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Expectations Impact on the Effectiveness of the Inflation-Real Activity Trade-Off

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Abstract

The current study takes place in the *Phillips* curve framework in which first, we look at determining econometrics models leading to characterize the dynamics of the main variables underlying the trade-off in uni-variate contexts. As a result, it appears that an adequate way to characterize the agents' expectations regarding the dynamics of these variables, is to consider a combination of some fixed levels (regimes) in the variables evolutions with an agents' adaptive beliefs notion. This expectation process is empirically captured by a *Markov* Switching Intercept Heteroskedastic - AutoRegressive ($MSIH - AR$) model. Finally, based on the implied expectations value of the variables, we show that the *Phillips* curve seems to disappear when the expected inflation rate's impact on its current value converges to its long-term value.

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1 Introduction

In a landmark paper, *Phillips* (1958) reported a strong inverse and relatively stable relationship over the last century between the unemployment rate and the rate of wage inflation in the United Kingdom. A few years later *Solow* (1960) highlights a similar correlation between the inflation and the unemployment rates based on United States data. A basic version of this relationship, which took the name of the *Phillips* curve in the macroeconomic literature, can be written as

$$\pi_t = \xi^{kpc}\psi_t + \varepsilon_t \tag{1}$$

where π_t, ψ_t and ε_t represent the inflation rate, a macroeconomic variable measuring the real economic activity and an error term with zero mean and constant variance. This *Phillips* curve, which reflects the basic Keynesian analysis, suggests the existence of a trade-off between changes in the aggregate price level and those in the real economic activity.

However, this curve was challenged in the late $60's$ as, for the Nobel *Milton Friedman* nominal variables cannot have permanent effects on real variables such that any Inflation - Real activity arbitration could only be exploited temporarily. Indeed, any macroeconomic policy would eventually lead to agents' behaviors changes. This monetarist perception of the trade-off leads to the following Augmented *Phillips* Curve

$$\pi_t = b_1^m \pi_t^* + \xi^m \left(\psi_t - \psi_t^*\right) + \varepsilon_t \tag{2}$$

in which, ψ_t^* represents a variable measuring the real activity to its natural level and π_t^* the adaptive expected inflation rate. This expected rate can also be regarded as the inflation target of monetary authorities. As reported in the literature, this first integration of agents' expectations in the debate appears very important. This last equation shows that an Inflation-Real activity relationship may only exist in the short term ($\xi^m \neq 0$ and $b_1^m \neq 0$). In long-term, when agents adjust their decisions, realized and expected rates of inflation should be equal ($b_1^m = 1$) to ensure a de facto equality between the current and expected (the natural level) values of the variable measuring the real activity. Then, the *Phillips* curve trade-off disappears ($\xi^m = 0$). Basically, any change in the nominal sphere of the economic system should lead to changes in the behaviors of the agents, that ultimately inhibit any possible impact on the real activity.

In the mid-$70's$, authors such as *Lucas* (1972*a*) extended the monetarist arguments by introducing the «revolutionary» hypothesis of the rational expectations. Taking into account this fundamental assumption of rationality upsets the whole macroeconomic analysis and even deeper vision of the trade-off. From this hypothesis, agents' decisions will reflect their immediate adjustments in response to changing economic environment within which they operate. Therefore, in the absence of nominal rigidities, the *Phillips* curve relationship disappears even in the short term. The activity fluctuations are real and above all their explanations seem to have no relationship with any interventionist policies.

In the early 80's, research that focused on the *Phillips* curve was made within a frame of systematic optimization behaviors of economic agents. In this context, the inflation rate dynamic is mainly studied under Time-Dependent models à la *Calvo* (1983). In this New-Keynesian *Phillips* Curve ($NKPC$) framework, the economic system consists of firms in monopolistic competition facing adjustment costs in their prices' set up. Formally, at each moment, each of these firms receives a signal[1] (a probability $(1-\alpha)$) to adjust its prices. This model is based on an asynchronous, non-global and non-random adjustment of all the firms' prices. For a representative firm, the decision to adjust its price will partially depend on the states of the economic environment in which it solves its profit maximization problem. This decision will be primarily influenced by the fact that the firm must wait for some periods before re-optimizing its price[2].

Under the strong assumptions of a zero steady state inflation rate and an instantaneous and costless reallocation of capital, the $NKPC$ will be written

$$\hat{\pi}_t = \tilde{\varrho}^{nkpc}\hat{\pi}_{t-1} + b_1^{nkpc}E_t\hat{\pi}_{t+1} + \xi^{nkpc}\hat{\psi}_t + \varepsilon_t \tag{3}$$

where, the current inflation rate is defined as a non-negative function ($\xi^{nkpc} \geq 0$, $b_1^{nkpc} \geq 0$) of the real marginal cost ($\hat{\psi}_t$) and the one period expected inflation rate ($E_t\hat{\pi}_{t+1}$). The coefficients b_1^{nkpc}, ξ^{nkpc} and $\tilde{\varrho}^{nkpc}$ are calculated as

$$\begin{aligned}
\tilde{\varrho}^{nkpc} &= \frac{\varrho}{1+\varrho\beta} \\
b_1^{nkpc} &= \frac{\beta}{1+\varrho\beta} \\
\xi^{nkpc} &= \frac{(1-\alpha)(1-\beta\alpha)}{\alpha(1+\omega\theta)(1+\varrho\beta)}
\end{aligned}$$

These coefficients depend on the degree of price rigidity (α), the real discount factor with which firms discount future real marginal costs (β), the elasticity of substitution between goods[3] (θ), the elasticity of the firm marginal cost on its own output[4] (ω) and an indexation parameter of current prices to past inflation (ϱ). The hat notations indicate that the variables are expressed in their log-deviations form from their steady-state values (e.g. $\hat{\pi}_t = \log(\pi_t/\bar{\pi})$).

In previous research, the inflation rate dynamic was mainly studied in this context of Time Dependent micro-based models à la *Calvo* (1980). These models are also based on the rational expectation hypothesis and the existence of frictions in the economy. Indeed, under this $NKPC$ label, the inflation dynamics are presented as a forward-looking phenomenon resulting from the optimizing

[1] By assumption, this probability is exogenous, single, identical for all firms and independent of the firms pricing history.

[2] Like in the *Taylor* (1980) model, prices are fixed for a predetermined time period and firms are constrained by periodic prices contracts. One of the features of the *Calvo* (1983) model is that it considers the length of individual contracts to be randomized while the average duration of price contracts is constant.

[3] Which determines the mark-up ($\frac{\theta}{\theta-1}$) that a firm can apply over its marginal costs.

[4] This parameter occurs in the equilibrium condition because there is no reallocation of capital between firms.

behaviors of economic agents. But, even though studies conducted by *Gali* & *Gertler* (1999), *Sbordone* (2002) have suggested almost a resurrection of the *Phillips* curve, one could observe that the $NKPC$ framework does not put to rest the debate surrounding the empirical effectiveness or the theoretical validity of this temporary arbitration.

The econometric weaknesses linked with these last results do not take away the theoretical doubts raised by monetarist or neoclassical approaches since the implied "rejections" of these Keynesian models call for a crucial need to review the way their Time Dependent framework considers the rational expectations process. Also, recalling that in the *Calvo* (1983) frame, firms are unable to adjust instantaneously their prices (even if they expect changes in their activity's environment), the $NKPC$ pricing approach can be perceived as inappropriate for describing the inflation rate dynamics. An adequate integration of the agents' expectations in these New Keynesian analyses[5] is clearly necessary.

One of the paper's goals is to start from a way of integrating these expectations that will enable desired values of the *Phillips* curve variables to possibly differ in sample periods, without being a consequence of continuous revisions of the agents expectations. In fact, continuous updating of these expectations could be perceived as a form of weakening towards agents' rationality since such revisions could only be justified through a frequent need for correction of errors that would be, in sense, systematic. From then on, it would be necessary to look at an empirical approach in which all the variables considered in the New Keynesian *Phillips* Curve equation could admit trends dynamics and this in spite of the «constraints» imposed by the rational expectations hypothesis.

Fundamentally, it would be about allowing the considered variable to be different from zero at steady state, while having a trend dynamics similar to those one can extract with a $Hodrick - Prescott$ filter ($Figure\ 1a - 1d$). An explicit consideration of this last point is needed in any empirical evaluation of the trade-off. Moreover, if the evolution of each of the *Phillips* curve variables is defined by

$$x_t = \mu\left(x_t\right) + \varepsilon_t$$

where ε_t is an $NID\left(0, \sigma_\varepsilon^2\right)$ shock and $\mu\left(x_t\right)$ a term which represents the systematic component (expected value) of a variable (x_t), it appears obvious that every study of the *Phillips* curve has to adequately characterize the evolution of this component.

Based on this fundamental limits, *Bakshi* & *al.* (2005), *Cogley* & *Sbordone* $(2005, 2008)$ or *Groen* & $Mumtaz$ (2008) examine the implications of a positive steady state inflation rate in the New Keynesian framework. Their studies lead to a second generation of New Keynesian models derived under the assumption of a non-zero steady state inflation rate.

Mathematically, the New Keynesian *Phillips* Curve with Positive Inflation

[5] *Ascari* (2004), *Sahuc* (2006), *etc.* show that the zero steady state inflation rate frame of analysis can only lead to bias in the New Keynesian *Phillips* Curve estimation so that the $NKPC$ models are actually presented as particularly restrictive when an analysis is done in a changing inflation environment which could affect the firms pricing decisions.

