost distortion smaller than in the Bagwell/Ramey case where advertising is equally effective among the incumbent types. Another interesting conclusion from Proposition 3 is that the equilibrium is no more located on $\psi\left(c^L\right)$, but on a hypothetical crestline $\psi\left(c^{LL}\right)$ located above $\psi\left(c^L\right)$, as shown in Figure 3. Also, a special feature of this equilbrium is that equlibrium profits rise with increasing difference in advertising effectiveness as it shifts the equilibrium closer to the low-cost incumbent's first-period maximizer. The sufficient condition for the equilibrium to exist results from Theorem 3 in Bagwell/Ramey, namely $\pi^L\left(P^0, A^0\right) \geq \hat{b}^L$, together with $\pi^L\left(\hat{P}, \hat{A}\right) > \pi^L\left(P^0, A^0\right)$.

Figure 3 provides an illustration of the equilibrium, which is located above the crestline $\psi\left(c^L\right)$. The figure is plotted by Maple using a numerical example [2]. The benchmark equlibrium, where only cost is private information, is located in point (1), while the multi-informational equilibrium is represented by point (2). The new equilibrium (2) is located closer to the low-cost incumbent first-period maximizer (4), and above crestline $\psi\left(c^L\right)$.

In summary, the unique undominated separating equilibrium, which is displayed in Figure 3, shows a distortion in cost and advertising effectiveness in such a way that the low-cost incumbent acts as if he has lower cost and a higher advertising effectiveness. Hypothetically curve $\psi\left(c^{LL}\right)$ runs above $\psi\left(c^L\right)$, and subtends at $\left(\hat{P}^L, \hat{A}^L\right)$, containing the low-cost equilibrium pair. The effect of equilibrium strategies on price and advertising outlays is inconclusive. By mimicking higher advertising effectiveness, the game results in an increase of advertising spend, but effect on price is unclear. Since $\psi\left(c^{LL}\right)$ runs above $\psi\left(c^L\right)$, and the equilibrium shifts upwards, prices close to monopoly level are possible. Thus, mimicking higher advertising effectiveness enables the incumbent to regain

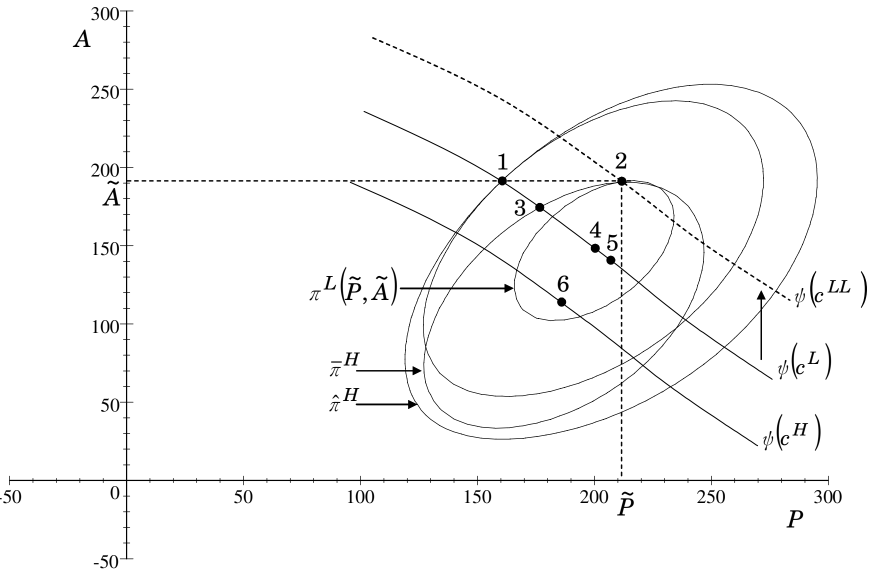

Figure 3: The undominated separating equilibrium under differently effective advertising. Legend: (1) (P^0, A^0), (2) $\left(\tilde{P}^L, \tilde{A}^L\right)$, (3) $\left(\hat{P}, \hat{A}\right)$, (4) $\left(P^L, A^L\right)$, (5) $\left(P^H, A^H \mid X^L\right)$, (6) $\left(P^H, A^H \mid X^H\right)$.

a fraction of his price decrement he incurs by deterring entry through signaling low cost.

