Securitization and moral hazard: Does security price matter?

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Abstract: This article analyses the effect of security price on the behaviour of bank securitization. We present a model of bank securitization in which security price together with liquid constraints create the incentive for banks to originate and sell assets backed securities to investors. Banks have a comparative advantage in locating and screening projects within their locality. Our results show that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broke in the US in 2007.

Keywords: securitization, security price, moral hazard

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1 Introduction

The financial crisis triggered by the US subprime mortgage sector in 2007 had an unprecedented negative impact on the real economy and on the banking sector. The use of securitization is considered a central issue, and it has provoked a number of discussions among the academics and regulators.

In economic theory, moral hazard is a situation in which a party insulated from risk behaves differently from how it would behave if it were fully exposed to the risk. Zandi (2009) described moral hazard as a root cause of the subprime mortgage crisis. He wrote: "...the risks inherent in mortgage lending became so widely dispersed that no one was forced to worry about the quality of any single loan."

Informational asymmetry in the securitization market is blamed for the origin of moral hazard (Ho and Sung (2012)). Several research has placed the emphasis on informational asymmetries in securitization markets, which they consider the main cause of the moral hazard of banks, such as lazy monitoring or screening (Dell’Ariccia et al., 2008; Berndt and Gupta, 2009; Purnanandam, 2009; Mian and Sufi, 2009; Keys et al., 2010).

However, one may ask whether informational asymmetry is the only root of evil in the disastrous impact of securitization. In this article, we measure the level of moral hazard as the number that banks invest on low profitable projects, securitize them and sell them at the secondary markets. We attempt to show a price motivation of moral hazard for securitization in banking sector. We try to link these increases in subprime securitization with the increases in security price. Our results show that show that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broke in the US in 2007.

The rest of this article proceeds as follows. Section 2 presents the related literature and section 3 presents the basic setup of the model. Then we analyse the equilibriums under two different pricing mechanisms in part four. To draw comparisons, we discuss some implications of our analysis in the subprime crisis in part five. The section six gives our conclusion.

2 Related Literature

Our study is mainly related to the literature on the motivation of credit risk transfer that leads to moral hazard. One main motivation for securitization is banks’ perspective on risk management, according to which banks use securitization to transfer or diversify credit risks. Duffee and Zhou (2001) show that a bank can use such swaps to temporarily transfer credit risks of their loans to others, reducing the likelihood that defaulting loans trigger the bank’s financial distress. However, they find that the introduction of a credit-derivatives market is not necessarily desirable because it can cause other markets for loan risk-sharing to break down. Different from their research, Allen and Carletti (2006) show that credit risk transfer can be beneficial when banks face uniform demand for liquidity. Wagner and Marsh (2006)’s analysis suggests that the incentive of banks to transfer credit risk is aligned with the regulatory objective of improving stability. Moreover, they find the transfer of credit risk from banks to non-banks to be more beneficial than CRT within the banking sector.
Another argument is that of the regulatory arbitrage associated with capital requirements. Given that capital is more costly than debt, the retention of a proportion of capital for loans in a balance sheet creates additional cost for banks. By taking this loan off their balance sheet, they can save their capital. Carlstrom and Samolyk (1995) shows that a loan sales market allows a banker having adequate capital to acquire profitable projects originated by a banker whose own capital is insufficient to support the additional risk. Calomiris and Mason (2004) show that the avoidance of capital requirements could be motivated either by efficient contracting or by safety net abuse. They find that securitizing banks set their capital relative to managed assets according to market perceptions of their risk, and seem not to be motivated by maximizing implicit subsidies relating to the government safety net when managing risk.

A third argument is related to the more resilient source of bank funds. With a constraint on funds, retaining a loan until maturity involves an opportunity cost if banks have other more profitable lending opportunities. By using securitization, banks can recuperate their funds earlier, and redeploy them in another investment project. Gorton and Pennacchi (1995) present a model of incentive-compatible loan sales and interpret why the amount of commercial and industrial loan sales outstanding had grown so fast. Parlour and Plantin (2008) characterize when a liquid secondary market for loans arises, when a liquid secondary loan market is socially desirable, and they provide testable predictions on the effect of the emergence of this market on prices and quantities in bond and primary loan markets.

