Implied correlation from VaR

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Abstract

Most of the methods used by financial institutions to implement value-at-risk models are based on the multivariate Gaussian distribution with a constant correlation matrix. In this paper we use VaR calculation in a reverse way to imply the correlation between asset price changes. The distribution of implied correlation under normality is also studied in order to take into account any bias and sampling error. Empirical results for US and UK equity markets show that implied correlation is not constant but tends to be higher for long positions than for short positions. This result is statistically significant and can be interpreted as departure from normality. Our test provides a new way – by focusing the tail dependence - to assess the model risk associated with quantitative methods based on normality in asset management and risk management areas.

JEL Classification: G12

Keywords: Implied Correlation, Model Risk, Normality, Value at Risk

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1. Introduction

Value at risk (VaR) is a risk measure that has been widely implemented by financial institutions. It measures the potential loss of a market position over a given time-period and for a given confidence level. For example, a 1-day 99% VaR of $1,000,000 means that over the next trading day, in one case over one hundred, the portfolio loss will be higher than $1,000,000. In order to compute the VaR of a portfolio, assumptions on the distribution of asset price changes are made (see Jorion, 2000; and Dowd, 2005; for a presentation of VaR methods).

One issue that arises on assuming a particular distribution that does not exactly match the data is model risk. In practice, most of the methods used by financial institutions to implement value-at-risk models are based on the multivariate Gaussian distribution with a constant correlation matrix. This is the case in risk management to assess market risk of trading positions and also in asset management when quantitative methods are used to derive optimal portfolios. This paper takes the reverse approach by inferring the correlation implied by VaR calculation. Financial engineers working in asset management and risk management areas have often used the term “implied correlation” to refer to the correlation based on VaR (by analogy with implied parameters used in derivatives markets). The correlation implied from VaR relates to the implied correlation implied from option prices (see Campa and Chang, 1998) that can be obtained when options are simultaneously traded on pairs of exchange rates or on a basket and each component for stocks. Implied correlation from options prices infer market expectation about the dependence structure of asset prices while implied correlation from VaR infer information about the dependence structure naturally contained in past data (historical VaR) or in a model (parametric VaR).

The implied correlation from VaR is computed for different probability levels for long and short positions. If asset returns were distributed according to a Gaussian distribution, the
VaR would not depend on the probability level and the type of position. It would remain constant whether ordinary market conditions (low probability level) or extraordinary market conditions (high level of probability) are considered, and whether bear markets (taken into account in the calculation of the VaR on long positions) and bull markets (short positions) are considered. Moreover, under the assumption of normality, implied correlation should be independent of any weighting scheme for the composition of asset portfolios, and of frequency of measurement. We examine these statements that are related to the recent literature that finds that correlation is not constant but tends to be higher during bear markets (especially market crashes) than during bull markets.\(^5\) For example, Longin and Solnik (1995) showed by using a GARCH methodology that the level of correlation depends on the level of volatility and on the market trend. To address potential bias and sample error of implied correlation from VaR calculations, the paper also studies the distribution of implied correlation under the hypothesis of normality.

Section 2 presents the VaR calculation. Section 3 explains how the implied correlation from VaR is computed. Section 4 presents the empirical results for the US and UK equity markets. Section 5 studies the distribution of implied correlation under normality. Section 6 summarizes the results as stylized facts and discusses applications in finance.

2. VaR calculation

In this section we consider a portfolio composed of two assets, asset 1 in proportion \(x_1\) and asset 2 in proportion \(x_2\) \((x_1 + x_2 = 100\%)\). The frequency used to measure the asset and portfolio price changes, which also corresponds to the holding period used to compute the VaR, is denoted by \(f\). The portfolio VaR is computed by two approaches: first, by considering the

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distribution of asset price changes of the whole portfolio and then compute the portfolio VaR as a quantile of this distribution; second, by considering the distribution of price changes of each asset, then compute the VaR for each asset and finally compute the portfolio VaR by using an aggregation formula.

**a) The portfolio approach**

In the portfolio approach, a time-series of the portfolio price changes is built from the time-series of each asset price: \( \Delta P_{\text{port}} = x_1 \cdot \Delta P_1 + x_2 \cdot \Delta P_2 \). The distribution of portfolio price changes is then built in order to compute the portfolio VaR, denoted by \( \text{VaR}_{\text{port}} \). In this approach the dependence between asset price changes is *implicitly* taken into account in the creation of the portfolio by building the time-series for \( \Delta P_{\text{port}} \).

