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Abstract

This paper analyzes the effects of non-atmospheric consumption externalities on optimal commodity taxation and on the social cost and optimal levels of public good provision. A negative consumption externality, by lowering the social cost of public good provision, may require the second-best level of public good provision to exceed the first-best level. If those households who are most important for building up the consumption reference level respond the least to commodity taxation, heterogeneity may imply an equity-efficiency tradeoff. This tradeoff is present *only* if the consumption externality is of the non-atmospheric type.

Keywords and Phrases: consumption externality, optimal commodity taxation, Pigou, public good provision, Ramsey rule.

JEL Classification Numbers: D62, H21, H41

1 Introduction

This paper addresses the effects of a non-atmospheric consumption externality on optimal commodity taxation (many person Ramsey rule), the social cost of public good provision (Pigovian rule property), and the optimal level of public good provision (Pigovian level property). The paper is motivated by the recent literature on consumption externalities and happiness.

Consumption externalities have attracted the attention of economists for centuries. Many classical economists, for instance, assumed that the quest for status — a consumption externality — is an important component of the pursuit of self-interest (Kern, 2001). Adam Smith (1759) in his *Theory of Moral Sentiments* wrote: "The poor man's son...when he begins to look around him, admires the condition of the rich. He finds the cottage of his father too small ... It appears in his fancy like the life of some superior rank of beings, and, in order to arrive at it, he devotes himself for ever to the pursuit of wealth and greatness." (Smith 1759, p. 181)[1] More recently, a rapidly growing body of literature addresses the paradoxical development of income and happiness. While real per capita disposable income has substantially increased over the last fifty years, there is no trend in subjective well-being. The fact that raising income of all does not increase happiness of all (Easterlin 1995) can be explained by a consumption externality: the average income or consumption level represents a point of reference (Brekke and Howarth 2002, Frank 1985 and 1999). A number of survey experimental studies confirm this explanation. Solnik and Hemenway (1998, 2005) present questions involving two states of the world. Both states are identical, except for one characteristic, e.g., income. In state A, an individual has a given income level that is lower than the average (others') income level. In state B, an individual has a *lower* absolute income level that exceeds the average income level. Nearly half of the respondents prefer state B over state A, which indicates the importance of a consumption reference level. Similar evidence is provided by survey experimental studies of Johansson-Stenman et al. (2002, 2006).[2]

Consumption externalities have shed light on the analysis of renewable resource extraction (Alvarez-Cuadrado and Van Long, 2011), on envy and inequality (Van Long and Alvarez-Cuadrado, forthcoming), on happiness (Easterlin 1995, Frank 1985, Frank 1999, Scitovsky 1992), economic growth (Brekke and Howarth 2002, Carroll *et al.* 1997, Liu and Turnovsky 2005), asset pricing (Abel 1999, Campbell and Cochrane

[1]Clearly, one can go back in time even much further. Plato, in *The Republic* (II) wrote: *Since ...appearance tyrannizes over truth and is lord of happiness, to appearance I must devote myself.*

[2]A number of further studies offer strong evidence of the existence of consumption externalities. Important contributions include Alpizar et al. (2005), Carlsson et al. (2007), Ferrer-i-Carbonell (2005), Luttmer (2005), McBride (2001), and Neumark et al. (1998).

1999, Dupor and Liu 2003), optimal tax policy over the business cycle (Ljungqvist and Uhlig 2000), optimal redistributive taxation (Aronsson and Johansson-Stenman 2010, Boskin and Sheshinski 1978, Layard 1980), and the excess burden (Wendner and Goulder 2008). Yet consumption externalities also have important implications for the optimal provision of public goods. Although other authors have raised this point[3], I know of no prior study that rigorously analyzes how a *non-atmospheric* consumption externality influences the optimal first-best and second-best *levels* of public good provision.[4] This is the focus of the present paper. I develop a theoretical model to examine a generalized Ramsey rule and optimal rules and levels of public good provision in the presence of a generalized consumption externality.

In the prior literature, it has been argued that the second-best level of public good provision is lower than the first-best level as long as the government's expenditures are financed by distortionary taxes, as suggested by Pigou (1947). This argument, however, is not always true. First, the fact that — under distortionary taxation — the social cost of public good provision may exceed the private cost does not necessarily imply that the second-best *level* of public good provision is lower than the first-best level. Second, an extensive literature argues that distortionary taxation need not inevitably raise the social cost of public good provision for a variety of reasons. Distortionary taxation may have desirable consequences for the income distribution (see e.g., King 1986, Batina 1990b, Gaube 2000). Next, if the private and public goods are Hicksian complements, an increase in the provision of the public good raises demand for the private good and thereby commodity tax revenue, which lowers the social cost of public good provision (Diamond and Mirrlees 1971, Atkinson and Stern 1974, King 1986, Batina 1990b). Furthermore, in a dynamic framework, distortionary taxation may improve the dynamic efficiency of the economy (Batina 1990a).[5] All of these effects lower the social marginal cost of a public good, which, in turn, potentially gives rise to a higher level of public good provision in the second-best optimum than in the first-best optimum.

This paper contributes to the prior literature in three ways. First, this paper identifies a negative consumption externality as a further source for "Pigovian level reversal." That is, the second-best level of public good provision may equal or even exceed the first-best level. Once available policy instruments include a poll transfer

[3]See Aronsson and Johansson-Stenman 2008, Wendner and Goulder 2008.

[4]As defined below, a consumption reference level gives rise to a consumption externality. The consumption reference level is some weighted average of individuals consumption levels. A consumption externality is non-atmospheric, if those weights differ among households. The term "atmospheric externality" was introduced by Meade (1952).

[5]Further important contributions to the discussion on the social cost and optimal levels of public goods include Chang (2000), Gaube (2005), Ng (1987), Stiglitz and Dasgupta (1971), Wildasin (1984), Wilson (1991).

(but not a poll tax), the second-best level of public good provision equals the first-best level whenever the revenue raised by applying the first-best commodity tax rate does not fall short of the revenue requirement for financing the optimal level of the public good. This case is the more likely the stronger is the negative consumption externality. Heterogeneity aside, "Pigovian level reversal," that is the second-best level of public good provision exceeds the first-best level, is also possible in an economy populated with homogeneous households with Cobb-Douglas preferences. For such an economy, without consumption externalities, Wilson (1991) demonstrates that the second-best level of public good provision is always lower than the first-best level. In contrast, in the presence of a negative consumption externality, the paper presents an example in which the second-best level of public good provision exceeds the first-best level.

Second, a negative consumption externality lowers the social cost of public good provision. We identify two cases under which a negative consumption externality gives rise to reversal of the Pigovian rule property. That is, due to the negative consumption externality, the social cost of public good provision, in the second best, is less than in the first best. These two cases involve a strong enough negative consumption externality, and a high enough demand for the public good.

Third, heterogeneity affects the second-best commodity tax rate. Regarding heterogeneity, the consumption externality does not introduce a wedge in case of an atmospheric externality. However, a non-atmospheric consumption externality introduces a wedge between the first-best and second-best commodity tax rates. Suppose those households who are most important for building up the consumption reference level respond the least to commodity taxation. Then, this kind of heterogeneity tends to reduce the optimal second-best commodity tax rate. In this case, the negative consumption externality can introduce a tradeoff between equity correction and externality correction. This type of equity-efficiency tradeoff only occurs in the presence of a non-atmospheric consumption externality.

Section 2 of this paper presents the economy's private and public sectors, introduces the consumption externality, and discusses the government's instruments under several restrictions. Section 3 develops a generalized many person Ramsey rule. Section 4 analyzes the effects of consumption externalities on the social cost and optimal levels of public good provision. In addition, the section presents an example of "Pigovian level reversal" for an economy with homogeneous households. Section 5 concludes the paper. The appendix contains proofs and mathematical results that support the analysis of the main text.

2 The economy

We consider an economy with a continuum of households $i \in I \equiv [0,1]$ with the distribution function $F(i)$. Households may differ with respect to some individual-specific attribute, such as preferences. The aim of allowing for such differences is to motivate the inclusion of consumption externalities based on a consumption reference level.

2.1 The private sector

We extend King's (1986) framework by introducing a consumption externality. A household has preferences over a consumption good, x, leisure, l, a pure public good, g, and a consumption reference level, $\bar{x}^r$, that is considered to be exogenous by an individual household:[6]

$$u = u(i, x, l, g, \bar{x}^r)\,. \tag{1}$$

A household's indirect utility function is given by:

$$v = v(i, q, g, y, \bar{x}^r)\,, \tag{2}$$

where q is the consumer price of the consumption good, and y is the household's full income (i.e., the value of labor endowment).[7] Equation (2) defines the maximum level of utility that a household can obtain, given the price of the consumption good, full income, the level of public good provision, *and* the consumption reference level.

The consumption reference level is given by:[8]

$$\bar{x}^r = \int_{i \in I} \zeta(i) x(i)\, \mathrm{d}F(i)\,, \quad \zeta(i) \geq 0\,, \bar{\zeta} = 1\,. \tag{3}$$

The consumption reference level, $\bar{x}^r$, is defined as a weighted mean of individual consumption levels, the weights being given by $\zeta(i)$. Without loss of generality, we set $\bar{\zeta} = 1$. If $\zeta(i) = 1$ for all $i \in I$, $\bar{x}^r = \bar{x} \equiv \int_{i \in I} x(i)\, \mathrm{d}F(i)$. In this case, the consumption reference level is simply the economy's mean consumption level. However, the weights $\zeta(i)$ may vary across households, in which case a given consumption quantity of some households contributes more to the consumption reference level than the same quantity consumed by other households. For example, consider $I_1 \subset I$ represents the

[6] The framework may be extended to any number of consumption goods. The results presented below are not affected by consideration of many consumption goods.

[7] We choose l to represent the numeraire good and set the wage rate equal to unity.

