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Abstract This paper presents a model of labour supply determination under job competition. In the presence of a positive rate of unemployment and increasing returns to labour, the level of labour supply chosen by each individual lies above the one that, at the offered wage, maximises utility. There is a unique strictly positive degree of job competition that is consistent with the optimal allocation. If labour supply is upward-sloping, increasing job competition raises the equilibrium level of activity and, when job competition causes production to exceed its optimal level, reducing output market competition leads to a welfare improvement.

Keywords: labour supply, job competition, welfare, product market competition. JEL-Codes: J22, D43, D60.
1 Introduction

Empirical evidence in industrialised countries has pointed to an increase in individual labour supply over the past two-three decades. Bluestone and Rose [6] note that, in the US, individual labour supply at each given real wage has increased over the period 1982-1996 allowing the economy to expand without price inflation in spite of low levels of unemployment. Costa [8] compares data on average hours worked per day in the 1890s, in 1973, and in 1991 in the US. Workers are sorted into wage groupings, from the lowest to the highest paid. It emerges that high earners reduce their time spent at work between 1890 and 1973 but increase it in the following years. A similar pattern for the UK has been highlighted by Green [12], who finds that since 1981 ”greater proportions of men and women have been working especially long hours” (Green [12], p. 61). Other studies have pointed out that this increase in individual labour supply does not reflect workers’ preferences. In particular, a survey conducted by Bielenski et al. [4] and covering 16 European countries reveals that in all countries full-time employees would prefer to work fewer hours. In other words, employees would like to trade earnings for leisure. Further evidence in this sense can be found in Baaijens and Schippers [3], Euwal and van Soest [11] and Hooker et al. [13].

The aim of this paper is to provide a model of labour supply that is consistent with these stylised facts. The standard approach to labour supply determination predicts that, given the prevailing wage rate, individuals choose their labour supply so as to maximise their utility function. By doing so, each individual disregards other individuals’ behaviour since, in a perfectly competitive labour market, all individuals looking for a job are certain to find one without delay.

However, the persistence of positive level of unemployment suggests the existence of imperfections in the labour markets that prevent them from clearing. Even the most competitive labour markets have some frictional unemployment. And, as long as economic cycles exist, each individual may expect to spend part of their working life unemployed.

In this paper we account for the effect of labour market imperfections on individuals’ behaviour. In particular, we assume that each individual at each point in time is either employed and working or unemployed and looking for a job. The probability that a given employed worker $j$ retains her job can be affected by the behaviour of the worker herself. Specifically, this probability is assumed to rise as labour supply of individual $j$ increases relative to the amount of labour supplied by all other individuals. This is justified by the presence of increasing returns to labour. By this it is meant that, for any individual worker, one of the following situations applies: (1) the worker’s
marginal productivity increases with the amount of labour she supplies; (2) the worker’s marginal productivity is constant but there are fixed costs of employment (such as insurance, recruitment, training, etc.); (3) the worker’s marginal productivity is decreasing but the fixed costs of employment are so large that, for all admissible equilibria, the worker’s average productivity increases with her labour supply. In all of these cases, firms strictly prefer to employ long-hours working individuals and a worker who supplies more labour than the average for any given real wage has a better than the average chance of keeping her job. In fact, if a firm decides to reduce the size of its workforce, it will first lay-off the least productive workers. And these, given increasing returns to labour, are the employees who supply the lowest levels of labour. Hence, firms rank employees according to the level of labour they supply. This gives rise to a rat race among individuals, each of them trying to improve their ranking by supplying more labour than the other individuals do. However, unlike in the original contribution of Akerlof [1], or in more recent papers like the one by Sampson [16], the rat race does not derive from workers’ desire to disclose/hide their unobservable characteristics (signalling). In fact, all workers are identical. A rat race emerges rather because individuals want to avoid to be laid-off as this entails being temporarily unemployed.

This job competition setting is nested into a general equilibrium framework and discussed under both perfectly and imperfectly competitive product markets. The main results of the paper can be summarised as follows: (a) there is a unique strictly positive degree of job competition consistent with the socially optimal level of labour supply, (b) if labour supply is increasing in wage, an increase (reduction) in job competition raises (lowers) the equilibrium level of activity; (c) a reduction in output market competition may lead to a welfare improvement.

These results are related to different fields of the economic literature. As for (a) and (b), analogous conclusions are derived in Moen [15]. In Moen’s paper firms rank prospective employees according to the level of education because more educated workers are more productive and there is rent sharing. This creates a rat race where workers invest in education partly in order to achieve a better ranking. It is shown that if there is little job competition workers underinvest in education, while if there is too much job competition they overinvest. Though in a different context, the same result emerges in this paper. The main difference is that individuals instead of choosing investment

\footnote{Such a set up is particularly relevant for firms which have a policy of annual staff dismissals. According to Hudson and YouGov [14], such a policy has been used by Microsoft and General Electric and more than three quarters of UK bosses think their companies would benefit from its adoption.}
in education choose labour supply. Result (c) is closely related to the literature on imperfectly competitive output markets. The novel feature is that an increase in product market competition may be detrimental. A result that is at odds with the general view that competition is always beneficial (see Dixon and Rankin [9] and Silvestre [17] for surveys and Blanchard and Giavazzi [5] and Ebell and Haefke [10] for models of product-labour market interaction.). This finding is a consequence of the fact that workers, by oversupplying labour, can cause production to lie above its optimal level. If overproduction occurs, lowering competition in the product market raises welfare as less competition implies less output. The imperfect competition literature, by contrast, usually assumes that, if employed, workers locate on their labour supply curve. Hence, overproduction never occurs so that reductions in output caused by a fall in competition have always a negative impact on welfare.

