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Cotter, John

University College Dublin

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JOHN COTTER[1]

University College Dublin

Abstract

Futures exchanges require a margin requirement that ensures their competitiveness and protects against default risk. This paper applies extreme value theory in computing unconditional optimal margin levels for a selection of stock index futures traded on European exchanges. The theoretical framework focuses explicitly on tail returns, thereby properly accounting for large levels of risk in measuring prudent margin levels. The paper finds that common margin requirements are sufficient for each contract, with the exception of the Norwegian OBX index, in providing equitable costs for traders. In addition, the paper shows the underestimation bias in margin levels that are calculated assuming normality. Differing margin requirements reflect the unconditional and conditional trading environments.

JEL Classification: G15

Keywords: Stock Index Futures; Extreme Value Theory; Margin Levels.

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[1] Tel.: +353-1-7068900; fax: +353 1 283 5482; e-mail: john.cotter@ucd.ie

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1. Introduction

The commercial strength of Futures Exchanges necessitates that there be a trade-off between optimising liquidity and prudence. The imposition of margins is the mechanism by which these objectives are met. The margin requirement represents a deposit that brokers, and consequentially traders enter into prior to trading. The margin is used to settle any profits or losses on a contract and to act as collateral against default. Default risk is incurred if the effect of the futures price change is at such a level that the investor's margin does not cover it, leading to non-payment by one of the parties to the contract.

The occurrence of default risk may lead to systemic problems thereby reducing investor confidence. Whilst a clearing house should guarantee compensation in the case of investor non-payment, this has not always been the situation as evidenced by defaults in Paris, Kuala Lumpar and Hong Kong over the past three decades (Booth et al, 1997).[2] At the same time, the total elimination of default risk through very high margins may not be desirable as it would discourage investor participation. Also, futures returns may be extremal in nature, that is specific values may be very large, and clearing houses would be reluctant to set the margin levels to cover these large returns for competitive reasons.[3]

[2] Individual high profile disasters including Barings and Orange County add to the lack of investor confidence in futures markets.

[3] Futures price changes, movements and returns are interchangeable for the purposes of this paper.

Instead, a compromise is set so that the margin covers a vast range of possible price movements with a relatively low probability that actual price changes exceed the margin. Thus an exchange's participants utility maximisation criteria involve similar preferences, that is, they want to safeguard the future of the exchange as well as having a competitive trading environment.

Whilst Duffie (1989) suggests that clearing houses impose margins based on a statistical analysis of price changes, actual margining systems have been introduced that also rely on other issues including open interest, volume of trade, concentration in futures positions and the margins of competing exchanges (Gay et al, 1986). Two alternative systems are Standard Portfolio Analysis of Risk (SPAN) and the Regulation T (Reg T). Kupiec and White (1996) compare these. They note that the former is based on simulating the effects of possible market conditions and imposing a margin that covers the resulting price changes, while the latter develops a strategic approach to the introduction of margins where margin size is based on a wide range of possible trading positions. Obviously both of these systems utilise subjective judgements as to the variables that should be included for simulation (SPAN) and the effects of particular trading positions on margin requirements (Reg T). A practical example of the use of judgement in margin setting is the approach taken in the London International Financial Futures Exchange through its clearing house, the London Clearing House. Here an extreme price movement is included in margin setting if it is believed that it may be repeated, and excluded if the view is that it will not be repeated (Vosper 1995). This leads to a lack of consistency which could be avoided if a statistical analysis of the variables that affect margin setting was adhered to. This paper focuses on the main proxy of futures volatility, namely their price changes, and provides a statistical mechanism to implement margin requirements on the basis of these price movements.

Two approaches to setting optimal margins in the finance literature have been implemented. First, development of economic models has taken place where the margin level is endogenously determined. For example, Brennan (1986) introduced a model for broker cost minimisation using the ratio between futures margins and the settlement costs due to losses. Second, and the focus of this paper, is the application of statistical guidelines that may evolve from a number of techniques. These procedures include parametric and non-parametric methods that rely on gaussian and non-normal assumptions for the underlying distribution of futures returns (Longin, 1999a; Dewachter and Gielens, 1999; Edwards and Neftci, 1988; and Gay et al, 1986).[4] The line of enquiry followed in this paper is statistical in nature relying on extreme value theory with non-parametric measures of the optimal margin level. Given, the true distribution of futures price changes being non-normal and in fact, unknown (Cotter and McKillop, 2000; Yang and Brorsen, 1993; and Hall et al, 1989), it is appropriate to examine the theoretical underpinnings of the asymptotic behaviour of a range of possible distributions. Leadbetter et al (1983) and Embrechts et al (1997) document a family of distributions that are separated into three distinguishing types, the assumption is that a sequence of values display asymptotic behaviour belonging either to a Gumbell, Frechet or Weibull distribution. In that the fitting of a sequence of price changes is not exact in nature, but rather, involves weak convergence, then the limiting distribution has non-parametric underlying

[4] The univariate analysis in this paper is similar to most of the decomposition of extreme price movements. Edwards and Neftci (1988) give an illustration of multivariate analysis.

assumptions. For this reason, a non-parametric tail statistic should be applied (Danielsson and de Vries, 1997c), and the Hill index is found to have the optimal estimation properties (Kearns and Pagan, 1997), and is therefore utilised in this study to generate measures of the margin levels. In addition, dynamic margin requirements accounting for current volatility levels are measured using a GARCH (1, 1) specification. Given the consistent finding of stochastic volatility for financial assets (Pagan, 1996), it is appropriate that each exchange incorporates the respective conditional distributional properties in their margin setting.

Whilst most individual exchanges have a common margin requirement regardless of the futures position, it may be necessary to distinguish between extreme price movements on either tail of the distribution of returns. Extreme price changes may not be symmetrical and this would require the imposition of separate margin requirements for long and short positions in order to optimise the trade-off between futures trading liquidity and their prudential control. Default due to extreme negative price changes occurs on a long position, whereas it occurs on a short position for extreme positive changes. Possible default through margin violation is illustrated for a short position in figure 1. As is evident small price changes are insufficient to cause default through margin exceedences whereas this is not the case for the very large positive returns. The non-parametric estimates of the optimal margin levels will be for left and right tail price movements, and a common measure that encompasses both sets of movements together. This latter measure is not necessarily an average of the positive and negative extreme price changes, but rather, a single margin level incorporating the most extreme price movements in its value regardless of its tail origin. Obviously, asymmetric tail returns impose inequitable margin levels on the two futures trading positions.

INSERT FIGURE 1

Two important issues in the determination of optimal margin requirements for a number of European stock index futures contracts are addressed in this paper. First, it applies extreme value theory and its asymptotic behaviour in the examination of large-scale price movements. It utilises the non-parametric Hill index to calculate the influence of extreme returns on margin requirements that should not be exceeded for a range of probability levels. Theoretically, the Hill index gives measures that correspond to the non-parametric nature of the extreme value distribution. Second, the paper examines whether the margin requirements on a long or short trading position should be different given the behaviour of upper and lower tail price movements. These tail movements might be significantly different and as a consequence, there should be two separate margin requirements in the interest of fairness to the trader.

The paper proceeds as follows. In section 2, extreme value theory is presented coupled with the non-parametric tail estimator that is used to generate probabilities of exceeding margin requirements given the price movements inherent in futures contracts. Justification of applying this approach in the context of margin setting is also given. Section 3 details the stylised facts of the stock index futures, and how these may influence the statistical approach taken in calculating margin requirements. Section 4 presents the empirical findings. Here, margin levels computed using extreme value theory and gaussian assumptions are presented for comparison.

Conditional margin requirements calculated using a GARCH (1, 1) model are discussed in the context of the Extreme Value findings. Also, an assessment is made on the issue of whether to have common margin levels or impose two separate requirements based on trading position for each exchange. Finally, a summary of the paper and some conclusions are given in section 5.

