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The role of high frequency intra-daily data, daily range and implied volatility in multi-period Value-at-Risk forecasting

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Abstract

In this paper, we assess the informational content of daily range, realized variance, realized bipower variation, two time scale realized variance, realized range and implied volatility in daily, weekly, biweekly and monthly out-of-sample Value-at-Risk (VaR) predictions. We use the recently proposed Realized GARCH model combined with the skewed student distribution for the innovations process and a Monte Carlo simulation approach in order to produce the multi-period VaR estimates. The VaR forecasts are evaluated in terms of statistical and regulatory accuracy as well as capital efficiency. Our empirical findings, based on the S&P 500 stock index, indicate that almost all realized and implied volatility measures can produce statistically and regulatory precise VaR forecasts across forecasting horizons, with the implied volatility being especially accurate in monthly VaR forecasts. The daily range produces inferior forecasting results in terms of regulatory accuracy and Basel II compliance. However, robust realized volatility measures such as the adjusted realized range and the realized bipower variation, which are immune against microstructure noise bias and price jumps respectively, generate superior VaR estimates in terms of capital efficiency, as they minimize the opportunity cost of capital and the Basel II regulatory capital. Our results highlight the importance of robust high frequency intra-daily data based volatility estimators in a multi-step VaR forecasting context as they balance between statistical or regulatory accuracy and capital efficiency.

Jel Classification: C13; C53; C58; G17; G21; G32

Keywords: Realized GARCH; Value-at-Risk; multiple forecasting horizons; alternative volatility measures; microstructure noise; price jumps.

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1. Introduction

Precise assessment of financial risks plays a crucial role for the viability of the financial institutions and the stability of the financial system as a whole. It helps minimizing the probability of extensive periods of financial distress which may be triggered by the failure of systemically important financial institutions. Obviously, the importance of accurate risk measurement and assessment is augmented during highly volatile periods, such as the recent 2007-2009 financial crisis, for which there is a widespread risk of global financial instability (Drakos *et al*., 2010).

This study concentrates on market risk which is defined as "the risk to a financial portfolio from movements in market prices such as equity prices, foreign exchange rates, interest rates, and commodity prices" (Christoffersen, 2003). The most popular market risk management tool in the financial services industry is the so called Value-at-Risk (VaR), which reflects an asset's market value loss not be exceeded over a specified holding period, with a specified confidence level (Alexander, 2008b). According to Giot and Laurent (2003b), the popularity of the VaR as a market risk measure can be attributed mainly to three reasons. First, VaR is relative simple to estimate – statistically, the $\alpha\%$ VaR is α-th quantile of the conditional returns distribution. Second, VaR is easy to communicate to higher level management as it encapsulates in a single quantity, either percentage or nominal amount, the potential portfolio losses. Third, the 1996 market risk amendment to the Basel Capital Accord and the Basel II regulatory framework, allows financial institutions to use their own internal VaR models for the calculation of market risk capital requirements (see also subsection 3.3) (BCBS, 1996a, 2006). Thus, the extant regulatory framework establishes the VaR as the benchmark method for market risk estimation.

The recent 2007-2009 global financial crisis and its subsequent widespread consequences in the real economy highlighted once again the key role of financial volatility in financial assets' risk management. During this turbulent period, characterized by extreme asset price movements and high volatility in financial markets, the majority of financial institutions failed to comply with the Basel Committee on Banking Supervision (BCBS) mandates regarding the accuracy of their VaR estimates (Campel and Chen, 2008). This example from the near past financial history underlines the need for accurate volatility measurement and forecasting and justifies the intensive research efforts on measuring, modeling and forecasting financial volatility during the last three decades .

In this study, we fill the gaps in the VaR related literature (see section 2) and we investigate the informational content of three alternative classes of volatility measures in terms of multi-period VaR forecasting using the S&P 500 stock index. The three volatility classes are: (i) Range-based volatility estimators that employ the daily range, i.e. the difference between the highest and lowest logarithmic prices within the trading day, and particularly the range estimator of Parkison (1980). (ii) Realized volatility estimators, that utilize high frequency intra-daily returns. In this category we use the realized variance (Andersen and Bollerslev, 1998; Andersen *et al*., 2001a), the realized bipower variation which is robust against price jumps (Barndorff-Nielsen and Shephard, 2004), the two time scale realized variance of Zhang *et al*. (2005) which accounts for the microstructure noise bias in the price process and the realized range of and Christensen and Podolskij (2007) and Martens and van Dijk (2007). (iii) Implied volatility as measured by the VIX implied volatility index (Giot, 2005; Giot and Laurent, 2007). Each of the abovementioned volatility estimators is based on different assumptions and informational sets and differs in terms of efficiency, consistency and probably the ability to forecast the unobserved volatility (Brownless and Gallo, 2010).

Here, we differentiate from previous works (see Section 2 for the related literature) and we concentrate solely on the ability of the alternative volatility measures to deliver accurate and efficient multi-step VaR forecasts. This practical approach for the evaluation of the informational content of the various volatility measures requires the use of analogous evaluation metrics. Thus, we do not restrict ourselves to only statistical accuracy evaluation of the VaR forecasts, i.e. via the (un)conditional coverage tests of Christoffersen (1998), but we also use metrics that account for the regulatory accuracy (Lopez, 1999) and the capital efficiency (Sharma *et al*., 2003) of the VaR estimates. Finally, we also evaluate the alternative volatility measures in a real-world setting utilizing the formula for the market risk capital requirements prescribed by the BCBS (1996a, 2006).

Modeling the alternative volatility measures is another important issue of concern. The most common approach is to use these volatility measures as lagged explanatory variables in a GARCH model (Bollerslev, 1986) i.e. a GARCH-X model (e.g. Engle, 2002; Blair *et al*., 2001; Giot, 2005; Fuertes *et al.*, 2009; Corrado and Truong, 2007). However, the GARCH-X model poses important limitations on our analysis as it can only produce day-ahead volatility or VaR forecasts. Therefore, we use the novel Realized GARCH model proposed by Hansen *et al.* (2011) and implemented in day-ahead VaR forecasts by Watanabe (2011), which is capable of generating multi-period forecasts (see subsection 3.1). The Realized GARCH model is a relative simple to estimate model that has the unique feature of joint modeling of realized volatility and conditional volatility of returns, through a bivariate equation approach. Thus, the Realized GARCH model eliminates the need for the two-step procedure usually implemented in the realized volatility – VaR studies (Giot and Laurent, 2004; Brownless and Gallo, 2010). The name of the model reflects its structure, which is similar to a GARCH model, and the fact that incorporates realized volatility measures. Of course the model can be easily extended to allow for alternative volatility measures.

Moreover, we follow Watanabe (2011) and we combine the Realized GARCH model with the skewed student distribution for the innovations process which captures both the fat tails and the asymmetry properties of the financial assets returns distribution (Fernadez and Steel, 1998; Lambert and Laurent, 2001). These attractive characteristics and the good empirical performance have popularized its use in day-ahead VaR applications (e.g. see Giot and Laurent, 2003a; Giot and Laurent, 2003b; Giot and Laurent, 2004 and Giot, 2005 among others). However, forecasting VaR at multi-period horizons is much more challenging than the day-ahead forecasts. The reason is that we are not aware of the analytical form of the multiple horizons returns density (Christoffersen, 2003). Hence, we utilize the numerical Monte Carlo simulation method in order to estimate the multi-period VaR forecasts (Christoffersen, 2003; Andersen *et al.*, 2006)

The rest of the paper is organized as follows. In Section 2 we provide the related literature, while in Section 3 we describe the econometric methodology and the VaR evaluation metrics used in this study. Section 4 presents the estimation results for the Realized GARCH model and the VaR forecasting evaluation results. Section 5 summarizes and concludes this paper.

2. Related literature

A plethora of volatility measures and models have been used and tested in VaR estimation and forecasting. The seminal Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982), which uses the past squared daily returns in order to model the conditional heteroscedasticity of financial assets returns, and its numerous extensions (e.g. see “Glossary to ARCH (GARCH)” by Bollerslev, 2010) have been widely used in the VaR literature (e.g. see Brooks and Persand, 2003; Angelidis *et* al., 2004; Kuester *et al*., 2006, Drakos *et al*., 2010 amongst others).