4 Conclusions

We have examined several related scenarios of entry deterrence in the context of private information and signaling through advertising. At first, we analyzed the behavior of a monopolist facing imminent entry by a firm being able to advertise prior to entry. The result of this case shows that a potential entrant can benefit from introductory advertising by reserving some future demand, and thereby stealing business from the incumbent. It turned out that competition is intensified, accompanied by decreasing pre-entry price and increasing pre-entry advertising, compared with the benchmark case where entrant advertising is absent. Despite entrant advertising can take off some second-period business from the incumbent, he is still able to deter entry, although it is more costly to do so. A potential challenger is unlikely to generate positive profits from undertaking pre-entry advertising, if the incumbent is able to play an entry deterrence strategy. Therefore, he does better to abstain from entry.

A specific issue of the introductory advertising scenario analysed here is that the entrant makes one of his key decisions (i.e. the advertising decision) without receiving a signal on the incumbent's type. However, the entrant can make a decision on expected profits, and complete information is revealed in the model before entry. Thus, the entrant is actually better equipped with information than in the real world, as it enables the entrant to abstain from market entry if competition with the incumbent turns out to be unprofitable, and thereby to avoid another financial loss in addition to the sunk advertising costs. Thus, the model can explain why an entrant should sometimes abstain from entry, even if he has carried out introductory advertising.

In a second scenario, we examined if the incumbent is able to render entry deterrence more profitable by imitating lower costs and greater advertising effectiveness. In the entry deterrence strategy with distortion of more than one information type, incumbent's advertising is increased, similar as in the benchmark outcome of one-type information signaling, but there is no limit pricing. By acting as if his costs were lower and advertising effectiveness would be greater, the incumbent is able to counterbalance the costs of overinvestment in advertising by maintaining his price level. This is a new prediction, showing that limit pricing is not a necessary entry deterrence strategy associated with overinvestment in advertising.

We have analysed two variants of Bagwell-Ramey entry game by consideration of entrant advertising on the one hand and different advertising effectiveness on the other. Both scenarios have effects on the high-cost imcumbent's profits resulting in location changes of the isoprofit curve $\overline{\pi}^H$. It is an interesting finding that the two scenarios have opposite effects on equlibrium profits, despite both scenarios act in a profit-reducing manner on the high-cost incumbent. While $\overline{\pi}^H$ is "expanding" in appearance of entrant advertising, it "shrinks" in case of different advertising effectiveness. The reason for this unexpected finding is caused by the different effects of advertising. Entrant advertising reduces second-period duopoly profits, while a reduction in advertising effectiveness reduces first-period monopoly profit. In the first case, profit in the same "profit range" is reduced, while in the second case the entire "profit range" of the high-cost incumbent is reduced. The latter effect causes the high-cost incumbent's level curves to run markedly lower, resulting in isoprofit curves to "shrink".

Appendix

Proof of Proposition 1. Since the proof of Proposition 1 is largely analogous to the proof of Theorem 2 in Bagwell and Ramey (1988) we constrict it to the main steps. At first we have to show monotonicity of the high-cost incumbent's profit function for all cost-levels $c_I < c_I^H$ on $\psi_I(c_I)$ in order to establish the intersection point of $\tilde{\pi}_I^H$ and $\psi_I(c_I)$ at $(P_I(\underline{c}_I), A_I(\underline{c}_I))$ as the necessary condition for the equilibrium. Following can be shown: $X_I(\underline{P}_I, \underline{A}_I) > X_I\left(\overline{P}_I, \overline{A}_I\right)$ with $(\underline{P}_I, \underline{A}_I) \equiv (P_I(\underline{c}_I), A_I(\underline{c}_I))$ and $\left(\overline{P}_I, \overline{A}_I\right) \equiv (P_I(\overline{c}_I), A_I(\overline{c}_I))$ and $\underline{c}_I < \overline{c}_I < c_I^H$. If using:

$$\left(\overline{P}_I - \overline{c}_I\right) X_I\left(\overline{P}_I, \overline{A}_I\right) - \overline{A}_I - (\underline{P}_I - \overline{c}_I) X_I(\underline{P}_I, \underline{A}_I) + \underline{A}_I > 0 \qquad \text{(A1)}$$

we can write by simple modification of A1:

$$\left(\overline{P}_I - c_I^H\right) X_I\left(\overline{P}_I, \overline{A}_I\right) - \overline{A}_I - \left(\underline{P}_I - c_I^H\right) X_I(\underline{P}_I, \underline{A}_I) + \underline{A}_I > 0 \qquad \text{(A2)}$$