[8] A bar is used to denote the mean level of a variable throughout. E.g., $\bar{z} \equiv \int_{i \in I} z(i) \mathrm{d}F(i)$. Subscripts are used to denote partial derivatives with respect to the subscripted variable(s).

set of all individuals with a "high" social status. If the consumption reference level depends only of members of this group, then $\zeta(i) > 0$ if $i \in I_1$, otherwise $\zeta(i) = 0$.

The consumption reference level gives rise to a negative or positive consumption externality. It may capture, for example, preferences related to conspicuous consumption or to a positive network externality. If $\zeta(i)$ is constant across households, the externality is said to be of the *atmospheric* type. In this case, any two individuals' consumption quantities are perfect substitutes as far as the consumption reference level is concerned. In contrast, if $\zeta(i)$ varies across households, the externality is said to be of the *non-atmospheric* type. In addition, we characterize consumption externalities according to the impact of the consumption reference level on indirect utility:

Definition 1 *A consumption externality is said to be* atmospheric *if* $\zeta(i) = \zeta(i')$ *for all* $i, i' \in I$. *A consumption externality is said to be* non-atmospheric *if* $\zeta(i) \neq \zeta(i')$ *for some* $i, i' \in I$.

A consumption externality is said to be negative *if* $v_{\bar{x}^r}(i, q, g, y, \bar{x}^r) < 0$. *A consumption externality is said to be* positive *if* $v_{\bar{x}^r}(i, q, g, y, \bar{x}^r) > 0$.

Consider the canonical example of a negative consumption externality: "keeping up with the Joneses" preferences. A keeping up with the Joneses externality is defined by: $\partial\,(u_x/u_l)/(\partial\,\bar{x}^r) > 0$ (see Dupor and Liu 2003). In this case, the consumption reference level raises the marginal utility of individual consumption relative to that of leisure. That is, for given (q, g, y), optimal individual consumption rises in the reference level: $x_{\bar{x}^r} > 0$. As one's consumption is subject to the keeping up with the Joneses externality, an additional unit of consumption not only satisfies one's direct benefit of consumption but also one's indirect benefit of keeping up with (or being better than) the Joneses. Consequently, a household is willing to give up more units of leisure for an additional unit of consumption in the presence of a keeping up with the Joneses externality.

It is important to emphasize that a negative consumption externality does not generally imply $x_{\bar{x}^r} > 0$, and a positive consumption externality does not generally imply $x_{\bar{x}^r} < 0$. Consider, for example, a positive network externality (a high mean level of cell phone users in a given country). In this situation, the marginal rate of substitution of consumption for leisure rises in $\bar{x}^r$, which implies $x_{\bar{x}^r} > 0$. Similarly, in case of a negative congestion externality (e.g., a high mean level of car users in a given road network), $\bar{x}^r$ lowers the marginal utility of consumption for leisure, and $x_{\bar{x}} < 0$. These examples suggest that a positive consumption externality is associated with $x_{\bar{x}^r} > 0$, while a negative consumption externality is associated with $x_{\bar{x}^r} < 0$.

But a keeping up with the Joneses externality represents a prominent exception to this rule.

Remark. A consumption externality may but need not imply that $x_{\bar{x}^r} \neq 0$. Consider, e.g., a utility function for which (x, l) is weakly separable from $\bar{x}^r$. In this case $x_{\bar{x}^r} = 0$, in spite of the fact that $v_{\bar{x}^r}(.) \neq 0$.

We define a household's money metric utility (equivalent income), $y_e(i)$, as the level of income which — at the reference values (q^R, g^R, $(\bar{x}^r)^R$) — yields the same level of utility as can be attained under $(q, g, \bar{x}^r)$:

$$y_e(i) \equiv e(i, q^R, g^R, (\bar{x}^r)^R, v) = f(i, q^R, g^R, (\bar{x}^r)^R, q, g, y, \bar{x}^r)\,, \tag{4}$$

where $f(.)$ is itself an indirect utility function for the household under consideration. Following King (1986), we will employ $f(.)$ in the social welfare function below. Considering conditional demand of the consumption good, Roy's identity — for given $\bar{x}^r$ — yields:

$$x(i, q, g, y, \bar{x}^r) = -\frac{\partial\, v(.)/\partial\, q}{\partial\, v(.)/\partial\, y} = -\frac{\partial\, f(.)/\partial\, q}{\partial\, f(.)/\partial\, y}\,. \tag{5}$$

Compensated demand (indexed by superscript c) is given by:

$$x^c(i, q, g, v, \bar{x}^r) = \frac{\partial\, e(.)}{\partial\, q}\,. \tag{6}$$

We finally state the Slutsky equation (again for given $\bar{x}^r$):

$$s_{xx}(i) \equiv \frac{\partial\, x^c(.)}{\partial\, q} = \frac{\partial\, x(.)}{\partial\, q} + \frac{\partial\, x(.)}{\partial\, y}\, x(.)\,. \tag{7}$$

In the following, we assume x to be weakly normal: $\partial\, x(.)/\partial\, y \geq 0$.

2.2 The public sector

An individual household considers the consumption reference level, $\bar{x}^r$, as given. The government, however, takes the impact of its policy instruments on the consumption reference level into account. The public sector controls the following instruments: a commodity tax $\tau = q - p$, the level of a public good, g, and lump sum taxes (transfers), $t(i) > 0$ ($t(i) < 0$), where p represents the constant marginal production cost of x. Per capita tax revenue, r, is given by: $r = t + \tau\, \bar{x}$, where $t = \int_{i \in I} t(i)\, \mathrm{d}\, F(i)$. We assume a constant average production cost of the public good, c. The government budget constraint is given by:

$$r = c\,g\,. \tag{8}$$

The government chooses its instruments so as to maximize an additively separable social welfare function, defined over individual levels of equivalent income:

$$SW = \int_{i \in I} W(y_e(i))\,\mathrm{d}F(i)\,, \tag{9}$$

where $W(.)$ is concave and increasing in $y_e(i)$. Concavity of $W(.)$ describes the degree of aversion to inequality in money metric utility levels, $y_e(i)$.[9]

The use of a social welfare function for determining an optimal set of government instruments may be criticized. In a companion paper, we study, in a similar framework, Pareto efficient taxation. In that paper, similar to Brekke and Howarth (2002, pp. 81–84), social comparisons are complemented by altruism (in the sense of concern for other people's utility).

Taking into consideration the resource constraint as well as (3), the Lagrangian of the government's maximization problem becomes:

$$\begin{aligned}\mathcal{L} = \int_{i \in I} W(y_e(i))\,\mathrm{d}F(i) + \lambda \left[\int_{i \in I} t(i)\,\mathrm{d}F(i) + \tau \int_{i \in I} x(i)\,\mathrm{d}F(i) - c\,g \right] \\ + \mu \left[\bar{x}^r - \int_{i \in I} \zeta(i) x(i)\,\mathrm{d}F(i) \right]\,,\end{aligned} \tag{10}$$

where μ denotes the Lagrange multiplier of the consumption reference level, and λ denotes the Lagrange multiplier of the resource constraint (measuring the social evaluation of an additional unit of government revenues). The government maximizes the Lagrangian (10) with respect to τ, g, $t(i)$, and $\bar{x}^r$. It might face one of the following three constraints:

$$t(i) = t\,, \tag{C1}$$

$$t(i) = t \leq 0\,, \tag{C2}$$

$$t(i) = t = 0\,, \tag{C3}$$

giving rise to second-best solutions. All constraints prevent the government from introducing *personalized* lump sum taxes or transfers. A poll tax is available, however, under constraint (C1). In addition, constraint (C2) restrains the government

[9] The use of equivalent income makes it possible to distinguish the cardinality of the social welfare function from the specific specifications of the indirect utility functions. That is, distributional concerns are channeled through the $W(.)-$ function rather than through concavity of the indirect utility function per se.

from imposing lump-sum *taxes* on households. Finally, constraint (C3) precludes the government from imposing any lump-sum taxes or transfers.

We now derive the necessary first-order conditions for this maximization problem, employing the facts that $\partial f/\partial q = \partial f/\partial \tau$, and $\partial f/\partial t = -\partial f/\partial y$.

$$\int_{i\in I} \frac{W(i)'}{\lambda}\, f_q(i)\,\mathrm{d}F(i) + \bar{x} + \tau \int_{i\in I} x_q(i)\mathrm{d}F(i) - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)x_q(i)\mathrm{d}F(i) = 0\,, \tag{11}$$

$$\int_{i\in I} \frac{W(i)'}{\lambda}\, f_g(i)\,\mathrm{d}F(i) + \tau \int_{i\in I} x_g(i)\mathrm{d}F(i) - c - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)x_g(i)\mathrm{d}F(i) = 0\,, \tag{12}$$

$$\int_{i\in I} \frac{W(i)'}{\lambda}\, f_y(i)\,\mathrm{d}F(i) - 1 + \tau \int_{i\in I} x_y(i)\mathrm{d}F(i) - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)x_y(i)\mathrm{d}F(i) = 0\,, \tag{13}$$

$$\int_{i\in I} \frac{W(i)'}{\lambda}\, f_{\bar{x}^r}(i)\,\mathrm{d}F(i) + \tau \int_{i\in I} x_{\bar{x}^r}(i)\mathrm{d}F(i) + \frac{\mu}{\lambda} - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)x_{\bar{x}^r}(i)\mathrm{d}F(i) = 0\,. \tag{14}$$

In addition, a first-best optimum satisfies $\mathrm{d}\mathcal{L}/\mathrm{d}t(i) = 0$:

$$\frac{W(i)'}{\lambda}\, f_y(i) + \tau\, x_y(i) - \frac{\mu}{\lambda}\,\zeta(i)x_y(i) = 1\,. \tag{15}$$

Equations (11) to (15) hold in a first-best optimum. In a second-best optimum, (11) , (12) and (14) are generally satisfied. Under all constraints, (15) does not hold, as no individualized lump sum taxes or transfers are available. In addition, under constraint (C2), first order condition (13) holds as a weak inequality ($\leq$). Under constraint (C3), (13) is not applicable.