Nonetheless, since the introduction of the ARCH-type models, many alternative volatility measures and models have been proposed for forecasting volatility and VaR. In particular, daily range volatility estimators (e.g. see Parkison, 1980; Garman and Klass, 1983; Rogers and Satchell, 1991) have also been employed in volatility (Chou, 2005; Li and Hong, 2011) and VaR (Brownless and Gallo, 2010) forecasting studies. In these studies the range-based volatility models outperform their GARCH counterparts. The intuition behind this result is that intraday price ranges contain more information than the squared returns, since the latter are computed from two arbitrary points in time, i.e. the closing prices (Chou *et al*., 2010). In Corrado and Truong (2007), the authors find that the daily range and the implied volatility, as measured by the VIX, VXO, VXN and VXD implied volatility indices, have similar volatility forecasting performance. Chou *et al*., (2010) provide a good review on range-based volatility estimators, models and their financial applications.

In the seminal papers of Andersen and Bollerslev (1998), Andersen *et al*. (2001a), Andersen *et al*. (2001b) and Barndorff-Nielsen and Shephard (2002) the authors propose the sum of squared intra-daily returns, the so called realized volatility, as an efficient and consistent estimator of the latent volatility and they establish its asymptotic properties. The strong theoretical foundations combined with the availability of high frequency intra-daily data initiated a frenzy of research on realized volatility modeling and forecasting. In most of the volatility or/and VaR forecasting empirical studies, the authors compare the ARCH-type to the realized volatility models, or in other words, they examine if there is incremental information in high frequency intra-daily returns compared to the daily squared returns.

In volatility forecasting studies the results are unequivocal; realized volatility models clearly outperform their ARCH-type counterparts (e.g. see Andersen *et al*., 2003; Koopman *et al*., 2005; Martens *et al*., 2009; Martens, 2002 amongst others). In VaR forecasting studies the results are somewhat mixed with some authors providing evidence in favor of the realized volatility models (Beltratti and Morana, 2005; Shao *et* al., 2009; Brownless and Gallo, 2010; Watanabe, 2011;

Louzis *et al*., 2011) and others reporting results in support of the ARCH-type models (Giot and Laurent, 2004; Angelidis and Degiannakis, 2008; Martens *et al*., 2009).

Brownless and Gallo (2010) are the first to emphasize on the ability of alternative realized volatility estimators to produce accurate day-ahead VaR forecasts. Using a P-spline multiplicative error model (MEM) (for MEM models see Engle, 2002; Engle and Gallo, 2006), the authors compare the predictive ability of realized variance, realized bipower variation, two time scale realized variance, realized kernels (Barndorff-Nielsen *et al*., 2008) and daily range for three NYSE stocks. Their results indicate that daily range and realized volatility estimators have comparable forecasting behavior with volatility estimators that are robust against microstructure noise performing relatively better. Moreover, Shao *et al.* (2009) provide evidence in favor of the realized range compared to the realized volatility estimators in daily VaR forecasts. Watanabe (2011) also report that the microstructure noise does not affect the accuracy of daily VaR forecasts for the S&P 500 stock index. Alternative volatility estimators have also been used in a volatility forecasting context. The majority of these studies favour the use of realized variation measures that employ absolute intraday returns and, thus, can mitigate the effect of price jumps (Forsberg and Ghysels, 2007; Ghysels *et al*., 2006; Ghysels and Sinko, 2006; Liu and Maheu, 2009; Fuertes *et al.*, 2009; Louzis *et al*., 2012).

Another vigorously researched area in financial economics is concerned with the informational content of implied volatility, which is deduced from options prices (see Alexander, 2008a). The majority of the studies in this area compare the volatility forecasts delivered by models utilizing implied volatility measures with the forecasts generated by models employing either daily range/squared returns or realized volatility estimators (e.g. see Blair *et al*., 2001; Christensen and Prabhala, 1998; Corrado and Miller, 2005; Corrado and Truong, 2007; Giot and Laurent, 2007 amongst others). Overall, implied volatility measures tend to outperform their historical volatility counterparts in terms of volatility forecasting (Poon and Granger, 2003; Giot, 2005). However, despite its good forecasting performance, implied volatility measures have not been broadly examined in risk management applications. The only exception is Giot (2005) who is the first to investigate the predictive ability of implied volatility indices (the old VIX, VXO and VXD corresponding to the S&P 500, S&P 100 and Nasdaq stock indices) in daily VaR forecasts. His empirical analysis shows that implied volatility indices can provide accurate day-ahead VaR forecasts when combined with the skewed student distribution.

Against this background, our work complements and extends previous empirical studies (e.g. Brownless and Gallo, 2010; Giot, 2005; Watanabe, 2011; Shao *et* al., 2009) for several aspects. First, this the first study that examines simultaneously the informational content of realized volatility, daily range and implied volatility measures in a VaR forecasting context. Second, with the exception Beltratti and Morana (2005), all other studies focus on day-ahead VaR forecasts. Here, we also investigate the predictive ability of the alternative volatility measures at weekly, biweekly and monthly horizons. Multi-step VaR estimates are quite important for both regulatory, as Basel II framework requires the computation of 1% VaR for a ten-days holding period, and internal risk management purposes. Third, we estimate the Realized GARCH using volatility measures other than the realized volatility. Particularly, we extend the Realized GARCH model in order to incorporate the realized bipower variation, the realized range, the daily range and the implied volatility measures. Fourth, we propose the use of a Monte Carlo simulation technique in conjunction with the Realized GARCH model in order to obtain the multiple horizon returns density. Finally, we investigate the out-of-sample VaR forecasting ability of the alternative volatility measures using eight years of the S&P 500 stock index which include the challenging 2007-2009 crisis period. The empirical results concerning this period are quite limited.

3. Volatility measures

In this section we briefly describe the three distinct categories of volatility measures employed in this study. The first category comprises the *daily range* volatility estimator of Parkison (1980) given by $RNG_t = \left(1/4\log 2\right)\ \left(\log H_t - \log L_t\right)^2$, where H_t and L_t are the high and low daily prices respectively. Range-based volatility estimators have certain appealing features in real-world applications. They are simple to estimate, more efficient than the squared returns volatility proxy and robust against the microstructure noise bias (e.g. see Parkison, 1980; Garman and Class, 1980; Alizadeh *et al*., 2002; Shu and Zhang, 2006).[1]

In the second category we examine volatility estimators that utilize ultra high frequency intra-daily data. The most popular estimator in this category is the *realized variance* (RV)

[1] Microstructure noise may take the form of bid-ask bounce, screen fighting, price discreteness and irregular trading (Fuertes *et al.*, 2009).

(Andersen and Bollerslev, 1998; Andersen et al., 2001a; Barndorff-Nielsen and Shephard, 2002) calculated as $RV_t = \sum_{j=1}^{M} \left(r_{t,j}\right)^2$, where M is the total number of intraday returns for each day t and $r_{t,j}$ is the *j*th intraday return of day t. Under certain semi-martingale assumptions and for $M \to \infty$, RV converges in probability to the quadratic variation of the price proces i.e. $RV_t \overset{p}{\to} QV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{t-1<s\leq t} \kappa^2(s)$, where the first part of the summation is the continuous path component, or integrated variance (IV_t) and the second part is the sum of squared jumps.

The *realized bipower variation* (RBV) of Barndorff-Nielsen and Shephard (2004) is a robust estimator against price jumps and is given by $RBV_t = (\pi/2)\sum_{j=k+1}^{M} \left|r_{t,j}\right|\left|r_{t,j-k}\right|$, where $k = 1$. The authors show that the RBV converges in probability to the continuous component of the quadratic variation of the price process, i.e. $RV_t \overset{p}{\to} IV_t = \int_{t-1}^{t} \sigma^2(s)ds$ and thus prove its immunity to price jumps.

The *two time scale realized variance* (TTS-RV) estimator of Zhang *et al.*, (2005) utilizes both high and low frequency intra-daily data combined with sub-sampling in order to eliminate the microstructure noise bias. Their estimator is $TTS\text{-}RV_t = (1/\Lambda)\sum_{\lambda}^{\Lambda} RV_{(\lambda),t} - (\bar{M}_f / M_f) RV_{f,t}$, where $RV_{(\lambda),t}$ is the low frequency realized variance estimator for the subsample λ, given that the full gird of the high frequency returns is partitioned into $\lambda = 1,...,\Lambda$, non-overlapping sub-grids (e.g. $\Lambda = 5$ if the low frequency is 5 minutes), $RV_{f,t}$ is the high frequency realized variance estimator using the full grid, $\bar{M}_f = (M_{(\lambda)} - \Lambda + 1)/\Lambda$ is the average number of observations in the subsamples and $M_{(\lambda)}$ is the total number of intraday observations in each subsample set λ. The rationale behind the TTS-RV estimator is that since the microstructure noise induced bias of the low frequency estimates is a function of the noise variance in the return processes, which is in turn consistently estimated by the high frequency realized variance estimator, we can use the latter in order to eliminate the low frequency estimator bias.

