Bayesian estimation of small-scale DSGE model of the Ukrainian economy

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BAYESIAN ESTIMATION OF SMALL-SCALE DSGE MODEL OF THE UKRAINIAN ECONOMY

In this article we try to introduce Bayesian methodology for the estimation of dynamic stochastic general equilibrium model of the Ukrainian economy. The resulting impulse response functions can be used for increasing the efficiency of monetary and fiscal policy interventions. In addition, we showed that technology is one of the most important factors contributing to the stable long-term growth path of the economic system of Ukraine.

Keywords: DSGE model, Bayesian estimation, monetary and fiscal policy.

Introduction and Literature Review

The goal of the proposed article is to introduce into the estimation of the dynamic stochastic general equilibrium (DSGE) model of the Ukrainian economy the method of Bayesian estimation.

Different frameworks have been proposed to model the economy of the state in general or to study the relation between specific macroeconomic variables in particular. Zagaglia [16] states that the majority of publications (e.g., Rudebusch and Wu [12] and Hördahl, Tristani, and Vestin [8]) are based on the reduced-form models which are not able to reveal micro foundations and deep reasons of underlying processes. This gap in the literature has been trying to fill by means of micro founded DSGE models, significant progress in development of which we observe during the last three decades. Since the seminal famous works on rational expectations modeling of Lucas [11], Kydland and Prescott [10] small model built on first principles with rational behavior of economic agents to the coherent complex structures of Christiano et al. [5], dynamic equilibrium theory has conducted a quantum leap in macroeconomic modeling. Recent achievement in estimation and construction of DSGE models force Central Banks of developed and emerging market economies (EMEs) to consider DSGE models for policy application and forecasting. DSGE models are powerful tool in the determination of sources of economic fluctuations
and allow finding the links between structural features of the economy and reduced-form of the parameters [15]. However, it was only recently DSGE model prove their practical usefulness in policymaking: Christiano et al. [5] showed that they could be applied effectively to monetary policy shocks analysis and Smets and Wouters [13] reveal the dominance of DSGE models in the forecasting ability over the classical wide-spread VAR models (estimated with Bayesian econometrics, that is, BVAR models).

The success of DSGE approach in modeling economic behavior was triggered to the large extent by the application of Bayesian econometrics used for the model estimation. It allowed to solve important problems which lied before DSGE modeling. First, unlike generalized method of moments (GMM) estimation, which is based on equilibrium relationships, the Bayesian analysis is system-based and fit the solved DSGE model to a vector of aggregate time series. Second, the estimation is based on the likelihood function generated by the DSGE model rather than, for instance, the discrepancy between DSGE model responses and VAR impulse responses. Third, prior distributions can be used to incorporate additional information into the parameter estimation.

In Ukraine much less attention is dedicated to the DSGE modeling. The example of one of the first attempts in DSGE models construction is represented by Bazhenova [1]. However, these models most often are simply calibrated but not estimated using Bayesian techniques.

We are trying to present DSGE model which can be used for the modeling of the Ukrainian economy. The novelty of our work and the main accent will be made on the application of Bayesian econometrics for the estimation of the model.

The rest of this article is organized as follows. First, prototypical economic framework is discussed in the third section. Secondly, short data analysis is presented in section four. Bayesian techniques are explained in the estimation methodology section. Finally, we present our results and conclusions.

**Standard New Keynesian Model**
In this section we are presenting standard New Keynesian model (NKM) à la Bernanke et al. [2] but without financial accelerator. The concept includes the behavior of households, which consume, save and work, intermediate firms, which rent capital and labor to produce intermediate commodities, final producers at monopolistic market, government, National Bank of Ukraine (NBU), the mechanisms of price stickiness, shocks and equilibrium relation.

We assume that the economy is populated by the agents who form the households. Each such economic entity consumes a set of differentiated goods and supply labor to firms. Based on the intertemporal preferences structure each period households decide how much to consume and invest so as to maximize their utility over households’ lifetime. Household’s utility function depends on three elements: consumed goods and services, utilities from leisure and money. The portfolio of assets includes currency and bonds. Summarizing, the representative household maximizes its intertemporal preferences over infinite period of time (we assume infinitely living agents):

$$
\max_{\{C_{t+k}, M_{t+k}, H_{t+k}\}} E \sum_{k=0}^{\infty} \beta^k \left( \ln(C_{t+k}) - \xi \ln \left( \frac{M_{t+k}}{P_{t+k}} \right) + \varphi \ln(1 - H_{t+k}) \right),
$$

where $\beta$ is the discount coefficient, $C_{t+k}$ is the monetary equivalent of consumed goods and services during the period $t+k$, $\frac{M_{t+k}}{P_{t+k}}$ is the real money balances at $t+k$, $H_{t+k}$ is the amount of hours worked.