$(NKPC-PI)$ equation can be written as follows

$$\begin{aligned}\hat{\pi}_t &= \tilde{\varrho}^{nkpc-pi}\hat{\pi}_{t-1} + b_1^{nkpc-pi}E_t\hat{\pi}_{t+1} + \xi^{nkpc-pi}\hat{\psi}_t \\ &+ b_2\sum_{j=2}^{\infty}\gamma_1^{j-1}E_t\hat{\pi}_{t+j} + \chi\left(\gamma_2-\gamma_1\right)\sum_{j=0}^{\infty}\gamma_1^{j}\left(E_t\hat{R}_{t+j,t+j+1} + E_t\hat{\gamma}_{y_{t+j+1}}\right) + \varepsilon_t\end{aligned} \tag{4}$$

This equation shows that the inflation rate $(\hat{\pi}_t)$ dynamics are explained by thier own expected dynamics $(E_t\hat{\pi}_{t+1},\ E_t\hat{\pi}_{t+j})$, the dynamics of the expected nominal discount rate $(E_t\hat{R}_{t+j,t+j+1})$, the expected output growth rate $(E_t\hat{\gamma}_{y_{t+j+1}})$ and the real marginal cost $(\hat{\psi}_t)$. The $NKPC-PI$ coefficients are functions of the structural parameters of the economy, i.e. $\Psi = [\alpha, \theta, \rho]$ and on the steady state inflation rate $(\bar{\pi} > 1)$. Formally, we have

$$\begin{aligned}\tilde{\varrho}^{nkpc-pi} &= \tfrac{\varrho}{\Delta} \\ b_1^{nkpc-pi} &= \tfrac{1}{\Delta}\left(\left(\tfrac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1 + \gamma_2\right) \\ \xi^{nkpc-pi} &= \tfrac{1}{\Delta}\left(\tfrac{(1-\alpha\varsigma_1)(1-\gamma_2)}{\alpha\varsigma_1(1+\theta\omega)}\right) \\ b_2 &= \tfrac{1}{\Delta}\left(\tfrac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\left(\tfrac{\theta(1-\varrho\gamma_1)+\varrho\gamma_1}{1+\theta\omega}\right)(\gamma_2-\gamma_1)\right) \\ \chi &= \tfrac{1}{\Delta}\left(\tfrac{1-\alpha\varsigma_1}{\alpha\varsigma_1(1+\theta\omega)}\right)\end{aligned}$$

and the intermediate terms are given by

$$\begin{aligned}\varsigma_1 &= \bar{\pi}^{(\theta-1)(1-\varrho)} \\ \varsigma_2 &= \bar{\pi}^{\theta(1+w)(1-\varrho)} \\ \tilde{\beta} &= \bar{R}\bar{\gamma}_y\bar{\pi} \\ \gamma_1 &= \alpha\tilde{\beta}\varsigma_1 \\ \gamma_2 &= \alpha\tilde{\beta}\varsigma_2 \\ \Delta &= 1 + \varrho\gamma_2 - \left(\tfrac{1-\alpha\varsigma_1}{\alpha\varsigma_1}\right)\phi_1 \\ \phi_1 &= \tfrac{\varrho\theta(\gamma_1-(1+\omega)\gamma_2)-\varrho\gamma_1}{1+\theta\omega} \\ \phi_2 &= \tfrac{1}{1+\theta\omega}\left(\gamma_2\left(1+\theta\omega\right) + \left(\gamma_2-\gamma_1\right)\left(\theta\left(1-\varrho\gamma_1\right)+\varrho\gamma_2\right)\right)\end{aligned}$$

The non-zero steady state hypothesis allows a "long term *Phillips* curve" characterized by an equation tying together the different steady state values of the $NKPC-PI$ variables, i.e.

$$\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varrho)}\right)^{\frac{1+\theta\omega}{1-\theta}}\left(\frac{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{1+\theta(1+w)(1-\varrho)}}{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{\theta-\varrho(\theta-1)}}\right) = \frac{\theta}{\theta-1}\left(1-\alpha\right)^{\frac{1+\theta\omega}{1-\theta}}\bar{\psi}$$

in which $\bar{\pi}$, $\bar{\psi}$, $\bar{\gamma}_y$ and $\bar{R}$ are these steady state values. It is to be noticed that that when the steady state inflation rate varies, the structure of the economy may be affected and in this case, the structural parameters may themselves vary[6].

[6] In a related work, we show that, even if there is structural changes in the economy, these changes remain infrequent and of small magnitude confirming the structural parameters stability.

To the extent that the steady state inflation rate can be non-zero, models from this $NKPC-PI$ framework suppose a possibly permanent arbitration resulting from a combination of the short term ($NKPC-PI$) and the long term ($NKPC_{SS}-PI$) equations. In short and medium terms, the effectiveness of the arbitration is possible because, like in the $NKPC$ framework, the $NKPC-PI$ approach combines the concepts of the new classical reasoning (rational expectations) and keynesian basis (nominal rigidities). However, unlike in the basic keynesian analysis, the *Phillips* curve is not unique so that we speak rather of arbitration with a prolonged persistence. During the transition of the economy to its steady state (time for a new trade-off), the economic system appears to follow a path characterized by a succession of inconstantly persistent moments of arbitration. As envisaged in the $NKPC$, the magnitude of the relationship between the inflation rate and the variable measuring the real activity will not necessarily remain the same throughout the time preceding the stationary state[7] where a new link ($NKPC_{SS}-PI$) is set up.

With this summary of the trade-off evolution, it appears clear that taking into account the agents' expectations seems essential to fully capture the possible Inflation-Real activity compromise in the short term, while its long term disappearance is almost certain. The speed of the economy's transition to its steady state highly depends on the expectations adjustments. In other words, the effectiveness of the *Phillips* curve depends on the agents' adaptation, which itself seems closely related to the expected effectiveness of the former trade-off situation. According to many researchers (*Samuelson* (2008), *Sims* (2008)), the agents' optimization behavior and their expectations modes (derived from their rationality) seem relatively clear in the keynesian framework, but how their rationality is introduced, defined and operated in the analysis should be clarified.

To address this problem, we first extend the results of *Boutahar* & *Gbaguidi* (2009) to characterize all the $NKPC-PI$ variables dynamics (section 1). Each of these variables is studied in a uni-variate context by using a *Markov* Switching Intercept Heteroscedastic - AutoRegressive ($MSIH-AR$) model. This approach permits us to consider the unconditional means of these variables as a series obeying the regimes' switching controlled by a *Markov* chain of order 1. This *Markov* switching framework allows to characterize the agents' expectations process and to take into account the non-linearities observed in the variables' dynamics. Conceptually, this approach seems to be the most adequate as the trend dynamics of a considered variable come from a random scheme. This first stage estimation represents the background of the empirical analysis of the *Phillips* curve. It then presents the expected values of the main variables that appear in the *Phillips* curve. Based on these expected dynamics, we estimate the different versions of the *Phillips* curve and highlight the contribution of the introduction of agents' expectations in the debate surrounding the Inflation-Real activity trade-off (section 2). For that purpose, the evolution of the *Phillips* curve coefficients are considered as time or state varying para-

[7] In short and medium terms, the slope of the $NKPC-PI$ can vary as $\xi^{nkpc-pi}=f(\alpha(\bar{\pi}))$.

meters. These estimates enable us to show that, as the agents' expectations converge to their rational long term values, the trade-off seems to disappear. A final section summarizes the main results and discusses further research.

2 Expected values of the NKPC-PI variables

Before considering the non-linear specifications, we present the data upon which our empirical study takes place and conduct a linear analysis as benchmark for the rest of this study.

2.1 Description of the data

To be able to reconsider previous results of the *Phillips* curve estimations (*Solow* (1968), *Friedman* (1968), *Gali* & *Gertler* (1999), *Cogley* & *Sbordone* (2005), *Groen* & *Mumtaz* (2008)), we focus on a database reflecting the best possible data used in these earlier studies. The main variables appearing in the *Phillips* curve debate are the inflation rate, a unit labor cost based measure of the real marginal cost, the nominal discount rate and the output growth rate. The sample period covers $T = 176$ quarters from $1960 : I$ to $2003 : IV$ for the U.S. economy.

The inflation rate is measured from the implicit price deflator as (P_t), recorded in the *Bureau of Labor Statistics* (*BLS*) database. From these data, we calculate this series as

$$\pi_t = 4 * [\ln(P_t) - \ln(P_{t-1})]$$

The real activity is measured by the real marginal cost. Assuming a $Cobb-Douglas$ production function, the real marginal cost (ψ_t) is proportional to the labor unit cost (ulc_t) as mathematically[8], we have

$$\psi_t = \ln\left(\frac{W_t N_t}{P_t Y_t}\right) - \ln(1-\kappa) = \ln(ulc_t) - \ln(1-\kappa)$$

where Y_t is the level of output in real terms, N_t is the total amount of labor input and W_t measure wages. Following *Cogley* & *Sbordone* (2005), the output elasticity to hours of work $(1-\kappa)$ in the production function is set equal to 0.6666 so that the strategic complementarities parameter equal to $\omega = \frac{\kappa}{1-\kappa} = 0.5001$.

Regarding the discount rate $(R_{t,t+1})$, we use the 3-Month Treasury Bill: Secondary Market Rate (i_t) from the *Federal Reserve Bank of St. Louis (FRED)* database. To construct $R_{t,t+1}$, we apply the formula: $R_{t,t+1} = \frac{i_t}{1+i_t}$, where i_t was divided by 100.

The output growth rate is calculated on the basis of a weighted sequence of the real Growth Domestic Product (GDP) expressed in 2000 dollars (seasonally adjusted at an annual rate) and recorded in the *NIPA-Table (*1.3.6*)*.

[8] This real marginal cost series is constructed according to *Groen* & *Mumtaz* (2008).

2.2 The linear-benchmark approach

As shown in *Tables* $1a-1d$, the series appears characterized by periods in which their average levels and their variability differ. Clearly this reveals non-linearity in their dynamics and these series appear to originate from an asymmetric law with a noticeably high flattening coefficient. Their trends or expected values dynamics enable us to foresee possible periods of instability.

In a linear framework, it appears that these series are generated by the following second order autoregressive processes with a Generalized Autoregressive Conditional Heteroskedasticity[9] $GARCH(1,1)$.

$$x_t = \phi_0 + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t$$
$$\sigma^2_{\varepsilon,t} = \omega_0 + \omega_1 \varepsilon^2_{t-1} + \omega_2 \sigma^2_{\varepsilon,t-1}$$

Tables $1e-1h$ give the results of these linear models estimation. One can see that all the variables (except the output growth rate) are characterized by a quite strong global persistence[10].

$$\hat{\phi}_1^{\pi_t} + \hat{\phi}_2^{\pi_t} = 0.8415$$
$$\hat{\phi}_1^{\psi_t} + \hat{\phi}_2^{\psi_t} = 0.9541$$
$$\hat{\phi}_1^{R_{t,t+1}} + \hat{\phi}_2^{R_{t,t+1}} = 0.9418$$
$$\hat{\phi}_1^{\gamma_{y_t}} + \hat{\phi}_2^{\gamma_{y_t}} = 0.4643$$

Their unconditional means are given by

$$\bar{\pi}^{Lin} = 0.0233$$
$$\bar{\psi}^{Lin} = -0.0087$$
$$\bar{R}^{Lin} = 5.9978$$
$$\bar{\gamma}_y^{Lin} = 0.0089$$

Nevertheless, in order to verify the instability of the parameters ϕ_i, $i = 0,1,2$, in these $AR(2)$ processes, we conduct stability tests (*Nyblom* (1989)) as described in *Hansen* (1990, 1992). The results of these tests[11] are described in *Table* $1i-1l$. From these tables and for all the variables, one can globally conclude a weak joint stability of parameters whereas we can't reject the null hypothesis for ϕ_i, $i = 0,1,2$ taken individually, except for the inflation rate

[9] On the basis of the *Akaike* and *Schwartz* information criteria, we can select linear specifications of two lags and the Q statistics of the squared residuals indicate the presence of $ARCH$ effects.

[10] Nevertheless, the Augmented $Dickey-Fuller$ tests show that these series can be considered as stationary.

[11] Noting that these stability tests strictly require the estimates of the models in their linear forms, it is a matter of testing the null hypothesis of the individual or collective stability of the parameters versus the alternative that they follow martingale processes. The L statistic corresponds to the case where only one parameter stability is tested while the L_c statistic corresponds to the case of joint parameters stability. These statistics follow non-standard laws which essentially depend on the number of tested parameters and the critical values are computed from the theoretical asymptotic distributions. The 5% critical values for these stability tests, taken individually and jointly, are respectively $vc_L = 0.47$ and $vc_{L_c} = 1.24$.