2. Theory and Methods

2.1. Extreme Value Theory

Margin requirements and measures of violation probabilities are developed using the theoretical framework of extreme value theory. The distributional assumptions are applicable through the maximum domain of attraction (MDA) allowing for approximation to certain distributional characteristics rather than belonging to a specific distribution (Leadbetter et al, 1983). Dealing with a sequence of futures returns, {R}, arranged in ascending order and expressed in terms of the maxima (M_n) of n random variables belonging to the true unknown cumulative probability density function F where

$$M_n = \max \{R_1, R_2,..., R_n\} \tag{1}$$

The corresponding density function of M_n is obtained from the cumulative probability relationship and this represents the probability of exceeding a margin level on a short position for n returns:

$$P_{short} = P\{M_n > r_{short}\} = P\{R_1 > r_{short}, \ldots, R_n > r_{short}\} = 1 - F_{max}^{\ n}(r_{short}) \tag{2}$$

r_{short} represents the margin level on a short position.

However, the random variables of interest to us are located at both upper and lower tails of the distribution $F^n(r)$ and whilst extreme value theory is usually notated for upper order statistics, it is equally applicable for lower order statistics. Lower tail price movements are relevant for margin requirements of a long position in a futures contract. The theoretical framework for examining sample minima tail statistics can easily be converted by applying the identity $\text{Min}\{R_1, R_2,..., R_n\} = -\text{Max}\{-R_1, -R_2,..., -R_n\}$. The corresponding probability expression for exceeding a margin level on a long position for n returns is:

$$P_{long} = P\{M_n < r_{long}\} = P\{R_1 < r_{long}, \ldots, R_n < r_{long}\} = F_{min}^{\,n}(r_{long}) \qquad (3)$$

r_{long} represents the margin level on a short position.

The rest of the extreme value theory framework is presented for the short position outlined in (2) but, as noted, it is equally applicable for a long position.[5]

2.1.1. Asymptotic Behaviour of Distribution

The Fisher-Tippett theorem is used to examine asymptotic behaviour of the distribution. From this theorem, there are three types of limit laws and these incorporate the extreme value distributions, namely the Gumbell (Λ), Frechet (Φ_α) and Weibull (ψ_α) distributions. The Fisher-Tippett theorem indicates that the maxima at the limit converges in distribution to H after normalising and centring. Formally this is expressed as

$$c_n^{-1}(M_n - d_n) \xrightarrow{d} H \qquad \text{for } c_n > 0, -\infty < d_n < \infty \qquad (4)$$

Where $\xrightarrow{d}$ represents convergence in distribution, c_n is a normalising constant and d_n is a centring constant that is determined as a particular quantile or related measure.

[5] The probability of exceeding a margin level for one return (for example, on a specific day in this study) can also be examined with expressions related to (2) and (3). For the short position, the probability of exceeding a margin level is $Prob_{short} = P\{R_i > r_{short}\}$, and the related expression for the long position in (3) is $Prob_{long} = P\{R_i < r_{long}\}$ where $i = \{1,\ldots, n\}$.

These extreme value distributions can be divided into three separate types depending on the value of their shape parameter, α. The classification of a Weibull distribution ($\alpha < 0$) includes the uniform example where the tail is bounded by having a finite right end point and is a short tailed distribution. The more commonly assumed class of distributions used for futures' price changes includes the set of thin tailed densities. This second classification of densities includes the normal and gamma distributions and these belong to the Gumbell distribution, having a characteristic of tails decaying exponentially. Of primary concern to the analysis of fat-tailed distributions is the Frechet classification, and examples of this type generated here are the Cauchy, student-t, ordinary frechet, and the pareto distributions. This important classification of distributions for extreme futures price movements has tail values that decay by a power function. A vast literature on financial returns (Longin, 1999b; Cotter, 1998; Danielsson and DeVries, 1997a, 1997b, 1997c; Kearns and Pagan, 1997; Venkataraman, 1997; Lux, 1996; and Koedijk et al, 1992) and on derivative first differences (Cotter and McKillop, 2000; Longin, 1999a; Hull and White, 1998; and Duffie and Pan, 1997) has recognised the existence of fat-tailed characteristics. For this reason the rest of the theory section deals with this Frechet type of extreme value distribution.

A single representation of the extreme value distributions is outlined in the Generalised extreme value distribution and this is as follows:

$$\begin{aligned} H_\gamma(r) &= \exp\left(-(1+\gamma r)^{(-1/\gamma)}\right) && \text{if } \gamma \neq 0, \text{ and} \\ &= \exp(-\exp(-r)) && \text{if } \gamma = 0, \end{aligned} \qquad (5)$$

where $1 + \gamma r > 0$, and $\gamma = 0$ is to be regarded as the limit of the distribution function as $\gamma \to 0$. Equation (5) is the Jenkinson-Von Mises representation of the generalised extreme value distribution. This simplified representation for the Frechet extreme value distribution focuses on a single parameter γ which has the following relationship with α:

Type II (Frechet): Φ_α for $\gamma = \alpha^{-1} > 0$

The necessary and sufficient conditions for a distribution to asymptotically converge on the Frechet type of extreme value distribution only requires its tail to have a regular variation at infinity property (Feller, 1971).

2.1.2. Maximum Domain of Attraction

For the fat-tailed case, returns do not have to exactly fit a particular set of distributional assumptions. Rather, our analysis assumes the return series have extreme values that are approximated by a Frechet type distribution, and this implies that the series belong to the maximum domain of attraction of the Frechet distribution. Parametric assumptions alone are required in estimating exact fits of a particular distribution. In contrast, measuring approximations of distributions utilises non-parametric frameworks. Formally we can denote the characteristic of belonging to the maximum domain of attraction (MDA) as

$$R_1, R_2, \ldots, R_n \text{ are stationary from } F \in \text{MDA } (H_\gamma) \qquad (6)$$

And in the specific case of a Frechet distribution approximation, (6) reduces to

$$F(r) = r^{-\alpha}L(r), \qquad r > 0 \qquad (7)$$

Where α has parametric assumptions, whereas L(r) is some slowly decaying function that is underpinned by non-parametric assumptions. While there is a general agreement on the existence of fat-tails for financial data, its exact form for all financial returns is unknown. For this reason, it is appropriate to deal with approximation of the Frechet distribution in the sense of being in the maximum domain of attraction.

2.2. Tail and Probability Estimators

Due to the semi-parametric specification of being in the maximum domain of attraction of the fat-tailed Frechet distribution, it is appropriate to apply non-parametric measures of our tail estimates. For example, from an analysis of different extremal statistics, Danielsson and de Vries (1997c) note that non-parametric measures offer an advantage over their parametric counterparts in that under non-gaussian conditions one obtains better bias and mean squared error properties. The non-parametric Hill index (1975) determines the tail estimates of the stock index futures, and is given as:

$$\gamma h = 1/\alpha = (1/m) \sum [\log r_{(n+1-i)} - \log r_{(n-m)}] \qquad \text{for } i = 1....m \qquad (8)$$

This tail estimator is asymptotically normal, $(\gamma - E\{\gamma\})/(m)^{1/2} \approx (0, \gamma^2)$ (Hall, 1982).

As this study is examining the probability of a sequence of returns exceeding a particular margin level relying on expressions (2) and (3), an empirical issue arises in determining the number of returns entailed in the tail of a distribution. Amongst the methods for finding the optimal threshold of where the tail of a distribution begins, we adopt the approach proposed by Phillips et al (1996). The optimal threshold value, M_n, which minimises the mean square error of the tail estimate, γ, is $m = M_n = \{\lambda n^{2/3}\}$ where λ is estimated adaptively by $\lambda = |\gamma_1/2^{1/2}(n/m_2(\gamma_1 - \gamma_2)|^{2/3}$.

One of the main focuses of this study is to not only determine optimal margin requirement for each futures' analysed, but also to determine if separate levels should be set for long and short trading exposure against the alternative of having a common margin regardless of trading position. To investigate this, the tail index estimator is used to determine each tail individually, and also to measure a common margin requirement encompassing the extreme price movements of both tails. The relative stability of the tail measures determines the optimal margin policy. Stability across the tails supports the hypothesis of having a common margin requirement regardless of trading position, and instability suggests the need for separate margin levels. Tail stability is tested using a statistic suggested by Loretan and Phillips (1994):

$$V(\gamma^+ - \gamma^-) = [\gamma^+ - \gamma^-]^2/[\gamma^{+2}/m^+ + \gamma^{-2}/m^-]^{1/2} \qquad (9)$$

for γ^+ (γ^-) is the estimate of the right (left) tail.

