Too much of a good thing? on the effects of limiting foreign reserve accumulation

Yan, Isabel K. and Kumhof, Michael

City University of Hong Kong, International Monetary Fund

September 2011

Too Much of a Good Thing? On the Effects of Limiting Foreign Reserve Accumulation*

Michael Kumhof, International Monetary Fund
Isabel Yan, City University of Hong Kong

Abstract

Some emerging economies have recently experienced large government surpluses and accelerating foreign exchange reserve accumulation far in excess of what would be implied by the literature on optimal reserves. China in particular has repeatedly stressed that there may be an upper limit to how many reserves it is willing to hold. Using a dynamic general equilibrium model, we show that the credible expectation of such a limit would lead to a balance of payments anti-crisis, which is characterized by an economic boom, real appreciation, growing demand for domestic currency, and domestic inflation, in the period prior to the limit being reached.

Keywords: Balance of payments anti-crises; foreign exchange reserves; foreign exchange intervention; inflation targeting; exchange rate targeting.

JEL Classification: F41, E52, E58, E63.

* Corresponding Author: Isabel Yan, Department of Economics and Finance, The City University of Hong Kong, Tat Chee Avenue, Hong Kong, email: efyan@cityu.edu.hk. Michael Kumhof, International Monetary Fund, Research Department/Modeling Unit, Washington, DC 20431, Tel.: 202 - 623 6769, Fax: 202 - 623 8291, email: mkumhof@imf.org.

There are surely limits on the tolerance of foreign investors for increased claims on the United States.[......]International debt accumulation at these rates cannot go on forever.[......]As a matter of arithmetic, any reduction in the U.S. current account deficit must be matched by reductions in current account surpluses elsewhere.

Lawrence H. Summers, Per Jacobsson Lecture, 2004.

1 Introduction

The recent debate about the desirability of a new world monetary system[1] has been motivated in large part by the perception that the existing system has contributed to the development of large current account imbalances that are costly to the surplus countries and therefore ultimately unsustainable. As discussed in Cook and Yetman (2011), Adams and Park (2009), Park (2007) and Dooley et al. (2004) in the context of developing Asia[2], part of their cost is due to the fact that the current account surpluses have led to explosive central bank foreign exchange accumulation, to the point that reserves now exceed, often by a wide margin, what is required for liquidity purposes or as an insurance device against sudden reversals in capital inflows (Flood and Marion (2002), Jeanne and Rancière (2006), Rodrik (2006), Summers (2006), Jeanne (2007), Alfaro and Kanczuk (2009) and Obstfeld, Shambaugh and Taylor (2010)).[3] As the reserve assets are low-yielding, with insufficient alternatives available on the required scale, this has large opportunity costs for the surplus countries. Rodrik (2006)

[1] Calls for changes have been made by, among others, Russia (BBC, 2009) and China (Bloomberg, 2009).

[2] Other countries have also experienced a massive increase in foreign exchange reserves since the late 1990s (Rodrik, 2006).

[3] Obstfeld, Shambaugh and Taylor (2010) point out that the fear of a sudden flight from domestic to foreign assets implies that the demand for international reserves should be proportional to M2, and that this can explain observed reserves in many emerging markets well during their sample, which ends in 2004. However, they find that in the last years of their sample a substantial fraction of China's reserves was left unexplained. Since that time Chinese reserves have more than quadrupled.

estimates the cost of reserves as the spread between the cost of private sector short-term foreign borrowing and the yield on the Central Bank's holdings of liquid foreign assets. This spread equals several percentage points in normal times, resulting in a social cost of reserve accumulation of close to 1 percent of GDP for many developing countries. Similarly, Hauner (2005) estimates an opportunity cost of international reserves of 0.2-0.6 percent of GDP in emerging countries.

In light of these difficulties, some central banks are looking for policy alternatives that curtail reserve accumulation and that rebalance demand and growth towards domestic sources.[4] The policy relevance of this concern with reserve levels is vividly illustrated by the recent experiences of China.

On March 20, 2007 China's central bank governor announced that his country would stop accumulating foreign exchange reserves. He was quoted in Reuters (2007) as stating "foreign exchange reserves in China are large enough. We do not intend to go further and accumulate reserves." As shown in Figure 1, this statement followed several years of very large and accelerating Chinese reserve gains. The central bank did not follow through on the statement and kept accumulating reserves throughout 2007. But in all subsequent debates this option has never been completely off the table. In 2009, as illustrated in Figure 2, China's reserve accumulation flattened out temporarily, with the country cutting its holdings of U.S. treasury securities by the sharpest amount in a decade around the end of 2009.[5] More recently, in an April 2011 speech at Tsinghua University, the central bank governor reiterated that China is making strong efforts to scale back its foreign exchange reserves, as over-accumulation of reserves leads to excess liquidity. In his view, China's reserves have already surpassed

[4] This need for a rebalancing of demand has also been stressed in IMF's policy advice in recent years. See International Monetary Fund (2005, 2009) for examples, where the latter mentions the need for emerging Asia to rebalance its demand towards domestic sources.

[5] See South China Morning Post (Feb 18, 2010).

the reasonable needs of China.[6]

This reflects an official concern that an existing policy regime may not be sustainable because it involves the accumulation of too much foreign exchange, certainly well in excess of anything that could be deemed socially optimal, either by the literature on optimal reserves or by policymakers' own assessments. At some point in time, which apparently has not yet been reached by China, the concern of the authorities may lead to stronger action in the form of decisive policies that impose an upper bound on reserves, and the public may then come to believe that these policies are indeed credible. The question, on which there has so far been very little work in the literature, is what this would imply for macroeconomic variables. This paper attempts to answer that question.

We suggest that it is useful to examine the theoretical implications of policy responses to excessive reserve accumulation through the lens of a familiar literature - the literature on first-generation balance of payments crises following Krugman (1979), Calvo (1987), the survey in Calvo and Vegh (1999), and Kumhof, Li and Yan (2007), which studies balance of payments crises under inflation targeting regimes. The critical difference between our paper and that literature is that in the latter the concern is with countries that experience the consequences of a central bank owning *too little* foreign exchange while our paper studies countries that accumulate what they perceive to be *too much* foreign exchange.

What these countries could then experience as reserves approach an upper limit is what we will refer to as a balance of payments *anti-crisis*. We will discuss that the upper limit is not arbitrary because the maximum possible limit is determined by the zero lower bound on nominal interest rates. We adopt the terminology anti-crisis because in terms of the dynamics of foreign

[6] See the Chinese version of the Wall Street Journal (April 19, 2011), available at http://chinese.wsj.com/BIG5/20110419/bch071424.asp.

exchange reserves it represents the opposite of a traditional balance of payments crisis. But it nevertheless represents a crisis in that it ends in the collapse of the previous monetary regime. This however is accompanied by another outcome that is desired by the authorities, a rebalancing of demand towards domestic sources.

In this paper we focus on the effects of such anti-crises on the domestic economy. We therefore study this problem in the context of a small open economy, without emphasizing the global repercussions. We assume that a central bank, similar to the Chinese announcements of 2007 and 2011, declares that it will continue accumulating reserves only up to an upper limit that is not too far above the existing level.[7] To account for the fact that the countries concerned may be pursuing a variety of different monetary regimes, we examine the cases of exchange rate targeting, CPI inflation targeting and domestic (nontradables) inflation targeting.

We assume that the event that ultimately leads to an anti-crisis is a favorable shock to the government budget. Specifically, we assume that the government starts to receive an additional tradables endowment equal to 1% of GDP, which leads to exponential growth in reserves.[8] This could for example represent the favorable fiscal consequences of higher productivity in the tradables sector. The policy response analyzed in this paper resembles the Chinese announcements of 2007 and 2011 - a government announcement that it will stop reserve accumulation at some maximum, regardless of the consequences for the sustainability of the monetary regime.

[7] In terms of describing pre-crisis dynamics, it is in fact sufficient to assume that markets perceive this intention or necessity on the part of the central bank. Adding uncertainty about the upper limit would change some details but not the main features of pre-crisis dynamics.

[8] According to the official data, fiscal surpluses were not the main driver of reserve accumulation for China. However, the official data may be misleading due to, for example, the manner in which they account for public enterprises. In a World Bank empirical study, Kuijs (2005) finds that "(The Chinese) Government saving is remarkably high compared to other countries, and is much higher than suggested by the headline fiscal data."

We show that this is enough to balance the budget endogenously if reserves are not already too large. Under inflation targeting, the announcement causes downward pressure on exchange rate depreciation, with goods and money demand increasing in an accelerating fashion due to the resulting reduction in inflationary distortions. The increase in real money demand, to the extent that it is accommodated by nominal money issuance in exchange for foreign currency, causes a final burst of reserve accumulation, the anti-crisis. Under inflation targeting this crisis is continuous while under exchange rate targeting it is instantaneous. The reserve gains are fastest, and the anti-crisis therefore happens earliest, under monetary regimes that imply the strongest commitment to intervene in foreign exchange markets by issuing money against foreign exchange, and therefore the strongest commitment against letting the nominal exchange rate appreciate. The ranking of regimes in terms of reserve gains is therefore, from fastest to slowest, exchange rate targeting followed by CPI inflation targeting and domestic inflation targeting.

For the government the effect of lower exchange rate depreciation is a reduction in seigniorage income that balances the budget. The point at which this happens is not arbitrary, because by uncovered interest parity exchange rate depreciation cannot drop arbitrarily low without violating the zero lower bound on nominal interest rates. We will therefore assume that the upper limit for reserves announced by the government is just below the maximum level of reserves at which lower seigniorage can still balance the budget.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 discusses model calibration and the solution algorithm. Section 4 discusses the dynamics of balance of payments anti-crises. Section 5 concludes. Technical details and a description of the solution algorithm are contained in the Technical Appendix accompanying the paper.

2 The Model

The economy consists of a government, a representative household, and representative tradables and nontradables producing firms. Real interest rates and international goods prices are exogenous and constant, and the latter are normalized to one. Purchasing power parity holds for tradable goods, while nontradables prices are flexible.

2.1 Households

Households maximize lifetime utility derived from their consumption of tradable goods c_t^T, nontradable goods c_t^N, and leisure $1 - h_t$, where 1 is the time endowment and h_t is hours worked or labor supply. The CES consumption aggregator is c_t, the elasticity of substitution between c_t^T and c_t^N is denoted by σ, and the quasi-share parameter of tradables is η. The personal discount rate is assumed to equal the constant real international interest rate r. We have the optimization problem

$$Max \quad \int_0^\infty \left[\gamma \ln c_t + (1-\gamma)\ln(1-h_t)\right] e^{-rt} dt \ , \tag{1}$$

$$c_t = \left(\eta^{\frac{1}{\sigma}} \left(c_t^T\right)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}} \left(c_t^N\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \ . \tag{2}$$

Labor is remunerated at the real wage rate $w_t = W_t/E_t$, where W_t is the nominal wage rate and E_t is the nominal exchange rate. Wages are equalized across the two sectors. Households own fixed capital stocks k^T and k^N with real returns $r_t^{k^T}$ and $r_t^{k^N}$. They also receive lump-sum transfers g_t from the government. Financial assets include nominal domestic currency money balances M_t, real international bonds b_t with constant real return r, and domestic currency government bonds Q_t, for which there is complete home bias, with nominal return i_t. The no-arbitrage condition between domestic and foreign currency denominated bonds, or uncovered interest parity, is given by

$$i_t = r + \varepsilon_t \ , \tag{3}$$

where $\varepsilon_t = \dot{E}_t / E_t$. Foreign and domestic currency denominated bonds are therefore perfect substitutes, and we can simplify further by assuming that domestic government bonds are in zero net supply at all times. Total real financial assets are then given by $a_t = b_t + m_t$, where $m_t = M_t / E_t$. After imposing the transversality condition $\lim_{t \to \infty} a_t e^{-rt} \geq 0$, households' lifetime budget constraint can be written as

$$a_0 + \int_0^\infty \left(w_t h_t + r_t^{k^T} k^T + r_t^{k^N} k^N + g_t \right) e^{-rt} dt \geq \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt \ , \tag{4}$$

where $e_t = E_t / P_t^N$ is the relative price of tradables, and P_t^N is the nominal price of nontradables. There is a cash-in-advance constraint on consumption

$$m_t \geq \alpha \left(c_t^T + \frac{c_t^N}{e_t} \right) \ . \tag{5}$$

We will assume and later verify that this constraint holds with equality at all times, which must be true as long as $i_t > 0 \ \forall t$. The household maximizes (1) and (2) subject to (4) and (5), taking as given $\left\{ w_t, r_t^{k^T}, r_t^{k^N}, g_t, E_t, P_t^N \right\}_{t=0}^\infty$. The multiplier of the lifetime budget constraint (4) is given by λ. Then the optimality conditions are (4) and (5) holding with equality, and the following first-order conditions:

$$c_t^T = \left(\frac{\eta}{1-\eta} \right) \left(\frac{c_t^N}{e_t^\sigma} \right) \ , \tag{6}$$

$$\frac{1-\gamma}{1-h_t} = \lambda w_t \ . \tag{7}$$

We can also derive the following condition relating aggregate consumption C_t to aggregate labor supply h_t (see the Technical Appendix):

$$w_t^{cpi} = \frac{W_t}{P_t} = \frac{1-\gamma}{\gamma} \frac{C_t (1 + \alpha i_t)}{1 - h_t} \ . \tag{8}$$

The consumption based price index P_t is given by

$$P_t = \left(\eta \left(E_t \right)^{1-\sigma} + (1-\eta) \left(P_t^N \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{9}$$

Letting $\pi_t = \dot{P}_t / P_t$ and $\pi_t^N = \dot{P}_t^N / P_t^N$ we also have the following relationship between inflation rates:

$$\pi_t \left(\eta e_t^{1-\sigma} + (1-\eta) \right) = \eta e_t^{1-\sigma} \varepsilon_t + (1-\eta) \pi_t^N . \tag{10}$$

2.2 Firms

The production functions of tradables and nontradables manufacturing firms are given by

$$y_t^T = \left(k^T \right)^{\rho^T} \left(h_t^T \right)^{1-\rho^T} , \tag{11}$$

$$y_t^N = \left(k^N \right)^{\rho^N} \left(h_t^N \right)^{1-\rho^N} , \tag{12}$$

where h_t^T and h_t^N are the respective labor inputs. Profit maximization implies standard first order conditions for labor.

