Regime Switching in a New Keynesian Phillips Curve with Non-zero Steady-state Inflation Rate

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Abstract

The main topic of this paper is to challenge the rational nature of the agents' expectations and the structural effectiveness of the behaviorally micro-based New Keynesian *Phillips* Curve ($NKPC$). Building on previous results, we model this trade-off between the U.S inflation rate and a Unit Labor Cost-based measure of the real activity through a $Markov$ Switching Intercept and Heteroscedastic - Vectorial AutoRegressive ($MSIH-VAR$) specification. This specification allows the adequate capture of the rationality in the agents' expectations process. It underlies a finite number of expected inflation rate regimes, which highlight the agents' adaptive beliefs on the achievements of these regimes. Moreover, the results confirm the structural stability of the $NKPC$ over the inflation rate regimes as its deep parameters seem to be unaffected by the regimes switching.

*Keywords: Inflation, New Keynesian Phillips Curve, Regime Switching

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1 Introduction

The recent modeling of the inflation rate dynamics, gathered under the New Keynesian *Phillips* Curve ($NKPC$), are based on micro-founding and on the existence of nominal rigidities in the economy. Within this framework, the inflation rate is shown as a forward-looking phenomenon, directly associated with the agents' rational expectations. However, according to some points of view, the way these optimization behaviors are introduced and exploited deserves some analytic details (*Samuelson* (2008), *Sims* (2008)).

In fact and until recently, the $NKPC$ modeling of the inflation rate dynamics postulates a zero steady state (expected) rate, so that the agents' expectations process seem to be based on the uniqueness and the constancy of the price level at long term. Even though in this frame, works of authors such as *Gali* & *Gertler* (1999), *Sbordone* (2002), etc. have enabled the almost "resurrection" of the Inflation - Real activity trade-off, its general approach would not have validated the existence of a stable link between the inflation rate and the main variables measuring the real activity dynamics. Many aspects, both theoretical and empirical, still remain object to controversies in recent macroeconomic literature.

In this context, a second generation of the $NKPC$ model has been derived assuming a non-zero steady state inflation rate. In this New Keynesian *Phillips* Curve with Positive steady state Inflation rate ($NKPC - PI$), the expected inflation rate is no longer constrained to a zero value and can be associated with different values during the period under study. This $NKPC - PI$ framework then enables us to consider a more flexible trend (expected) of inflation rate dynamics and offers new possibilities to integrate the agents' expectations mechanism in the analysis.

Under these new aspects, the first empirical studies (*Cogley* & *Sbordone* (2005)) of the $NKPC-PI$ are mainly based on Time Varying Parameter - Vectorial AutoRegressive ($TVP-VAR$) reduced forms to characterize the agents' expectations. However, this kind of empirical approach is one among many to estimate the *Phillips* curve. Also, to validate this $TVP - VAR$ approach, it is important to postulate a theoretical framework within which each variable of the $NKPC-PI$ model could have a varying trend or be unstable at their steady states. *Cogley* & *Sbordone* (2008) derived such a "Time Varying" $NKPC-PI$ model by log-linearizing the system around a time varying steady state inflation rate. But, as they noted : « *When trend inflation varies over time, we have to take a stand about the evolution of agents' expectations: we therefore replace the assumption of rational expectation with one of subjective expectations and make appropriate assumptions on how beliefs evolve over time*».

Consequently, one of the current points of all the inflation rate dynamics studies in this $NKPC-PI$ framework consists of defining an econometric frame in which the different variables of the model, either individual or collective, could admit underlying trends dynamics in spite of the «constraints» imposed by the rational expectation hypothesis. Taking into account that such trend dynamics must necessarily be in line with the criterion of non-systematic review of the agents' expectations as any continuous expectations review (conceptually assume in the background of the TVP specification) weakens the rational nature of the expectation process, as it could only be justified through a systematic need of error corrections.

In what follows, we deal with this problem by proposing an econometric framework within which the $NKPC-PI$ model conceptually fits. In our point of view, this framework permits us to maintain the rational expectation assumption. It additionally enables the expected values of the different variables of the $NKPC-PI$ model to potentially delay at any time, without being a consequence of ongoing expectations revisions. We particularly wish to allow the inflation rate to be constant and non-zero at its steady-state while having a trend dynamics as highlighted by a $Hodrick-Prescott$ filter . Basically, this problem is intimately linked to the critical core of the *Phillips* curve trade-off traditionally made by authors[1] such as *Phelps* (1967), $Friedman$ (1968) or $Lucas$ (1972a).

Relying on previous results (*Boutahar* & *Gbaguidi* (2009)), but also on multivariate findings such as those of *Cogley* & *Sbordone* (2005, 2008) or *Groen* & $Mumtaz$ (2008), we propose a $Markov$ Switching Intercept Heteroscedascic - Vectorial AutoRegressive ($MSIH-VAR$) model to characterize the agents' expectation process. The advantage of such a reduced form is that it enables a description of the main empirical changes whilst taking into account agents expectations in the $NKPC-PI$ model and fundamentally, as noted by *Groen* & $Mumtaz$ (2008) : «*we prefer the Markov switching VAR model as the auxiliary model for our structural estimation over the time varying parameter VAR framework, as it allows us to identify the VAR parameters that correspond with the different inflation regimes in an objective, data-driven manner. Also, it treats the shifts in the inflation process as stochastic, which potentially could be non-monotonic, and its probabilistic selection of the inflation regimes is in our view more compatible with the theoretical framework*».

Following these theoretical and technical views, we check the effectiveness of the $NKPC-PI$ trade-off, showing that the dynamics of the trend inflation rate can be adequately described by the $Markov$ switching model. In such a model, the unconditional mean of the inflation rate varies randomly between a fixed number of regimes within a probabilistic frame controlled by a first order $Markov$ chain. The matrix of the probabilities of transition between the regimes can be perceived as a system of beliefs of the rational agents on the possible achievements of their inflationary expectations.

Also, the invariability of the regimes allow us to ensure the rational nature of the agents expectations, insofar as it guaranteed the non-systematic revision of these expectations. The New Keynesian agents' expectations are then characterized by a combination of the variables fixed levels with the agents' adaptive beliefs[2] empirically captured by the $Markov$ switching model. In addition, the

[1] For all those *Nobel* Prize winners, the optimizing behavior of the agents must be able to evolve adaptively to the states of the economic environment without imposing a fundamental reappraisal of the rationality principle.

[2] These beliefs can be qualified as adaptive ones because the transition probabilities depend

time varying beliefs technically come to balance the values of the inflation rate rationally expected and make it possible to generate a trend rate dynamic. The regime switching framework maintains the assumption of rational expectations while reinforcing it with a (notional) system of adaptive beliefs.

The paper is organized as follows. In the first section, we briefly present the derivation of the $NKPC-PI$ model highlighting the main consequences of the inclusion of a non-zero steady state inflation rate. The second section is devoted to the empirical aspects and, as *Cogley* & *Sbordone* (2005) and *Groen* & *Mumtaz* (2008), we reconsider a two stage approach to estimate the $NKPC-PI$ model. In the first stage, we compare different $MS-VAR$ specifications to find the one that enables the most appropriate characterization of agents' expectations. We consider specifications in which firstly, the unconditional means of the $NKPC-PI$ varies ($MSM-VAR$) secondly, all the parameters ($MSH-VAR$) and finally, only the intercepts ($MSIH-VAR$) of the vectorial autoregressive model are affected by the switching. In each of these regime switching specifications, we admit that all the variables are able to evolve between a finite numbers ($m \leq 3$) of regimes according to the $Markov$ chain. In the second stage, we exploit the restrictions imposed by the theoretical $NKPC-PI$ model on the first stage selected reduced forms to estimate the structural parameters of the economy. The idea of this stage is to build a measure of the adequacy of the theoretical model to the observed data. The results of this second stage distance minimization problem are comparable to those of recent studies (*Cogley* & *Sbordone* (2005), *Groen* & $Mumtaz$ (2008)). Finally, the third section presents our conclusions.

2 The New Keynesian Phillips Curve with Positive steady-state Inflation model

In order to have the non-neutralities surrounding the basic Keynesian framework, we postulate the existence of economic frictions in the form of prices rigidity. More precisely, the $NKPC-PI$ leans on the hypothesis of monopolistic competition between firms in a non-frequent and unpredictable context with price adjustments exploited in the *Calvo* (1983) way. The particularity of the *Calvo* (1983) model comes from the fact that it starts with the idea of randomized price adjustments. It then considers individual duration of the price rigidity as being random, while the average duration of all the price contracts remains constant.

In this context, the $NKPC-PI$ model is technically derived considering an economy in which the technology of production (common to all the firms) is of $Cobb-Douglas$ type and the capital is not instantaneously reallocated between the firms[3]. Furthermore, at each moment, firms have a probability $(1-\alpha)$ of

on the former states of the economic system.

[3] This last hypothesis means that the marginal cost of a firm that does not adjust its price differs from marginal cost through the economy.

modifying their prices. This probability is the same for all the firms regardless of their historical decisions.