The remainder of the paper is organised as follows. The model with increasing returns to labour and no job competition is outlined in section 2. Section 3 introduces job competition. Section 4 discusses the possible equilibrium outcomes and the impact of an increase in job competition. Section 5 analyses the link between job competition and product market competition. Section 6 provides an example, while section 7 considers the case of a backward-bending labour supply. Section 8 contains final comments.

2 Increasing returns, social optimum and market outcome

We consider an economy in which, at each point in time, some workers are laid-off and become unemployed, while other workers leave unemployment and start a new job. Once a worker is made redundant, she begins searching for a new job. After a certain time period, she finds work and becomes employed again. So, each individual at every point in time is in either one of two states: employed or unemployed. The latter state occurs with probability \(1 - \epsilon\), where \(\epsilon \in (0, 1)\) is the fraction of the total workforce in employment at each point in time. \(\epsilon\) is decreasing in both the number of workers that lose their job at each point in time and the time it takes to find a new job. The fact that \(\epsilon\) is strictly smaller than 1 implies that finding a job always takes some time. Hence, the rate of unemployment is always strictly positive. Workers are homogeneous, infinitely living and maximise the expected present discounted value of utility with a discount rate of \(r > 0\). Specifically, worker \(j\) chooses consumption, \(c_j\), and labour supply, \(n_j\), so as to maximise
The social optimum problem consists in finding a pair \((n_j, b)\) that maximises individual utility. (1), (2), (3) and setting \(\pi = 0\) yield the following objective function for the social planner

\[
\epsilon U(c_j, n_j) + (1 - \epsilon)U(b + \pi, 0)
\]

where \(b\) is real unemployment benefit and \(\pi\) is real profit. As usual \(U(\cdot)\) is continuous, twice differentiable, concave, increasing in consumption and decreasing in labour supply. The budget constraint if worker \(j\) is employed is given by

\[
c_j = wn_j + \pi - tb
\]

where \(w\) is real wage and \(t := (1 - \epsilon)/\epsilon\).

Production takes place within firms. These may differ in size, i.e., in the number of workers they employ. Otherwise they are identical. Labour is the only input. The level of output produced by worker \(j\), \(f_j\), is given by

\[
f_j = f(n_j).
\]

The production function of a firm employing \(m > 0\) workers is therefore equal to \(\sum_{j=1}^{m} f_j\). (3) is continuous and the maximum amount of labour worker \(j\) can supply is normalised to 1. Increasing returns to labour are assumed over the relevant range, that is, \(\partial f_j/\partial n_j > f_j/n_j > 0\quad \forall n_j \in (0,1]\). This assumption implies one of the following scenarios: (1) increasing marginal productivity of labour (i.e., \(\partial^2 f_j/\partial n_j^2 > 0\)); (2) constant marginal productivity of labour and fixed costs of employment, such as insurance, training, etc. (e.g., \(f_j = \alpha n_j - v, \alpha > v > 0\)); (3) decreasing marginal productivity of labour and sufficiently large fixed costs of employment (e.g., \(f_j = n_j^\beta - v, \beta \in (0,1), v \in [1-\beta,1)\)).

The social optimum problem consists in finding a pair \((n_j,b)\) that maximises individual utility. (1), (2), (3) and setting \(\pi = 0\) yield the following objective function for the social planner

\[
\epsilon U(f(n_j) - tb, n_j) + (1 - \epsilon)U(b, 0)
\]

Define \(n^* \in (0,1)\) and \(b^* \geq 0\) as the values of \(n_j\) and \(b\) that maximise (4). Assuming concavity of the objective function, \(n^*\) and \(b^*\) satisfy the following two equations

\footnote{Under scenario (3), if the fixed costs are not large enough (e.g., \(f_j = n_j^\beta - v, \beta \in (0,1), v \in (0,1-\beta)\)), increasing returns to labour will arise only if \(n_j \in (0,s)\), where \(s < 1\). In this case, the results of this paper are unaffected only if no equilibrium exists where \(n_j > s\). One way to rule out equilibria where \(n_j > s\) is to assume that the level of wage individuals require to supply more than \(s\) units of labour is so large that firms would make negative profits.}
\[ U_1(f(n_j) - tb, n_j) \frac{\partial f(n_j)}{\partial n_j} + U_2(f(n_j) - tb, n_j) = 0 \]  
\[ U_1(f(n_j) - tb, n_j) - U_1(b, 0) = 0 \]

where \( U_i(.) \) denotes the derivative of \( U(.) \) with respect to its \( i \)-th argument. We call the pair \((n^*, b^*)\) the (socially) optimal allocation.