2.3 Government

The government receives a flow endowment of tradable goods $\{d_t\}_{t=0}^{\infty}$ that is normalized to zero in the initial steady state. Government policy consists of a specification of the path of lump-sum transfers $\{g_t\}_{t=0}^{\infty}$ and of a monetary policy rule. For the latter we consider exchange rate targeting, CPI inflation targeting and domestic inflation targeting. The initial target inflation rates under the respective regimes are denoted by $\bar{\varepsilon}$, $\bar{\pi}$ and $\bar{\pi}^N$. These target growth rates are assumed to have been consistent with fiscal solvency under a previous path of endowments and transfers. However, after the arrival of information at time 0 about a more favorable path of endowments $\{d_t\}_{t=0}^{\infty}$, they become too high to prevent ongoing foreign exchange reserve accumulation that would be unbounded without an ultimate change of policy. The eventual steady state target nominal growth rates will be determined by a balanced budget requirement for the government.

For each of the three monetary regimes, we assume that central bank accommodation of changes in real money demand at time 0 ensures a smooth path of the targeted price variable. Exchange rate targeting is therefore fully defined by a continuous target path for the nominal anchor:

$$E_t = E_0 e^{\bar{\varepsilon} t} \; . \tag{13}$$

For inflation targeting, we follow the literature in assuming that the central bank follows a nominal interest rate rule that responds to deviations of inflation from its target. But we replace inflation deviations with deviations of the price path from its targeted path. This avoids indeterminacy under flexible prices. The target price paths $\tilde{P}_t$ and $\tilde{P}_t^N$ are formulated as in (13). Furthermore, the nominal interest rate is raised one for one with the current rate of exchange rate depreciation. For CPI inflation targeting, we therefore have the rule

$$i_t^P = r + \varepsilon_t + \xi^P (P_t - \tilde{P}_t) \; , \; \xi^P > 0 \; , \tag{14}$$

$$\tilde{P}_t = P_0 e^{\bar{\pi} t} \; , \tag{15}$$

and similarly for domestic inflation targeting

$$i_t^{P_N} = r + \varepsilon_t + \xi^{P_N} (P_t^N - \tilde{P}_t^N) \; , \; \xi^{P_N} > 0 \; , \tag{16}$$

$$\tilde{P}_t^N = P_0^N e^{\bar{\pi}^N t} \; . \tag{17}$$

Let x_t be the government's foreign exchange reserves. Then its budget constraint is

$$\dot{x}_t = r x_t + \dot{m}_t + \varepsilon_t m_t + d_t - g_t = r x_t + \mu_t m_t + d_t - g_t \; , \tag{18}$$

where $\mu_t m_t$ is the amount of seigniorage collected by the central bank. At times of discrete upward jumps in real money balances between t^- and t, we can decompose that jump as $\Delta m_t = \Delta m_t^M + \Delta m_t^E > 0$. Here Δm_t^M are jumps in real money balances that are due to jumps in nominal money balances, that is, $\Delta m_t^M = (M_t - M_{t-})/E_t$, while Δm_t^E are jumps in real money balances that are due to downward jumps in the nominal exchange rate, that is, $\Delta m_t^E = M_{t-}(1/E_t - 1/E_{t-})$. The former are associated with central bank acquisition of foreign exchange, so that $x_t - x_{t-} = \Delta m_t^M$, but the latter are not associated with changes in foreign exchange reserves. There is a similar decomposition for continuous increases in real money balances $\dot{m}_t = \mu_t m_t - \varepsilon_t m_t > 0$ around the time of the anti-crisis. Real money balances can increase either through an increase of the nominal money supply ($\mu_t m_t > 0$) or through nominal exchange rate appreciation alone ($\varepsilon_t m_t < 0$). These two effects can be shown to be substitutes – ceteris paribus, a faster expansion in nominal money balances slows down the appreciation of the nominal exchange rate.

The government may at any time announce a maximum level of foreign exchange reserves beyond which it will stop reserve accumulation and instead allow the rate of exchange rate depreciation to adjust. This maximum level is not relevant before time 0, when the government budget is balanced. But when the fiscal revenue shock hits at time 0 we assume that an upper limit for reserves $\bar{x}$ is part of the announced government policy package. We have

$$x_t \leq \bar{x} \quad \forall t \ . \tag{19}$$

In addition we impose the transversality condition $\lim_{t\to\infty} (x_t - m_t) e^{-rt} = 0$ to obtain the government's infinite horizon budget constraint from (18) as follows:

$$x_0 + \int_0^\infty (i_t m_t + d_t - g_t) e^{-rt} dt = m_0 \ . \tag{20}$$

2.4 Equilibrium

In equilibrium, households and firms maximize their objective functions, the government follows the policy rules set out in the previous subsection, and labor and goods markets clear:

$$h_t = h_t^T + h_t^N \ , \tag{21}$$

$$c_t^N = y_t^N \ . \tag{22}$$

Combining (4) and (20), and denoting the economy's overall net foreign assets by $f_t = b_t + x_t$, the economy's overall resource constraint can then be derived as

$$f_0 + \int_0^\infty \left(y_t^T + d_t\right) e^{-rt} dt = \int_0^\infty c_t^T e^{-rt} dt \ , \tag{23}$$

with current account

$$\dot{f}_t = r f_t + y_t^T + d_t - c_t^T \ . \tag{24}$$

Furthermore, for the two inflation targeting regimes, in equilibrium it must be true that $i_t^P = i_t$ and $i_t^{P_N} = i_t$. Together with the uncovered interest parity condition (3) this implies

$$P_t = P_0 e^{\bar{\pi} t} \ , \tag{25}$$

$$P_t^N = P_0^N e^{\bar{\pi}^N t} \ . \tag{26}$$

These are analogous to equation (13) for exchange rate targeting, and amount to exact price level targeting.

2.5 Government Revenue Shock

Assume that the economy is in an initial steady state (subscript ss) with constant net foreign assets f_{ss}, foreign exchange reserves x_{ss}, endowment flow d_{ss}, and with a balanced budget. In this steady state all rates of price change are equal to the initial target growth rate of the nominal anchor. We assume that $f_{ss} = 0$ and $\varepsilon_{ss} = \pi_{ss} = \pi_{ss}^N = 0$. Therefore the budget is simply

$$g_{ss} - d_{ss} = rx_{ss} \ . \tag{27}$$

For simplicity we assume that $d_{ss} = 0$. Now assume that the government experiences a permanent increase in its endowment flow from d_{ss} to $\bar{d}$ at time 0, with $\bar{g} = g_{ss}$, but that it keeps the target growth rate of its nominal anchor at 0 under all three monetary regimes. We therefore have $\bar{g} - \bar{d} < rx_{ss}$. By (18) this generates an accumulation of foreign exchange reserves that would ultimately be unbounded in the absence of the constraint (19). The constraint therefore becomes binding within finite time τ. At that time the monetary regime collapses in an anti-crisis, and the economy reaches its final (subscript τ) steady state. The time τ is endogenous. Given that the constraint (19) is binding for all $t \geq \tau$, the budget must be balanced through lower seigniorage income from that time onwards,

$$\bar{g} - \bar{d} = r\bar{x} + \varepsilon_\tau m_\tau \ , \tag{28}$$

where $\varepsilon_\tau < 0$. It is shown in the Technical Appendix that under CPI and domestic inflation targeting ε_t must be continuous for all $t > 0$ including τ.

As in the traditional balance-of-payments crisis literature, the crisis is made inevitable by the inflexibility of other fiscal instruments, in this case an inability to change transfers sufficiently to offset the change in the endowment, combined with an exogenous limit on reserves. In both cases it is recognized that these assumptions are ad-hoc, but that at the same time they may be realistic as a description of certain countries, and that the model should therefore be judged on whether it provides useful predictions for the behavior of key variables. For the traditional literature this has certainly been the case. For the mechanism outlined in our paper the same could come to happen if the situation in China ever develops to the point where the policy announcements concerning reserves are followed by decisive policy action that makes them credible in the eyes of the public. As in our model, China has been unable to stop the accumulation of reserves through the use of other fiscal instruments, as evidenced by continued albeit slowed reserve growth in 2009 despite an unprecedented fiscal stimulus package in the same year.

3 Model Solution

3.1 Parameter Values

Where available, parameters are calibrated based on Chinese data. Other parameter values are assigned based on the literature for developing countries. The time unit for calibration of stock-flow ratios is a quarter.

As is common in the monetary business cycle literature, we assume a value of 3% per annum (p.a.) for the real international interest rate r. Given our assumption of zero initial inflation, the nominal interest rate i_{ss} therefore also equals 3% p.a. The inverse velocity α is set equal to the average ratio of the real monetary base to quarterly output in China over the period 2000Q1 through 2007Q4, implying $\alpha = 1.55$. The quasi-share parameter for tradables consumption η is set equal to $\eta = 0.5$, while the elasticity of substitution between tradables and nontradables is $\sigma = 0.5$, based on the evidence discussed in Mendoza (2005). Several of the remaining parameters are calibrated based on a normalization of output and asset stocks in the initial steady state. We normalize $f_{ss} = 0$, $y^*_{ss} = c^*_{ss} = 1$ and $y_{ss} = c_{ss} = 1$. By (6), and given our choice of $\eta = 0.5$, this implies an initial relative price of tradables $e_{ss} = 1$. This in turn implies that the initial share of tradables in consumption is equal to 0.5 in our baseline. The proportion of time spent working in the initial steady state is $h_{ss} = 1/3$. Labor income shares are assumed to equal 60%, by setting $\rho^T = \rho^N = 0.4$. As for the price variables, the initial CPI price level is normalized to one, $P_{ss} = 1$.

Then the formula for the CPI determines the price levels P_N and E, and the cash-in-advance constraint determines the levels of real and nominal balances.

Initial inflation rates are set to zero, and initial central bank net foreign exchange reserves x_{ss} are set equal to China's average ratio of net foreign exchange reserves to annual output in 2006, which equals 22%. Because initial annual output equals 8, this implies $x_{ss} = 0.22 \times 8 = 1.76$. Government transfers are set equal to the interest earnings on these reserves so as to balance the budget by (27), which requires $g_{ss} = 0.0132$. The new and permanently higher government tradables endowment is assumed to be $\bar{d} = 0.02$, which equals 1% of overall output and 2% of tradables output. Finally, the upper limit on foreign exchange reserves is fixed at 24% of initial annual output, or $\bar{x} = 1.92$. This means that the higher endowment alone would take reserves to their upper limit within eight quarters. In practice the limit will be reached faster because of a combination of compound interest and increases in money demand at time 0 and during the anti-crisis.

3.2 Solution Method

To compute the paths of all variables we adopt a nested shooting algorithm for the CPI and domestic inflation targeting cases, because these cases involve complicated transitions to a new steady state. The general strategy is to iterate over the marginal value of lifetime wealth λ and the initial exchange rate jump ε_0 to ensure that, given the policy announced at time 0, equilibrium paths satisfy both the economy's overall resource constraint (23) and the government's lifetime budget constraint (20), the latter combined with the upper bound on foreign exchange reserves (19). The steps of the algorithm are described in detail in the Technical Appendix.[9]

[9] Computation of the exchange rate targeting case is much simpler, as it involves simple step paths for all variables. Details are also provided in the Technical Appendix.

4 Anti-Crises

Figure 3 presents solution paths for our policy experiment, with broken lines denoting exchange rate targeting (ET), solid lines denoting CPI inflation targeting (CPIT), and dotted lines denoting domestic inflation targeting (DIT). For most variables the plots show percent deviations from their initial steady state values, except for ratios to GDP, which are shown in percentage point deviations from their original steady state values, interest and inflation rates, which are shown in percent per annum, and foreign exchange reserves, which are shown in levels.

The three monetary regimes share a number of features. At time 0, households learn that the government will receive a permanently higher endowment of tradables equal to 1% of the initial GDP. They know through the policy announcement that this windfall will not be shared with them immediately by way of higher transfers, but instead will be shared over their lifetimes by way of lower inflation, after an initial period in which the government saves the extra revenue without a significant reduction in inflation.

This pattern, as seen in Figure 3a, has two effects on household behavior. First, there is an immediate positive wealth effect as households anticipate future benefits by consuming more and working less today. Second, when inflation eventually does decline, households increase consumption even further. But this is due to a reduction in inflationary distortions rather than a further wealth effect. This means that at that time, to satisfy the extra demand, labor supply actually rises to slightly above its original level. In the long run, both consumption and labor are therefore above their initial steady state values. This reflects not only the positive endowment shock and the foreign asset accumulation experienced during the transition, but also the smaller inflationary distortions in the new steady state. The end result is a rebalancing of demand towards domestic sources rather than exports, which is reflected in a closing of the current account surplus gap. This type of rebalancing is exactly what IMF policy advice in recent years has been recommending to China, see footnote 4.

