Job-search and FDI in a two-sector general equilibrium model

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Abstract: The purpose of this paper is to extend the Fields’ (1989) multi sector job-search model by introducing international trade and capital. Two types of capital are considered: fixed capital and mobile capital. The effects of search intensity and the inflow of foreign capital on the volume and the rate of urban unemployment and on the social welfare are also examined in both of the two cases. The main finding is: more efficient on-the-job search from the rural sector raises unemployment rate when capital is mobile between the two sectors. This is counterproductive to the standard result.

Keywords: Job search, foreign capital, unemployment rate, ex-post labour, ex-ante labour, general equilibrium.

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1. Introduction

Job search is an integral part of the labour market. The idea of job search was first introduced by Burdett (1978). Originally, the search theory was formulated to analyse unemployment. The idea of job search has been incorporated in the models of McCall (1970), Fields (1975, 1989), Majumder (1975), Stark (1982), Adam and Cletus (1995), Postel-Vinay and Robin (2002), Dolado et al. (2009), Hussey (2005), Sheng and Xu (2007), Flinn and Mabli (2008), Arseneau and Chugh (2009), Macit (2010).

search and matching framework. Macit (2010) develop a New Keynesian model in search and matching structure with firing costs and he shows how labour market institutions affect the wage and inflation dynamics.

Another important concept in job search is the ‘graduation theory’, according to which, it is beneficial to remain in the urban informal sector\(^1\) and search part time for a highly paid job in the urban formal sector. However, this theory fails in the following circumstances: if the urban formal sector directly recruits from the rural sector (see Majumder, 1975), if urban informal sector workers prefer self employment to an urban formal sector job (see Squire, 1981) or if urban informal sector workers think of an urban informal sector job as a permanent source of income (see Sethuraman, 1981). Moreover, most of the theoretical models on the graduation theory adopts partial equilibrium analysis. Yet there are many factors, such as an imbalance of supply and demand in the analysis of the development of a Less Developed Country (LDC), intersectoral linkages etc., that partial equilibrium analysis cannot address. So, it is desirable to conduct a more general equilibrium analysis of job-search in order to highlight the process of job searching.

This paper builds on the two-sector labour market model of Fields (1975,1989), by introducing capital and international trade into Fields’ framework. Fields (1989) argues that distinguishing between ex-ante and ex-post allocation of the labour force is important for understanding the effects on the unemployment rate. He finds that as long as rural migrants have a positive probability of finding a job in the urban sector, ex-ante and ex-post labour forces will differ, affecting the unemployment rate. In particular, Fields (1989) argues that in this setup, “given a constant agricultural wage, a more efficient on-the-job search from agriculture lowers the urban unemployment rate in equilibrium”. However, unlike Fields (1989), we assume a flexible and market determined rural sector wage. The present paper examines the role of search intensity and the inflow of foreign capital on

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\(^1\) The ILO/UNDP employment mission report on Kenya (1972) suggested some characteristics of the informal sector as easy entry, reliance on indigenous resources, family ownership of enterprises, small scale of operation, low productivity, labour intensive technology, unregulated market, lack of govt. support etc.
unemployment and on social welfare. In particular, it attempts to show that in a job-search model for a small open economy with two factors of production, labour and capital, some of the results obtained in Fields get altered dramatically if capital is mobile across the sectors.

2. The model

The paper builds a two-sector job-search model for a small open economy. The two sectors are the rural sector (Sector 1) and the urban sector (Sector 2). $X_1$ is the export good which is produced in Sector 1 and $X_2$ is the import good, produced in Sector 2. The assumption of small open economy gives constant product prices in each sector. In the existing theoretical literature on trade and development the developing countries are considered as capital scarce and abundant in the supply of labour. Naturally, these economies are considered to be the exporters of labour-intensive (agricultural) commodities and importers of capital-intensive manufacturing commodities.

The two sectors use both labour and capital as inputs. The production function of all the sectors are subject to the Law of constant return to scale and diminishing marginal productivity to each input. All the markets are competitive and in the long run equilibrium, product price is matched exactly by the unit cost of production in each sector. Capital is specific to each sector. The rural sector uses domestic capital and the urban sector uses foreign capital. So, we have different rentals on capital in the two sectors.

The urban formal sector’s wage rate is institutionally given. Urban unemployment exists in our stylized economy as urban job seekers devote full time for searching urban jobs and all of them do not get high paid urban jobs. The unsuccessful urban job seekers stay in the urban

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2 In this model we assume away the other factors like educational skills and innovation activities. These factors lead to externality.
sector being unemployed.