Two related margin levels measures are generated based on the non-parametric tail index estimates. The first measure allows us to determine the probability of exceeding a certain price movement. From this, the setting of optimal margin requirements can be made based on an examination of the violation probability for a range of price movements in association with the trade-off between optimising liquidity and prudence for an exchange's contract. The non-parametric measure detailing the probability, p, of exceeding a certain large price change, r_p for any tail measure:

$$r_p \quad = \quad r_t(M/np)^{\gamma} \qquad (10)$$

Using (10) a related non-parametric measure examines the margin level that would not be violated for particular extreme price movements, r_p at different probabilities, p:

$$p \quad = \quad (r_t/r_p)^{1/\gamma}M/n \tag{11}$$

This will be used to compare the margin levels associated with a normal distribution at similar confidence levels.

2.3. Dynamic Risk Measurement

The theoretical underpinnings presented in the previous sub-sections focus on the unconditional distribution of futures returns. Complementary, but different information can be attained from analysing the conditional distribution. This analysis will give futures' markets' clearing houses information on the volatility levels around the current period, t, as opposed to that of the full sample period available with the unconditional distribution. Thus margin setters will forecast risk for time t + 1, and make margin requirement decisions which incorporate the level of risk currently inherent in the respective futures contracts. In its favour, the technique recognises that the risk environment is dynamic as opposed to static. This implies that margin levels will incorporate any impact the current economic climate would have on their values, for instance increasing volatility levels resulting in greater margin levels due to incidences of investor default.

Given a vast set of empirical evidence in favour of time varying volatility (see Pagan, 1996; for a comprehensive review), this paper focuses on modelling conditional volatility levels through a stochastic process. The major breakthrough in modelling financial time series volatility has been through GARCH processes whose specification mirrors many of the characterististics of the data. A GARCH (1,1) model is applied to determine the effects of time varying volatility, and is capable of dealing with the clustering of futures returns. Here, the unconditional variance is assumed normal, whereas the conditional variance is allowed to be time-varying and is given as follows:

$$\sigma^2_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\sigma^2_{t-1} \tag{12}$$

with the variance modelled by past values of itself, σ^2_{t-1}, and the noise process process, ε_{t-1}^2.

However, there are shortcomings with dynamic estimation of extreme quantile and probability risk measures relative to the Extreme Value approach and these should be noted. For instance, GARCH specifications are specifically set up to predict common volatility levels, but have relatively poor tail properties, where the extreme values are located (Danielsson and de Vries, 1997a). This is because the stochastic model's strength is in accounting for volatility clustering, whereas extreme values do not actually cluster (Cotter, 1999; and Danielsson and de Vries, 1997a). In addition, GARCH models are outperformed by Extreme Value methods due to the lack of distributional assumptions which the latter relies on, thereby avoiding model misspecification problems such as constant unconditional variances in the time series.

3. Data Description

3.1. Futures Contracts

Primary stock index futures are selected from a number of European exchanges. The time period of focus varies by contract as the analysis deals with price movements since their inception. A brief description of these contracts is given in appendix 1 detailing the contracts chosen, the exchanges on which they trade, and the respective time period of analysis. The distinct time periods of analysis may result in differing empirical findings. There are a number of reasons for this including the effect that a changing trading environment may have on the contracts analysed. For example, events during the 1980's were different form the 1990's and futures trading during the former decade may have different risk and return characteristics as a consequence. However, the purpose of this paper is to provide insights into the margin levels for European bourses, regardless of specific time periods, so the longest period of analysis possible for each contract is chosen. In addition, any impact from separate sample sizes on the extreme value findings are counteracted by using the tail method of Phillips et al (1996). Here, each tail measure is based on an optimal tail threshold, m, and this is specific to the respective stock index contract.

Whilst there are a number of methods of generating the time series of prices with different qualities, consecutive series of futures price series are developed using each nearest contract to maturity and upto the last trading day for the period before the delivery month. Price changes are measured by the first difference of the natural logarithm of closing day quotes. Figure 2 represents a plot of the French CAC40 returns and as can be seen the largest positive and negative price movements occur around the Gulf crises in 1991, and volatility tends to be greater towards the end of the decade.[6] Whether any of the price movements exceed the margin requirements imposed on traders remains to be seen in our non-parametric analysis of the tail returns in the empirical results section.

INSERT FIGURE 2 HERE

[6] Given the extensive data set, figures are presented for a selection of contracts rather than all contracts. The full set of figures are available on request.

3.2. Preliminary Statistics

A preliminary description of the price movements for all the contracts is given in table 1. The full set of returns follow the usual stylised facts about futures data, namely, the price movements do not belong to a normal distribution using a Kolmogorov-Smirnov test, there is negative skewness present (Norwegian OBX contract excepted), and also leptokurtosis. Thus, statistical determination of margin levels based on the normal distribution is inappropriate, and would lead to inadequate requirements that would test the integrity of the exchanges prudential controls given the excess kurtosis. Leptokurtosis is demonstrated by a fat-tail characteristic, and this is most evident for the OBX index. This finding may be due to the influence of the most extreme outliers of the contracts analysed.

INSERT TABLE 1 HERE

The consideration of using one margin level as opposed to giving individual requirements for the two trading positions may be examined to a certain extent by relying on the skewness, maximum, minimum, mean and median values. Here, the conclusion is ambiguous as negative price movements appear to be more pronounced given the skewness coefficient and the minimum values generally outweigh the positive ones, can be contrasted by the observation that the positive returns outweigh their negative counterparts in the mean values. A more detailed description of the price movements around both tails is given by the upper and lower values that represent ten percent of the returns respectively. Extreme European stock index futures returns do have different characteristics from the full set of returns in agreement with the excess kurtosis and non-normality being more pronounced for the former values. Figures 3 - 5 details this clearly for the German DAX contract as the deviation from the normal distribution as given by the straight line is much greater for the tail values than for the full data set. Although, it is still unclear as to the extent of any divergence in the characteristics of the two tails.

INSERT FIGURES 3 - 5 HERE

3.3. Dependency in Returns

The dependency structure of financial returns is generally cited as the source of the many stylised facts including non-normality and excess kurtosis (Taylor, 1986). However, dependency in the ordinary returns is found to be usually very small (Ding and Granger, 1996). In this study we conclude with this finding by examining the sample correlation of returns for all the futures contracts. An example of the general lack of dependence is shown in figures 6 - 9 for the CAC40 contract where autocorrelation coefficient is plotted over thirty lags for the full series returns as well as three subsets. The subsets involve similar time periods broken down by the number of observations: 0 - 900 (figure 7), 901 - 1800 (figure 8), and 1801 - 2672 (figure 9). While the pattern of dependency is not uniform across the subsets, the relatively weak first order dependence is common for the different time periods analysed. Further analysis of futures returns dependency looking at squared and absolute values indicate very strong volatility persistence, and these findings provide a source of the non-normality present in European stock indexes (Cotter, 1999). These findings, the first and second order dependence, should not be underestimated as the margin measures computed using the commonly applied gaussian assumptions do not take account of these characteristics. The effects of these attributes are analysed by comparing normal and extreme value estimates in the empirical findings section. In addition, the lack of independence suggests benefits from conditional modelling of short run returns. The dynamics are introduced in the GARCH specification.

INSERT FIGURES 6 - 9 HERE

Summarising these stylised facts, we find that stock index futures exhibit significant skewness, kurtosis and non-normality. In addition, the contracts all have a fat-tailed characteristic, that becomes more apparent when focusing explicitly on the tail returns. Finally, first moment dependence is not the cause of the stylised facts, as shown by the sample autocorrelation of the full sample of futures returns, and substantiated by sub-sample results. Rather, it is more likely that second moment dependence causes volatility persistence that is synonymous with fat-tails.

4. Empirical Findings

4.1. Non-Parametric Tail Estimates

In order to determine the probability of a margin level being exceeded, non-parametric tail estimates are generated with the Hill index, and these are given in table 2. The table shows the optimal number of tail returns and non-parametric Hill index estimates for lower, upper and both tails, corresponding to the shape parameter used in the calculation of a long, short and common margin requirements. The optimal number of returns in each tail appears to be reasonably constant, hovering around the five percent mark for each contract. For example, the number of returns in an optimal sequence for the lower tail (100) of the CAC40 index represents 3.74 percent of all returns analysed.