(*Boutahar* & *Gbaguidi* (2009)). Also, one can reject the variance stability only for the inflation and output growth rates[12].

However, in this linear framework, the intercepts are the possible source of non-linearity. To capture this possible non-linearity, the recourse to models with pure or partial parameter instabilities seems necessary. From these $AR(2)$ representations, we then introduce the $Markov$ Switching specification to capture these non-linearity.

2.3 The Markov Switching approach

The purpose of this specification is that the variables evolve between m regimes (levels) which are controlled by a probability law. We consider a $Markov$ Switching Intercept Heteroskedastic – AutoRegressive ($MSIH - AR$) type of model in which the intercept parameters characterizing each variable dynamics have the possibility to change at each date according to the $Markov$ chain. We estimate $MSIH(m) - AR(2)$ models defined as

$$x_t = \phi_{0_{S_t=k}} + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t$$

where $(\varepsilon_t \mid S_t = k) \sim IIN\left(0, \sigma^2_{\varepsilon_{S_t=k}}\right)$, $k = 1, 2, ..., m$ and S_t is a first order $Markov$ chain with transition matrix defined as

$$P = \begin{bmatrix} p_{11} & ... & p_{m1} \\ ... & ... & ... \\ p_{1m} & ... & p_{mm} \end{bmatrix}$$

where $p_{ij} \geq 0$ and $\sum_{j=1}^{m} p_{ij} = 1$, $\forall i, j \in \{1, ..., m\}$. In this specification, we suppose that the intercept ($\phi_{0_{S_t=k}}$) and the variance ($\sigma^2_{\varepsilon_{S_t=k}}$) change with the regimes S_t given the information ($I_{t-1} = (x_{t-1}, ..., x_1)$) available in the beginning of the period t. Those terms vary according to the probability matrix P and the terms $p_{i,j} = P(S_t = i \mid S_{t-1} = j)$ measure the probability that a variable x_t switch from a level j at date $t-1$ to a level i at date t. In this context, the unconditional mean of a considered variable ($x_t = \{\pi_t, \psi_t, R_{t,t+1}, \gamma_{y_t}\}$) can be measured by

$$\bar{x}_t^{MSIH} = \frac{\phi_{0_{S_t=k}}}{1 - \phi_1 - \phi_2}, \; k = 1, 2, ..., m$$

The estimation of this model is accomplished by the method of maximum likelihood and according to the procedure proposed by $Hamilton$ (1989). The idea is to estimate the probability that an observation x_t has been generated by a regime k and therefore at time t, the intercept and the variance are in a state $S_t = k$, $k = 1, 2, 3$. This estimation takes the form of a conditional probability

[12] It seems like the linear model can adequately characterize the discount rate and the real marginal cost dynamics so that their expected values could be consider constant.

$P(S_t = k \mid I_t; \Theta)$ and can be written as

$$\begin{aligned} P(S_t = k \mid I_t; \Theta) &= P(S_t = k \mid x_t,\ I_{t-1}; \Theta) \\ &= \frac{P(S_t = k, x_t \mid I_{t-1}; \Theta)}{f(x_t \mid I_{t-1}; \Theta)} \\ P(S_t = k \mid I_t; \Theta) &= \frac{P(S_t = k \mid I_{t-1}; \Theta) * f(x_t \mid S_t = k, I_{t-1}; \Theta)}{\sum_{i=1}^{3} P(S_t = k \mid I_{t-1}; \Theta) * f(x_t \mid S_t = k, I_{t-1}; \Theta)} \end{aligned} \tag{5}$$

where $\Theta = \left(\phi_{0_{S_t=k}},\ \phi_1,\ \phi_1,\ \sigma^2_{\varepsilon_{S_t=j}},\ p_{11},\ p_{12},\ p_{21},\ p_{22},\ p_{31},\ p_{33}\right)$ is the set of parameters to estimate and $f(x_t \mid S_t = k, I_{t-1}; \Theta)$ represents the density of the conditional system of states $S_t = k$. Essentially, it requires a filtering procedure which can be more readily visible when the expression (5) is rewritten under the following compact form

$$\hat{\xi}_{t|t} = \frac{f(x_t \mid S_t = k, I_{t-1}; \Theta) \odot \hat{\xi}_{t|t-1}}{1_3' \left(f(x_t \mid S_t = k, I_{t-1}; \Theta) \odot \hat{\xi}_{t|t-1}\right)}$$

where $\hat{\xi}_{t|t}$ is a vector of conditional probabilities containing the predictions of the analyst about the possibility that the observation π_t has been generated by a regime k. The k-th element of this vector represents $P(S_t = k \mid I_{t-1}; \Theta)$. The term $1_3'$ denotes an (3×1) vector all of whose elements are unity. The estimates and the optimal predictions for each date t in the sample are described by the following recursive algorithm

$$\begin{cases} \hat{\xi}_{t|t} = \frac{\eta_t \odot \hat{\xi}_{t|t-1}}{1_3'\left(\eta_t \odot \hat{\xi}_{t|t-1}\right)} \\ \hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t} \end{cases} \tag{6}$$

where η_t represents the vector of conditional densities of which the k-th element is given by

$$f(x_t \mid S_t = k, I_{t-1}; \Theta) = \frac{1}{\sqrt{2\pi}\sigma_{\varepsilon_{S_t=k}}} \exp\left(-\frac{\varepsilon_t' \varepsilon_t}{2\sigma^2_{\varepsilon_{S_t=k}}}\right)$$

The log-likelihood function is

$$\ln L^{MSI}(\Theta) = \sum_{t=1}^{T} \ln\left(f(x_t \mid I_{t-1}; \Theta)\right)$$

and is maximized by using the system (6), given an initial value of the vector of conditional probabilities $(\hat{\xi}_{1|0})$ and a vector of initial parameters (Θ_0).

First of all, we use some estimations to select the number m of regimes for each of the variables in the $MSIH(m) - AR(2)$ class of models. The results[13]

[13] The statistic AIC is calculate as

$$AIC = -2 * LnL + 2 * l$$

where l is the number of parameters to be estimated in the model.

are presented in the *Tables* $2a - 2d$. Given the $MSIH(m) - AR(2)$ estimates presented in *Tables* $3a - 3d$, we have the following results for the main variables surrounding the *Phillips* curve debate.

- **Inflation rate:** The $MSIH(3) - AR(2)$ model seems to be adequate to characterize the inflation rate dynamics[14]. The unconditional means of the inflation rate calculated from this $MSIH(3) - AR(2)$ estimates are given by $\bar{\pi}_1^{MSIH} = 0.0927$, $\bar{\pi}_2^{MSIH} = 0.0357$ and $\bar{\pi}_3^{MSIH} = 0.0139$. Thereby, the last state ($\hat{\phi}_{0_{S_t=3}} = 0.0074$, the most frequently visited) characterizes about 45% of the observations, whereas the first one, associated with a higher mean ($\bar{\pi}_1^{MSIH}$), appears as an exception because it only covers 26 quarters out of the 174 of the sample. The probability of being in this first state is estimated as $\hat{P}(S_t = 1) = 0.1488$ and naturally, we see that this state is effective over the course of the quarters of «hyper-inflation», namely during the years 74-76 and 80-82. Once in this state, the probability of remaining there is given by $\hat{p}_{11} = 0.9126$ and the probabilities of leaving this regime are given by $\hat{p}_{21} = 0.0317$ and $\hat{p}_{31} = 0.0000$. The probabilities of staying in the other regimes are higher ($\hat{p}_{22} = 0.9418$, $\hat{p}_{33} = 0.9753$) than $\hat{p}_{11}$. In such a case, we can say that the first regime of a high expected inflation rate captures particular dates of this variable dynamics. The probabilities of leaving regimes 2 and 3 to reach the first one are $\hat{p}_{12} = 0.0874$ and $\hat{p}_{13} = 0.0000$. It appears that there is no direct transition between the regimes of low and high expected inflation rate. The second regime ($\bar{\pi}_2^{MSI}$) which covers 71 quarters and can be associated with the average of the inflation rate represents an «intermediate» regime between the two others. We can see that the probabilities to switch between the regimes 2 and 3 are close to each other ($\hat{p}_{23} = 0.0265$ and $\hat{p}_{32} = 0.0247$) and smaller than $\hat{p}_{12}$ and $\hat{p}_{21}$. Consequently, each regime can be perceived as «persistent» because if the inflation starts in its low regime then it will certainly switch to the intermediate level where it will be more attracted by the high inflation rate regime than the low rate regime. *Figure* $2a$ illustrates these observations. To check the adequacy of this specification, a panel of tests[15] based on the score method is executed. The results[16] of these tests are given in *Table* $4a$. They indicate the instability of the variance of the residuals but not at the level of other parameters of the model in its global specification. Nevertheless, all the parameters of the model appear stable when each state is taken individually except in the last one. The absence of residuals autocorrelation, residuals heteroskedasticity and the hypothesis of a 1st order *Markov* are

[14] According to the likelihood value, the $MSIH(3) - AR(2)$ specification is preferable to the linear model (*Garcia* (1998)) and the $MSIH(2) - AR(3)$ model. Also, *Kang & al.* (2009) investigate the existence and timing of changes in U.S. inflation persistence using an unobserved components model of inflation with *Markov* switching parameters. Their results support using a model with three regimes to capture all of the serial correlation and heteroskedasticity in the inflation rate data.

[15] For more extensive details concerning these tests, see *Hamilton* (1996).

[16] The 5% critical values are given between [.] the $p - values$ are given between {.}.

not rejected. Consequently, the $MSIH(3) - AR(2)$ specification for the inflation rate appears to be more adequate. Its expected dynamics are illustrated in *Figure* 3*a* and its predicted dynamics are given by *Figure* 4*a*.

- **Real marginal cost**: The results suggest that the linear model can be considered as adequate even if the $MSIH(2) - AR(2)$ specification is preferred to the $MSIH(3) - AR(2)$. The unconditional means calculated from the $MSIH(2) - AR(2)$ specification are $\bar{\psi}_1^{MSIH} = 0.0028$ and $\bar{\psi}_2^{MSIH} = -0.0017$. These regimes reflect opposite values of the real marginal cost series. The probabilities of remaining in each of these two regimes are $p_{11} = 0.7578$ and $p_{22} = 0.8827$ while the probabilities of leaving these regimes are given by $p_{12} = 0.2422$ and $p_{21} = 0.1173$. *Figure* 2*b* illustrates the probabilities of being in each regime at t given the information at $t-1$. These probabilities vary significantly across the sample indicating that the estimated regimes are not stable. Globally and when each state is picked up individually, the results of the adequacy tests (*Table* 3*b*) indicate that all the parameters can be considered stable in this specification. The absence of residuals autocorrelation, heteroskedasticity and the hypothesis of a 1st order *Markov* chain are not rejected. The expected real marginal cost dynamics associated with the $MSIH(2) - AR(2)$ specification is illustrated in *Figure* 3*b* and the real marginal cost predicted by this specification is given by *Figure* 4*b*.

