Commitment and Optimal Incentive

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Abstract: We propose an extended principal-agent model considering employee commitment and describe how to motivate committed agent, who not only shows regard for his own income but also cares the organizational benefit. The principal also would like to provide support to such an agent and his utility depends on both the final profit and the payoff to the agent. There are some interesting insights into the characteristic of optimal contracts: First, commitment is an effective motivator and committed employee needs less monetary inducement to perform his job well than one who not. More specifically, undifferentiated pay is sufficient in incentivizing committed agent to implement high effort in some cases. Second, commitment and wage differential are substitutable to each other in the optimal incentive compensation design. Third, commitment is not always good for organizational efficiency when the increase in employee commitment relies on the principal’s support. Our model's finding is consistent with employee incentive in some organizations, and also help to incentive mechanism design under wages differential constraints and understanding excessive compensation.

Key Words:  Commitment, Organizational support, Optimal Incentive, Contract
Introduction

Employee commitment is a familiar topic in management research. Many scholars, such as Mowday, Porter & Steers (1982); McElroy, Morrow, Power & Iqbal (1993); Meyer & Allen (1997); Becker, Billings, Eveleth & Gilbert (1996) have deepened people’s understanding of it. There are various forms through which employee commitment can be analyzed and deconstructed. Organization commitment is the most typical among all the concepts and the earliest to be widely studied. In this paper, an explanation of commitment will be made on the basis of organizational commitment. As described by Mowday, Porter & Steers (1982), committed employees are characterized as loyal and productive, and identify with organizational goals and organizational values. They are dedicated to the organization and put forth extra effort on its behalf. Consequently a wide array of desirable behavioral outcomes, such as good job performance, a high attendance rate, was found to be associated with employee commitment (e.g. Becker (1992); Becker, Randall & Riegel (1995); Blau & Boal (1987); Meye & Allen (1997)). Moreover, due to its potential benefits for organizations, commitment is typically valued by its practitioners, as managers prefer loyal and devoted employees (Paula C. Morrow, 1983). The above studies have suggested or implied the significance of the connection between employee commitment and organizational benefit and have shown that committed employees care about organizational benefit. Therefore, having a sense of commitment can affect the utility function of employees. They consider not only their own benefit but also the benefit of the organization. Furthermore, employees make a trade-off between their own benefit and organizational benefit depending on their commitment level. It is clear that employee commitment is linked to behaviors that tend to benefit the organization.
Since the publishing of a series of ground-breaking papers, including research by Spence and Zeckhauser (1971), Ross (1973), Mirrlees (1974, 1976) and Holmstrom (1979), the basic principal–agent model has been greatly developed to solve the problem of moral hazard. A method of using the principal-agent model to deduce optimal contracts becomes increasingly widely applied to inspire employees. However, optimal contracts in the classic model may sometimes be limited in actual operation. That is, for one thing, almost all optimal compensation schemes are based on variables (such as output or profits) that are observable. But in some cases the output or profits are hard to evaluate and measure in practical management. Additionally, although economic theorists have put forth their best efforts to construct optimal compensation for workers, the use of optimal compensation schemes in practice is not without its drawbacks (Prendergast (1999); Gibbons (1998)). For example, Holmstrom and Milgrom (1991) indicate that in a multitasking environment, workers will be inspired to over-perform on well rewarded tasks and to underperform on poorly rewarded tasks. Lazear and Rosen (1981) and Lazear (1989) show that workers may have incentive to sabotage one another when the payment is based on relative performance. George Becker (2000) also stresses that incentive contracts may induce gaming, noncooperative behavior and dysfunctional consequences when an organization has to use the distorted performance measure, since it is hard to find “good” performance measures in most cases. Therefore, in the aforementioned cases we need to rely instead on employee commitment to improve incentive, as commitment has been proven to be effective in inspiring employees. Bringing the concept of commitment into the principal-agent model can deepen our understanding of managerial policies such as incentive strength and optimal payment. We argue that employee commitment is also an important source of motivation and has a marked effect on optimal compensation schemes.
Inculcating employee commitment is critical to enhance an organization’s efficiency.

In the conventional principal-agent model, the agents’ utility function only describes their own benefit. However, committed employees’ behavior is inconsistent with this utility function in classic principal-agent theory, because individuals not only care for their own payoff but also for organizational benefit. In some other economic theories, such as altruism and reciprocity theories, the benefit of others appears as a positive component in an individual’s utility function (Andreoni, 1990), or the principle of reciprocity acts as a constraint on traditional individual utility maximization (Sugden, 1984). While most of the literature referring to altruism and reciprocity theories discusses the public goods provision problem, it does not consider the moral hazard problem at the micro-level. Moreover, commitment is somewhat different than altruism and reciprocity. Firstly, altruism theories predict a negative relationship between the contributions of others and those of the individual (Charles R. Plott, Vernon L. Smith, 2008). In contrast, commitment theories predict a positive relationship between the contributions of others and those of an individual. Secondly, the principle of reciprocity states that an individual must contribute when others are contributing, thus no cheap or “free rides” are permitted (Rachel T.A. Croson, 2007). However, commitment does not ask for the condition of others’ contributions, thus allowing for “side-bets” (Becker, 1960).

The goal of this paper is to extend the classic principal-agent model, by incorporating commitment, in which the principal and agent’s utility depends not only on wage but also on commitment. The model illustrates that commitment is an effective source of incentive under some conditions. We integrate the notion of commitment in our model, where a committed agent’s utility also includes the benefit of an organization. As with regard to the principal, his utility is not only
related to his own profit but also related to the agent’s payoff, due to
organizational support. Under the action of commitment, the agent’s utility
changes instead to the sum of his own utility, which comes from payoff, and the
expected profits multiplied by the commitment coefficient. In turn, the principal’s
utility changes to the sum of his own utility which comes from profit, and also the
agent’s payoff multiplied by the support coefficient. We further will deduce
optimal compensation. Then we will discuss how commitment affects the
motivation of the agent and the structure of optimal compensation. Combined
with the reality of management, we give an analysis of employee incentives in
various situations. The model which is discussed in this paper overcomes some
limitations of the compensation contract deduced from the classic model, which
must be solely carried out by wage differentials to inspire high employee effort.
On one hand, the optimal compensations of this model indicate that if employee
incentive is going to function well, it should not rely solely on monetary
compensation schemes, but should also rely on employee commitment and
organizational support. An employee who is committed to the organization needs
less monetary incentives to excel in job performance than one who is not
committed. On the other hand, the optimal contracts are inclined to pay an
undifferentiated wage when employee commitment is high enough. Additionally,
the employee commitment can also reduce the gap between differentiated wages
when they need to be set. We find that commitment and wage differential are
virtually interchangeable in the optimal incentive design. These characteristics of
optimal compensation are useful in management practice. Managers need to have
a better understanding of how to incentivize committed employees and what is the

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1 Employee commitment and organizational support is a pair of relative concepts in our paper. They have
been proved to coexist in many studies. There is usually a positive relationship between them. Organizations
staffed by committed employees often support their employees. For committed employees’ potential benefits
to organizations, we argue that the ability of organizations to hire workers who commit to them and the
creation of such commitment are central to organizational efficiency.
optimal compensation to committed employees, and make a distinction among different cases, avoiding the loss of committed employees.

The remainder of the paper is organized as follows. In Section 1, we discuss employee commitment in more detail and how commitment affects work incentives. In Section 2, we lay out the basic model and study optimal contracts. Section 3 explores applications of the model, and Section 4 offers our main conclusions.

1 Commitment and Incentives

Commitment is regarded as an attitudinal variable for most management scholars, which is characterized by an enduring psychological attachment (e.g. Mark John Somers and Dee Birnbaum, 1998). It can be conceptualized as a construct with multiple foci, such as organization commitment, supervisor commitment, occupational commitment, workgroup commitment. Organization commitment has been researched most widely and deeply amongst all of the constructs. A case is made here with reference to organizational commitment. It is necessary to explain that if another focus of commitment is substituted for organization, the interaction mechanism between commitment and incentive is nearly the same.