There is some research on price fluctuations and shocks on aggregate economic performance (Jalles, 2009). However, there is few research that explicitly analyse the link between security price and moral hazard in bank securitization behaviour. Our research concentrates on the effect of price have on the behaviour of banks’ choice of securitised loan. We introduce a two period model to show that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious.

The framework builds on the work of Bernanke and Gertler (1987) and Samolyk (1989). Based on these work, Carlstrom and Samolyk (1995) develop a general equilibrium model in which localized information creates the incentive for some banks to originate investments and sell them to other investors rather than fund them on theft balance sheets. We follow this idea and focus the relationship between security price and moral hazard.

3 The basic setting and the benchmark model

3.1 The model setting

There is an economy made up of individuals who can be described as bankers, depositors and investors. The economy lasts two periods. There are several segmented markets and there is one banker per region. In each market, there is an unlimited supply of safe projects available to all individuals and each bank also has $N$ local risky investment opportunities. The safe projects yield a risk free rate return of $R^f$ in the next period. The risky project cost $1$ in period 0 and yields two possible outcomes in period 1. If the project fails, it will yields $\theta_l$; If the project success, it will yields $\theta_u$. The probability that the project success is $\pi_i$ (Given project $i$ in market $j$). Each local banker
ranks the success probabilities of his local projects from high to low, \( \pi_{1,j} < \pi_{2,j} < \ldots < \pi_{N,j} \). It is assumed that \( \theta_L < R_f < \theta_H \), so that the banker will be willing to hold both safe and risky projects.

In period 0, a banker \( j \) in market \( J \) takes his endowment of bank capital, \( w^b_j \), as given. We assume that \( w^b_j \) is identical across the markets. Without loan sales, bankers invest their endowments and attract local deposits to fund their on-balance-sheet portfolios. In period 0, the representative depositor in each market receives an endowment, \( w_d \), which is identical across markets.

In period 1, the bank takes his endowment of bank capital, \( w^b \), as given. We assume that \( w^b \) is identical across all banks. Without loan sales, bankers invest their endowments and attract local deposits to fund their on-balance-sheet portfolios.

Bankers are risk neutral and possess an information technology that enables them to screen the ex-ante quality and monitor the ex post performance of certain risky projects. Depositors and other bankers cannot observe the quality of a given banker's projects. All individuals know the distribution of the success probabilities for risky projects (\( \pi_i \sim U[0,1] \)).

The banker can offer to sell a pool loan projects, \( L_p \), to other banks or investors. The bank is rational and it chooses the \( n \) best projects to hold on-balance-sheet. And securitise the next best project available.

The securitised loan, \( L_p = \{n+1, \ldots, n+L_p\} \), sell for a per-project price \( P \). At the same time, the bank may purchase a pool of loans, \( L_{p,k} \), from other banks in market. Let the \( L_{p,k} = \{n_k+1, \ldots, n_k+L_{p,k}\} \) denote the securities the bank buy from other banks. We assume that a loan purchaser accrues all of the profits from the transaction and the price of the loan sales is \( P(L_{p,k}) = 1 \). The loan purchaser obtains the maximum rents from the transaction.

In period 0, the bank maximizes expected period 1 profits of

\[
\Pi = \text{Max} \left\{ \sum_{i=1}^{n} \pi_i \theta_H + (1 - \pi_i) \theta_L + R^f s + E(R(L_{p,k}))l_{p,k} - R^d w^d \right\}
\]

where \( E(R(L_{p,k})) = \frac{1}{L_{p,k}} \sum_{i=1}^{L_{p,k}} E(\pi_{m,i} \theta_H - \theta_L) + \theta_L \cdot n \) is the number of risky projects originated and funded on-balance-sheet. And \( s \) is the bank's investment in safe projects. The bank maximizes Eq. (1) subject to the portfolio-balance constraint,

\[
\text{Constraint in period 0: } w^d + w^b + (P(L_p) - 1)L_p = n + s + P(L_{p,k})l_{p,k}
\]

The left side of Eq. (2) indicates that any proceeds from loan sales net of origination costs, \( (P(L_p) - 1) \), augment bank capital and deposits as a source of funds for on-balance-sheet activities.