- **Discount rate**: The linear model appears to be the best one can use to characterize the discount rate dynamics. The $MSIH(2) - AR(2)$ estimates (*Table* 3*c*) confirm this result as the calculated unconditional means in each regime, i.e. $\bar{R}_1^{MSIH} = 0.0547$ and $\bar{R}_2^{MSIH} = 0.0518$, are close to each other. One can also notice that once in each of these states, the probabilities of remaining there are close to 1, given by $\hat{p}_{11} = 0.9173$ and $\hat{p}_{22} = 0.9594$ so that, these regimes can be considered as a unique one. However, we note that the infrequent switching between these two regimes are observed during the reported inflation «crisis» episodes from the end of the 60's to the mid 80's and another episode[17] in 2001. *Figure* 2*c* illustrates the evolution of these probabilities of being in each regime at t given the information at $t-1$. The results of the specification adequacy (*Table* 4c) indicate a global stability of the all the parameters even when each state is picked up individually. The presence of residuals autocorrelation cannot be rejected while, residuals heteroskedasticity and the hypothesis of a *Markov* chain of the 1st order can. Consequently, the $MSIH(2) - AR(2)$ specification for the discount rate does not appear to be the most adequate one. The expected rate dynamics associated with this $MSIH(2) - AR(2)$ specification is illustrated in *Figure* 3*c* and the predicted rate is given by *Figure* 4*c*.

[17] This last episode may reflect the *September* 11 terrorist attack.

- **Output growth rate**: Almost clearly, it seems like there is no break and, like in the discount rate case, the two regimes detected by the $MSIH(2) - AR(2)$ specification ($\bar{\gamma}_{y,1}^{MSIH} = 0.0087$ and $\bar{\gamma}_{y,2}^{MSIH} = 0.0080$) are close. The probabilities of remaining in each of these regimes can be assimilated to one ($p_{11} = 0.9894$ and $p_{22} = 1.0000$) while the probabilities of leaving them are close to zero ($p_{12} = 0.0106$ and $p_{21} = 0.0000$). The «high» growth regime takes place during the years 1973 to 1983 as shown by *Figure* 2*d*. The results of the adequacy tests are given in *Table* 3*d*. The dynamics of the expected output growth rate associated with this $MSIH(2) - AR(2)$ specification are illustrated in *Figure* 3*d* and the predicted series by this specification are given by *Figure* 4*d*.

In summary, the results indicate that only the inflation rate switches between three clearly identified regimes. In the real marginal cost case, even if the linear specification seems to be the preferred one, the $MSIH - AR$ specification identifies two distinct regimes and some frequent switches between these regimes. The discount rate and the output growth rate seem to be adequately characterized by the linear specification. Based on this first stage results, we estimate the different versions of the *Phillips* curve according to different econometric specifications. The choice of these specifications is based on the theoretical background of each version of the Inflation-Real activity trade-off as discussed in the introduction of this paper.

3 Estimation of the Phillips curve

Insofar as we try to measure the impact of expectations on the theoretical validity of the trade-off, we focus on empirical aspects of the different versions of the *Phillips* curve presented in the introduction of this paper. We estimate these major versions of the post-keynesian views of the trade-off assuming that all the coefficients in these versions could be time or states varying. Building on the expected values of the main variables, one can consider the following econometric approaches.

3.1 Classical estimation of the Keynesian trade-off

As a benchmark version of the trade-off, we estimate the Keynesian *Phillips* Curve. This basic version, described by equation (KPC), is estimated assuming an intercept term and a Generalized Autoregressive Conditional Heteroskedasticity $GARCH(1,1)$ process as

$$\pi_t = c + \xi^{kpc}\psi_t + \varepsilon_t$$
$$\sigma^2_{\varepsilon,t} = \omega_0 + \omega_1\varepsilon^2_{\varepsilon-1} + \omega_2\sigma^2_{\varepsilon,t-1}$$

The results (*Table* 5) indicate that the KPC trade-off is weakly effective over the sample period as $\xi^{kpc} = 0.0787$ is only significative at 12%.

3.2 Time Varying Parameters estimation of the monetarist trade-off

Recalling that in the monetarist vision of the trade-off, agents are assumed to adaptively make their expectations, we estimate the following Time Varying Parameters - Augmented *Phillips* Curve

$$\begin{gathered}\pi_t = b_{1,t}^m \pi_t^* + \xi_t^m \left(\psi_t - \psi_t^*\right) + \varepsilon_t \\ b_{1,t}^m = b_{1,t-1}^m + \eta_{b_1,t} \\ \xi_t^m = \xi_{t-1}^m + \eta_{\xi,t}\end{gathered}$$

In this framework, the expected inflation rate is calculated based on the $MSIH(3) - AR(2)$ estimates ($\pi_t^* = \pi_t^{MSIH}$) and the natural real activity is extracted from the $MSIH(2) - AR(2)$ estimates of the real marginal cost ($\psi_t^* = \psi_t^{MSIH}$).

The results, obtained using a linear *Kalman* filter procedure, are given by *Table* 6. The extracted filtered series of the *APC* coefficients $b_{1,t}^m$ and ξ_t^m are illustrated in *Figures* 5*a* and 5*b*. One can show that before 1969, the expected inflation rate impact on the current rate evolves under the frontier line ($b_{1,t}^m < 1$). After this year, this series converge to this long term value. The coefficient measuring the real marginal cost impact on the inflation rate (ξ_t^m) is positive except during the years 1965-1970 and $1972 - 1974$ which can be considered as a period during which the *Phillips* curve was temporally ineffective. After 1975, the series tend to the estimated *KPC* value (ξ_t^{kpc}) even if the coefficient $b_{1,t}^m$ stays close to its long term value**.** The effectiveness of this monetarist trade-off is weaker than the keynesian one.

When we estimate the *APC* equation mesuring the expected rate of inflation with the one period lag series ($\pi_t^* = \pi_{t-1}$), the expected rate impact never reaches its frontier line even if it is close to it after 1973 (*Figure* 5*c*). The coefficient $\xi_t^m \mid \pi_{t-1}$ is positive before 1973 but stays negative after this year (*Figure* 5*d*).

Put together, these results seem to confirm the intuition behind the monetarist view of the trade-off as they globally highlight an advent of the long-term conditions impulse by the agents' myopic correction.

3.3 Two-steps estimation of the New Keynesian Phillips Curves

The main problem which is raised in estimating equations (3 and 4) is related to the presence of expectations terms ($E_t\,[.]$). In order to respond to this problem, we follow a two steps strategy to estimate the New Keynesian *Phillips* Curves coefficients. In the New Keynesian framework, it is almost assumed that the inflation rate is a stationary process. But, one can assume a possible long memory process in the inflation rate or in all the main *NKPC* variables' dynamics.

3.3.1 The Fractional Integrated - Vectorial AutoRegressive reduced forms estimation

In a first step of these estimations of the New Keynesian *Phillips* Curves coefficients, we study the dynamics of the variables considering a Fractional Integrated - Vectorial AutoRegressive ($FI-VAR$) reduced form. This reduced form permits us to investigate the inflation persistence hypothesis. Also, building on the evidence of asymmetries in the evolution of the inflation rate, we combine the techniques based on fractional integration with the results of the non-linear model estimations outline in the first stage of this study.

In this Fractional Integration framework, we first fit $ARFIMA(1,d,0)$ univariate models to the series based on demeaned data and using maximum likelihood procedure. The demeaned data are calculated using the sample mean of the series for the real marginal cost ($\bar{\psi}^{Lin}$), the discount rate ($\bar{R}^{Lin}$) and the output growth rate ($\bar{\gamma}_y^{Lin}$). In the inflation rate case, we used the $MSIH\,(3)-AR\,(2)$ means calculated for each of the three regimes, (i.e. $\bar{\pi}_1^{MSIH}=0.0927$ from $1974:I$ to $1982:I$, $\bar{\pi}_2^{MSIH}=0.0357$ in the periods $1966:II-1973:IV$ and $1982:II-1992:I$ and finally $\bar{\pi}_3^{MSIH}=0.0139$ in the periods $1960:I-1966:III$ and $1992:II-2003:IV$).

Considering these inflation rate regimes subdivisions of the sample, we estimate a $FI-VAR$ model with one lag to reduce the number of parameters to be estimated. The model can be written as

$$A\,(L)\,D\,(L)\,Z_t=\varepsilon_t \tag{7}$$

$$D\,(L)=\begin{bmatrix}(1-L)^{d_{\hat{\pi}_t}} & 0 & 0 & 0\\ 0 & (1-L)^{d_{\hat{\psi}_t}} & 0 & 0\\ 0 & 0 & (1-L)^{d_{\hat{R}_{t,t+1}}} & 0\\ 0 & 0 & 0 & (1-L)^{d_{\gamma_{y_{t,t+1}}}}\end{bmatrix}$$

where $Z_t=\left(\hat{\pi}_t,\hat{\psi}_t,\hat{R}_{t,t+1},\hat{\gamma}_{y_{t,t+1}}\right)'$, $A\,(L)=I_4-A_1L$, $\varepsilon_t=\left(\varepsilon_t^{\pi},\varepsilon_t^{\psi},\varepsilon_t^{R},\varepsilon_t^{\gamma_y}\right)'$ with $E\,(\varepsilon_t)=0$ and $Var\,(\varepsilon_t)=\Sigma$.

This fractional departures from the linear VAR specification have very different long-run implications as in equation (7), each variable in Z_t can be non-stationary but non-explosive depending on the values of the differencing parameters $d_.$. When these parameters are equal to 0.5 the variables are non-stationary and the non-stationarity increasing towards $d_.=1$, Z_t can be viewed as becoming "more non-stationary", but it does so gradually. Non-linearity and the order of integration of inflation rates can, therefore, be considered as a key point to understand the dynamics of the inflation rate and to measure the expectations impact on the Inflation - Real activity trade-off. Noting that fractional integration and non-linearity are issues which are intimately related (*Diebold* & *Inoue* (2001), *Davidson* & *Teräsvirta* (2002), *Caporale* & *Gil – Alana* (2008), etc.), we take into account the analysis of the order of integration of the variables in the first stage *Markov* Switching Intercept Heteroskedastic - AutoRegression

framework. To estimate this model, we follow the procedure described by *Sela & Hurvich* (2009).

3.3.2 The structural parameters estimation

In a second step, we run the estimation of the structural parameters using the cross-equation restrictions that the model requires for the considered reduced form. Specifically, the estimation performed in the first step offers a set of $FI-VAR$ coefficients describing the data through these reduced forms which, combined with the restrictions imposed by the theoretical model, lead to a moment conditions F that capture the gap between the data and the model.