INSERT TABLE 2 HERE

All the tail estimates range between two and four with the exception of the lower tail estimate for the Portuguese PSI20 contract. These values correspond to the general conclusions made for time series of financial returns, namely that they have fat-tail characteristics. The goodness of fit of each equity index being associated with the Frechet distribution is confirmed in table 2 using a difference in means statistic given in Koedijk and Kool (1992). Specifically all tail values are significantly positive, corresponding to the requirement that $\gamma = \alpha^{-1} > 0$. In addition previous studies on financial returns have distinguished different fat-tailed distributions on the basis of the tail estimates. For example, Hill estimates with a value less than or equal to two have stable paretian characteristics of which the cauchy and normal distributions have values of one and two respectively, whereas, GARCH related specifications have values greater than 2 (Ghose and Kroner, 1995). Formally, this is tested using the difference in means statistic for a number of hypotheses including belonging to the stable paretian family of distributions ($H_o < 2$), and having GARCH characteristics ($H_o > 2$). Results for the common tail estimate indicate that there is support for the stable paretian hypothesis being rejected for all of the futures contracts with the exception of the PSI20 index, whereas the GARCH hypothesis is never rejected.

The most important finding presented in table 2 is the statistic examining whether there is any significant difference in the lower and upper tail estimates. These tail estimates are compared to determine whether the policy of imposing a single margin requirement regardless of trading position is fair to operators of each exchange. Using (9), the findings indicate that while the right tail estimators are always greater than their left tail counterpart, a common margin requirement is sufficient based on a five percent significance level in each case with the exception of the OBX index. Regardless of this conclusion, margin requirements are calculated separately for the tails to emphasise any differences that do exist in these measures. Whereas, the Hill estimates are reasonably stable across the tails of a contract, they differ significantly between contracts, implying that the margin requirements should be distinct for different indexes. Taking the widest divergence of the common Hill estimate as an example, CAC40's 3.89 estimate is significantly different (test statistic 3.47) from PSI20's 2.32 estimate using (9). The implications of this are that there should be size differences in the margin requirements of these respective contracts.

4.2. Extreme Value Margins

Rather than discuss the attributes of margin requirements for each futures contract analysed, the rest of the results focus on two questions. First, what is the probability of price movements exceeding a certain percentage given the risk inherent in each contract? A range of very large price changes are presented and these can be thought of as margin levels that would be violated at certain probabilities. Second, what are the margin levels required to cover a range of extreme price changes under the assumptions of a normal and Frechet distribution? This will demonstrate the statistical development of margin levels using normality vis-à-vis extreme value theory. The latter method explicitly assumes the existence of fat-tail returns for stock index futures.

Table 3 presents findings for the probability of common extreme price movements ranging from ten to fifty percent being exceeded. The related findings for the long and short positions are in appendix 2. The results indicate a number of characteristics about the risk inherent in futures contracts. For instance, if a very large margin level of fifty percent is imposed, the probability of it being violated on any individual is very low. For example, the probability of exceeding a price change of fifty percent in table 3 is 0.009 for the PSI index, the contract with the most inherent risk. As we can see, the probability of violating a price movement increases as the price change decreases, as with the PSI index, a ten percent change would be exceeded involving a probability of 0.3750. This may appear to be negligible but over a trading year, the probability of such a price movement is 98.25 percent assuming there are 262 trading days. Notwithstanding the other considerations such as open interest that are used by clearinghouses in the development of margin levels, consideration of this issue using statistical analysis of price movements alone indicates that large margin levels can be exceeded albeit at a low probability. Similarly, to put the analysis of large price movements in the context of extreme value theory, Gumbel (1958) describes a motivation for examining these changes of random variables as a consideration to the event that:

> "Il est impossible que l'improbable n'arrive jamais"
>
> (It is impossible that the improbable will never happen)
>
> (quote taken from Gumbel (1958), pp. 201)

Another result that can be deduced from table 3 is that margin levels should diverge for the respective contracts given that the risk characteristics differ. Continuing with the example of the CAC40 and PSI20 indexes mentioned in terms of the Hill estimates, the probability of violating a ten percent price change (regardless of it occurring on a long or short position over a year's trading) is significantly greater for the Portuguese (98.25%) contract than for its French counterpart (6.90%). This implies that margin requirements should not be imposed on the basis of contract type, but rather, further analysis must be undergone to diagnose the risk characteristics of each futures contract. Related to this is the comparison of margin requirements based on trading positions. In appendix 2, the probability of extreme returns being exceeded on a long position is greater than for a short position. Thus, downside risk is greater than its upside counterpart. Taking the case where there are significant differences between the risk inherent in long and short positions using the tail index as an example, the probability for the OBX contract of exceeding a return of ten percent over a year is almost twenty four percent for a negative price change, in comparison to almost six percent for a positive price change. This implies that in the interests of equity for traders, the margin requirements of a long position should be greater than that of a short position.

4.3. Extreme Value and Gaussian Margins Compared

Turning to the analysis of distributions that may be applied in the context of margin setting. The comparison of the margin levels required for a range of price changes under the assumption of normality, and after making allowances for the fat-tail characteristic of stock index futures using extreme value theory, are presented in table 4.[7] The results incorporate the common margin requirement, so ninety eight percent covers all eventualities with the exception of a two percent default, that is one percent long and short respectively. In order to demonstrate other fat-tail measures, the commonly cited fat-tailed distribution, the student-t, is used to generate margin requirements with degrees of freedom equalling the respective Hill estimates. These

[7] Statistical margin estimation assuming normality is the standard benchmark process.

student-t results are larger than the other measures due to parsimonious degrees of freedom.

Dealing with a gaussian and extreme value comparison, it is clear that the former method underestimate the true margin requirement for any price movement, and that this becomes more pronounced as you try to cover the possibility of smaller exposures. This indicates that the fat-tailed characteristic has greater implications as you move to greater extremes. Taking the DAX contract as an example to illustrate this point, we can see that the requirement to cover ninety eight percent of the price movements calculated under the assumption of a normal (Frechet) distribution is 2.97% (4.20%), and that this increases to 4.75% (16.98%) in order to protect the investor from 99.98 percent of all price changes that could occur for this contract.

4.4. Conditional Margin Requirements

Moving away from Extreme Value Theory, conditional margin requirements based on a GARCH (1, 1) model are presented in table 5. The use of the stochastic volatility models incorporates dynamics into the margin setting procedure, thereby reflecting the current volatility levels in the trading environment. In general, the margin requirements are calculated taking account of respective future's exchange volatility surrounding February 28, 1999.[8] In table 5, the respective volatility environment at this time is indicated in terms of the margin levels needed to protect the exchanges against default. The profile of the margin requirements for each contract changes from switching to conditional from the unconditional trading environment. For example, in contrast to the Extreme Value findings in table 4, the Portuguese PSI20

[8] In the case of the Danish KFX and Swiss contracts, the GARCH model examines conditional volatility levels around 19 December 1998 and 1 July respectfully.

contract is no longer the most volatile contract analysed at the 99.98% confidence interval. In its place, the trading environment for the Norwegian OBX contract is such that it incurs the most volatility, requiring margin levels in excess of 20%. Thus, the benefits of incorporating dynamics into margin setting is illustrated as the respective exchanges have distinct factors influencing their trading environment and these are reflected in their current volatility structure.

Finally, in terms of size, the conditional margin requirements in table 5 are between those of the Gaussian and Extreme Value measures in table 4. The underestimation vis-à-vis the Extreme Value measures is because a GARCH (1, 1) model is unconditionally normal, thereby suffering from the same misspecification as the pure gaussian estimates. However, the GARCH estimates are an improvement on the gaussian measures, as they update the margin requirements according to the current volatility levels. The extent of this improvement over gaussian measures is more pronounced at the more common volatility levels, for example within 98 percentiles.

5. Summary and Conclusion

This paper examines the calculation of margin requirements for a range of circumstances for European stock index futures. Each exchange's clearinghouse must impose margins given the relationship of securing the safety of the exchange against large price movements for contracts and encouraging investor participation in trading. Low margins discourages (encourages) the former (latter), whereas high margins discourages (encourages) the latter (former). The emphasis is on the statistical calculation of margin levels focusing on extreme price movements that are located in the tail rather than in the entire distribution as it is these price changes that margin requirements are meant to combat against.