2.3 Consumption externalities

The multiplier $\frac{\mu}{\lambda}$ reflects the consumption externality's social harm ($\mu > 0$) or benefit ($\mu < 0$), measured in terms of government tax revenue. Notice that $\mu > 0$ (that $\mu < 0$) reflects a negative (positive) consumption externality. First order condition (14) provides insight into the nature of $\frac{\mu}{\lambda}$.

Let $\epsilon \equiv \int_{i\in I} \frac{W(i)'}{\lambda}\, f_{\bar{x}^r}(i)\,\mathrm{d}F(i)$ denote the *direct* impact of the consumption externality on social welfare.[10] Moreover, in order to capture a non-atmospheric externality, we consider the covariance between $\zeta(i)$ and $x_{\bar{x}^r}(i)$. Let this covariance be expressed by $\phi(\zeta, x_{\bar{x}^r}) = \int_{i\in I} \zeta(i)\, x_{\bar{x}^r}(i)\mathrm{d}F(i) - 1\,\bar{x}_{\bar{x}^r}$.[11] E.g., if $\phi(\zeta, x_{\bar{x}^r}) > 0$, those households whose consumption levels increase the most in response to a marginal rise in the reference level are the ones who are associated with the highest weights $\zeta(i)$. Clearly,

[10] By "direct," we mean that ϵ does not account for any responses in $x(i)$ due to a marginal increase in $\bar{x}^r$, i.e., for $x_{\bar{x}^r}(i)$. Clearly, as shown below, μ also accounts for the *indirect* effects due to $x_{\bar{x}^r}(i)$.

[11] More generally, we express all covariances by ϕ. Let $\phi(m,n)$ denote the covariance between variables $m(i)$ and $n(i)$. Then, $\phi(m,n) \equiv \mathrm{Cov}\,(m(i), n(i)) = \int_{i\in I} (m(i) - \bar{m})(n(i) - \bar{n})\,\mathrm{d}F(i) = \int_{i\in I} m(i)\,n(i)\,\mathrm{d}F(i) - \bar{m}\,\bar{n}$.

$\phi(\zeta, x_{\bar{x}^r}) = 0$ in two cases: in case of an atmospheric externality, and in case of a utility function that is separable in $\bar{x}^r$, in which case $x_{\bar{x}^r}(i) = 0$. In view of this notation, (14) yields:

$$\frac{\mu}{\lambda} = \frac{-\epsilon - \tau\, \bar{x}_{\bar{x}^r}}{1 - \bar{x}_{\bar{x}^r} - \phi(\zeta, x_{\bar{x}^r})}\,, \tag{16}$$

which reveals the components the consumption externality's social harm or benefit depend on. First, consider a utility function for which (x, l) is weakly separable from $\bar{x}^r$. In this case $x_{\bar{x}^r}(i) = 0$, and so are $\bar{x}_{\bar{x}^r}$ and $\phi(\zeta, x_{\bar{x}^r})$. As a consequence: $\frac{\mu}{\lambda} = -\epsilon$.

Second, suppose $x_{\bar{x}^r}(i) \neq 0$ but $\zeta(i) = \zeta(i') = 1$ for all $i, i' \in I$. That is, $\phi(\zeta, x_{\bar{x}^r}) = 0$, and $\frac{\mu}{\lambda} = \frac{-\epsilon - \tau\, \bar{x}_{\bar{x}^r}}{1 - \bar{x}_{\bar{x}^r}}$. To fix ideas, suppose $\epsilon < 0$ (a negative consumption externality) and $\bar{x}_{\bar{x}^r} > 0$, as is the case for keeping up with the Joneses preferences. In this case, $\frac{\mu}{\lambda}$ captures also the responses of all $x(i)$ due to a rise in $\bar{x}^r$, i.e., $\bar{x}_{\bar{x}^r}$. The numerator is reduced by the fact that $\bar{x}_{\bar{x}^r} > 0$ raises government revenue. At the same time, the denominator is also reduced by the fact that $\bar{x}_{\bar{x}^r} > 0$. That is, $\bar{x}_{\bar{x}^r} \neq 0$ may exercise an ambiguous affect on $\frac{\mu}{\lambda}$.

Third, suppose $\phi(\zeta, x_{\bar{x}^r}) > 0$. If those households whose consumption levels increase the most in response to a marginal rise in the reference level are the ones who are most important, according to $\zeta(i)$, $\frac{\mu}{\lambda}$ is increased further. That is, multiplier $\frac{\mu}{\lambda}$ depends on the direct externality, as captured by ϵ, on the indirect externality, as captured by $\bar{x}_{\bar{x}^r}$, as well as on non-atmospheric contributions, as captured by $\phi(\zeta, x_{\bar{x}^r})$.

2.4 Net social marginal utility of income

Taking into account the consumption externality, the net social marginal utility of income becomes:

$$b(i) = \frac{W(i)'}{\lambda} f_y(i) + \tau\, x_y(i) - \frac{\mu}{\lambda} \zeta(i) x_y(i)\,, \tag{17}$$

$$\bar{b} \equiv \int_{i \in I} b(i) \mathrm{d}F(i) = \int_{i \in I} \frac{W(i)'}{\lambda} f_y(i) + \tau\, x_y(i) - \frac{\mu}{\lambda} \zeta(i) x_y(i) \mathrm{d}F(i)\,. \tag{18}$$

In (17), the first and second terms on the right hand side represent the conventional net social marginal utility of income. The additional term on the right hand side accounts for the externality. By assumption, $x_y(i) > 0$. Thus, the net social marginal utility of income is lowered (is raised) by a negative (positive) consumption externality in proportion to $\frac{\mu}{\lambda}$. Equation (18) shows the mean level of the net social marginal utility of income, $\bar{b}$.

In a first-best optimum, by (13) and (15), $b(i) = \bar{b}^* = 1$.[12] In a second-best optimum, under (C1), $\bar{b}^{**} = 1$, as (13) holds. Under restriction (C2), however the

[12]In what follows, an asterisk refers (two asterisks refer) to the first-best (second-best) allocation.

social cost of a rise in t (at $t = 0$) is not larger than the social benefit (in terms of government revenue): $\bar{b}^{**} \leq 1 = \bar{b}^{*}$. Under restriction (C3), $\bar{b}^{**} \lesseqgtr 1 \Leftrightarrow t^{*} \gtreqless 0$.

3 Ramsey rule

As shown in the appendix, first order condition (11) yields the generalized many person Ramsey rule:

$$\frac{-(\tau - \frac{\mu}{\lambda})\,\bar{s}_{xx}}{\bar{x}} = 1 - \bar{b} - \frac{\phi(b,x)}{\bar{x}} - \frac{\mu}{\lambda}\,\frac{\phi(\zeta, s_{xx})}{\bar{x}}\,, \tag{19}$$

where $\bar{s}_{xx} \equiv \int_{i \in I} s_{xx}(i)\,\mathrm{d}F(i)$. The optimal (approximate) proportionate change in compensated demand of the consumption good is proportional to $\bar{b}$, the covariances $\phi(b,x)$, $\phi(\zeta, s_{xx})$, and the externality, $\frac{\mu}{\lambda}$.

Lemma 1 *In a first-best optimum, $\tau^{*} = \frac{\mu}{\lambda}$.*

Proof. See appendix. ‖

The first-best (Pigovian) tax rate, τ^{*}, equals the externality term. Specifically, $\tau^{*} > 0$ ($\tau^{*} < 0$) in the presence of a negative (positive) consumption externality. Regarding the second-best tax rate(s), Ramsey rule (19) reveals two important results, one for the case of homogeneous households and the other for the case of heterogeneous households.

Proposition 1 (Homogeneous Households) *Consider a negative consumption externality ($\mu > 0$).*
(i) Under constraint (C1)*: $\tau^{**} = \frac{\mu}{\lambda} = \tau^{*}$, as $\bar{b}^{**} = 1$.*
(ii) Under constraint (C2)*: If $t^{*} \leq 0$ then $\tau^{**} = \tau^{*} = \frac{\mu}{\lambda}$ and $t^{**} = t^{*}$. If $t^{*} > 0$ then $\tau^{**} > \tau^{*}$ and $t^{**} = 0$.*
(iii) Under constraint (C3)*: If $t^{*} < 0$ ($t^{*} > 0$) then $\tau^{**} < \tau^{*}$ ($\tau^{**} > \tau^{*}$).*

Proof. See appendix. ‖

The sign of t^{*} depends on both the revenue requirement for financing the optimal level of the public good and the revenue earned from applying the (corrective) first-best tax rate on consumption.[13] Statement (i) shows that, as long as the second-best constraint (C2) is not binding — that is, as long as the optimal lump sum tax is negative — first-best and second-best optima coincide. If the revenue earned from applying the (corrective) first-best tax rate on consumption exceeds the revenue requirement for

[13] To determine the sign of t^{*}, we also need the Pigovian rule, which is derived below.

financing the optimal level of the public good, the excess revenue is rebated to all households as a lump sum transfer. Once, however, the constraint becomes binding, $\bar{b}^{**} < 1$, implying that a rise in the lump sum tax in order to increase government revenues would be welfare improving. In the second-best optimum, no lump sum *tax* can be introduced. Instead, the tax on consumption good x is raised beyond its first-best rate.

Statement (ii) considers a constraint in addition to (C2): no lump sum transfers are available. If this constraint binds, $\bar{b}^{**} > 1$, implying that a rise in the lump sum *transfer* would be welfare improving, as the "corrective revenue" exceeds the revenue requirement for financing the public good. As no lump sum *transfers* can be introduced, the tax on the consumption good is reduced to a rate below its first-best rate.

A corollary of Proposition 1 concerns the case of a positive consumption externality. If $\mu < 0$, $\tau^* < 0$, according to Lemma 1. Consequently, $t^* > 0$. In this case, both constraints (C2) and (C3) are binding, and $\tau^{**} > 0 > \tau^*$.

In the following, we consider heterogeneous households. If the social marginal utility of income is decreasing in income and the consumption level is increasing in income, covariance ϕ_{bx} will be negative. This is the main case considered below.