Starting from the $FI-VAR(1)$ estimates and considering equation (4), one can express the conditional expectations of the deviations of the variables relative to their steady states as[18]

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) &= e'_\pi A_1 \tilde{Z}_{t-1} \\ E\left(\hat{\psi}_t \mid \tilde{Z}_{t-1}\right) &= e'_\psi A_1 \tilde{Z}_{t-1} \\ E\left(\hat{R}_{t,t+1} \mid \tilde{Z}_{t-1}\right) &= e'_R A_1 \tilde{Z}_{t-1} \\ E\left(\hat{\gamma}_{y_{t,t+1}} \mid \tilde{Z}_{t-1}\right) &= e'_{\gamma_y} A_1 \tilde{Z}_{t-1} \end{aligned}$$

Under the assumption that $E\left(\varepsilon_t \mid \tilde{Z}_{t-1}\right) = 0$, we are able to obtain the conditional expectations of each variable by projecting the left and right terms of equation (4) on $\tilde{Z}_{t-1} = D(L) Z_{t-1}$, i.e.

$$E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = e'_\pi A_1 \tilde{Z}_{t-1} = f(A_1, \Psi) \tilde{Z}_{t-1}$$

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = \tilde{\rho}^{nkpc-pi} e'_\pi \tilde{Z}_{t-1} + \xi^{nkpc-pi} e'_\psi A_1 \tilde{Z}_{t-1} + b_1^{nkpc-pi} e'_\pi A_1^2 \tilde{Z}_{t-1} \\ + b_2 e'_\pi \gamma_1 (I - \gamma_1 A_1)^{-1} A_1^3 \tilde{Z}_{t-1} + \chi(\gamma_2 - \gamma_1) e'_R (I - \gamma_1 A_1)^{-1} A_1 \tilde{Z}_{t-1} \\ + \chi(\gamma_2 - \gamma_1) e'_{\gamma_y} (I - \gamma_1 A_1)^{-1} A_1^2 \tilde{Z}_{t-1} \end{aligned}$$

where e_k terms are column vectors of value 1 at the position corresponding to the variable k and 0 elsewhere and are used to select separately each of the four variables in the vector Z_t.

We then obtain a first set of moment conditions that capture the difference between data and model as

$$F_{1,S_t}(A_1, \Psi) = e'_\pi A_1 - f(A_1, \Psi)$$

[18] Their empirical steady state equivalents are given by

$$\begin{aligned} \bar{\pi} &= \exp\left(\bar{\pi}^{MSI}\right) \\ \bar{\psi} &= \exp\left(\bar{\psi}^{Lin}\right) \\ \bar{R} &= \bar{R}^{Lin} \\ \bar{\gamma}_y &= \exp\left(\bar{\gamma}_y^{Lin}\right) \end{aligned}$$

Similarly, one can use the $NKPC_{SS} - PI$ equation to form the second set of moment conditions linking the steady state values of all the model variables

$$\begin{aligned} & F_{2,S_t}(A_1, \Psi) \\ = & \left(1 - \alpha\bar{\pi}^{(\theta-1)(1-\rho)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left(\frac{1 - \alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{1+\theta(1+w)(1-\rho)}}{1 - \alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{\theta-\rho(\theta-1)}}\right) - \frac{\theta(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}}{\theta - 1}\bar{\psi} \end{aligned}$$

These two sets of moment conditions define an overall distance measure that enables us to judge the adequacy of the model to the data

$$\digamma_{S_t}(\Psi) = \left(F'_{1,S_t} F'_{2,S_t}\right)'$$

so that, the model fits the data, if and only if, there is a vector of structural parameters (Ψ) that solves the following constrained minimization problem

$$\underset{\Psi}{Min} \digamma_{S_t}(\Psi)' \digamma_{S_t}(\Psi)$$

subject to $\alpha \in]0, 1[$, $\varrho \in [0, 1]$ and $\theta \in [0, +\infty[$.

The FI-VAR estimation of the NKPC model

Based on the first stage and the previous results but also on previous works (*Cogley* & *Sbordonne* (2005), *Groen* & *Mumtaz* (2008)), we estimate the first generation of the New Keynesian *Phillips* Curve given by equation (3) following the two-steps strategy. In the first step, the $FI - VAR$ estimation[19] procedure starts with the univariate estimated differencing parameters (*Tables* $7a - 7c$) for the all variables and setting all the initial off-diagonal elements of A_1 and Σ to zero. From these univariate estimates, the bi-variate $FI - VAR(1)$ results are reported in *Tables* $8a - 8c$.

One can show that in the second regime, the inflation rate has a larger differencing parameter than in the other two. In this skepticism regime, the inflation rate differencing parameter is quite close to 0.5 implying a long memory in the series. In the real marginal cost case, the highest differencing parameter is associated to the third regime. These results imply that, when price start to increase from the third regime to the second one (decrease from the first regime to the second one), reflecting the departure from the optimism (pessimism) regime to reach the skepticism one, the agents become more concerned by the level of the inflation rate. Similarly, when the economy moves to the pessimism regime, firms seem to pay more attention to the level of the real marginal cost.

[19] Note that in this $NKPC$ context, the $FI - VAR$ model will be written as

$$A(L)\, D(L)\, Z_t = \varepsilon_t$$

$$D(L) = \begin{bmatrix} (1-L)^{d_{\hat{\pi}_t}} & 0 \\ 0 & (1-L)^{d_{\hat{\psi}_t}} \end{bmatrix}$$

where $Z_t = \left(\hat{\pi}_t, \hat{\psi}_t\right)'$, $A(L) = I_2 - A_1 L$, $\varepsilon_t = \left(\varepsilon_t^{\pi}, \varepsilon_t^{\psi}\right)'$ with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \Sigma$.

The corresponding results of the distance minimization[20], obtained from the bi-variate $FI-VAR$ reduced form estimates, are presented in *Table* 9. The results indicate that the parameter measuring the degree of price ridgidity is estimated as $\hat{\alpha}_{S_t=1} = 0.2500$, $\hat{\alpha}_{S_t=2} = 0.8500$ and $\hat{\alpha}_{S_t=3} = 0.9500$. The indexation parameter is estimated as $\hat{\varrho}_{S_t=1} = 0.0010$, $\hat{\varrho}_{S_t=2} = 0.7500$ and $\hat{\varrho}_{S_t=3} = 0.0010$. Then, firms that do not receive the signal to optimize their prices have a weak and quasi-negligible tendency to index them on the past inflation. Finally, the parameter that measures the degree of substitution between goods is estimated as $\hat{\theta}_{S_t=k} \in [9.8580, 36.4040]$. This estimated degree of substitution between goods imply a mark-up of about $\vartheta_{S_t=k} = 11\%$.

The corresponding $NKPC$ coefficients are computed for each inflation rate regimes and associated with each of the $FI-VAR$ autoregressive coefficients. *Figures* 6*a*−6*b* show these *Phillips* curve coefficients (b_1^{nkpc} and ξ^{nkpc}). We note that during episodes of oil and monetary shocks ($1973-1976$ and $1979-1982$), $\xi^{nkpc} < 0$ so that the *Phillips* curve seems to disappear. This reversal of the Inflation - Real activity arbitration clearly marks the renewal of the arbitration vision initiated by authors such as *Phelps* (1967), *Friedman* (1968) and *Lucas* (1972*a*). The impact of the expected inflation rate is high ($b_1^{nkpc} > 1$) during these years.

The FI-VAR estimation of the NKPC-PI model

In this New Keynesian *Phillips* Curve with Positive steady state Inflation framework, the $FI-VAR(1)$ estimation procedure starts with the univariate

[20] Note that in this simplified version of the trade-off, the conditional expectations of the deviations of the two variables relative to their steady states are given by

$$E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) = e'_\pi A_1 \tilde{Z}_{t-1}$$
$$E\left(\hat{\psi}_t \mid \tilde{Z}_{t-1}\right) = e'_\psi A_1 \tilde{Z}_{t-1}$$

and their empirical steady state equivalents are given by

$$\bar{\pi} = \exp\left(\bar{\pi}^{MSI}\right)$$
$$\bar{\psi} = \exp\left(\bar{\psi}^{Lin}\right)$$

Also, the resulting conditional expectations of each of these two variables are given by

$$\begin{aligned} E\left(\hat{\pi}_t \mid \tilde{Z}_{t-1}\right) &= e'_\pi A_1 \tilde{Z}_{t-1} \\ &= f\left(A_1, \Psi\right) \tilde{Z}_{t-1} \\ &= \tilde{\rho}^{nkpc} e'_\pi \tilde{Z}_{t-1} + b_1^{nkpc} e'_\pi A_1^2 \tilde{Z}_{t-1} + \xi^{nkpc} e'_\psi A_1 \tilde{Z}_{t-1} \end{aligned}$$

and the unique set of moment conditions that capture the restrictions implied by the theoretical model on the set of parameters describing data via the reduced form will be written as

$$F_{S_t}\left(A_1, \Psi\right) = e'_\pi A_1 - f\left(\Psi\right)$$

In this $NKPC$ framework, we then solve the following constrained minimization problem

$$\underset{\Psi}{Min} F_{S_t}\left(\Psi\right)' F_{S_t}\left(\Psi\right)$$

to obtain the estimated structural parameters.

estimated differencing parameters ($Tables$ $10a - 10c$) for the all variables and setting all the initial off-diagonal elements of A_1 and Σ to zero. The multivariate results are reported in $Tables$ $11a - 11c$.

Considering the univariate results, we note that, like in the $NKPC$ case, the differencing parameters are low in the optimism regime indicating that all the series have short memories. When the economy enter in the skepticism regime, we note that the differencing parameters of the inflation and the output growth rates increase to reach values close to 0.5 indicating that these series have long memories. In this second regime where the expected inflation rate is at an intermediate level, agents seem to be extremely concerned by the output growth dynamics as as $\hat{d}_{\gamma_{y_t}} = 0.49$. This last result could indicate that agents are questioning the monetary authority's credibility in its fight against the inflation. In the pessimism regime, firms seem to be almost attentive to the real marginal cost evolution and extremely concerned by the dynamics of the discount rate. The differencing parameters of these two last series are $\hat{d}_{\hat{\psi}_t} = 0.3496$ and $\hat{d}_{\hat{R}_{t,t+1}} = 0.46$. Clearly, the agents are examining the monetary authority's decisions in these medium and high inflation rate regimes.

Considering the multivariate case, the results are globally the same as in the univariate one. However, in the pessimism regime, in addition to the real marginal cost and the discount rate, we note that firms continue to attentively look at the inflation rate dynamics as its estimated differencing parameter is $\hat{d}_{\hat{\pi}_t} = 0.47$. Note that we consider a measure of an inflation gap so that our estimated differencing parameters mesure the persistence of this inflation gap. As suggested by $Cogley$, $Primiceri$ & $Sargent$ (2010) : *"this inflation gap is weakly persistent when the effects of shocks decay quickly and that it is strongly persistent when they decay slowly. When the effects of past shocks die out quickly, future shocks account for most of the variations in the inflation gap, pushing our measure (of the differencing parameter) close to zero. But when the effects of decay slowly, they account for a higher proportion of the near-term mouvements, pushing our measure of persistence closer to"* 0.5. Our results then suggest that the inflation gap's persistence has changed over time.