The market outcome differs. Define \( \tilde{n}_j := \tilde{n}_j(w) \) as the labour supply function of individual \( j \) and \( \phi(w, n_j, b) \) as the first derivative of the expected utility function (1) with respect to \( n_j \) after insertion of the budget constraint, that is

\[ \phi(w, n_j, b) := U_1(wn_j + \pi - tb, n_j) + U_2(wn_j + \pi - tb, n_j) \]  

Then, by the very definition of \( \tilde{n}_j \), the following must hold

\[ \phi(w, \tilde{n}_j, b) = 0. \]  

As for the output market, as a benchmark we assume that competition among firms ensures that goods are priced according to their average cost, so that profit is zero\(^3\). Since workers are homogenous and, therefore, supply in equilibrium the same amount of labour, the zero profit condition can be expressed on each worker as

\[ w = f(n_j) \frac{n_j}{n_j} \]  

where \( n_j \) is independent of \( j \) as all individuals are identical. To simplify the analysis we make the following assumptions

**Assumption 1** \( \phi_{12} \phi_1 - \phi_{11} \phi_2 \leq 0 \) for \( n_j = \tilde{n}_j \) and \( \forall w > w \), where \( w \) is defined through \( U(w \tilde{n}_j - tb, \tilde{n}_j) = U(b, 0) \).

**Assumption 2** Define \( \theta := n_j^2 \frac{\partial^2 f_j}{\partial n_j^2} - 2n_j(n_j \frac{\partial f_j}{\partial n_j} - f_j) \). \( \theta \) is continuous and either \( \theta \neq 0 \) \( \forall n_j \in (0, 1] \) or \( \theta = 0 \) \( \forall n_j \in (0, 1] \).

**Assumption 3** \( \phi_1 > 0 \) for all \( w > w \).

Assumption 1 implies that \( \tilde{n}_j \) is concave. From Assumption 2 follows that the zero-profit curve (9) is either everywhere concave, everywhere convex or linear. Assumption 3 states that \( j \)'s labour supply is increasing in wage. This

\(^3\)The number of firms in the market is indeterminate as it depends on the firms’ sizes, which can vary, and the total number of workers populating the economy.
last assumption will be relaxed in section 7.

Assumptions 1 to 3 are pretty standard. Additively separable functions such as \( c_j - n_j^\gamma \), \( \gamma > 2 \), fulfill Assumptions 1 and 3. Assumption 2 applies to Cobb-Douglas production functions and linear production functions with fixed costs.

Assumptions 1 and 2 imply that, ignoring the polar cases \( n_j = 0 \) and \( n_j = 1 \), the economy has, in general, either zero, one, or two equilibria\(^4\). Given Assumption 3, in the latter case one equilibrium is stable and the other one is not.

An example of the two equilibria case is depicted in Figure 1. Since all individuals are identical, the economy is fully described by the labour supply of a single individual \( j \) and the zero-profit condition (9). \( S \) denotes \( j \)'s labour supply, and corresponds therefore to \( \tilde{n}_j \), while \( T \) is the zero-profit curve (9). The lower equilibrium, \( A \), is unstable, while the higher one, \( B \), is stable. To see this, consider that for any point lying to the right (left) of the zero-profit line firms make positive (negative) profit for every hired worker. So, there, they increase (decrease) demand for labour causing real wage to rise (fall). Similarly, \( j \) increases (reduces) her labour supply at any point to the left (right) of her labour supply schedule.

The single equilibrium case can be obtained either by shifting \( S \) to the left until it becomes tangent to \( T \) or by rotating it until one equilibrium vanishes. If \( j \)'s labour supply schedule lies completely above the zero-profit line, no equilibrium exists.

Note that, wherever the economy is located in the real wage-labour supply space, it always converges towards an equilibrium. In fact, given the directions of the adjustment process as represented by the arrows in Figure 1, the economy, sooner or later, enters one of the regions labelled I, II, and III. Once the economy is in one of these regions it will remain within it until an equilibrium is reached. Specifically, if the economy is in II, it will move within this area until it settles in \( B \). The same applies when the economy is in III. If, instead, the economy is in I, it stays in that region while moving towards the \( n_j = 0 \) equilibrium.

\( A \) and \( B \) correspond to different levels of individual labour supply not to different levels of employment. In other words, in both \( A \) and \( B \) an equal fraction \( \epsilon \) of the total workforce is employed. The difference is that in \( A \) each employed individual works fewer hours than in \( B \).

We conclude this section by pointing out that the market delivers a suboptimal outcome as neither \( n^A \) nor \( n^B \) maximise individual utility \( U(f(n_j)) - \)

\(^4\)Equilibria are more than two only if for some interval labour supply schedule and zero-profit curve coincide. In this case the number of equilibria is infinite.
$tb^*, n_j$). In fact, both $n^A$ and $n^B$ satisfy

$$U_1(f(n_j) - tb^*, n_j) \frac{f(n_j)}{n_j} + U_2(f(n_j) - tb^*, n_j) = 0 \quad (10)$$

under the participation constraint

$$U(f(n_j) - tb^*, n_j) \geq U(b^*, 0) \quad (11)$$

The left-hand-side of (10) is always smaller than the left-hand-side of (5), so that neither $n^A$ nor $n^B$ can be a solution of (5). Finally, concavity of (4) implies that $n^*$ lies to the right of $n^B$, that is, $n^* \in (n^B, 1)$.