When a representative firm receives the signal to adjust its prices, it takes into account the probability of not having other opportunities during the following periods and adjusts these prices on the basis of its expectations of future states of the economic environment within which it operates. Following *Cogley & Sbordone* (2005), the decision-making problem of a representative firm (i), which is to maximize its expected profit under the constraint of satisfying its demand, can be written as follows

$$\underset{X_t(i)}{Max}\Pi(i) = E_t \sum_{j=0}^{\infty} \alpha^j R_{t,t+j} \left\{ \frac{(X_t(i)\,\Xi_{tj})}{P_{t+j}^{-\theta}}^{1-\theta} Y_{t+j} - TC_{t+j,t}(i) \right\}$$

where $R_{t,t+j}$, X_t, P_{t+j}, Y_{t+j} and $TC_{t+j,t}(i)$ are the nominal discount rate between periods t and $t+j$, the firm relative price (relative to the aggregate level), the production and total costs (in nominal terms) at $t+j$ for the optimizing firm. The function Ξ_{tj} is defined as

$$\Xi_{tj} = \begin{cases} 1 & if\ j=0 \\ \prod_{k=0}^{j-1} \pi_{t+k}^{\varrho} & if\ j \geq 1 \end{cases}$$

and captures the fact that non-optimized prices evolve according to an indexation criterion linked to past inflation. This indexation can alternatively be formalized as

$$P_t(i) = \pi_{t-1}^{\varrho} P_{t-1}(i)$$

The First Order Condition (FOC) for the representative firm that adjusts its price can be written as

$$\begin{array}{c} E_t \sum_{j=0}^{\infty} \alpha^j R_{t,t+j} \prod_{k=1}^{j} \gamma_{y_{t+k}} \prod_{k=1}^{j} \pi_{t+k}^{\theta} \prod_{k=0}^{j-1} \pi_{t+k}^{-\varrho(\theta-1)} * \\ \left(x_t^{1+\theta w} - \frac{\theta}{\theta-1} \psi_{t+j} \prod_{k=1}^{j} \pi_{t+k}^{1+\theta w} \prod_{k=0}^{j-1} \pi_{t+k}^{-\varrho(1+\theta w)} \right) \end{array} = 0 \qquad (1)$$

where $\gamma_{y_t} = \frac{Y_t}{Y_{t-1}}$, $\pi_t = \frac{P_t}{P_{t-1}}$, $x_t = \frac{X_t}{P_t}$ and $\psi_t = \frac{MC_t}{P_t}$ represent the gross rate of output growth, the gross rate of inflation, the relative price and the aggregate real marginal cost of the optimizing firm, respectively.

At the steady state characterized by non-zero inflation ($\bar{\pi} \neq 1$), the equilibrium condition above can be rewritten as

$$\bar{x}^{1+\theta\omega} = \frac{\theta}{\theta-1} \left(\frac{1-\alpha\bar{R}\bar{\gamma}_y \bar{\pi}^{\theta-\varrho(\theta-1)}}{1-\alpha\bar{R}\bar{\gamma}_y \bar{\pi}^{1+\theta(1-\varrho)(1+\omega)}} \right) \bar{\psi}$$

This last equation gives the link between the long-term values of the relative price ($\bar{x}$) and the real marginal cost ($\bar{\psi}$) of the firm. Moreover, the price dynamics at the aggregate level can be described by the following equation

$$1 = \left[(1-\alpha)\, x_t^{1-\theta} + \alpha \pi_{t-1}^{\varrho(1-\theta)} \pi_t^{-(1-\theta)} \right] \qquad (2)$$

Combining equations (1) and (2) in their log-linearized forms (eg $\hat{\pi}_t = \log(\pi_t/\bar{\pi})$) around the non-zero steady state inflation rate, one can obtain the following $NKPC - PI$ equation

$$\begin{aligned}\hat{\boldsymbol{\pi}}_t =& \tilde{\varrho}\hat{\pi}_{t-1} + b_1 E_t \hat{\pi}_{t+1} + b_2 \textstyle\sum_{j=2}^{\infty} \gamma_1^{j-1} E_t \hat{\pi}_{t+j+2} + \xi \hat{\psi}_t \\ &+ \chi(\gamma_2 - \gamma_1) \textstyle\sum_{j=0}^{\infty} \gamma_1^j \left(E_t \hat{R}_{t+j,t+j+1} + E_t \hat{\gamma}_{y_{t+j,t+j+1}} \right) + \varepsilon_t\end{aligned} \qquad (NKPC - PI)$$

with

$$\tilde{\varrho} = \frac{\varrho}{\Delta}$$
$$b_1 = \frac{1}{\Delta}\left(\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1 + \gamma_2\right)$$
$$b_2 = \frac{1}{\Delta}\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\left(\frac{\theta(1-\varrho\gamma_1)+\varrho\gamma_1}{1+\theta\omega}\right)(\gamma_2-\gamma_1)\right)$$
$$\xi = \frac{1}{\Delta}\left(\frac{(1-\alpha\varsigma_1)(1-\gamma_2)}{\alpha\varsigma_1(1+\theta\omega)}\right)$$
$$\chi = \frac{1}{\Delta}\left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1(1+\theta\omega)}\right)$$

where, we have the following intermediate terms

$$\varsigma_1 = \bar{\pi}^{(\theta-1)(1-\varrho)}$$
$$\varsigma_2 = \bar{\pi}^{\theta(1+w)(1-\varrho)}$$
$$\tilde{\beta} = \bar{R}\bar{\gamma}_y\bar{\pi}$$
$$\gamma_1 = \alpha\tilde{\beta}\varsigma_1$$
$$\gamma_2 = \alpha\tilde{\beta}\varsigma_2$$
$$\Delta = 1 + \varrho\gamma_2 - \left(\frac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1$$
$$\phi_1 = \frac{\varrho\theta(\gamma_1-(1+\omega)\gamma_2)-\varrho\gamma_1}{1+\theta\omega}$$
$$\phi_2 = \frac{1}{1+\theta\omega}\left(\gamma_2(1+\theta\omega) + (\gamma_2-\gamma_1)(\theta(1-\varrho\gamma_1)+\varrho\gamma_2)\right)$$

Naturally, the assumption of a non-zero steady state inflation rate leads to a "long term *Phillips* curve" characterized by an equation tying together the different steady state values of the variables ($\bar{\pi}$, $\bar{\psi}$, $\bar{\gamma}_y$ and $\bar{R}$) given by

$$\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varrho)}\right)^{\frac{1+\theta\omega}{1-\theta}}\left(\frac{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{1+\theta(1+w)(1-\varrho)}}{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{\theta-\varrho(\theta-1)}}\right) = \frac{\theta}{\theta-1}(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}\bar{\psi} \qquad (NKPC_{SS} - PI)$$

According to the $NKPC - PI$ equation, the inflation rate fluctuations are mainly explained by the expectations made on the evolution of almost all the variables of interest, i.e. the inflation rate itself ($\hat{\boldsymbol{\pi}}_t$), the real marginal cost ($\hat{\psi}_t$), the output growth rate ($\hat{\gamma}_{y_{t,t+1}}$) and the nominal discount rate ($\hat{R}_{t,t+1}$). The parameters in the previous equations are the main structural parameters of the $NKPC - PI$ economy. They measure the degree of price rigidity (α), the indexation (ϱ) to past inflation and the *Dixit−Stiglitz* elasticity of substitution (θ) between differentiated goods, respectively.

3 Regime switching in the NKPC-PI

The main problem which is raised in estimating this kind of equation is related to the presence of expectations terms ($E_t[.]$). Essentially, it is about how we consider the expectations of the four variables have to follow their underlying trends dynamics, on the condition that the structural parameters of the economy ($\Psi = [\alpha, \varrho, \theta]$) remain constant. In order to resolve this problem and to estimate the structural parameters, we follow a two-stage strategy.

As explained in the introduction, in the first stage, we model the dynamic properties of the variables in a reduced form, using $Markov$ Switching - Vectorial AutoRegressive ($MS-VAR$) models to analyze the inflation rate dynamics. We choose the $MS-VAR$ representation to characterize as fully as possible the agents' expectation process and more precisely, to take into account possible variations of the four variable trends dynamics. As part of the VAR representation, several regime switching specifications may be considered in the class of $MS-VAR$ models.

In the second stage, we use the cross-equation restrictions that the $NKPC-PI$ model requires for the selected reduced forms to construct a measure of the gap between the model and the data. Specifically, the given estimates of the first stage offer a set of parameters describing the data via the reduced forms. Combined with the theoretical restrictions imposed on these first stage estimates, one can express moment conditions which have to be minimized to recover the structural parameters.

3.1 Description of the data

To be able to reconsider the previous $NKPC-PI$ estimation results (*Cogley & Sbordone* (2005), $Groen$ & $Mumtaz$ (2008)), we chose to build a database reflecting the best possible data as used by these earlier studies. Then, as *Cogley & Sbordone* (2005) our sample period covers $T = 176$ quarters from $1960 : I$ to $2003 : IV$.

Likewise, we calculate the output growth rate on the basis of a weighted sequence of the real GDP expressed in 2000 dollars (seasonally adjusted at an annual rate) and recorded in the National Income and Product Accounts *NIPA Table (*1.3.6*)*. Regarding the discount rate ($R_{t,t+1}$), we use the 3-Month Treasury Bill: Secondary Market Rate (i_t) from the *Federal Reserve Bank of St. Louis (FRED)* database. To construct $R_{t,t+1}$, we apply the formula: $R_{t,t+1} = \frac{i_t}{1+i_t}$, where i_t was divided by 100.