The results of the distance minimization, presented in $Table$ 12, indicate instability in the price stickiness through the estimated inflation rate regimes. This parameter is estimated as $\hat{\alpha}_{S_t=1} = 0.0010$, $\hat{\alpha}_{S_t=2} = 0.6004$ and $\hat{\alpha}_{S_t=3} = 0.0509$. As reported by $Groen$ & $Mumtaz$ (2008), a week instability tendency can be associated to this parameter. The estimated probabilities of prices non-adjustment are then much more varying when one considers both the presence of long memory and regimes switching in the inflation rate dynamics. We observe that in the second regime, the prices are much more rigid than in the other two regimes illustrating the skepticism in the economic environment.

The indexation parameter is estimated as $\hat{\varrho}_{S_t=1} = 0.0010$, $\hat{\varrho}_{S_t=2} = 0.8002$ and $\hat{\varrho}_{S_t=3} = 0.0010$. Globally, firms that do not receive the signal to re-optimize their prices have a weak and quasi-negligible tendency to index those prices on the past inflation except in the second regime. This results confirm those of $Cogley$ & $Sbordone$ (2005) who estimate $\varrho^{CS} = 0$. However, in the skepticism

regime, where the differencing parameters of the inflation and the output growth rates are the highest, firms that do not have the opportunity to re-optimize their prices are much more backward looking than in the other two regimes. The fact that this parameter can be non-zero seems to confirm results obtained by many other studies performed in the $NKPC$ with a zero steady state inflation rate (*Gali* & *Gertler* (1999), *Giannoni* & *Woodford* (2003)). For most of these studies, this indexation parameter is significant and estimated between 0.2 and 1. The existence of a non-zero indexation degree can capture the observed persistence of the inflation rate additionally to what is detected by the $FI-VAR$ model. This result is also highlighted by *Groen* & *Mumtaz* (2008) who estimate $\varrho^{GM} \in [0.65, 0.95]$.

Finally, the parameter that measures the degree of substitution between the goods is estimated as $\hat{\theta}_{S_t=k} \in [12.8080, 57.0505]$. These results remain fairly close to the values estimated by *Cogley* & *Sbordone* (2005) and those of *Groen* & *Mumtaz* (2008).

The $NKPC - PI$ coefficients are derived from these estimated structural parameters computed for each inflation rate regimes and associated with each of the $FI-VAR$ autoregressive coefficients. *Figures* $7a-7b$ show the *Phillips* curve coefficients $b_1^{nkpc-pi}$ and $\xi^{nkpc-pi}$, reflecting the evolution of the expected inflation rate impact and the effectiveness of the Inflation-Real activity trade-off respectively.

We note that during episodes of oil and monetary shocks, $\xi^{nkpc-pi} \to 0$ challenging the *Phillips* curve in these periods but less consistently than in the $NKPC$ case. Also, some non-negligible challenges (possibly associated to the *NBER* recessions episodes and the *September* 11 events) are highlighted and, as one can expect, the impact of the expected inflation rate is high ($b_1^{nkpc-pi} > 1$) during these years.

4 Conclusion

In this paper, we undertake some econometric inquiries into the dynamics of the main variables involved in modeling the New Keynesian *Phillips* Curve. We have shown that the inflation rate appears to be generated by an autoregressive process of the second order with constant lag coefficients and an unconditional mean, oscillating between three different regimes that could be assimilated to three expected targets of the U.S. inflation rate. According to this $MSIH(3) - AR(2)$ specification, the expected inflation rate evolves between these regimes controlled by a *Markov* chain. This latter could be perceived as a system of beliefs formed by the agents on the three presumed fulfillments of their inflationist expectations. These beliefs can be qualified as adaptive since the probabilities of switching from one regime to another are conditional on the previous states of the economy. In the real marginal cost case, the $MSIH(2) - AR(2)$ specification does not appear to be the most adequate one to characterize its dynamics. The linear model seems to be the most adequate specification to characterize this variable dynamics. For the discount rate and

the output growth rate, the expected rates estimated by the $MSIH(2) - AR(2)$ are almost constant so that the best specification to characterize these variables dynamics appears to be the linear one.

From this first empirical stage findings, we conducted empirical analysis around the famous bridge between the nominal and the real economic spheres associated to the *Phillips* curve. Our results are supportive of regimes' persistence inflation hypothesis, implying that shocks have a permanent effect in some of the regimes (like the skepticism or pessimism ones), but have finite lives in the optimism regime.

The results of this study show how the introduction of agents' expectations in the different versions of the trade-off has affected its empirical effectiveness and helped highlight the nuances between the three main visions of the evolution of the economic system. Globally, the results schematize some of the main aspects of the divergence between the classical, the monetarist and the keynesian views on the theoretical validity of the *Phillips* curve.

Annex

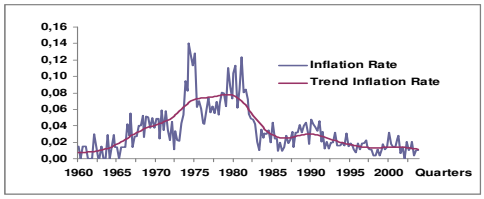

Figure 1*a* : *Hodrick – Prescott trend inflation rate*

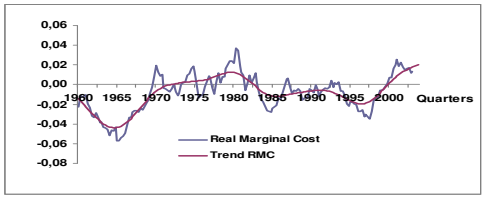

Figure 1*b* : *Hodrick – Prescott trend real marginal cost*

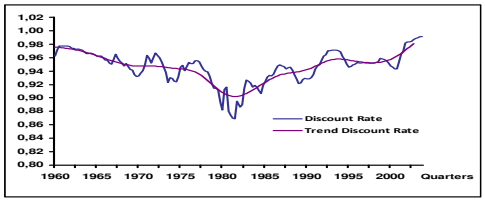

Figure 1*c* : *Hodrick – Prescott trend discount rate*

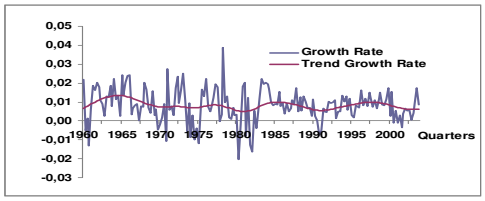

Figure 1*d* : *Hodrick – Prescott trend output growth rate*

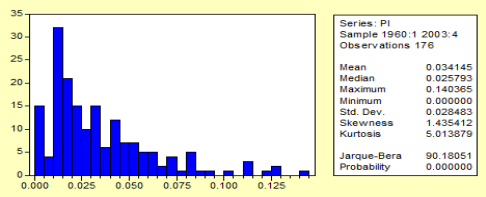

Table 1*a* : *Descriptive statistics of the inflation rate*

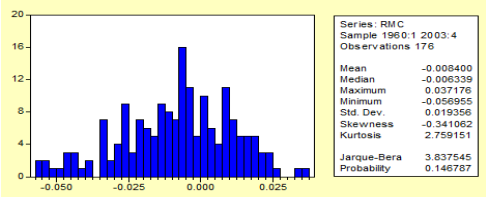

Table 1*b* : *Descriptive statistics of the real marginal cost*

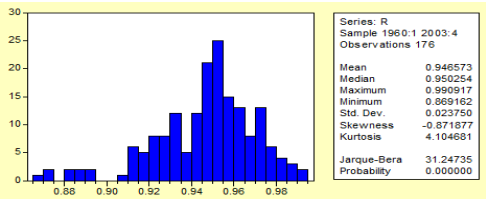

Table 1*c* : *Descriptive statistics of the discount rate*

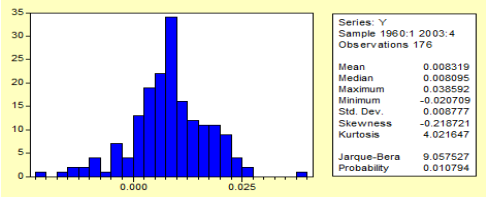

Table 1*d* : *Descriptive statistics of the output growth rate*

	Estimates [*p−values*]
$\hat{\phi}_0$	0.0037 [0.0094]
$\hat{\phi}_1$	0.3749 [0.0000]
$\hat{\phi}_2$	0.4666 [0.0000]
$\hat{\omega}_0$	$6.4 \cdot e^{-6}$ [0.2628]
$\hat{\omega}_1$	0.1969 [0.0042]
$\hat{\omega}_2$	0.7873 [0.0000]
$lnL^{Lin}_{\pi_t} = 502.0563$	

Table 1*e* : *Estimates of the linear model for the inflation rate*

	Estimates [*p−values*]
$\hat{\phi}_0$	−0.0004 [0.3933]
$\hat{\phi}_1$	1.1250 [0.0000]
$\hat{\phi}_2$	−0.1709 [0.0283]
$\hat{\omega}_0$	$2.2 \cdot e^{-6}$ [0.5852]
$\hat{\omega}_1$	0.0334 [0.5616]
$\hat{\omega}_2$	0.8994 [0.0000]
$lnL^{Lin}_{\psi_t} = 654.9954$	

Table 1*f* : *Estimates of the linear model for the real marginal cost*

	Estimates [*p−values*]
$\hat{\phi}_0$	0.9613 [0.0000]
$\hat{\phi}_1$	1.3847 [0.0000]
$\hat{\phi}_2$	−0.4429 [0.0000]
$\hat{\omega}_0$	$9.2 \cdot e^{-7}$ [0.0857]
$\hat{\omega}_1$	0.3396 [0.0009]
$\hat{\omega}_2$	0.6840 [0.0000]
$lnL^{Lin}_{R_{t,t+1}} = 691.4020$	

Table 1*g* : *Estimates of the linear model for the discount rate*

	Estimates [*p−values*]
$\hat{\phi}_0$	0.0048 [0.0000]
$\hat{\phi}_1$	0.2061 [0.0211]
$\hat{\phi}_2$	0.2582 [0.0021]
$\hat{\omega}_0$	$2.2 \cdot e^{-6}$ [0.2294]
$\hat{\omega}_1$	0.1735 [0.0145]
$\hat{\omega}_2$	0.8012 [0.0000]
$lnL^{Lin}_{\gamma_{y,t}} = 601.5058$	

Table 1*h* : *Estimates of the linear model for the output growth rate*

	Estimates (*Std.dev*)	L
$\hat{\phi}_0$	0.0039 (0.0018)	0.2643*
$\hat{\phi}_1$	0.4888 (0.0798)	0.0726*
$\hat{\phi}_2$	0.3984 (0.0693)	0.0735*
$\hat{\sigma}^2_\varepsilon$	0.0002 ($3.5 \cdot e^{-5}$)	1.0018