And the banks face a regulation constraints:
\[ \theta_L (n + l_{p,k}) + R^f s \geq R^d w^d \]  

(3)

Because depositors cannot observe the ex post returns on a bank’s risky investments, the bank must offer a return on deposits that is not contingent on the return on bank projects. Deposit contracts must also offer a return that is greater than or equal to the opportunity cost of funds. These two considerations imply that

\[ R^d \geq R^f \]  

(4)

There are infinite quantity investors in the markets whose risk taking is influenced by investor sentiment. Sentiment can reflect either biased expectations or institutional preferences and constraints, such as the demand by foreigners or money market funds for securitised loan.

For expository simplicity, it is assumed that whenever an agent is indifferent between a nonzero asset position and not trading at all, the former is chosen for the reason to keep the market share or build a strong client relationship.

3.2 The benchmark model: No loan sales and purchases

When loan sales and purchases are prohibited, \( l_{p,k} = 0 \) and \( l_p = 0 \). So in the beginning period, the banks contract with depositors and choose \( n \) and \( s \) to maximize Eq.(1), subject to constraints (2), (3) and (4). And we can get our Lemma 1.

**Lemma 1:** When loan sales and purchases are prohibited, the banks maximize utility when:

\[ \pi_n (\theta_H - \theta_L) + \theta_L = R^d \]  

if the bank is not constraint and \( \lambda_1 = 0 \)

\[ \pi_n (\theta_H - \theta_L) + \theta_L = R^d + \lambda_1 (R^d - \theta_L) \]  

if the bank is constraint and \( \lambda_1 > 0 \)

\[ R^d = R^f \]

where the Lagrange multiplier \( \lambda_1 \) is positive when (3) is binding for the profit-maximizing level of risky investments.

Proof: Please refer to the appendix.

Lemma 1 shows that the profit-maximizing choice of \( n \) is equivalent to the choice of a cutoff success probability for investment in local risky projects.

Given the results in Lemma 1 and Eq.(2), we can calculate \( n \) yields the associated constraint on the on-balance-sheet funding of risky projects,

\[ n \leq \frac{R^f w^b}{R^f - \theta_L} \equiv n_c \]

\( n_c \) is the maximum number of risky projects that the banker’s capital can support. Facing a draw of project opportunities, the bank invests in the best local projects available subject to this constraint. As
\( n_c < n_f \) where \( R_{n_f} = R^f \), the bank can’t fund all risky project opportunities in his locality which is not socially optimal.

In next section, we will show that under low pricing mechanism the economy can achieve social optimal through securitization by the constraint bank and the moral hazard problem is not so serious while under high pricing mechanism the economy tend to be over investment through securitization and the moral hazard problem is serious.

4 Equilibria under two pricing mechanism

We consider two pricing scenarios between bank \( i \) and bank \( j \). We assume there is no pure arbitrage. Different liquidity constraints mean that beneficial exchange can occur via a loan sale by constrained bank \( i \) to unconstrained bank \( j \) as long as

\[
E(R(L_{p,k}))/R^f \geq P(L_{p,k}) \geq 1
\]

In either scenario, the problem is that of calculating the equilibrium exante expected utility of the banks for any given price, and then studying the bankers’ optimal asset choice.