A1 added with:

$$(\underline{P}_I - \underline{c}_I) X_I(\underline{P}_I, \underline{A}_I) - \underline{A}_I - \left(\overline{P}_I - \underline{c}_I\right) X_I\left(\overline{P}_I, \overline{A}_I\right) + \overline{A}_I > 0$$

yields $X_I(\underline{P}_I, \underline{A}_I) > X_I\left(\overline{P}_I, \overline{A}_I\right)$, which proves monotonicity and thereby it proves that an intersection point of $\psi_I(c_I)$ and $\tilde{\pi}_I^H$ indeed exists.

The next step is to show that the low-cost incumbent's equilibrium choice is located on $\tilde{\pi}_I^H$. We can show this by contradiction. We know that $(\underline{P}_I, \underline{A}_I)$ is located on $\tilde{\pi}_I^H$. Assuming $\left(\tilde{P}_I^L, \tilde{A}_I^L\right)$ is not located on $\tilde{\pi}_I^H$ such that $\pi_I^H\left(\tilde{P}_I^L, \tilde{A}_I^L\right) < \tilde{\pi}_I^H$ then we can write:

$$\left(\tilde{P}_I^L - c_I^H\right) X_I\left(\tilde{P}_I^L, \tilde{A}_I^L\right) - \tilde{A}_I^L - \left(\underline{P}_I - c_I^H\right) X_I(\underline{P}_I, \underline{A}_I) + \underline{A}_I < 0$$

However, by definition $\left(\tilde{P}_I^L, \tilde{A}_I^L\right)$ is the equilibrium combination not dominated by another strategy. Therefore, $\left(\tilde{P}_I^L, \tilde{A}_I^L\right)$ maximizes the c_I^L-incumbent's profit on $\tilde{A} \cap \tilde{B}$ and we have to write:

$$\left(\tilde{P}_I^L - c_I^L\right) X_I\left(\tilde{P}_I^L, \tilde{A}_I^L\right) - \tilde{A}_I^L - \left(\underline{P}_I - c_I^L\right) X_I(\underline{P}_I, \underline{A}_I) + \underline{A}_I \geq 0 \qquad \text{(A3)}$$

But this cannot be if $\pi_I^H\left(\tilde{P}_I^L, \tilde{A}_I^L\right) < \tilde{\pi}_I^H$. Since $\left(P_I^L, A_I^L\right) \notin \tilde{H}$ we have:

$$\left(\tilde{P}_I^L - c_I^L\right) X_I\left(\tilde{P}_I^L, \tilde{A}_I^L\right) - \tilde{A}_I^L - \left(\underline{P}_I - c_I^L\right) X_I\left(\underline{P}_I, \underline{A}_I\right) + \underline{A}_I < 0$$

which is a contradiction and proves that A3 must be correct. Otherwise $\left(\tilde{P}_I^L, \tilde{A}_I^L\right)$ is not an equilibrium.

Finally, we can complete the proof by showing that the unique undominated equilibrium is located at $\left(\underline{P}_I, \underline{A}_I\right)$, i.e. $\left(\underline{P}_I, \underline{A}_I\right) = \left(\tilde{P}_I^L, \tilde{A}_I^L\right)$. Choosing $\left(P_I, A_I\right)$ such that $\pi_I^H\left(P_I, A_I\right) = \tilde{\pi}_I^H$ we can write:

$$\left(\underline{P}_I - c_I^H\right) X_I\left(\underline{P}_I, \underline{A}_I\right) - \underline{A}_I - \left(P_I - c_I^H\right) X_I\left(P_I, A_I\right) + A_I = 0 \tag{A4}$$

and:

$$\left(\underline{P}_I - \underline{c}_I\right) X_I\left(\underline{P}_I, \underline{A}_I\right) - \underline{A}_I - \left(P_I - \underline{c}_I\right) X_I\left(P_I, A_I\right) + A_I > 0 \tag{A5}$$

Adding A4 and A5 yields: $\left(c_I^H - \underline{c}_I\right)\left[X\left(\underline{P}_I, \underline{A}_I\right) - X_I\left(P_I, A_I\right)\right] > 0$. However, using A4 we get by simple modification:

$$\left(\underline{P}_I - c_I^L\right) X_I\left(\underline{P}_I, \underline{A}_I\right) - \underline{A}_I - \left(P_I - c_I^L\right) X_I\left(P_I, A_I\right) + A_I$$

$$= \left(c_I^H - \underline{c}_I\right)\left[X_I\left(\underline{P}_I, \underline{A}_I\right) - X_I\left(P_I, A_I\right)\right] > 0$$

which is a contradiction to A4, and therefore proves that $\left(\underline{P}_I, \underline{A}_I\right)$ uniquely maximizes $\pi_I^L\left(P_I, A_I\right)$ on $\tilde{\pi}_I^H$, i.e. $\left(\tilde{P}_I^L, \tilde{A}_I^L\right)$ is the unique undominated equilibrium.