Besides, assuming a $Cobb-Douglas$ production function, the real marginal cost (ψ_t) is proportional to the labor unit cost (ulc_t) as mathematically, we have

$$\psi_t = \ln\left(\frac{W_t N_t}{P_t Y_t}\right) - \ln(1-\kappa) = \ln(ulc_t) - \ln(1-\kappa)$$

where Y_t is the level of output in real terms, N_t is the total amount of labour input, W_t measures the wages. Following *Cogley & Sbordone* (2005), the output elasticity to hours of work $(1-\kappa)$ in the production function is set equal to 0.6666 so that the strategic complementarities parameter equal to $\omega = \frac{\kappa}{1-\kappa} = 0.5001$. This real marginal cost series is constructed according to $Groen$ & $Mumtaz$ (2008). The data came from the *NIPA Tables (*1.1.5 and 1.12*)*. The inflation rate is measured from the implicit price deflator as (P_t), recorded by the *Bureau of Labor Statistics (BLS)* database. From this data, we compute this series as

$$\pi_t = 4 * [\ln(P_t) - \ln(P_{t-1})]$$

3.2 MS-VAR reduced forms

Let, $Z_t = \left(\pi_t,\ R_{t,t+1},\ \psi_t,\ \gamma_{y_{t,t+1}}\right)'$ the vector of the variables, m the numbers of regimes, p the lag order of the VAR in the various MS specifications which will be considered and $\varepsilon_t = \left(\varepsilon_t^{\pi}, \varepsilon_t^{R}, \varepsilon_t^{\psi}, \varepsilon_t^{\gamma_y}\right)'$ the vector of errors.

In each of these specifications, $S_t = \{1, ..., m\}$ is defined as an unobservable variable that can take m values (regimes) according to a first order $Markov$ chain so that, when $S_t = 1$ all the variables are simultaneously (at time t) in the first of the m regimes. Also, in all the $MS(m) - VAR(p)$ specifications, the transition matrix is defined as

$$P = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{m1} \\ p_{12} & p_{22} & \cdots & p_{m2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1m} & p_{2m} & \cdots & p_{mm} \end{pmatrix}$$

with $p_{ij} = P(S_{t+1} = i \mid S_t = j) \geq 0$ and $\sum_{j=1}^{m} p_{ij} = 1$ for $i, j = 1, ..., m$.

In this context, we consider the following $MS(m) - VAR(p)$ specifications.

- **The linear specification**

$$Z_t = A_0 + A_1 Z_{t-1} + ... + A_p Z_{t-p} + \varepsilon_t \qquad (VAR)$$

in which, $\varepsilon_t \sim N(0, \Sigma)$. Under this $MS(1) - VAR(p)$ specification, the variance-covariance matrix of errors (Σ) and all the coefficients $(A_0, A_1, ..., A_p)$ are assumed constant. From this benchmark model, three non-linear specifications allowing three different trend dynamics for each of the four variables could be considered.

- **The MSM**(m) **− VAR**(p) **specification** (*Hamilton* (1989))

In this specification, only the unconditional means $(\mu_{S_t=k},\ k = \{1, ..., m\})$ of the variables are subject to the regimes switching, i.e.

$$Z_t = A_1\left(Z_{t-1} - \mu_{S_t=k}\right) + ... + A_p\left(Z_{t-p} - \mu_{S_t=k}\right) + \varepsilon_t \qquad (MSM - VAR)$$

where $\varepsilon_t \sim N(0, \Sigma)$, the variance-covariance matrix of errors (Σ) and all the autoregressive coefficients $(A_1, ..., A_p)$ are assumed constant or identical in each regime.

- **The general MSH**(m) **− VAR**(p)

In this second non-linear specification[4], all the VAR coefficients (including the variance-covariance matrix) are subject to the regimes switching, i.e.

$$Z_t = A_{0,S_t=k} + A_{1,S_t=k} Z_{t-1} + ... + A_{p,S_t=k} Z_{t-p} + \varepsilon_t \qquad (MSH - VAR)$$

where $\varepsilon_t \sim N\left(0, \Sigma_{S_t=k}\right)$ and $\Sigma_{S_t=k}$ the covariance matrix varies from one regime ($k = \{1, ..., m\}$) to another. $A_{0,S_t=k}$ is the intercepts vector, while the matrices $A_{1,S_{t=k}}, ..., A_{p,S_{t=k}}$, each of size (4×4), contain the autoregressive coefficients of the VAR.

- **The MSIH**(m) **– VAR**(p) **specification**

Finally, in the third non-linear specification, only the intercepts $(A_{0,S_t=k})$ and the covariance matrix $(\Sigma_{S_t=k})$ are assume to switch.

$$Z_t = A_{0,S_t=k} + A_1 Z_{t-1} + ... + A_p Z_{t-p} + \varepsilon_t \qquad (MSIH - VAR)$$

It is evident that each of these specifications allows us to consider a particular characteristic of the trends of the variables. In what follows, we estimate these specifications and identify the one which gives the appropriate characterization of the agents' expectations. The selection procedure is achieved comparing the usual information criteria (AIC, BIC and LIL) and making the diagnostic check of the residuals associated with each of these specifications.

3.3 The adequate MS-VAR specification

First of all, in the linear context (*Table* 1), a model with two lags ($p = 2$) seems more appropriate than specifying a single lag in the vectorial autoregressive coefficients that describe the joint dynamics of the variables. Secondly, according to the information criteria associated with the estimates of each non-linear specification (*Table* 2), the three non-linear specifications could be used to econometrically characterize the agents' expectations. In fact, the AIC criterion select the $MSH(3)-VAR(2)$ specification while the BIC and the LIL criterion select the $MSM(2) - VAR(2)$ and the $MSIH(2) - VAR(2)$, respectively.

Globally and whatever the number ($m \leq 3$) of regimes considered, the results argue in favor of the $MSIH-VAR(2)$ reduced form. Then, one can assume that only the intercepts are subjected to the regimes switching and the autoregressive coefficients (A_1 and A_2) can be considered constant[5]. Furthermore, in this $MSIH - VAR(2)$ framework, the model with three regimes ($m = 3$) appears slightly better than the model with two regimes. This is confirmed by the residuals diagnostics as the three regimes specification captures most of the

[4] This is the specification assumed by *Groen* & *Mumtaz* (2008) who considers an $MS(2)-VAR(1)$ model.

[5] Noting that, in a related work, we show, using the *Nyblum* (1989) tests in univariate context, that the stability of the autoregressive coefficients can not be rejected. Then, one can assume the stability or the constancy of the autoregressive coefficients of the VAR. Moreover, *Sims* (2001), *Stock* (2001) and *Pivetta* & *Reis* (2007) argue that inflation persistence is approximately unchanged.

serial autocorrelation and the heteroskedasticity in the main variable of the model, i.e. the inflation rate ($Figures\ 1a - 1d$).

In addition, the results of the $MSIH(3) - VAR(2)$ model ($Table\ 3b$) show that some estimated regimes for some of the variables are basically the same. Then, one could estimate some constrained $MSIH - VAR(2)$ models. In a related work, we show that the unconditional means of both the discount and the output growth rates could be considered as constant, while a model with two regimes ($MSIH(2) - AR(2)$) seems slightly preferable to a linear model to describe the real marginal cost dynamics. On the other hand, the inflation rate seems to be adequately described by a three regimes switching model. So, to complete this first stage of the $NKPC - PI$ estimation, we consider a $MSIH(\pi(3), \psi(2)) - VAR(2)$ and a $MSIH(\pi(3)) - VAR(2)$ as constrained models. In the first constrained model, the unconditional mean of the inflation rate and the real marginal cost series varies between three and two regimes, respectively. In the second one, only the unconditional mean of the inflation rate varies between three regimes during the period under study. The selection criteria associated to these two constrained models ($Table\ 4$) are close to those of the $MSIH(2) - VAR(2)$ and $MSIH(3) - VAR(2)$ specifications so that these two constrained models could also be relevant to characterize the variables dynamics.

Taken together, these results indicate that we cannot particularly rely on one of these specifications. Each of them might be suitable for capturing the non-linearity in the variables dynamics. Yet, we can consider all these $MSIH - VAR(2)$ specifications to characterize the agents' expectations process.

3.4 MSIH-VAR(2) estimates

Recalling that all the $MS - VAR$ specifications were estimated by the maximum likelihood method and the $Hamilton$ (1989) filtering procedure, the $MSIH - VAR(2)$ results are summarized in $Table\ 3a - 3d$. In terms of the regimes (unconditional means) visited by the inflation rate in each of the four $MSIH - VAR(2)$ specifications, the results are given in $Table\ 5$. The dynamics of the unconditional means of the inflation rate associated to each of these specifications are shown in $Figure\ 2a - 2d$. These graphs show that the unconditional mean of the inflation rate varied greatly in the years $68 - 72$, $74 - 76$, $78 - 83$ and $01 - 02$, periods covering the oil and the monetary shocks. Such variations may be associated with episodes of major structural change in the U.S. economy initiated or accompanied by changes in the agents' beliefs. This finding is also observable through the dynamics of the probabilities of being in each regime given the past information as illustrated in $Figure\ 3a - 3d$.

