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2011

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Current Version: October, 2010

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Abstract

This paper analyzes the secret reservation price in eBay auctions. Under the assumptions of secret and public reservation price, we derive the optimal bidding function for the bidders. For the seller, we solve for the equilibrium reservation price. We argue that the choice of secret reservation price is rational for the seller, as they can generate higher revenue in certain conditions. Our model predicts that, under endogenous entry, secret reservation price leads to higher revenue since it attracts more bidders to the auction. This effect is more noticeable for luxury goods. However, secret and public reservation prices generate identical revenue for the seller if entry is exogenous. Furthermore, the results are supported by empirical works such as Elyakime et al. (1997), Bajari (2002) and Lucking-Reiley et al. (2006).

Keywords: eBay, auctions, secret reservation prices, endogenous entry, mechanism design

1. Introduction

Ever since the origin of internet auctions in 1995, web-based auctions have developed at a remarkable rate. One of the largest consumer-oriented auction sites is eBay, where individual sellers list their items for vending, and individual buyers bid on the items. Nowadays, eBay is one of the largest marketplaces in the world with an annual transactions over 60 billion dollars.

Internet auctions represent a rich topic for research. One of the most interesting features of eBay's auction mechanism is *secret reservation price*. Unlike the standard auction setting, in which the reservation price is announced to all the bidders before submitting their biddings, eBay has an option for sellers to keep reservation price hidden. The reservation price is not disclosed to the bidders, but kept as secret in the auction. The reservation price is disclosed only *after* all the bidders submit their bidding. This type of mechanism is simply referred as secret reservation price.

Since secret reservation price is ubiquitous in eBay, the study of it has a realistic significance to both market agents and researchers. To the agents, it is still left unexplained how the bidders should bid and how the sellers should choose the optimal level of reservation price. More importantly, because secret and public reservation price divide payoffs to the sellers and bidders unevenly, the study of secret reservation price is also crucial to the mechanism designer. The central planner is capable to decide which reservation price, hidden or public, maximize the social welfare. To the researchers, the study of secret reservation explains the rationality behind this common practice: why sellers choose to keep reservation price hidden, and when should they do so?

The importance of secret reservation price attracted the attention of researchers for years. Elyakime, Laffont, Loisel, and Vuong (1994) provide a fundamental theoretical framework on this issue. However, their model fails to justify the common practice of secret reservation price: their model predicts that, for the sellers, a public reservation price is superior to the secret reservation price. A possible reason of this dilemma is that they only consider the case of first price, private-value auctions with risk-neutral bidders.

Many subsequent scholars, attempting to rationalize the secret reservation price, modified the original assumptions in Elyakime et al.(1994). They found some support for this widely observed practice. Vincent (1994) deviates from the assumption of private-value auctions and extends the model to common-value auctions. He demonstrated an example in which secret reservation price can increase ex ante expected profit for the seller in common value auctions. Huagang Li and Guofu Tan (2000) provide an alternative insight in this subject. In their research, they adopt the assumption of risk-averse bidder. They concluded that, as the number of the risk-averse buyer increases, the hidden reserve price is likely to dominate in a private value auction.

In my research, I try to extend the analysis of Elyakime, Laffont, Loisel, and Vuong (1994) by adding endogenous entry assumption into the model. Endogenous entry means that the number of bidders is no longer determined outside of the model. Potential participants are able to decide whether to enter the auction after knowing their private values of the auction item. Besides, my model also generalizes Elyakime et al.(1994)'s work to second-price auctions.

The main results of this research show that the choice of secret reservation price is rational for the seller, as they can generate higher revenue in certain conditions. Under endogenous entry, secret reservation price attracts more bidders to the auction and thus lead to higher revenue, especially for luxury items.

The remaining parts are organized as follows. Section 2 will scrutinize the institutional details of eBay auctions. Section 3 examines the eBay auction mechanism, 4 discusses the possible effect of endogenous entry. In section 5, I provide a brief discussion of the empirical results and some of my theoretical predictions. The conclusion is in section 6.

2. Institutional Details of eBay Auctions

eBay provides a central market for consumers and sellers to trade using auctions. Sellers pay a fee whereas bidders pay no fee, and sellers also choose an auction type for the sale of their goods. They set a starting bid, a minimum bid increment and specify the duration of the auction. They also have the option to set a secret reserve price. If they do, during the auction process eBay indicates whether the reserve price has been met or not, and if the reserve price has not been met at the end of the auction, sellers have the option of not selling the item.