Proposition 2 (Heterogeneous Households) *In the presence of a negative consumption externality, suppose constraint* (C1) *holds.*
(i) Suppose $\phi(\zeta, s_{xx}) = 0$*. Then* $\phi(b, x) \lesseqgtr 0 \Leftrightarrow \tau^{**} \gtreqless \tau^*$.
(ii) Suppose $\phi(b, x) = 0$*. Then* $\phi(\zeta, s_{xx}) \lesseqgtr 0 \Leftrightarrow \tau^{**} \gtreqless \tau^*$.

Proof. Under constraint (C1) a poll tax is available, thus, $\bar{b}^{**} = 1$. Ramsey rule (19) becomes:

$$\frac{-(\tau - \frac{\mu}{\lambda})\, \bar{s}_{xx}}{\bar{x}} = -\frac{\phi(b, x)}{\bar{x}} - \frac{\mu}{\lambda}\, \frac{\phi(\zeta, s_{xx})}{\bar{x}}\,. \tag{19'}$$

In the presence of a negative consumption externality, $\frac{\mu}{\lambda} > 0$. ‖

The proposition shows how the second-best commodity tax rate responds to the heterogeneity of households. It is well known that the second-best commodity tax rate responds to heterogeneity in net social marginal utilities of income. If, e.g., consumption is concentrated among those households with a low social valuation, as captured by $b(i)$, covariance $\phi(b, x) < 0$. The welfare cost of imposing the commodity tax is lowered by the fact that those households that pay a large share of the commodity tax revenue are associated with a low social valuation. Consequently, $\tau^{**} > \tau^*$.

Proposition 2 also shows that *only* in case of a non-atmospheric consumption externality, heterogeneity has an impact on the deviation of the second-best commodity tax rate from the first-best one. If the consumption externality is of the atmospheric type, $\phi(\zeta, s_{xx}) = 0$, and τ^{**} does not deviate from τ^* (given $\phi(b, x) = 0$).[14]

If the consumption externality is of the non-atmospheric type, that is, $\phi(\zeta, s_{xx}) \neq 0$, the second-best commodity tax rate is further affected by heterogeneity with respect to $\zeta(i)$. The weight $\zeta(i)$ can be interpreted as social weight attached to "externality generator" i. If, e.g., the highest reductions in compensated demand due to the imposition of the commodity tax are concentrated among those households with the highest weights $\zeta(i)$, covariance $\phi(\zeta, s_{xx}) < 0$. In this case, given $\phi(b, x) = 0$, Proposition 2 shows that $\tau^{**} > \tau^*$. Compared to an economy with homogeneous households, heterogeneity adds an additional benefit to commodity taxation. The households whose consumption levels decline the most due to the imposition of the commodity tax, also contribute the most to the buildup of the consumption reference level. This additional benefit lowers the distortionary cost of commodity taxation, implying $\tau^{**} > \tau^*$.

It does not seem to be unreasonable to consider the following case: $\phi(b, x) < 0$, $\phi(\zeta, s_{xx}) > 0$. We interpret $\phi(b, x) < 0$ as concern for equity correction. That is, households with a low consumption level are those with a high social valuation (in terms of the net social marginal utility of income). Similarly, we interpret $\phi(\zeta, s_{xx}) > 0$ as concern for externality correction. With $\phi(\zeta, s_{xx}) > 0$, households whose consumption contributes most to the buildup of the consumption reference level are those who respond the least to the imposition of the commodity tax. Consider, e.g., a status good. Status goods probably contribute more to the buildup of consumption reference levels than non-status goods. In such an example, $\phi(\zeta, s_{xx}) > 0$ suggests that the imposition of the commodity tax lowers demand of the status good by less than demand of a non-status good.

Corollary 1 *In the presence of a negative consumption externality, suppose constraint* (C1) *holds. Consider $\phi(b, x) < 0$, and $\phi(\zeta, s_{xx}) > 0$. Then, the concern for equity tends to raise the second-best commodity tax above the first-best rate. Regarding externality correction, heterogeneity tends to lower the second-best commodity tax below the first-best rate.*

The corollary follows directly from Proposition 2. The two purposes of the commodity tax under (C1) are: equity correction and externality correction. The corollary shows that these objectives may be in conflict to each other, once $\phi(b, x) < 0$, $\phi(\zeta, s_{xx}) > 0$. In other words, there is a tradeoff between equity correction and externality correction. A marginal increase in the commodity tax rate (beyond its first-best level) advances equity via redistribution. Given the equity correction purpose, (19') requires $\tau^{**} > \tau^*$.

[14] Clearly, the consumption externality does have an impact on τ^{**}, but $\tau^{**} = \tau^* = \frac{\mu}{\lambda}$.

However, once $\phi(\zeta, s_{xx}) > 0$, a marginal increase in the commodity tax rate imposes an additional cost due to the fact that those households who account the most for $\bar{x}^r$ reduce their demand by the least amount. Given the externality correction purpose, (19') requires $\tau^{**} < \tau^*$. A lowering of τ does not much affect behavior of those households that contribute a lot to the consumption externality. A lowering of τ, however, gives rise to less distortionary behavior among those households who respond the most to τ. The following table summarizes important results.

Table I. Second-best versus first-best commodity tax rate

	$\phi(b,x) = 0$ $\phi(\zeta, s_{xx}) = 0$	$\phi(b,x) < 0$ $\phi(\zeta, s_{xx}) = 0$	$\phi(b,x) = 0$ $\phi(\zeta, s_{xx}) > 0$	$\phi(b,x) < 0$ $\phi(\zeta, s_{xx}) > 0$
C1	$\tau^{**} = \tau^*$	$\tau^{**} > \tau^*$	$\tau^{**} < \tau^*$	$\gtrless$
C2 ($t^* > 0$)	$\tau^{**} > \tau^*$	$\tau^{**} > \tau^*$	$\gtrless$	$\gtrless$
C3 ($t^* < 0$)	$\tau^{**} < \tau^*$	$\gtrless$	$\tau^{**} < \tau^*$	$\gtrless$

Notes. A negative consumption externality ($\mu > 0$) is considered. The case C3 ($t^* > 0$) is identical to the case C2 ($t^* > 0$). Entry "$\gtrless$" means the relationship between first-best and second-best tax rate is ambiguous.

We conclude this section with three brief remarks. First, if $\phi(\zeta, s_{xx}) < 0$, equity and externality correction reinforce each other. This seems to be an unlikely case, however, as it requires status-goods to be more price elastic than non-status goods. Second, if one of the constraints (C2) or (C3) bind in addition to (C1), the second-best effects discussed in relation to Proposition 1 apply in addition to those discussed in relation to Proposition 2. Third, in case of a positive consumption externality, the role of $\phi(\zeta, s_{xx})$ is reversed. Moreover, with a positive consumption externality, $\tau^* < 0$, which implies $t^* > 0$. As a consequence, both second-best constraints (C2) and (C3) are binding.

4 Pigovian rule- and Pigovian level property

For a given level of utility, $\bar{v}$, the willingness to pay for a marginal unit of g is given by:

$$\omega(i, q, g, y, \bar{x}^r) \equiv -\frac{dy}{dg}|_{\bar{v}} = \frac{v_g(i, q, g, y, \bar{x}^r)}{v_y(i, q, g, y, \bar{x}^r)},$$

where the latter term follows from differentiating the equation $v(.) = \bar{v}$ with respect to g. Considering the willingness to pay, Roy's identity — for given $\bar{x}^r$ — yields:

$$\omega(i, q, g, y, \bar{x}^r) = \frac{\partial\, v(.)/\partial\, g}{\partial\, v(.)/\partial\, y} = \frac{\partial\, f(.)/\partial\, g}{\partial\, f(.)/\partial\, y}\,, \tag{20}$$

and the compensated willingness to pay is given by:

$$\omega^c(i, q, g, v, \bar{x}^r) = -\frac{\partial\, e(.)}{\partial\, g}\,. \tag{21}$$

We finally state the Slutsky equation (again for given $\bar{x}^r$):

$$s_{xg}(i) \equiv \frac{\partial\, x^c(.)}{\partial\, g} = \frac{\partial\, x(.)}{\partial\, g} - \frac{\partial\, x(.)}{\partial\, y}\,\omega(.)\,. \tag{22}$$

The sign of the Slutsky term is indeterminate. As we assume x to be weakly normal, if the private and public goods are Marshallian substitutes then they are Hicksian substitutes, $s_{xg}(i) < 0$. Moreover, (22) shows that Hicksian complementarity implies Marshallian complementarity. Young's theorem implies: $s_{xg}(i) = \partial^2 e(.)/[\partial\, q\, \partial\, g] = \partial^2 e(.)/[\partial\, g\, \partial\, q] = -\partial\, \omega^c(.)/\partial\, q$. That is, Hicksian substitutability (complementarity) between x and g implies that the compensated willingness to pay for the public good rises (declines) in q.

In the following, we employ regularity assumption:

Assumption 1

$$\operatorname{sign}\left[\frac{\partial\ \bar{\omega}^c(q, v, g, \bar{x}^r)}{\partial\ z}|_{\bar{x}^r\ \mathit{fixed}}\right] = \operatorname{sign}\left[\frac{d\,\bar{\omega}^c(q, v, g, \bar{x}^r)}{d\, z}\right]\ , \quad z \in \{q, v, g\}\,.$$

The left hand side of Assumption 1 captures the partial effect of a change in $z \in \{q, v, g\}$, for a given consumption reference level. The right hand side of Assumption 1 captures the total effect of a change in z, taking into account the implied change in the consumption reference level. The assumption prevents the indirect externality effect, stemming from $\bar{x}^r_z$, from dominating the direct (partial) effect of a change in $z \in \{q, v, g\}$.