The following notations are used in the model:

\( X_1 \) = level of output produced in Sector 1; \( X_2 \) = level of output produced in Sector 2; \( a_{ji} \) = amount of the jth input required to produce one unit of the ith commodity; \( L^k \) = ex-ante amount of labour in the kth job-search strategy, \( k = 1, 2; L_i \) = ex-post level of employment in ith sector;

\( P_1 \) = 1 (commodity 1 is the numeraire); \( P_2 \) = world price of commodity 2; \( P_2^* = (1 + t)P_2 \) = tariff-inclusive domestic price of commodity 2; \( t \) = ad-valorem rate of tariff; \( W_1 \) = rural wage rate; \( W_2^* \) = exogenously fixed urban wage rate; \( R_1 \) = rate of return on domestic capital; \( R_2 \) = rate of return on foreign capital; \( \rho \) = probability of getting urban jobs; \( \phi \) = efficiency on-the-job search in the rural sector; \( L \) = total labour endowment in the economy; \( K_D \) = stock of domestic capital in the economy; \( K_F \) = inflow of foreign capital in the economy; \( U \) = level of urban unemployment; \( \mu \) = rate of urban unemployment; \( D_i \) = domestic demand for the ith goods; \( M \) = demand for the importable goods; \( Y \) = national income at domestic prices; \( ^\wedge \) = proportional change.

The general equilibrium structure of the model is as follows.

The competitive profit conditions are given by the price unit cost equality:

\[
W_1 a_{L1} + R_1 a_{K1} = 1 \tag{1}
\]

\[
W_2^* a_{L2} + R_2 a_{K2} = (1 + t)P_2 = P_2^* \tag{2}
\]

The probability of getting urban formal sector job is:

\[
\rho = a_{L2} X_2 / (\phi L^1 + L^2) \tag{3}
\]

where \((\phi L^1 + L^2)\) is the total number of job seekers.
It is assumed that each worker searches for urban formal sector jobs, perhaps, because of its highest paying potentials. We consider two different job-search strategies: The first strategy describes full time jobs search as remaining unemployed at the beginning. We find this type of job search in Harris–Todaro (1970), Harberger (1971), Mincer (1976), Gramlich (1970), Stiglitz (1982) and Mcdonald and Solow (1985) and Fields (1989). If a person, searching full time for urban formal sector jobs, becomes successful, he can earn high urban formal wage with a specific probability of getting urban formal sector job and earns zero, as unemployed if he becomes unsuccessful. The second strategy is to remain in the rural sector and search part time for urban formal sector jobs. In this strategy, the success gives high paid urban formal sector jobs, while failure means to remain in the rural sector and earn rural wage.

In the case of job-search, a person may get job in the sector where he does not stay at the beginning. Thus, the number of ex-ante job searchers differs from the ex-post labour force. For this reason, Fields (1989) distinguishes between the ex-ante allocation of labour among different search strategies and the ex-post allocation of labour among different sectors. Each search strategy has expected income. In equilibrium, the expected income from the two strategies would be equal. Thus, the allocation of labour force among the two strategies is given by:

$$\rho W_2^* = \phi \rho W_2^* + (1 - \phi \rho)W_1$$  \hspace{1cm} (4)

The number of people searching urban formal sector jobs from the rural sector is $L_1$. Out of $L_1$; $\phi \rho L_1$ people get employment in the urban formal sector. Thus, the ex-post number of workers in the rural sector is:

$$a_LX_1 = L_1(1 - \phi \rho)$$  \hspace{1cm} (5)

The fixed amount of total labour force in the economy is not fully employed. The ex-ante and the ex-post endowments of labour are given by the following equations:

$$L_1 + L_2 = L$$  \hspace{1cm} (6)
The endowments of capital are fixed and fully employed. The full employment of domestic as well as foreign capital is given by:

\[ a_{K1}X_1 = K_D \]  

\[ a_{K2}X_2 = K_F \]  

The welfare of this small open economy is national income at domestic prices, which is given as follows.\(^3\) Foreign capital income is completely repatriated.

\[ Y = X_1 + P^*_2X_2 + tP_2M - R_2K_F \]  

or, \[ Y = (W_1a_{L1}X_1 + R_1K_D) + (W_2a_{L2}X_2 + R_2K_F) + tP_2M - R_2K_F \]  

or, \[ Y = W_1a_{L1}X_1 + W_2a_{L2}X_2 + R_1K_D + tP_2M \]  

In Equation (10.1) \( W_1a_{L1}X_1 \) and \( W_2a_{L2}X_2 \) are the wage incomes of the workers in the two sectors, respectively. \( R_1K_D \) denotes the rental income of domestic capital. Finally, \( tP_2M \) is the amount of tariff revenue of the government from the import of commodity 2 which is completely transferred to the consumers as lump-sum payments.