The sufficient condition for the equilibrium is given by $\pi_I^L\left(P_I^L, A_I^L\right) - \pi_D^L\left(A_I^L, A_E^{\infty}\right) \geq \pi_I^H\left(P_I^H, A_I^H\right) - \pi_D^H\left(A_I^H, A_E^{\infty}\right)$. If this is the case, then $\left(\underline{P}_I^L, \underline{A}_I^L\right)$ is located within $\tilde{E}$ with certainty and therefore it is element of $\tilde{H} \cap \tilde{E}$, as required. The proof in brief is as follows. Starting from:

$$\pi_I^L\left(\underline{P}_I, \underline{A}_I\right) + \delta\pi_D^L\left(P_I^L, A_I^L\right) - \pi_I^L\left(P_I^L, A_I^L\right) - \delta\pi_D^L\left(A_I^L, A_E^{\infty}\right) \tag{A6}$$

we would expect that this expression is positive or at least equals zero since $\pi_I^L\left(\hat{P}_I^L, \hat{A}_I^L\right) \geq \hat{\pi}_I^L$. Subtraction of A6 by $\pi_I^H\left(\underline{P}_I, \underline{A}_I\right) - \hat{\pi}_I^H$, which actually equals zero, as well as recombination of terms and cancelling down leads to: $(c_I^H - c_I^L) X\left(\underline{P}_I, \underline{A}_I\right) - X_I\left(P_I^L, A_I^L\right)$. Since the adding of the following two inequations, which are valid due to monotonicity:

$$\left(\underline{P}_I - \underline{c}_I\right) X_I\left(\underline{P}_I, \underline{A}_I\right) - \underline{A}_I - \left(P_I^L - \underline{c}_I\right) X_I\left(P_I^L, A_I^L\right) + A_I^L > 0$$

$$\left(P_I^L - c_I^L\right) X_I\left(P_I^L, A_I^L\right) - A_I^L - \left(\underline{P}_I - c_I^L\right) X_I\left(\underline{P}_I, \underline{A}_I\right) + \underline{A}_I > 0$$

yields following inequation: $(c_I^L - \underline{c}_I)\left[X\left(\underline{P}_I, \underline{A}_I\right) - X_I\left(P_I^L, A_I^L\right)\right] > 0$, we can immediately conclude that A6 must be positive too. This proves that $\left(\underline{P}_I, \underline{A}_I\right) \in \hat{E}$. Thus, together with the result from the proof that $\left(\underline{P}_I, \underline{A}_I\right) = \left(\hat{P}_I^L, \hat{A}_I^L\right)$ we have shown that $\left(\underline{P}_I, \underline{A}_I\right)$ is the unique undominated separating equilibrium, which will be chosen by the low-cost incumbent in the context of preentry advertising ($A_E^1 > 0$). ■

Proof of Proposition 2. If entry is profitable under prior beliefs, it is easy to see that the challenger always carries out advertising under our requirements as $\partial\pi_E/\partial A_E^1 > 0$. Since pooling is excluded in this case, the entrant's advertising level may be high or very small but always exceeds zero. Let us pick up $(P_I^0, A_I^0) \equiv \left(P_I\left(c_I^0\right), A_I\left(c_I^0\right)\right)$ such that $\pi_I^H\left(P_I^0, A_I^0\right) = \overline{\pi}_I^H = (1-\delta)\,\pi_I^H\left(P_I^H, A_I^H\right) + \delta\pi_D^H\left(A_I^H, 0\right)$ and assume that $\pi_I^H\left(P_I^0, A_I^0\right) \leq \hat{\pi}_I^H = \pi_I^H\left(\underline{P}_I, \underline{A}_I\right)$ and $A_E^1 > 0$. Then:

$$(1-\delta)\,\pi_I^H\left(P_I^H, A_I^H\right) + \delta\pi_D^H\left(A_I^H, 0\right) = \pi_I^H\left(P_I^0, A_I^0\right)$$

$$\leq (1-\delta)\,\pi_I^H\left(P_I^H, A_I^H\right) + \delta\pi_D^H\left(A_I^H, A_E^1\right)$$

yielding:

$$\pi_D^H\left(A_I^H, 0\right) \leq \pi_D^H\left(A_I^H, A_E^1\right) \tag{A7}$$

However, we know that $\partial\pi_D^H/\partial A_E^1 < 0\ \forall A_E^1 > 0$. Hence, A7 is s a contradiction, which proves that $\left(\underline{P}_I, \underline{A}_I\right)$, the game's equilibrium under $A_E^1 > 0$, must be located on an isoprofit curve that gives less profit to the incumbent than $\overline{\pi}_I^H$. Due to monotonicity this proves that Condition 1 is satisfied for cost-level $\underline{c}_I$ that must be smaller than c_I^0.

Proof of Proposition 3. We first show that the equilibrium is always located on the crestline $\psi\left(c^L\right)$ in the case that there is no difference in advertising effectiveness among the incumbent types. A maximum of $\pi^i(P, A)$, $i \in \{L, H\}$, is characterized by:

$$\nabla\left[(P - c^i)X(P, A) - A\right] = \overrightarrow{0}$$

Let (P^0, A^0) be the monopoly first-period maximizer for cost level c^0. Assuming that in such an equilibrium the isoprofit curves of $\pi^i(P, A) = (P - c^i)X(P, A) - A$, $i \in \{L, H\}$, are tangent to each other, then the gradients are parallel at this location and we have:

$$\nabla\pi^L(P^0, A^0) = \lambda\nabla\pi^H(P^0, A^0)$$

$$\nabla[P^0 X(P^0, A^0) - A^0] - c^L\nabla X(P^0, A^0)$$

$$= \lambda\nabla[P^0 X(P^0, A^0) - A^0] - c^H\lambda\nabla X(P^0, A^0)$$

Modification yields:

$$(1 - \lambda)\,\nabla[P^0 X(P^0, A^0) - A^0] - \left(c^L - \lambda c^H\right)\nabla X(P^0, A^0) = \overrightarrow{0}$$

Since $\left(c^L - c^H\right)\nabla X(P^0, A^0) = \overrightarrow{0}$ and, because of $c^L \neq c^H$, $\nabla X(P^0, A^0) = \overrightarrow{0}$ if $\lambda = 1$, we exclude $\lambda = 1$. After modification we get:

$$\nabla[P^0 X(P^0, A^0) - A] - \frac{c^L - \lambda c^H}{1 - \lambda}\nabla X(P^0, A^0) = \overrightarrow{0}$$

where $(c^L - \lambda c^H)/(1 - \lambda)$ equals c^0. Thus, we have:

$$\nabla\left[(P^0 - c^0)X(P^0, A^0) - A^0\right] = \overrightarrow{0}$$

Thus, the osculation point of $\pi^H(P^0, A^0)$ and $\pi^L(P^0, A^0)$ is located exactly on the crestline $\psi\left(c^L\right)$ at the the location where (P^0, A^0) is the maximizer for cost level c^0. This proves that the equilibrium is always located on the crestline $\psi\left(c^L\right)$ if both incumbent types have equally effective advertising.

Secondly, we show that the equilibrium is not located on the crestline if advertising effectiveness differs between the two incumbent types. Because of $\partial X^L/\partial A > \partial X^H/\partial A$ for all A, the first-period maximizers for the types under the same cost level c^0 cannot be identical. Therefore, (P^0, A^0) cannot be the equilibrium under our specification $X^H(P,A) = X^L(P,A) - bA$. Define $\left(\hat{P}, \hat{A}\right) = \left(P(\hat{c}), A(\hat{c})\right)$ being the first-period maximizer for cost level $\hat{c}$ and intersection point of $\hat{\pi}^H$ and $\psi\left(c^L\right)$. Assume $\left(\hat{P}, \hat{A}\right)$ being the osculation point of $\hat{\pi}^H$ and $\pi^L(\tilde{P}^L, \tilde{A}^L)$, then the gradients of both isoprofit curves are parallel at this point, and we have:

$$\nabla\pi^L(\hat{P}, \hat{A}) = \lambda\nabla\pi^H(\hat{P}, \hat{A})$$

$$\nabla[\hat{P}X^L(\hat{P}, \hat{A}) - \hat{A}] - c^L\nabla X^L(\hat{P}, \hat{A})$$

$$= \lambda\nabla[\hat{P}\left(X^L(\hat{P}, \hat{A}) - b\hat{A}\right) - \hat{A}] - c^H\lambda\nabla\left(X^L(\hat{P}, \hat{A}) - b\hat{A}\right)$$