4.1 Generalized Pigovian rule

The first order condition for the public good provision (12), combined with Roy's identity (20) and the Slutsky equation for the public good (22), yields the generalized many person Pigovian rule (the derivation is shown in the appendix):

$$-\frac{(\tau-\frac{\mu}{\lambda})}{\bar{\omega}}\frac{\partial\,\bar{\omega}^c}{\partial\,q}=\frac{(\tau-\frac{\mu}{\lambda})\,\bar{s}_{xg}}{\bar{\omega}}=\left[\frac{c}{\bar{\omega}}-\bar{b}\right]-\frac{\phi(b,\omega)}{\bar{\omega}}+\frac{\mu}{\lambda}\,\frac{\phi(\zeta,s_{xg})}{\bar{\omega}}\,, \tag{23}$$

where covariances are denoted by $\phi(.)$, as above. The Slutsky term $\bar{s}_{xg}$ is positive (negative) if the private and public goods are Hicksian complements (substitutes). The left hand side of the Pigovian rule represents the negative optimal (approximate) proportionate change in the compensated willingness to pay due to the implementation of the commodity tax. The derivative $\partial\,\bar{\omega}^c/(\partial\,q)$ is positively related with $\phi(b,\omega)$, and — in the presence of a negative *non-atmospheric* consumption externality — negatively related with $\phi(\zeta,s_{xg})$.

In the following, we refer to the derivative $\partial\,\bar{\omega}^c/(\partial\,q)$ as *marginal willingness to pay*. Proposition 3 considers the effects of heterogeneity on the marginal willingness to pay. Specifically, the proposition focuses on the second-best constraint (C2) along with a *negative* consumption externality.

Proposition 3 *Consider a negative consumption externality and suppose that the second-best constraint* (C2) *binds:* $\tau^{**}>\frac{\mu}{\lambda}>0$.

(i) If $\phi(b,\omega)>0$ *(if* $\phi(b,\omega)<0$*), heterogeneity raises (lowers) the marginal willingness to pay.*

(ii) If $\phi(\zeta,s_{xg})>0$ *(if* $\phi(\zeta,s_{xg})<0$*), heterogeneity lowers (raises) the marginal willingness to pay.*

Proposition 3 follows directly from the generalized Pigovian rule (23). If $\phi(b,\omega)>0$, those households with the highest social evaluation have the highest willingness to pay. In this case, the marginal social benefit of an additional unit of public good supply is increased by heterogeneity. First order condition (A.3) then requires the marginal willingness to pay to be more positive or less negative, which tends to raise the optimal level of public good provision. Consider, for example, the case that the private and the public goods are Hicksian substitutes ($\bar{s}_{xg}<0$). Then the higher supply of the public good is associated with a lower demand of the private good, that is, with a higher distortionary consumption tax rate.

If the consumption externality is non-atmospheric, $\phi(\zeta,s_{xg})\neq 0$. We discuss two cases: $\phi(\zeta,s_{xg})>0$ (Case 1), and $\phi(\zeta,s_{xg})<0$ (Case 2). In Case 1, suppose $s_{xg}(i)>0$. That is, a higher level of public good supply increases demand of the private good. Positivity of the covariance requires that this increase is concentrated among those households whose consumption contributes the most to the buildup of the consumption reference level. As we consider a negative consumption externality, this heterogeneity reduces the optimal change in the compensated willingness to pay due to the implementation of the commodity tax. This tends to lower public good supply.

In Case 2, suppose $s_{xg}(i) < 0$. Insofar as a higher level of public good supply decreases demand of the private good, negativity of the covariance requires that this decrease is concentrated among those households whose consumption contributes the most to the buildup of the consumption reference level. As we consider a negative consumption externality, heterogeneity raises the optimal change in the compensated willingness to pay due to the implementation of the commodity tax, which tends to increase public good supply.

We summarize those cases in Corollary 2 and refer to $\phi(b, \omega)$ as heterogeneity with respect to the willingness to pay, and to $\phi(\zeta, s_{xg})$ as heterogeneity with respect to the *marginal* willingness to pay.

Corollary 2 *Consider a negative non-atmospheric consumption externality and suppose that the second-best constraint* (C2) *binds:* $\tau^{**} > \frac{\mu}{\lambda} > 0$.

(i) Heterogeneity has an impact on the Pigovian rule via the consumption externality only in case of a non-atmospheric externality.

(ii) If $\operatorname{sign} \phi(b, \omega) = -\operatorname{sign} \phi(\zeta, s_{xg})$*, heterogeneity with respect to the willingness to pay and heterogeneity with respect to the* marginal *willingness to pay reinforce each other.*

(iii) If $\operatorname{sign} \phi(b, \omega) = \operatorname{sign} \phi(\zeta, s_{xg})$*, there is a tradeoff between heterogeneity with respect to the willingness to pay and heterogeneity with respect to the* marginal *willingness to pay.*

One may interpret $\phi(b, \omega) > 0$ as concern for equity. Corollary (2) points out that a negative non-atmospheric consumption externality introduces an equity-efficiency tradeoff, once $\phi(\zeta, s_{xg}) > 0$. As discussed above, this is the case when the public and private goods are complements, and those households whose consumption contributes the most to the buildup of the consumption reference level, respond the most to public good supply.

4.2 Pigovian rule property

In this subsection, we discuss the Pigovian rule property in the presence of consumption externalities. That is, we investigate the impact of a consumption externality on the first-best- and second-best average willingness to pay for the public good. Based on the results of this subsection, we consider the impact of consumption externalities on first-best- and second-best levels of public good provision in the proceeding subsection.

Definition 2
The Pigovian rule property is said to hold if $\int_{i\in I} \omega^{**}(i)\mathrm{d}F(i) > \int_{i\in I} \omega^{*}(i)\mathrm{d}F(i)$.

The definition of the Pigovian rule property implies that the social cost of public good provision in the second-best exceeds the social cost in the first-best. It is important to recognize that it is mistaken to assume that a higher social cost of public good provision (in the second-best as compared to the first-best) generally lowers the optimal *level* of public good provision. That is, the Pigovian rule property does not generally imply $g^{**} < g^{*}$, as shown below.

Combining (23) with Ramsey rule (19) yields an expression relating the willingness to pay to the *social* cost of providing the public good:

$$\int_{i\in I} \omega(i)\,\mathrm{d}F(i) = c - \left(\tau - \frac{\mu}{\lambda}\right)\bar{\omega}\left[\frac{\bar{s}_{xx}}{\bar{x}} + \frac{\bar{s}_{xg}}{\bar{\omega}}\right] + H\,, \tag{24}$$

$$\text{with } H \equiv \bar{\omega}\left[\left(\frac{\phi(b,x)}{\bar{x}} - \frac{\phi(b,\omega)}{\bar{\omega}}\right) + \frac{\mu}{\lambda}\left(\frac{\phi(\zeta, s_{xx})}{\bar{x}} + \frac{\phi(\zeta, s_{xg})}{\bar{\omega}}\right)\right],$$

where c represents the *private* cost of public good provision. The term $-\left(\tau - \frac{\mu}{\lambda}\right)\bar{\omega}\left[\frac{\bar{s}_{xx}}{\bar{x}}\right]$ represents the Pigou effect. It shows the indirect cost of financing the public good in the absence of nondistortionary means of taxation. Clearly, a negative consumption externality lowers the Pigou effect. The term $-\left(\tau - \frac{\mu}{\lambda}\right)\bar{\omega}\left[\frac{\bar{s}_{xg}}{\bar{\omega}}\right]$ captures the compensated Diamond-Mirrlees provision effect. If the private and public goods are Hicksian complements, an increase in the provision of the public good raises demand for the private good, and thereby commodity tax revenue. In case of Hicksian complementarity, the provision effect lowers the social cost of public good provision.[15] The term H accounts for heterogeneity of households. $H < 0$ lowers the social cost of public good provision. A negative consumption externality lowers the social cost of public good provision — via heterogeneity — if $\left(\frac{\phi(\zeta,s_{xx})}{\bar{x}} + \frac{\phi(\zeta,s_{xg})}{\bar{\omega}}\right) < 0$.

Lemma 2 *In the first-best optimum,* $\bar{\omega}^{*} = \int_{i\in I} \omega^{*}(i)\,\mathrm{d}F(i) = c$.

Proof. See appendix. ‖

The lemma shows the Samuelson rule, according to which the "sum" of the marginal

[15] Hicksian complementarity and normality of private consumption imply Marshallian complementarity between x and g. Bradford and Hildebrandt (1977) argue that such complementarities exist between, e.g., air safety and air travel, traffic network and private cars, lighthouses and private boating, public tennis courts and tennis rackets, or national defense and ownership of private property.

rates of substitution of g for l equals the marginal rate of transformation between those goods.

We define $\hat{s}_{xg} \equiv -\bar{s}_{xx}\bar{\omega}/\bar{x} > 0$. In (24), if $\bar{s}_{xg} < \hat{s}_{xg}$, the term in square brackets is negative (dominated by the Pigou effect). If if $\bar{s}_{xg} > \hat{s}_{xg}$, there is strong Hicksian complementarity between the public and private good, and the term in square brackets is positive (dominated by the provision effect). If $\bar{s}_{xg} = \hat{s}_{xg}$ the provision effect counterbalances the Pigou effect, and the term in square brackets equals zero. In this case, the sign of H alone determines whether or not the Pigovian rule property holds.

In the absence of heterogeneity, the sign of $\bar{s}_{xg}$ plays a major role for whether or not the Pigovian rule property holds. Consider, e.g., $\tau^{**} > \frac{\mu}{\lambda} > 0$. If $\bar{s}_{xg} < 0$, the Pigovian rule property always holds. By the contrapositive, if $\bar{\omega}^{**} < \bar{\omega}^{*}$, then $\bar{s}_{xg} > 0$. That is, a reversal of the Pigovian rule property requires complementarity between the private and the public good. More generally, the proceeding two propositions offer conditions for which the Pigovian rule property holds (is reversed).