![Figure 1: Market equilibrium](image)

### 3 Job competition

In this section we relax the implicit assumption that workers cannot affect their probability of becoming unemployed. Specifically, we assume that, because of increasing returns, firms rank employees according to the level of labour they supply. This means that worker $j$ can raise her individual
expected utility by expanding her relative level of labour supply. By doing so she enjoys three benefits: firstly, she reduces any management’s incentive to replace her with a new employee\(^5\); secondly, she reduces the probability of being laid-off in case of downsizing; thirdly, she lowers the production costs of the firm she is working for thereby reducing the probability of liquidation and job loss. We argue therefore that a rat race takes place among workers, each of them trying to lower their expected unemployment spells by supplying more labour than other workers do. We shall denote this process as job competition. Job competition implies a change in worker \(j\) expected utility function (1). This takes now the following form

\[
\frac{\epsilon\Omega_j^* U(c_j, n_j) + (1 - \epsilon\Omega_j^*) U(b + \pi, 0)}{r}
\]

\(\Omega_j\) is a continuous, twice differentiable function of \(n_j\) and \(n_{-j}\), where the latter denotes labour supply of all workers except \(j\) and, like \(n_j\), is bounded between 0 and 1. So, \(\Omega_j := \Omega(n_j, n_{-j})\) and \(n_{-j} \in [0, 1]\). \(\Omega_j\) has the following characteristics: \(\Omega_j^* \in (0, \frac{1}{2}]\) \(\forall n_j \in [0, 1]\) and \(\forall n_{-j} \in [0, 1]\); \(\Omega_j(v, v) = 1\) \(\forall v \in [0, 1]\); \(\partial \Omega_j/\partial n_j > 0\) and \(\partial \Omega_j/\partial n_{-j} < 0\).

The parameter \(\rho\) accounts for the level of job competition. The larger is \(\rho\) the higher is the impact of \(\Omega_j\) on \(j\)’s expected utility function. \(\rho = 0\) implies that there is no competition for jobs.

Insert (2) into (12) and assume that the resulting expression is concave in \(j\)’s labour supply and that worker \(j\) maximises it taking \(n_{-j}\) as given. Then, optimal labour supply of individual \(j\) must satisfy the following condition

\[
\rho \psi(w, n_j, n_{-j}, b) + \phi(w, n_j, b) = 0
\]

where

\[
\psi(w, n_j, n_{-j}, b) := \frac{\partial \Omega_j(n_j, n_{-j})}{\partial n_j} \left[ U\left(wn_j - tb + \pi, n_j\right) - U(b + \pi, 0)\right] / \Omega_j(n_j, n_{-j})
\]

A comparison between (13) and (8) makes clear that, for every level of real wage and every level of labour supplied by the other workers, individual \(j\), in general, supplies more labour than under no job competition. In fact, \(\psi(w, n_j, n_{-j}, b)\) is non-negative, so that (13) implies \(\phi(w, n_j, b) \leq 0\). Job competition has no impact on \(j\)’s labour supply only when the participation constraint is binding. In this case, workers are indifferent between the two states, employed and unemployed, and have therefore no reason to compete

\(^5\)The more labour a given individual supplies, the less likely is that anyone replacing her will supply a larger amount of labour.
for jobs. As a result, \( \psi(w, n_j, n_{-j}, b) \) becomes equal to zero and (13) collapses to (8).

So, if \( \rho > 0 \) and the participation constraint is slack, \( j \)'s labour supply in Figure 1 shifts to the right. Hence, if there are equilibria, they must lie either to the right of \( n^B \) or to the left of \( n^A \), i.e., the market can in principle attain the optimal allocation \( n^* \). Further, the following proposition holds

**Proposition 1** Define \( \hat{n}_j := \hat{n}_j(\rho, w, n_{-j}) \) as \( j \)'s labour supply (and, therefore, the solution to (13))\(^6\). Then

\[
\frac{d\hat{n}_j}{d\rho} > 0 \quad \forall \rho \geq 0, \quad \forall n_{-j} \in [0, 1].
\]

Proposition 1 stems from the fact that (12) is concave. In order to have a fairly well-shaped labour supply schedule we extend Assumption 1 to the job competition case. So

**Assumption 4** \( (\rho \psi_{12} + \phi_{12}) (\rho \psi_1 + \phi_1) - (\rho \psi_{11} + \phi_{11}) (\rho \psi_2 + \phi_2) \leq 0 \) for \( n_j = \hat{n}_j, \forall w > \bar{w}, \forall \rho \geq 0, \) and \( \forall n_{-j} \in [0, 1] \).

Analog to Assumption 1, Assumption 4 ensures that \( j \)'s labour supply is concave for each level of job competition and each level of labour supplied by the other individuals. As for the relationship between \( \hat{n}_j \) and \( w \), the following proposition holds

**Proposition 2**

\[
\frac{d\hat{n}_j}{dw} > 0 \quad \forall \rho \geq 0, \quad \forall n_{-j} \in [0, 1].
\]

Proposition 2 states that labour supply is upward-sloping. In fact, from total differentiation of \( j \)'s optimality condition (13) follows that the sign of \( d\hat{n}_j/dw \) is equal to the sign of \( \rho \psi_1 + \phi_1 \) (15)

Since \( \psi_1 > 0, \rho \geq 0, \) and, given Assumption 3, \( \phi_1 > 0, \) (15) is always positive and so is \( d\hat{n}_j/dw \). Hence, labour supply is increasing under job competition as it is in the absence of it.