The *realized range* (RR) of Christensen and Podolskij (2007) and Martens and van Dijk (2007) is the 'realized version' of Parkison's range volatility estimator and is calculated as $RR_t = \left(1/4\log 2\right)\sum_{j=1}^{M}\left(\log h_{t,j} - \log l_{t,j}\right)^2$, where $h_{t,j}$ and $l_{t,j}$ are the high and low prices of the *j*th time interval respectively. In Martens & van Dijk (2007), the authors propose the use of a scaling factor in order to adjust the RR estimator for the microstructure noise bias. The scaling factor is a ratio of the average value of daily range, RNG_t, and the average value of RR and the adjusted RR is given by $RR_t = \left(\sum_{k=1}^{66} RNG_{t-k} \Big/ \sum_{k=1}^{66} RR_{t-k} \right) RR_t$. Martens and van Dijk (2007) show that the scaled RR outperforms the TTS-RV estimator in terms of efficiency.

Finally, in the third category, we assess the informational content of the *implied volatility* (IV) measure which is deduced from options prices. We follow Giot (2005), Giot and Laurent (2007) and Becker *et al*. (2009) and we use the VIX index, provided by the Chicago Board of Options Exchange (CBOE), as a model-free IV measure. The VIX index is computed from a number of put and call options on the S&P 500 index over a wide range of strike prices and is designed to provide market's expectation for the level of the S&P 500 volatility over the next 30 (22) calendar (trading) days(CBOE, 2003).[2] Since VIX is reported in an annualized standard deviation form, the daily IV measure is given by $IV_t = (1/365)\left(VIX_t\right)^2$ (Giot, 2005; Giot and Laurent, 2007).

4. Econometric methodology

4.1. The Realized GARCH model

The VaR forecasts are generated using the recently proposed Realized GARCH of Hansen *et al*. (2011). Assuming that $r_t = \log\left(P_t / P_{t-1}\right)$ are the daily logarithmic returns, where P_t is the closing price of day *t*, the AR(1)- logarithmic Realized GARCH(1,1) is defined as:

$$r_t = c + \phi_1 r_{t-1} + \sqrt{h_t} z_t \text{, with } z_t \sim i.i.d\ skst(0,1,\xi,v) \tag{1}$$

$$\tilde{h}_t = \omega + \beta \tilde{h}_{t-1} + \gamma \tilde{x}_{t-1} \tag{2}$$

$$\tilde{x}_t = \kappa + \pi \tilde{h}_t + \tau_1 z_t + \tau_2 \left(z_t^2 - 1\right) + \varepsilon_t \text{, with } \varepsilon_t \sim i.i.d\ N(0,\ \sigma_\varepsilon^2) \tag{3}$$

[2] We use the new VIX index of CBOE released on September 22, 2003. For technical details for the construction of the VIX index see CBOE White Paper (2003).

where z_t are distributed as a standardized skewed student (skst) distribution with $\nu > 2$ and $\xi > 0$ being the degrees of freedom and the asymmetry parameter respectively (Fernadez and Steel, 1998; Lambert and Laurent, 2001), $\tilde{h}_t = \log(h_t)$ and $\tilde{x}_t = \log(x_t)$ with $x_t = RNG_t, RV_t, RBV_t, TTS\text{-}RV_t, RR_t$ or IV_t being the volatility measures presented in section 2. The parameter τ_1 captures the asymmetric impact of negative shocks on volatility process, i.e. the leverage effects, and is expected to be negative, while τ_2 captures the size effects or volatility clustering i.e. the fact that large shocks tend to be followed by large shocks, and is expected to be positive. Finally, the errors in equation (3), ε_t, are normally distributed and mutually independent with z_t.

We chose to model the conditional mean in equation (1), i.e. $E\left(r_t \middle| \mathcal{F}_{t-1}\right) = c + \phi_1 r_{t-1}$, as an *AR(1)* process in order to account for any autocorrelation in the returns series while we follow Watanabe (2011) and we assume the skewed student distribution for the innovations distribution, which captures both the fat tails and the asymmetry properties of the financial assets returns density. Moreover, the use of the logarithms ensures the positivity of the conditional volatility estimates and retains the ARMA structure of the '*GARCH equation*' or equation (2).

Nevertheless, the key feature of the Realized GARCH model is equation (3) or '*measurement equation*'. It relates the volatility measures, x_t, with the latent conditional variance, $h_t = Var\left(r_t \middle| \mathcal{F}_{t-1}\right)$, and enables the joint modelling of volatility measures and returns, which is a very important aspect for empirical VaR applications. Moreover, in contrast with the GARCH-X model, the Realized GARCH model can produce multi-period volatility – and consequently VaR – forecasts via the measurement equation. For example, assume that we want to forecast the log conditional variance for $t+2$ i.e. $\tilde{h}_{t+2} = \omega + \beta\tilde{h}_{t+1} + \gamma\tilde{x}_{t+1}$. Hence, we need a forecast for $\tilde{h}_{t+1}$ which is easily given by $\tilde{h}_{t+1} = \omega + \beta\tilde{h}_t + \gamma\tilde{x}_t$ and a forecast for $\tilde{x}_{t+1}$ which is given by the measurement equation as follows: $\tilde{x}_{t+1} = \kappa + \pi\tilde{h}_{t+1} + \tau_1 z_{t+1} + \tau_2\left(z_{t+1}^2 - 1\right) + \varepsilon_{t+1} = \kappa + \pi\left(\omega + \beta\tilde{h}_t + \gamma\tilde{x}_t\right) + \tau_1 z_{t+1} + \tau_2\left(z_{t+1}^2 - 1\right) + \varepsilon_{t+1}$. Finally, the measurement equation accounts for possible biases of the alternative volatility measures. In general, we expect that an unbiased estimator of the true daily volatility will produce estimates of κ close to zero and of π close to one (see also the discussion in subsection 4.1).

All three equations' parameters are estimated simultaneously by maximizing the joint log likelihood of the Realized GARCH model, i.e.:

$$\log \mathcal{L}\left(r_t, x_t; \boldsymbol{\theta}\right) = \sum_{t=1}^{\mathrm{T}} \log f^{skst}\left(r_t \left| x_{t-1}; \boldsymbol{\theta}\right.\right) + \log f^{n}\left(x_t \left| x_{t-1}, r_t; \boldsymbol{\theta}\right.\right) \tag{4}$$

$$= \sum_{t=1}^{\mathrm{T}} \log \Gamma\left(\frac{\nu+1}{2}\right) - \log \Gamma\left(\frac{\nu}{2}\right) - 0.5 \log\left[\pi\left(\nu-2\right)\right] + \log\left(\frac{2}{\xi+\xi^{-1}}\right)$$

$$+ \log(s) - 0.5\left\{\log\left[\exp\left(\tilde{h}_t\right)\right] + \left(1+\nu\right)\log\left[1 + \frac{\left(sz_t+m\right)^2}{\nu-2}\xi^{-2Ind_t}\right]\right\}$$

$$-0.5\left[\log(2\pi) + \log(\sigma_\varepsilon^2) + \frac{\varepsilon_t^2}{\sigma_\varepsilon^2}\right]$$

where $\boldsymbol{\theta} = \left(c, \phi_1, \omega, \beta, \gamma, \kappa, \pi, \tau_1, \tau_2, \sigma_\varepsilon, \nu, \xi\right)'$ is the parameters vector, $f^{skst}\left(r_t \left| x_{t-1}; \boldsymbol{\theta}\right.\right)$ and $f^{n}\left(x_t \left| x_{t-1}, r_t; \boldsymbol{\theta}\right.\right)$ is the skst and the normal density function respectively, $\Gamma\left(.\right)$ is the gamma function, $m = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$ and $s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$ are the mean and the standard deviation of the non-standardized skst distribution respectively and Ind_t equals 1 if $z_t \geq -m/s$ and -1 otherwise.

We maximize the log likelihood function in equation (4) using the numerical optimization routines of OxMetrics 6. In particular, we implement the MaxSQPF maximization routine which utilizes the feasible sequential quadratic programming algorithm of Lawrence and Tits (2001). The standard errors of the parameters estimates, i.e. $\hat{\boldsymbol{\theta}}$, are approximated by $se(\hat{\boldsymbol{\theta}}) \cong \sqrt{diag\left[\left(-\left.\frac{\partial^2 \mathcal{L}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}\right)^{-1}\right]}$, where the second derivative matrix of the log likelihood is also calculated numerically (Hamilton, 1994). However, note that for the case of normally distributed innovations, z_t, Hansen *et al*. (2011) provide closed form solutions for the hessian matrix.