Following Bernanke et al. [2], we would like to highlight households’ budget constraint:

$$
C_t = W_t H_t - T_t + \Pi_t + R_t D_t - D_{t+1} + \frac{M_{t+1} - M_t}{P_t},
$$

where $W_t$ is the wage, $T_t$ is the lump sum tax, $\Pi_t$ are the dividends received by the households from the enterprises, they owned, $D_t$ are the households’ deposits, $R_t$ is the deposits interest rate.
Now we can set up Lagrangian function, which will summarize the households’ problem of utility maximization within existing budget constraint. The first order conditions are the following:

\[
\frac{1}{C_t} = E_t \left( \beta \frac{1}{C_{t+1}} \right) R_{t+1}, \quad (3)
\]

\[
W_t \frac{1}{C_t} = \varrho \frac{1}{1-H_t}, \quad (4)
\]

\[
\frac{M_t}{P_t} = \xi C_t \left( \frac{1-R^n_{t+1}}{R^n_{t+1}} \right)^{-1}, \quad (5)
\]

where \(R^n_{t+1}\) is the nominal interest rate, \(R^n_{t+1} = \frac{R_{t+1}P_{t+1}}{P_t}\).

Productions sector is represented by two subgroups: companies producing intermediate homogenous commodities that are used by other group to produce final heterogeneous goods and services (basically, it is the final output of the country). The production function of representative intermediate producer is assumed to have constant return-to-scale technology and has Cobb-Douglas form with capital, labor and technology as the inputs:

\[
Y_t = A_t K_t^\alpha H_t^{1-\alpha}, \quad (6)
\]

where \(Y_t\) is the output at period \(t\), \(K_t\) is the capital used during the period \(t\), which is rented beforehand at \(t-1\), \(H_t\) is the labor force, \(A_t\) is the exogenous technology, \(\alpha\) is the parameter.

The capital evolves based on the following rule:

\[
K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1-\delta) K_t, \quad (7)
\]

where \(\Phi(\bullet)\) is the increasing concave function: \(\Phi'(\bullet) > 0\), \(\Phi''(\bullet) < 0\), \(I_t\) is the investment level at \(t\), \(\delta\) is the depreciation rate.

Then we can derive the expression for the price of capital, \(Q_t\):

\[
Q_t = \left( \Phi \left( \frac{I_t}{K_t} \right) \right)^{-1}, \quad (8)
\]

In addition, return to capital, which is used during the period \(t+1\), equals to:
where \( X_t \) is the marginal mark-up.

Final goods and services produced by the continuum of final firms are aggregated into final output using Dixit and Stiglitz [6] production function:

\[
Y_t^f = \int_0^1 Y_t(z) \frac{1}{\tau} dz, \quad (10)
\]

where \( Y_t^f \) is the final GDP, \( Y_t(z) \) is the output sold by \( z \)-th final producer, \( \epsilon \) is the elasticity of substitution.

Firms set prices to maximize the present discounted value of future stream of profits. Following Calvo [3], we may assume that prices are staggered, i.e. they are Calvo-sticky and follow Calvo-process. Staggered price adjustment generates price inflexibility in equilibrium and makes monetary policy effective to control aggregate demand and, consequently, to affect prices and output in the short run.

Fiscal policy is conducted by the government and can be described by the following equation:

\[
G_t = \frac{M_t - M_{t-1}}{P_t} + T_t, \quad (11)
\]

Monetary policy is implemented by targeting the nominal interest rate. Specifically, it may be assumed that the monetary authority uses a Taylor [14] rule reaction function:

\[
\frac{R^n_t}{R^n_{t-1}} = \left( \frac{R^n_{t-1}}{R^n} \right)^{\gamma_x} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_{\Pi}} \left( \frac{Y_t}{Y} \right)^{\gamma_Y}, \quad (12)
\]

where nominal interest rate is set by the Central Bank depending on the past ratio of nominal rate to its long-term value, \( \frac{R_{t-1}}{R^n} \), ratio of inflation to the target level of inflation, \( \frac{\Pi_t}{\Pi} \), ratio of GDP to its long-term level \( \frac{Y_t}{Y} \); \( \gamma_x \) and \( \gamma_Y \) are parameters.
The equilibrium condition for goods market clears when the demand from the households, investment demand from the firms and the government expenditure can be met by the production of the firms. So the aggregate demand as a sum of three mentioned elements is equated to the aggregate supply curve:

\[ Y_t = C_t + I_t + G_t, \quad (13) \]