Having derived the relationship between \( \hat{n}_j \) and \( \rho \) and between \( \hat{n}_j \) and \( w \), we now discuss the impact of other workers’ labour supply on \( \hat{n}_j \). Total differentiation of (13) yields the following proposition

\(^6\)Hence, \( \hat{n}_j(0, w, n_{-j}) = \hat{n}_j(w) \).
Proposition 3

\[
\text{sign} \left[ \frac{d\hat{n}_j}{dn_{-j}} \right] = \text{sign} \left[ \Omega_j \frac{\partial^2 \Omega_j}{\partial n_j \partial n_{-j}} - \frac{\partial \Omega_j}{\partial n_j} \frac{\partial \Omega_j}{\partial n_{-j}} \right].
\]

So, depending on the characteristics of \( \Omega_j \), \( \hat{n}_j \) either increases, decreases, or remains unchanged in response to a rise in \( n_{-j} \). The first case corresponds to assuming strategic complementarities while the second one implies the existence of strategic substitutabilities.\(^7\)

Individual labour supply under job competition can be depicted as in Figure 2. The solid lines correspond to those of Figure 1. Each dotted line can be interpreted as \( j \)'s labour supply for some given combination of \( \rho \) and \( n_{-j} \). In particular, if we keep \( n_{-j} \) constant and vary \( \rho \), we obtain for every level of job competition a different labour supply curve, whereby the higher is \( \rho \), the more rightward lies the supply curve. The same applies if, instead of \( \rho \), \( n_{-j} \) is varied. In this case, however, the more external curves do not correspond necessarily to a higher \( n_{-j} \). For this to be the case we need to assume strategic complementarities (\( d\hat{n}_j/dn_{-j} > 0 \)). On the contrary, \( j \)'s labour supply at each real wage would decrease after a rise in \( n_{-j} \) in the presence of strategic substitutabilities (\( d\hat{n}_j/dn_{-j} < 0 \)). So, all curves lying to the right of \( S \) in Figure 2 can be interpreted in two different ways. Either they reflect different levels of job competition at a given \( n_{-j} \), in which case the more outward is a curve, the higher is \( \rho \). Or they correspond to different levels of \( n_{-j} \) for a given \( \rho \), in which case a more outward schedule implies a larger \( n_{-j} \) under strategic complementarities and a smaller \( n_{-j} \) under strategic substitutabilities.

Notably, the parameter \( \epsilon \) affects \( j \)'s labour supply only via the tax rate \( t \). Still, it plays a crucial role in the model as the fact that \( \epsilon \) is smaller than one implies that the labour market is unable to instantaneously match every job seeker with a vacancy. And this is sufficient to give rise to job competition.

4 Equilibrium analysis

The equilibrium analysis follows the lines of the discussion carried out in section 2 for the no-job competition case. Since all workers are identical, their objective functions are concave and there is a common single wage, only symmetric equilibria can arise. That is, in equilibrium, all individuals supply the same amount of labour so that \( \hat{n}_j = n_{-j} \) must hold. If \( \hat{n}_j \) is independent of \( n_{-j} \), the results are qualitatively the same as in the

\(^7\)See Cooper and John [7].
Figure 2: Labour supply with and without job competition

no-job competition case as for every $\rho$ there is a unique labour supply. So, for any given $\rho$, there are still at most two equilibria. One equilibrium belongs to $L := [0, n^A]$ and is unstable and the other one belongs to $H := [n^B, 1]$ and is stable. For $\rho = 0$, the high equilibrium corresponds to $n^B$ and the low equilibrium to $n^A$. As $\rho$ is increased, $j$’s labour supply shifts outwards so that the low equilibrium moves towards the lower bound of $L$ while the high equilibrium moves towards the upper bound of $H$.

If $\hat{n}_j$ is instead a function of $n_{-j}$, the location of $j$’s labour supply depends on both $\rho$ and $n_{-j}$. In this case all possible equilibria still belong to either $L$ or $H$. However, for each value of $\rho$, there are potentially infinite different equilibria. In fact, for any given $\rho$, the optimality condition (13) allows for an unbounded number of symmetric solutions.

Like in the no-job competition case and whatever the relationship between $\hat{n}_j$ and $n_{-j}$, the system always converges towards an equilibrium. The argument is the same used in section 2. That is, wherever the economy is located, sooner or later, it moves into one of the regions labelled I, II, and III in Figure 1. And from there to an equilibrium. The only difference is that, if $\hat{n}_j$ depends on $n_{-j}$, the boundary of these regions represented by $j$’s labour supply moves inwards and outwards while the economy adjusts towards an
equilibrium. The same argument can be exploited to predict the effect of a change in $\rho$ even without knowing the relationship between $\hat{n}_j$ and $n_{-j}$. Specifically

**Proposition 4** The impact of a change in $\rho$ depends on where the economy is located at the time of the change. In particular

(a) if at any equilibrium level of $n_j$ in $L$ $\rho$ is increased (decreased), the economy converges towards a new equilibrium in $H$ ($n_j = 0$);

(b) if at any equilibrium level of $n_j$ in $H$ $\rho$ is increased (decreased), the economy converges towards a new equilibrium in $H$ characterised by a larger (smaller) individual labour supply.