4.2. Value-at-Risk forecasting methodology

VaR reflects the asset's market value loss over the time horizon *h*, that is not expected to be exceeded with probability $1-\alpha$, i.e. $\Pr\left(r_{t+1:t+h} \leq VaR^{\alpha}\left(t+h|t\right)\middle|\mathcal{F}_t\right)=1-\alpha$, where $r_{t+1:t+h}$ is the cumulative asset's return over the period *h* and α is the significance or coverage level. Hence, α% VaR is the α-th quantile of the conditional returns distribution defined as: $VaR^{\alpha}\left(t+h|t\right) = F_{t+h}^{-1}\left(\alpha|\mathcal{F}_t\right)$, where F^{-1} is the returns inverse cumulative distribution function. Assuming that the returns process is described in equations (1)-(3), the α% next day's ($h=1$) VaR is given by: $VaR^{\alpha}\left(t+1|t\right)=\hat{c}+\hat{\phi}_1 r_t+\sqrt{\exp\left(\hat{\omega}+\hat{\beta}\tilde{h}_t+\hat{\gamma}\tilde{x}_t\right)}c_{\alpha,\hat{v},\hat{\xi}}^{skst}$, with:

$$c_{\alpha,v,\xi}^{skst}=\begin{cases}\left\{(1/\hat{\xi})c_{\alpha,\hat{v}}^{st}\left[\left(\alpha/2\right)\left(1+\hat{\xi}^2\right)\right]-\hat{m}\right\}/\mathrm{s} \;\text{ if }\; \alpha<1/(1+\hat{\xi}^2)\\ \left\{-\hat{\xi}c_{\alpha,\hat{v}}^{st}\left[\left((1-\alpha)/2\right)\left(1+\hat{\xi}^{-2}\right)\right]-\hat{m}\right\}/\mathrm{s} \;\text{ if }\; \alpha\geq 1/(1+\hat{\xi}^2)\end{cases} \tag{5}$$

where ^ denotes the maximum likelihood estimates obtained from maximizing (4) and $c_{\alpha,v}^{st}$ denotes the quantile function of the standardized Student-t density function (see Lambert and Laurent, 2001 and Giot and Laurent, 2003a).

In the absence of a closed form solution for the multi-period returns density, we rely on a Monte Carlo (MC) simulation approach for the multiple horizons VaR forecasts ($h>1$). The process is described in the following steps (Christoferssen, 2003, Andersen *et al*., 2006):

1. Set $h = 1$.
2. Produce the conditional variance forecasts for $t+h$: $\hat{h}_{t+h}=\exp\left(\hat{\omega}+\hat{\beta}\tilde{h}_{t+h-1}+\hat{\gamma}\tilde{x}_{t+h-1}\right)$. If $h = 1$ these forecasts are readily available, otherwise are obtained from step 7 below.
3. Generate $\breve{\alpha}_{i,h}$, $i=1,2,...,1,000$ random draws from the uniform distribution.
4. Replace α with $\breve{\alpha}_{i,h}$ in equation (5) and use the estimates $\hat{v}$ and $\hat{\xi}$ to generate $\breve{z}_{i,h}$ drawn from the $skst(0,1,\hat{v},\hat{\xi})$ distribution.

5. Create the hypothetical returns for $t+h$ as: $\breve{r}_{i,t+h} = \hat{c} + \hat{\phi}_1 r_{t+h-1} + \sqrt{\hat{h}_{t+h}}\,\breve{z}_{i,t+h}$.

6. Create the volatility measures forecasts for $t+h$ as: $\breve{x}_{i,t+h} = \hat{\kappa} + \hat{\pi}\tilde{h}_{t+h} + \tau_1 \breve{z}_{i,h} + \tau_2 \left(\breve{z}_{i,h}^2 - 1\right)$.

7. Create the conditional variance forecasts for $t+h+1$ as: $\breve{\tilde{h}}_{i,t+h+1} = \hat{\omega} + \hat{\beta}\tilde{h}_{t+h} + \hat{\gamma}\breve{\tilde{x}}_{i,t+h}$. These forecasts are inputs for the next step, i.e. step 2, while the volatility measures forecasts, $\breve{\tilde{x}}_{i,t+h}$, are available from the previous step.

8. Set $h = h+1$ and repeat steps 1 to 7 until $h = 20$.

Thus, for each i, $i = 1,2,...,1{,}000$, we generate $h = 1,2,...,20$ MC simulation paths of hypothetical returns from which we calculate the h-day cumulative hypothetical returns as $\breve{r}_{i,t+1:t+h} = \sum_{l=1}^{h} \breve{r}_{i,t+l}$, where h = 5, 10 and 20 for the weekly, biweekly and monthly forecast horizons. In this way we generate a hypothetical distribution of h-day returns generated by the process described in equations (1)-(3). Next, we calculate the h-day VaR as: $VaR^{\alpha}\left(t+h \middle| t\right) = Quantile\left\{\left\{\breve{r}_{i,t+1:t+h}\right\}_{i=1}^{1,000}, \alpha\right\}$.

4.3. VaR evaluation measures

The VaR evaluation measures implemented here build on the "failure process" described by the following indicator function $I_t = \mathcal{I}_{\left\{r_{t+1:t+h} < VaR^{\alpha}\left(t+h|t\right)\right\}}$, which takes the value of 1 if $r_{t+1:t+h} < VaR^{\alpha}\left(t+h \middle| t\right)$ and zero otherwise. We expect that an accurate VaR model will generate a *failure rate* (FR) i.e. $\hat{\alpha} = n_1 / n$, where n_1 and n are the number of exceptions and the sample size respectively, close to the predetermined coverage level, α.

Christoffersen's (1998) unconditional coverage test examines statistically if $\hat{\alpha} = \alpha$. Under the null hypothesis of accurate unconditional coverage, i.e. $E\left(I_t\right) = \alpha$ and given the assumption of independence between the exceptions, the *likelihood ratio* (LR) test is:

$$LR^{uc} = 2\log\left(\left(1-\hat{\alpha}\right)^{n_0} \hat{\alpha}^{n_1} / \left(1-\alpha\right)^{n_0} \alpha^{n_1}\right) \sim \chi^2\left(1\right) \tag{6}$$

The complementary conditional coverage test proposed by Christoffersen (1998) is a joint test of correct unconditional coverage and first order independence of the failure process against a first order Markov failure. The corresponding LR test is:

$$LR^{cc} = 2\log\left(\left(1-\hat{p}_{01}\right)^{n_{00}} \hat{p}_{01}^{n_{01}} \left(1-\hat{p}_{11}\right)^{n_{10}} \hat{p}_{11}^{n_{11}} / \left(1-\alpha\right)^{n_0} \alpha^{n_1}\right) \sim \chi^2\left(2\right) \quad (7)$$

where $p_{ij} = \Pr\left(I_t = i \middle| I_{t-1} = j\right)$ estimated as $\hat{p}_{ij} = n_{ij} / \sum_{j=0}^{1} n_{ij}$, with $i, j = 0,1$ and n_{ij} is the number of transitions from state i to state j. Note that, for both tests, the null hypothesis is rejected if the VaR model generates too many or too few exceptions while for the conditional coverage a VaR model may also be rejected if it generates too clustered exceptions.

For the h-step ahead forecasts, we follow Diebold *et al*. (1998) and Beltratti and Morana (2005) and we employ a test based on Bonferroni bounds. In particular, the series of exceptions in multi-step forecasts are inherently correlated and thus the coverage tests cannot be applied directly. Hence, we subsample the full series of exceptions in order to produce identically and independently distributed subseries of the form $\{I_1, I_{1+h}, I_{1+2h}, ...,\}$, $\{I_2, I_{2+h}, I_{2+2h}, ...,\}$, …, $\{I_h, I_{2h}, I_{3h}, ...,\}$. Then, the (un)conditional coverage test with size bounded by q is applied on each of the h subseries with size q/h i.e. we perform h tests with size q/h. A VaR model is rejected if it produces a p-value smaller than the q/h significance level for any of the subseries.

The statistical accuracy of a VaR model is prerequisite for a functional risk management system but it does not reassure the efficiency or the regulatory accuracy of the VaR estimates. Hence, we also employ a complementary set of evaluation statistics that reflect both regulators and risk managers' preferences.

Specifically, adhering to the Basel Committee's guidelines, supervisors are not only concerned with the number of failures of a VaR model, but also with the magnitude of these failures (BCBS, 1996a, 1996b). Thus, we use the *regulatory loss function* (RLF) of Lopez (1999) which considers both the number of exceptions and their magnitude and is given by: $RLF_t = \left\{1 + \left[r_{t+1:t+h} - VaR^{\alpha}\left(t+h\middle|t\right)\right]^2\right\} \mathcal{I}_{\left\{r_{t+1:t+h} < VaR^{\alpha}\left(t+h\middle|t\right)\right\}}$.