It is generally agreed that the mechanism used by eBay resembles a second-price auction. At any time, eBay shows the current standing bid of the auction, which is the current second highest bid, and the transaction price is the second highest bidding; in the special cases where there is no bid or only one bid, the current standing bid is the starting bid. When a bidder submits a bid, he/she knows the current standing bid, the identity of the seller, the starting and ending time of the auction and a description of the item. However, the exact amount of each bid is not revealed until the end of the auction. The final price is the second highest bid plus a specified minimum increment. (Anwar et.al.2004)

3. eBay Auctions with Exogenous Entry

As argued before, second price private value auction typically depicts the secret reservation price mechanism used in eBay. To capture these features, in this section, we set up a model of second price auctions with a hidden reservation price.

3.1 The Model

First of all, we specify the mechanism of the second price auction with a hidden reservation price. There is one risk neutral seller tries to sell a single object to n bidders in an auction. The valuation of each bidder is $v_1, v_2, v_3 \ldots v_n$, which are private information for the bidder i. Further, we assume they are independently draw from [0,1] with a PDF of $f(\cdot)$ and CDF of $F(\cdot)$; the seller's valuation is v_0, which is also draw from [0,1] but with a possible different PDF of $g(\cdot)$ and CDF if $G(\cdot)$; The buyers are assumed to be risk adverse, and their von Neumann-Morgenstern utility function is $u(\cdot)$. Again, we need $f(\cdot)$ and $g(\cdot)$ are continuously differentiable; and $u(\cdot)$ is strictly increasing, concave, and continuously differentiable.

The timing of the game is described as follows: in the period zero, natural assigns a private valuation of v_i to each bidder and t to the seller. In the first period, bidders submit their biddings $b_1, b_2, \ldots b_n$ and the seller write down the reservation price r in sealed envelopes; in the second period, all biddings are open and reservation price is revealed. The object is given to the bidder with highest bidding if it is higher than the reservation price. The winner pays the higher price between the second highest bidder and the reservation price.

Before formally examine the bidding functions, an interesting institutional feature of the model is notable. "Since the seller tenders a sealed bid, which is opened at the same time as the buyers' sealed bids, the seller is not much different from a buyer except that his objective function and eventually the distribution of his valuation are different." (Elyakime, Laffont, Loisel, and Vuong ,1994). Formally, consider two types of auctions: the second price, private value auction with a secret reservation price r satisfying all the above assumptions, denoted by (G1); and another no reservation price, second price auction with the same bidders as(G1) *and* an additional bidder $b_{n+1} = r$ (her bidding is r), denoted by (G2).

After we see (G1) and (G2) generate the equivalent payoff for the bidders, their strategy can be characterized by proposition 1.

***Proposition* 1** In the second price, private value auctions with secret reservation price, the weakly dominate bidding strategy for the bidders is to bid his/her true valuation.

Proof: Without loss of generality, consider the bidder i. If her bidding b_i is the highest among all biddings ($b_i > b_j, \forall j = 1,2,..i-1, i+1,..n$), and larger than r ($b_i > r$), then in (G1), she is the winner and pays $max\{b_1, b_2, ...b_{i-1}, b_{i+1}...b_n, r\}$. Equivalently, in (G2), bidder i is also the winner, and pays the *second* highest amount among $\{b_1, b_2, ...b_n, r\}$. It is straightforward to see that the bidder i make the same amount of payment in both (G1) and (G2) if she wins. On the other hand, if she loses the auction, nothing is paid in both cases, obviously. This argument leads to the conclusion that for any bidder 1,2,…n, (G1) and (G2) gives the same results.

To the bidders, the mechanism (G1) is identical to the case in which seller joins the auction as an additional bidder, so the strategy of bidders will also be the same. As a result, the strategy for the bidders is given as

$$b(v_i) = v_i, \forall i = 1,2,...n \tag{12}$$

3. 2 Comparison of Revenue

Without solving the problem for the sellers, we can see from proposition 1 that, in the second price auction, secret reservation price provides the same revenue as announced reservation price. Because bidders' strategy is the same in both cases, the seller's revenue should also be identical.