**b) The risk factor approach**

In the risk factor approach, the distribution of price changes of each asset (more generally called “risk factors”) is first estimated in order to compute the individual VaR of each asset, denoted by \( \text{VaR}_1 \) and \( \text{VaR}_2 \). The portfolio VaR is then computed by using an aggregation formula linking the individual VaR and the portfolio weights (more generally called “risk sensitivities”). A classical aggregation formula used in practice (JP Morgan, RiskMetrics, 1995) computes the portfolio VaR denoted by \( \text{VaR}_{\text{agg}} \) for a given \( f \) as follows:

\[
\text{VaR}_{\text{agg}} = \sqrt{x_1^2 \cdot \text{VaR}_1^2 + x_2^2 \cdot \text{VaR}_2^2 + 2 \cdot x_1 \cdot x_2 \cdot \rho_{12} \cdot \text{VaR}_1 \cdot \text{VaR}_2}
\]  

In this approach the dependence between asset price changes is *explicitly* taken into account by the correlation coefficient between asset price changes, denoted by \( \rho_{12} \).

This approach is also theoretically justified in the case of normality (in theory meaning that there is no sampling error or induced statistical bias). If asset price changes are distributed
according to a Gaussian distribution, then the two approaches to compute the VaR lead exactly to the same result.

3. Implied correlation

In this section we use the VaR calculation derived above in a reverse manner in order to estimate the implied correlation between asset price changes. By using the formula given in Equation (1) in a reverse way to compute the implied correlation, we infer the market information about the dependence structure contained in the portfolio VaR.

By assuming that the portfolio approach and the risk factor approach lead to the same VaR results, we compute the correlation coefficient in the aggregation formula that equates the aggregated VaR and the portfolio VaR. The correlation implied from VaR is given by:

\[
\rho_{12} = \frac{2 \cdot x_1 \cdot x_2 \cdot \text{VaR}_1 \cdot \text{VaR}_2}{\left(\text{VaR}_{\text{port}}\right)^2 - x_1^2 \cdot \left(\text{VaR}_1\right)^2 - x_2^2 \cdot \left(\text{VaR}_2\right)^2}
\]

(2)

If asset price changes are distributed according to a Gaussian distribution, then the correlation implied from VaR should be constant and equal to Pearson’s correlation. Especially, implied correlation should be autonomous from the following parameters:

- The probability used to compute the VaR: \(p\),
- The weights used to build the portfolio: \(x_1\) and \(x_2\),
- The type of position: long or short,
- The frequency used to measure asset price changes: \(f\) (under the i.i.d. assumption).

Under normality, the implied correlation does not depend on the parameters listed above and is simply equal to the classical Pearson correlation coefficient. Whether these propositions hold true is empirically tested in the next section.
4. Empirical results

The correlation implied from VaR is computed for a portfolio comprising the S&P 500 and FTSE 100 indexes. Data are closing index values over the time-period from January 1, 1995 to December 31, 2005. Two frequencies are used to measure price changes: daily (2,871 observations) and weekly (575 observations). Different probability levels are used to compute the VaR: 80%, 95.45%, 98.46%, 99.23%, 99.62% and 99.81% (for daily VaR) and 75%, 92.31%, 96.15% and 98.08% (for weekly VaR). These probability levels correspond to average waiting time-periods of 1 week, 1 month, 1 quarter, 1 semester, 1 year and 2 years (for daily VaR) and 1 month, 1 quarter, 1 semester and 1 year (for weekly VaR). Two types of position are used: long positions and short positions in both indexes. Different weights are chosen to build the portfolios: (25%, 75%), (50%, 50%) and (75%, 25%). The individual and portfolio VaR are computed with historical distributions.

Table 1 gives the correlation implied from daily VaR (Panel A) and from weekly VaR (Panel B). A graphical representation is also given in Figure 1. Results indicate that the implied correlation tends to depend on the type of position (long or short), on the probability level used to compute the VaR and on the frequency used to measure price changes. However, the same pattern of implied correlation is obtained when portfolio weights vary.