Given previous findings of futures price changes being associated with fat-tails, extreme value theory and the limiting Frechet distribution is applied in the calculation of the risk characteristics of the futures analysed. The tail indexes are measured using the non-parametric Hill estimates and this is appropriate given the semi-parametric nature of the relationship of a set of futures price changes and the Frechet distribution. Using the Hill estimate, two questions in relation to margin requirements are addressed. First, what is the probability of exceeding a range of margin levels on an individual day or over a year? Second, what margin requirement would be imposed to protect investors from a range of extreme price movements under the assumption of the normal and Frechet distributions?

The paper makes a number of interesting findings. First, common margin requirements are sufficient for each stock index futures with the exception of the OBX contract. For the Norwegian contract, the risk characteristics of the upper tail is different from that of the lower tail to such a degree, that in the interest of fairness to traders, long margins should be greater that short ones. Second, the margin requirements of stock index futures varies across contracts with the PSI contract being the most risky, and the CAC40 being the least. Again, separate margins should be imposed for these contracts reflecting the risk inherent in the respective indexes. Third, assumptions of normality impose smaller margins that using extreme value theory, and this becomes more pronounced as you try to protect against returns further out on the tail of a distribution. These normal based measures underestimate the margin requirements as they assume an exponential tail decline and they should not be used in the statistical calculation of margin requirements of futures sequences. Fourth, conditional margin requirements differ from unconditional ones as they reflect the current volatility environment facing each of the stock index futures.

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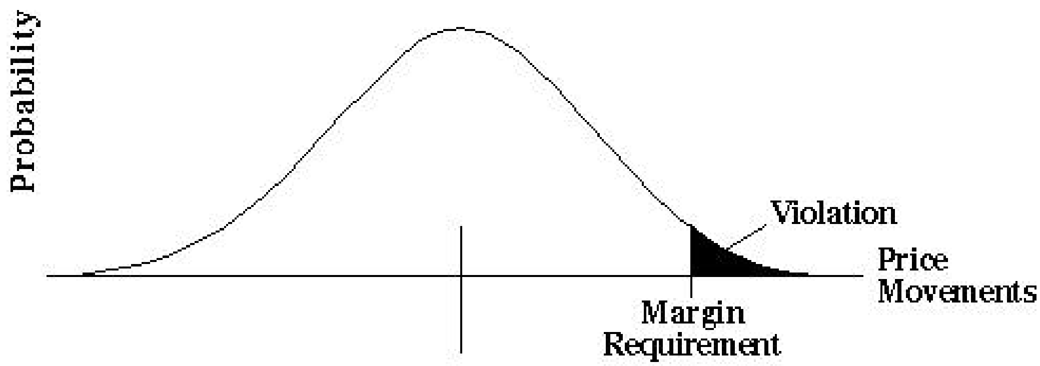
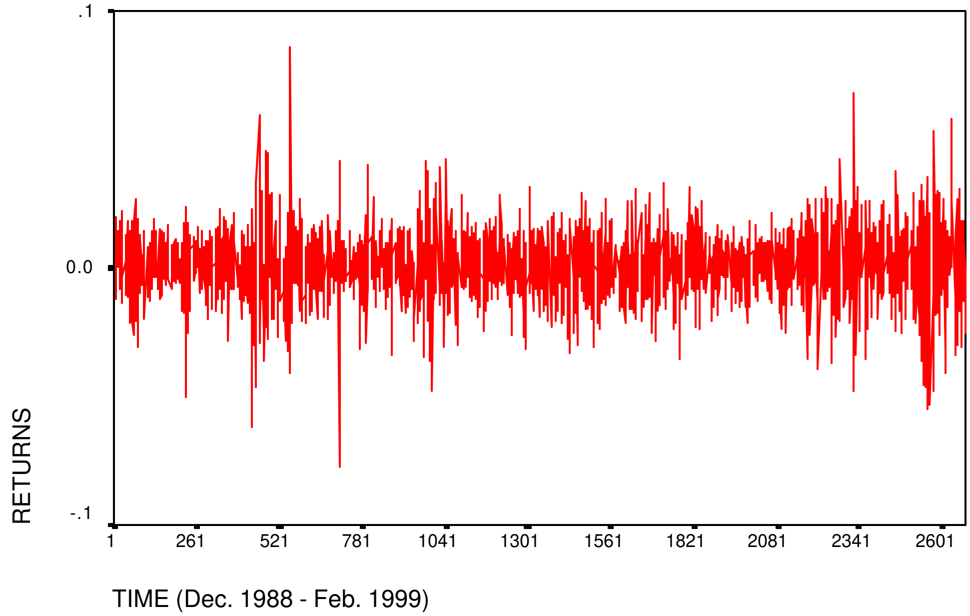
Figure1: Margin Requirements for a Short Position and a Distribution of Returns



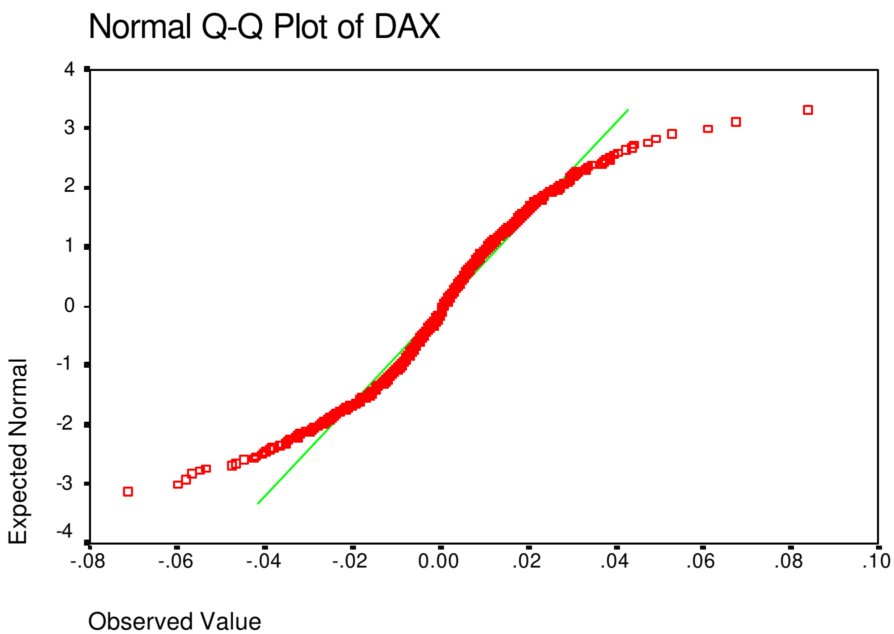
This figure illustrates a distribution of futures returns with special emphasis on the short position. At the upper tail of the distribution, a certain margin requirement is identified. Any price movement in excess of this margin requirement, given by the shaded area, represents a violation or default by the investor.

Figure 2: Time Series Plot for CAC40 Futures Log Return Series



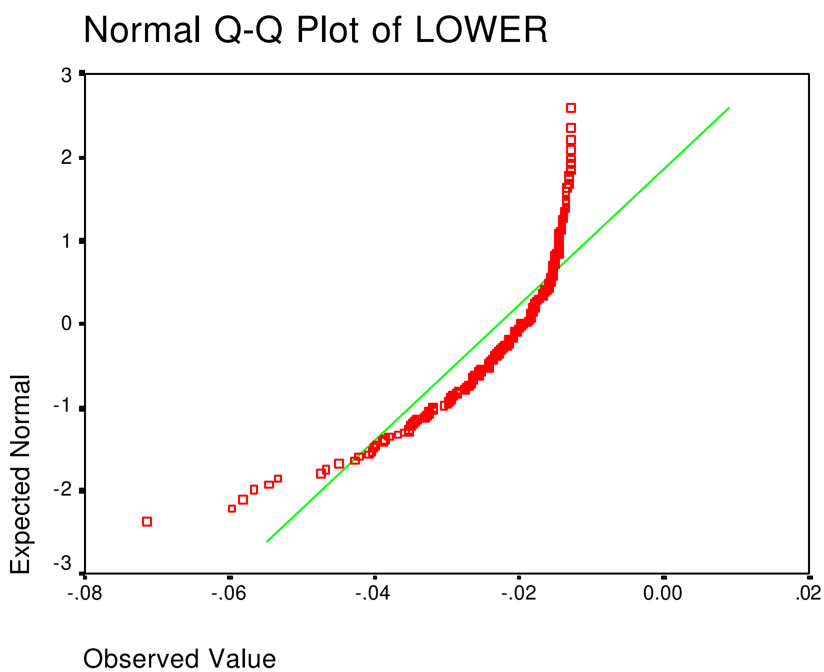
This Figure shows a time series plot of one of the futures index returns, namely the CAC40 contract. The vertical axis measures returns with a possible range between –10 and +10 percent, and the horizontal axis measures time between December 1988 and February 1999. Whilst returns throughout the time period of analysis tend to be close to zero, there are periods where the returns more fluctuate more widely. For example during the Gulf crises in 1991, some of the return spikes are in excess of 5 percent.