In the *firm loss function* (FLF) the non-exception days are penalized according to the opportunity cost of capital held by the firm for risk management purposes: $FLF_t =$

$\left\{1+\left[r_{t+1:t+h}-VaR^{\alpha}\left(t+h|t\right)\right]^{2}\right\}\mathcal{I}_{\left\{r_{t+1:t+h}<VaR^{\alpha}\left(t+h|t\right)\right\}}-cVaR^{\alpha}\left(t+h|t\right)\mathcal{I}_{\left\{r_{t+1:t+h}>VaR^{\alpha}\left(t+h|t\right)\right\}}$, where c is the firm's opportunity cost of capital (Sarma *et al*., 2003). Thus, an otherwise accurate model producing a limited number of small magnitude violations may be highly inefficient as high VaR estimates entail additional opportunity costs.

Finally, we adopt a real-world evaluation metric which is prescribed by the BCBS in the Basel II regulatory framework and it refers to the calculation of the market risk capital (MRC) requirements (BCBS, 2006, 718, LXXVI). The formula for the MRC requirements is a widely accepted method for evaluating alternative VaR models (e.g. see Ferreira and Lopez, 2005; Lopez, 1999; Sajjad et al., 2008 and the discussion therein) and is given by:

$$MRC_{t|t-1}=\max\left[VaR\$^{0.01}\left(t+10|t-1\right),\frac{k}{60}\sum_{i=1}^{60}VaR\$^{0.01}\left(t+10|t-i\right)\right] \qquad (8)$$

where $VaR\$^{0.01}\left(t+10|t-1\right)=P_{t-1}\left[1-\exp\left(VaR^{0.01}\left(t+10|t-1\right)\right)\right]$ is the 1% VaR in dollars for a ten days holding period (Ferreira and Lopez, 2005), k is a multiplier set by the BCBS's traffic light system. Specifically, the value of k is based on the number of 1% daily VaR exceptions over the previous 250 trading days. If the model produces 4 or less violations, then it is considered sufficiently accurate and the multiplier k takes its minimum value of 3 (green zone or green light models). If the model generates between 5 and 9 violations over the previous trading year then it is placed in the yellow zone. It is also considered acceptable for regulatory purposes, with k being set to 3.4, 3.5, 3.65, 3.75 or 3.85, for the corresponding exceptions in the interval $[5,9]$. A red zone or red light model is one which generates 10 or more exceptions and then k takes its maximum value of 4. In this case, the regulators can reject the VaR model and put a request to the financial institution to revise their risk management systems.

The predictive ability of the alternative volatility measures in terms of the QLF, the FLF and the MRC is also assessed via Hansen's (2005) Superior Predictive Ability (SPA) test which examines whether the null hypothesis that the benchmark model is not outperformed by any of its competitors is rejected or not. The forecasting performance of the benchmark model, model 0, with respect to model k is deduced from the loss function differential: $f_{t,k}=l_{0,t}-l_{k,t}$, where

$k = 1 \ldots j$ is the total number of competing counterparts. Under the null hypothesis and assuming stationarity for $f_{t,k}$, we expect that on average the forecasting loss function of the benchmark model will be smaller, or at least equal to that of model k. Thus, the null hypothesis can be stated as: $H_0: \max\limits_{k=1\ldots l} \mu_k = E\left(f_{t,k}\right) \leq 0$ and is tested through the following test statistic: $T_n^{SPA} = \max\limits_{k=1\ldots l} \frac{\sqrt{n}\overline{f}_k}{\sqrt{\mathrm{var}\left(\sqrt{n}\overline{f}_k\right)}}$, where $\overline{f}_k = \left(1/n\right)\sum_{t=1}^{n} f_{t,k}$ and $\mathrm{var}\left(\sqrt{n}\overline{f}_k\right)$ is the variance of $\sqrt{n}\overline{f}_k$. Both $\mathrm{var}\left(\sqrt{n}\overline{f}_k\right)$ and the test statistic *p*-values are consistently estimated via stationary bootstrapping.

5. Empirical analysis

5.1. The data set and estimation results

The (intra-)daily data set was obtained from Tick Data and consists of previous tick interpolated prices for the S&P 500 stock index over an approximately thirteen year period, from 1.1.1997 to 09.30.2009 ($T = 3{,}196$ trading days). For all realized volatility measures we use the standard five minutes sampling frequency (e.g. see Andersen *et al.*, 2001a) while for the TTS-RV estimator we use both one (high) and five (low) minutes sampling frequencies. All realized volatility estimators are based on six and a half (6.5) trading hours per day, from 08:30 to 15:00, which are interpreted as $M = 78\ (390)$ intraday returns for the five (one) minutes sampling frequency. The daily VIX index was downloaded from the CBOE site.

Table 1 reports the daily returns and volatility measures distributional properties. The return series exhibits negative skewness and fat tails, as expected, justifying the use of the skst distribution in equation (1). The average values of all volatility measures are relatively close fluctuating between 1.128 and 1.623. The RBV is the least noisy estimator among realized and range-based volatility estimators followed by the TTS-RV and RR estimators. All volatility measures are in line with the lognormality assumption since their logarithms (not shown here) are approximately normal. Figure 1 displays the daily prices, returns and volatility measures of the S&P 500 index.

[Insert Table 1 about here]

[Insert Figure 1 about here]

In Table 2, we present the Realized GARCH model maximum likelihood estimation results for the full sample, i.e. from 1.1.1997 to 09.30.2009. Overall, almost all coefficients estimates are statistically significant and their sign and magnitude are in accordance with the theoretical assumptions and the previously reported estimation results of Hansen *et al*. (2011) and Watanabe (2011). Starting form the conditional mean specification, we see that the autoregressive parameter, $\hat{\phi}_1$, is negative and statistically significant implying a negative autocorrelation in the returns series. The skst distribution parameters estimates $(\hat{\nu}, \hat{\xi})$ indicate fat tails and negative skewness ($\hat{\xi} < 1$) for the *z*'s density, while their estimates do not change significantly with use of alternative volatility measures. The estimates of τ_1 and τ_2 in the measurement equation align with the theory of leverage effects and volatility clustering.

Nonetheless, the most interesting results emerge from the estimation of κ and π parameters in the measurement equation. Specifically, across volatility measures the estimates of π are close to 1, $\hat{\pi} \approx 1$, which is an expected result since all volatility measures are '*roughly*' proportional to the unobserved conditional variance (Hansen *et al*., 2011). However, for all realized and range-based volatility measures the estimates of κ are much lower than zero indicating that volatility estimators that utilize intraday data are biased. This result can be simply explained from the fact that these measures are computed employing only the 6.5 active trading hours (08:30-15:00 in our case) of day *t*, whereas the conditional variance, h_t, refers to a 24 hours time period which spans from the closing time (15:00) of day $t-1$ to the closing time (15:00) of day *t*, since daily returns, r_t, are calculated using close-to-close prices (see Hansen et al., 2011 for a related discussion). Moreover, the RNG and the RBV estimators have the lowest estimates of κ. One possible explanation is that the theoretical foundation of Parkison's RNG estimator is based on the restrictive assumption of zero drift geometric Brownian motion which may result in biased estimates in real-world settings. Indeed, the authors in Alizadeh *et al*. (2002), Brand and Diebold (2003) and Shu and Zang (2006), based on simulation results, find evidence of downward bias for the RNG estimator. Furthermore, the RBV estimator is robust against jumps implying that on average is lower or equal to the quadratic variation of the price process as estimated by the RV (see section 3). Consequently, the κ estimates for the RBV are expected to be lower compared to those of the RV estimator. On the contrary, when we use the IV as a volatility proxy the estimation of κ is very close to zero ($\hat{\kappa} = 0.052$) and marginally statistically significant at a 10% significance level. This evidence indicates that the IV is almost an unbiased estimator of the daily conditional volatility. From the GARCH equation results it is also obvious that the IV measure has the greatest impact on future volatility ($\hat{\gamma} = 0.862$) compared to its realized/range counterparts while the persistence is also high with the persistence parameter being approximately 0.97 across volatility measures and very close to the estimation results reported by Hansen *et al*. (2011).

In Figure 2, we illustrate the Monte Carlo simulated returns using the RR volatility estimator for the last day of our sample i.e. 09/30/2009. The graph compares the simulated returns with the normal density of equal variance. For all four forecasting horizons the simulated returns are negatively skewed and possess fat tails relative to the normal density implying the inappropriateness of Gaussian assumption for the VaR applications.