The details of the dynamics of anti-crises under the three monetary regimes differ in that they happen instantaneously under exchange rate targeting, while under inflation targeting they happen continuously over a period of one to two quarters. This difference is best understood in terms of the time profiles of the inflationary distortions. By equations (8) and (3), the key variable is the rate of exchange rate depreciation ε_t. Under exchange rate targeting this is held constant by the central bank, both before and, at a different level, after τ. There is therefore a discrete reduction in distortions and thus an increase in real variables at time τ. Under inflation targeting, ε_t becomes endogenous and, as shown in the Technical Appendix, continuous after time 0. Inflationary distortions are therefore allowed to increase in a gradual fashion before time τ. As a result, all real variables approach their post-crisis values in a continuous fashion.

Both consumption and production patterns depend on the evolution of the relative price of tradables e, which falls both initially and again later during the anti-crisis. The reason is that households now demand more of all goods, but unlike tradables, nontradables are not in perfectly elastic supply. This drives up their relative price, which helps to stimulate additional production as real wages fall in terms of nontradables and rise in terms of tradables. This causes employment to move to the nontradables sector, as illustrated in Figure 3b. The final outcome is a Dutch disease phenomenon, as non-endowment tradables output falls by about one third of the increase in the tradables endowment (0.7% versus 2% of initial tradables output).

Comparing tradables consumption and output in Figure 3a, we observe that the initial gap between the two equals less than half of the increase in the endowment. This is because prior to the anti-crisis the endowment supports mostly additional reserve accumulation rather than additional consumption. As a result the country at this stage starts to run a current account surplus equal to around 0.4% of GDP. But during the anti-crisis consumption increases sharply as inflationary distortions are eliminated, and in the final steady state the current account is again balanced. At this stage the gap between tradables consumption and output equals slightly over 2% of GDP, the size of the endowment increase plus interest on the accumulated reserves.

Figure 3c shows how this pattern is reflected in the government budget and in its stock of foreign exchange reserves. The government gains reserves that ultimately hit the upper bound for several reasons. Firstly, there is of course the increase in endowment income. But secondly, money demand also increases, both on impact and then again during the anti-crisis. By the cash-in-advance constraint this is due to an increase in consumption, and the amount of that increase is nearly independent of the monetary regime.

The extent to which this causes reserves to increase is however a function of the monetary regime, as discussed above in the paragraph following equation (18). The main differences between regimes consist of the degree of commitment to let the money supply expand, or of the degree of commitment against letting the exchange rate appreciate, in order to defend the target of monetary policy. A monetary regime that intervenes less, and lets the exchange rate appreciate earlier and more, thereby collecting a lower inflation tax before the anti-crisis, experiences smaller reserve gains during the entire transition to the eventual collapse. Its anti-crisis therefore takes place later. The differences between nominal money issuance versus nominal exchange rate appreciation between the three monetary regimes can be seen clearly in Figure 3d. We observe that intervention is weakest under domestic inflation targeting, intermediate under CPI inflation targeting, and strongest under exchange rate targeting.

We start our discussion with domestic inflation targeting. Here the commitment against letting the exchange rate appreciate is weakest. The successive reductions in the relative price of tradables are therefore accomplished by an appreciation of the nominal exchange rate rather than by an increase in the nontradables price. This raises the real value of existing money balances sufficiently to require much smaller additional money issuance and therefore reserve gains to satisfy the successive increases in money demand. This is true both at the outset and during the anti-crisis.

Under CPI inflation targeting, the commitment to intervene is stronger and reserve gains are larger. The reason is that, while permitting exchange rate appreciation under domestic inflation targeting does not directly affect the targeted inflation rate, the same appreciation does lead to a negative deviation from a CPI inflation target. A further increase of the money supply, via central bank purchases of foreign exchange, is therefore required to induce nontradables inflation and to limit exchange rate appreciation.

Under exchange rate targeting, the central bank's commitment to intervene is strongest, given its complete commitment against letting the exchange rate appreciate. Therefore the entire increase in money demand, both initially and during the anti-crisis, leads to instantaneous stock money issuance against a stock acquisition of foreign exchange reserves, $\Delta m = \Delta m^M$. As a result, the upper limit on reserves is reached most quickly, after 5.5 quarters, a full 1.5 quarters earlier than under domestic inflation targeting.

The bottom half of Figure 3c shows the flow budgetary implications of anti-crises. We note that the seigniorage gains under domestic inflation targeting are indeed very much smaller than under CPI inflation targeting, because additional money issuance $\dot{m}$ is barely sufficient to offset the drop in the inflation tax εm. The final steady state is characterized by a reduction in the seigniorage to GDP ratio by slightly more than one percentage point. This is of course precisely what is required to offset the one percentage point (of initial GDP) higher endowment income and the increased interest income on a two percentage point (of initial GDP) larger stock of reserves.

Finally we turn to Figure 3d for a closer look at the dynamics of inflation. The first observation is that exchange rate depreciation comes to a halt at -2.7%, meaning that the nominal interest rate, shown in Figure 3a, reaches a new steady state of 0.3%, just above its zero lower bound. This emphasizes that the upper reserve limit announced by the government cannot be arbitrarily large. The zero lower bound on nominal interest rates imposes an upper limit on foreign exchange reserves, because once reserves get beyond that limit, even the largest feasible reduction in seigniorage will not stabilize government asset dynamics. We have calibrated the example on the realistic premise that a monetary authority would not want to drive its nominal interest rate all the way down to zero.

The second observation pertaining to Figure 3d concerns the behavior of nontradables inflation under CPI inflation targeting. In the monetary business cycle literature it is generally argued that nontradables inflation targeting has advantages over CPI inflation targeting in terms of stabilizing the business cycle. But in practice some version of CPI inflation targeting is still the most widely used monetary regime. As discussed above, under this regime the central bank fights exchange rate appreciation prior to time τ through higher money growth that induces nontradables inflation. Therefore, as long as central banks can be described as following, at least implicitly, a version of CPI inflation targeting, an increase in the domestic component of inflation would be consistent with an incipient balance of payments anti-crisis.

5 Conclusion

This paper addresses a relatively new phenomenon for many emerging markets, a concern either by the government or by financial markets with excessive rather than insufficient government foreign exchange reserves, often accompanied by exchange rate appreciation and higher domestic inflation. The paper explores the nature of the underlying fiscal problem and its dependence on the monetary regime.

We show that one way to understand these episodes is by looking at them through the lens of the literature on first-generation balance-of-payments crises. The major difference is that the crises, or rather the anti-crises, that concern us feature an upper rather than a lower limit on foreign exchange reserves. Moreover, this upper limit is not arbitrary but rather depends on the ability of seigniorage adjustments to stabilize asset dynamics. That ability encounters its natural limit in the desire of the monetary authority to stay away from the zero lower bound on nominal interest rates.

We have shown that the end-phase of such anti-crises is characterized by an economic boom accompanied by a further acceleration in the accumulation of foreign exchange reserves, and by nominal interest rates approaching their zero lower bound. While exchange rate depreciation drops sharply at that time, domestic inflation can rise sharply if the government targets CPI inflation. Monetary regimes that imply the strongest commitment to intervene in the foreign exchange market to prevent exchange rate appreciation, such as exchange rate targeting and to a lesser extent CPI inflation targeting, experience the most rapid reserve accumulation and therefore the quickest onset of the anti-crisis.

References

[1] Adams, C. and Park, D. (2009), "Causes and Consequences of Global Imbalances: Perspective from Developing Asia", *Asian Development Review*, **26(1)**, 19-47.

[2] Alfaro, L. and Kanczuk, F. (2009), "Optimal Reserve Management and Sovereign Debt", *Journal of International Economics*, **77**, 23-36.

[3] BBC (2009), "Putin Urges Reserve Currency Move", January, available at http://news.bbc.co.uk/2/hi/business/davos/7857005.stm.

[4] Bloomberg (2009), "BRIC Reserve Currency Plans Won't Undermine Dollar, RBC Says", July, available at http://www.bloomberg.com/apps/news?pid=20601086&sid=aOSpiNsVYSnU.

[5] Calvo, G.A. (1987), "Balance of Payments Crises in a Cash-in-Advance Economy", *Journal of Money, Credit and Banking*, **19(1)**, 19-32.

[6] Calvo, G.A., Vegh, C.A. (1999), "Inflation Stabilization and BOP Crises in Developing Countries", Ch. 24 in: J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Volume 1C. Elsevier, Amsterdam, North Holland.

[7] Cook, D. and Yetman, J. (2011), "Expanding Central Bank Balance Sheets in Emerging Asia: A Compendium of Risks and Some Evidence", Working Paper.

[8] Dooley, M., Folkerts-Landau, D. and Garber, P. (2004), "The Revised Bretton Woods System: The Effects of Periphery Intervention and Reserve Management on Interest Rates and Exchange Rates in Center Countries", NBER Working Paper No. 10332.

[9] Flood, R. and Marion, N. (2002), "Holding International Reserves in an Era of High Capital Mobility", IMF Working Papers, WP/02/62.

[10] Hauner, D. (2005), "A Fiscal Price Tag for International Reserves", *International Finance*, **9(2)**, 169-195.

[11] International Monetary Fund (2005), "Globalization and External Imbalances", Chapter III in April 2005 World Economic Outlook.

[12] International Monetary Fund (2009), "Global Economy Beyond the Crisis—Challenges Over the Medium Term", Meeting of G-20 Deputies, September 3, 2009, London, U.K, available at http://www.imf.org/external/np/g20/090309.htm.

[13] Jeanne, O. and Rancière, R. (2006), "The Optimal Level of International Reserves for Emerging Market Countries: Formulas and Applications", IMF Working Papers, WP/06/229.

[14] Jeanne, O. (2007), "International Reserves in Emerging Market Countries: Too Much of a Good Thing?", *Brookings Papers on Economic Activity*, **38(1)**, 1-55.

[15] Krugman, P. (1979), "A Model of Balance-of-Payments Crises", *Journal of Money, Credit and Banking*, **11(3)**, 311-325.

[16] Kuijs, L. (2005), "Investment and Saving in China", World Bank Policy Research Working Paper No. 3633.

[17] Kumhof, M., Li, S. and Yan, I. (2007), "Balance of Payments Crises under Inflation Targeting", *Journal of International Economics*, **72(1)**, 242-264.

[18] Mendoza, E. (2005), "Real Exchange Rate Volatility and the Price of Nontradables in Sudden-Stop-Prone Economies", NBER Working Paper No. 11691.

[19] Obstfeld, M., Shambaugh, J. and Taylor, A. (2010), "Financial Stability, the Trilemma, and International Reserves", *American Economic Journal — Macroeconomics* (forthcoming).

[20] Park, D. (2007), "Beyond Liquidity: New Uses for Developing Asia's Foreign Exchange Reserves", Economics and Research Department Working Paper No. 109, Asian Development Bank, Manila.

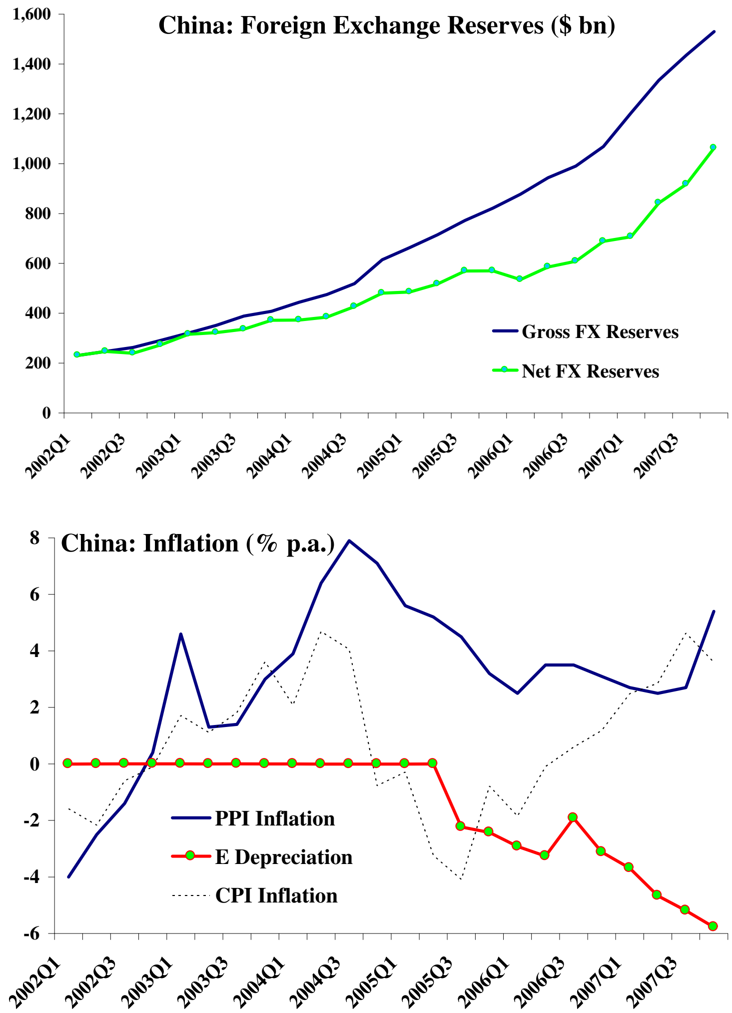
[21] Reuters (2007), "Update 1 - China to Stop Accumulating Foreign Reserves - Zhou", available at http://www.reuters.com/bondsNews/idUSN2035119120070320.