$L_c = 1.2652$

Table 1i : Parameters stability tests for the inflation rate

	Estimates (*Std.dev*)	L
$\hat{\phi}_0$	−0.0003 (0.0005)	0.1777*
$\hat{\phi}_1$	1.1272 (0.0720)	0.3413*
$\hat{\phi}_2$	−0.1788 (0.0762)	0.2974*
$\hat{\sigma}^2_\varepsilon$	$3.1 \cdot e^{-5}$ ($3.4 \cdot e^{-6}$)	0.3906*

$L_c = 0.9303^*$

Table 1j : Parameters stability tests for the real marginal cost

	Estimates (*Std.dev*)	L
$\hat{\phi}_0$	0.0468 (0.0370)	0.2079*
$\hat{\phi}_1$	1.2031 (0.0954)	0.2263*
$\hat{\phi}_2$	−0.2527 (0.0968)	0.2237*
$\hat{\sigma}^2_\varepsilon$	$4.2 \cdot e^{-5}$ ($1.1 \cdot e^{-5}$)	0.4164*

$L_c = 0.9472^*$

Table 1k : Parameters stability tests for the discount rate

	Estimates (*Std.dev*)	L
$\hat{\phi}_0$	0.0049 (0.0012)	0.1257*
$\hat{\phi}_1$	0.2427 (0.0779)	0.1086*
$\hat{\phi}_2$	0.1626 (0.0887)	0.1181*
$\hat{\sigma}^2_\varepsilon$	$6.8 \cdot e^{-5}$ ($9.9 \cdot e^{-6}$)	0.8715

$L_c = 1.1136^*$

Table 1l : Parameters stability tests for the output growth rate

	LnL	l	AIC
Linear Model	502.0563	6	-992.1126
$MSI(2)-AR(2)$	507.0065	8	-998.0130
$\mathbf{MSI(3)-AR(2)}$	513.5232	14	$\mathbf{-999.0464}$

Table 2a : Selection of the number of regimes for the inflation rate

	LnL	l	AIC
Linear Model	654.9954	6	$\mathbf{-1297.9908}$
$MSI(2)-AR(2)$	655.3994	8	-1294.7988
$MSI(3)-AR(2)$	644.4619	14	-1260.9238

Table 2b : Selection of the number of regimes for the real marginal cost

	LnL	l	AIC
Linear Model	691.4013	6	$\mathbf{-1370.8026}$
$MSI(2)-AR(2)$	683.5897	8	-1351.2000
$MSI(3)-AR(2)$	$n.c$	14	$n.c$

Table 2c : Selection of the number of regimes for the discount rate

	LnL	l	AIC
Linear Model	601.5331	6	-1191.0662
$\mathbf{MSI(2)-AR(2)}$	606.2054	8	$\mathbf{-1196.4108}$
$MSI(3)-AR(2)$	589.6142	14	-1151.2284

Table 2d : Selection of the number of regimes for the output growth rate

State (j)	**n°obs/state**	$\hat{\mathbf{P}}(S_t = k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	26	0.1488 (0.1616)	0.0491 (0.0114)	0.1782 (0.0766)	0.2924 (0.0751)	0.0005 (0.0003)
2	71	0.4106 (0.2546)	0.0189 (0.0042)	0.1782 (0.0766)	0.2924 (0.0751)	0.0001 (0.0000)
3	78	0.4406 (0.3357)	0.0074 (0.0017)	0.1782 (0.0766)	0.2924 (0.0751)	0.0001 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.0929)}{0.9126} & \underset{(0.0295)}{0.0317} & \underset{(0.0089)}{0.0000} \\ \underset{(0.0954)}{0.0874} & \underset{(0.0402)}{0.9418} & \underset{(0.0375)}{0.0247} \\ \underset{(0.0080)}{0.0000} & \underset{(0.0269)}{0.0265} & \underset{(0.0316)}{0.9753} \end{bmatrix} \qquad lnL^{MSI}_{\pi_t} = 513.5232$$

Table 3a : Estimates of the $MSI(3) - AR(2)$ model for the inflation rate

State (j)	**n°obs/state**	$\hat{\mathbf{P}}(S_t = k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	57	0.3264 (0.3332)	0.0027 (0.0016)	1.0455 (0.1278)	−0.0898 (0.1153)	0.0000 (0.0000)
2	117	0.6736 (0.3332)	−0.0017 (0.0020)	1.0455 (0.1278)	−0.0898 (0.1153)	0.0000 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.2185)}{0.7578} & \underset{(0.1757)}{0.1173} \\ \underset{(0.2185)}{0.2422} & \underset{(0.1757)}{0.8827} \end{bmatrix} \qquad lnL^{MSI}_{\psi_t} = 655.3994$$

Table 3b : Estimates of the $MSI(2) - AR(2)$ model for the real marginal cost

State (j)	**n°obs/state**	$\hat{\mathbf{P}}(S_t = k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	57	0.3284 (0.1532)	0.0328 (0.0135)	1.4404 (0.0671)	−0.4751 (0.0676)	0.0001 (0.0000)
2	117	0.6716 (0.1532)	0.0330 (0.0141)	1.4404 (0.0671)	−0.4751 (0.0676)	0.0000 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.0505)}{0.9172} & \underset{(0.0244)}{0.0405} \\ \underset{(0.0505)}{0.0828} & \underset{(0.0244)}{0.9595} \end{bmatrix} \qquad lnL^{MSI}_{R_{t,t+1}} = 683.5897$$

Table 3c : Estimates of the $MSI(2) - AR(2)$ model for the discount rate

State (j)	**n°obs/state**	$\hat{\mathbf{P}}(S_t = k)$	$\hat{\phi}_{0_{S_t=k}}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\sigma}^2_{\varepsilon_{S_t=k}}$
1	94	0.0000 (0.5304)	0.0048 (0.0012)	0.2214 (0.0728)	0.2312 (0.0773)	0.0001 (0.0000)
2	80	1.0000 (0.5304)	0.0044 (0.0009)	0.2214 (0.0728)	0.2312 (0.0773)	0.0000 (0.0000)

$$\hat{P} = \begin{bmatrix} \underset{(0.0109)}{0.9894} & \underset{(0.0056)}{0.0000} \\ \underset{(0.0109)}{0.0106} & \underset{(0.0056)}{1.0000} \end{bmatrix} \qquad lnL^{MSI}_{\gamma_{y,t}} = 606.2054$$

Table 3d : Estimates of the $MSI(2) - AR(2)$ model for the output growth rate

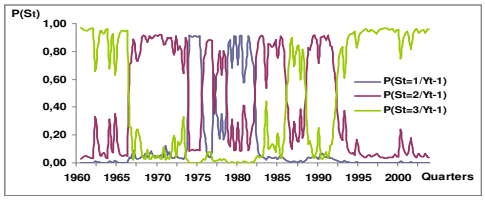

Figure 2a : $MSI(3) - AR(2)$ regimes probabilities for the inflation rate

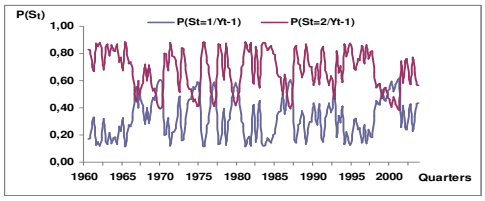

Figure 2b : $MSI(2) - AR(2)$ regimes probabilities for the real marginal cost

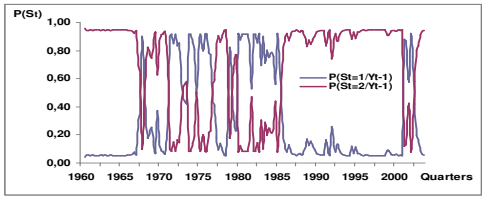

Figure 2c : $MSI(2) - AR(2)$ regimes probabilities for the discount rate

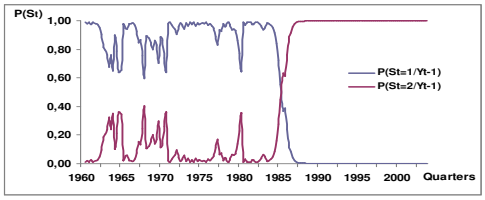

Figure 2d : $MSI(2) - AR(2)$ regimes probabilities for the output growth rate

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.7361	0.3344	1.8652
[1.6800]	[1.0100]	[1.0100]

Tests for parameters stability in state		
State 1	*State 2*	*State 3*
0.1942	0.5225	2.1669
[1.2400]	[1.2400]	[1.2400]

Test of serially uncorrelated Markov chain	
Wald-test	*F-test*
1371.2596	323.1129
{0.0000}	{0.0000}

Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
0.6395	0.6727	2.0582
{0.7620}	{0.7328}	{0.0611}

Table 4a : Adequation tests of the $MSI(3) - AR(2)$ model for the inflation rate

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.1993	0.3139	0.5361
[0.7500]	[0.7500]	[0.7500]

Tests for parameters stability in state	
State 1	*State 2*
0.6563	1.0752
[1.2400]	[1.2400]

Test of serially uncorrelated Markov chain	
Wald-test	*F-test*
4.0937	3.9290
{0.0430}	{0.0491}

Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
0.1210	0.1362	6.1520
{0.9748}	{0.9687}	{0.0001}

Table 4b : Adequation tests of the $MSI(2) - AR(2)$ model for the real marginal cost

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.2886	0.1857	0.3503
[0.7500]	[0.7500]	[0.7500]

Tests for parameters stability in state	
State 1	*State 2*
0.5702	0.7003
[1.2400]	[1.2400]

Test of serially uncorrelated Markov chain	
Wald-test	*F-test*
192.2238	184.4907
{0.0000}	{0.0000}

Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
1.3280	0.3756	0.4628
{0.2617}	{0.8258}	{0.7629}

Table 4c : Adequation tests of the $MSI(2) - AR(2)$ model for the discount rate

Tests for parameters stability		
Markov	*mu*	*Res-cov*
0.2022	0.0943	0.3758
[0.7500]	[0.7500]	[0.7500]

Tests for parameters stability in state	
State 1	*State 2*
0.5649	0.3362
[1.2400]	[1.2400]

Test of serially uncorrelated Markov chain	
Wald-test	*F-test*
5978.1938	5737.6917
{0.0000}	{0.0000}

Misspecification tests based on conditional scores		
Autocorr	*ARCH*	*Markov*
1.5096	1.9186	0.3875
{0.2017}	{0.1097}	{0.8174}

Table 4d : Adequation tests of the $MSI(2) - AR(2)$ model for the output growth rate