[Insert Table 2 about here]

[Insert Figure 2 about here]

5.2. VaR forecasting evaluation results

We use a rolling window of approximately five years or 1,250 trading days in order to produce the out-of-sample VaR forecasts from 12.20.2000 to 09.30.2009.[3] Table 3 presents the FR and the p-values for the (un)conditional coverage tests for a 5% and 1% coverage level. We follow Beltratti and Morana (2005) and we report the lowest p-value obtained by the *h* (un)conditional coverage tests performed for each of the *h* subseries of exceptions. We reject the null hypothesis at a 0.05 significance level if the tests produce a p-value lower than 0.05/*h*. For instance, for the monthly forecasts, i.e. for $h = 20$, the null hypothesis of correct (un)conditional coverage is rejected if the LR test generates p-value lower than 0.0025.

[3] For this period we generate 1,946 daily, 1,941 weekly, 1936 biweekly and 1926 monthly out-of-sample VaR forecasts.

The most striking feature of Table 3 is that all volatility measures, with the exception of the RNG for the 5% day-ahead VaR, can produce VaR forecasts with correct (un)conditional coverage at a 5% significance level, across forecasting horizons and quantiles. This implies that a risk manager will be indifferent, in terms of statistical accuracy, among the volatility measures examined here. However, a closer examination of the results reveals some interesting points. First, for the one and ten day(s)-ahead VaR forecasts and for the 1% quantile, which bear the greatest practical interest (see in the subsection 3.3 the BCBS mandates for the MRC requirements), the TTS-RV, RBV and IV (only for the one day ahead horizon) are the best performers. The IV measure tends to be over-conservative when we examine the ten days ahead predictions. Moreover, the results for the one day-ahead forecasts are in line with the findings of Giot (2005) and Brownless and Gallo (2010) who report good day-ahead VaR performance using either range/realized or implied volatility estimators. Second, the IV is the overall best performing volatility measure, as it ranks first in sixteen out of the twenty four cases across forecasting horizons, quantiles and evaluation metrics. In addition, the IV measure forecasting performance improves, on average, as the forecasting horizon increases, with monthly FRs produced by the IV model being much closer to the predetermined coverage level than those produced by its counterparts. Although this is an expected result, as the VIX index is computed in order to deliver market's volatility prospects over the subsequent trading month, it is a unique empirical finding in market risk literature. Finally, RNG and RR seem to be the weaker performers especially in longer-term forecasting horizons.

[Insert Table 3 about here]

Table 4 summarizes the results for the average RLF which takes into account both the number and the magnitude of the VaR failures. We also report the SPA test results in order to discern which of the models cannot be outperformed by its counterparts in terms of the RLF metric. The empirical findings are in alignment with the ones presented in Table 3. In particular, the RNG volatility estimator has the poorest performance as it generates the highest average RLF across almost all forecasting horizons and quantiles. As a consequence the null hypothesis of the SPA test is rejected at a 5% significance level across all (monthly) forecasting horizons for the

5% (1%) quantile.[4] On the other hand, IV model seems to have an adequate performance across forecasting horizons as it does not reject the SPA test hypothesis, but for the 5% day-ahead VaR predictions. The IV is again the best performing measure for the longest (monthly) forecasting horizon since it minimizes the RLF for both coverage levels. With the exception of the RR and the RBV for the 5% bi-weekly and monthly VaR forecasts respectively, in all other cases the realized volatility measures behave quite well as they are not outperformed by any of their counterparts. Nonetheless, the RV model has the most consistent behaviour as it ranks first for the one, five and ten-days ahead forecasts irrespective of the quantile used.

[Insert Table 4 about here]

The picture is different regarding the efficiency of the VaR forecasts as measured by the average FLF. The empirical findings, presented in Table 5, suggest that when we account for the opportunity cost of capital, the IV and the RV measures are the worst performers. They generate the overall highest average FLF and reject the SPA test hypothesis irrespective of the horizon examined, with the only exception being the 5% day-ahead and monthly VaR forecasts for the IV and the RV measure respectively. On the contrary, RNG, TTS-RV, RBV and RR models produce the most efficient weekly, biweekly and monthly VaR forecasts as they are not outperformed by any other volatility measure according to the SPA test results (the only exception is the RR model for the 5% biweekly VaR forecasts). For the 5% and 1% day-ahead VaR forecasts RR and RNG respectively are the only historical volatility measures that do not reject the null hypothesis of the SPA test. A possible explanation is that these volatility estimators are robust against either the microstructure noise bias or the price jumps and thus, they can mitigate extreme or noisy price movements. Consequently, the models incorporating these volatility measures produce moderate VaR estimates that help minimize the opportunity cost of capital.

[Insert Table 5 about here]

[4] In the spirit of Sarma et al. (2003) for the 5% quantile we do not include the RNG model in the SPA test as it does not pass the unconditional coverage tests (see Table 3)

The results for the MRC requirements, presented in Table 6, confirm the aforementioned findings. The RNG volatility estimator is the worst performer in terms of regulatory accuracy as it is the only volatility measure that produces red light days. All other volatility proxies comply with the regulators accuracy rules. In terms of efficiency, the RR and RBV volatility measures generate the lowest regulatory capital with the other two realized volatility estimators lagging closely behind. The highest regulatory capital is generated by the IV measure indicating its relative inefficiency. The SPA test results also confirm these findings. Figure 3 shows the market risk capital requirements estimates for the six alternative volatility measures. The graph reveals that the capital requirements increase considerably during the 2007-2009 crisis and that the IV measure generates the highest regulatory capital reserves especially from 2003 to 2007 and after the end of 2009.

[Insert Table 6 about here]

[Insert Figure 3 about here]

Table 7 summarizes the average performance of the alternative volatility measures across forecasting horizons and quantiles. Overall, the best performing volatility measures are the RR and the RBV as they manage to combine statistical accuracy, regulatory compliance and capital efficiency, while the TTS-RV is also a good alternate. The IV measure behaves very well in terms of accuracy (both statistical and regulatory), especially in long term forecasts, but it tends to produce inefficient VaR estimates. The RV measure has similar behavior with the IV measure, while the RNG volatility estimator demonstrates inferior forecasting performance.

[Insert Table 7 about here]

6. Conclusions

We complement and extend the previous VaR literature by examining the informational content of daily range, realized variance, realized bipower variation, two time scale realized variance, realized range and implied volatility in a multi-step VaR forecasting context. In our analysis we use the recently proposed Realized GARCH model of Hansen et al. (2011) which allows for the joint modelling of alternative volatility measures and returns. The Realized GARCH model is combined with the skewed student distribution, which captures the fat tails and the asymmetry of returns distribution, and a Monte Carlo simulation methodology for the multi-step VaR forecasts.

Based on the S&P 500 stock index and on an approximately eight years of out-of-sample forecasts, including the turbulent period 2007-2009, we find that almost all volatility measures can produce statistically accurate multi-step VaR forecasts. The only exception is the daily range for the 5% day-ahead VaR forecasts. Our empirical findings are in accordance with Giot (2005) and indicate that the implied volatility measure is a good alternative volatility estimator for market risk applications, especially for longer term (monthly) VaR predictions.

The results for a loss function that reflects regulators' preferences are in alignment with the statistical accuracy results. However, when we employ a loss function that considers the opportunity cost of capital the results are slightly different. Now, daily range and high frequency data volatility estimators that are robust against either the microstructure noise bias or the price jumps generate the most efficient VaR estimates that minimize the opportunity cost of capital.

A real-world application based on Basel II regulatory framework confirms the above mentioned findings. In particular, all volatility measures, except for the daily range, comply with the regulators' mandates regarding the number of exceptions during the previous trading year. Moreover, the adjusted realized range (Martens and van Dijk, 2007) and the realized bipower variation (Barndorff-Nielsen and Shephard, 2004), which are robust against microstructure noise and price jumps respectively, minimize the market risk capital requirements and thus, the released idle capital can be used in more efficient and productive ways.

Therefore, a risk manager or a regulator who emphasizes on statistical and regulatory precision of the VaR estimates will be indifferent among the realized or the implied volatility measures examined here and perhaps he will choose the implied volatility for the monthly forecasting horizons. Nonetheless, a risk manager who is more concerned with efficiency issues, without disregarding the importance of statistical accuracy and regulatory compliance, he will concentrate on realized range and realized bipower variation measures.

Our empirical results give evidence in favour of robust high frequency intra-daily data volatility estimators since they balance between statistical or regulatory accuracy and capital efficiency. However, further research is required in order to gain more insights on the informational content of alternative volatility measures in multi-period VaR forecasting using other stock indices or asset classes such as stocks, bonds, currencies or commodities.