We assume the economy is disturbed by three types of shocks:

\[ G_t = G_{t-1} \rho e^{\sigma e_t} \quad \text{and} \quad (14) \]
\[ A_t = A_{t-1} \rho e^{\sigma e_t}, \quad (15) \]

where \( \sigma \) denotes standard deviation of the variable, \( \rho \) are the parameters, \( e_t \) and \( e_{at} \) are i.i.d. variables with \( N(0,1) \). The number of shocks should be equal to the number of observables (data series used for the model estimation) so as to avoid singularity problem. In addition, we add the exogenous shock to the monetary policy rule by multiplying (12) by \( e^{\sigma e_w} \).

The DSGE model is linearized using first-order Taylor expansion, obtaining a linear rational expectation (LRE) model (lower-case letters denote the deviations from the steady state, upper-case letters without time subscript denotes steady state values; for more details of derivations see Bernanke et al. [2]:

**Aggregate demand:**

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t, \quad (16) \]

\[ c_t = -r_{t+1} + E_t(c_{t+1}), \quad (17) \]
\[ r_{t+1} = (1 - \zeta)(y_{t+1} - k_{t+1} - x_{t+1}) + \zeta q_{t+1} - q_t, \quad (18) \]
\[ q_t = \varphi(i_t - k_t), \quad (19) \]

**Aggregate supply:**

\[ y_t = a_t + \alpha k_t + (1 - \alpha) h_t, \quad (20) \]
\[ y_t - h_t - x_t - c_t = \eta^{-1} h_t, \quad (21) \]
\[ \pi_t = \kappa(-x_t) + \beta E_t(\pi_{t+1}), \quad (22) \]
\[ \pi_t^e = r_t + E_t(\pi_{t+1}), \quad (23) \]
Evolution of capital:
\[ k_{t+1} = \delta i_t + (1 - \delta) k_t , \quad (24) \]

Monetary policy rule:
\[ r_t = \rho r_{t-1} + \xi \pi_{t-1} + \epsilon_i , \quad (25) \]

Shocks:
\[ g_t = \rho g_{t-1} + \epsilon_g , \quad (26) \]
\[ a_t = \rho_a a_{t-1} + \epsilon_a , \quad (27) \]

where \( \zeta = \frac{1 - \delta}{1 - \delta + \alpha \frac{Y}{K}} \), \( \varphi = \left( \Phi \left( \frac{I}{K} \right) \right)^{-1} \), \( \eta = 1 - H \), \( \kappa = \left( \frac{1 - \theta}{\theta} \right) (1 - \theta \beta) \).

Data and calibration

The model described above will be estimated on quarterly data of the Ukrainian economy for the period 2002Q1-2010Q3. To keep the estimation as simple as possible we will use three time series: GDP, consumption and NBU discount rate. We apply X-12-ARIMA filter of U.S. Census Bureau to eliminate seasonality in the data.

In addition, we use linear detrending to eliminate stationarity in the GDP and consumption series.

Based on the quarterly data for the period 2002-2010 long-term ratio of consumption to GDP equals to 59%, long-term ratio of investment to GDP is 23%, long-term ratio of government expenditures to GDP is 18%. The other variables and parameters are calibrated similar to Bernanke et al. [2]: ratio of capital to GDP is 10, \( X = 1.1 \), \( \beta = 0.95 \), \( \alpha = 0.35 \), \( \delta = 0.025 \), \( \rho = 0.8 \), \( \rho_a = 0.99 \), \( \rho_g = 0.95 \), \( \varphi = 0.25 \), \( \theta = 0.75 \).

Bayesian econometrics

Having calibrated the model, we already are able to study the relationships between the variables it describes. However, we can go further and try to incorporate directly the data for the estimation of the model parameters. To do this, we should rely on the methodology developed within Bayesian econometrics.
Bayesian econometrics is based on a simple probability rule. Let assume that there exist some data generating process (DGP) which produces observables (GDP, consumption and NBU discount rate) selected for DSGE model. The matrix of data may be denoted by $\Psi$. Since this is the sample data, $\Psi$ can be considered as a random multidimensional variable. Secondly, we are interesting in the parameters which describe the relations between the variables from the linearized system of equations. The vector of parameters is $\chi$. In Bayesian econometrics $\chi$ is considered as a random variable in contrast to its chief competitor frequentist (classical) econometrics where population parameters are considered as nonrandom. From Bayesian point of view, we are interesting in unknown $\chi$ (model parameters) given the known information $\Psi$ (data). In terms of Bayes formula we can get:

$$p(\chi | \Psi) = \frac{p(\Psi | \chi) * p(\chi)}{p(\Psi)}, \quad (28)$$

or probability kernel can be expressed as:

$$p(\chi | \Psi) \propto p(\Psi | \chi) * p(\chi), \quad (29)$$

where $p(\chi | \Psi)$ is posterior density of $\theta$ given $\Psi$, $p(\Psi | \chi)$ is the likelihood function and $p(\chi)$ is prior density of $\chi$ (based on the values of calibrated parameters).