[22] Rodrik, D. (2006), "The Social Cost of Foreign Exchange Reserves", *International Economic Journal*, **20(3)**, 253-266.

[23] South China Morning Post (2010), "China Sells US$ 34bn in US Treasuries", February 18.

[24] Summers, L. (2004), "The U.S. Current Account Deficit and the Global Economy", Per Jacobsson Lecture.

[25] — (2006), "Reflections on Global Account Imbalances and Emerging Markets Reserves Accumulation", L. K. Jha Memorial Lecture, Reserve Bank of India, Mumbai, March 24.



Note: Both gross and net foreign exchange reserves are shown. The latter deduct from gross reserves the central bank bond liabilities issued for the purpose of sterilizing reserve accumulation. The model treats such bonds as perfect substitutes for international bonds.

Figure 1: China's Reserves and Inflation
(2000Q1-2007Q4)

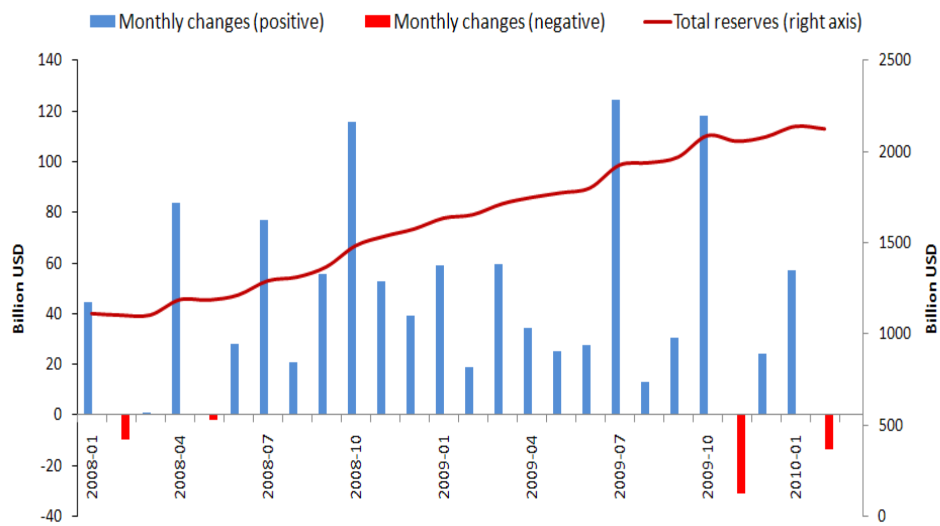


Figure 2a: China's Total Reserves (Jan 2008 onwards)

Source: Bloomberg

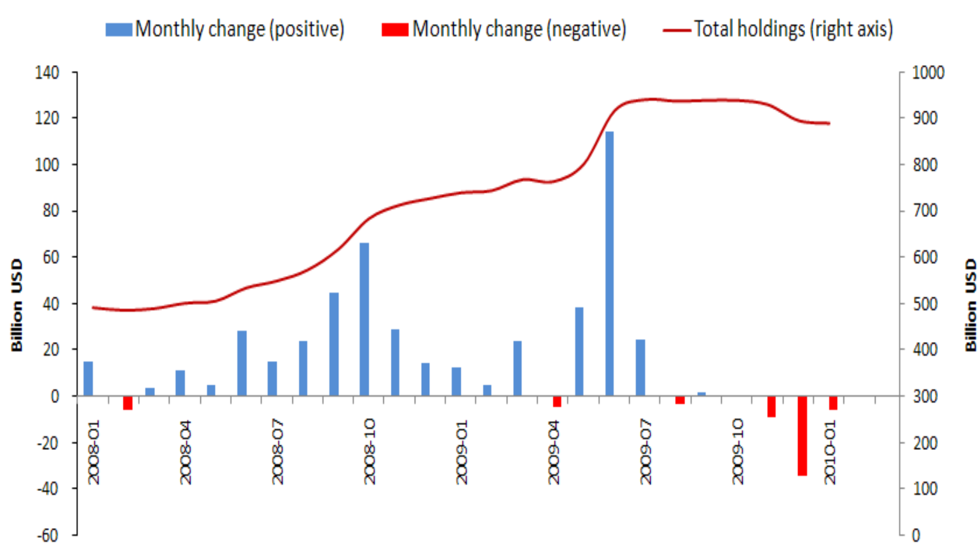


Figure 2b: China's Holding of U.S. Treasuries (Jan 2008 onwards)

Source: United States Treasury

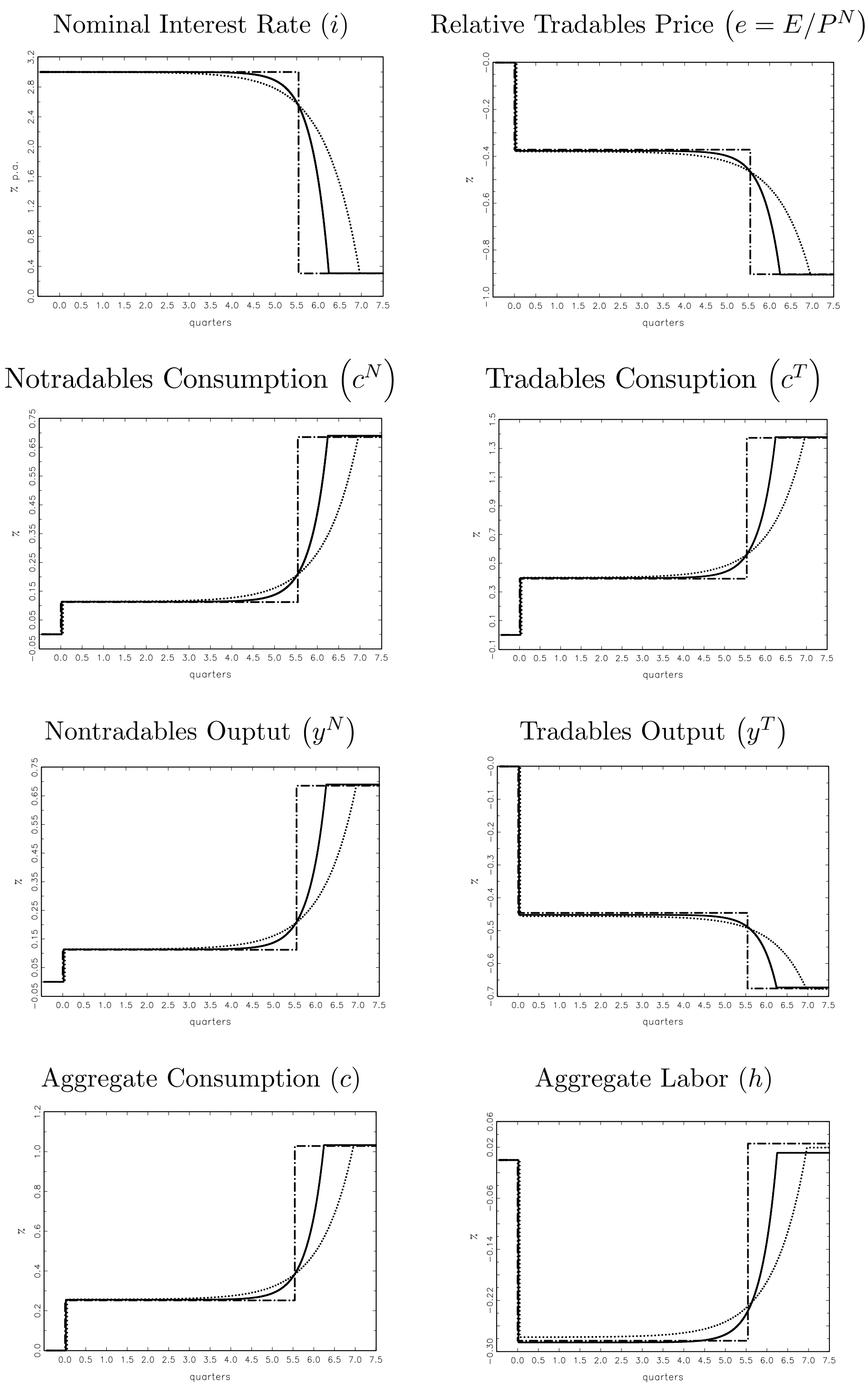


Figure 3a: Anti-Crisis – Overview

ET=_ _, CPIT=—, DIT=...

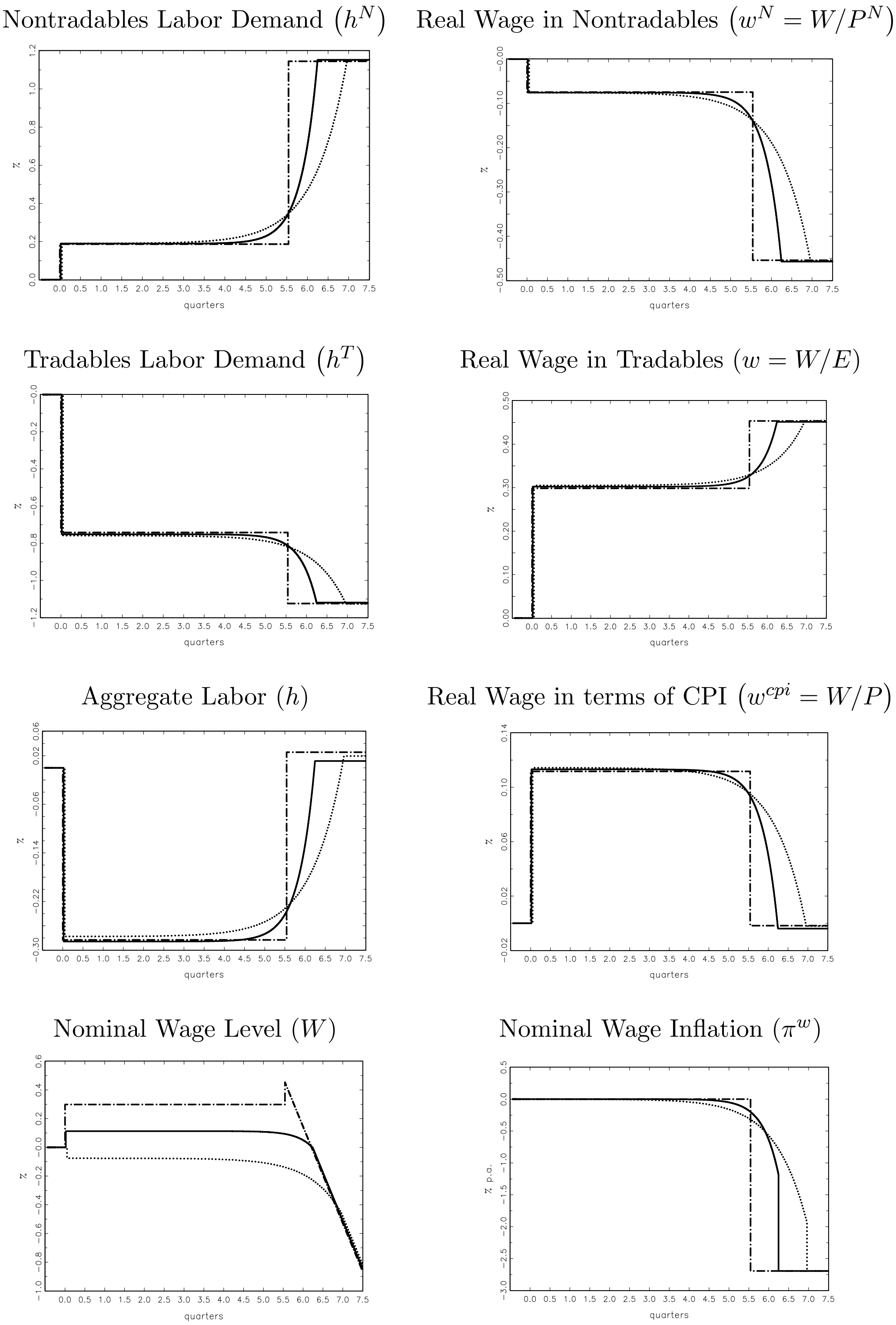


Figure 3b: Anti-Crisis – Labor Market

ET=_ _, CPIT=—, DIT=...