Considering first results for daily VaR, implied correlation appears to be higher than the Pearson correlation for long positions and lower than the Pearson correlation for short positions. The results also indicate that the implied correlation for long and short positions tends to diverge as the probability level used to compute the VaR increases (that is when we look further in the

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6 The average waiting time-period for a given quantile (VaR level) represents the time we have to wait on average to observe a price change greater than VaR. As explained by Longin (2000), the concept of waiting time-period is more meaningful than a probability, especially for high levels of risk. For example, the difference in probability between 99.62% and 99.81% appears very small while translated in terms of waiting time-period, the associated daily VaR events occur on average every year and every two years respectively, and is easier to understand and relate to.
distribution tails). For example, for an equally-weighted portfolio (50%, 50%), for a probability level of 99.62% (corresponding to an average waiting time-period of 1 year), it is equal to 0.470 for the VaR on a long position and 0.333 for the VaR on a short position while the Pearsons correlation is equal to 0.416. For a probability level of 99.81% (corresponding to an average waiting time-period of 2 years), it is equal to 0.555 for the VaR on a long position and 0.141 for the VaR on a short position. These results are in line with those obtained by Longin and Solnik (2001), which find that the correlation for long positions is always higher than the correlation for short positions. While Longin and Solnik (2001) use extreme value theory in order to estimate the extreme correlation, we use a more practical approach based the risk measure VaR.

Implied correlation also depends on the frequency used to measure price changes. It is higher for weekly price changes than for daily price changes: 0.643 instead of 0.401 (average computed on the Pearsons correlation values are respectively 0.692 and 0.416). Due to the lower number of weekly observations (divided by 5 compared to daily observations), the pattern of correlation implied from weekly VaR appears more erratic but once again, for high probability levels (or equivalently long average waiting time-periods), implied correlation is higher for long positions than for short positions. For example, for the equally-weighted portfolio and for a probability level of 99.08% (corresponding to an average waiting time-period of 1 year), the implied correlation is equal to 0.624 for the VaR on a long position and to 0.486 for the VaR on a short position.

A similar pattern of implied correlation is found for different portfolio weights. For example, for the highest probability level, implied correlation from daily and weekly VaR of long positions is systematically higher than those of short positions.

The difference between the implied correlation from VaR and the classical Pearsons correlation has found several explanations. First, as highlighted by recent studies\(^7\), conditional

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correlation may be biased. Although for a Gaussian distribution the implied correlation from VaR is in theory equal to the Pearsons correlation for any probability level, there may still be a bias due to the use of the historical method and to the limited number of observations. Second, the difference between the implied correlation from VaR and the classical Pearsons correlation may not be statistically significant as the sampling error could be quite important for small samples. Third, it may be interpreted as a departure from normality. These issues are investigated in the next section as we study the statistical distribution of implied correlation under the null hypothesis of normality.

5. Statistical distribution of implied correlation

The distribution of the implied correlation from VaR is computed by Monte Carlo simulations. For a given frequency, we simulate a series of asset returns drawn from a bivariate Gaussian distribution with means, standard deviations and correlation equal to their historical values. Then, for given portfolio weights, we build a series of portfolio returns. Finally, for a given probability level, we compute $VaR_1$, $VaR_2$ and $VaR_{port}$ and then the implied correlation $\rho_{12}$. We run this simulation procedure 100 000 times in order to get a precise estimate of the distribution of implied correlation. This exercise is reproduced for the different frequencies, portfolio weights and probability levels used to compute the VaR considered in the empirical study. Identical results are obtained for long and short positions due to the simulation techniques used (variance reduction). This simulation study will give us relevant information such as the bias and the dispersion representing the sampling error.

Note that although the simulated returns are drawn from a Gaussian distribution, we still use the historical method to compute the VaR as done in the empirical study. This procedure presents two noises. First, there is a noise due to the simulation procedure of the data as for each simulation the Pearsons correlation obtained from simulated returns is different from the correlation value used to simulate the returns. Second, there is a noise due to the historical
method used to compute the VaR and then the implied correlation. Note that if the variance-covariance method to compute VaR was used instead of the historical method, then for each simulation, the implied correlation from VaR would be exactly equal to the Pearsons correlation. As a result, our test is more conservative as it is more difficult to reject the null hypothesis that the implied correlation from VaR is equal to the Pearsons correlation.

Table 2 gives some basic statistics (mean, standard deviation and the 90% centered confidence interval) of the distribution of implied correlation from VaR under the hypothesis of normality. Figure 2 gives a graphical representation of the simulated distribution for various average waiting time-periods (or equivalently probability levels). Simulation results show that there is a small bias, which tends to disappear as the number of observations increases. For an equally-weighted portfolio, the mean of the simulated implied correlation from weekly VaR (using 575 observations) lies between 0.726 and 0.743 according to the probability level used to compute VaR. Compared with the correlation value used to simulate the data of 0.692, this shows that the bias due to the VaR method and the limited number of data is around 0.04. The mean of the simulated implied correlation from daily VaR (using 2,871 observations, five times more than for the weekly VaR) lies between 0.413 and 0.422 according to the probability level used to compute VaR. Compared with the correlation value used to simulate the data of 0.416, this shows that with many more observations the bias is now very small: less than 0.006. The dispersion measured by the standard deviation and the 90% centered confidence interval of the distribution tends to increase with the probability level. For example, for a position on an equally-weighted portfolio and a probability level of 95.45% (waiting time-period of one month), the standard deviation is equal to 0.049 and the 90% centered confidence interval to [0.342; 0.504]. For a probability level of 99.62% (waiting time-period of one year), the standard deviation is equal to 0.110 (almost double) and the 90% centered confidence interval to [0.248; 0.609] (much wider). Note that with the variance-covariance method, the mean of the simulated