So, whether there are strategic complementarities, substitutabilities or $\hat{n}_j$ is independent of $n_{-j}$ is irrelevant when it comes to determine the effect of an increase in $\rho$. To see this and why Proposition 4 holds, consider Figure 2. Suppose that the economy is in equilibrium at A. If we increase $\rho$, $j$’s labour supply schedule shifts to the right, for example to $S^k$. At the current wage, $w^A$, $j$ as well as all other individuals increase their labour supply, firms make positive profit, and real wage starts to rise. The economy moves therefore inside the region labelled II in Figure 1 and will stay there until a new equilibrium has been reached in $H$. Since all individuals increase their labour supply, $n_{-j}$ rises in the process. If $\hat{n}_j$ is independent of $n_{-j}$, the economy will settle at D on $S^k$. However, if $\hat{n}_j$ is increasing in $n_{-j}$ (strategic complementarities), $j$’s labour supply will move outwards during the adjustment process and the economy will settle somewhere to the right of D, for example in F. By contrast, if $\hat{n}_j$ is decreasing in $n_{-j}$ (strategic substitutabilities), $j$’s labour supply will move inwards during the adjustment process and the economy will settle somewhere to the left of D, for example, in E. If the economy actually settles in E and $\rho$ is decreased, $j$’s labour supply shifts to the left and the economy moves into the region labelled III in Figure 1. And from there to a new equilibrium between B and E.

A similar reasoning can be applied to any equilibrium point in $L$ and $H$ and leads to the results summarised in Proposition 4. So, the difference between strategic complementarities and strategic substitutabilities is that the former exacerbate the impact of an increase in job competition while the latter dampen it.

Finally, there is a unique value of $\rho$ that is compatible with the optimal allocation $(n^*, b^*)$. This value is denoted by $\overline{\rho}$ and is given by

$$\overline{\rho} = -\frac{\phi\left(f(n^*_n, n^*_b, b^*)\right)}{\psi(n^*_n, n^*_b, b^*)}.$$  \hspace{1cm} (16)
(16) is derived from (13) and (9) and does not require either Assumption 3 or Assumption 4. Given the definition of $\psi$ (see (14)), (16) implies that $\bar{p}$ is decreasing in the gap between the utility of holding a job and that of being unemployed. This is because the larger is the loss of utility associated with becoming unemployed, the keener are workers to retain their jobs. Hence, the larger the amount of labour they are willing to supply. Since labour supply is positively correlated to $\rho$ (see Proposition 1), any increase in the difference between the utility of holding a job and that of being unemployed must be matched by a decrease in job competition if labour supply is to be kept unchanged.

Note that setting $\rho = \bar{p}$ may not be sufficient to achieve the optimal allocation as equilibria different from $n^*$ could emerge. In fact, the right-hand-side of (16) may be equal to $\bar{p}$ for values of $n_j$ and $n_{-j}$ different from $n^*$. However, if $\hat{n}_j$ is independent of $n_{-j}$, $n^*$ is the only stable equilibrium compatible with $\bar{p}$.

5 Product market competition and job competition

This section discusses the relationship between job competition and product market competition. We assume that, due to output market imperfections, in equilibrium profits are strictly positive. In particular, firms are assumed to price their output at the constant mark-up $1/\lambda$, $\lambda \in (0,1]$, over the average cost of production. The smaller is $\lambda$, the larger is the mark-up and the higher is firms’ market power. $\lambda = 1$ corresponds to the case we have considered so far, i.e., the one in which firms have no market power. $\lambda$ is to be interpreted as a measure of product market competition: the larger is $\lambda$, the more competitive is the product market. The mark-up rule implies a profit share of $1 - \lambda$ (hence, more competition means less profit) and yields the following real wage equation

$$w = \lambda \frac{f(n_j)}{n_j}$$

(17) nests as a special case the zero-profit curve (9) and yields, in the real wage-labour supply space, a different constant-profit schedule for each $\lambda$.

8One possible way to justify this pricing rule is to assume collusion among incumbent firms and a shadow cost of entry proportional to output. This latter assumption is used by Blanchard and Giavazzi [5] to derive the long-run equilibrium of a monopolistically competitive economy in which firms make positive profits.
Specifically, as $\lambda$ decreases, the constant-profit curve shifts to the right lowering wage for each given amount of worked hours. This means that lowering/raising product market competition has the same effect as lowering/raising job competition. In fact, shifting the constant-profit schedule outwards is the same as shifting the labour supply curve inwards. So

**Proposition 5** At any equilibrium point, a change in $\lambda$ has the same qualitative effect of a change in $\rho$ (see Proposition 4).

Although the impact on $n_j$ is qualitatively the same whether we change $\rho$ or $\lambda$, the income distribution effects are different. In fact, an increase in $\lambda$ reduces the profit share in the economy. By contrast, a rise in $\rho$ leaves the profit share unchanged. Since in both cases production increases, firms strictly prefer a rise in $\rho$ to a rise in $\lambda$, while workers' preferences are the opposite. We illustrate this point by means of Figure 3.

Suppose the economy is in equilibrium at $Q$ and, for simplicity, that $j$‘s labour supply depends only on $w$ and $\rho$. The social planner can achieve the optimal allocation $n^*$ either by increasing job competition\(^9\) or by raising product market competition. In the former case, $j$‘s labour supply shifts from $S^1$ to $S^2$ and the economy ends up in $Z$. In the latter case, the constant-profit line shifts from $T^1$ to $T^2$ and the economy settles in $V$. Although both $V$ and $Z$ are consistent with the optimal allocation $n^*$, they entail a different income distribution. In fact, the profit share, $1 - \lambda$, is larger in $Z$ than in $V$. Hence, firms prefer $Z$, while workers prefer $V$.