Studies on commitment over the previous decades have flourished. The most accepted definition of organization commitment is regarded as an emotional attachment to an organization that includes acceptance of organizational values and a desire to remain with the organization, which was put forth by Porter et al. (1974) in initial studies. Arnon E. Reichers (1985, p.468) reconfirms the view of Porter et al. (1974) that commitment consists of “a.) a belief in and acceptance of organizational goals and values, b.) the willingness to exert effort towards organizational goal accomplishment, and c.) a strong desire to maintain
organizational membership”. Meyer and Allen (1984, 1997) consider this definition as one dimension of organization commitment, which was termed affective commitment to the organization. In addition, Mowday, Porter and Steers (1982)’s study is also consistent with the earlier definition. They present this interpretation using the term attitudinal commitment, and maintain that it reflects how individuals identify with an organization, and that they are willing to work on its behalf. They relate attitudinal commitment to behavioral commitment and suggest there is a cyclical relationship between the two. Apart from these, numerous investigations about the development of the OCQ (organizational commitment Questionnaire) construct their work on the basis of the aforementioned widely-used understanding of the term, such as Allen and Meyer (1990), Olive (1990), Brown (1996), Meyer, Allen and Topolnytsky (1998), Meyer and Herscovitch (2001), and Aaron Cohen (2007). All of these studies on organization commitment consistently suggested that the core of commitment is the individual’s care for the benefit of the organization. Therefore, commitment is useful to economists because it suggests a natural means by which an individual’s preference can vary.

Individuals who commit to their organization always identify with the goals and values of that organization. They regard themselves as an insider of the organization, and enjoy being a member of it as such. Allen N. J. and Meyer J. P. (1990) argued similarly that they truly feel as if the organization’s problems are their own and as if they are “part of the family”. Mael F. A., & Ashforth B. E. (1992) also noticed that employees are interested in how others view their organization and consider the success of organization as their own. Begley & Czajka (1993) suggest that committed employees might experience a stronger sense of honor than those who are less committed. When someone praises the organization, it is perceived as a personal compliment. When someone criticizes
the organization, on the other hand, it feels like a personal insult. Organizations have a great deal of personal meaning for committed employees. They feel a strong sense of belonging to their organization. It is obvious that the benefits related to the organization will contribute to committed employees’ utility. Their utility will increase as the benefit of the organization increases, and decrease as the benefit of organization diminishes. Therefore, we can soundly consider that commitment plays an important role on the employee’s utility, similar to the work provided by George Akerlof & Rachel Kranton (2005), who added identity to the worker’s utility function. We try to combine both the economic and commitment components of utility to yield a formula that summarizes our discussion of these employees’ utility.

We argue that organization commitment which is characterized by the employee’s care about the benefit of organization will change the employee’s preference and affect his utility function. Considering commitment, the employee’s utility will not only rely on his own benefit but also on the benefit of the organization. That is, the employee’s utility includes two parts, one brought about by his wage and the other brought about by the profit of organization. We use $w$ to denote the employee’s wage, use $\beta$ to measure the employee commitment level, and use $\pi$ to denote the profit of organization. We can express the employee’s utility function as follows:

$$U^a = U^a_1(w) + \beta U^a_2(\pi)$$ (1)

The utility function in the formula above reflects that a committed employee is motivated not only by monetary compensation but also by his commitment. The tradeoff between maximizing his own benefit and organization’s profits is dependent upon the employee commitment.

Organizational support is a construct which is closely related to organization commitment. They are coexisting or simultaneous in organization for the long
Organizational support and organization commitment are interdependent and interact with each other, even at times caused and effected by one another. Scholars engaged in organizational behavior research termed the phrase Perceived Organizational Support (POS) in their literature. Earlier work by Buchanan (1974) described organizational support simply as the organization recognizing employees’ contributions and fulfilling promises to them. In subsequent research, Eisenberger et al. (1990) develop more precisely the content of organizational support, specifying three-fold the ways through which the organization: a.) provides employees with needed support, b.) values employees’ contribution, and c.) cares about employees’ well-being. The theoretical work of both Buchanan (1974) and Eisenberger, Fasolo and Davis-LaMastro (1990) observed a positive relationship between POS and organization commitment. Further, in their research on commitment, Meyer, Allen, and Gellatly (1990) also show that organizational “dependability” enhances organization commitment by the organization’s sharp willingness to fulfill its promises towards employees. Some other empirical studies have also shown that perceived organizational support was strongly related with organizational commitment. For example, O’ Driscoll and Randall (1999) have demonstrated that perceived organizational support was significantly linked with organizational commitment, as examined by the study of samples of dairy workers in Ireland and New Zealand. The verification of Organizational Commitment develops when interrelated Organizational Support is increased, as

\[ \text{term}^2 \]

Many studies on organizational behavior use two potent theories to explain this phenomenon. One is social exchange theory; the other is role identity theory. The former states that the transactional quality of social interactions is based on the fact that individual actors are self interested and instrumental. Actors are motivated to enhance their own rewards, and their concern about others’ rewards is contingent on whether that concern serves their own self-interest. It means the stable employee-organization relation is also based on both parties caring about each other. And both parties are likely to pursue more social exchange, not only economic exchange. The latter states that the individual, on a certain role, who must be able to rely on the reciprocity and exchange relation with its counter role (Haslam, 2001). From this perspective, interrelated individuals perform unique but integrated activities, and the meaning and expectations are tied to each of these roles, regarding performance and the relationships between the roles. That is, the employee and organization are a pair of counter roles, who are tied to each other, and care more or less about each other’s benefit. It is typically regarded as the Organizational Support and employee Commitment.
seen in numerous literature in both theoretical research and empirical studies, such as those mentioned above (e.g. O’Driscoll and M. Randal (1999); Eisenherger, Fasolo and Davis-LaMastro (1990)). To summarize, most of the studies found that a stable level of employee commitment coexists with the support of the organization. It is thus logical to assume a close relationship between organizational support and employee’s organizational commitment based on evidence from a large body of research. Therefore, with regard to the employee commitment, we must at the same time take into consideration the effect of organizational support.

Considering the significant relationship between commitment and organizational support as previously demonstrated, we must now define more specifically the meaning of organizational support. The term is regarded as describing the extent to which the organization values employees’ contributions and cares about their well-being (Eisenberger, Fasolo, & Davis LaMastro, 1990; Eisenberger, Huntington, Hutchison, & Sowa, 1986). We take this definition as the basis for our understanding of organizational support. Whitener (2001) also agrees with this definition, elaborating that an organization’s care and concern about the well-being of an employee will convey information about the organization’s benevolence and good will leading to perceptions of its trustworthiness in the eyes of the employee. The intrinsic meaning of organizational support as is to “fulfill its exchange obligation of noticing and rewarding employee efforts made on its behalf” (Eisenberger et al., 1990, p. 57). This idea expressly unifies the above descriptions, and thus the concentrated expression of organizational support can focus on the concern with employee’s benefit, organizational support may be expressed as that organization’s preference include caring about the employee’s benefit. The utility function of organization contains two parts. One part is the utility from the profit of an organization, and
another part is the utility from the benefit of the employee. Specifically, the latter consists of the utility from payoff and the disutility from the cost. We use $v$ to denote the employee’s utility from payoff, use $\alpha$ to denote the organization’s support, and use $g$ to measure the cost of the employee’s effort level. We can express the organization’s utility function as follows:

$$ U^p = U^p_1(\pi - w) + \alpha U^p_2(v, g) $$  \hspace{1cm} (2) 

When both the organization and the employee's utility function are analyzed as above, it is obvious that commitment and organizational support will affect incentive's effectiveness. Further, it will affect the design of optimal compensation. We argue that the optimal compensation paid to committed employees depends on both the agent’s commitment level and principal’s support level. We will lay out the extended principal-agent model considering commitment and study optimal contracts in the following section.

2 The Model

2.1 The Environment and Preferences

A “firm” consists of a risk-neutral principal and a risk-averse agent. The principal needs the agent to carry out a project. The project’s outcome can be “success” or “failure”: if the outcome is “success”, the principal receives the project profit $\pi$, $\pi > 0$; if the outcome is “failure”, the project profit is 0.

The probability of the success outcome is dependent on the effort supplied by the agent, $e$, at a cost, $g(e)$. Effort has two levels, “high” or “low”, $e \in \{e_H, e_L\}$. Effort is unobservable and hence non-contractible. If the effort supplied by the agent is “high”, the probability of the success outcome is $p_1$. In contrast, if the effort supplied by the agent is “low”, the probability of the success outcome is $p_2$, $p_1 > p_2$. 