According to these $MSIH - VAR(2)$ specifications, the third regime, associated with low inflation rates $\bar{\pi}_{S_t=3} \in [0.0188, 0.0259]$ covers the bulk of the sample. It can be characterized as a regime of "optimism" and can be associated with the basic environment of the economy. In contrast, the first regime associated with high inflation rates $\bar{\pi}_{S_t=1} \in [0.0619, 0.0829]$ appears as an exception because it only covers $25 - 45$ out of the 176 quarters of the sample. This state can be regarded as a regime of "pessimism" or a "confidence crisis" regime in the dynamic of the economy. Between these two extreme regimes, it is possible to consider a third regime associated with an intermediate inflation rate level $\bar{\pi}_{S_t=2} \in [0.0290, 0.0641]$. The inclusion of this regime of "skepticism" appears important for us to understand the inflation rate dynamics, as it seems to be a transitory regime between the other two.

From $Figure\ 3a - 3d$, we can see that the probability of being in this intermediate regime, given the past information ($P(S_t = 3 \mid I_{t-1})$), reached its peak shortly before or during episodes of confidence crisis. We note that during these transitional quarters, the economy is subjected to an attraction force that pulls the system to the "pessimism" regime, even if it does not remain in this last regime, except during the crisis quarters. It seems that during the periods of transition between the third and the first regimes, the agents believe that the monetary policy is temporarily out of discretion in the favor of the real activity stimulation; away from a fight against inflation. Therefore, this intermediate regime captures transient episodes in the dynamic of the agents' beliefs. It is somehow a sign announcing inflationary episodes, possibly resulting from questioning the credibility of the regulatory policies. The coming of this regime that announces the episodes of rising inflation rates confirms periods of structural change (oil price shocks or monetary policy) identified in the macroeconomic literature.

4 Estimation of the Phillips curve

Given these first stage results, we run the estimation of the structural parameters using the cross-equation restrictions that the model requires for the four $MSIH - VAR$ reduced forms previously selected. Specifically, the estimate made in the first stage offers a set of estimated coefficients ($A_{0,S_t=k}$, A_1 and A_2). Combining these estimates with the restrictions imposed by the theoretical model, leads to moment conditions $F_{1,S_t}\left(A_{0,S_t=k}, A_1, A_2, \Psi\right)$ and $F_{2,S_t}\left(A_{0,S_t=k}, A_1, A_2, \Psi\right)$ that capture the gap between data and model.

Starting with a centered reduced form $MSIH\,(m) - VAR(2)$ model,

$$\tilde{Z}_t = A_1\tilde{Z}_{t-1} + A_2\tilde{Z}_{t-2} + \varepsilon_t$$

where $\tilde{Z}_t = Z_t - (I - A_1 - A_2)^{-1} A_{0,S_t=k}$ and defining $z_t = \left(\tilde{Z}_t, \tilde{Z}_{t-1}\right)'$, we have in a $VAR(1)$ form

$$z_t = \mathbf{A}z_{t-1} + \epsilon_t$$

with

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 \\ I & O \end{bmatrix}$$
$$\epsilon_t = (\varepsilon_t, 0, 0, 0, 0)'$$

where I is the identity matrix of dimension 4 and O is a (4×4) matrix of zeros. We can express the conditional expectations of the deviations of the four variables relative to their steady states as[6]

$$\begin{aligned}E\left(\hat{\pi}_t \mid z_{t-1}\right) &= e'_\pi \mathbf{A} z_{t-1}\\ E\left(\hat{\psi}_t \mid z_{t-1}\right) &= e'_\psi \mathbf{A} z_{t-1}\\ E\left(\hat{R}_{t,t+1} \mid z_{t-1}\right) &= e'_R \mathbf{A} z_{t-1}\\ E\left(\hat{\gamma}_{y_{t,t+1}} \mid z_{t-1}\right) &= e'_{\gamma_y} \mathbf{A} z_{t-1}\end{aligned}$$

where e_k terms are column vectors of value 1 at the position corresponding to the variable k and 0 elsewhere and are used to select separately each of the four variables in the vector Z_t.

Under the assumption that $E\left(\epsilon_t \mid z_{t-1}\right) = 0$, we are able to obtain the conditional expectations of each variable by projecting the left and right terms of the $NKPC - PI$ equation on z_{t-1}, i.e.

$$E\left(\hat{\pi}_t \mid z_{t-1}\right) = e'_\pi \mathbf{A} z_{t-1} = f\left(\mathbf{A}, \Psi\right)$$

$$\begin{aligned}E\left(\hat{\pi}_t \mid z_{t-1}\right) = {} & \tilde{\rho} e'_\pi z_{t-1} + \xi e'_\psi \mathbf{A} z_{t-1} + b_1 e'_\pi \mathbf{A}^2 z_{t-1} + b_2 e'_\pi \gamma_1 \left(I - \gamma_1 \mathbf{A}\right)^{-1} \mathbf{A}^3 z_{t-1}\\ & + \chi\left(\gamma_2 - \gamma_1\right) e'_R \left(I - \gamma_1 \mathbf{A}\right)^{-1} \mathbf{A} z_{t-1} + \chi\left(\gamma_2 - \gamma_1\right) e'_{\gamma_y} \left(I - \gamma_1 \mathbf{A}\right)^{-1} \mathbf{A}^2 z_{t-1}\end{aligned}$$

We obtain the first set of moment conditions that capture the difference between data and model and the restrictions implied by the theoretical model on the set of parameters describing data via the reduced form follows as

$$F_{1,S_t}\left(A_{0,S_t=k}, A_1, A_2, \Psi\right) = e'_\pi \mathbf{A} - f\left(\mathbf{A}, \Psi\right)$$

Similarly, one can use the $NKPC_{SS} - PI$ equation to form the second set of moment conditions linking the steady state values of all the model variables

$$\begin{aligned}& F_{2,S_t}\left(A_{0,S_t=k}, A_1, A_2, \Psi\right)\\ = {} & \left(1 - \alpha \bar{\pi}^{(\theta-1)(1-\rho)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left(\frac{1 - \alpha \bar{R} \bar{\gamma}_y \bar{\pi}^{1+\theta(1+w)(1-\rho)}}{1 - \alpha \bar{R} \bar{\gamma}_y \bar{\pi}^{\theta-\rho(\theta-1)}}\right) - \frac{\theta\left(1-\alpha\right)}{\theta - 1}^{\frac{1+\theta\omega}{1-\theta}} \bar{\psi}\end{aligned}$$

These two sets of moment conditions define an overall distance measure that enables us to judge the adequacy of the model to data

$$\digamma_{S_t}\left(\Psi\right) = \left(F'_{1,S_t} F'_{2,S_t}\right)'$$

The model fits the data if and only if there is a vector of structural parameters (Ψ) that solves the following constrained minimization problem

$$\underset{\Psi}{Min} \digamma_{S_t}\left(\Psi\right)' \digamma_{S_t}\left(\Psi\right)$$

subject to $\alpha \in \left]0,1\right[$, $\varrho \in [0,1]$ and $\theta \in [0, +\infty[$.

[6] Their empirical steady state equivalents are given by

$$\begin{aligned}\bar{\pi} &= \exp\left(e'_\pi \left(I - \mathbf{A}\right)^{-1} A_{0,S_t=k}\right)\\ \bar{R} &= \left(e'_R \left(I - \mathbf{A}\right)^{-1} A_{0,S_t=k}\right)\\ \bar{\psi} &= \exp\left(e'_\psi \left(I - \mathbf{A}\right)^{-1} A_{0,S_t=k}\right)\\ \bar{\gamma}_y &= \exp\left(e'_{\gamma_y} \left(I - \mathbf{A}\right)^{-1} A_{0,S_t=k}\right)\end{aligned}$$

4.1 Structural parameters estimates

The results of the distance minimizations[7], obtained by the method of grid variations, are presented in *Table* 6. Overall, they are comparable to those of previous studies (*Cogley* & *Sbordone* (2005), *Groen* & *Mumtaz* (2008)). Our results indicate that the parameter measuring the degree of price ridgidity is estimated as $\hat{\alpha}_{S_t=k} \in [0.0259, 0.0509]$. We note that there is generally a high stability in the price ridgidity through the estimated inflation rate regimes. *Cogley* & *Sbordone* (2005) and *Groen* & *Mumtaz* (2008) results confirm this tendency since $\alpha^{CS} = 0.60$ and $\alpha^{GM} \in [0.05, 0.40]$ but the estimated probabilities of price non-adjustment are instead much lower than those estimated by these authors. From these estimates, prices vary more often than indicated by micro-evidences (*Bils* & *Klenow* (2004)). The price ridgidity associated is $\tau = \frac{1}{1-\alpha} \in [1.03, 1.05]$ quarters, i.e. around one quarter compared to $\tau^{BK} \in [4.4, 5.5]$ months, $\tau^{CS} = 4.1$ months. This last result could be explained by the fact that firms adjust their expectations following an adaptive beliefs system and as a consequence, prices change more frequently or more endogenously than indicated by the *Cogley* & *Sbordone* (2005) estimates[8].