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Table 1 Distributional properties of daily returns and alternative volatility measures of the S&P 500 stock index

	Mean	St.Dev	Skewness	Kurtosis	Max	Min
Daily returns (%)	0.011	1.344	-0.089	6.770	10.714	-9.789
RNG	1.277	2.769	8.153	88.455	42.807	0.021
RV	1.335	2.718	10.857	208.011	75.364	0.053
RBV	1.128	2.288	9.337	133.231	51.707	0.047
TTS-RV	1.181	2.531	14.940	396.058	84.474	0.049
RR	1.286	2.612	11.394	215.305	71.759	0.058
IV	1.623	1.619	4.160	25.068	17.913	0.268

Notes: RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is VIX implied volatility index.

Table 2 Maximum likelihood estimation results for the AR(1)- log Realized GARCH(1,1) model

	RNG	RV	RBV	TTS-RV	RR	IV
c	0.004	0.008	0.011	0.007	0.009	0.000
	(0.251)	(0.551)	(0.736)	(0.472)	(0.589)	(0.028)
ϕ_1	-0.027*	-0.078***	-0.101***	-0.100***	-0.104***	-0.058***
	(-1.936)	(-4.448)	(-5.799)	(-5.718)	(-5.969)	(-4.122)
ω	0.134***	0.162***	0.238***	0.228***	0.196****	-0.044
	(13.328)	(11.990)	(14.576)	(13.883)	(12.402)	(-1.639)
β	0.773***	0.621***	0.596***	0.573***	0.554***	0.124***
	(62.810)	(36.587)	(33.641)	(31.709)	(29.882)	(8.376)
γ	0.192***	0.361***	0.383***	0.411***	0.431***	0.862***
	(17.799)	(19.905)	(20.418)	(20.892)	(21.048)	(33.563)
κ	-0.677***	-0.442***	-0.615***	-0.546***	-0.448***	0.052*
	(-26.664)	(-16.304)	(-22.205)	(-19.943)	(-16.031)	(1.781)
π	1.046***	0.982***	0.985***	0.973***	0.974***	0.979***
	(49.498)	(37.322)	(36.764)	(37.722)	(36.742)	(40.640)
τ_1	-0.024*	-0.096***	-0.129***	-0.121***	-0.131***	-0.090***
	(-1.657)	(-10.136)	(-13.841)	(-13.517)	(-15.689)	(-47.603)
τ_2	0.289***	0.098***	0.077***	0.080***	0.056***	0.008***
	(26.099)	(16.680)	(13.914)	(14.928)	(11.963)	(11.240)
σ_ε	0.642***	0.474***	0.473***	0.450***	0.427***	0.070***
	(79.771)	(79.823)	(79.650)	(79.689)	(79.661)	(79.580)
ν	11.875***	12.924***	13.040***	12.438***	12.406***	11.424***
	(5.131)	(4.797)	(4.758)	(5.012)	(5.073)	(5.421)
ξ	0.895***	0.886***	0.883***	0.880***	0.878***	0.891***
	(41.048)	(40.477)	(40.439)	(40.326)	(40.259)	(40.968)
Persistence:						
$\beta+\pi\gamma$	0.975	0.976	0.974	0.973	0.974	0.967
$\log\mathcal{L}$	-7,870	-6,859.	-6,846	-6,694	-6,521	-6,784

Notes: The t-statistics are in parenthesis. *, ** and *** indicates statistical significance at 10%, 5% and 1% significance level respestovely. For the estimation we use the full sample from 1.1.1997 to 09.30.2009. RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is VIX implied volatility index.

Table 3 Failure rates and Christoffersen's (un)conditional coverage tests results

Volatility measures	Daily (h = 1) FR(%)	UC	CC	Weekly (h = 5) FR(%)	UC	CC	Bi-weekly (h = 10) FR(%)	UC	CC	Monthly (h = 20) FR(%)	UC	CC
5% Coverage level												
RNG	6.22	**0.017**	0.058	5.92	0.096	0.190	5.66	0.177	0.134	7.16	0.078	0.196
RV	4.98*	0.975*	0.996	5.77	0.096	0.190	4.12	0.089	0.206	7.21	0.035	0.100
TTS-RV	5.09	0.860	0.894	6.08	0.096	0.190	4.27	0.089	0.206	7.36	0.078	0.100
RBV	5.04	0.942	0.997*	6.13	0.061	0.122	4.07	0.196	0.358*	7.31	0.078	0.123
RR	5.19	0.702	0.877	5.82	0.096	0.190	5.36*	0.292*	0.224	7.16	0.035	0.107
IV	5.65	0.195	0.232	4.84*	0.114*	0.214*	3.45	0.034	0.097	5.97*	0.170*	0.125*
1% Coverage level												
RNG	1.13	0.571	0.662	1.49	0.153	0.120	1.65	0.018	0.051	1.96	0.020	0.041*
RV	0.82	0.416	0.629	1.44	0.153	0.316*	0.77	0.455*	0.738*	2.16	0.020	0.041*
TTS-RV	0.98*	0.916*	0.402	1.49	0.066	0.117	0.98*	0.455*	0.738*	2.27	0.003	0.007
RBV	1.03	0.903	0.437	1.54	0.066	0.156	0.93	0.455*	0.742*	2.42	0.003	0.007
RR	0.82	0.416	0.629	1.65	0.066	0.120	1.80	0.018	0.051	2.37	0.003	0.007
IV	0.98*	0.916*	0.825*	1.08*	0.317*	0.083	0.72	0.455*	0.738*	1.65*	0.094*	0.041*

Notes: All forecasts are generated by the Realized GARCH model. RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is VIX implied volatility index. FR denotes the failure rate in percentage points, UC and CC are the p-values for the Christoffersen's unconditional and conditional coverage tests respectively. The bold faced figures indicate rejection of the null at 0.05/h significance level. The table reports the lowest p-value across the h sub-series of exceptions. The asterisk (*) indicates the best performing model i.e. the model with the closest FR to the prespecified coverage level (α) and the highest p-value.

Table 4 Average regulatory loss function (RLF) and superior predictive Ability (SPA) test results

Volatility measures	Daily (h = 1) RLF	Daily (h = 1) SPA-test	Weekly (h = 5) RLF	Weekly (h = 5) SPA-test	Bi-weekly (h = 10) RLF	Bi-weekly (h = 10) SPA-test	Monthly (h = 20) RLF	Monthly (h = 20) SPA-test
	5% Coverage level							
RNG	0.116	-	0.444	**0.046**	1.143	**0.024**	2.865	**0.013**
RV	0.082*	0.783*	0.383*	0.958*	0.658*	0.960*	2.492	0.325
TTS-RV	0.084	0.320	0.415	0.051	0.704	0.116	2.578	0.063
RBV	0.082	0.646	0.400	0.118	0.671	0.498	2.595	**0.010**
RR	0.084	0.284	0.391	0.493	1.089	**0.009**	2.609	0.104
IV	0.117	**0.013**	0.426	0.184	0.726	0.173	2.321*	0.858*
	1% Coverage level							
RNG	0.019	0.066	0.082	0.141	0.405	0.100	1.152	**0.049**
RV	0.013*	0.974*	0.068*	0.977*	0.178*	0.917*	0.806	0.645
TTS-RV	0.015	0.232	0.076	0.115	0.195	0.314	0.816	0.602
RBV	0.015	0.226	0.068	0.311	0.178	0.803	0.810	0.734
RR	0.013	0.651	0.065	0.116	0.382	0.055	0.822	0.512
IV	0.018	0.168	0.126	0.132	0.244	0.087	0.744*	0.836*

Notes: All forecasts are generated by the Realized GARCH model. RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is VIX implied volatility index. For the SPA test we show the p-values. The bold faced figures indicate rejection of the null at a 0.05 significance level. The asterisk indicates the best performing model i.e. the model with the lowest RLF and the highest SPA p-value.

Table 5 Average firm loss function (FLF) and superior predictive Ability (SPA) test results

	5% Coverage level							
Volatility	Daily (h = 1)		Weekly (h = 5)		Bi-weekly (h = 10)		Monthly (h = 20)	
measures	FLF	SPA-test	FLF	SPA-test	FLF	SPA-test	FLF	SPA-test
RNG	2.684	-	5.965	0.346	8.825	0.072	13.499	0.141
RV	2.829	**0.006**	6.024	**0.000**	8.591	**0.001**	13.364	0.072
TTS-RV	2.818	**0.022**	5.948	0.424	8.481*	0.725*	13.278*	0.957*
RBV	2.821	**0.030**	5.933	0.802	8.499	0.473	13.300	0.690
RR	2.818	0.052	5.932*	0.803*	8.837	**0.028**	13.289	0.795
IV	2.745*	0.571*	6.315	**0.000**	9.242	**0.000**	14.391	**0.000**
	1% Coverage level							
RNG	4.178*	0.616*	8.800*	0.615	12.509	0.373	17.943	0.080
RV	4.380	**0.000**	8.913	**0.000**	12.534	**0.000**	17.864	**0.000**
TTS-RV	4.371	**0.000**	8.832	0.150	12.405	0.510	17.726	0.106
RBV	4.364	**0.000**	8.803	0.794*	12.381*	0.924*	17.666	0.771
RR	4.375	**0.001**	8.806	0.695	12.526	0.252	17.658*	0.858*
IV	4.437	**0.000**	9.484	**0.000**	13.714	**0.000**	20.276	**0.000**

Notes: All forecasts are generated by the Realized GARCH model. RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is VIX implied volatility index. For the SPA test we show the p-vlaues. The bold faced figures indicate rejection of the null at a 0.05 significance level. The asterisk indicates the best performing model i.e. the model with the lowest FLF and the highest SPA p-value.