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If the project profit is $\pi$, the principal pays to agent $w_1, w_1 \geq 0$; if not, the project profit is 0, the principal pays to agent $w_2$, which implies that the agent has to be given payment level at least of $w_2$ every period, irrespective of performance, $w_2 \geq 0$. And we assume $w_i \geq w_2$.

Consider the agent who has commitment and the utility function as equation (1). More specifically, we can think of that the utility of such agents depends on their own payoff, and the expected organizational benefit. We will refer to the parameter $\beta$ as the agents’ commitment level, $\beta \geq 0$. Although their utility still depends positively on their own income and negatively on effort, they are motivated not only by their own income, but also by caring intrinsically about the benefit of the organization. The utility function of the agent is comprised of three parts. The first part is the utility from private income which is noted by $v(w)$, satisfying $v'(w) > 0$, $v''(w) < 0$, $v(0) = 0$, and also Inata condition $\lim_{w \to -\infty} v'(w) = +\infty$, and $\lim_{w \to +\infty} v'(w) = 0$. The second part is the disutility from the cost of effort which is denoted by $g(e)$ . The third part is the utility from commitment, which is expressed by the product of $\beta$ and the benefit of the organization. Based on the above assumptions, the agent’s utility function can be expressed as

$$U^a = p_i v(w_i) + (1 - p_i)v(w_2) - g(e) + \beta p_i \pi, i = 1, 2.$$

Consider the principal who is the representative of the firm. The principal would like to support the agent and has a utility function as equation (2). More specifically, we can think of that the utility of such principal depends on both the
final profit and the payoff of the agent. We assume the parameter $\alpha$ as the support level of principal, $\alpha \geq 0$. Based on the above assumptions, the principal’s utility function is

$$U^\rho = p_i(\pi - w_i) + (1 - p_i)(-w_2) + \alpha[p_i v(w_i) + (1 - p_i)v(w_2) - g(e)], i = 1,2.$$

The principal’s optimal contracting problem solves:

$$\max_{w_1, w_2} p_i(\pi - w_i) + (1 - p_i)(-w_2) + \alpha[p_i v(w_i) + (1 - p_i)v(w_2) - g(e)], i = 1,2 \quad (3)$$

Subject to:

$$\begin{align*}
  p_i v(w_i) + (1 - p_i)v(w_2) - g(e) + \beta p_i \pi & \geq \bar{u} , i = 1,2 \quad (PC) \\
  p_i v(w_i) + (1 - p_i)v(w_2) - g(e) + \beta p_i \pi & \geq p_i v(w_i) + (1 - p_i)v(w_2) - g(e) + \beta p_i \pi \\
  i = 1,2, e \in \{e_{\mu}, e_L\} \quad (IC)
\end{align*}$$

The equation (PC) is the participation constraint of the agent. We take the agent’s reservation payoff $\bar{u} \geq 0$ to be exogenously given. The equation (IC) is the incentive-compatibility constraint, which stipulates that the effort level maximizes the agent’s utility given $(w_1, w_2)$.

We consider how the principal optimally implements each of the two possible levels of $e$ below, as the method in Hart and Holmstrom (1987), Holmstrom and Milgrom (1991). We first solve the optimal compensation scheme when implementing low effort. Second, we solve the optimal compensation scheme when implementing high effort.

2.2 Implementing low effect $e_L$

Suppose the principal wishes to implement effort level $e_L$. In this case, $p_i = p_2$, and substituting this into the principal’s optimal contracting problem (3), and the two constraints. Letting $\lambda \geq 0$, $\mu \geq 0$ denote the multipliers on
constraints (PC) and (IC), respectively, \( w_1, w_2 \) must satisfy the following Kuhn-Tucker first-order condition\(^3\):

\[
v'(w_1) = \frac{p_2}{\alpha p_2 + \lambda p_2 + \mu p_2 - \mu p_1} \\
v'(w_2) = \frac{1 - p_2}{(\alpha + \lambda + \mu)(1 - p_2) - \mu(1 - p_1)}
\] (4) (5)

The following proposition 1 characterizes the optimal contract when the principal wishes to implement effort level \( e_L \). The proof of proposition 1 is presented in the Appendix.

**PROPOSITION 1:** Suppose the principal wishes to implement effort level \( e_L \). Letting \( \beta^*_{e_L} = \frac{g(e_L) + \bar{u} - v(w^*)}{\pi(p_1 - p_2)} \) and \( \beta^* = \frac{g(e_L) - g(e_h)}{\pi(p_1 - p_2)} \), the optimal contract is characterized by the following:

(a) When \( \beta < \beta^* \), the optimal contract is:

\[
\begin{align*}
\begin{cases}
w_1 = w_2 = v^{-1}\left(\frac{1}{\alpha}\right) = w^*, & \text{if } \beta > \beta_{L}^* \\
&w_1 = w_2 = v^{-1}[\bar{u} + g(e_L) - \beta p_2 \pi], & \text{if } \beta \leq \beta_{L}^*
\end{cases}
\end{align*}
\]

(b) When \( \beta \geq \beta^* \), the optimal contract does not exist.

There are some interesting insights into the role of the agent’s commitment in changing the level of optimal incentive pay. First, equilibrium solution does not exist when the agent’s commitment level is higher than a certain value (\( \beta \geq \beta^* \)). An employee would not engage in low effort work when his commitment is high enough.

Second, when implementing \( e_L \), the optimal compensation contract is characterized by undifferentiated wages, i.e. \( w_1 = w_2 \), irrespective of performance.

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\(^3\) The Kuhn-Tucker first-order condition is derived by examining

\[
\frac{\partial L}{\partial w_1} = -p_2 + \alpha p_2 v'(w_1) + \lambda p_2 v'(w_1) + \mu p_2 v'(w_1) - \mu p_1 v'(w_1) = 0
\]

\[
\frac{\partial L}{\partial w_2} = -(1 - p_2) + \alpha(1 - p_2)v'(w_2) + \lambda(1 - p_2)v'(w_2) + \mu(1 - p_2)v'(w_2) - \mu(1 - p_1)v'(w_2) = 0
\]
Consistent with the management practices, this case does not require pay-for-performance. It is worth mentioning that the equilibrium solution is always undifferentiated wages when implementing \( e_L \) in the classic principal-agent model, because the utility of agent at low effort is always higher than that at high effort under fixed wages and the incentive-compatibility constraint does not work. However, in our model, even with the condition of undifferentiated wages, the incentive-compatibility constraint may bind sometimes here. Because that the utility function of the agent includes the part, \( \beta p_1 \pi \), and \( p_1 > p_2 \). Therefore, the utility of the agent at low effort may be lower than that at high effort when the commitment is high enough.

Thirdly, when the agent’s commitment \( \beta \) is low enough (\( \beta \leq \beta_L^* \) or \( \beta < \beta^* \)), the optimal payment is \( w_1 = w_2 = v^{-1}[\bar{u} + g(e_L) - \beta \bar{p}_2 \pi] \). The participation constraint of agent is binding in this case. The optimal compensation is similar to the equilibrium solution of the classic principal-agent model. The undifferentiated payment is in relation with the agent’s commitment. The optimal wage will decrease when the agent’s commitment increases, while the optimal wage will increase when the agent’s commitment decreases. Specifically, when the agent’s commitment decreases to 0, the optimal payment is the same to the equilibrium solution of the classic principal-agent model.

Fourth, as the agent’s commitment is increasing into the interval \( \beta_L^* < \beta < \beta^* \), the undifferentiated payment will only depend on the principal’s support. The wage will increase as the principal’s support is increasing. It implies that the principal pays to agent entirely according to his willingness to support.
Both of the two constraints do not play a role on this optimal compensation solving. The principal maximizes his utility without any constraints here, which is impossible in the classic principal-agent model. That is, although the principal can go on with reducing the wage which could also be accepted by the agent, he would not like to do it because of his support to the agent. We can call this optimal compensation “Willingness Payment”. Obtaining the “Willingness Payment” calls for the agent’s commitment to not be lower than $\beta_L^*.$

We now offer a corollary of the above proposition, which are useful in understanding the effect of the principal’s support $\alpha$ on incentive wages design.

**COROLLARY 1:** Suppose the principal wishes to implement effort level $e_L$. As the principal’s support $\alpha$ is increasing, the agent’s commitment $\beta$ more likely fall in the interval $\beta > \beta_L^*$. And the possibility of obtaining equilibrium solution $w^*$ is increasing.