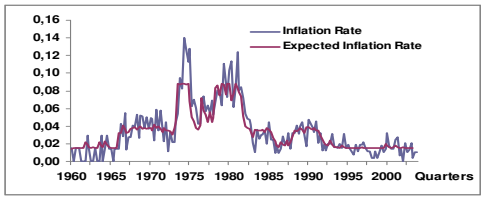

Figure 3a : $MSI\,(3) - AR(2)$ expected inflation rate

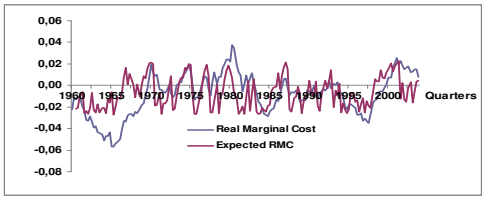

Figure 3b : $MSI\,(2) - AR(2)$ expected real marginal cost

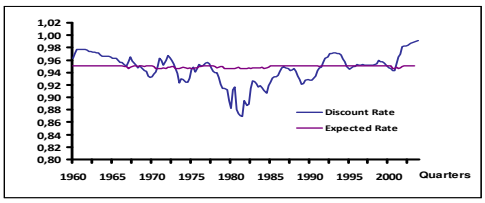

Figure 3c : $MSI\,(2) - AR(2)$ expected discount rate

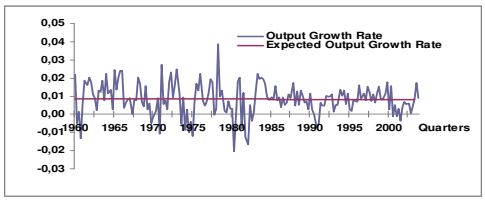

Figure 3d : $MSI\,(2) - AR(2)$ expected output growth rate

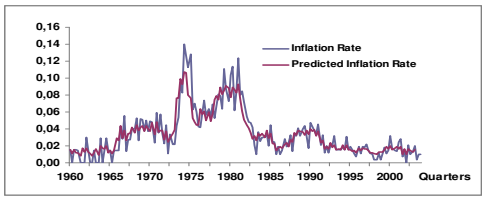

Figure 4a : Inflation rate as described by the $MSI\,(3) - AR\,(2)$ specification

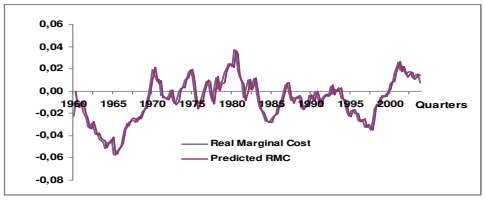

Figure 4b : $MSI\,(2) - AR\,(2)$ predicted real marginal cost

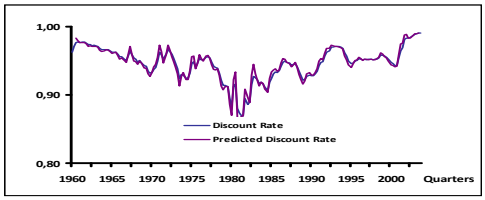

Figure 4c : Discount rate as described by the $MSI\,(2) - AR\,(2)$ specification

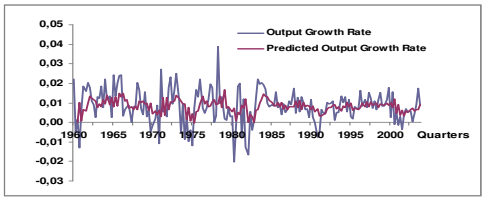

Figure 4d : $MSI\,(2) - AR\,(2)$ predicted output growth rate

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.020560	0.001489	13.80881	0.0000
RMC	0.078766	0.049736	1.583683	0.1133
		Variance Equation		
C	2.59E-05	1.91E-05	1.351911	0.1764
ARCH(1)	0.440608	0.120539	3.655309	0.0003
GARCH(1)	0.533628	0.124963	4.270273	0.0000

R-squared	-0.204524	Mean dependent var	0.034145
Adjusted R-squared	-0.232700	S.D. dependent var	0.028483
S.E. of regression	0.031624	Akaike info criterion	-5.039496
Sum squared resid	0.171015	Schwarz criterion	-4.949426
Log likelihood	448.4757	Durbin-Watson stat	0.306373

Table 5 : Estimates for the KPC model

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-8.431297	0.089729	-93.96400	0.0000
	Final State	Root MSE	z-Statistic	Prob.
B1	1.053917	0.028252	37.30439	0.0000
KSI	0.051449	0.061573	0.835571	0.4034

Log likelihood	467.4473	Akaike info criterion	-5.361463
Parameters	1	Schwarz criterion	-5.343308
Diffuse priors	2	Hannan-Quinn criter.	-5.354098

Table 6 : Estimates of the Augmented Phillips Curve

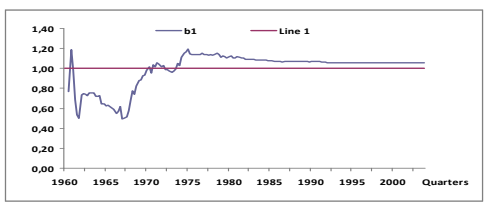

Figure 5a : Dynamic of the expected inflation rate impact when $\pi_t^* = \pi_t^{MSI}$

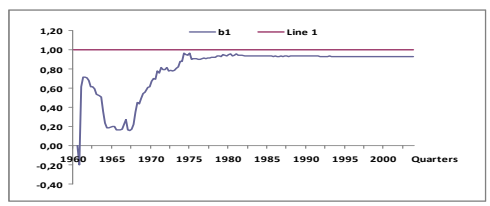

Figure 5b : Dynamic of the expected inflation rate impact when $\pi_t^* = \pi_{t-1}$

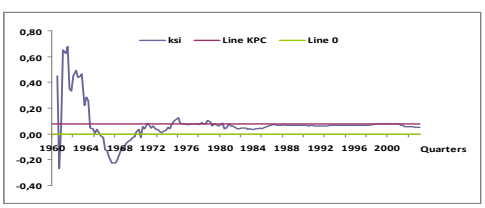

Figure 5c : Dynamic of the real marginal cost impact when $\pi_t^* = \pi_t^{MSI}$

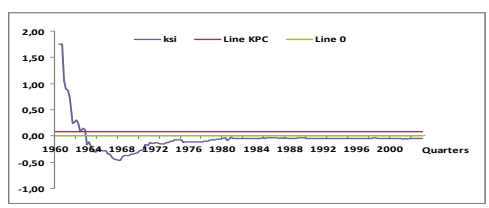

Figure 5d : Dynamic of the real marginal cost impact when $\pi_t^* = \pi_{t-1}$

	Inflation	*Real Marg. Cost*
ϕ_1	0.1489 $(1.04e^{-1})$	0.6118 $(7.61e^{-2})$
σ_ε^2	0.0005 $(1.89e^{-8})$	$1e^{-4}$ $(2.43e^{-8})$
d	0.2819 $(4.47e^{-2})$	0.3496 $(4.77e^{-2})$
$\ln L$	110.5020	145.8362

Table 7a : $FI-AR(1)$ estimates based on the first inflation rate regime

	Inflation	*Real Marg. Cost*
ϕ_1	0.7977 $(2.92e^{-2})$	0.8050 $(1.95e^{-2})$
σ_ε^2	0.0002 $(1.43e^{-9})$	$1e^{-4}$ $(6.30e^{-10})$
d	0.4199 $(4.26e^{-2})$	0.2001 $(4.49e^{-2})$
$\ln L$	261.7809	312.5373

Table 7b : $FI-AR(1)$ estimates based on the second inflation rate regime

	Inflation	*Real Marg. Cost*
ϕ_1	0.0017 $(5.33e^{-2})$	0.9000 $(8.85e^{-4})$
σ_ε^2	$1e^{-4}$ $(7.06e^{-10})$	$1e^{-4}$ $(6.06e^{-9})$
d	0.1075 $(3.42e^{-2})$	0.1157 $(1.34e^{-3})$
$\ln L$	302.5667	311.8143

Table 7c : $FI-AR(1)$ estimates based on the third inflation rate regime

	Maximum Likelihood *with regression approximation*
A_1	0.0983 $(1.73e^{-1})$ 0.7739 $(6.34e^{-2})$ 0.0645 $(5.97e^{-1})$ 0.3210 $(8.67e^{-1})$
Σ	$4.29e^{-4}$ $(6.77e^{-6})$ $5.80e^{-6}$ $(3.20e^{-6})$ $5.80e^{-6}$ $(3.20e^{-6})$ $1.00e^{-4}$ $(4.15e^{-6})$
d	0.2456 $(2.85e^{-2})$ 0.4671 $(1.80e^{-3})$
$\ln L$	259.6892

Table 8a : $FI-VAR(1)$ estimates in the first inflation rate regime

	Maximum Likelihood *with regression approximation*
A_1	0.8062 $(2.31e^{-1})$ 0.0412 $(3.41e^{-1})$ -0.0239 $(6.59e^{-2})$ 0.8041 $(9.19e^{-1})$
Σ	$2.05e^{-4}$ $(1.48e^{-6})$ $-6.23e^{-7}$ $(1.44e^{-6})$ $-6.23e^{-7}$ $(1.44e^{-6})$ $1.00e^{-4}$ $(6.18e^{-6})$
d	0.4488 $(5.91e^{-3})$ 0.1976 $(6.38e^{-2})$
$\ln L$	574.5333

Table 8b : $FI-VAR(1)$ estimates in the second inflation rate regime

	Maximum Likelihood *with regression approximation*
A_1	-0.0390 $(4.72e^{-2})$ 0.1005 $(1.41e^{-1})$ -0.0634 $(5.46e^{-3})$ 0.8930 (2.1616)
Σ	$1.00e^{-4}$ $(9.48e^{-7})$ $-2.02e^{-5}$ $(1.84e^{-6})$ $-2.02e^{-5}$ $(1.84e^{-6})$ $1.04e^{-4}$ $(2.70e^{-6})$
d	0.1285 $(2.57e^{-2})$ 0.1221 $(2.71e^{-2})$
$\ln L$	616.7422

Table 8c : $FI-VAR(1)$ estimates in the third inflation rate regime

	α	ρ	θ
$S_t = 1$	0.250	0.001	9.858
$S_t = 2$	0.850	0.750	24.606
$S_t = 3$	0.950	0.001	36.404

Table 9 : Structural parameters based on the $FI-VAR$ estimates

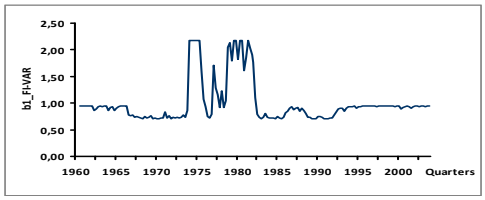

Figure 6a : Dynamic of the expected inflation rate impact based on the $FI-VAR$ estimates

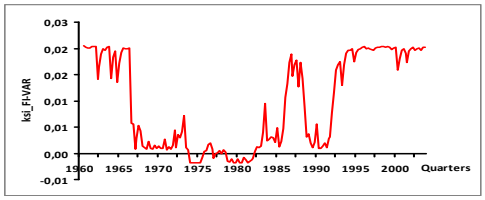

Figure 6b : Dynamic of the real marginal cost impact based on the $FI-VAR$ estimates

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