The posterior combines prior distribution and the likelihood function. First of all we should define how to compute them and then we can run the optimization algorithm.

The prior does not depend on the data. It means that it contains information about $\chi$, which is not derived from the data directly or derived before seeing the data. For each parameter, we want to estimate, the prior should be specified. The prior specification can be conducted in the form of its distribution (normal, gamma, normal-gamma, beta, Wishart, their inverses, etc.) with corresponding parameters (all moments, e.g., mean, variance, $3^{rd}$ moment, $4^{th}$ moment). If we do not estimate some parameter and use only its calibrated value, we can specify for it almost non-informative prior, e.g., expressed by uniform distribution with a wide range.
want to give more weight to the data and less to the calibrated value, then in the prior low variance should be defined.

The likelihood function is related to DGP and shows the probability of receiving $\Psi$ given $\chi$. To derive the likelihood, we can take into account the linearized model, assume that shocks are normally distributed and notice that its state space representation is similar to the Kalman filter:

$$s_t = Ast_{t-1} + Bsh_t, \quad (30)$$

$$st_t = Cst_{t-1} + Dsh_t, \quad (31)$$

where $A$, $B$, $C$ and $D$ are parameters, $sh_t$ is the combined set of shocks to observables, $sh_t \sim N(0,I)$. Now with either “pen and pencil” or using computer we can find the likelihood function based on the procedures developed for the Kalman filter (for more details see, for example, [7]).

Finally, we should select the optimization algorithm to determine (24). While there are other alternatives, Metropolis-Hastings (MH) algorithm is typically used for optimization and it can be represented by the outlined three steps [9, p. 93]:

0. Running the initial draw $\chi^{(0)}$ and evaluating $p(\Psi|\chi^{(0)})$ and $p(\chi^{(0)})$.

1. From the candidate generating density, $q(\chi^{(i-1)},\chi^{(*)})$, candidate draw $\chi^{(*)}$ should be taken. As a rule, random walk (RW) process is used to migrate from the previous parameters to the new ones:

$$\chi^{(*)} = \chi^{(i-1)} + o, \quad (32)$$

where $o \sim N(0,\Sigma_o)$.

2. Evaluating $p(\Psi|\chi^{(*)})$ and $p(\chi^{(*)})$.

3. Calculating acceptance probability (forces the parameters to move from the region of low posterior probability to the higher):

$$ac = \min \left( \frac{p(\Psi|\chi^{(*)})q(\chi^{(*)},\chi^{(i-1)})}{p(\Psi|\chi^{(i-1)})q(\chi^{(i-1)},\chi^{(*)})}, 1 \right) = \text{in case of RW MH} = \min \left( \frac{p(\Psi|\chi^{(*)})}{p(\Psi|\chi^{(i-1)})}, 1 \right), \quad (33)$$
The derivation of this formula, which guarantees that the resulting vector of parameters converges to the posterior, is given, for example, in [4].

4. Iterations from 1 to 3 predetermined number of times.

At the end, we should receive the posterior distributions of the parameters we wanted to estimate.

Results

We are using Dynare/Matlab to run the estimation of the model. To keep the exposition simple and easy for analysis we offer to estimate with Bayesian econometrics one parameter $\varphi$ from the equation (19), which describes the relations between price of capital, demand for investment and demand for capital. This parameter can be considered as the elasticity of investment-to-capital ratio to Tobin’s $q$ in the steady state. It determines the degree of capital adjustment costs and if $\varphi = 0$, then there are no installation costs. Bernanke et al. [2] recommend to consider for $\varphi$ the $(0,0.5)$ range. We select for $\varphi$ the normal density prior $\mathcal{N}(0.25,1)$ and truncated it at zero.

MH algorithms were replicated 20,000 times. To achieve the efficiency of MH procedure two parallel chains of estimation were launched. The first half of draws were dropped since these values may be too far from the convergence region. Average acceptance rate lies in the (0.3-0.4) region, which allows not to accept or reject too often candidate draws.

On the figure 1 we plot the prior and posterior densities.