4.1 Price formation mechanism

Price of securitized loan is determined by the choice of the infinite quantity investors whoes risk taking is influenced by investor sentiment. Investor sentiment can come from a variety of sources, such as shifts in psychology, regulatory rules, or demand for a particular asset class that is otherwise unrelated to fundamental payoffs (Shleifer and Vishny, 2010). For example, if some investors such as foreigners, insurance companies, or money market funds demand AAA-rated bonds for reasons beyond the fundamental economics of payoffs, and are willing to pay substantially more for such bonds than for almost equally safe bonds, we think of this as investor sentiment (Caballero and Krishnamurthy, 2009; Gorton and Metrick, 2009; Shleifer and Vishny, 2010). Such demand can be fueled by loose monetary policy or by evidence of a default history (Kaplan and Stein, 1993) or when the price appreciation of houses made mortgage defaults relatively rare (similar as the subprime crisis). At the other end of the spectrum, bad fundamental news can cause investors to dump securities when they lose confidence in their valuation models (Caballero and Krishnamurthy, 2008). Except the sentiment, speculative trading plays a role in determining the asset price, Mei et al.(2009) find that trading caused by investors’ speculative motives can help explain a significant fraction of the price difference between the dual-class shares. But the pricing forming mechanism is very similar to the investor sentiment. We consider the two extreme pricing mechanisms influenced by the investor sentiment:

1. \( P(L_{p,k}) = 1 \) (Pessimistic investor sentiment)

2. \( P(L_{p,k}) = E(R(L_{p,k}))/R^f \) (optimistic investor sentiment)

In fact, the price can be reduced below 1 when the market broke down and can be increased above \( E(R(L_{p,k}))/R^f \) which we call it as “price bubble”. For simplicity, we exclude these two situations.

4.2 Equilibrium under low price (\( P(L_{p,k}) = 1 \))
In this pricing mechanism, the purchase bank accrues all the profits from the transaction. The loan purchaser obtains the maximum rents from the transaction. Facing the security price, the bank maximizes (1), subject to the constraints (2) and (3). By solving the problem, we can get

The optimal condition for the unconstraint bank is

\[ \pi_{n+k}(\theta_H - \theta_L) + \theta_L = R^d \tag{5} \]

with \( I_{p,k} > 0 \) and \( I_p = 0 \). And the optimal condition for the constraint bank is \( I_p > 0 \) and \( I_{p,k} = 0 \) (Please refer to the appendix). A constrained bank will sell loan while unconstrained banks purchase loans, they will continue to do so until profits are maximized. Constrained banks continue to fund \( n_l \) loans on-balance-sheet and sells loan to unconstrained banks and investors. The unconstraint banks cannot verify the profitable remaining projects, based on his assessment, he expects to receive a pool of projects in which the marginal project included has an expected return equal to the risk-free rate. We can summary in Proposition 1.

**Proposition 1:** When security price is low, the constrained bank continues to fund loans on-balance-sheet and sells projects to unconstrained bank and investors. The unconstrained bank will buy loans from the constrained banks until regulation constraints bind. The two types of banks can share risk with each other and the moral hazard problem is not serious.

Proof: Please refer to the appendix.

This pricing scenario implies a relatively simple equilibrium allocation. Since loan sales only occur so that on average the efficient number of projects are invested in, the number of projects invested in by an unconstrained bank will in general equal \((n_c + I_{p,k})\). The economy can achieve social optimal,

\[
\frac{n + 1}{\text{on-balance sheet projects of unconstrained banks}} + \frac{n_c}{\text{on-balance sheet projects of constraint banks}} \approx N^* \]

### 4.3 Equilibrium under high price

\( P(L_{p,k}) = E(R(L_{p,k})) / R^f \)

In this pricing mechanism, the sell bank accrues all the profits from the transaction.

Facing the security price, the bank maximizes (1), subject to the constraints (2) and (3). Solving the problem, we can proof that regardless of whether the regulation constraint binds or not, the optimal condition for the unconstraint bank is \( \frac{\partial L}{\partial I_p} = R^d \left( E(R(L_{p,k})) / R^f - 1 \right) > 0 \) which means the more loan sales the more profit the unconstraint bank can get.