After modification, and exclusion of $\lambda = 1$, we get:

$$\nabla\left[(\hat{P}X^L(\hat{P}, \hat{A}) - \hat{A}\right] - \frac{c^L - \lambda c^H}{1-\lambda}X^L(\hat{P}, \hat{A}) + \frac{\nabla\left(\hat{P} - c^H\right)b\hat{A}}{1-\lambda} = \vec{0}$$

where $(c^L - \lambda c^H)/(1-\lambda)$ equals $\hat{c}$, yielding:

$$\nabla\left[(\hat{P} - \hat{c})X^L(\hat{P}, \hat{A}) - \hat{A}\right] + \frac{\nabla\left(\hat{P} - c^H\right)b\hat{A}}{1-\lambda} = \vec{0}$$

As $\lambda \neq 1$, and $\left(\hat{P} - c^H\right)$ and $b\hat{A}$ are positive numbers, the osculation point of $\pi^L(\tilde{P}^L, \tilde{A}^L)$ and $\pi^H(\hat{P}, \hat{A}) = \hat{\pi}^H_I$, is not located on the crestline $\psi\left(c^L\right)$, but above of it.

Thirdly, we show that the equilibrium price-advertising pair provides a higher profit to the low-cost incumbent than $\pi^L(P^0, A^0)$, the equilibrium profit under equally effective advertising. Assuming $\pi^H\left(\hat{P},\hat{A}\right) = (1-\delta)\,\pi^H\left(P^H,A^H\right) + \delta\pi_D^H = \hat{b}^H \cap \psi\left(c^L\right)$ is fulfilled for pair $\left\{\left(\hat{P},\hat{A}\right)\right\} = \left(P(\hat{b}), A(\hat{b})\right)$ with $c^L > \hat{b}$. From the proof of Proposition 1 we know that the unique undominated separating equilibrium $\left(\hat{P}^L,\hat{A}^L\right)$ must be located on $\hat{b}^H$. Let the first-period maximizer of c^0 under sales schedule X^L be $\left(P(c^0), A(c^0)\right) = (P^0, A^0)$ and that of c^H under X^H be $\left(P\left(c^H\right), A\left(c^H\right)\right) = \left(P^H, A^H\right)$. Assume that (P^0, A^0) is located on an isoprofit curve closer to $\left(P^H, A^H\right)$ than $\left(\hat{P},\hat{A}\right)$, then:

$$\pi^H\left(P^0, A^0 \middle| X^H\right) - \pi^H\left(\hat{P},\hat{A} \middle| X^H\right) > 0$$

Due to $\pi^H\left(\hat{P},\hat{A} \middle| X^H\right) = \hat{b}^H$ we can write:

$$\pi^H\left(P^0, A^0 \middle| X^H\right) - \pi^H\left(P^H, A^H \middle| X^H\right)$$

$$+\,\delta\left[\pi^H\left(P^H, A^H \middle| X^H\right) - \pi_D^H\right] > 0$$

If $\delta = 0$ then the term in brackets is zero and the whole term is clearly smaller than zero, which is a contradiction. In this case we have $\hat{b}_I > c_I^0$. As the term in parantheses increases in δ, we need to verify if $\delta = 1$ may let the inequation be valid. We yield:

$$\pi^H\left(P^0, A^0 \middle| X^H\right) - \pi_D^H > 0$$

Due to $\delta = 1$ and $\pi^H\left(P^0, A^0 \middle| X^L\right) = \overline{\pi}^H$ we know that:

$$\pi^H\left(P^0, A^0 \middle| X^L\right) - \pi_D^H = 0$$

where π_D^H is equal in both situations of advertising effectiveness as first-period advertising does not have an effect on second-period sales. Thus, substituting zero yields:

$$\pi^H\left(P^0, A^0 \middle| X^L\right) - \pi^H\left(P^0, A^0 \middle| X^H\right) < 0$$

Using profit functions, we can write:

$$(P^0 - c^H)\left[X^H\left(P^0, A^0\right) + bA^0\right] - A^0 - (P^0 - c^H)X^H\left(P^0, A^0\right) + A^0 < 0$$

yielding:

$$\left(P^0 - c^H\right) bA^0 < 0$$

which is a contradiction since $\left(P^0 - c^H\right)$ and bA^0 are positive numbers. This proves that $\hat{c} > c^0$, i.e. $\hat{c}^H$ subtends $\psi\left(c^L\right)$ at a point closer to $\left(P^L, A^L\right)$ than (P^0, A^0).