Proposition 5 and the example of Figure 3 suggest a negative relationship between the value of $\rho$ compatible with the optimal allocation $(n^*, b^*)$ and the degree of product market competition. And this is indeed the case. When the output market is not perfectly competitive, the optimal value of $\rho$, that is, the only value $\rho^*$ consistent with the optimal allocation, is given by

$$
\rho^* = -\frac{\phi \left( \lambda f(n^*_j), n^*_j, b' \right)}{\psi \left( n^*_j, n^*_j, b' \right)}.
$$

(18)

where $b' := b^* - \pi = b^* - \epsilon(1 - \lambda)f(n_j)$. Clearly, when $\lambda = 1$, $\rho^* = \bar{\rho}$. Like in the case of a perfectly competitive output market, setting $\rho$ equal to its optimal value is not sufficient to ensure that the economy will converge towards $n^*$, as multiple equilibria can exist for the same value of $\rho$.

As expected, $d\rho^*/d\lambda < 0$. That is, there is a negative correlation between the

\(^9\)One way the social planner may affect $\rho$ is through labour market regulation. For example, reducing hiring/firing costs may raise job competition as it makes it easier for firms to lay off/recruit workers.
optimal level of job competition and the degree of product market competition. The reason is that an increase in $\lambda$ raises real wage inducing individuals to supply more labour. To offset this effect a decrease in job competition is therefore necessary. So, if the economy has reached the optimal allocation, an increase in either job or output market competition lowers welfare. Hence, the relationship between output market competition and welfare can be negative.

6 A special case

In this section we present an example. Assume that utility and production functions take on the following form

$$U(c_j, n_j) = c_j - \frac{n_j^q}{\gamma}$$  \hspace{1cm} (19)$$

$$f_j = Z \frac{n_j^a}{\alpha}$$ \hspace{1cm} (20)$$
with \( \gamma > 2, \alpha > 1, \gamma > \alpha \) and \( Z \in (0,1) \). The socially optimal level of individual labour supply is

\[
n^* = Z^{\frac{1}{\gamma - \alpha}}
\]

Since (6) is met for every level of \( b \), there is no specific optimal level of employment compensation. The zero-profit market outcome is expressed by

\[
\tilde{n} = \left( Z^{\gamma \alpha} \right)^{\frac{1}{\gamma - \alpha}} < n^*
\]

that is, the market outcome is suboptimal.

Let us now introduce job competition and assume that \( \Omega_j \) takes on the following form

\[
\Omega_j = \begin{cases} 
(n_j/n_{-j})^\rho & \text{if } n_j \leq (1/\epsilon)^{1/\rho} n_{-j} \\
1/\epsilon & \text{otherwise}
\end{cases}
\]

Then there are neither strategic complementarities nor strategic substitutabilities, i.e., \( d\tilde{n}_j/dn_{-j} = 0 \). So, for each \( \rho \), there is only one labour supply curve. Moreover, for each degree of job competition, there is only one market equilibrium.

The level of job competition that leads to the optimal allocation is given by

\[
\rho^* = \left( 1 - \frac{\lambda}{\alpha} \right) / \left( \frac{\lambda}{\alpha} - \frac{1}{\gamma} - \frac{b}{\epsilon} Z^{\frac{\gamma - 1}{\alpha \gamma}} \right) > 0
\]

and setting \( \rho = \rho^* \) is a sufficient condition to achieve the optimal allocation. It is easy to see that the following inequalities hold

\[
\frac{\partial \rho^*}{\partial b} > 0 \quad \frac{\partial \rho^*}{\partial Z} < 0 \quad \frac{\partial \rho^*}{\partial \lambda} < 0 \quad \frac{\partial \rho^*}{\partial \epsilon} < 0
\]

i.e., \( \rho^* \) is increasing in \( b \) and decreasing in \( A, \lambda, \) and \( \epsilon \).

The negative relationship between job and product market competition is consistent with Proposition 4 and 5. These state that an increase/decrease in either \( \lambda \) or \( \rho \) raises/lowers the activity level. Hence, if the latter is at its social optimum, a rise in \( \lambda \) must be accompanied by a decrease in \( \rho \) and vice-versa. The economic intuition behind the other results is as follows.

The optimal level of job competition is increasing in \( b \) because an increase in unemployment compensation reduces the difference between the employed workers’ utility and that of those receiving benefits. Individuals are therefore induced to restrict their labour supply.

On the contrary, the larger is \( Z \), i.e., the more productive is labour, the
smaller should be competition for jobs. In fact, the more productive is labour, the smaller is the relative size of the tax burden ($tb$) for employed individuals and the larger is the gap between their utility and that of those who are unemployed.

Finally, the optimal level of job competition increases when $\epsilon$ falls. This means that an increase in expected unemployment (a reduction in $\epsilon$) leads unambiguously to a decrease in labour supply. The reason is that the only effect of a decrease in $\epsilon$ is an increase in taxation to finance unemployment benefits and, therefore, a reduction in the employed workers’ utility. Being employed becomes consequently less attractive and the social planner must increase competition for jobs to avoid a fall in the level of activity\textsuperscript{10}.