The Proof is simply put as below: Since $\beta_L^* = [g(e_L) + \bar{u} - v(w^*)]/(1/ p_2 \pi),$ and $w_1 = w_2 = v^{-1}(1/ \alpha) = w^*$, we have when $\alpha$ increases, $w^*$ will increase, and $\beta_L^*$ will decrease. It means the possibility of $\beta > \beta_L^*$ increases. So the possibility of obtaining equilibrium solution $w^*$ increases.

In conclusion, suppose the principal wishes to implement effort level $e_L$, the characteristics of equilibrium solutions are undifferentiated wages. There are three important properties of the solutions. First of all, the undifferentiated wages will increase as the principal’s support is increasing when the agent’s commitment higher than a certain level. Secondly, the undifferentiated payment is related to the agent’s commitment when the agent’s commitment is lower than a certain level, and the wage will decrease when the agent’s commitment is increasing. Thirdly, the possibility of obtaining the “willingness wage” will be increased as the principal’s support $\alpha$ is increasing. Besides, we also deduce that there does not exist an equilibrium solution when the agent’s commitment is too high here.
2.3 Implementing high effect $e_h$

Suppose the principal wishes to implement effort level $e_h$. In this case, $p_i = p_1$, and substitute this into the principal’s optimal contracting problem (3), and the two constraints. Also letting $\lambda \geq 0$, $\mu \geq 0$ denote the multipliers on constraints (P C) and (I C), respectively, $w_1$ and $w_2$ must satisfy the following Kuhn-Tucker first-order condition:\(^4\)

\[
\frac{1}{v'(w_i)} = \alpha + \lambda + \mu - \frac{p_2}{p_1}
\]

(6)

\[
\frac{1}{v'(w_2)} = \alpha + \lambda + \mu - \frac{1-p_2}{1-p_1}
\]

(7)

The following proposition 2 characterizes the optimal contract when the principal wishes to implement effort level $e_h$. The proofs of proposition 2 are presented in the Appendix.

In order to state proposition 2 clearly, we define $\beta = \beta_{H}$, when satisfy $p_i / v'(\hat{w} \times (1-p_i))/v'(\hat{w}) = \alpha$. In here, $v(\hat{w}) = \bar{v} - [p_2 g(e_H) - p_1 g(e_L)]/(p_1 - p_2)$, and $p_i / v'(w_i) + (1-p_i)/v'(w_2) = \alpha$, the function $p_i / v'(w_i) + (1-p_i)/v'(w_2)$ is monotone decreasing about $\beta$, so it exists the only solution to satisfy the equation.

We also define $w_1 = \bar{w}_1$ and $w_2 = \bar{w}_2$. They are the solutions when satisfy both of the equations $p_i / v'(w_i) + (1-p_i)/v'(w_2) = \alpha$ and $v(\bar{w}_i) - v(\bar{w}_2) = [g(e_H) - g(e_L)]/(p_1 - p_2)$.

**PROPOSITION 2:** Suppose the principal wishes to implement effort level $e_H$, letting $\beta'_{H} = [g(e_H) + \bar{v} - v(w')] / p_1 \pi$ and $\beta' = [g(e_H) - g(e_L)] / \pi(p_1 - p_2)$, the optimal contract is characterized by the following:

1. When $\beta > \beta'$, the optimal compensation is as below.

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\(^4\) The Kuhn-Tucker first-order condition is derived by examining

\[
\partial L / \partial w_1 = -p_1 + \alpha p_1 v'(w_1) + \lambda p_1 v'(w_1) + \mu p_1 v'(w_1) - \mu p_2 v'(w_1) = 0
\]

\[
\partial L / \partial w_2 = -(1-p_1) + \alpha (1-p_1)v'(w_2) + \lambda (1-p_1)v'(w_2) + \mu (1-p_1)v'(w_2) - \mu (1-p_2)v'(w_2) = 0
\]
\[
\begin{align*}
  w_1 &= w_2 = v^{-1}\left(\frac{1}{\alpha}\right) = w^* \quad \text{if } \beta > \beta_n^*
  \\
  w_1 &= v^{-1}(\bar{u} + g(e_U) - \beta p_1 \pi) \quad \text{if } \beta \leq \beta_n^*
\end{align*}
\]

(2) When \( \beta \leq \beta^* \), the optimal compensation is as below.

\[
\begin{align*}
  w_1 &= \bar{w}_1, \quad w_2 = \bar{w}_2 \quad \text{if } \beta \geq \bar{\beta}_n \\
  w_1 &= v^{-1}(\bar{u} - (1 - p_2)g(e_U) - (1 - p_1)g(e_L) - \beta \pi), \quad \text{if } \beta < \bar{\beta}_n \\
  w_2 &= v^{-1}(\bar{u} - \frac{p_2 g(e_U) - p_1 g(e_L)}{p_1 - p_2})
\end{align*}
\]

The first part of the proposition 2 shows that when the agent’s commitment is high enough (\( \beta > \beta^* \)), undifferentiated wages are set irrespective of performance. In this situation, if the agent’s commitment is higher than a certain level (\( \beta_n^* \)), the wage payment is only in relation to the principal’s support. If the agent’s commitment is lower than a certain level (\( \beta_n^* \)), the wage payment is in relation to the expected profit of organization and agent’s commitment. The undifferentiated wage will decrease if the expected profit of the organization and the agent’s commitment are increasing. It is impossible to obtain the equilibrium solution characterized by undifferentiated wages when implementing high effort in the classic principal-agent model. However, it could attain here when the incentive-compatibility constraint does not bind, even if the participation constraint also does not bind, because of the effect of the principal’s support and the agent’s commitment. This is due to that the agent always enjoys implementing high effort when the agent’s commitment high enough, or although the constraints still allows the payment to go on to decrease, the principal does not want to do that and would rather pay the “willingness wage”. 

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The second part of Proposition 2 shows that when the agent’s commitment is lower than a certain level \((\beta \leq \beta^*)\), the differentiated wages are linked to the performance. In the case of differentiated wages, when \(\beta < \tilde{\beta}_H\), the optimal payment \(w_1\) changes not only rely on the expected profit of organization but also rely on the agent’s commitment. The optimal payment \(w_1\) will decrease when \(\beta\) or \(p_1\pi\) is increasing. When the agent’s commitment level belongs to the interval, \(\tilde{\beta}_H \leq \beta \leq \beta^*\), low wage \(\bar{w}_2\) is lower than \(w^*\) and high wage \(\bar{w}_1\) is higher than \(w^*\).

We have \(\beta^*\) deduced from the incentive-compatibility constraint, which corresponds to the lowest commitment level required by the condition of the agent’s utility with high effort surpassing that with low effort under undifferentiated wages. \(\beta^*_H\) corresponds to the lowest commitment level which ensures implementing high effort of the agent under the principal’s “willingness wage”.

Proposition 2 summarizes two different cases of implementing high effort level. When the agent’s commitment is higher than the lowest commitment level which satisfies the incentive-compatibility constraint, the agent always chooses high effort no matter which kind of undifferentiated wages. It is only needed to consider the participation constraint in this situation. If the agent’s commitment is higher than the lowest commitment level which ensures implementing high effort under the principal’s “willingness wage”, the optimal compensation is always the “willingness wage”. If not, the optimal compensation is the undifferentiated wages when the participation constraint is binding. When the agent’s commitment
is lower than the lowest commitment level which satisfies the incentive-compatibility constraint, the optimal compensation is always differentiated wages. That is, in this situation, only differentiated wages can satisfy the incentive-compatibility constraint.

We now offer two corollaries of the proposition 2, which are useful in understanding the effect of the principal’s support $\alpha$ on incentive wages design.

**COROLLARY 2:** Suppose the principal wishes to implement effort level $e_H$. As the principal’s support $\alpha$ increasing, agent’s commitment $\beta$ are more likely fall into the interval $\beta > \beta_H^*$. And the possibility of obtaining the equilibrium solution $w^*$ is increasing.

The Proof is simple as below: Since $\beta_H^* = \left[ g(e_H) + \bar{v} - v(w^*) \right] / p_i \pi$, and $w_1 = w_2 = v^{-1}(1/\alpha) = w^*$, we have when $\alpha$ is increasing, $w^*$ will increase, and $\beta_H^*$ will decrease. It means that the possibility of $\beta_H^* \leq \beta^*$ is increasing, and also means that the possibility of $\beta > \beta_H^*$ is increasing. So the possibility of obtaining equilibrium solution $w^*$ is increasing.