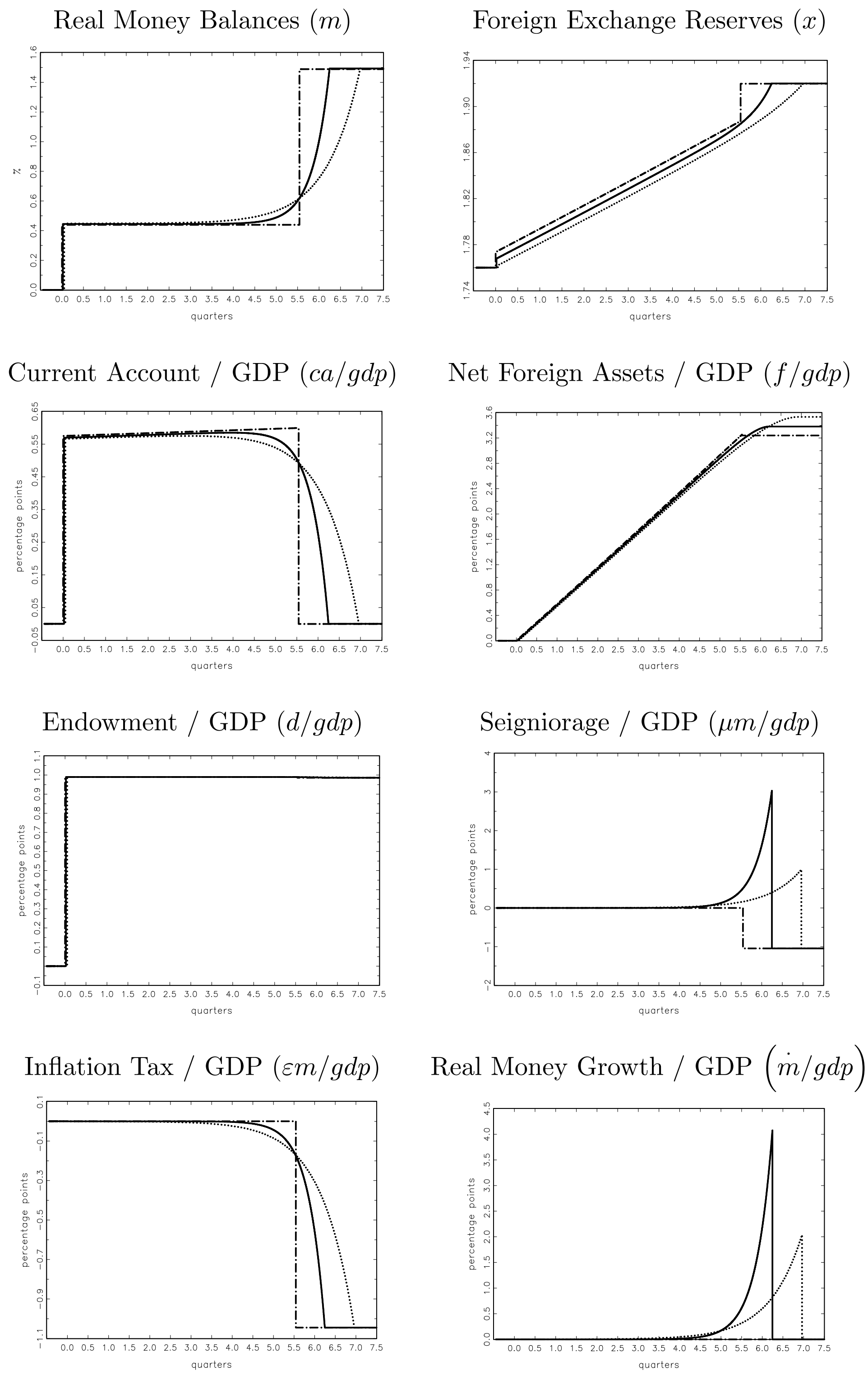


Figure 3c: Anti-Crisis – Government Budget

ET=_ _, CPIT=—, DIT=...

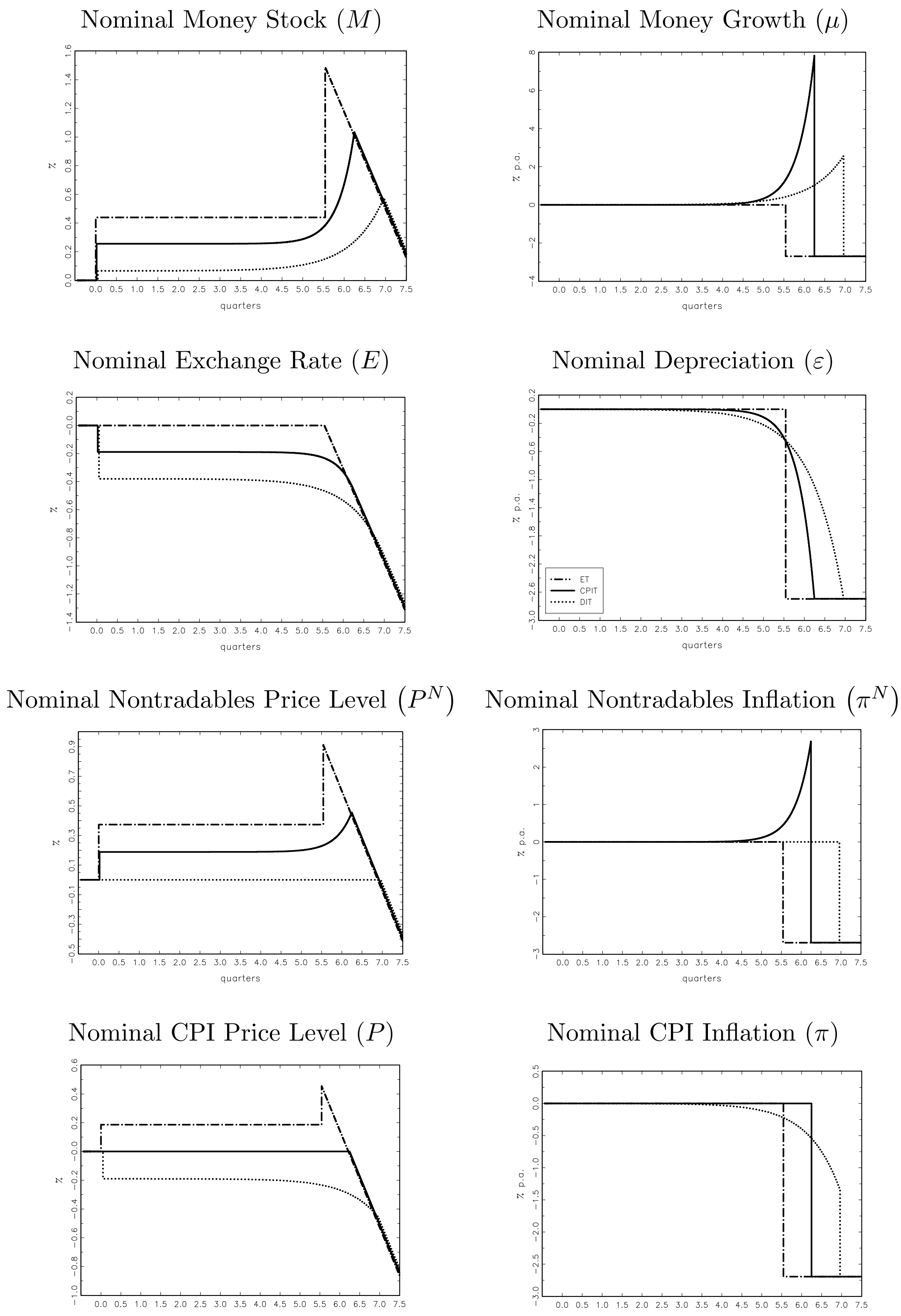


Figure 3d: Anti-Crisis – Price Levels and Inflation Rates

ET=_ _, CPIT=—, DIT=...

Technical Appendix

Too Much of a Good Thing? On the Effects of Limiting Foreign Reserve Accumulation

1 INTRODUCTION

The following note first derives the conditions characterizing the economy's competitive equilibrium, and then outlines the nested shooting algorithm we use to compute the solution paths of the model.

1.1 Competitive Equilibrium

1.1.1 Household FOC

The household's maximization problem is:

$$Max \quad \int_0^\infty \left[\gamma \ln\left(\left[\eta^{\frac{1}{\sigma}}(c_t^T)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right) + (1-\gamma)\ln(1-h_t)\right] e^{-rt}dt$$

$$+\lambda\left[a_0 + \int_0^\infty \left(w_t h_t + r_t^{k^T} k^T + r_t^{k^N} k^N + g_t\right) e^{-rt}dt - \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t\right) e^{-rt}dt\right]$$

subject to

$$m_t \geq \alpha\left(c_t^T + \frac{c_t^N}{e_t}\right)$$

Define D_t as

$$D_t \equiv \left[\eta^{\frac{1}{\sigma}}(c_t^T)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{\frac{\sigma-1}{\sigma}}\right] \tag{1}$$

Then we get the following FOC:

- FOC for Tradables Consumption c_t^T:

$$\left[\frac{\gamma}{c_t}\left[\eta^{\frac{1}{\sigma}}(c_t^T)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1} \eta^{\frac{1}{\sigma}}(c_t^T)^{-\frac{1}{\sigma}}\right] = \lambda(1+\alpha i_t) \quad \text{, or}$$

$$\frac{\gamma}{D_t}\eta^{\frac{1}{\sigma}}(c_t^T)^{-\frac{1}{\sigma}} = \lambda(1+\alpha i_t)$$

$$c_t^T = \left(\frac{\gamma\eta^{\frac{1}{\sigma}}}{\lambda(1+\alpha i_t)D_t}\right)^\sigma \tag{2}$$

- FOC for Nontradables Consumption c_t^N:

$$\left[\frac{\gamma}{c_t}\left[\eta^{\frac{1}{\sigma}}(c_t^T)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1} (1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{-\frac{1}{\sigma}}\right] = \frac{\lambda(1+\alpha i_t)}{e_t} \quad \text{, or}$$

$$\frac{\gamma}{D_t}(1-\eta)^{\frac{1}{\sigma}}(c_t^N)^{-\frac{1}{\sigma}} = \frac{\lambda(1+\alpha i_t)}{e_t}$$

- MRS between c_t^T and c_t^N:

$$\left(\frac{\eta}{1-\eta}\right)^{\frac{1}{\sigma}}(c_t^T)^{-\frac{1}{\sigma}} = e_t(c_t^N)^{-\frac{1}{\sigma}}$$

$$\left(\frac{1-\eta}{\eta}\right)c_t^T = e_t^{-\sigma}c_t^N$$

$$c_t^T = \left(\frac{\eta}{1-\eta}\right)\left(\frac{c_t^N}{e_t^\sigma}\right) \tag{3}$$

- FOC for Labor h_t:

$$\frac{1-\gamma}{1-h_t} = \lambda w_t \tag{4}$$

1.1.2 Firm FOC

- Tradable Goods:

$$y_t^T = \left(k^T\right)^{\rho^T}\left(h_t^T\right)^{1-\rho^T} \tag{5}$$

$$\left(1-\rho^T\right)\left(\frac{k^T}{h_t^T}\right)^{\rho^T} = w_t \tag{6}$$

$$\rho^T\left(\frac{h_t^T}{k^T}\right)^{1-\rho^T} = r_t^{k^T}$$

- Nontradable Goods:

$$y_t^N = \left(k^N\right)^{\rho^N}\left(h_t^N\right)^{1-\rho^N} \tag{7}$$

$$\left(1-\rho^N\right)\left(\frac{k^N}{h_t^N}\right)^{\rho^N} = e_t w_t \tag{8}$$

$$\rho^N\left(\frac{h_t^N}{k^N}\right)^{1-\rho^N} = e_t r_t^{k^N}$$

- Notes:
  - $w_t = W_t/E_t$ is the real wage in terms of tradables.
  - The real returns on the two capital stocks $r_t^{k^T}$ and $r_t^{k^N}$ are similarly expressed in terms of tradables. Their FOC are redundant.

1.1.3 Market Clearing and Resource Constraints

- Cash-in-Advance Constraint:

$$m_t = \alpha \left(c_t^T + \frac{c_t^N}{e_t} \right) \tag{9}$$

- Nontradables Market Clearing:

$$c_t^N = y_t^N \tag{10}$$

- Current Account:

$$\dot{f}_t = r f_t + y_t^T + d_t - c_t^T \tag{11}$$

- Government Flow Budget Constraint:

$$\dot{x}_t = r x_t + \dot{m}_t + \varepsilon_t m_t + d_t - g_t \tag{12}$$

1.1.4 CPI Price Index and Inflation Rates

- Aggregate price index:

$$P_t = \left(\eta \left(E_t \right)^{1-\sigma} + (1-\eta) \left(P_t^N \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- Divide both sides by $(1-\eta)^{\frac{1}{1-\sigma}} P_t^N$:

$$\frac{1}{(1-\eta)^{\frac{1}{1-\sigma}}} \frac{P_t}{P_t^N} = \left(\frac{\eta}{(1-\eta)} e_t^{1-\sigma} + 1 \right)^{\frac{1}{1-\sigma}}$$

- Take logs and differentiate:

$$\pi_t - \pi_t^N = \frac{1}{\left(\frac{\eta}{(1-\eta)} \left(e_t \right)^{1-\sigma} + 1 \right)} \frac{\eta}{(1-\eta)} e_t^{1-\sigma} \frac{\dot{e}_t}{e_t}$$

- Use $\pi_t^N = \varepsilon_t - \dot{e}_t / e_t$:

$$\frac{\dot{e}_t}{e_t} = \left(\pi_t - \varepsilon_t + \frac{\dot{e}_t}{e_t} \right) \left(1 + \frac{1-\eta}{\eta} e_t^{\sigma-1} \right)$$

- Equivalently:

$$\frac{\dot{e}_t}{e_t} \left(\frac{1-\eta}{\eta} e_t^{\sigma-1} \right) = \left(1 + \frac{1-\eta}{\eta} e_t^{\sigma-1} \right) \left(\varepsilon_t - \pi_t \right)$$

- We obtain the following dynamic equation, which will be used as one of the transition equations of our dynamic system:

$$\frac{\dot{e}_t}{e_t} = \left(\frac{\eta}{1-\eta} e_t^{1-\sigma} + 1 \right) (\varepsilon_t - \pi_t) \tag{13}$$

- To get a relationship between inflation rates we again use $\dot{e}_t / e_t = \varepsilon_t - \pi_t^N$:

$$(1-\eta)\left(\varepsilon_t - \pi_t^N\right) = \left(\eta e_t^{1-\sigma} + (1-\eta)\right)(\varepsilon_t - \pi_t)$$

- We obtain the following relationship, which will be used to define the relationships between inflation rates at discrete jumps:

$$\pi_t \left(\eta e_t^{1-\sigma} + (1-\eta)\right) = \eta e_t^{1-\sigma} \varepsilon_t + (1-\eta)\pi_t^N \tag{14}$$

1.2 Monetary Rules

1.2.1 The Rules

- Monetary rule for Exchange Rate Targeting ("ET"):

$$E_t = E_{0+} e^{\bar{\varepsilon} t} \quad . \tag{15}$$

- Monetary rule for CPI Inflation Targeting ("CPIT"):

$$i_t^P = r + \varepsilon_t + \xi^P (P_t - \tilde{P}_t) \quad , \tag{16}$$

$$\tilde{P}_t = P_{0+} e^{\bar{\pi} t} \quad . \tag{17}$$

- Monetary rule for Domestic Inflation Targeting ("DIT"):

$$i_t^{P_N} = r + \varepsilon_t + \xi^{P_N} (P_t^N - \tilde{P}_t^N) \quad , \tag{18}$$

$$\tilde{P}_t^N = P_{0+}^N e^{\bar{\pi}^N t} \quad . \tag{19}$$

1.2.2 Interpretation of the two IT Rules

1. They imply, together with UIP, exact price level targeting:

$$P_t = \tilde{P}_t \quad \forall t \geqq 0+ \tag{20}$$

$$P_t^N = \tilde{P}_t^N \quad \forall t \geqq 0+ \tag{21}$$

2. The definition of the target price level as starting from time 0+ allows for initial price level jumps without affecting the interest rate, i.e. the key in going from time 0− to time 0+ is monetary accommodation alone.