Proposition 4 *Suppose $H = 0$. The Pigovian rule property holds in the following two cases:*
(i) $\bar{s}_{xg} < \hat{s}_{xg}$ and $\tau^{**} - \frac{\mu}{\lambda} > 0$,
(ii) $\bar{s}_{xg} > \hat{s}_{xg}$ and $\tau^{**} - \frac{\mu}{\lambda} < 0$.

The proposition follows directly from (24). In statement (i), the public and private goods are either Hicksian substitutes or Hicksian complements with $\bar{s}_{xg} < \hat{s}_{xg}$. If they are Hicksian substitutes, the provision effect adds to the social cost of public good provision. A marginal increase in public good provision lowers the compensated demand for the private good — thereby it lowers the commodity tax revenue. If the private and public goods are Hicksian complements (with $\bar{s}_{xg} < \hat{s}_{xg}$), the provision effect lowers the social cost of public good provision, but it is dominated by the Pigou effect that raises the social cost. Statement (i) is restricted to $\tau^{**} - \frac{\mu}{\lambda} > 0$. According to Proposition 1, $\tau^{**} - \frac{\mu}{\lambda} > 0$ holds under both second-best restrictions (C2) and (C3) as long as $t^{*} > 0$. Considering a negative consumption externality, the first-best, corrective commodity tax revenue is lower than the revenue required to finance the optimal level of the public good. Therefore $\tau^{**} > \tau^{*} = \frac{\mu}{\lambda}$ introduces a distortion whose social cost is not fully compensated for by the provision effect.

In statement (ii), $\tau^{**} < \tau^{*}$, which can only occur under second-best restriction (C3) when $t^{*} < 0$. In this case, a marginal increase of the commodity tax lowers the distortion created by the consumption externality. That is, the Pigou effect *lowers* the social cost of public good provision, and it dominates the provision effect. As $\tau^{**} < \tau^{*}$, the first-best, corrective commodity tax revenue exceeds the revenue required to finance the optimal level of the public good. But the government has no lump sum taxes or transfers available to rebate the "excess revenue."

In Proposition (4)(i), the social cost of public good provision is lowered by a negative consumption externality, as only a part of commodity taxation is distortionary. The opposite is true in case of a positive consumption externality. According to Proposition (4)(ii), if the provision effect dominates, the Pigovian rule property can only hold if $\tau^{**} - \frac{\mu}{\lambda} < 0$. In this case, again, a negative consumption externality lowers the social cost of public good provision.

Heterogeneity, i.e., $H \neq 0$, adds to the effects discussed so far. One case merits a note. Suppose, $\tau^{**} = \frac{\mu}{\lambda}$. Then, the Pigovian rule property is satisfied (reversed) if $H > 0$ (if $H < 0$).

Proposition 5 *Suppose $H = 0$. The Pigovian rule property is reversed in the following two cases:*
(i) $\bar{s}_{xg} > \hat{s}_{xg}$ and $\tau^{**} - \frac{\mu}{\lambda} > 0$,
(ii) $\bar{s}_{xg} < \hat{s}_{xg}$ and $\tau^{**} - \frac{\mu}{\lambda} < 0$.

The reasoning parallels the one given for Proposition 4. In statement (i), e.g., the Pigou effect raises the cost of public good provision. It is, however, dominated by the provision effect, so that the social cost of public good provision is lower than the private cost. Table II sums up the results concerning the Pigovian rule property.

Table II. Pigovian rule property under $H = 0$

	$\tau^{**} - \frac{\mu}{\lambda} > 0$ (C2, C3: $t^* > 0$)	$\tau^{**} - \frac{\mu}{\lambda} = 0$ (C2: $t^* \leq 0$; C3: $t^* = 0$)	$\tau^{**} - \frac{\mu}{\lambda} < 0$ (C3: $t^* < 0$)
$\bar{s}_{xg} < \hat{s}_{xg}$	$\bar{\omega}^{**} > \bar{\omega}^*$	$\bar{\omega}^{**} = \bar{\omega}^*$	$\bar{\omega}^{**} < \bar{\omega}^*$
$\bar{s}_{xg} = \hat{s}_{xg}$	$\bar{\omega}^{**} = \bar{\omega}^*$	$\bar{\omega}^{**} = \bar{\omega}^*$	$\bar{\omega}^{**} = \bar{\omega}^*$
$\bar{s}_{xg} > \hat{s}_{xg}$	$\bar{\omega}^{**} < \bar{\omega}^*$	$\bar{\omega}^{**} = \bar{\omega}^*$	$\bar{\omega}^{**} > \bar{\omega}^*$

NOTES. $\bar{\omega}^* \equiv \bar{\omega}(q^*, y, g^*, \bar{x}^{r\,*})$, $\bar{\omega}^{**} \equiv \bar{\omega}(q^{**}, y, g^{**}, \bar{x}^{r\,**})$.

Two cases can lead to a reversal of the Pigovian rule property. First, a high negative consumption externality — $\tau^{**} < \frac{\mu}{\lambda}$ — together with the unavailability of lump-sum transfers (C3). Second, a high enough demand for the public good together with the unavailability of lump-sum taxes such that $\tau^{**} > \frac{\mu}{\lambda} = \tau^*$.

4.3 Pigovian level property

We are ready to discuss the Pigovian *level* property in the presence of consumption externalities. That is, we investigate the impact of a consumption externality on the first-best- and second-best levels of public good provision, based on the results of the previous subsection.

Definition 3
The Pigovian level property is said to hold *(to be* reversed*) if $g^{**} < g^*$ (if $g^{**} > g^*$).*

The Pigovian rule property does not generally imply the Pigovian level property. As demonstrated by Chang (2000), the linkage between the social cost of public good provision and its optimal level critically depends on the Hicksian complementarity (substitutability) between public and private goods.

Proposition 6 (Pigovian Level Property) *Suppose $H = 0$.*
Let the economy fulfill Assumption 1, $\bar{\omega}_g^c < 0$, $\bar{\omega}_v^c \geq 0$, and $\bar{s}_{xg} \in [0, \hat{s}_{xg}]$.
If $\tau^{**} - \frac{\mu}{\lambda} > 0$, then $g^{**} < g^*$.

Proof. See appendix. ||

Proposition 6 provides sufficient conditions for the Pigovian level property to hold. Conditions $\bar{\omega}_v^c \geq 0$ and $\bar{\omega}_g^c < 0$ require the public good to be (weakly) normal with declining marginal benefit. As $\bar{s}_{xg} \in [0, \hat{s}_{xg}]$, the private and public goods are (weak) Hicksian complements, and the compensated willingness to pay for the public good does not increase in τ. Restriction $\tau^{**} - \frac{\mu}{\lambda} > 0$ is satisfied under second-best conditions (C2) and (C3), when $t^* > 0$. That is, the first-best commodity tax revenue is lower than the revenue required to finance the optimal level of the public good. Therefore $\tau^{**} > \tau^*$.

If $\bar{s}_{xg} \in [0, \hat{s}_{xg})$, both the Pigovian rule and level properties hold. In fact, in this case, the Pigovian rule property implies the Pigovian level property. This is a generalization of Chang's (2000, p.88) result. Under the conditions of Proposition 6, $g^{**} \geq g^*$ implies $\bar{\omega}^{c**} \leq \bar{\omega}^{c*}$, which Chang (2000) termed the *linkage property.* However, as in fact $\bar{\omega}^{c**} > \bar{\omega}^{c*}$, it follows: $g^{**} < g^*$.

In the presence of a negative consumption externality, under second-best restriction (C3), the Pigovian level property fails to hold if the first-best commodity tax revenue exceeds the revenue required to finance the optimal level of the public good. In this case, the Pigovian level property is reversed, and the second-best level of public good provision exceeds the first-best level.

Proposition 7 (Reversal of Pigovian Level Property) *Suppose $H = 0$. Let the economy fulfill Assumption 1, $\bar{\omega}_g^c < 0$, $\bar{\omega}_v^c = 0$, and and $\bar{s}_{xg} \in [0, \hat{s}_{xg}]$. If $\tau^{**} - \frac{\mu}{\lambda} < 0$, then $g^{**} > g^*$.*

Proof. See appendix. ‖

The proposition provides a counterexample to the Pigovian level property. The prior literature already identified a positive provision effect (Diamond and Mirrlees 1971), a dynamic efficiency effect (Batina 1990a), and heterogeneity (Gaube 2000) as possible sources for reversal of the Pigovian level property. The proposition adds a further source to the list: a negative consumption externality.

In the presence of a negative consumption externality, $\tau^* = \frac{\mu}{\lambda} > 0$. If the corrective revenue exceeds the funds required to finance the first-best level of the public good, $t^* < 0$. Under the second-best restriction (C3), however, no lump sum taxes or transfers are available to the public sector. As a consequence, $\tau^{**} < \tau^*$, as shown by Proposition 1. In this situation, a marginal increase of the commodity tax lowers the distortion introduced by the negative consumption externality, and it is optimal to provide a higher than the first-best level of the public good. Table III summarizes the results.

Table III. Pigovian level property under $H = 0$

		$\tau^{**} - \frac{\mu}{\lambda} > 0$ (C2, C3: $t^* > 0$)	$\tau^{**} - \frac{\mu}{\lambda} = 0$ (C2: $t^* \leq 0$; C3: $t^* = 0$)	$\tau^{**} - \frac{\mu}{\lambda} < 0$ (C3: $t^* < 0$)
$\bar{s}_{xg} < 0$	$\bar{\omega}_v^c \geq 0$	$\gtreqless$	$g^{**} = g^*$	$\gtreqless$
$\bar{s}_{xg} \in [0, \hat{s}_{xg}]$	$\bar{\omega}_v^c = 0$	$g^{**} < g^*$	$g^{**} = g^*$	$g^{**} > g^*$
	$\bar{\omega}_v^c > 0$	$g^{**} < g^*$	$g^{**} = g^*$	$\gtreqless$
$\bar{s}_{xg} > \hat{s}_{xg}$	$\bar{\omega}_v^c \geq 0$	$\gtreqless$	$g^{**} = g^*$	$\gtreqless$

Notes. $\bar{\omega}_v^c = \bar{\omega}_v^c(q, v, g, \bar{x}^r)$. Entry "$\gtreqless$" means the relationship between first-best and second-best level of public good provision is ambiguous.