The indexation parameter is estimated as $\hat{\varrho}_{S_t=k} \in [0.0010, 0.2008]$ and firms that do not receive the signal to re-optimize their prices have a weak and quasi-negligible opportunity to index them on the past inflation. Building on these results, prices may possibly change each quarter in contrast to results obtained by *Cogley* & *Sbordone* (2005) who estimated $\varrho^{CS} = 0$. Nevertheless, we find that the estimated values of the indexation parameters are rather low and often zero. As suggested by *Kozicki* & *Tinsley* (2003) or *Ireland* (2007), shifts in trend inflation rate or the central bank's inflation target can substitute for the backward-looking terms in the *Phillips* curve in explaining inflation persistence. However, the fact that this parameter can also be non-zero seems to confirm results obtained by many other studies performed in a *NKPC* with a zero steady state inflation rate (*Gali* & *Gertler* (1999), *Giannoni* & *Woodford* (2003)). For most of these studies, this indexation parameter is significantly estimated between 0.2 and 1. The existence of a non-zero indexation degree can capture the persistence observed in the U.S. inflation rate dynamics. This result is also highlighted by *Groen* & *Mumtaz* (2008) who estimate $\varrho^{GM} \in [0.65, 0.95]$. Thus, the fact that the $NKPC-PI$ model initially takes trend inflation rate dynamics into account does not make it possible to directly exclude its backward-looking component.

Finally, the parameter that measures the degree of substitution between

[7] We suppose $\theta \in [0, 60]$ to conduct these distance minimizations.

[8] Moreover, it emerges an inverse relationship between the steady state inflation rate level and the degree of prices stickiness as

$$\alpha_{S_t=k} = f\left(\bar{\pi}_{S_t=k}\right) \ with \ \frac{\partial f}{\partial \bar{\pi}} \leq 0$$

Thus, as trend inflation rate rises, the degree of prices stickiness decreases. This inverse relationship confirms the findings of many others studies (*Ball* & *al.* (1998), *Bakhshi* & *alii.* (2007)).

goods is estimated as $\hat{\theta}_{S_t=k} \in [34.9292, 58.5252]$. This result remains fairly close to the values estimated by *Groen* & *Mumtaz* (2008) but somewhat higher when compared to those of *Cogley* & *Sbordone* (2005). We also note that this last parameter seems to be less stable than the other two. This "instability" could indicate a time-varying elasticity of demand for each good. This estimated degree of substitution between goods implies a mark-up of about $\vartheta_{S_t=k} \in [1\%, 4\%]$ and an increasing relationship between the degree of substitution and the steady state inflation rate seems extractable.

4.2 Estimates of the NKPC-PI coefficients

The $NKPC-PI$ coefficients are derived from the estimated structural parameters and they are computed for each of the inflation rate regimes associated with each of the $MSIH(m)-VAR(2)$ specifications. As explained above, we do not focus on any of these models specifically since any one of them can sufficiently characterize the agents' expectations process. The emphasis is rather put on the average values of these $MSIH(m)-VAR(2)$ estimated coefficients.

Figures $4a-4d$ show that the coefficient ξ, reflecting the effectiveness of the Inflation - Real activity trade-off, moves inversely to the trend inflation rate from almost 0.30 to 0.08. Our estimates of the impact of the real marginal cost on the current inflation rate are almost higher than those reported by *Cogley* & *Sbordone* (2005, 2008) and *Groen* & *Mumtaz* (2008). But, as noted by *Ireland* (2007, *p.* 41) : " estimates of this magnitude can be reconciled with greater frequencies of adjustment of individual prices, if additional plausible sources of real rigidities (in the sense of *Ball* & *Romer* (1990)) are taken into account ". Compared to the $TVP-VAR$ reduced form considered by *Cogley* & *Sbordone* (2005, 2008), these added sources of real rigidities could come from the fact that, in our $MSIH-VAR$ reduced forms, all the VAR coefficients (except the intercepts) are maintained as constant. However, regarding this coefficient ξ, our estimates confirms those of *Dufour* & *al.* (2010) who consider alternative structural New Keynesian inflation equations concerning the average duration of prices in the U.S. to evaluate the precision of *Calvo* parameter estimates.

Also, we note that

$$\xi = g\left(\bar{\pi}_{S_t=k}\right) \ with \ \frac{\partial g}{\partial \bar{\pi}_{S_t=k}} < 0$$

During episodes of high inflation rate, the trade-off tends to disappear, challenging the *Phillips* curve consistently within these identified periods that marked the renewal of the visions of the arbitration initiated by authors such as *Phelps* (1967), *Friedman* (1968) and *Lucas* (1972*a*). We also note that a non-negligible challenge (possibly associable to the *September* 11 events) is highlighted during late 2001 - early 2002.

The others coefficients vary in the same direction as the steady state inflation rate. Particularly, the impacts of the expected inflation rate are quite significant, ranging on average between $b_1 \in [0.20, 0.40]$ and $b_2 \in [0.09, 0.30]$. In contrast, as highlights by *Cogley* & *Sbordone* (2005) and *Groen* & *Mumtaz* (2008), the coefficient measuring the joint impact of the discount and the output growth rates is almost zero and therefore negligible $\chi \in [0.001, 0.008]$.

5 Conclusion

The present article offers an adequate way of integrating the agents' rational expectations that enable desired values of the $NKPC - PI$ variables to differ during the period under study, without being a consequence of continuous revisions in the agents' expectations as "constrained" by the rational expectations hypothesis. We capture the non-linearity of the series in a $Markov$ Switching Intercept - VAR framework. This framework implies that the unconditional means of the variables to obey the regimes switching controlled by a $Markov$ chain of order 1. These switchings are dictated by a matrix of transition probabilities that could be perceived as a system of beliefs formed by the agents on the presumed fulfillments of their inflationist expectations. These beliefs can be qualified as adaptive since the probabilities of the switching from one regime to another are conditional on the previous states of the economy.

As a result, it appears that the agents' expectations emerge from combinations of some fixed levels (regimes) detected in the variables dynamics with an agents' adaptive beliefs process empirically captured by the $MSIH - VAR$ models. Conceptually, this approach seems to be the most adequate one to characterize the variables dynamics as their trends dynamics come from a random scheme. More fundamentally, we allow the inflation rate to move from zero at its steady state, while having a trend dynamics as illustrated in $Figure$ 2.

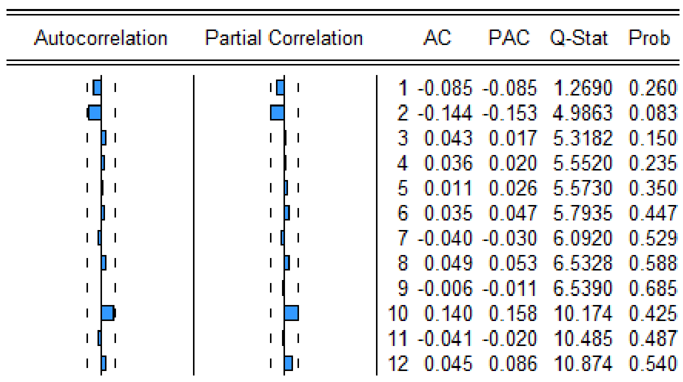
Taking into account these identified trends, we conducted empirical analysis around the famous bridge between the nominal and the real economics spheres associated to the *Phillips* curve. Our results indicate whether there is a trade-off that is deeply entrenched in micro-founded behavior of the agents. More specifically, as the structural parameters of the economy seem to be globally unaffected by the regimes' switching, the variation in the $NKPC - PI$ coefficients is only due to variations of the steady state inflation rate. The observed time or state variations in the Inflation - Real activity trade-off could then be solely due to changes in the monetary policy regime. These results confirm those of $Kozicki$ & *Tinsley* (2003), *Cogley* & *Sbordone* (2005, 2008) or $Ireland$ (2007) among others. It seems like switches in the trend inflation rate can substitute for the intrinsic inflation persistence as introduced in previous $NKPC$ models. Finally, as suggested by $Ireland$ (2007), our estimates also "*provide some support for stories told previously by Blinder* (1982) *and* $Mayer$ (1998)*, which attribute the rise in the U.S. inflation during the 1960s and 1970s to a systematic tendency for Federal Reserve policy to translate short-run price pressures set off by adverse supply-side shocks into more persistent movements in the inflation rate itself*".