Figure 3: Q-Q Plot of DAX Log Returns Series



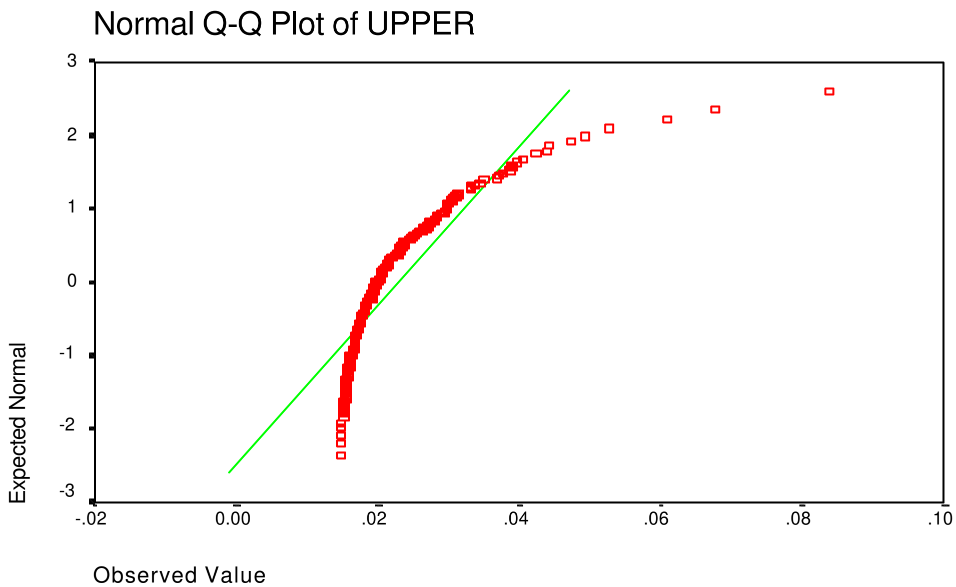
This figure plots the quantile of the empirical distribution of the full set of DAX futures index returns against the normal distribution. The plot shows whether the distribution of the DAX returns match a normal distribution. The straight line represents a gaussian quantile plot whereas the curved line represents the quantile plot of the empirical distribution of the DAX contract. If the full set of DAX returns followed a normal distribution, then its quantile plot should match the gaussian plot and also be a straight line. The extent to which these DAX returns diverge from the straight line indicates the relative lack of normality.

Figure 4: Q-Q Plot of Lower Tail DAX Log Returns Series



This figure plots the quantile of the empirical distribution of the lowest 10 percent of DAX futures index returns against the normal distribution. The plot shows whether the distribution of the DAX returns match a normal distribution. The straight line represents a gaussian quantile plot whereas the curved line represents the quantile plot of the empirical distribution of the DAX contract. If the lower set of DAX returns followed a normal distribution, then its quantile plot should match the gaussian plot and also be a straight line. The extent to which these DAX returns diverge from the straight line indicates the relative lack of normality.

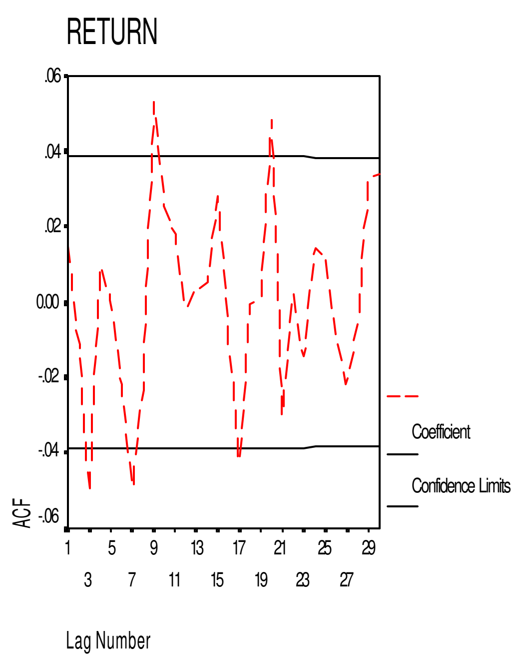
Figure 5: Q-Q Plot of Upper Tail DAX Log Returns Series

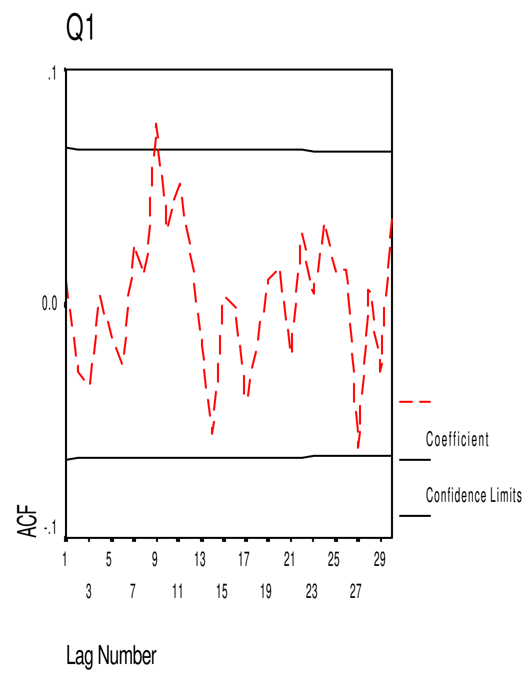


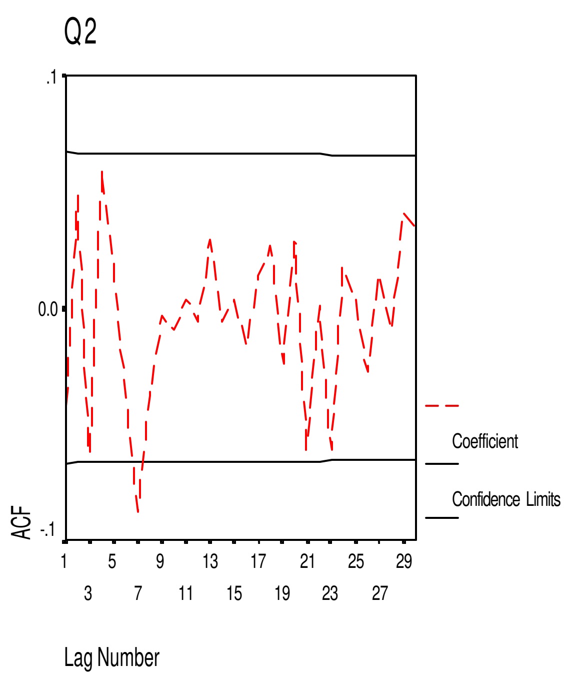
This figure plots the quantile of the empirical distribution of the highest 10 percent of DAX futures index returns against the normal distribution. The plot shows whether the distribution of the DAX returns match a normal distribution. The straight line represents a gaussian quantile plot whereas the curved line represents the quantile plot of the empirical distribution of the DAX contract. If the highest set of DAX returns followed a normal distribution, then its quantile plot should match the gaussian plot and also be a straight line. The extent to which these DAX returns diverge from the straight line indicates the relative lack of normality.