Pigovian level reversal may occur in the presence of a negative consumption externality only. Only then the corrective revenue may exceed the revenue required to finance the optimal level of public good provision.

As discussed above, heterogeneity adds to the effects discussed so far. Suppose, $\tau^{**} = \frac{\mu}{\lambda}$. Then, the Pigovian level property is satisfied (reversed) if $H > 0$ (if $H < 0$).

Proposition 7 is illustrated by a Cobb-Douglas example below. In the context of an economy *without* consumption externalities, Wilson (1991) demonstrates that the

Pigovian level property holds if preferences can be represented by a Cobb-Douglas utility function and the public good is normal.[16] The example shown below demonstrates that — in the context of an economy *with a negative* consumption externality — the Pigovian level property may be reversed, even for preferences that can be represented by a Cobb-Douglas utility function.

4.4 Pigovian level reversal with homogeneous households

Consider an economy with identical households whose preferences are represented by:

$$u(x,l,g,\bar{x}^r) = \left[x\left(\frac{x}{\bar{x}^r}\right)^{\frac{\gamma}{1-\gamma}}\right]^{\alpha} l^{1-\alpha} + \beta \ln g\,, \quad 0<\alpha<1\,, \tag{25}$$

where $\beta > 0$ represents the strength of the desire for the public good, and $0 < \gamma < 1$ introduces a negative consumption externality. The strength of the consumption externality increases in γ. As $\gamma > 0$, a rise in the consumption reference level lowers utility. Thereby, (25) represents an example of a negative consumption externality.[17]

Let $\hat{\alpha} \equiv \alpha/[1-(1-\alpha)\,\gamma]$. The indirect utility function becomes:

$$v(q,y,g,\bar{x}^r) = \left[\frac{(1-\alpha)(1-\gamma)\,\hat{\alpha}}{\alpha}\right]^{1-\alpha} \left[\frac{\hat{\alpha}}{q}\right]^{\alpha} \left[\frac{\hat{\alpha}\,y}{q\,\bar{x}^r}\right]^{\frac{\alpha\,\gamma}{1-\gamma}} y + \beta \ln g\,.$$

Roy's identity yields: $x(q,y,g,\bar{x}^r) = \hat{\alpha}\,y/q$, and $\bar{x}^r = x$ (as individuals are identical), where strong separability implies independence of $x(.)$ from g.

Calculation of the Hicksian demands yields the following expenditure function: $e(q,v,g,\bar{x}^r) = \alpha^{-\hat{\alpha}}\,[(1-\alpha)(1-\gamma)]^{-(1-\hat{\alpha})}\,q^{\hat{\alpha}}\,(\bar{x}^r)^{\hat{\alpha}\,\gamma}\,[1-(1-\alpha)\gamma]\,[v-\beta\,\ln g]^{\hat{\alpha}(1-\gamma)/\alpha}$, and the compensated willingness to pay, $-\partial\, e(.)/\partial\, g$, becomes:

$$\begin{aligned}\bar{\omega}^c(q,v,g,\bar{x}^r) =& \alpha^{-\hat{\alpha}}\,(1-\alpha)^{-(1-\hat{\alpha})}\,(1-\gamma)^{\hat{\alpha}}\,q^{\hat{\alpha}}\,(\bar{x}^r)^{\hat{\alpha}\,\gamma}\,[v-\beta\,\ln g]^{-\hat{\alpha}\,\gamma}\,\beta\,g^{-1}\,,\\ =& \frac{\beta\,(1-\gamma)^{\alpha}\,q^{\alpha}}{\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\,g}\,,\end{aligned}$$

where the latter term takes into account that $\bar{x}^r = x(.) \equiv x^c(.)$, with utility appropriately defined. As required by Proposition 7, $\bar{\omega}^c_g < 0$, and $\bar{\omega}^c_v = 0$.[18]

[16] Indeed, Wilson (1991) demonstrates that the Pigovian level property holds in the case of CES utility, given that the ad valorem commodity tax rate does not exceed 100 per cent.

[17] In the context of a keeping up with the Joneses externality, γ represents the marginal degree of positionality (Johansson-Stenman et al., 2002).

[18] In the example, $\bar{\omega}^c_q > 0$. Thus $0 > \bar{s}_{xg} \notin [0,\hat{s}_{xg}]$. However, $\bar{s}_{xg} \in [0,\hat{s}_{xg}]$ is a sufficient, not a necessary condition to generate reversal of the Pigovian level property.

Let second-best restriction (C3) apply. From the Lagrangian:

$$\mathcal{L} = \frac{\hat{\alpha}}{\alpha} \frac{\alpha^\alpha \left[(1-\alpha)(1-\gamma)\right]^{1-\alpha}}{q^\alpha} y + \beta \ln g + \lambda \left[t + (q-1) \frac{\hat{\alpha}\,(y-t)}{q} - c\,g \right] ,$$

the first- and second-best solutions can easily be derived. In the Lagrangian, it is taken into account that $\bar{x}^r = x$, and the constant marginal production cost of the consumption good is set equal to $p = 1$.

In the first-best optimum: $\tau^* = \gamma/(1-\gamma) \Leftrightarrow q^* = 1/(1-\gamma)$, and the optimal public good provision is given by:

$$g^* = \frac{\beta}{\alpha^\alpha \,(1-\alpha)^{1-\alpha}\, c} \, .$$

In the second-best optimum:

$$g^{**} = \frac{1}{\left[q\,(1-\gamma)\right]^{1-\alpha}} \frac{\beta}{\alpha^\alpha \,(1-\alpha)^{1-\alpha}\, c} = \frac{1}{\left[q\,(1-\gamma)\right]^{1-\alpha}}\, g^* \, ,$$

with the implication that: $g^{**} \lesseqgtr g^* \Leftrightarrow q^{**} \gtreqless q^* = (1-\gamma)^{-1}$.

The reversal of the Pigovian level property occurs when $q^{**} < q^*$. Whether or not $q^{**} < q^*$ depends on the strength of the negative consumption externality, γ, as well as on the strength of the preference for the public good, β. Intuitively, the higher γ (thereby the corrective tax revenue) and the lower β (thereby the revenue requirement for financing the public good) the more likely is the reversal of the Pigovian level property. Formally:

$$\beta \gtreqless \hat{\beta} \Leftrightarrow g^{**} \lesseqgtr g^* \, , \quad \hat{\beta} \equiv \gamma\, \hat{\alpha}\, \alpha^\alpha \,(1-\alpha)^{1-\alpha}\, y \, .$$

In the absence of a consumption externality $\gamma = \hat{\beta} = 0$. If $\beta > 0$, $g^{**} < g^*$, as demonstrated by Wilson (1991). In the presence of a negative consumption externality, however, if $\beta < \hat{\beta}$, the corrective first-best revenue exceeds the revenue requirement for financing the public good. As a consequence, $\tau^{**} < \tau^*$, and $g^{**} > g^*$.

5 Conclusions

This paper addresses the effects of consumption externalities on optimal commodity taxation, the social cost of public good provision, and the optimal level of public good provision. Several of the paper's results deserve comments.

First, not taking heterogeneity effects into account for the moment, the paper shows that in the presence of a negative consumption externality, it is mistaken to assume that the second-best level of provision of the public good necessarily *differs* from the first-best level. Once available policy instruments include a poll transfer (but not a poll tax), $g^{**} = g^{*}$ whenever the revenue raised by applying the first-best commodity tax rate does not fall short of the revenue requirement for financing the optimal level of the public good. This case is the more likely the stronger is the negative consumption externality.

If the revenue raised by applying the first-best commodity tax rate is lower than the revenue requirement for financing the optimal level of the public good, a negative consumption externality still lowers the social cost of public good provision and thereby tends to raise the optimal level of provision. Intuitively, the commodity tax not only serves a revenue raising purpose but also an externality correcting purpose. For this reason, the externality lowers the Pigou effect, thus, the social cost of public good provision.

A negative consumption externality is a potential source for the reversal of the Pigovian level property. In the presence of a negative consumption externality, the second-best level of public good provision may exceed the first-best level, once lump sum taxes (transfers) are not available to the public sector. If the preference for the public good is low relative to the strength of the consumption externality, the first-best corrective revenue may exceed the revenue requirement for financing the optimal public good level. In this case, the commodity tax lowers the distortion introduced by the consumption externality, and the second-best level of public good provision exceeds the first-best level. This result even holds true for an economy populated with homogeneous households with Cobb-Douglas preferences.

Second, it is mistaken to assume that heterogeneity unambiguously raises the commodity tax for reasons of equity. In the presence of a non-atmospheric negative consumption externality, the paper identifies an equity-efficiency tradeoff due to heterogeneity. The covariance between the social valuations of households and their consumption levels, if negative, tends to raise the optimal commodity tax rate for equity reasons. However, if $\phi(\zeta, s_{xx}) > 0$, the fact that those households who are most important for building up the consumption reference level respond the least to commodity taxation tends to reduce the optimal commodity tax rate. In this case, there is a tradeoff between equity correction and externality correction.

Third, again, suppose available policy instruments include a poll transfer but not a poll tax. Then, heterogeneity, as captured by $\phi(\zeta, s_{xx})$, affects the relation between first-best and second-best commodity tax only in case of a non-atmospheric consumption externality. Once the consumption externality is of the atmospheric type, $\phi(\zeta, s_{xx}) = 0$, and there is no *direct* impact of the consumption externality on the relation between first-best and second-best commodity tax rate.

Fourth, the specification of the consumption reference level is quite general in this paper. Individuals may contribute — via their own consumption levels — in a variety of ways. It is possible that just a subset of individuals makes up the consumption reference level. Every individual may contribute differently to the buildup of the consumption reference level. Of course, the formulation includes the case that the consumption reference level equals the average consumption level of society. In addition, every individual may attach a different weight to the reference level in its utility function. The limitation to be emphasized is that the makeup of the consumption reference level is identical across households. While the reference level may be weighed differently across households, everybody agrees on the contributions of households, ζ_i, to the given reference level. This specification rules out situations, in which one group's consumption reference level differs from that of another group. One question for future research may be to capture an even more general specification for consumption externalities.