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8 See Jorion (2000) and Dowd (2005) for a discussion of the efficiency of the historical method.
The simulation results can be used to test the null hypothesis of normality. More precisely, we test the equality between the implied correlation from VaR and the Pearsons correlation. For a probability level of 99.62% (waiting time-period of one year), the null hypothesis is rejected at the 5% confidence level if the implied correlation is below 0.248 or above 0.609. In Table 1 we indicate with an asterisk when the implied correlation from VaR is statistically different from the Pearsons correlation at the 5% confidence level. For the daily frequency and the highest probability level, the null hypothesis of normality is always rejected for the short position. For example, for a short position on an equally-weighted portfolio, the implied correlation is equal to 0.141 well below the Pearsons correlation of 0.416.

These results are in contrast with those obtained by Longin and Solnik (2001) as we find here that the correlation for long positions is higher than but not statistically different from the Pearsons correlation and that the correlation for short positions is lower than and statistically different from the Pearsons correlation. In Longin and Solnik (2001), correlation for long positions is always higher than and statistically different from the correlation of extremes obtained under normality, and correlation for short positions is most of the time lower than but not statistically different from the correlation of extremes obtained under normality. This may be attributed to the differences in data frequency and time-period covered by the studies.

6. Summary

The empirical results presented in this paper can be summarized as the following stylized facts:
1) Implied correlation tends to deviate from the Pearson's correlation and to be higher for long positions than for short positions. From a statistical point of view this effect is especially significant for short positions, although the sampling error when estimating the correlation can be quite large. This deviation can be seen as evidence of departure from normality.

2) Implied correlation tends to increase with the probability level for long positions. Implied correlation tends to decrease with the probability level for short positions.

3) Implied correlation tends to decrease over all with the frequency of price changes.

4) Implied correlation behaves in a similar way for different portfolio weights.

Finally, by thinking of applications of this research, the framework developed in this paper can be used in financial institutions (asset management and risk management departments) to evaluate model risk. The test on the implied correlation from VaR allows one to know the limits of the usual techniques based on the Gaussian distribution with constant correlation and the need to develop a specific model for the dependence in the tails. Although most of the existing statistical tests look at the distribution tails, our test focuses on the tail dependence by considering the correlation.

References:


Table 1. Implied correlation from VaR for portfolios invested in S&P 500 and FTSE 100 indexes.

Panel A. Correlation implied from daily VaR.

<table>
<thead>
<tr>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
<th>Portfolio weights</th>
<th>Portfolio weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(25%, 75%)</td>
<td>(50%, 50%)</td>
<td>(75%, 25%)</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
</tr>
<tr>
<td>80% (1 week)</td>
<td>0.305</td>
<td>0.380</td>
<td>0.308</td>
</tr>
<tr>
<td>95.45% (1 month)</td>
<td>0.461</td>
<td>0.411</td>
<td>0.496</td>
</tr>
<tr>
<td>98.46% (1 quarter)</td>
<td>0.455</td>
<td>0.566</td>
<td>0.425</td>
</tr>
<tr>
<td>99.23% (1 semester)</td>
<td>0.352</td>
<td>0.369</td>
<td>0.481</td>
</tr>
<tr>
<td>99.62% (1 year)</td>
<td>0.633*</td>
<td>0.536</td>
<td>0.470</td>
</tr>
<tr>
<td>99.81% (2 years)</td>
<td>0.542</td>
<td>0.140*</td>
<td>0.555</td>
</tr>
</tbody>
</table>

Panel B. Correlation implied from weekly VaR.