**COROLLARY 3:** Suppose the principal wishes to implement effort level $e_H$. As the principal’s support $\alpha$ is increasing, agent’s commitment $\beta$ more likely fall into the interval $\beta \geq \bar{\beta}_H$, in which it always can obtain equilibrium solution $w_1 = \bar{w}_1$, $w_2 = \bar{w}_2$. And the possibility of obtaining this equilibrium solution will increase.

The proof is simple as below: Since $\beta = \bar{\beta}_H$ satisfy $p_i / v'(\hat{w}_1) + (1 - p_i) / v'(\hat{w}_2) = \alpha$, and $p_i / v'(w_1) + (1 - p_i) / v'(w_2)$ is a monotone decreasing function about $\beta$, so when $\alpha$ is increasing, $w_i$ will increase, and
\( \tilde{\beta}_{H} \) also will decrease. It means that the possibility of \( \beta \geq \tilde{\beta}_{H} \) is increasing. So the possibility of obtaining equilibrium solution \( w_{1} = \bar{w}_{1}, w_{2} = \bar{w}_{2} \) will increase.

In conclusion, contrasting the results of the first part of proposition 2 with the second part of proposition 2, it yields interesting insights into the role of agent’s commitment in the changing of the pattern of incentive pay, and its role in changing the level of optimal incentive pay. The equilibrium solution to implement high effort is also undifferentiated wage irrespective of performance when the agent’s commitment is high enough. The equilibrium solution to implement high effort is differentiated wage when the agent’s commitment is lower than a certain level, however, the disparity between differentiated wages will reduce when the employee commitment is increasing.

2.4 Summary of the optimal contract

Given the preceding analysis, which kind of effort that the principal should choose to inspire depends on the dispersion of the expected utilities under different effort levels. There are two cases here\(^5\). When the agent’s commitment is higher than the lowest commitment level which satisfies the incentive-compatibility constraint, the principal’s optimal choice is inspiring high effort, and the optimal contract is undifferentiated wage. When the agent’s commitment is lower than the lowest commitment level which satisfies the incentive-compatibility constraint, the principal’s optimal choice depends on the comparison of expected utilities under two effort levels. This is related to the

\(^{5}\) It is obviously deduced from proposition 1 and 2.
expected profit, wage, probability of different profit, the principal’s support, the cost of effort, and also the agent’s commitment. Although we cannot precisely point out the principal’s optimal choice, we have clearly analyzed the relationship between commitment and optimal compensation. The characteristics of optimal compensation in each possible effort level are enough to illustrate the relations between the agent’s commitment and payment, which is exactly the central question that concerns us.

We summarize several important results from the optimal compensation payment when implementing either of the two possible effort levels. First of all, the characteristics of optimal payment illustrate that commitment is an effective motivator. In the classic principal-agent model, the principal must rely on the wage differential to inspire the agent to exert high effort. But here, undifferentiated wage also can inspire the agent to exert high effort when the agent’s commitment is high enough. Moreover, the agent would not implement low effort when his commitment is high enough.

Second, commitment and wage differential are substitutable to each other in the optimal incentive compensation design. Commitment can compress the wage differential. Compared to the classic principal-agent model, our model can realize effective incentive without expanding the wage differential. A committed employee needs less monetary inducement to perform his job well than one who has no commitment. Commitment helps to solve the incentive problem that differentiated wage could not resolve. Moreover, the differentiated wages usually incur efficiency loss induced by more income risks, given the risk aversion
preference of the agent. It means the principal needs to pay more expected wages to risk-averse agents. More than that, it is difficult to set effective differentiated pay in many situations of practical management. For example, differentiated wages are always based on observable performance. When the performance is hard to observe or the real profit is difficult to evaluate, it is impossible to set differentiated pay accurately and efficiently. More than that, differentiated pay may have some negative impacts, bringing about such occurrences as over-competition and interpersonal disharmony. This point of view is supported by Lazear (1989), who demonstrated that workers have the incentive to sabotage one another when compensation incentive intensity is tight on the basis of relative performance. In the above cases, we need pay more attention to the function of commitment to achieve more effective incentive scheme. Conversely, when it is hard to inculcate and promote employee commitment, we need to emphasize more the effectiveness of differentiated pay.

Third, commitment is not always good for organizational efficiency when the increase in employee commitment totally relies on the principal’s support. That is, when employee commitment is higher than the lowest commitment level which satisfies the participation constraint, the optimal compensation is the principal’s “Willingness wage”. Even though the constraint allows the payment to decrease even lower than “Willingness wage”, but due to the principal’s support, the optimal payment is not lower than “Willingness wage”. Furthermore, the “Willingness wage” will increase as the principal’s support is increasing, which means efficiency loss. So, although the organization’s support always improves
employee commitment and then may help to improve organizational efficiency, it may reverse in this situation.

3 Applications

The benchmark for our analysis is the case where the principal provides support to the agent and the agent is committed to the principal. The model describes how to motivate committed agents, and explains why committed agents are met with great favor in all kinds of organizations. A number of recruitment methods in recent years have leaned toward screening employee commitment, and investments have leaned toward developing employee commitment not only in for-profit but also in nonprofit organizations (Paul Iles, Christopher Mabey, Ivan Robertson, 1990; Greguras Gary J., Diefendorff James M., 2009). The model developed here is well placed to consider employee motivation. In this section, we discuss several main contexts in which the ideas apply. We begin with a discussion of a kind of organization staffed by committed employees. We then discuss the case of wages differential constraints. Finally, we discuss excessive welfare and some other issues.

3.1 Nonprofit Organizations

Nonprofit organizations are often staffed by committed agents. They aim to provide collective goods centered around a specific goal, such as Environmental Protection, Charity, and Relief Agencies. Specifically, we refer to organizations which are focused on humanitarian efforts, such as the Red Cross, Doctors without Borders, and others. Workers employed in these sectors are typically
committed employees, who inevitably identify with the organization’s goals and values, and are willing to be participants within the organization, and therefore put forth extra effort to achieve organizational goals (José Alatrista, James Arrowsmith, 2004). Burton A. Weisbrod (1988) observes that workers in nonprofit organizations have quite different motivations from those who work in for-profit organizations, and they also have different work goals. Specifically, the goals of workers in nonprofit organizations are more directly linked to the goals of the organization as a whole. Furthermore, the nonprofit organization model, which is developed by Glaeser (2002) states more precisely that employees cared directly about the output of their organization. All of the above research emphasized that employees in nonprofit organizations are different from those in the for-profit sector, and thus means of incentivizing these two types of employees will be quite different.

Many studies on nonprofit organizations support our argument. On one hand, workers employed in nonprofit organizations tend to earn lower wages than in other types of organizations. Preston (1989) analyzed the wages of employees working in different types organizations in the United States. He found that managers and professionals employed by nonprofits earned 20% less than their counterparts in for-profits, after controlling human capital, demographic structure, occupation, flexibility and rigidity of work schedules. Other studies such as Linden, Pencil, and Studley (1989) and Studley (1989), which focused solely on lawyers, also found that the fixed wages in nonprofit law firms are almost 40% lower than in private practices. This number is as high as 66% for New York City
lawyers working in large non-profits, compared with those employed in for-profit organizations. A later investigation controlling employee gender, choice of curriculum, and academic performance, Frank (1996) similarly found that the salary of graduates who were employed in nonprofit organizations is about 59% less on average than those who were employed in for-profit organizations, based on an employment survey conducted on Cornell University graduates. In view of all of the above evidence, Femida Handy & Eliakim Katz (1998) summarize that nonprofit organizations tend to pay their employees a lower wage than for-profit organizations. On the other hand, Sherwin Rosen (1986) suggested that different types of organizations may have different preferences in choosing wage differentials. His theoretical viewpoint has become dominant throughout the field. Many empirical studies show that the wage differential in nonprofit organizations is markedly different from other types of organizations (Laura Leete, 2000, 2001).