Finally, we show that $\pi^L\left(\hat{P}, \hat{A}\right) - \pi^L(P^0, A^0) > 0$. From the proof above we know that $\pi^H\left(\hat{P}, \hat{A}\right) - \pi^H(P^0, A^0) > 0$. Thus, we can write:

$$(\hat{P} - c^H)X^H\left(\hat{P}, \hat{A}\right) - \hat{A} - (P^0 - c^H)X^H\left(P^0, A^0\right) + A^0 > 0$$

$$(P^0 - c^0)X^H\left(P^0, A^0\right) - A^0 - (\hat{P} - c^0)X^H\left(\hat{P}, \hat{A}\right) + \hat{A} > 0$$

where the second inequality is valid due to monotonicity of $g : \left[0, c^H\right] \to \Re, c \longmapsto \pi\left(\psi(c) \mid c^H\right) = \left[\psi_1(c) - c^H\right] X[\psi(c)] - \psi_2(c)$ by intermediate value theorem using auxiliary function $\pi\left(\psi(c) \mid c^H\right)$, as shown in the proof of Theorem 2 in Bagwell/Ramey. Adding the two inequalities yields:

$$\left(c^H - c^0\right)\left[X^H\left(P^0, A^0\right) - X^H\left(\hat{P}, \hat{A}\right)\right] > 0 \tag{A8}$$

giving $X^H(P^0, A^0) > X^H\left(\hat{P}, \hat{A}\right)$ because of $c^H > c^0$. Assume $\pi^H(P^0, A^0) - \pi^H\left(\hat{P}, \hat{A}\right) > 0$, then:

$$(P^0 - c^L)\left[X^H\left(P^0, A^0\right) + bA^0\right] - A^0 - (\hat{P} - c^L)\left[X^H\left(\hat{P}, \hat{A}\right) + b\hat{A}\right] + \hat{A} > 0 \tag{A9}$$

$$(\hat{P} - \hat{c})\left[X^H\left(\hat{P}, \hat{A}\right) + b\hat{A}\right] - \hat{A} - (P^0 - \hat{c})\left[X^H\left(P^0, A^0\right) + bA^0\right] + A^0 > 0$$

where the second inequality is true due to monotonicity of $\pi\left(\psi(c) \mid c^L\right)$. Adding yields:

$$\left[X^H\left(\hat{P}, \hat{A}\right) - X^H\left(P^0, A^0\right)\right] + b\left(\hat{A} - A^0\right) > 0$$

which is wrong due to A8 and because of $A^0 > \hat{A}$, where the latter follows from the Dorfman-Steiner equation. Thus, A9 is a contradiction. This shows that $\pi^L\left(\hat{P}, \hat{A}\right) - \pi^L(P^0, A^0) > 0$, and thus $\pi^L\left(\hat{P}^L, \hat{A}^L\right) > \pi^L(P^0, A^0)$, which supports a separating equilibrium with properties as shown above. ■ ■

Notes

[1] Note that we actually cannot speak of an "intersection point" of $\overline{\pi}^H$ and $\psi\left(c^L\right)$ at $\left(\hat{P},\hat{A}\right) = \left(P\left(\hat{c}\right), A_I\left(\hat{c}\right)\right)$ here. The reason is that $\psi\left(c^L\right)$ at point $\left(\hat{P},\hat{A}\right)$ is running far above of $\overline{\pi}^H$ in the three-dimensional space, i.e. $\pi\left(\hat{P},\hat{A}|\hat{c}\right) > \overline{\pi}^H$ at $c_I = \hat{c}_I$. We keep the term "intersection" nevertheless, and thereby mean intersection in the two-dimensional space spanned by variables P and A.

[2] Maple by Maplesoft, 615 Kumpf Drive, Waterloo, Ontario, Canada, N2V 1K8. The example can be requested from the authors.

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