Pigovian level reversal occurs when the preference for the public good is weak relative to the strength of the negative consumption externality. Whether or not this condition is satisfied is an empirical question. Empirical estimates of the parameters governing both the preference for the public good and the strength of the consumption externality — β and γ in the example presented in section 4 — are scarce and unreliable, at best. Another question for future research then is to determine empirical estimates for those parameters.

I hope this study contributes, in a significant way, to the discussion on the theoretical effects of consumption externalities on the social cost and optimal levels of public good provision and fosters future discussions in this research field.

Appendix

A. Generalized many person Ramsey rule. In first order condition (11), consider Roy's identity (5), and the definition of the social marginal utility of income (17):

$$-\int_{i\in I} b(i)\, x(i)\, \mathrm{d}F(i) + \int_{i\in I} [\tau - \frac{\mu}{\lambda}\, \zeta(i)][x_y(i)x(i) + x_q(i)]\, \mathrm{d}F(i) + \bar{x} = 0\,. \tag{A.1}$$

Considering the Slutsky equation (7) along with the covariances yields:

$$-\,\bar{b}\,\bar{x} - \phi(b, x) + \bar{x} + \tau\,\bar{s}_{xx} - \frac{\mu}{\lambda}(\phi(\zeta, s_{xx}) + \bar{s}_{xx}) = 0\,. \tag{A.2}$$

Rearranging terms yields Ramsey rule (19). $\|$

B. Proof of Lemma 1.
(i) Homogeneous households. The right hand side of Ramsey equation (19) equals $1-\bar{b}$, which equals zero by first order condition (13). |
(ii) Heterogeneous households. Employing (17), the right hand side of (19) equals zero by the definition of $b(i)$. $\|$

C. Proof of Proposition 1.
(i) If $t^* \leq 0$, constraint (C2) is not binding. If $t^* > 0$, constraint (C2) binds, implying $\bar{b}^{**} < 1$. Thus, the right hand side of (19) is positive. As a consequence, $\tau^{**} > \frac{\mu}{\lambda} = \tau^*$.
(ii) If constraint (C3) binds, then either $t^* > 0$ or $t^* < 0$. In the former case, argument (i) applies. In the latter case, $\bar{b}^{**} > 1$, which, in light of Ramsey rule (19), implies: $\tau^{**} < \frac{\mu}{\lambda} = \tau^*$. $\|$

D. Generalized many person Pigovian rule. In first order condition (12), consider Roy's identity for the public good (20), and the Slutsky term (22):

$$\int_{i\in I} \frac{W'(.)}{\lambda} f_y(i)\omega(i)\mathrm{d}F(i) + \tau \int_{i\in I} s_{xg}(i)\mathrm{d}F(i) + \tau \int_{i\in I} x_y(i)\omega(i)\mathrm{d}F(i) - c$$
$$- \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)\, s_{xg}(i)\mathrm{d}F(i) - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)\, x_y(i)\,\omega(i)\mathrm{d}F(i) = 0\,.$$

Next, consider the definition of the social marginal utility of income (17):

$$\int_{i\in I} b(i)\omega(i)\mathrm{d}F(i) + \tau\bar{s}_{xg} - \frac{\mu}{\lambda}\int_{i\in I} \zeta(i)\, s_{xg}(i)\mathrm{d}F(i) = c\,.$$

Consider covariances $\phi(b,\omega)$, $\phi(\zeta, s_{xg})$:

$$\bar{b}\,\bar{\omega} + \phi(b,\omega) + \left(\tau - \frac{\mu}{\lambda}\right)\bar{s}_{xg} - \frac{\mu}{\lambda}\,\phi(\zeta, s_{xg}) = c\,. \tag{A.3}$$

Rearranging terms yields Pigovian rule (23). $\|$

E. Proof of Lemma 2. Employing definition (17) in (A.2) yields: $\int_{i\in I}\omega(i)\,\mathrm{d}F(i) = c - \left(\tau - \frac{\mu}{\lambda}\right)\,\bar{\omega}\,\left[\frac{\bar{s}_{xx}}{\bar{x}} + \frac{\bar{s}_{xg}}{\bar{\omega}}\right]$. By Lemma 1, $\tau^* = \frac{\mu}{\lambda}$. Thus: $\int_{i\in I}\omega^*(i)\mathrm{d}F(i) = c$. $\|$

F. Proof of Proposition 6. Assumption $\bar{s}_{xg} \in [0, \hat{s}_{xg}]$ implies: $\bar{\omega}^{**} \geq \bar{\omega}^*$ (see Table II). More specifically, consider $\bar{\omega}^{c\,**} \equiv \bar{\omega}^c(q^{**}, g^{**}, v^{**}, \bar{x}^{r\,**}) = \bar{\omega}(q^{**}, g^{**}, y, \bar{x}^{r\,**}) \equiv \bar{\omega}^{**}$ as well as $\bar{\omega}^{c\,*} \equiv \bar{\omega}^c(q^*, g^*, v^*, \bar{x}^{r\,*}) = \bar{\omega}(q^*, g^*, y, \bar{x}^{r\,*}) \equiv \bar{\omega}^*$, where $v^* \equiv v(q^*, g^*, y, \bar{x}^{r\,*})$, and $v^{**} \equiv v(q^{**}, g^{**}, y, \bar{x}^{r\,**})$. That is, $\bar{\omega}^{**} \geq \bar{\omega}^*$ implies $\bar{\omega}^{c\,**} \geq \bar{\omega}^{c\,*}$. We distinguish two cases: Case A with $\bar{s}_{xg} \in [0, \hat{s}_{xg})$; Case B with $\bar{s}_{xg} = \hat{s}_{xg}$.

Case A.
(i) As $0 \leq \bar{s}_{xg} < \hat{s}_{xg}$, $\bar{\omega}^c(q^{**}, v^{**}, g^{**}, \bar{x}^{r\,**}) > \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, by Proposition (4).
(ii) $\bar{\omega}^c_q(q, v, g, \bar{x}^r) \leq 0$ and $\bar{\omega}^c_v(q, v, g, \bar{x}^r) \geq 0$. The inequalities $\tau^{**} - \frac{\mu}{\lambda} > 0 \Rightarrow q^{**} > q^*$ and $v^{**} \leq v^*$ imply: $\bar{\omega}^c(q^{**}, v^{**}, g^*, \bar{x}^{r\,**}) \leq \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, for the *same* level of the public good g^*. While, in general, $\bar{x}^{r\,*} \neq \bar{x}^{r\,**}$, Assumption 1 ensures that indirect externality effects ($\bar{x}^r_q$, $\bar{x}^r_v$, $\bar{x}^r_g$) do not reverse this inequality.
(iii) From step (i), however, we know: $\bar{\omega}^{c\,**} > \bar{\omega}^{c\,*}$. Given $\bar{\omega}^c_g < 0$, this inequality can only be satisfied if: $g^{**} < g^*$. |

Case B.
(i) As $\bar{s}_{xg} = \hat{s}_{xg}$, $\bar{\omega}^c(q^{**}, v^{**}, g^{**}, \bar{x}^{r\,**}) = \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$.
(ii) $\bar{\omega}^c_q(q, v, g, \bar{x}^r) < 0$, and $\bar{\omega}^c_v(q, v, g) \geq 0$. Inequalities $q^{**} > q^*$ and $v^{**} \leq v^*$ imply: $\bar{\omega}^c(q^{**}, v^{**}, g^*, \bar{x}^{r\,**}) < \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, for the *same* level of the public good (g^*).
(iii) From step (i), however, we know: $\bar{\omega}^{c\,**} = \bar{\omega}^{c\,*}$. Given $\bar{\omega}^c_g < 0$, this equality can only be satisfied if: $g^{**} < g^*$. ‖

G. Proof of Proposition 7. We distinguish two cases, A and B.
Case A: $\bar{s}_{xg} \in [0, \hat{s}_{xg})$.
(i) As $0 \leq \bar{s}_{xg} < \hat{s}_{xg}$, $\bar{\omega}^c(q^{**}, v^{**}, g^{**}, \bar{x}^{r\,**}) < \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, see Table II.
(ii) $\bar{\omega}^c_q(q, v, g, \bar{x}) \leq 0$. As $q^{**} < q^*$, $\bar{\omega}^c(q^{**}, v^{**}, g^*, \bar{x}^{r\,**}) \geq \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, for the *same* level of the public good g^* (notice that $\bar{\omega}^c_v = 0$). Assumption 1 ensures that indirect externality effects ($\bar{x}^r_q$, $\bar{x}^r_v$, $\bar{x}^r_g$) do not reverse this inequality.
(iii) From step (i), however, we know: $\bar{\omega}^{c\,**} < \bar{\omega}^{c\,*}$. Given $\bar{\omega}^c_g < 0$, this inequality can only be satisfied if: $g^{**} > g^*$. |

Case B: $\bar{s}_{xg} = \hat{s}_{xg}$.
(i) As $\bar{s}_{xg} = \hat{s}_{xg}$, $\bar{\omega}^c(q^{**}, v^{**}, g^{**}, \bar{x}^{r\,**}) = \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$.
(ii) $\bar{\omega}^c_q(q, v, g, \bar{x}) < 0$. $q^{**} < q^* \Rightarrow \bar{\omega}^c(q^{**}, v^{**}, g^*, \bar{x}^{r\,**}) > \bar{\omega}^c(q^*, v^*, g^*, \bar{x}^{r\,*})$, for the *same* level of the public good (g^*).
(iii) From step (i), however, we know: $\bar{\omega}^{c\,**} = \bar{\omega}^{c\,*}$. Given $\bar{\omega}^c_g < 0$, this equality can only be satisfied if: $g^{**} > g^*$. ‖

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