***Proposition* 2** In the second price, private value auctions, secret reservation price and public reservation generates identical revenue for the seller.

Proof: Followed from proposition 1 directly.

Given all bidders reveal her true valuations, the seller's payoff $V(r, v_0)$ is

$$V(r,v_0) = n(n-1)\int_r^1 (v-v_0)[1-F(v)]F^{n-2}(v)dF(v) + n(r-v_0)[1-F(r)]F^{n-1}(r) \tag{13}$$

The first part of (3) captures the payoff when reservation price r is less than the second highest bidding. In this case, the payment received from bidders is the second highest bidding.

The second part of (2) calculates the payoff when reservation price *r* is larger than the second highest bidding but still less than the highest price bidding. In this case, the payment received from bidders is just *r*. Maximizing (11) with respect to *r*, we can derive the optimal secret reservation price. Then, we can solve for the Bayesian Nash equilibrium for this game.

$$r^* = \frac{n(n-2)(1+v_0) - \sqrt{(n-2)^2 n^2 (v_0 - 1)^2 + 4v_0}}{2[n(n-1)-1]} \tag{14}$$

To sum up, in an eBay auction with secret reservation price, the bidders submit the bidding equal to their private valuation, but the seller does not always set secret reservation price equivalent to her private valuation. Most importantly, with exogenous entry, both public and secret reservation price provide the same revenue for the seller, no matter how risk-averse the bidders are.

4. eBay Auctions with Endogenous Entry

The standard independent valuation auction assumes the number of bidders is public information, and it is determined outside the model. Obvious, this is not a realistic assumption. The seller may know the number of *potential* bidders. However, since participation incurs some cost, e.g. transaction cost, opportunity cost and so on, whether to participate in the auction is an important decision for the bidders. The bidder may not want to participate because the cost can overweight the expected payoff in the auction. Empirically, Bajari(2002) find that in eBay, the number of bidders is related to the time of bids, seller's reputation, and the existence of secret reservation price. He concludes that the participation of the bidders is endogenous. In this section, our motivation is trying to relax the assumption of exogenous number of bidders and to see whether our earlier analysis is robust to the assumption of endogenous entry. We first consider the participation decision for the bidders, and then compare the seller's revenue generated by the public reservation price with secret ones.

As Menezes and Monteiro(2000), we assume the number of *potential* bidder to be *n*, and bidders face participation costs $c \in (0,1)$.[1] This cost can be any transaction cost, opportunity

[1] Prof. Kunimoto(Brown University, PhD) raises an interesting concern on this assumption. It is true that every bidder may have different opportunity cost to participate in the auction. However, it is better to think *c* here as a fixed cost, and all variable cost are included in the term v_i i.e. bidder's private valuation for the item. One the other hand, if *c* here is the entry fee into the auction, then our model is so simple that it does not consider seller can choose the optimal *c*.

cost and so on. The timing of the game with a secret reservation price is as follows: In the period zero, natural assigns a private valuation of v_i to each bidder and v_0 to the seller. In the first period, potential bidders decide whether to enter the auction given her own valuation v_i. If entered, the actual bidders submit their biddings $b_1, b_2, \ldots b_I$ and the seller write down the reservation price r in sealed envelopes. If not enter, bidders receive zero. The remaining part game is the same as the standard second price sealed-bid auctions. The timing of a public reservation is similar to the secret reservation price case. Rather than with a reservation price r in sealed envelope, the seller announce the r publicly before bidder decide whether to join the auction.

***Lemma* 1** For any bidder chooses to participate in the second price sealed-bid auction, either in one with secret reservation price or announced reservation price, biding his/her true private valuation is the weakly dominate strategy.

Proof: For any bidders enter the auction with secret reservation price, Proposition 1 applies. For any bidders enter the auction with public reservation price, standard proof of second price auction applies.[2] In either of these two situations, bidding truthfully is the weakly dominate strategy.

Lemma 1 describes the bidding strategy for the bidders after entering the auction. The next problem is to determine the participation condition for the bidders. Denote v_ρ as the cut-off value, namely, only bidders with a private valuation greater than a certain cut-off point actually bid in these auctions.[3] To simplify our analysis, we assume uniform distribution of private valuation v_i on [0,1].