$Models$	LnL	AIC
$VAR(1)$	2394.686	-27.1392
VAR(2)	**2423.060**	$\mathbf{-27.4374}$

Table 1 : Selection of the VAR lags

$Models$	AIC	BIC	LIL
$MSM(2)-VAR(2)$	-27.8414	$\mathbf{-25.3555}$	-27.4584
$MSM(3)-VAR(2)$	-27.8027	-24.9344	-27.3608
$MSI(2)-VAR(2)$	-28.3141	*−25.3502*	$\mathbf{-27.8575}$
$MSI(3)-VAR(2)$	*−28.4438*	-24.6194	*−27.8546*
$MS(2)-VAR(2)$	-28.2521	-23.7584	-27.5598
$MS(3)-VAR(2)$	$\mathbf{-28.5524}$	-21.6685	-27.4919

Table 2 : Selection of the MS−VAR(2) specification



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.085	-0.085	1.2690	0.260
		2	-0.144	-0.153	4.9863	0.083
		3	0.043	0.017	5.3182	0.150
		4	0.036	0.020	5.5520	0.235
		5	0.011	0.026	5.5730	0.350
		6	0.035	0.047	5.7935	0.447
		7	-0.040	-0.030	6.0920	0.529
		8	0.049	0.053	6.5328	0.588
		9	-0.006	-0.011	6.5390	0.685
		10	0.140	0.158	10.174	0.425
		11	-0.041	-0.020	10.485	0.487
		12	0.045	0.086	10.874	0.540

Figure 1a : Correlogram of the $MSI(2)-VAR(2)$ residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.029	0.029	0.0901	0.764
		2	-0.013	-0.014	0.1086	0.947
		3	0.002	0.002	0.1089	0.991
		4	0.111	0.111	1.4410	0.837
		5	0.008	0.001	1.4477	0.919
		6	0.029	0.032	1.5415	0.957
		7	0.141	0.141	3.7423	0.809
		8	-0.102	-0.126	4.9090	0.767
		9	0.011	0.024	4.9236	0.841
		10	-0.001	-0.011	4.9238	0.896
		11	-0.107	-0.149	6.2553	0.856
		12	0.006	0.046	6.2592	0.902

$Figure\ 1b: Correlogram\ of\ the\ MSI(3) - VAR(2)\ residuals$

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.175	0.175	5.4278	0.020
		2	0.037	0.007	5.6751	0.059
		3	0.110	0.106	7.8564	0.049
		4	0.103	0.069	9.7856	0.044
		5	0.048	0.017	10.195	0.070
		6	0.060	0.040	10.857	0.093
		7	0.199	0.174	18.113	0.011
		8	0.072	0.001	19.080	0.014
		9	0.052	0.030	19.579	0.021
		10	0.131	0.087	22.772	0.012
		11	0.020	-0.052	22.848	0.019
		12	-0.022	-0.040	22.936	0.028

$Figure\ 1c: Correlogram\ of\ the\ MSI(2) - VAR(2)\ squared\ residuals$

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.043	-0.043	0.1881	0.665
		2	-0.036	-0.038	0.3275	0.849
		3	-0.037	-0.040	0.4723	0.925
		4	0.015	0.011	0.4977	0.974
		5	-0.042	-0.044	0.6868	0.984
		6	-0.039	-0.044	0.8542	0.991
		7	0.012	0.006	0.8714	0.997
		8	0.012	0.006	0.8876	0.999
		9	-0.042	-0.043	1.0871	0.999
		10	-0.029	-0.033	1.1853	1.000
		11	0.015	0.006	1.2110	1.000
		12	-0.028	-0.034	1.2997	1.000

$Figure\ 1d: Correlogram\ of\ the\ MSI(3)-VAR(2)$
$squared\ residuals$

$\textbf{State}\,(k)$	$\textbf{n}^\circ\textbf{obs/state}$	$\bar{\mathbf{P}}\,(S_t=k)$	$A_{0,S_t=k}$
1	45	$\underset{(0.1080)}{0.2409}$	$\begin{bmatrix} \underset{(0.0070)}{0.0105} \\ \underset{(0.0039)}{-0.0020} \\ \underset{(0.0030)}{-0.0060} \\ \underset{(0.0039)}{0.0114} \end{bmatrix}$
2	129	$\underset{(0.1080)}{0.7591}$	$\begin{bmatrix} \underset{(0.0043)}{0.0012} \\ \underset{(0.0013)}{-0.0007} \\ \underset{(0.0016)}{-0.0042} \\ \underset{(0.0022)}{0.0099} \end{bmatrix}$

$$A_1 = \begin{bmatrix} \underset{(0.0818)}{0.3234} & \underset{(0.2236)}{0.8119} & \underset{(0.2192)}{0.3932} & \underset{(0.1663)}{0.1604} \\ \underset{(0.0307)}{0.0272} & \underset{(0.0909)}{1.3612} & \underset{(0.0762)}{-0.0338} & \underset{(0.0613)}{0.1074} \\ \underset{(0.0350)}{0.0395} & \underset{(0.0978)}{-0.0724} & \underset{(0.0971)}{1.2470} & \underset{(0.0683)}{0.1904} \\ \underset{(0.0465)}{0.0103} & \underset{(0.1420)}{-0.0021} & \underset{(0.1373)}{-0.1894} & \underset{(0.1091)}{0.1260} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \underset{(0.0904)}{0.4223} & \underset{(0.2216)}{-0.7482} & \underset{(0.2105)}{-0.2941} & \underset{(0.1563)}{0.1603} \\ \underset{(0.0234)}{0.0350} & \underset{(0.0939)}{-0.4142} & \underset{(0.0792)}{0.0348} & \underset{(0.0569)}{0.0882} \\ \underset{(0.0298)}{-0.0311} & \underset{(0.1026)}{0.1204} & \underset{(0.0950)}{-0.2767} & \underset{(0.0709)}{-0.0006} \\ \underset{(0.0483)}{-0.0345} & \underset{(0.1424)}{-0.0683} & \underset{(0.1332)}{0.1181} & \underset{(0.1008)}{0.1267} \end{bmatrix}$$

$$P = \begin{bmatrix} \underset{(0.0786)}{0.8327} & \underset{(0.0274)}{0.0531} \\ \underset{(0.0786)}{0.1673} & \underset{(0.0274)}{0.9469} \end{bmatrix}$$

$\ln L = 2525.330$

Table 3a : Estimates of the $MSI(2){-}VAR(2)$ model

State (k)	**n°obs/state**	$\bar{\mathbf{P}}\left(S_t = k\right)$	$A_{0,S_t=k}$
1	25	$\underset{(0.0555)}{0.1340}$	$\begin{bmatrix} \underset{(0.0113)}{0.0164} \\ \underset{(0.0053)}{-0.0043} \\ \underset{(0.0024)}{-0.0029} \\ \underset{(0.0042)}{0.0063} \end{bmatrix}$
2	132	$\underset{(0.1033)}{0.7702}$	$\begin{bmatrix} \underset{(0.0039)}{-0.0010} \\ \underset{(0.0011)}{-0.0002} \\ \underset{(0.0016)}{-0.0054} \\ \underset{(0.0020)}{0.0107} \end{bmatrix}$
3	17	$\underset{(0.0957)}{0.0958}$	$\begin{bmatrix} \underset{(0.0090)}{0.0095} \\ \underset{(0.0141)}{-0.0027} \\ \underset{(0.0091)}{-0.0063} \\ \underset{(0.0215)}{0.0126} \end{bmatrix}$

$$A_1 = \begin{bmatrix} \underset{(0.0850)}{0.2992} & \underset{(0.2659)}{0.7523} & \underset{(0.2115)}{0.3900} & \underset{(0.1550)}{0.4328} \\ \underset{(0.0286)}{0.0070} & \underset{(0.0895)}{1.2979} & \underset{(0.0637)}{-0.0306} & \underset{(0.0614)}{0.1174} \\ \underset{(0.0317)}{0.0671} & \underset{(0.0776)}{-0.0519} & \underset{(0.0950)}{1.3871} & \underset{(0.0705)}{0.3846} \\ \underset{(0.0552)}{-0.0689} & \underset{(0.1827)}{0.0266} & \underset{(0.1497)}{-0.4048} & \underset{(0.1144)}{-0.1005} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \underset{(0.0962)}{0.4206} & \underset{(0.2478)}{-0.6833} & \underset{(0.2030)}{-0.2986} & \underset{(0.1652)}{0.0857} \\ \underset{(0.0190)}{0.0587} & \underset{(0.0912)}{-0.3487} & \underset{(0.0614)}{0.0484} & \underset{(0.0471)}{0.0740} \\ \underset{(0.0345)}{-0.0484} & \underset{(0.0686)}{0.0797} & \underset{(0.1011)}{-0.3787} & \underset{(0.0690)}{-0.0079} \\ \underset{(0.0546)}{0.0306} & \underset{(0.1733)}{-0.0583} & \underset{(0.1394)}{0.3058} & \underset{(0.1043)}{0.1381} \end{bmatrix}$$

$$P = \begin{bmatrix} \underset{(0.3473)}{0.5292} & \underset{(0.0559)}{0.0819} & \underset{(0.2508)}{0.0000} \\ \underset{(0.6311)}{0.0000} & \underset{(0.0736)}{0.8756} & \underset{(0.4722)}{1.0000} \\ \underset{(0.6520)}{0.4708} & \underset{(0.0829)}{0.0425} & \underset{(0.4429)}{0.0000} \end{bmatrix}$$