1.3 Key Resource Constraints

Two aggregate constraints will be imposed during the following iteration procedures to ensure consistency of dynamic solution paths with

- aggregate solvency vis-a-vis the rest of the world.
- the upper bound on government foreign exchange reserves $\bar{x}$.

These two constraints will be used to pin down two unknowns and one additional variable implied by these unknowns:

- The initial jump in exchange rate depreciation $\Delta\varepsilon_0$.
- The length of time τ until foreign exchange reserves reach their upper bound.
- The implied value for the multiplier λ.

The variables $\Delta\varepsilon_0$ and λ are required to solve for the initial conditions of the economy. The procedure involves a nested loop over these two variables that ensures they are consistent with the two aggregate constraints.

The resource constraints are as follows:

- Economy's resource constraint:

$$f_0 + \int_0^{\tau} \left(y_t^T + d_t\right) e^{-rt} dt + \frac{e^{-r\tau}}{r} \left(y_\tau^T + d_\tau\right) = \int_0^{\tau} c_t^T e^{-rt} dt \ + \frac{e^{-r\tau}}{r} c_\tau^T \quad . \tag{22}$$

- Government's resource constraint:

$$\left(x_0 + \triangle x_0\right) e^{r\tau} + \int_0^{\tau} \left(\dot{m}_s + \varepsilon_s m_s - g_s\right) e^{r(\tau - s)} ds = x_\tau = \bar{x} \quad . \tag{23}$$

There is also a time 0 resource transfer that depends on whether the government allows the nominal money supply to jump at that time to accommodate changes in real money demand. We can decompose a change in real money balances into a portion that is due to jumps in the exchange rate at time 0, Δm_0^E, and another portion that is due to increases in demand for nominal balances at the new exchange rate, Δm_0^M:

$$m_{0+} - m_{0-} = \Delta m_0^E + \Delta m_0^M \quad , \tag{24}$$

$$\Delta m_0^E = M_{0-}\left(\frac{1}{E_{0+}} - \frac{1}{E_{0-}}\right) \quad , \tag{25}$$

$$\Delta m_0^M = \frac{(M_{0+} - M_{0-})}{E_{0+}} = \frac{dM_0}{E_{0+}} \quad . \tag{26}$$

These two components have different implications for the evolution of foreign exchange reserves x. The exchange rate induced component (25) does not have any effect on foreign exchange reserves on impact, but it affects reserves over time as it changes the amount of seigniorage the government needs to raise to service its liabilities. The other component (26) , however, results in an inflow of foreign exchange reserves to the central bank at time 0:

$$x_{0+} - x_{0-} = \Delta m_0^M \quad . \tag{27}$$

2 OVERVIEW OF THE ITERATION PROCEDURE

We first solve for the initial steady state of the economy, using the time subscript $0-$ for that period. Starting from time 0, the government receives increased endowments $\bar{d} > d_{0-} = 0$. This is inconsistent with the continuing zero inflation (or exchange rate depreciation) target at the inherited level of foreign exchange reserves. There are level jumps in some variables at time 0, and we therefore refer to the moment immediately after the announcement of the new policy as $0+$. To compute the paths of all variables until and after the end of the zero inflation regime, we adopt (i) a nested shooting algorithm for the CPI and DIT inflation targeting cases because these cases involve complicated transitions to a new steady state, and (ii) a simultaneous equation system solution for the ET exchange rate targeting case, which involves step paths for all variables.

2.1 General Strategy for Inflation Targeting

As both λ and ε jump at time $0+$, we need to adopt the following general strategy:

1. Solve the system again at $0+$ based on guesses about the magnitude of those jumps.

2. Use the $0+$ values to initialize our computation of dynamic transitions.

3. Check the consistency of the resulting dynamic paths with the two aggregate resource constraints (22) and (23).

4. Start again at (i) with updated guesses if the consistency conditions are not satisfied.

2.2 Specific Procedure

Specifically, we adopt the following procedure:

1. Step 1: Solve for all variables at time $0-$.

2. Outside loop over initial guess for $\Delta\varepsilon_0^{(b)}$.

   - We make an initial guess of the jump in exchange rate depreciation $\Delta\varepsilon_0^{(b)}$.
   - b means the b^{th} round of iterations over $\Delta\varepsilon_0$.
   - In general the initial jumps will be extremely close to zero, which causes numerical challenges. For this reason, our updating of the guess for $\Delta\varepsilon_0^{(b)}$ is generally done manually.

3. Inside loop over initial guess for $\lambda^{(b_i)}$, which depends on $(\triangle\varepsilon_0^{(b)}, \tau)$.

   - For any given guess $\Delta\varepsilon_0^{(b)}$, we make an additional guess of $\lambda^{(b_i)}(\triangle\varepsilon_0^{(b)}, \tau)$.
   - b_i means the i^{th} round of iterations in the loop for λ that is inside the b^{th} outer loop of iterations over $\Delta\varepsilon_0$.
   - τ is the final point of the anti-crisis.

4. Step 2: For any set of values λ and ε in the loop, i.e. for any combination of $\triangle\varepsilon_0^{(b)}$ and $\lambda^{(b_i)}(\triangle\varepsilon_0^{(b)}, \tau)$, we compute the values of all other variables at time $0+$ $\left(c_{0+}^T, c_{0+}^N, e_{0+}, D_{0+}, w_{0+}, h_{0+}^T, h_{0+}^N, y_{0+}^T, y_{0+}^N, \varepsilon_{0+}\right)$.

5. Step 3: Starting from these initial values, we compute the dynamic paths of all variables.

6. Steps 4-7: We check the consistency of the computed paths with (22) and (23). If there are discrepancies, we update $\triangle\varepsilon_0^{(b)}$ and $\lambda^{(b_i)}(\triangle\varepsilon_0^{(b)}, \tau)$ and start again at Step 2.

3 STEP 1: SOLVING FOR VARIABLES AT TIME 0-

We make the following assumptions for initial consumption and net foreign assets:

$$\begin{aligned} c^T_{0-} &= c^N_{0-} = 1 \quad , \\ f_0 &= 0 \quad . \end{aligned} \tag{c^T_{0-}, c^N_{0-}}$$

Because of the time 0 current account balance equation, the nontradables market clearing condition, and a zero government endowment, we also have:

$$y^T_{0-} = y^N_{0-} = 1 \quad , \tag{y^T_{0-}, y^N_{0-}}$$

$$e_{0-} = \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{\sigma}} \quad , \tag{e_{0-}}$$

$$D_{0-} = \eta^{\frac{1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}} \quad . \tag{D_{0-}}$$

We assume that

$$h^T_{0-} + h^N_{0-} = h_{ss} = \frac{1}{3} \quad . \tag{28}$$

This can be combined with two further conditions derived from the production functions and labor demands:

$$\begin{aligned} \text{Prod. Fct. :} \quad & 1 = \left(k^T\right)^{\rho^T} \left(h^T_{0-}\right)^{1-\rho^T} \\ \text{Labor FOC:} \quad & \left(1-\rho^T\right)\left(k^T\right)^{\rho^T} \cdot \left(h^T_{0-}\right)^{-\rho^T} = w_{0-} \\ \text{Combined:} \quad & \left(1-\rho^T\right) = w_{0-} h^T_{0-} \end{aligned} \tag{29}$$

$$\begin{aligned} \text{Prod. Fct. :} \quad & 1 = \left(k^N\right)^{\rho^N} \cdot \left(h^N_{0-}\right)^{1-\rho^N} \\ \text{Labor FOC:} \quad & \left(1-\rho^N\right)\left(k^N\right)^{\rho^N} \cdot \left(h^N_{0-}\right)^{-\rho^N} = e_{0-} w_{0-} \\ \text{Combined:} \quad & \left(1-\rho^N\right) = e_{0-} w_{0-} h^N_{0-} \end{aligned} \tag{30}$$

From (29) and (30), we have

$$h^N_{0-} = \frac{\left(1-\rho^N\right)}{\left(1-\rho^T\right)} \frac{h^T_{0-}}{e_{0-}}$$

Substitute this into (28) to get

$$h^T_{0-} = \frac{h_{ss}}{\left(1 + \frac{(1-\rho^N)}{(1-\rho^T)} \frac{1}{e_{0-}}\right)} \tag{h^T_{0-}}$$

$$h^N_{0-} = h_{ss} - h^T_{0-} \tag{h^N_{0-}}$$

$$w_{0-} = \frac{\left(1-\rho^{T}\right)}{h_{0-}^{T}} \tag{w_{0-}}$$

The capital stocks consistent with output normalization at one can be computed as follows:

$$k^{T} = \left(h_{0-}^{T}\right)^{\frac{\rho^{T}-1}{\rho^{T}}} \quad , \tag{k^T}$$

$$k^{N} = \left(h_{0-}^{N}\right)^{\frac{\rho^{N}-1}{\rho^{N}}} \quad . \tag{k^N}$$

These two values are held constant in all subsequent calculations.

Two simultaneous equations derived from the first order conditions for tradables consumption and for labor supply can be used to solve for λ_{0-} and the preference parameter γ. The latter has to be endogenously solved for so as to be consistent with our assumption of aggregate steady state labor being equal to 1/3, equation (28):

$$\left\{\frac{\gamma}{\left[1+\left(\frac{1-\eta}{\eta}\right)^{\frac{1}{\sigma}}\right]\lambda_{0-}\left(1+\alpha(r+\varepsilon_{0-})\right)}\right\}^{\sigma} = 1 \quad , \tag{λ_{0-}}$$

$$\frac{1-\gamma}{\lambda_{0-}w_{0-}} = 1-h_{ss} \quad . \tag{γ}$$

The initial conditions for all inflation rates are simply as follows:

$$\varepsilon_{0-} = \pi_{0-}^{N} = \pi_{0-} = 0 \quad . \tag{$\varepsilon_{0-}, \pi_{0-}, p_{0-}$}$$

Similarly, for interest rates:

$$i_{0-}^{P} = i_{0-}^{P_N} = i_{0-}^{E} = r \quad . \tag{$i_{0-}^{P}, i_{0-}^{P_N}, i_{0-}^{E}$}$$

We adopt the following normalization of the aggregate price level, which can be brought about through an appropriate level of nominal money supply:

$$P_{0-} = \widetilde{P}_{0-} = 1 \quad . \tag{$P_{0-}, \widetilde{P}_{0-}$}$$

Then a simultaneous solution of the following two equations determines the remaining price levels E_{0-} and P_{N_0-}:

$$1 = \left(\eta\left(E_{0-}\right)^{1-\sigma} + (1-\eta)\left(P_{0-}^{N}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad , \tag{E_{0-}}$$

$$E_{0-} = e_{0-}P_{0-}^{N} \quad . \tag{P_{0-}^N}$$

Also, we have

$$\widetilde{E}_{0-} = E_{0-} \quad , \tag{$\widetilde{E}_{0-}$}$$

$$\tilde{P}_{0-}^{N} = P_{0-}^{N} \quad . \tag{$\tilde{P}_{0-}^N$}$$

Finally, we can solve for inital real and nominal money balances as:

$$m_{0-} = \alpha c^T_{0-}(1 + e^{\sigma-1}_{0-} \cdot \frac{1-\eta}{\eta}) \quad , \tag{m_{0-}}$$

$$n_{0-} = \alpha c^N_{0-}(1 + e^{1-\sigma}_{0-} \cdot \frac{\eta}{1-\eta}) \quad , \tag{n_{0-}}$$

$$M_{0-} = E_{0-} m_{0-} \tag{M_{0-}}$$

4 STEP 2: SOLVING FOR VARIABLES AT TIME 0+ (based on iterated guess for λ and ε)

4.1 System of Equations

The computation is done by first solving the following simultaneous system of 10 equations.