**Proposition 2:** When security price is high, both the constrained bank and the unconstrained bank continues to fund loans on-balance-sheet and sells projects to investors. The banks tend to security high risky projects which is not socially optimal. The two types of banks cannot share risk with each other and the moral hazard problem is serious.

Proof: Please refer to the appendix.
The assessment of the expected return on unfunded projects is inferred from observed portfolio behavior. Therefore, if \[ P(L_{p,k}) = E(R(L_{p,k})) / R^f \] > 1, unconstrained bankers may have the incentive to mimic constrained bankers. This would involve funding some unprofitable local projects in order to appear to have received a good draw of investment opportunities.

Since loan sales occur for both types of banks, both the constraint banks and the unconstraint banks tend to over investment on low profit project and sell it at the secondary market. The economy cannot achieve social optimal,

\[
\left( n + l^c \right) + \left( n + l^u \right) \equiv N^H > N^*
\]

### 4.4 Comparison

Without securitization, the constraint banks can only invest in \( n_c \) project and with \((n_f - n_c)\) socially profitable projects uninvested. The number of invested projects including both the constraint banks’ and unconstraint banks’ is \( N_c \) which is not socially optimal. Under the low pricing mechanism, the number of invested projects in the economy is around socially optimal number \( N_c \). The level of moral hazard is low. Under the high pricing mechanism, the number of invested projects in the economy is \( N^H \) which is above the socially optimal number \( N_c \). The banks in the economy tend to over invest on low profitable projects which is not socially optimal. The difference between \( N^H \) and \( N_c \), \((N^H - N_c)\) can be considered as the level of the moral hazard.

![The level of moral hazard](image)

**Figure 1** Moral hazard under different prices

### 5 Implications

While securitization has revolutionized fixed income markets and brought billions of dollars of revenues to the banks, for many investors, even some institutional investors, this process can be opaque and filled with problems of asymmetric information and moral hazard. Our main results concludes from the theory model is that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious.

The explosive growth of the private, non–government-sponsored enterprise (GSE) backed mortgage-backed securities (MBS) market lies at the heart of the 2007–2009 global financial crisis. This market both fueled and was fuelled by the expansion of subprime credit and the housing boom(He et al., 2011). Our results fit the subprime crisis well in some aspects.
The market for MBS may have tilted toward scenario “Optimistic investor sentiment” particularly for large issuers when this new and rapidly growing market boomed between 2004 and 2006. Figure 1 plots the median fraction of AAA tranches of MBS, sorted by issuing year and issuer size. The median fraction of financing in AAA tranches sold by large and small issuers is quite similar in 2000 (just above 96 percent for the median deal) but then trends downward for both groups of securities as the housing and MBS markets grow.

The gap increases over time, peaking at about 10 percentage points in 2006, the height of the boom. Moreover, the incentive toward favoritism ought to be stronger during market booming periods.

![Figure 1 Fraction of AAA rating securities](image1)

Notes: “Big issuer” means that the market share of the issuer falls into the top 10 percent of the market share distribution in that year; “Small issuer” refers to the rest of the sample.
Source: Bloomberg

Nonetheless, the frantic pace of asset price increases could not be sustained indefinitely, and when prices slipped, the financial feedback loops worked in reverse. By the end of 2006, housing sales had weakened, prices were declining and mortgage payments fell behind which we can consider as “Pessimistic investor sentiment”. Mounting losses continued in 2008, bringing down Bear Stearns, Lehman Brothers, Merrill Lynch and AIG, and the icy breezes spread across Europe. The federal government took over AIG and renationalized Fannie Mae and Freddie Mac (dealers in over half of all secondary mortgages). Subprime securitization plummeted (Figure 3).

![Figure 2 Media Monthly prices of MBS](image2)

Notes: The price history starts from the month of origination until the security stops trading or April 2009 (whichever comes first).
Source: Bloomberg

Figure 2 Media Monthly prices of MBS

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6 Conclusions

This paper has shown that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broke in the US in 2007.