$\boxed{\ln L = 2554.612}$

Table 3b : Estimates of the $MSI(3){-}VAR(2)$ model

State (k)	**n°obs/state**	$\bar{\mathbf{P}}(S_t = k)$	$A_{0,S_t=k}$
1	37	$\underset{(0.1480)}{0.2040}$	$\begin{bmatrix} \underset{(0.0075)}{0.0239} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{0.0027} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$
2	71	$\underset{(0.2187)}{0.4376}$	$\begin{bmatrix} \underset{(0.0039)}{0.0080} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{-0.0017} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$
3	66	$\underset{(0.1277)}{0.3584}$	$\begin{bmatrix} \underset{(0.0040)}{0.0044} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{-0.0017} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$

$$A_1 = \begin{bmatrix} \underset{(0.0879)}{0.2593} & \underset{(0.2280)}{0.7467} & \underset{(0.2003)}{0.3396} & \underset{(0.1660)}{0.0479} \\ \underset{(0.0208)}{0.0331} & \underset{(0.0708)}{1.3598} & \underset{(0.0499)}{-0.0486} & \underset{(0.0316)}{0.1070} \\ \underset{(0.0377)}{0.0178} & \underset{(0.1079)}{-0.0428} & \underset{(0.1077)}{1.2374} & \underset{(0.0737)}{0.1344} \\ \underset{(0.0454)}{-0.0032} & \underset{(0.1435)}{0.0180} & \underset{(0.1395)}{-0.2174} & \underset{(0.1112)}{0.2027} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \underset{(0.0869)}{0.4046} & \underset{(0.2219)}{-0.7184} & \underset{(0.1883)}{-0.2767} & \underset{(0.1619)}{0.0996} \\ \underset{(0.0186)}{0.0111} & \underset{(0.0666)}{-0.4370} & \underset{(0.0507)}{0.0753} & \underset{(0.0359)}{0.0601} \\ \underset{(0.0312)}{-0.0533} & \underset{(0.1013)}{0.0550} & \underset{(0.1018)}{-0.3054} & \underset{(0.0824)}{-0.0338} \\ \underset{(0.0419)}{-0.0278} & \underset{(0.1379)}{-0.0108} & \underset{(0.1291)}{0.1681} & \underset{(0.1123)}{0.2192} \end{bmatrix}$$

$$P = \begin{bmatrix} \underset{(0.0847)}{0.8325} & \underset{(0.0389)}{0.0000} & \underset{(0.0713)}{0.0954} \\ \underset{(0.0077)}{0.0000} & \underset{(0.0710)}{0.9050} & \underset{(0.0706)}{0.1160} \\ \underset{(0.0858)}{0.1675} & \underset{(0.0718)}{0.0950} & \underset{(0.0926)}{0.7886} \end{bmatrix}$$

$\boxed{\ln L = 2537.130}$

Table 3c : Estimates of the MSI($\pi(3)$,$\psi 2$)−VAR(2) model

$\mathbf{State}\,(k)$	$\mathbf{n^\circ obs/state}$	$\bar{\mathbf{P}}\,(S_t=k)$	$A_{0,S_t=k}$
1	37	$\underset{(0.1196)}{0.2121}$	$\begin{bmatrix} \underset{(0.0067)}{0.0215} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{-0.0004} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$
2	95	$\underset{(0.0953)}{0.5412}$	$\begin{bmatrix} \underset{(0.0039)}{0.0072} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{-0.0004} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$
3	43	$\underset{(0.0692)}{0.2466}$	$\begin{bmatrix} \underset{(0.0037)}{0.0031} \\ \underset{(0.0000)}{0.0019} \\ \underset{(0.0000)}{-0.0004} \\ \underset{(0.0000)}{0.0048} \end{bmatrix}$

$$A_1 = \begin{bmatrix} \underset{(0.0828)}{0.2403} & \underset{(0.2316)}{0.7298} & \underset{(0.2077)}{0.4075} & \underset{(0.1793)}{0.0378} \\ \underset{(0.0289)}{0.0070} & \underset{(0.0707)}{1.4539} & \underset{(0.0655)}{-0.0411} & \underset{(0.0474)}{0.0925} \\ \underset{(0.0340)}{0.0592} & \underset{(0.0874)}{-0.0217} & \underset{(0.1078)}{1.2329} & \underset{(0.0716)}{0.1135} \\ \underset{(0.0534)}{0.0038} & \underset{(0.1549)}{-0.0480} & \underset{(0.1469)}{-0.2351} & \underset{(0.0906)}{0.1501} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \underset{(0.0967)}{0.4194} & \underset{(0.2208)}{-0.6946} & \underset{(0.2159)}{-0.2885} & \underset{(0.1393)}{0.2105} \\ \underset{(0.0207)}{0.0240} & \underset{(0.0653)}{-0.5206} & \underset{(0.0658)}{0.0597} & \underset{(0.0523)}{0.0499} \\ \underset{(0.0326)}{-0.0658} & \underset{(0.0844)}{0.0248} & \underset{(0.1090)}{-0.2829} & \underset{(0.0663)}{-0.0718} \\ \underset{(0.0441)}{-0.0067} & \underset{(0.1478)}{0.0407} & \underset{(0.1396)}{0.1599} & \underset{(0.0924)}{0.1935} \end{bmatrix}$$

$$P = \begin{bmatrix} \underset{(0.0979)}{0.8286} & \underset{(0.0438)}{0.0672} & \underset{(0.0740)}{0.0000} \\ \underset{(0.0775)}{0.0272} & \underset{(0.1232)}{0.5337} & \underset{(0.1361)}{1.0000} \\ \underset{(0.1089)}{0.1442} & \underset{(0.1246)}{0.3992} & \underset{(0.1210)}{0.0000} \end{bmatrix}$$

$\ln L$=2536.879

Table 3d : Estimates of the $MSI(\pi(3))-VAR(2)$ model

$Models$	AIC	BIC	LIL
$MSI\,(\pi\,(3)\,,\psi\,(2))-VAR(2)$	**−28.3463**	**−24.9522**	**−27.8234**
$MSI\,(\pi\,(3))-VAR(2)$	−28.3434	−24.9493	−27.8205

Table 4 : Selection of the contrained $MSI(3)-VAR(2)$ specification

$Regimes \diagdown Models$	$MSI(2)$	$MSI(3)$	$MSI(\pi\,(3)\,,\psi\,(2))$	$MSI(\pi\,(3))$
$\bar{\pi}_{S_t=1}$	0.0619	0.0690	0.0829	0.0735
$\bar{\pi}_{S_t=2}$	0.0259	0.0253	0.0189	0.0188
$\bar{\pi}_{S_t=3}$		0.0641	0.0290	0.0310

Table 5 : Unconditional means of the inflation rate calculated from the $MSI-VAR(2)$ specifications

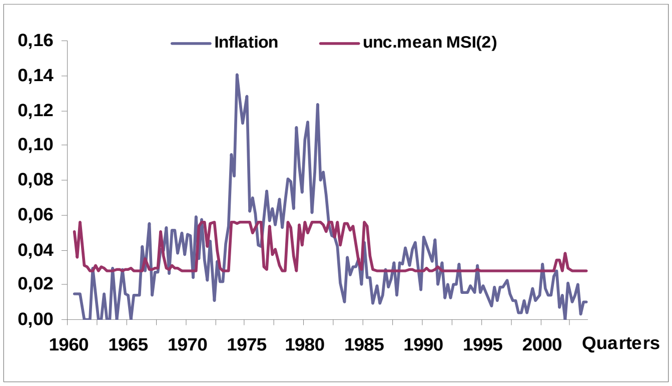


Figure 2a : Evolution of the $MSI(2)-VAR(2)$ expected inflation rate

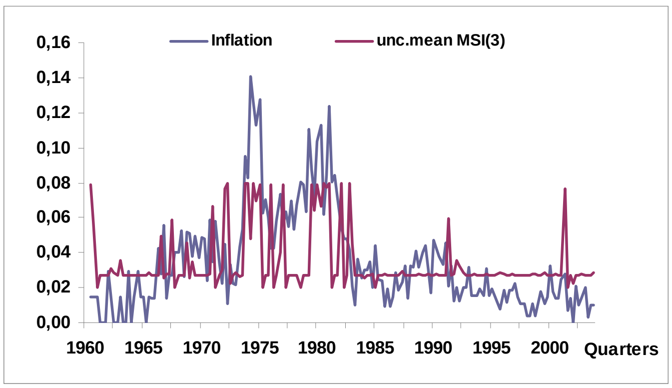
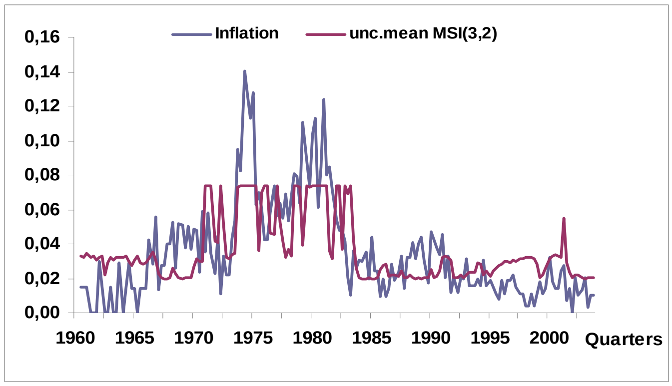
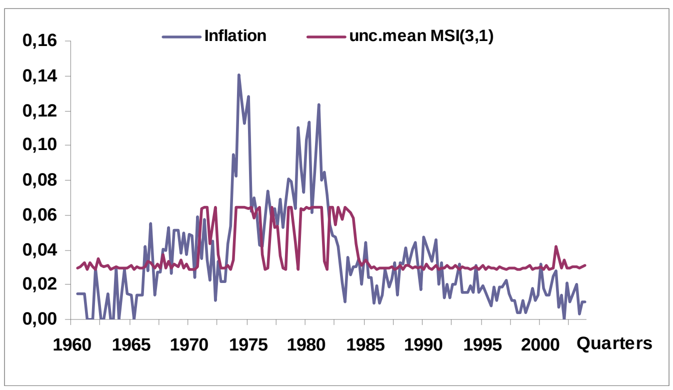