![Figure 1. Prior and posterior densities of $\varphi$.](image)

Note: Solid gray line denotes the prior, solid black line denotes the posterior, dashed line is the posterior mode.
The results of the posterior estimation are summarized in the table 1.

**Table 1. Posterior parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>0.692</td>
</tr>
<tr>
<td>Mean</td>
<td>0.681</td>
</tr>
<tr>
<td>Median</td>
<td>0.681</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.070</td>
</tr>
</tbody>
</table>

In comparison to the prior the main difference of the posterior is its higher value of the mode and smaller variance. Such high value of the mode can be explained by high volatility of investment with respect to output and large installation costs of capital.

In addition, we test the economy for the reaction of selected variables to three types of innovations: monetary, government expenditures and technology shocks. On the figure 2, the impulse response functions to the negative unanticipated monetary innovation are shown. The reaction of output, consumption, investment, inflation and labor employed is humped-shaped. Initially the deviation from the steady state increases with the decreasing speed and after reaching the maximum, it starts decreasing also with the decreasing speed.
**Fig. 2.** Impulse response functions: impulse – monetary shock. All panels: horizontal axis – quarters, vertical axis – logarithms of deviations from the steady states.

The increase of the government expenditures has positive effect of the output, which after immediate increase slowly converges to its steady state value. The impulse has a negative influence on the consumption; however, the perturbation is relatively small in comparison to the total output. In the case of investment, we observe crowding out effect caused by increasing unanticipated government expenditures. Labor employed has similar to the output behavior, while inflation oscillates around its steady state converging to zero.

![Impulse response functions](attachment:image.png)

**Fig. 3.** Impulse response functions: impulse – government expenditures shock. All panels: horizontal axis – quarters, vertical axis – logarithms of deviations from the steady states.

Technology shock has positive effect on all variables except inflation. The effects are much more stable in comparison to the monetary and government innovations. Only inflation converges to its long-term value after 18 quarters, while
for other variables it takes more time to return to their steady states. The shock to these variables is more persistent than temporary. This is due to the assumed close to the unity value of the parameter $\rho_a = 0.99$, which provides such a picture.

![Graphs of impulse response functions showing output, consumption, investment, inflation, and labor over time.](image)

**Fig. 4.** Impulse response functions: impulse – technology shock. All panels: horizontal axis – quarters, vertical axis – logarithms of deviations from the steady states.

**Conclusions**

In this article we introduced Bayesian estimation to small-scale simple DSGE model of Ukrainian economy. As it has been already mentioned, this methodology is considered to be superior to the existing alternatives. It allows to combine prior information with the data while estimating the parameters of the linearized system of equations. However, we should mention that Bayesian methodology has its own weaknesses. First, prior information often is not well justified. In the presented model, the prior for the elasticity of investment-to-capital ratio to the Tobin’s q appeared to be far away from the resulting posterior. Second, replication of the estimation based on the Bayesian econometrics sometimes cannot be achieved due to
the probabilistic nature of MH algorithm. Third, reality requires introducing higher than first-order Taylor expansion. As a result, the model becomes non-linear, which significantly complicates the application of Bayesian methodology.

Estimation results in the form of impulse response functions can form valuable recommendations to the monetary and fiscal policy authorities. Interest rate and government expenditures can be used to smooth short-term fluctuations of the Ukrainian economy. The precise form of the reaction of output and its subelements, inflation and employment to the monetary and fiscal shocks can be helpful to the country management for determining the exact values of NBU interest rate and government expenditures which should be set to achieve some predetermined goals. One of the most important conclusion of the resulting model is the persistence of the shocks influence. Monetary and fiscal policy shocks have much shorter period of influence than technology one. The influence of the latter is relatively permanent. It means that for the stable long-term development Ukraine should pay more attention to the investment into technological progress. Other instrument should be used more intensively for short-term management.

To conclude, the model has a large potential for the further development. More equations can be added. It will make possible to model the behavior of economic agents in more details and to understand the relations between the variables deeper, in particular within the monetary and fiscal policy transmission mechanisms.

References

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БАЙЄСІВСЬКА ОЦІНКА НЕВЕЛИКОЇ МОДЕЛІ ДСЗР ЕКОНОМІКИ УКРАЇНИ

У даній статті ми розглядаємо методологію Байєсівської оцінки динамічної стохастичної моделі загальної рівноваги для економіки України. Отримані функції відгуків можуть бути корисними для підвищення ефективності монетарних та фіскальних інтервенцій. Крім того, ми показали, що технології є одним з найбільш важливих факторів, що впливають на стабільний довгостроковий ріст економіки України.

Ключові слова: модель ДСЗР, Байєсівська оцінка, монетарна та фіскальна політика.