Table 6 Basel II market risk capital requirements

Volatility measures	Basel II zones			Basel II Capital requirements	
	Green (%)	Yellow (%)	Red (%)	Mean	SPA test
RNG	69.104	28.597	2.300	268.800	-
RV	81.250	18.750	0.000	263.968	**0.000**
TTS-RV	75.531	24.469	0.000	265.351	**0.000**
RBV	80.483	19.517	0.000	263.412*	0.515*
RR	75.531	24.469	0.000	263.567	0.479
IV	84.375	15.625	0.000	294.383	**0.000**

Notes: RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is the VIX implied volatility index. The table presents the percentage of days during the out of sample forecasting period that the model is placed in the green, yellow and red zone according to the Basel traffic light system, the average daily capital requirements and Superior Predictive Ability (SPA) test p-values. The bold faced figures indicate rejection of the null at a 0.05 significance level.

Table 7 Summary of the empirical results

Volatility measures	Statistical Accuracy (Coverage tests)	Regulatory Accuracy (RLF)	Capital Efficiency (FLF)	Basel II market risk regulatory capital: Accuracy (Number of exceptions during the previous trading year)	Basel II market risk regulatory capital: Efficiency (Minimization of the regulatory capital)
RNG	**Yes**	No	**Yes**	No	No
RV	**Yes**	**Yes**	No	**Yes**	No
TTS-RV	**Yes**	**Yes**	**Yes**	**Yes**	No
RBV	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
RR	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
IV	**Yes**	**Yes**	No	**Yes**	No

Notes: RNG is the daily range, RV is the realized variance, TTS-RV is the two time scale realized variance, RBV is the realized bipower variation, RR is the realized range and IV is the VIX implied volatility index. The table shows the average performance of the volatility measures across forecasting horizons and quantiles. A volatility measure is considered as inadequate (No) if it fails in 4 or more out of the total 8 forecasting schemes examined here, i.e. 4 forecasting horizons and 2 quantiles. For the Basel II market risk regulatory capital the results are based solely on Table 6.

Figure 1. S&P 500 stock index daily prices, returns (%), realized, range and implied variance measures

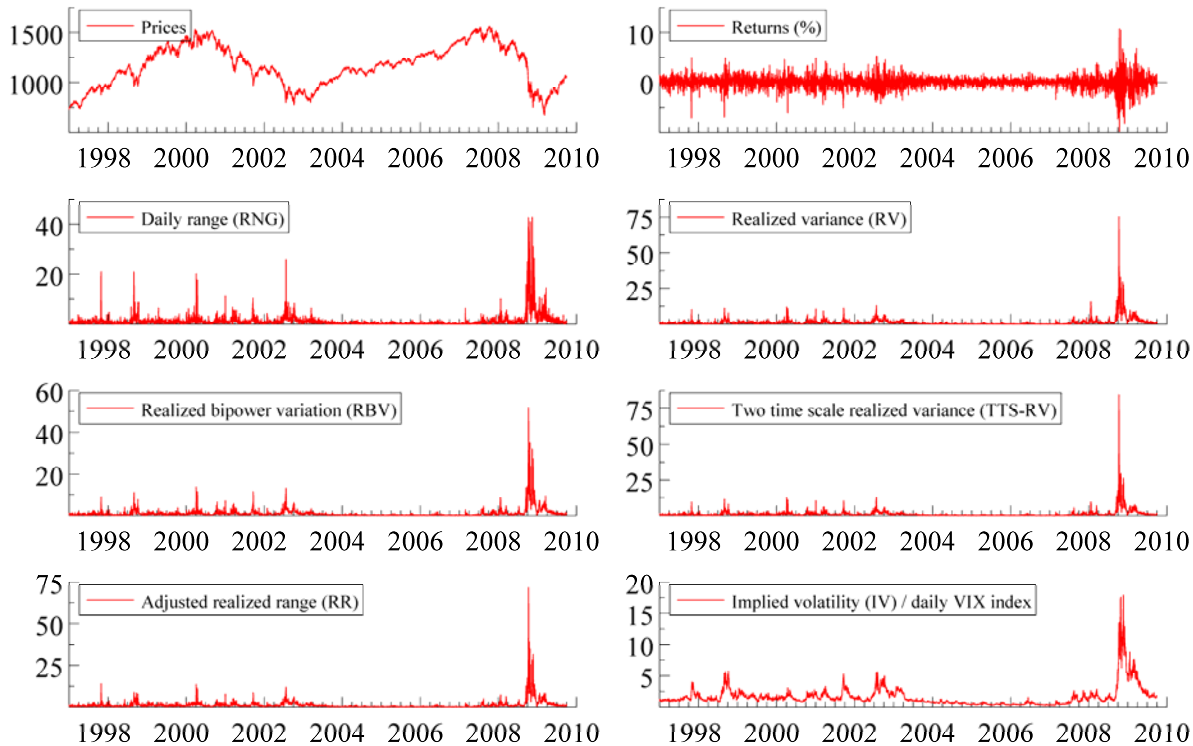

Figure 2. Monte Carlo simulated returns using the realized range (RR) volatility measure for the 09/30/2009

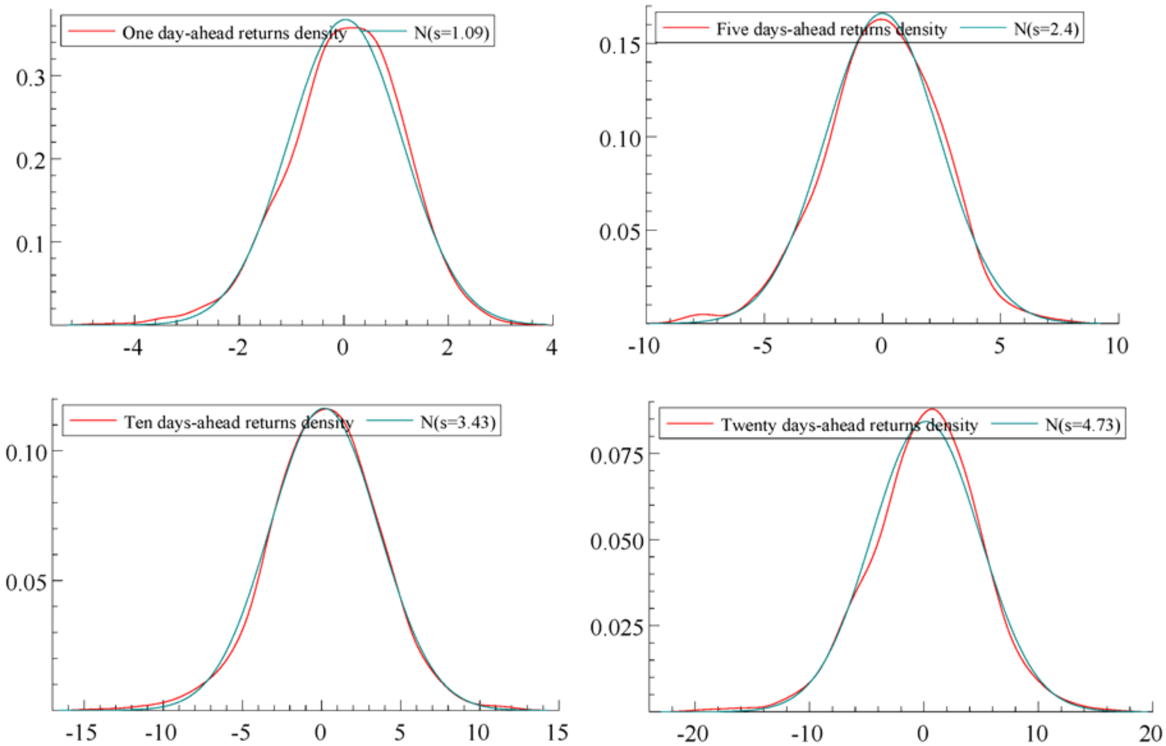

Figure 3. Basel II market risk capital requirements