Figure 2b : Evolution of the $MSI(3)-VAR(2)$ expected inflation rate



$Figure\ 2c: Evolution\ of\ the$
$MSI(\pi\,(3)\,,\psi\,(2)) - VAR(2)\ expected\ inflation\ rate$



$Figure\ 2d: Evolution\ of\ the\ MSI(\pi\,(3)) - VAR(2)$
$expected\ inflation\ rate$

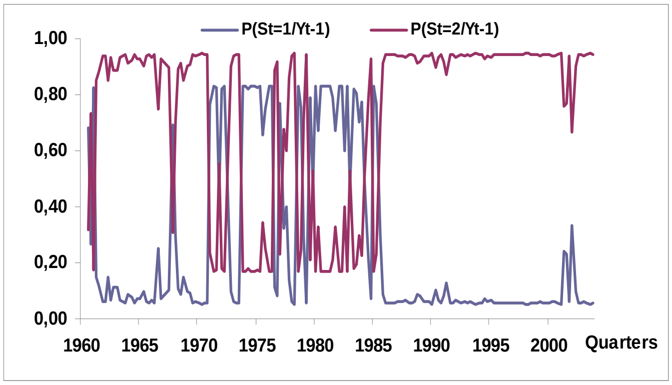


Figure 3a : $MSI(2) - VAR(2)$ *marginal probabilities*

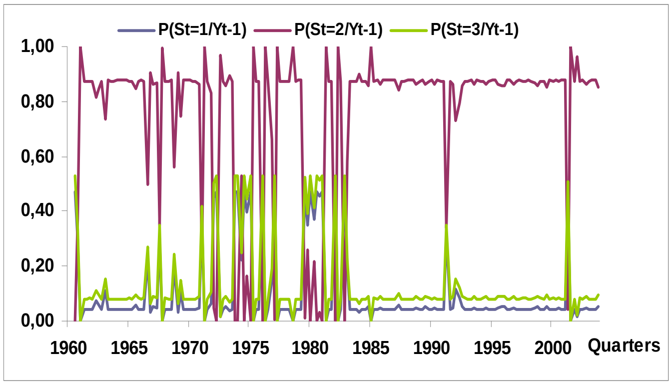


Figure 3b : $MSI(3) - VAR(2)$ *marginal probabilities*

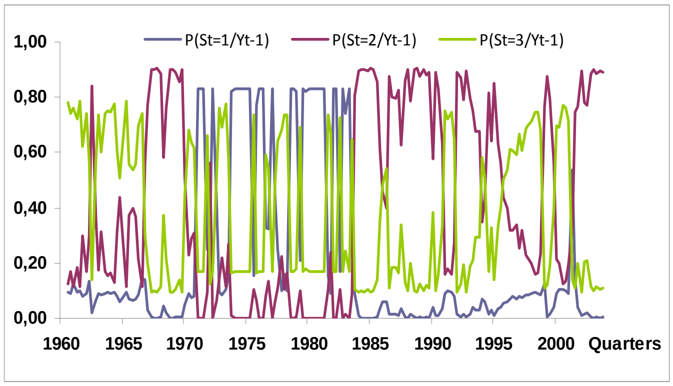


Figure 3c : $MSI(\pi\,(3)\,,\psi\,(2)) - VAR(2)$ *marginal probabilities*

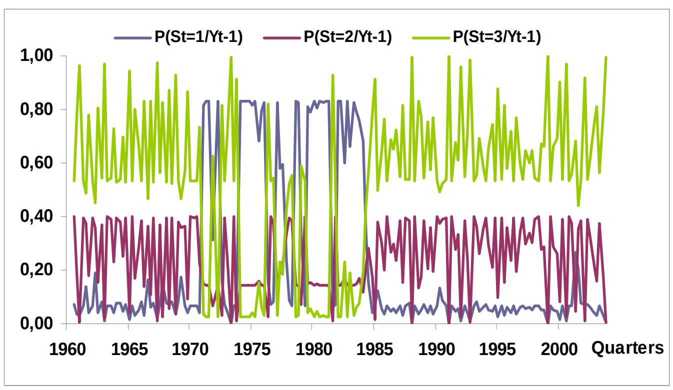


Figure 3d : $MSI(\pi\,(3)) - VAR(2)$ *marginal probabilities*

	$MSI(2)$	$MSI(3)$	$MSI(\pi(3), \psi(2))$	$MSI(\pi(3))$
$\alpha_{S_t=1}$	0.026	0.026	0.026	0.026
$\alpha_{S_t=2}$	0.026	0.026	0.051	0.051
$\alpha_{S_t=3}$		0.026	0.026	0.026
	$MSI(2)$	$MSI(3)$	$MSI(\pi(3), \psi(2))$	$MSI(\pi(3))$
$\varrho_{S_t=1}$	0.0010	0.200	0.001	0.001
$\varrho_{S_t=2}$	0.0010	0.001	0.075	0.075
$\varrho_{S_t=3}$		0.001	0.001	0.001
	$MSI(2)$	$MSI(3)$	$MSI(\pi(3), \psi(2))$	$MSI(\pi(3))$
$\theta_{S_t=1}$	46.727	36.404	34.929	37.878
$\theta_{S_t=2}$	58.525	58.525	58.525	58.525
$\theta_{S_t=3}$		40.828	58.525	58.525

Table 6 : Estimates of the Structural Parameters

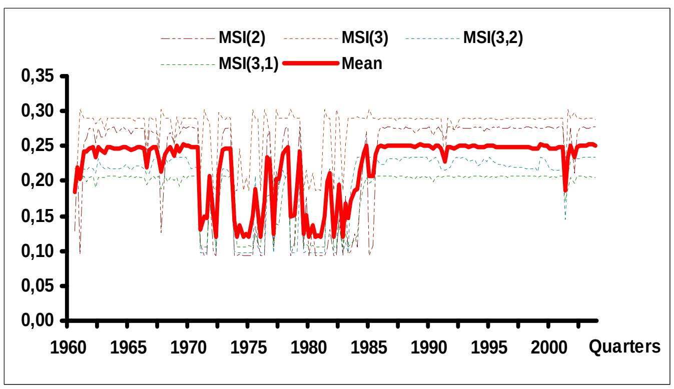
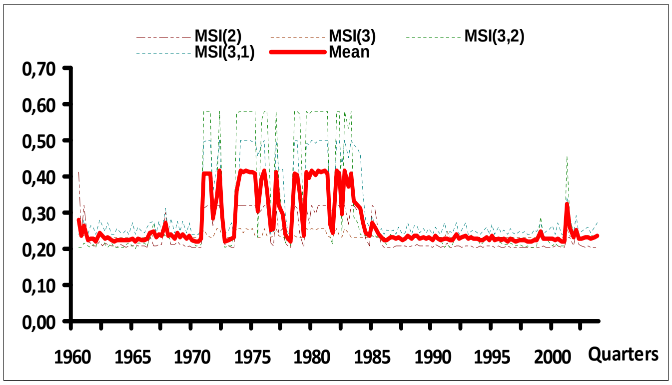
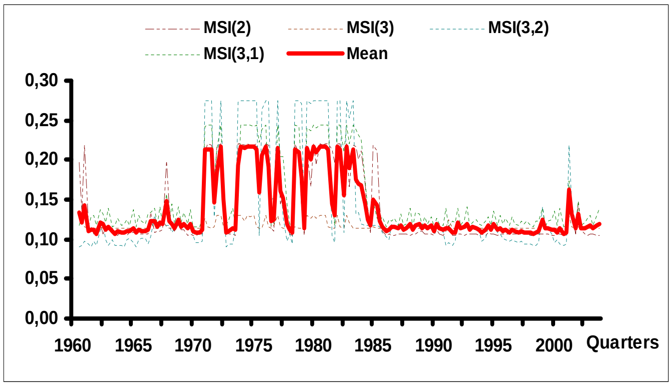


Figure 4a : Dynamic of the real marginal cost impact $(\xi(\Psi, \bar{\pi}))$



$Figure\ 4b: Dynamic\ of\ the\ expected\ inflation\ rate\ impact\ (b_1\,(\Psi,\bar{\pi}))$



$Figure\ 4c: Dynamic\ of\ the\ over-one\ period\ expected\ inflation\ rate\ impact\ (b_2\,(\Psi,\bar{\pi}))$

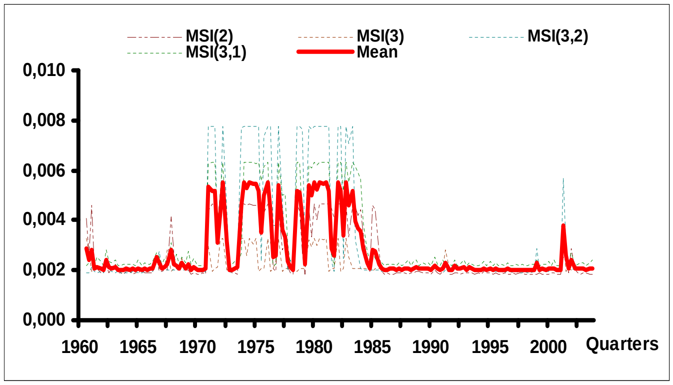


Figure 4d : Dynamic of the discount and the output growth rates impacts $(\chi(\Psi, \bar{\pi}))$

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