Note that since there is no specific optimal level of unemployment benefits, the social planner has an additional degree of freedom. That is, even if $\rho$ were fixed, an optimal allocation could still be attainable by choosing the appropriate level of unemployment compensation.

\section{Backward-bending labour supply}

In this section we relax Assumption 3 and suppose instead that, in the absence of job competition, labour supply bends backwards once a certain level of real wage $\tilde{w}$ is reached. This case is depicted in Figure 4. Like A in Figure 1, A\textsuperscript{1} is unstable. However, unlike B, B\textsuperscript{1} may not be stable either. In fact, if the economy is out of equilibrium, it will not necessarily converge. Specifically, unlike in the upward-sloping labour supply case, if the economy is in either II or III, it does not necessarily stay within these regions. Instead, it may keep moving inside and outside of them and around B\textsuperscript{1} (or even possibly towards A\textsuperscript{1}) without ever converging to an equilibrium.

While Proposition 1 is unaffected by the relaxation of Assumption 3 (labour supply increases in $\rho$ independently of its shape), Proposition 2 no longer holds. In fact, for $w > \tilde{w}$, $\phi_1$ is negative so that (15) turns out to be negative for low values of $\rho$. In particular, there is a value of $\rho$, $\tilde{\rho} := -\phi_1/\psi_1$, such that, if $w > \tilde{w}$, $j$’s labour supply is increasing in wage if $\rho > \tilde{\rho}$ and is decreasing otherwise. This means that if under no-job competition labour supply is decreasing in $w$, it may be increasing when individuals compete for jobs.\textsuperscript{10}

\textsuperscript{10}This somehow counterintuitive result hinges crucially on the assumption that $\rho$ is independent of $\epsilon$. Nevertheless, it may seem reasonable to assume that when the probability of being unemployed increases, competition for jobs becomes fiercer. This is equivalent to say that a decrease in $\epsilon$ has a positive direct impact on $\rho$. If this is true, a fall in $\epsilon$ leads ceteris paribus to more job competition and possibly to a larger labour supply. In this case the social planner would have to take measures apt to reduce the level of competition for jobs.
jobs (see Figure 5). Hence, econometric tests showing a positive elasticity of labour supply may be misleading, as the true relationship between labour supply and wage may well be negative\textsuperscript{11}.

Proposition 3 maintains its validity while most of Proposition 4 applies only in the short term. In fact, while the immediate effect of a change in $\rho$ is the one described in Proposition 4, the long term impact is in most cases unknown. Specifically, the only statement that can be made is that if the economy is in equilibrium in $L$, a decrease in $\rho$ moves it towards a lower equilibrium.

As for Proposition 5, whether the effects of a change in job competition and of a change in product market competition are the same depends on where the economy is located. Specifically, the short term impact of a change in $\lambda$ is the same as the one caused by a change in $\rho$ if the economy is in equilibrium in $L$, while it is the opposite if the economy is in equilibrium in $H$. In the latter case, the short term effect of a reduction (increase) in $\lambda$ is an increase (reduction) in labour supply while the immediate effect of a reduction (increase) in $\rho$ is a fall (rise) in labour supply. The long term effect of a change

\textsuperscript{11}For estimates of labour supply elasticities see Ashenfelter and Card [2].
in product market competition is instead ambiguous unless the change consists in a reduction in $\lambda$ that occurs when the economy is in equilibrium in $L$. In this case, the economy converges towards a lower equilibrium level of activity.

Finally, for each level of product market competition there is still a unique degree of job competition that is consistent with the optimal allocation. However, the optimal degree of job competition is now increasing in $\lambda$. In fact, since at the social optimum both $\phi$ and $\phi_1$ are negative, the right hand side of (18) is increasing in $\lambda$ so that $\rho^*$ rises when product market competition is increased instead of decreasing as it was the case when labour supply was upward-sloping. The economic explanation mirrors the one given under Assumption 3 (see end of section 5). The difference is that now an increase in real wage induces a reduction instead of an expansion in labour supply. Hence, more, not less, job competition is needed to compensate for the effect of an increase in $\lambda$.

Figure 5: Labour supplies. $S^4 := S(\rho = 0); S^5 := S(\rho > \tilde{\rho})$. 
8 Conclusion

Empirical studies have highlighted two stylised facts: first, individual labour supply has increased in many industrialised countries; second, individuals do not seem to locate on their labour supply schedule, in the sense that they work more hours than they would like to at the offered wage. This paper has provided a model that is consistent with this empirical evidence. The model makes two key assumptions: first, there is frictional unemployment and, second, there are increasing returns to labour. These two assumptions imply that individuals compete for jobs. Specifically, in order to reduce their probability of being laid-off, individuals expand their labour supply above the level which would be consistent with their preferences. We established that there is a unique strictly positive level of job competition that is compatible with the optimal allocation. If labour supply is upward sloping, an increase (reduction) in either job or product market competition raises (lowers) the equilibrium level of labour supply. Hence, if labour supply exceeds its optimal level, lowering competition in either the product or the job market raises welfare. Finally, the socially optimal allocation is consistent with infinite combinations of the degrees of job and product market competition. Each of such combinations implies a different income distribution.
References