<table>
<thead>
<tr>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
<th>Portfolio weights</th>
<th>Portfolio weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(25%, 75%)</td>
<td>(50%, 50%)</td>
<td>(75%, 25%)</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
</tr>
<tr>
<td>75% (1 month)</td>
<td>0.711</td>
<td>0.874</td>
<td>0.595</td>
</tr>
<tr>
<td>92.31% (1 quarter)</td>
<td>0.967*</td>
<td>0.734</td>
<td>0.808</td>
</tr>
<tr>
<td>96.15% (1 semester)</td>
<td>0.536</td>
<td>0.709</td>
<td>0.428</td>
</tr>
<tr>
<td>98.08% (1 year)</td>
<td>0.790</td>
<td>0.447</td>
<td>0.624</td>
</tr>
</tbody>
</table>

Note: this table gives the correlation implied from daily VaR (Panel A) and from weekly VaR (Panel B) for long and short positions on portfolios invested in the S&P 500 and FTSE 100 indexes. The VaR is computed by the historical method. Different probability levels (or equivalently average waiting time-periods) are used to compute the VaR. Different weights are chosen to build the portfolios. The asterisk indicates that the implied correlation from VaR is statistically different from the Pearsons correlation at the 5% confidence level (see Table 2). The estimation period is from January 1, 1995 to December 31, 2005.
Table 2. Distribution of the implied correlation from VaR under normality.

Panel A. Correlation implied from daily VaR.

<table>
<thead>
<tr>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>80% (1 week)</td>
<td>(50%, 50%)</td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.413</td>
<td>0.056</td>
</tr>
<tr>
<td>95.45% (1 month)</td>
<td></td>
<td>0.422</td>
<td>0.049</td>
</tr>
<tr>
<td>98.46% (1 quarter)</td>
<td></td>
<td>0.420</td>
<td>0.061</td>
</tr>
<tr>
<td>99.23% (1 semester)</td>
<td></td>
<td>0.420</td>
<td>0.073</td>
</tr>
<tr>
<td>99.62% (1 year)</td>
<td></td>
<td>0.420</td>
<td>0.091</td>
</tr>
<tr>
<td>99.81% (2 years)</td>
<td></td>
<td>0.421</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Panel B. Correlation implied from weekly VaR.

<table>
<thead>
<tr>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
<th>Probability (waiting period)</th>
<th>Portfolio weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>75% (1 month)</td>
<td>(50%, 50%)</td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.743</td>
<td>0.137</td>
</tr>
<tr>
<td>92.31% (1 quarter)</td>
<td></td>
<td>0.729</td>
<td>0.100</td>
</tr>
<tr>
<td>96.15% (1 semester)</td>
<td></td>
<td>0.727</td>
<td>0.106</td>
</tr>
<tr>
<td>98.08% (1 year)</td>
<td></td>
<td>0.726</td>
<td>0.119</td>
</tr>
</tbody>
</table>

Note: this table gives some basic statistics (mean, standard deviation and the 90% centered confidence interval) of the distribution of implied correlation from daily VaR (Panel A) and from weekly VaR (Panel B) for positions on portfolios composed of the S&P 500 and FTSE 100 indexes. Different probability levels (or equivalently average waiting time-periods) are used to compute the VaR. The VaR is computed by the historical method. Different weights are chosen to build the portfolios. The distribution of implied correlation from VaR is computed under the hypothesis of normality for returns. It is obtained by Monte Carlo simulation by assuming that returns are drawn from a bivariate Gaussian distribution with means, standard deviations and correlation equal to their historical values. The estimation period is from January 1, 1995 to December 31, 2005.
Figure 1. Implied correlation from VaR for portfolios invested in S&P 500 and FTSE 100 indexes.

Note: this figure represents the correlation implied from VaR for long and short positions on an equally-weighted portfolio invested in the S&P 500 and FTSE 100 indexes. Different average waiting time-periods (or equivalently probability levels) are used to compute the VaR. The VaR is computed by the historical method. The semi-dotted line represents the Pearson correlation and the dotted lines the lower and upper bounds of the 90% centered confidence level for the implied correlation from VaR under the hypothesis of normality. The estimation period is from January 1, 1995 to December 31, 2005.
Figure 2. Distribution of implied correlation from VaR under normality.

Note: this figure represents the distribution of implied correlation from VaR for positions on an equally-weighted portfolio invested in the S&P 500 and FTSE 100 indexes. Different average waiting time-periods (or equivalently probability levels) are used to compute the VaR. The VaR is computed by the historical method. In each case the distribution of implied correlation from VaR is computed under the hypothesis of normality for returns. It is obtained by Monte Carlo simulation by assuming that returns are drawn from a bivariate Gaussian distribution with means, standard deviations and correlation equal to their historical values. The estimation period is from January 1, 1995 to December 31, 2005.