The classical model demonstrates that the principal always needs to pay higher wages and expand wage differentials to inspire high effort. In reality, nonprofit organizations often pay lower wages than for-profit organizations, and also do not set strong incentives for their employees. We can therefore see that the classic principal agent model does not explain the phenomenon cited above. Our framework in this capacity can provide an explanation and shed some light onto previous discrepancies. Most studies in labor literature may ascribe small wage differentials to the character of organizational culture, lack of funding and non-profit orientation (Rosemarie Emanuele, Susan H. Higgins, 2000; J. Cheverton, 2007). However, our model illustrates the main reason for this phenomenon is the
existence of commitment, which can inspire employees to implement high effort in these organizations, and thus they do not need such large wage differentials. We can thus see that commitment is an effective motivator for employees in these organizations, and monetary incentives alone are not sufficient in motivating a higher caliber of work. Our model also explains in particular how to motivate those employees who care about the organization’s benefit in nonprofit organizations. Although nonprofit organizations often attract employees who identify with their goals and values, however, if the organizations ignore support for their committed agents in the long term, they also may face difficulty in recruiting more committed agents or could also lower the motivation of those already employed in the organization.

3.2 Wages differential constraints

External forces have at times placed constraints on wage differentials. This can occur for a variety of reasons. First, the law may impose high income tax in some countries, especially in the high welfare states of Europe. The strength of labor unions who constrain wage differentials is another example of the influence of an external force. Organizations in those contexts are unable to carry out high-strength efficiency wages. For example, Mahmood Arai (1994) maintained through his research that the wage differentials of the Swedish labor market are not as large as those in the U.S. Second, nearly all efficiency wage schemes should be based on performance measures that are observable. However, we cannot often find performance measures that perfectly coincide with the agent’s effort, which may present us with serious problems. As George Becker (2000)
stresses, incentive contracts may result in negative behavioral consequences when an organization has to use the distorted performance measure. Moreover, many performance measures cannot be precisely evaluated. An example of the difficulties in evaluating performance measure was demonstrated by Holmstrom (1982), Baker (1992) and McLaughlim (1994), who state that agents’ effort is hard to indicate in teamwork. If we must use subjective evaluation, other problems may arise, such as discrimination (Bentley MacLeod, 2003). We can deduce therefore the wage differentials is restricted by performance measure, which itself may be distorted, and also due to its inability to be precisely evaluated. Third, efficiency wage carried out through wage differentials may also bring forth some negative effects in actual operation, for example, over incentive. A series of problems such as vicious competition, disharmony, and noncooperation may occur due to this practice. As mentioned in the previous section of this paper, the work of Holmstrom and Milgrom (1991), Lazear and Rosen (1981) and Lazear (1989) support this argument, discussing the problems of excessive incentive in multiple task and tournament situations. In order to avoid these negative effects on organization, wage differentials may be used in restriction. The three reasons cited above reveal that it is imperative in some situations for organizations to pay attention to inculcating employee commitment. It may be the main reason why some organizations take employee commitment seriously while others do not.
3.3 Excessive Compensation

According to our model, when employee commitment reaches a certain level, the principal will pay the willingness wage to the agent, but this willingness wage is not necessarily so high, thus resulting in excessive compensation. According to generally accepted principles of management, one of the most important reasons to improve an employee’s well-being is to enhance the employee’s motivation. Many companies have attached importance to rewarding employees in recent years. They carry out versions of welfare programs, and increasingly, kinds of profit sharing schemes have been used to encourage employees. Organizations spare no effort in their quest to further employees’ well-being and anchor their hope on inspiring the employee to work hard. The enhancement of employees’ welfare clearly reflects the increase of organizational support.

However, our model reveals that such organizational support which is mainly pushed forward by enhancing employees’ welfare is not always effective for improving organizational efficiency. This occurs due to the fact that if increasing employees’ welfare can improve employee commitment, it is effective in inspiring the employee, while if not, it is not effective. In actuality, when the employees’ welfare has already been at a relatively high level, its increase cannot improve the employee commitment effectively, and cannot push forward organizational efficiency any more. Thus when the increase of employee commitment occurs at a slower rate than the increase of organizational support, this leads a situation in which the increase of employees’ welfare is not very effective in inspiring employees, but instead only increases the expenditure of the
organization. Increasing employees’ welfare is insignificant for improving organizational efficiency in this case, and it is a waste of the organization’s resources.

Although organizational support plays a central role in arousing employee commitment in most cases, it may also sometimes lead to excessive compensation. There are many non-effective employee welfare programs in practice. Take Total Reward for example, which is a complete salary system of return to employees, offered by the America Compensation Association (Robert L. Heneman, 2002). Total Reward is theoretically quite useful in inspiring employee commitment as it is been effective in implementing organizational support mechanisms. However, some cases of its application in practice have already been found to be ineffective in improving organizational efficiency (A. Verbruggen, 2006). Our finding may explain the discrepancy between theory and practice in some cases, and reveal the root cause of these practical failures, which is that increased compensation plays no role in improving incentive, and is also useless for organizational efficiency. Our model describes the mechanisms of organizational support that best encourage employee commitment, and is therefore a practical tool in determining which level of support is necessary in any specific case. It further provides a good reference for designing the optimum welfare system for the organization.

3.4 Other Issues

Our model also can be expanded to other cases in which the employee’s utility is related to organizational profit. For instance, we can interpret the
commitment parameter in our model as the employee’s share of the organization’s benefit. The usage of this new interpretation will account for Partnership, Employee Stock Ownership, Occupational Pension Systems, Profit-Sharing Programs and so forth. In start-up companies, the creation of a company often begins with an effective entrepreneurial team who demonstrate characteristics of industriousness, loyalty, solidarity and cooperation under a high risk environment. These team members are so passionate and highly motivated because they share the benefits of the organization after its success, and are simultaneously deeply committed to the company. Although given a low fixed wage in the initial stage of the company's development, the members will always work hard and are incentivized by the increasing potential benefit of the company. It can also feasibly explain why people take pleasure in engaging in pioneering ventures, even to the extent of removing themselves from other potential chances for high salaries in low-risk situations.

4 Concluding Remarks

In recent years, an increasing number of researchers working on employee incentive have incorporated psychological characteristics into economic models as sources of intrinsic motivation. Commitment is typically included as one form of intrinsic motivation. Regarded as a psychological attachment to the organization, commitment prominently changes the employee's preference, which is demonstrated through the employee’s care about the benefit of the organization via his utility function. From basic sociological theories including the principle of
social exchange and reciprocity, we can deduce that organizational support usually coexists with commitment.

This paper provides a simple model to explain how employee commitment has an effect on optimal compensation and how to incentivize committed employees. There are three main results that can be deduced from our model. First, we can see that commitment is an effective motivator. Committed employees are more inclined to work hard, and need less financial incentive than non-committed employees. When sufficient commitment exists, a fixed wage may be enough to inspire employees to put forth high effort. Second, commitment can be a substitute for wage differentials in playing the role of an incentive. Wage differentials can be reduced in the optimal incentive design when employee commitment exists. So, a more compressed optimal compensation scheme will be provided to committed employees. Third, an increase in commitment may even have some negative effects on organizational efficiency when the principal pays a "willingness wage" to the agent. In this situation, "willingness wage" loses its effect on incentive and is beyond the minimum necessary wage used to inspire high effort.

These findings provide a potentially instructive way to cover the shortage of explicit incentive schemes, which solely rely on financial incentive that may undermine the real benefit of an organization in the long run. The advantage of our model is in considering commitment as an intrinsic motivation and deducing its potential effect on optimal compensation, which is more consistent with actual situations in management. Therefore, our model well-explains why it is
unnecessary to use wage differentials to inspire employee in nonprofit organizations, and also provides a viable solution in the case of restricted wage differentials. Moreover, this model reminds organizations to be careful to avoid excessive welfare, as in many cases increased welfare is misperceived to also increase employee motivation.

Further discussions of this model may proceed in at least two ways. In the first, we have seen that the commitment parameter in our model is exogenous. In future work, it would be of great interest to develop the commitment parameter as endogenous, to describe the generation of commitment in more detail, and to understand how commitment interacts with the governance and culture of organizations. The second way of developing the analysis of this model is to conduct empirical research and illustrate quantitative evidence. Both psychological questionnaire surveys and methods of experimental economics can be utilized to push forward these related research endeavors. In this paper and in future discussion and analyses of this model, we must always consider methods of inspiring committed employees as a main focus.