In the case of public reservation price, suppose the public reservation price is given as r. if participate, the potential bidder with a private valuation v_i receives

$$\pi_i^A(v_i) = -c + v_i\left(\frac{v_\rho - r}{1-r}\right)^{n-1} + (n-1)\int_{v_\rho}^{v_i}(v_i - x)\left(\frac{x-r}{1-r}\right)^{n-2}\frac{1}{1-r}\,dx \tag{19}$$

The first term $-c$ is the participation cost. The second term calculates the expected payoff when there is only one bidder taking part in the auction. In this case, the only bidder receives

[2] One proof is given in Vijay Krishna, *Auction Theory*, Academic Press, 2002.

[3] See Menezes and Monteiro(2000) for proof and more details. Special Thanks to Prof. Maxim Sinitsyn (McGill University) for precious comments on this assumption

the item for free. The last term is the expected payoff when there is more than one bidder in the auction.

Similarly, in the case of secret reservation price, suppose the secret reservation price is given as r = b(v_0). The potential bidder with a private valuation v_i receives

$$\pi_i^S(v_i) = -c + \int_0^1 \{v_i(\frac{v_\rho - b(v_0)}{1-b(v_0)})^{n-1} + (n-1)\int_{v_\rho}^{v_i}(v_i - x)(\frac{x-b(v_0)}{1-b(v_0)})^{n-2}\frac{1}{1-b(v_0)}dx\}dv_0 \quad (20)$$

We can see that, in the hidden reservation price case, there is one more integral compared to the public case. The difference between secret and public reservation price is that, in the case of announced reservation price, r is known; but in the secret case, it is not. So the bidders have to make an estimation of the reservation price when it is hidden.

The participation condition requires that $\pi_i^A(v_\rho) = \pi_i^S(v_\rho) = 0$. Thus, in the public reservation price case, the participation condition is

$$v_\rho^P(\frac{v_\rho^P - r}{1-r})^{n-1} = c \quad (21)$$

In the case of hidden reservation price, the participation condition is

$$v_\rho^S\int_0^1(\frac{v_\rho^S - b(v_0)}{1-b(v_0)})^{n-1}dv_0 = c \quad (22)$$

(21) indicates that for each r, there will be an associated cut-off value $v_\rho(r)$. This is due to the fact that bidders observe the reservation price r before deciding whether to participate. Thus, their strategies depend on what they observed. However, (22) shows that, in the case of secret reservation price, there will be only one cut-off value v_ρ for any v_0. This is because the bidders have no information about what the reservation price truly is. Obviously, (21) and (22) show that the participation condition is disparate for public and hidden reservation price. Consequently, proposition 2 is invalid under the assumption of endogenous entry, since the number of participated bidders can be different in these two cases.

Please recall that one of our primary tasks in this section is to evaluate the number of participants under both hidden and public reservation price assumptions. We also want to see why auctions with a high value item tend to choose secret reservation price. The lemma 2 below answers this question.

***Lemma* 2** There exist a $\tau > 0$, such that for any $v_0 > \tau$, $v_\rho^{\ P} > v_\rho^{\ S}$

Proof: The proof of proposition 3 has two steps. First we would love to show as $v_0 \to 1$, $r \to 1$. The second step involves showing $v_\rho^{\ P} \to 1$ as $r \to 1$. First, we must have $v_0 \le r \le 1$. This is due to the fact that, if $v_0 > r$, the seller has a positive probability to sell the item but making negative revenue. This is definitely irrational for the seller. As a result, by squeeze theorem, as $v_0 \to 1$, $r \to 1$. Second, because $v_\rho^{\ P}(\frac{v_\rho^{\ P} - r}{1-r})^{n-1} = c$, and $0 < c < 1$, it must be $v_\rho^{\ P}(v_\rho^{\ P} - r)^{n-1} \to 0$ as $r \to 1$. The condition $v_\rho^{\ P} > r$ rule out the possibility that $v_\rho^{\ P} \to 0$. Therefore, we should have $\lim_{r \to 1} v_\rho^{\ P}(r) = 1$. The above two steps implies $\lim_{v_0 \to 1^-} v_\rho^{\ P}(v_0) = 1$. Hence, $\forall \varepsilon > 0, \exists \delta > 0$ such that if $1 - v_0 < \delta$, $1 - v_\rho^{\ P} < \varepsilon$. Let $\varepsilon = 1 - v_\rho^{\ S}$ and $\tau = 1 - \delta$, then $\exists \tau$ such that if $v_0 > \tau$, $v_\rho^{\ P} > v_\rho^{\ S}$. ▮