$$\varepsilon_{0+} = \triangle \varepsilon_0^{(b)} \quad , \tag{31}$$

$$c_{0+}^T = c_{0+}^N \cdot e_{0+}^{-\sigma} \cdot \frac{\eta}{1-\eta} \quad , \tag{32}$$

$$c_{0+}^T = \left\{ \frac{\gamma \cdot \eta^{\frac{1}{\sigma}}}{D_{0+} \lambda^{(b_i)} (1 + \alpha(r + \varepsilon_{0+}))} \right\}^{\sigma} \quad , \tag{33}$$

$$D_{0+} = \eta^{\frac{1}{\sigma}} (c_{0+}^T)^{\frac{\sigma-1}{\sigma}} + (1-\eta)^{\frac{1}{\sigma}} (c_{0+}^N)^{\frac{\sigma-1}{\sigma}} \quad , \tag{34}$$

$$y_{0+}^T = \left(k^T\right)^{\rho^T} \cdot \left(h_{0+}^T\right)^{1-\rho^T} \quad , \tag{35}$$

$$y_{0+}^N = \left(k^N\right)^{\rho^N} \cdot \left(h_{0+}^N\right)^{1-\rho^N} \quad , \tag{36}$$

$$\left(1-\rho^T\right) \left(k^T\right)^{\rho^T} \cdot \left(h_{0+}^T\right)^{-\rho^T} = w_{0+} \quad , \tag{37}$$

$$\left(1-\rho^N\right) \left(k^N\right)^{\rho^N} \cdot \left(h_{0+}^N\right)^{-\rho^N} = e_{0+} w_{0+} \quad , \tag{38}$$

$$h_{0+}^T + h_{0+}^N = 1 - \frac{1-\gamma}{\lambda^{(b_i)} w_{0+}} \quad , \tag{39}$$

$$c_{0+}^N = y_{0+}^N \quad . \tag{40}$$

The values of the variables at time 0+ become the initial values of the dynamic paths in the transition period $t \in [0, \tau)$, which is computed in Step 3 below. Based on these 10 dynamic paths, the paths of a further 9 variables can be computed in accordance with the following sets of equations. Except for the first two equations, (41) and (42), these equation sets differ depending on

(a) Our assumptions about the initial central bank accommodation of changes in money demand.

(b) The monetary regime.

Below we therefore present initial conditions for each combination of (a) and (b) above.

4.2 Net Foreign Assets and Real Money Balances

$$f_{0+} = f_{0-} = 0 \quad , \tag{41}$$

$$m_{0+} = \alpha \left(c^T_{0+} + \frac{c^N_{0+}}{e_{0+}} \right) \quad . \tag{42}$$

4.3 Central Bank Foreign Exchange Reserves

The results here depend on whether the central bank is assumed to accommodate initial jumps in real money demand. We distinguish two cases:

- Full central bank accommodation of initial jump in real money demand (smooth nominal exchange rate):

$$x_{0+} - x_{0-} = (m_{0+} - m_{0-}) \quad . \tag{43}$$

- Smooth levels of targeted price variables.

$$x_{0+} - x_{0-} = \frac{M_{0+} - M_{0-}}{E_{0+}} \quad . \tag{44}$$

4.4 Price Variables

For price variables, initial jumps also depend on the monetary regime. The next subsection considers the full accommodation case corresponding to (43), and the subsection following it considers the case of smooth target price levels corresponding to (44). The equations in each case are derived from the following two equations for the CPI price level and CPI inflation:

$$P_t = \left(\eta \left(E_t \right)^N + (1 - \eta) \left(P^N_t \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{45}$$

$$\left(\eta e_t^{1-\sigma} + (1 - \eta) \right) \pi_t = (1 - \eta) \pi^N_t + \eta e_t^{1-\sigma} \varepsilon_t \tag{46}$$

4.4.1 Price Variables - Full Accommodation

1. Common Equations Across Monetary Regimes: The Price Levels

$$E_{0+} = E_{0-} \quad , \tag{47}$$

$$P^N_{0+} = \frac{E_{0+}}{e_{0+}} \quad , \tag{48}$$

$$P_{0+} = \left(\eta \left(E_{0+}\right)^{1-\sigma} + (1-\eta)\left(P^N_{0+}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad , \tag{49}$$

$$M_{0+} = m_{0+}E_{0+} \quad . \tag{50}$$

2. CPIT:

$$\pi_{0+} = \pi_{0-} = \bar{\pi} = 0 \quad , \tag{51}$$

$$\pi^N_{0+} = \frac{\eta}{\eta-1} e^{1-\sigma}_{0+}\varepsilon_{0+} \quad \text{(follows from (46) and (51)).} \tag{52}$$

3. DIT:

$$\pi^N_{0+} = \pi^N_{0-} = \bar{\pi}^N = 0 \quad , \tag{53}$$

$$\pi_{0+} = \frac{\eta e_{0+}^{1-\sigma}}{\eta e_{0+}^{1-\sigma} + (1-\eta)} \varepsilon_{0+} \quad \text{(follows from (46) and (53)).} \tag{54}$$

4. ET:

$$\varepsilon_{0+} = \varepsilon_{0-} = \bar{\varepsilon} = 0 \quad \text{(this replaces (31))}, \tag{55}$$

$$\pi_{0+} = \pi_{0-} = 0 \quad , \tag{56}$$

$$\pi^N_{0+} = \pi^N_{0-} = 0 \quad . \tag{57}$$

4.4.2 Price Variables - Smooth Target Price Levels

1. CPIT:

$$P_{0+} = P_{0-} = 1 \quad , \tag{58}$$

$$\pi_{0+} = \pi_{0-} = \bar{\pi} = 0 \quad , \tag{59}$$

$$P^N_{0+} = \left(\eta e_{0+}^{1-\sigma} + (1-\eta)\right)^{\frac{1}{\sigma-1}} \quad \text{(follows from (45) and (58))}, \tag{60}$$

$$\pi^N_{0+} = \frac{\eta}{\eta-1} e_{0+}^{1-\sigma} \varepsilon_{0+} \quad \text{(follows from (46) and (59))}, \tag{61}$$

$$E_{0+} = e_{0+} P^N_{0+} \quad , \tag{62}$$

$$M_{0+} = m_{0+} E_{0+} \quad . \tag{63}$$

2. DIT:

$$P^N_{0+} = P^N_{0-} \quad , \tag{64}$$

$$\pi^N_{0+} = \pi^N_{0-} = \bar{\pi}^N = 0 \quad , \tag{65}$$

$$E_{0+} = e_{0+} P^N_{0+} \quad , \tag{66}$$

$$P_{0+} = \left(\eta \left(E_{0+}\right)^{1-\sigma} + (1-\eta)\left(P^N_{0+}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad \text{(follows from (45))}, \tag{67}$$

$$\pi_{0+} = \frac{\eta e_{0+}^{1-\sigma}}{\eta e_{0+}^{1-\sigma} + (1-\eta)} \varepsilon_{0+} \quad \text{(follows from (46) and (65))}, \tag{68}$$

$$M_{0+} = m_{0+} E_{0+} \quad . \tag{69}$$

3. ET:

$$\varepsilon_{0+} = \varepsilon_{0-} = \bar{\varepsilon} = 0 \quad \text{(this replaces (31))}, \tag{70}$$

$$\pi_{0+} = \pi_{0-} = 0 \quad , \tag{71}$$

$$\pi^N_{0+} = \pi^N_{0-} = 0 \quad , \tag{72}$$

$$E_{0+} = E_{0-} \quad , \tag{73}$$

$$P^N_{0+} = \frac{E_{0+}}{e_{0+}} \tag{74}$$

$$P_{0+} = \left(\eta \left(E_{0+}\right)^{1-\sigma} + (1-\eta)\left(P^N_{0+}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad \text{(follows from (45))}, \tag{75}$$

$$M_{0+} = m_{0+} E_{0+} \quad . \tag{76}$$

5 STEP 3: SOLVING FOR DYNAMIC PATHS (based on iterated guess for λ and ε)

Based on the same guess for λ and ε that was used in computing initial (time 0+) values in Step 2, we now compute the dynamic paths of all variables. Using the values at time 0+ as initial values of a system of differential equations, we solve that system using the **Runge-Kutta Method (RK4)** (see Judd p. 345 for details).

5.1 CPI Targeting

- Differentiate (3):

$$\frac{\dot{c}_t^T}{c_t^T} = \frac{\dot{c}_t^N}{c_t^N} - \sigma \frac{\dot{e}_t}{e_t} \tag{c_t^T}$$

- Differentiate (2):

$$\frac{\dot{c}_t^T}{c_t^T} = -\sigma \left[\frac{\dot{D}_t}{D_t} + \frac{\alpha \dot{\varepsilon}_t}{(1+\alpha(r+\varepsilon_t))} \right] \tag{ε_t}$$

- Differentiate (1):

$$\dot{D}_t \equiv \eta^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} (c_t^T)^{\frac{\sigma-1}{\sigma}} \left(\frac{\dot{c}_t^T}{c_t^T} \right) + (1-\eta)^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} (c_t^N)^{\frac{\sigma-1}{\sigma}} \left(\frac{\dot{c}_t^N}{c_t^N} \right) \tag{D_t}$$

- Differentiate (4):

$$\frac{\dot{h}_t^T + \dot{h}_t^N}{(1-h_t^T-h_t^N)} = \frac{\dot{w}_t}{w_t} \tag{w_t}$$

- Differentiate (6):

$$-\rho^T \frac{\dot{h}_t^T}{h_t^T} = \frac{\dot{w}_t}{w_t} \tag{h_t^T}$$

- Differentiate (5):

$$\frac{\dot{y}_t^T}{y_t^T} = (1-\rho^T) \frac{\dot{h}_t^T}{h_t^T} \tag{y_t^T}$$

- Differentiate (8):

$$-\rho^N \frac{\dot{h}_t^N}{h_t^N} = \frac{\dot{e}_t}{e_t} + \frac{\dot{w}_t}{w_t} \tag{h_t^N}$$

- Differentiate (7):

$$\frac{\dot{y_t^N}}{y_t^N} = (1-\rho^N)\frac{\dot{h_t^N}}{h_t^N} \tag{y_t^N}$$

- Differentiate (10):

$$\frac{\dot{c_t^N}}{c_t^N} = \frac{\dot{y_t^N}}{y_t^N} \tag{c_t^N}$$

- Current Account (11):

$$\dot{f_t} = rf_t + y_t^T + d_t - c_t^T \tag{f_t}$$

- Government Flow Budget Constraint (12):

$$\dot{x}_t = rx_t + \dot{m}_t + \varepsilon_t m_t + d_t - g_t \tag{x_t}$$

- Cash-in-Advance (9):

$$\dot{m}_t = \alpha\left(\dot{c_t^T} + \frac{\dot{c_t^N}}{e_t} - \frac{c_t^N}{e_t}\frac{\dot{e_t}}{e_t}\right) \tag{m_t}$$

- From CPI differentiation (13):

$$\frac{\dot{e_t}}{e_t} = \left(\frac{\eta}{1-\eta}e_t^{1-\sigma}+1\right)(\varepsilon_t - \bar{\pi}) \tag{e_t}$$

This is a system of 13 differential equations with 13 unknown functions of time: c_t^T, c_t^N, e_t, D_t, ε_t, w_t, h_t^T, h_t^N, y_t^T, y_t^N, f_t, $x_t$and m_t. Equation (w_t) can be replaced according to the following derivation:

- First, combine (w_t) with (h_t^N) and (f_t):

$$\frac{\dot{h_t^T} + \dot{h_t^N}}{1-h_t^T-h_t^N} = \frac{\dot{w}_t}{w_t} = -\rho^N\frac{\dot{h_t^N}}{h_t^N} - \left(\frac{\eta}{1-\eta}(e_t)^{1-\sigma}+1\right)(\varepsilon_t - \bar{\pi})$$

- Transform this to get

$$\dot{h_t^T} = \left(1-h_t^T-h_t^N\right)\left[-\rho^N\frac{\dot{h_t^N}}{h_t^N} - \left(\frac{\eta}{1-\eta}(e_t)^{1-\sigma}+1\right)(\varepsilon_t - \bar{\pi})\right] - \dot{h_t^N} \tag{77}$$

- Next, combine (h_t^T) with (h_t^N) to get:

$$-\rho^T\frac{\dot{h_t^T}}{h_t^T} = \frac{\dot{w}_t}{w_t} = -\rho^N\frac{\dot{h_t^N}}{h_t^N} - \left(\frac{\eta}{1-\eta}(e_t)^{1-\sigma}+1\right)(\varepsilon_t - \bar{\pi})$$

- Transform this to get:

$$\dot{h_t^T} = h_t^T \left[\frac{\rho^N}{\rho^T} \frac{\dot{h_t^N}}{h_t^N} + \frac{1}{\rho^T} \left(\frac{\eta}{1-\eta} (e_t)^{1-\sigma} + 1 \right) (\varepsilon_t - \bar{\pi}) \right] \tag{78}$$

- Combining (77) and (78) gives:

$$\begin{aligned} &\left(1 - h_t^T - h_t^T\right) \left[-\rho^N \frac{\dot{h_t^N}}{h_t^N} - \left(\frac{\eta}{1-\eta} (e_t)^{1-\sigma} + 1 \right) (\varepsilon_t - \bar{\pi}) \right] - \dot{h_t^N} \\ = \quad & h_t^T \left[\frac{\rho^N}{\rho^T} \frac{\dot{h_t^N}}{h_t^N} + \frac{1}{\rho^T} \left(\frac{\eta}{1-\eta} (e_t)^{1-\sigma} + 1 \right) (\varepsilon_t - \bar{\pi}) \right] \end{aligned}$$