**APPENDIX**

**PROOF OF PROPOSITION 1**

Suppose the principal wishes to implement effort level $e_L$. The principal’s optimal contracting problem solves:

$$\max_{w_1, w_2} p_2 (\pi - w_1) + (1 - p_2) (-w_2) + \alpha [p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L)]$$

Subject to:

$$p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi \geq \bar{u}$$

(P C)
\[ p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi \geq p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi \]  
\text{(IC)}

As mentioned in section 2.2, we let $\lambda \geq 0$, $\mu \geq 0$ denote the multipliers on constraints (PC) and (IC), respectively. We have Lagrange Equation as follow:

\[ L = p_2 (\pi - w_1) + (1 - p_2)(-w_2) + \alpha [p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L)] + \lambda [p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi - \mu] + \mu [p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi + \mu ([p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi])

The solutions of the principal’s optimal contracting problem, $w_1$ and $w_2$, must satisfy the following Kuhn-Tucker first-order condition:

\[ \partial L / \partial w_1 = -p_2 + \alpha p_2 v'(w_1) + \lambda p_2 v'(w_1) + \mu p_2 v'(w_1) - \mu p_1 v'(w_1) = 0 \]
\[ \partial L / \partial w_2 = -(1 - p_2) + \alpha (1 - p_2) v'(w_2) + \lambda (1 - p_2) v'(w_2) + \mu (1 - p_2) v'(w_2) - \mu (1 - p_1) v'(w_2) = 0 \]

These conditions can simplify as equations (4) and (5) as mentioned in section 2.2.

To prove Proposition 1, we distinguish four cases according to either the participation constraint or incentive-compatible constraint binding or not.

(a) When $\lambda = 0$, $\mu = 0$, the optimal contract must satisfy the following three conditions.
\[ p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi > \bar{u} \]  
\text{(1.1)}
\[ p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi > p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi \]  
\text{(1.2)}
\[ v'(w_1) = v'(w_2) = \frac{1}{\alpha} \]  
\text{(1.3)}

Substituting $\lambda = 0, \mu = 0$ into the Kuhn-Tucker first-order condition, we get equation (1.3). From (1.3), we get the optimal compensation $w_1 = w_2 = v'^{-1}(1 / \alpha) = w^*$. Substituting $w^*$ into (1.1), we have $v(w^*) - g(e_L) + \beta p_2 \pi > \bar{u}$. Letting
\[ \beta^*_L = \frac{1}{p_2 \pi} [g(e_L) + \bar{u} - v(w^*)] = \frac{1}{p_2 \pi} [g(e_L) + \bar{u} - v(v'^{-1}(1 / \alpha))] \]
and \( \beta^* = [g(e_H) - g(e_L)]/\pi(p_1 - p_2) \).

We can infer \( \beta > \beta_L^* \) from (1.1), and \( \beta < \beta^* \) from (1.2).

So, if \( \beta_L^* < \beta < \beta^* \), the optimal contract is \( w_1 = w_2 = v^{-1}(1/\alpha) = w^* \). If \( \beta_L^* \geq \beta^* \), the optimal contract does not exist.

It shows that if the equilibrium solution exist, the optimal contract is undifferentiated wage, irrespective of performance, when \( \beta_L^* < \beta < \beta^* \).

(b) When \( \lambda = 0, \mu > 0 \), the optimal contract must satisfy the following conditions.

\[
P_2v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2 \pi > \bar{u}
\]

\[
P_2v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2 \pi = p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H) + \beta p_1 \pi
\]

\[
\frac{1}{v'(w_1)} = \alpha + \mu(1 - p_1/p_2)
\]

\[
\frac{1}{v'(w_2)} = \alpha + \mu(p_1 - p_2)/(1 - p_2)
\]

Since \( p_1 > p_2 \), we have \( \mu(1 - p_1/p_2) < 0 \), and \( \mu((p_1 - p_2)/(1 - p_2)) > 0 \).

So we can infer \( w_1 < w_2 \) from equations (1.5) and (1.6), which contradicts with the assumption \( w_1 \geq w_2 \).

So this case cannot exist.

(c) When \( \lambda > 0, \mu = 0 \), the optimal contract must satisfy the following conditions.

\[
P_2v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2 \pi = \bar{u}
\]

\[
P_2v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2 \pi = p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H) + \beta p_1 \pi
\]

\[
v'(w_1) = v'(w_2) = \frac{1}{\alpha + \lambda}
\]

From (1.8), we can infer \( w_1 = w_2 \), so we get

\[
v(w_1) = v(w_2) = \bar{u} + g(e_L) - \beta p_2 \pi \text{ from (1.7), and } \beta < \beta^* \text{ from (1.4)}.
\]

As \( \beta \geq 0 \) and \( v(w) \geq 0 \), we get \( 0 \leq \beta \leq [\bar{u} + g(e_L)]/p_2 \pi \) from

\[
v(w_1) = v(w_2) = \bar{u} + g(e_L) - \beta p_2 \pi.
\]
Since (1.8) \( v'(w_1) = v'(w_2) = 1/(\alpha + \lambda) < 1/\alpha \), we can infer \( w_1 = w_2 > w^* \).

And combine this to the condition \( \beta^*_L = [g(e_L) + \bar{u} - v(w^*)]/p_2\pi \), so we can deduce that \( \beta \leq \beta^*_L \). It is obvious that \( \beta^*_L \leq [\bar{u} + g(e_L)]/p_2\pi \).

So we have: if \( \beta < \beta^* \leq \beta^*_L \) or \( \beta \leq \beta^*_L < \beta^* \), the optimal contract is \( w_1 = w_2 = v^{-1}[\bar{u} + g(e_L) - \beta p_2\pi] \). All of the above cases potentially satisfy \( \beta \leq [\bar{u} + g(e_L)]/p_2\pi \). The optimal contract relies on the expected profit and agent’s commitment.

(d) When \( \lambda > 0, \mu > 0 \), the optimal contract must satisfy the following conditions.

\[
p_v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2\pi = \bar{u}
\]

\[
p_v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2\pi = p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H) + \beta p_1\pi
\]

\[
\frac{1}{v'(w_1)} = \alpha + \lambda + \mu(1 - \frac{p_1}{p_2})
\]

\[
\frac{1}{v'(w_2)} = \alpha + \lambda + \mu(\frac{p_1 - p_2}{1 - p_2})
\]

Similarly as (b), we can infer \( w_1 < w_2 \) from (1.9) and (1.10), which contradicts with the assumption \( w_1 \geq w_2 \).

So this case cannot exist.

Summarize the above four cases, the optimal contract is characterized by the proposition 1 when suppose that the principal wishes to implement effort level \( e_L \).

Proof end.

**PROOF OF PROPOSITION 2**

Suppose the principal wishes to implement effort level \( e_H \). The principal’s optimal contracting problem under moral hazard solves:

\[
\text{Max}_{w_1, w_2} p_1(\pi - w_1)(1 - p_1)(-w_2) + \alpha[p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H)]
\]

Subject to:

\[
p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H) + \beta p_1\pi \geq \bar{u}
\]
\[ p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi \geq p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi \]

As mentioned in section 2.3, we also let \( \lambda \geq 0 \), \( \mu \geq 0 \) denote the multipliers on constraints (P C) and (I C), respectively. We have Lagrange Equation as follow:

\[
L = p_1(\pi - w_1) + (1 - p_1)(-w_2) + \alpha[p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H)] + \\
\lambda[p_1v(w_1) + (1 - p_1)v(w_2) - g(e_H) + \beta p_1\pi - \bar{u}] + \mu[p_1 v(w_1) + (1 - p_1)v(w_2) - g(e_L) + \beta p_2 \pi] \\
- [p_2v(w_1) + (1 - p_2)v(w_2) - g(e_L) + \beta p_2 \pi]}
\]

The solutions of the principal’s optimal contracting problem, \( w_1 \) and \( w_2 \), must satisfy the following Kuhn-Tucker first-order condition:

\[
\partial L / \partial w_1 = -p_1 + \alpha p_1 v'(w_1) + \lambda p_1 v'(w_1) + \mu p_1 v'(w_1) - \mu p_2 v'(w_1) = 0, \\
\partial L / \partial w_2 = -(1 - p_1) + \alpha(1 - p_1)v'(w_2) + \lambda(1 - p_1)v'(w_2) + \mu(1 - p_1)v'(w_2) - \mu(1 - p_2)v'(w_2) = 0.
\]

These conditions can simplify as equations (6) and (7) as mentioned in section 2.3. We distinguish four cases according to either the participation constraint or incentive-compatible constraint binding or not.