Our paper complements and contributes to the growing literature on the anatomy of the housing crisis and the role of MBS markets in this crisis, as well as incentive problems of financial service industries generally.

A limitation of the framework is the restriction of omitting the information asymmetry. For studying the interaction of risk-sharing and information-based trading like securities, it is too restrictive. However, there is large range of research in this area (Campbell and Kracaw, 1980; Fulghieri and Lukin, 2001; Inderst and Mueller, 2006; etc.)

Another limitation of the work is the pricing mechanism. For some technology reasons, we cannot model the price as detail as Rahi (1996) which shows that the resulting equilibrium is fully revealing, with all private information being transmitted through prices. We use the investor sentiment instead this process.

References

Appendix

Proof of the Lemma 1

From Eq.(2), we can get \( w^d = n + s - w^b \). Substituting it in to Eq.(1) and (2), we can construct the Lagrange function:

\[
L_1(n,s) = \sum_{i=1}^{n} [\pi_i \theta_H + (1-\pi_i) \theta_L] + R^f s - R^d (n + s - w^b) - \lambda_i [R^d (n + s - w^b) - \theta_L n - R^f s]
\]

where \( \lambda_i \) is the Lagrange Multiplier.

First order condition:

For \( n > 0 \), we can get \( \frac{\partial L_1(n,s)}{\partial n} = 0 \) which shows

\[
\pi_n (\theta_H - \theta_L) + \theta_L + \lambda_i \theta_L = R^d (1 + \lambda_i) \text{, i.e.}
\]

\[
\pi_n (\theta_H - \theta_L) + \theta_L + \lambda_i (R^d - \theta_L) \quad (A_1)
\]

For \( 1 + \lambda_i > 0 \), we can get \( R^f \leq R^d \) which together with Eq.(4) shows that

\[
R^f = R^d
\]

Proof of proposition 1

As \( P(I_p) = P(I_{p,k}) = 1 \), Eq.(2) can be simplified:

\[
w^d + w^b = n + s + l_{p,k}
\]

We can construct the Lagrange function:

\[
L_2 = \sum_{i=1}^{n} [\pi_i \theta_H + (1-\pi_i) \theta_L] + R^f s + E(R(L_{p,k}))l_{p,k} - R^d (n + s + l_{p,k} - w^b)
\]

\[
- \lambda_2 [R^d (n + s + l_{p,k} - w^b) - R^f s - \theta_L (n + l_{p,k})]
\]

where \( \lambda_2 \) is the Lagrange Multiplier.

First order condition with respect to \( n, s \) and \( l_{p,k} \):
\[
\frac{\partial L_2}{\partial n} = \pi_n (\theta_H - \theta_L) + \theta_L - R^d - \lambda_2 (R^d - \theta_L) \leq 0 \quad \text{with} \quad \frac{\partial L_2(n,s)}{\partial n} = 0 \quad \text{if} \quad n > 0
\]

We can get similar results as in Lemma 1

\[
\pi_n (\theta_H - \theta_L) + \theta_L + \lambda_2 \theta_L = R^d (1 + \lambda_2)
\]

The optimal \( s \) also shares the same result as in lemma 1.

\[
l_{p,k} : \frac{\partial L_2}{\partial l_{p,k}} = \pi_{n+k} (\theta_H - \theta_L) + \theta_L - R^d - \lambda_2 (R^d - \theta_L) \leq 0 \quad \text{i.e.}
\]

\[
\pi_{n+k} (\theta_H - \theta_L) + \theta_L (1 + \lambda_2) - R^d (1 + \lambda_2) \leq 0 \quad \text{with} \quad \frac{\partial L_2}{\partial l_{p,k}} = 0 \quad \text{if} \quad l_{p,k} > 0
\]

If the regulation constraint doesn’t bind, i.e. \( \lambda_2 = 0 \), the bank maximizes his utility when

\[
\pi_{n+k} (\theta_H - \theta_L) + \theta_L - R^d = 0 \quad \text{and} \quad l_{p,k} > 0.
\]

So the optimal condition for the unconstraint bank is

\[
\pi_{n+k} (\theta_H - \theta_L) + \theta_L = R^d (A_2)
\]

with \( l_{p,k} > 0 \) and \( l_p = 0 \).