Figure 6: Sample Autocorrelation for full set (RETURN) of French CAC40 Returns
Figure 7: Sample Autocorrelation for first subset (Q1) of French CAC40 Returns
Figure 8: Sample Autocorrelation for second subset (Q2) of French CAC40 Returns
Figure 9: Sample Autocorrelation for third subset (Q3) of French CAC40 Returns

Figures 6 – 9 represent the plots of the autocorrelation function for 30 lags of the CAC40 index returns. Whilst figure 6 deals with the full sample of the contract's returns, the other plots show subsets of the full sample focusing on consecutive returns. Specifically, these are returns numbered 0 – 900 (Q1 – figure 7), 901 – 1800 (Q2 - figure 8) and 1801-2672 (Q3 - figure 9) from the full data set. Confidence limits are provided to determine whether any particular lag is significantly different from zero. Any autocorrelation value in excess of the confidence interval indicates a significantly dependent lag.







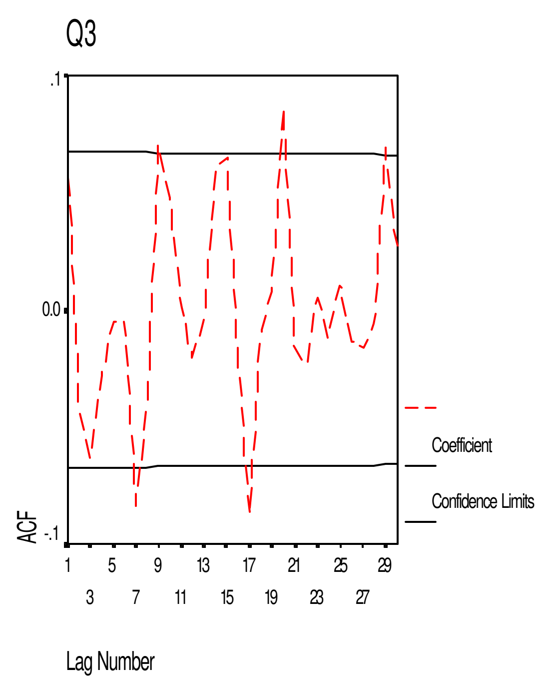


Table 1: Summary Statistics for Stock Index Futures

Contract	Mean[a]	Minimum[a]	Maximum[a]	Interquartile Range [a]	Skewness	Kurtosis	Normality
BEL20	0.06	-5.26	5.48	0.99	-0.11†	4.40	0.07
Lower	-1.66	-5.26	-0.95	0.64	-2.07	5.27	0.18
Upper	1.65	1.08	5.48	0.57	2.94	10.62	0.22
KFX	0.04	-7.80	6.65	1.11	-0.33	5.19	0.08
Lower	-2.09	-7.80	-1.22	0.86	-2.61	9.41	0.20
Upper	2.06	1.30	6.65	0.73	2.39	7.18	0.19
CAC40	0.04	-7.74	8.63	1.41	-0.08†	3.36	0.05
Lower	-2.27	-7.74	-1.40	1.02	-2.07	5.97	0.17
Upper	2.23	1.49	8.63	0.93	3.01	13.41	0.20
DAX	0.06	-12.85	8.38	1.24	-0.56	8.31	0.08
Lower	-2.30	-12.85	-1.27	1.10	-3.91	26.24	0.20
Upper	2.29	1.48	8.38	0.87	2.85	11.98	0.19
AEX	0.06	-7.70	7.28	1.06	-0.33	5.94	0.08
Lower	-2.04	-7.70	-1.09	1.11	-2.04	5.12	0.19
Upper	1.98	1.21	7.28	0.84	2.71	9.54	0.20
MIF30	0.08	-7.84	7.07	1.78	-0.06†	2.27	0.06
Lower	-2.96	-7.84	-1.84	1.11	-2.07	5.25	0.17
Upper	3.14	2.12	7.07	1.04	1.80	3.11	0.18
OBX	0.04	-19.55	21.00	0.66	0.32	97.69	0.18
Lower	-1.93	-19.55	-0.97	0.80	-6.91	64.85	0.29
Upper	1.92	1.11	21.00	0.66	9.06	97.95	0.32
PSI20	0.13	-11.55	6.96	1.34	-0.87	7.95	0.11
Lower	-2.89	-11.55	-1.45	1.80	-2.49	7.79	0.22
Upper	2.96	1.78	6.96	1.53	1.46	1.93	0.18
IBEX35	0.07	-10.84	7.25	1.54	-0.49	4.72	0.07
Lower	-2.68	-10.84	-1.60	1.03	-2.67	9.29	0.21
Upper	2.62	1.76	7.25	0.90	2.26	5.81	0.19
OMX	0.06	-11.92	10.81	1.61	-0.27	8.93	0.07
Lower	-2.75	-11.92	-1.60	0.99	-3.23	12.31	0.24
Upper	2.75	1.68	10.81	1.00	3.11	11.74	0.23
SWISS	0.08	-9.09	7.08	0.95	-0.50	9.30	0.06
Lower	-1.58	-9.09	-0.93	0.75	-4.65	34.45	0.22
Upper	1.65	1.08	7.08	0.69	3.61	21.65	0.21
FTSE100	0.05	-16.72	8.09	1.21	-1.18	18.78	0.05
Lower	-1.89	-16.72	-1.13	0.75	-6.84	71.49	0.26
Upper	1.90	1.24	8.09	0.66	2.98	14.81	0.19

The summary statistics are presented for each future's index as well as the lower and upper 10 percent of that contract's returns. The mean, minimum and maximum values represent the average, lowest and highest returns respectively. The interquartile range gives the spread between the 75th and 25th percentiles. The skewness statistic is a measure of distribution asymmetry with symmetric returns having a value of zero. The kurtosis statistic measures the shape of a distribution vis-à-vis a normal distribution with a gaussian density function having a value of zero. Normality is formally examined with the Kolmogorov-Smirnov test which indicates a gaussian distribution with a value of zero.

a) statistics are expressed in percentages.

b) † represents insignificant at the five percent level whereas all other skewness, kurtosis and normality coefficients are significant different from zero.

Table 2: Optimal Tail Estimates for Stock Index Futures

Contract	m^-	γ^-	m^+	γ^+	m^*	γ^*	$\gamma^+ - \gamma^-$
BEL20	63	2.81	68	2.85	92	3.20	0.07
		(0.35)		(0.35)		(0.33)	
KFX	72	2.65	78	3.01	111	2.98	0.78
		(0.31)		(0.34)		(0.28)	
CAC40	100	2.97	103	3.33	137	3.89	0.83
		(0.30)		(0.33)		(0.33)	
AEX	95	2.72	95	2.93	141	2.95	0.51
		(0.28)		(0.30)		(0.25)	
DAX	83	2.92	83	3.07	116	3.30	0.33
		(0.32)		(0.34)		(0.31)	
MIF30	55	3.31	55	3.66	77	3.41	0.53
		(0.45)		(0.49)		(0.39)	
OBX	71	2.04	70	2.89	113	2.47	2.02•
		(0.24)		(0.35)		(0.23)	
PSI20	41	1.91	43	2.38	57	2.32	0.99
		(0.30)		(0.36)		(0.31)	
IBEX35	74	2.62	78	3.35	118	3.21	1.50
		(0.30)		(0.38)		(0.30)	
OMX	88	2.59	98	2.77	177	2.67	0.46
		(0.28)		(0.28)		(0.20)	
FTSE100	126	2.99	134	3.36	204	3.21	0.93
		(0.27)		(0.29)		(0.22)	
SWISS	72	2.81	84	2.89	103	3.12	0.17
		(0.33)		(0.32)		(0.31)	

Hill tail estimates, γ are calculated for lower, upper and both tails for each stock index future. The symbols -, +, * represent the lower, upper and both tails respectively. The optimal number of values in the respective tails, m, is calculated following the method proposed by Phillips et al (1996). Standard errors are presented in parenthesis for each tail value. Tail stability is calculated in the last column with the symbol • representing significant different upper and lower tail values at the five percent level.