(a) When \( \lambda = 0 \), \( \mu = 0 \), the optimal contract must satisfy the following conditions.

\[ p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi > \bar{u} \quad (2.1) \]

\[ p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi > p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi \quad (2.2) \]

\[ v'(w_1) = v'(w_2) = \frac{1}{\alpha} \quad (2.3) \]

Substituting \( \lambda = 0, \mu = 0 \) into the Kuhn-Tucker first-order condition, we get equation (2.3). We can infer the equilibrium solution is \( w_1 = w_2 = v^{-1}(1/\alpha) = w^* \) from (2.3). Substituting \( w^* \) into (2.1), we have \( v(w^*) - g(e_H) + \beta p_1 \pi > \bar{u} \). Letting \( \beta_H^* = [g(e_H) + \bar{u} - v(w^*)]/p_1 \pi \), we can deduce \( \beta > \beta_H^* \) from (2.1), and \( \beta > \beta^* \) from (2.2).
So, when $\beta > \max\{\beta^*, \beta_H^*\}$, the optimal contract is $w_1 = w_2 = v^{-1}(1/\alpha) = w^*$. It shows that in this case, the optimal contract is undifferentiated wage, irrespective of performance. Compare to proposition 1, we can find that agent chooses different effort lever depend on his own commitment level under the same payment. The principal’s support level decided the optimal pay level totally.

(b) When $\lambda > 0$, $\mu = 0$, the optimal contract must satisfy the following conditions.

$$p_1 v(w_1) + (1-p_1)v(w_2) - g(e_H) + \beta p_1 \pi = \bar{u} \tag{2.4}$$

$$p_1 v(w_1) + (1-p_1)v(w_2) - g(e_H) + \beta p_1 \pi > p_2 v(w_1) + (1-p_2)v(w_2) - g(e_L) + \beta p_2 \pi \tag{2.2}$$

$$v'(w_1) = v'(w_2) = \frac{1}{\alpha + \lambda} \tag{2.5}$$

Since (2.5), we can infer $w_1 = w_2$. Substituting this into (2.4), we can get $v(w_1) = v(w_2) = \bar{u} + g(e_H) - \beta p_1 \pi$. As $\beta \geq 0$ and $v(w) \geq 0$, we can get $0 \leq \beta \leq [\bar{u} + g(e_H)]/ p_1 \pi$.

We also can deduce $\beta > \beta^*$ from (2.2) as the case (a).

Since $v'(w)$ is a strictly decreasing function, and combine with equations (2.3) and (2.5), we can infer $w_1 = w_2 > w^*$. And for $v(w)$ is a strictly decreasing function about $\beta$, and $\beta_H^* = [g(e_H) + \bar{u} - v(w^*)]/ p_1 \pi$, we have $\beta \leq \beta_H^*$ in this case. It is obviously that $\beta \leq \beta_H^* \leq [\bar{u} + g(e_H)]/ p_1 \pi$.

So, when $\beta^* < \beta \leq \beta_H^*$, the equilibrium solution is $w_1 = w_2 = v^{-1}([\bar{u} + g(e_H) - \beta p_1 \pi])$.

(c) When $\lambda > 0$, $\mu > 0$, the optimal contract must satisfy the following four conditions simultaneously.

$$p_1 v(w_1) + (1-p_1)v(w_2) - g(e_H) + \beta p_1 \pi = \bar{u} \tag{2.4}$$

$$p_1 v(w_1) + (1-p_1)v(w_2) - g(e_H) + \beta p_1 \pi > p_2 v(w_1) + (1-p_2)v(w_2) - g(e_L) + \beta p_2 \pi \tag{2.6}$$
\[
\frac{1}{v'(w_1)} = \alpha + \lambda + \mu - \mu \frac{p_2}{p_1}
\]
(2.7)

\[
\frac{1}{v'(w_2)} = \alpha + \lambda + \mu \frac{1 - p_2}{1 - p_1}
\]
(2.8)

We solve equations (2.4) and (2.6), and get

\[
\begin{align*}
v(w_2) &= \bar{u} - \frac{p_2 g(e_H) - p_1 g(e_L)}{p_1 - p_2} \\
v(w_1) &= \bar{u} - \frac{(1 - p_2) g(e_H) - (1 - p_1) g(e_L) - \beta \pi}{p_1 - p_2}
\end{align*}
\]
(2.9) (2.10)

Since \( v(w_1) \geq v(w_2) \), we can deduce \( \beta \leq \beta^* \) from equation

\[v(w_1) - v(w_2) = [g(e_H) - g(e_L)]/(p_1 - p_2) - \beta \pi ,\]
which is inferred from equation (2.6).

Solving equations (2.7) and (2.8), we get

\[
\begin{align*}
\mu &= \frac{(1 - p_1) p_1}{p_1 - p_2} \left( \frac{1}{v'(w_1)} - \frac{1}{v'(w_2)} \right) \\
\lambda &= \frac{1}{v'(w_1)} - \alpha - \mu + \mu \frac{p_2}{p_1}
\end{align*}
\]
(2.11) (2.12)

Substituting (2.11) into (2.12), and since \( \lambda > 0 \), we get

\[
\frac{p_1}{v'(w_1)} + 1 - p_1 > \alpha
\]
(2.13)

Letting \( p_1 / v'(w_1) + (1 - p_1) / v'(w_2) = \alpha \), and we mark \( w_1 = \hat{w}_1, \ w_2 = \hat{w}_2 \) and \( \beta = \tilde{\beta}_H \) in this situation. Since \( p_1 / v'(w_1) + (1 - p_1) / v'(w_2) \) is a monotone decreasing function about \( \beta \). Therefore, we can infer \( \beta < \tilde{\beta}_H \) from (2.13).

So we have when \( \beta < \tilde{\beta}_H \) and \( \beta \leq \beta^* \), the optimal contract is

\[
\begin{align*}
w_2 &= v^{-1}(\bar{u} - p_2 g(e_H) - p_1 g(e_L)) \\
w_1 &= v^{-1}(\bar{u} - (1 - p_2) g(e_H) - (1 - p_1) g(e_L) - \beta \pi)
\end{align*}
\]

(d) When \( \lambda = 0, \ \mu > 0 \), the optimal contract must satisfy the following conditions.

\[
p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi > \bar{u}
\]
(2.1)

\[
p_1 v(w_1) + (1 - p_1) v(w_2) - g(e_H) + \beta p_1 \pi = p_2 v(w_1) + (1 - p_2) v(w_2) - g(e_L) + \beta p_2 \pi
\]
(2.6)
\[
\frac{1}{v'(w_1)} = \alpha + \mu(1 - \frac{p_2}{p_1}) \\
\frac{1}{v'(w_2)} = \alpha + \mu(\frac{p_2 - p_1}{1 - p_1})
\] (2.14) (2.15)

We also get \( \beta \leq \beta^* \) from equation (2.6) as the case (c).

Combine (2.1) with (2.4), we get
\[
v(w_2) > \bar{u} - \frac{p_2 g(e_H) - p_1 g(e_L)}{p_1 - p_2} \quad (2.16)
\]
\[
v(w_1) > \bar{u} - \frac{(1 - p_2) g(e_H) - (1 - p_1) g(e_L)}{p_1 - p_2} - \beta \pi \quad (2.17)
\]

Combine (2.14) with (2.15), we get
\[
\frac{p_1}{v'(w_1)} + \frac{1 - p_1}{v'(w_2)} = \alpha \quad (2.18)
\]

Since \( \beta = \tilde{\beta}_1 \), satisfy \( p_1 \frac{v'(w_1)}{v'(w_1)} + (1 - p_1) \frac{v'(w_2)}{v'(w_2)} = \alpha \), combine with (2.16) and (2.17), and \( p_1 \frac{v'(w_1)}{v'(w_1)} + (1 - p_1) \frac{v'(w_2)}{v'(w_2)} \) is a monotone decreasing function about \( \beta \). So we have \( \beta \geq \tilde{\beta}_1 \) here.

Therefore, when \( \beta \leq \beta^* \) and \( \beta \geq \tilde{\beta}_1 \), the optimal contract is: \( w_1 = \bar{w}_1 \), \( w_2 = \bar{w}_2 \), as defined in section 2.3.

Proof end.

References