Since a highly valued item requires a high reservation price, making it public will deter many potential bidders from bidding. On the other hand, by keeping it secret, the sellers will avoid this trouble. Thus, secret reservation price auctions attract more participants than the auctions with a public reservation price. In other words, lemma 3 means that if the item in the auction is highly valued, then the secret reservation price attracts more participants than public reservation price.

After solving the cut-off value, we can compare the expected revenue for the sellers under both mechanisms. Proposition 3 summaries the findings as below.

***Proposition* 3** Let π^S, π^P be the seller's revenue in the secret reservation price auction and pubic reservation price respectively. If we assume uniform distribution of private valuation v_i on [0,1], then given others equal, there exist a $\tau > 0$, such that for any $v_0 > \tau$, $\pi^S > \pi^P$

Proof: First, it is easy to see from (13) that as the number of participant *n* increase, seller's revenue raises. Therefore, from lemma 3, it follows that given others equal, there exist a $\tau > 0$, such that for any $v_0 > \tau$, $\pi^S > \pi^P$

To sum up, we include the assumption of endogenous entry into your model in this section. If the value of the selling item is high, secret reservation price attracts a larger number of participants. Thus, by keeping the reservation price secret, the seller can gain more revenue.

Lastly, these predictions are congruent with the empirical evidence: in eBay auctions, the more valuable the item is, the more likely it is sold under secret reservation price. [4]

5. Empirical Evidences

Recent popularity of online auctions provides an unprecedented opportunity for empirical auction studies, because the institutional features of eBay auctions are analogous to the second price auction with a secret reservation price. (Anwar and McMillan, 2004; Ockenfels, 2002; Lucking-Reiley et al., 2006).

One type of research focuses on testing the assumption of endogenous entry. Bajari(2002) tests this assumption in eBay coin auctions. They run a regression of a few variables on the number of participated bidders. The results show that a low minimum bid and a high book value significantly increase the number of bidders entering into an auction. They argue that "this is consistent with a model in which bidding is a costly activity, as in Harstad (1990) and Levin and Smith (1994) where bidders enter an auction until expected profits are equal to the costs of participating." In short, Bajari(2002) it provide an empirical foundation for our assumption of endogenous entry.

Another type of study concentrates on the comparison of the sell's revenue. Lucking-Reiley et al.(2006) examine the determinant of auction prices in eBay. Specifically, they study the precious coin auctions. Through regression analysis, they conclude that secret reservation price can increase the revenue of the seller by approximately 15%. Meanwhile, public reservation price merely increase seller's revenue by 0.01%. This implies that the seller can gain significantly higher revenue by making reservation price secret. It directly supports our model predictions. If the auction items are valuable, then a high public reservation price will deter bidders from entry. Therefore, a secret reservation price brings higher revenue for the seller than a public one.

To sum up, there are empirical evidences that support our assumption of endogenous entry. Moreover, our model predicts similar results as those observed in real data.

[4] See next section for more details

6. Conclusion

This paper analyzes the secret reservation price in eBay auctions. Under the assumptions of secret and public reservation price, we derive the optimal bidding function for the bidders. For the seller, we solve for the equilibrium reservation price. We argue that the choice of secret reservation price is rational for the seller, as they can generate higher revenue in certain conditions. The main results are that secret and public reservation price generate identical revenue for the seller if entry is exogenous. Otherwise, secret reservation price attracts more bidders to the auction and thus lead to higher revenue. Furthermore, the results are supported by empirical works such as Elyakime et al.(1997), Bajari(2002) and Lucking-Reiley et al.(2006).

Our model predicts that secret reservation price generates higher revenue for the seller under certain conditions. However, there are still many other types of auctions to consider. For example, is secret reservation price profitable in common-value auctions? Are endogenous entry and risk-averse the only reasons for the profitability of secret reservation price? After all, it is only the first attempt to justify the use of secret reservation price. Further research is needed to address all of these questions.

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