Table 3: Exceedence Probability for Different Common Margin Levels

Contract	50%	40%	30%	20%	10%
BEL20	0.0002	0.0004	0.0009	0.0031	0.0267
KFX	0.0005	0.0010	0.0024	0.0079	0.0622
CAC40	0.0001	0.0001	0.0004	0.0018	0.0263
AEX	0.0006	0.0011	0.0025	0.0084	0.0646
DAX	0.0003	0.0006	0.0015	0.0058	0.0574
MIF30	0.0005	0.0010	0.0028	0.0110	0.1172
OBX	0.0017	0.0030	0.0062	0.0168	0.0932
PSI20	0.0090	0.0151	0.0294	0.0752	0.3750
IBEX35	0.0005	0.0011	0.0028	0.0101	0.0940
OMX	0.0025	0.0046	0.0100	0.0294	0.1875
FTSE100	0.0002	0.0004	0.0009	0.0034	0.0316
SWISS	0.0001	0.0003	0.0007	0.0026	0.0229

The values in this table represent the probability of each contract's returns exceeding a certain margin requirement, for example, 10 percent on any single day. Values are expressed in percentages.

Table 4: Common Margin Requirements to cover Extreme Price Movements

Contract	Method	98%	99%	99.80%	99.90%	99.98%
BEL20	*Normal*	2.15	2.38	2.86	3.04	3.45
	Student-t	4.20	5.40	9.44	11.95	20.52
	Extreme	3.10	3.88	6.52	8.16	13.72
KFX	*Normal*	2.47	2.73	3.27	3.48	3.95
	Student-t	7.37	10.51	23.63	33.45	74.84
	Extreme	3.93	4.96	8.53	10.76	18.48
CAC40	*Normal*	2.93	3.25	3.89	4.14	4.70
	Student-t	5.72	7.36	12.86	16.28	27.96
	Extreme	3.92	4.69	7.09	8.48	12.83
AEX	*Normal*	2.61	2.89	3.46	3.69	4.18
	Student-t	7.80	11.12	25.01	35.40	79.20
	Extreme	3.95	5.00	8.62	10.91	18.82
DAX	*Normal*	2.97	3.28	3.93	4.19	4.75
	Student-t	5.78	7.44	13.00	16.45	28.26
	Extreme	4.20	5.19	8.45	10.43	16.98
MIF30	*Normal*	3.88	4.30	5.15	5.48	6.22
	Student-t	7.57	9.73	17.02	21.54	37.00
	Extreme	5.34	6.54	10.48	12.83	20.56
OBX	*Normal*	2.83	3.14	3.76	4.00	4.54
	Student-t	8.47	12.08	27.17	38.45	86.03
	Extreme	3.83	5.07	9.72	12.86	24.66
PSI20	*Normal*	3.78	4.19	5.02	5.34	6.06
	Student-t	11.31	16.11	36.25	51.31	114.80
	Extreme	6.55	8.83	17.68	23.85	47.74
IBEX35	*Normal*	3.46	3.83	4.58	4.88	5.53
	Student-t	6.73	8.66	15.15	19.17	32.93
	Extreme	4.79	5.94	9.81	12.17	20.09
OMX	*Normal*	3.67	4.07	4.87	5.19	5.88
	Student-t	10.53	15.00	33.74	47.76	106.86
	Extreme	5.35	6.93	12.65	16.40	29.95
FTSE100	*Normal*	2.56	2.83	3.39	3.61	4.09
	Student-t	4.98	6.41	11.20	14.17	24.35
	Extreme	3.41	4.23	6.99	8.67	14.31
SWISS	*Normal*	2.13	2.36	2.83	3.01	3.42
	Student-t	4.16	5.35	9.36	11.84	20.34
	Extreme	2.99	3.73	6.24	7.79	13.04

The values in this table represent the margin requirements needed to cover a range of extreme price movements for each contract, for example, 98% of all movements. The associated margin requirements are calculated relying on extreme value theory, gaussian and student-t distributions. Student-t degrees of freedom are given by the Hill tail estimates, γ^*, which are also incorporated in the extreme value estimates. Values are expressed in percentages.

Table 5: Common Conditional Margin Requirement to cover Extreme Price Movements

Contract	98%	99%	99.80%	99.90%	99.98%
BEL20	2.61	3.15	4.82	5.29	5.39
KFX	3.30	3.89	5.80	7.03	7.62
CAC40	3.28	4.10	5.70	6.43	8.16
AEX	3.11	3.95	5.91	6.59	7.49
DAX	3.51	4.13	6.09	7.18	10.93
MIF30	4.56	5.35	6.94	7.83	7.94
OBX	2.94	3.93	6.61	11.63	20.51
PSI20	4.60	5.55	8.28	9.32	10.44
IBEX35	3.88	5.38	7.05	7.50	9.96
OMX	4.32	5.63	9.97	10.69	11.47
FTSE100	2.77	3.43	5.01	6.88	11.01
SWISS	2.33	2.87	3.87	5.57	8.43

The values in this table represent the margin requirements needed to cover the price movements accounting for current volatility levels. A GARCH (1, 1) specification describes the time-varying volatility and is given as $\sigma^2_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\sigma^2_{t-1}$. Values are expressed in percentages.

Appendix 1: Details of Futures Contracts Analysed

Contract	Exchange	Time Period
BEL20	Belgium - Futures and Options Exchange	Dec. 1993 - March 1999
KFX	Denmark - Guarantee Fund for Options & Futures	June 1992 - Dec. 1998
CAC40	France - Marche a Terme International de France	Dec. 1988 - March 1999
AEX	Holland - Amsterdam Exchange	Dec. 1988 - March 1999
DAX	German - Eurex	Dec. 1990 - March 1999
MIF30	Italy - Italian Derivatives Market	Dec. 1994 - March 1999
OBX	Norway - Oslo Stock Exchange	Dec. 1992 - March 1999
PSI20	Portugal - Bolsa de Dernados de Porto	Oct. 1996 - March 1999
IBEX35	Spain - Mercado De Futuros Financieros	June 1992 - March 1999
OMX	Sweden - The OMLX exchange	Mar. 1990 - March 1999
FTSE100	United Kingdom - London International Financial Futures Exchange	June 1984 - March 1999
SWISS	Switzerland - Swiss Options and Financial Futures Exchanges	Dec. 1990 - June 1997

This table outlines the origin and time period of the futures contracts analysed. Datastream provided the data.

Appendix 2: Exceedence Probability for Different Long and Short Margin Levels

Long Margin Level					
Contract	**50%**	**40%**	**30%**	**20%**	**10%**
BEL20	0.0002	0.0004	0.0010	0.0030	0.0209
KFX	0.0007	0.0012	0.0027	0.0078	0.0492
CAC40	0.0003	0.0007	0.0016	0.0053	0.0416
AEX	0.0006	0.0011	0.0025	0.0074	0.0487
DAX	0.0005	0.0009	0.0021	0.0070	0.0526
MIF30	0.0003	0.0006	0.0017	0.0064	0.0633
OBX	0.0034	0.0054	0.0097	0.0221	0.0909
PSI20	0.0130	0.0200	0.0346	0.0750	0.2820
IBEX35	0.0014	0.0026	0.0055	0.0159	0.0980
OMX	0.0016	0.0028	0.0060	0.0171	0.1028
FTSE100	0.0002	0.0003	0.0008	0.0028	0.0219
SWISS	0.0002	0.0004	0.0008	0.0025	0.0179
Short Margin Level					
BEL20	0.0002	0.0003	0.0008	0.0025	0.0181
KFX	0.0002	0.0004	0.0010	0.0034	0.0276
CAC40	0.0001	0.0002	0.0006	0.0023	0.0229
AEX	0.0003	0.0005	0.0012	0.0038	0.0288
DAX	0.0002	0.0005	0.0012	0.0040	0.0340
MIF30	0.0001	0.0003	0.0009	0.0038	0.0482
OBX	0.0002	0.0004	0.0010	0.0031	0.0228
PSI20	0.0039	0.0066	0.0131	0.0344	0.1791
IBEX35	0.0002	0.0004	0.0009	0.0036	0.0371
OMX	0.0010	0.0018	0.0040	0.0125	0.0852
FTSE100	0.0001	0.0001	0.0003	0.0012	0.0127
SWISS	0.0002	0.0003	0.0007	0.0023	0.0167

The values in this table represent the probability of each contract's returns exceeding a certain margin requirement on any single day, for example, 10 percent. Long and short margin levels are calculated separately incorporating the respective optimal tail values, m^- and m^*. Values are expressed in percentages.