- Transform this once to get:

$$\dot{h_t^N} \left[-\rho^N \frac{\left(1 - h_t^T - h_t^N\right)}{h_t^N} - 1 - \frac{h_t^T}{h_t^N} \frac{\rho^N}{\rho^T} \right] = \left(1 - h_t^N - h_t^T \left(1 - \frac{1}{\rho^T} \right) \right) \left[\left(\frac{\eta}{1-\eta} (e_t)^{1-\sigma} + 1 \right) (\varepsilon_t - \bar{\pi}) \right]$$

- Transform once again to get:

$$\dot{h_t^N} = \frac{-\left(1 - h_t^N - h_t^T \left(1 - \frac{1}{\rho^T}\right)\right)}{\left[1 + \frac{\rho^N}{h_t^N} \left(1 - h_t^N - h_t^T \left(1 - \frac{1}{\rho^T}\right)\right)\right]} \left[\left(\frac{\eta}{1-\eta} (e_t)^{1-\sigma} + 1 \right) (\varepsilon_t - \bar{\pi}) \right] \tag{79}$$

5.2 Domestic Inflation Targeting

The system of equations is identical to that for CPI Targeting, except for the last equation (e_t), which is replaced by

$$\frac{\dot{e_t}}{e_t} = \left(\varepsilon_t - \bar{\pi}^N\right) \quad . \tag{e}$$

5.3 Continuity under Inflation Targeting

The following argument shows that under CPI inflation targeting ε_t must be continuous for all $t > 0$ including τ. The path of E_t must be continuous to rule out arbitrage opportunities, and the path of P_t is continuous by the inflation targeting policy. By (45) this means that the path of P_t^N is also continuous, and therefore so is $e_t = E_t/P_t^N$. Now assume that h_t^T is not continuous. Then by (6) w_t is discontinuous, and by (8) h_t^N is discontinuous. The production functions (5) and (7) then imply discontinuous jumps in output. These jumps will generally be such that the ratio y_t^T/y_t^N is discontinuous. The market clearing condition for nontradables (10) and the current account (11) therefore imply discontinuous jumps in consumption such that c_t^T/c_t^N is discontinuous. But this contradicts the continuity of e_t by equation (3). We therefore conclude that h_t^T is continuous, and by repeating the above chain of arguments the same is true for $w_t, h_t, y_t^T, y_t^N, c_t^T, c_t^N$, and also for D_t. Finally, the continuity of c_t^T, c_t^N and D_t, by the uncovered interest parity condition and (2), implies continuity of ε_t, and by the real money demand conditions it implies continuity of m_t, n_t and M_t. This continuity result is important for the computational solution, as it allows us to construct solution paths such that all variables remain at the values they reach at the end of the anti-crisis. A similar argument can be constructed for domestic inflation targeting.

5.4 Exchange Rate Targeting

For the exchange targeting case, all variables remain constant in $[0^+, \tau]$, and again after τ, except for x and f. For this case we therefore do not use the iterative procedure, as we only need to solve for a small number of piece-wise constant solution paths plus the time τ and the value of the multiplier λ. The reader can therefore skip to Step 7 for an explanation of the computation of this case.

6 STEP 4: SOLVE FOR τ (based on iterated guess for λ and ε)

- Based on our guess for λ and ε (formally $\varepsilon_0^{(b)}, \lambda^{(b_i)}(\triangle\varepsilon_0^{(b)}, \tau)$) we have computed a pre-τ solution path for each variable in Step 2 and Step 3. We now use those paths to compute the terminal time τ. We know that τ must satisfy three conditions: (i) $\dot{x}_\tau = 0$, (ii) $x_\tau = \bar{x}$, and (iii) consistency with the aggregate resource constraint (22). Checking the first condition is equivalent to checking (i') $\varepsilon_\tau m_\tau = \bar{g} - \bar{d} - r\bar{x}$.

- **Steps 4 and 5 represent the "inside loop" over λ that ensures condition (iii) while checking $\varepsilon_\tau m_\tau = \bar{g} - \bar{d} - r\bar{x}$. Step 6 represents the "outside loop" over $\Delta\varepsilon_0$ that ensures condition (ii).**

- We check $\varepsilon_\tau m_\tau = \bar{g} - \bar{d} - r\bar{x}$ by computing the path of $\varepsilon_t m_t$. It can easily be shown to be monotonically increasing in ε_t and therefore monotonically increasing over time, with an initial value of zero. Let the solution for time τ be denoted as $\tau^{(b^*)}$. It is given by:

$$\tau^{(b^*)} = \left\{ t : \varepsilon_t m_t = \bar{g} - \bar{d} - r\bar{x} \right\} \quad . \tag{80}$$

7 STEP 5: ITERATE OVER λ

- This step ensures that our λ is consistent with the infinite horizon transversality condition (22) for the overall economy.

- We use the first order condition for tradables consumption (2), the economy's lifetime resource constraint (22), and the $\tau^{(b^*)}$ computed in Step 4 to compute a value for λ, denoted as $\hat{\lambda}$. This will in general not equal the $\lambda^{(b_i)}$ that we assumed in Steps 2-4. We therefore update our guess for lambda as $\lambda^{(b_{i+1})} = f(\hat{\lambda})$.

- In the simplest case, $\lambda^{(b_{i+1})} = \hat{\lambda}$, and we have

$$\lambda^{(b_{i+1})} = \left[\frac{\eta\gamma^\sigma \left\{ \int_0^{\tau^{(b^*)}} \left[D_t(1 + \alpha r + \alpha\varepsilon_t) \right]^{-\sigma} e^{-rt} dt + \left[D_{\tau^{(b^*)}}(1 + \alpha r + \alpha\varepsilon_{\tau^{(b^*)}}) \right]^{-\sigma} \frac{e^{-r\tau^{(b^*)}}}{r} \right\}}{f_0 + \int_0^{\tau^{(b^*)}} \left(y_t^T + d_t \right) e^{-rt} dt + \left(y_{\tau^{(b^*)}}^T + d_{\tau^{(b^*)}} \right) \frac{e^{-r\tau^{(b^*)}}}{r}} \right]^{\frac{1}{\sigma}} . \tag{81}$$

- We return to Step 2 and start again.

- We iterate until the $\lambda^{(b_{i+1})}$ from the $(i+1)^{th}$ iteration is the same, within a small tolerance level, as the $\lambda^{(b_i)}$ from the previous iteration. We call the solution $\lambda^{(b^*)}(\triangle\varepsilon_0^{(b)}, \tau^{(b^*)})$), with associated anti-crisis time $\tau^{(b^*)}$, and with associated paths for all variables. The path for central bank foreign exchange reserves $\{x_t\}_{t=0+}^{\infty}$ is the basis for Step 6.

- At this point our solution only depends on the guess for $\Delta\varepsilon_0$, which needs to be consistent with the condition $x_{\tau^{(b^*)}} = 0$.

8 STEP 6. CHECK WHETHER $x_{\tau^{(b^*)}} = \bar{x}$

- Using the path $\{x_t\}_{t=0+}^{\infty}$ from Step 5, we check whether $x_{\tau^{(b^*)}} = \bar{x}$.
- If not, change the initial guess $\triangle\varepsilon_0^{(b)}$.
- Make a new guess for $\lambda^{(b_i)}$ also.
- Return to Step 2 and start again.
- Repeat until $x_{\tau^{(b^*)}(\triangle\varepsilon_0^{(b*)},\lambda^{(b^*)}(\triangle\varepsilon_0^{(b*)},\tau^{(b^*)}))} = \bar{x}$.
- Theoretically, the value of $\triangle\varepsilon_0^{(b)}$ can be solved directly using the system of equations at time 0+. However, this generates computational problems because this value is generally extremely close to zero. We therefore find the solution for $\triangle\varepsilon_0^{(b)}$ using a "manual" grid search.

9 STEP 7: DETERMINE VALUES AFTER T

9.1 Inflation Targeting (CPI and DIT)

The above conditions ensure consistency of values at τ with (i) $\varepsilon_\tau m_\tau = \bar{g}-\bar{d}-r\bar{x}$, (ii) $x_\tau = \bar{x}$ (which together imply consistency with the government resource constraint (23)), and (iii) consistency with the aggregate resource constraint (22). Values after τ can therefore simply be set equal to those at τ.

9.2 Exchange Rate Targeting

Unlike in the previous two cases, there are jumps in the variables at time T for the exchange targeting case. We therefore need to solve for the values of the variables after time T at the new steady state. We compute all equilibrium quantities in one shot by solving the following system of 24 simultaneous equations:

1. The levels equations at time 0+:

$$\varepsilon_{0+} = \bar{\varepsilon} \quad , \tag{ε_{0+}}$$

$$c_{0+}^T = c_{0+}^N \cdot e_{0+}^{-\sigma} \cdot \frac{\eta}{1-\eta} \quad , \tag{e_{0+}}$$

$$c_{0+}^T = \left\{ \frac{\gamma \cdot \eta^{\frac{1}{\sigma}}}{D_{0+}\lambda(1+\alpha(r+\varepsilon_{0+}))} \right\}^{\sigma} \quad , \tag{c_{0+}^*}$$

$$D_{0+}=\eta^{\frac{1}{\sigma}}(c^T_{0+})^{\frac{\sigma-1}{\sigma}}+(1-\eta)^{\frac{1}{\sigma}}(c^N_{0+})^{\frac{\sigma-1}{\sigma}} \quad , \tag{D_{0+}}$$

$$y^T_{0+}=\left(k^T\right)^{\rho^T}\cdot\left(h^T_{0+}\right)^{1-\rho^T} \quad , \tag{y^*_{0+}}$$

$$y^N_{0+}=\left(k^N\right)^{\rho^N}\cdot\left(h^N_{0+}\right)^{1-\rho^N} \quad , \tag{y_{0+}}$$

$$\left(1-\rho^T\right)\left(k^T\right)^{\rho^T}\cdot\left(h^T_{0+}\right)^{-\rho^T}=w_{0+} \quad , \tag{ℓ^*_{0+}}$$

$$\left(1-\rho^N\right)\left(k^N\right)^{\rho^N}\cdot\left(h^N_{0+}\right)^{-\rho^N}=e_{0+}w_{0+} \quad , \tag{ℓ_{0+}}$$

$$h^T_{0+}+h^N_{0+}=1-\frac{1-\gamma}{\lambda w_{0+}} \quad , \tag{w_{0+}}$$

$$c^N_{0+}=y^T_{0+} \quad , \tag{c_{0+}}$$

$$m_{0+}=\alpha\left(c^T_{0+}+\frac{c^N_{0+}}{e_{0+}}\right) \quad . \tag{m_{0+}}$$

2. The levels equations at time τ:

$$\varepsilon_\tau=\frac{\bar{g}-\bar{d}-r\bar{x}}{m_\tau} \quad , \tag{ε_τ}$$

$$c^T_\tau=c^N_\tau\cdot e^{-\sigma}_\tau\cdot\frac{\eta}{1-\eta} \quad , \tag{e_τ}$$

$$c^T_\tau=\left\{\frac{\gamma\cdot\eta^{\frac{1}{\sigma}}}{D_\tau\lambda(1+\alpha(r+\varepsilon_\tau))}\right\}^\sigma \tag{λ}$$

$$D_\tau=\eta^{\frac{1}{\sigma}}(c^T_\tau)^{\frac{\sigma-1}{\sigma}}+(1-\eta)^{\frac{1}{\sigma}}(c^N_\tau)^{\frac{\sigma-1}{\sigma}} \tag{D_τ}$$

$$y^T_\tau=\left(k^T\right)^{\rho^T}\left(h^T_\tau\right)^{1-\rho^T} \tag{y^T_τ}$$

$$y^N_\tau=\left(k^N\right)^{\rho^N}\left(h^N_\tau\right)^{1-\rho^N} \tag{y^N_τ}$$

$$\left(1-\rho^T\right)\left(k^T\right)^{\rho^T}\left(h^T_\tau\right)^{-\rho^T}=w_\tau \tag{h^T_τ}$$

$$\left(1-\rho^N\right)\left(k^N\right)^{\rho^N}\left(h^N_\tau\right)^{-\rho^N}=e_\tau w_\tau \tag{h^N_τ}$$

$$h^T_\tau+h^N_\tau=1-\frac{1-\gamma}{\lambda w_\tau} \tag{w_τ}$$

$$c^N_\tau=y^N_\tau \tag{c^N_τ}$$

$$m_\tau=\alpha\left(c^T_\tau+\frac{c^N_\tau}{e_\tau}\right) \quad . \tag{m_τ}$$

3. Resource constraints consistency conditions:

   - Lifetime Budget Constraint:

$$\left(1-e^{-r\tau}\right)\left(y^T_{0+}+d_{0+}-c^T_{0+}\right)+e^{-r\tau}\left(y^T_\tau+d_\tau-c^T_\tau\right)=0 \quad . \tag{c^T_τ}$$

- Central Bank Foreign Exchange Reserves (x_{0-} and m_{0-} are known from Step 1):

$$\left(x_{0-} + m_{0+} - m_{0-}\right) e^{r\tau} + \left(m_\tau - m_{0+}\right) + \frac{e^{r\tau} - 1}{r} \left(\bar{d} - \bar{g}\right) = \bar{x} \quad . \; (\tau)$$