If the regulation constraint binds, i.e. \( \lambda_2 > 0 \), \( \pi_{n+k} (\theta_H - \theta_L) + \theta_L (1 + \lambda_2) - R^d (1 + \lambda_2) < 0 \) and \( l_{p,k} = 0 \).

So the optimal condition for the constraint bank is \( l_p > 0 \) and \( l_{p,k} = 0 \).

Proof of proposition 2

As \( P(l_p) = P(l_{p,k}) = E(R(L_{p,k}))/R^{f} \), Eq.(2) can be simplified:

\[
w^d + w^b + (E(R(L_{p,k}))/R^{f} - 1)l_p = n + s + E(R(L_{p,k}))/R^{f}l_{p,k}
\]

We can construct the Lagrange function:

\[
L_3 = \sum_{i=1}^{n} \{ \pi_i \theta_H + (1 - \pi_i) \theta_L \} + R^f s + E(R(L_{p,k}))l_{p,k}
\]

\[- R^d \left[ n + s + E(R(L_{p,k}))/R^{f}l_{p,k} - w^b - (E(R(L_{p,k}))/R^{f} - 1)l_p \right]
\]

\[- \lambda_3 \left[ R^d \left[ n + s + E(R(L_{p,k}))/R^{f}l_{p,k} - w^b - (E(R(L_{p,k}))/R^{f} - 1)l_p \right] - R^{f}s - \theta_L (n + l_{p,k}) \right]
\]

where \( \lambda_3 \) is the Lagrange Multiplier.
First order condition with respect to $l_p$ and $l_{p,k}$:

$$l_p: \frac{\partial L_3}{\partial l_p} = R^d (E(R(L_{p,k}))) / R^f - 1 + \lambda_3 R^d (E(R(L_{p,k}))) / R^f - 1 \leq 0 \text{ with } \frac{\partial L_3}{\partial l_p} = 0 \text{ if } l_p > 0$$

If the regulation constraint doesn’t bind, i.e. $\lambda_3 = 0$, the optimal condition for the unconstraint bank is

$$\frac{\partial L_3}{\partial l_p} = R^d (E(R(L_{p,k}))) / R^f - 1 > 0$$

which means the more loan sales the more profit the unconstraint bank can get.

If the regulation constraint binds, i.e. $\lambda_3 > 0$, the optimal condition for the unconstraint bank is

$$\frac{\partial L_3}{\partial l_p} = (\lambda_3 + 1) R^d (E(R(L_{p,k}))) / R^f - 1 > 0$$

which also means that the more loan sales the more profit the constraint bank can get.

$$l_{p,k}: \frac{\partial L_3}{\partial l_{p,k}} = \pi_n + k (\theta_H - \theta_L) + \theta_L - \left[ \pi_n + k (\theta_H - \theta_L) + \theta_L \right] \frac{R^d}{R^f} - \lambda_3 \left[ \pi_n + k (\theta_H - \theta_L) + \theta_L \right] \frac{R^d}{R^f} \leq 0$$

with $\frac{\partial L_3}{\partial l_{p,k}} = 0 \text{ if } l_{p,k} > 0$

If the regulation constraint doesn’t bind, i.e. $\lambda_3 = 0$, $\left[ \pi_n + k (\theta_H - \theta_L) + \theta_L \right] (1 - \frac{R^d}{R^f}) = 0$ and $l_{p,k} > 0$.

If the regulation constraint binds, i.e. $\lambda_3 > 0$, $\left[ \pi_n + k (\theta_H - \theta_L) + \theta_L \right] (1 - \frac{R^d}{R^f} - \lambda_3) < 0$ and $l_{p,k} = 0$.

So under the high pricing mechanism, both the constraint banks and the unconstraint banks tend to over investment on low profit project and sell it at the secondary market.