The domestic demand for the two goods is:

\[ D_1 = D_1(P^*_2, Y) \]  

where \( D_{11} = (\partial D_1 / \partial P^*_2) < 0 \); and, \( D_{12} = (\partial D_1 / \partial Y) > 0 \)

\(^3\) It has been rightly pointed out by one of the two anonymous referees that considering national income (or per capita national income) as an indicator of welfare may sometimes be misleading. This is because despite a substantial increase in national income a lion’s share of the population may not at all be benefited if there exists a high degree of income inequality among various groups of the population in the economy. In such a situation the welfare measure of Sen (1974), defined as the per-capita income multiplied by one minus the Gini-coefficient of the income distribution, is an appropriate measure of welfare of the different groups of population. Keeping this limitation in mind we, however, continue to measure social welfare in terms of national income as our prime objective is not to focus on income inequality.
and \( D_2 = D_2(P^*_2, Y) \)  

(12)

where \( D_{21} = (\partial D_2 / \partial P^*_2) < 0 \); and, \( D_{22} = (\partial D_2 / \partial Y) > 0 \)

The import demand for the commodity 2 is:

\[ M = D_2 - X_2 \]  

(13)

Using Equations (4), (5), (6) into Equation (3) we get,

\[ W_1a_{L1}X_1 = \rho LW^*_2 - W^*_2a_{L2}X_2 \]  

(3.1)

Using Equations (3.1), (4), (5), (6), (12) and (13) into Equation (10.1) we get,

\[ Y = \rho W^*_2L + R_1K_F + tP_2[D_2(P^*_2, Y) - X_2] \]  

(10.2)

We can determine \( R_2 \) from Equation (2), given \( W^*_2, P_2, t \). Thus, we get \( a_{K2} \). Then, Equation (9) gives \( X_2 \), given \( K_F \). Now, from Equation (1) we get \( R_1 \) as a function of \( W_1 \); i.e., \( R_1 = g(W_1) \), where \( g' < 0 \). Thus, \( a_{K1} \) is also a function of \( W_1 \). Equation (4) shows that \( \rho \) also depends on \( W_1 \); i.e., \( \rho = h(W_1), h' > 0 \).  

Solving Equations (3.1) and (8) we get \( W_1 \) and \( X_1 \) given \( K_F \). Then, we get \( R_1 \) and \( \rho \). Next, \( U \) is obtained from Equation (7), given \( L_1 \). \( L_1 \) is obtained from Equation (5) and \( L^2_2 \) from Equation (6). We find \( Y \) from Equation (10.2). Finally, we get \( D_1, D_2, M \) from Equations (11), (12) and (13) respectively.

\[ \frac{d \rho}{dW_1} = \left( \frac{(1 - \varphi \rho)}{\varphi W_1 + (1 - \varphi)W^*_2} \right) > 0 \]

This implies that given the urban wage and the job search intensity, if the rural wage rate rises, \( \rho \) has to increase to maintain expected income equalization from the two search strategies.
3. Comparative static:

We are now going to examine the effects of search intensity and foreign capital inflow on urban unemployment and national income at domestic prices.

We are going to use some more symbols which are as follows.

\( \theta_{ji} \) = distributive share of the \( j \) th factor in \( i \) th sector with \( j = L, K \); and, \( i = 1, 2 \); \( \lambda_{ji} \) = allocative share of the \( j \) th factor in \( i \) th sector; \( \sigma_i \) = elasticity of factor substitution in \( i \) th sector.

Total differential of Equation (4) yields:

\[
\dot{\rho} = \{ (W_2^*/W_1) - 1 \} \varphi \rho \dot{\varphi} + (1 - \varphi \rho) \dot{W}_1
\]

(Total 14)

Totally differentiating Equations (3.1) and (8) we get the following results\(^5\):

\[
\left[ \lambda_{L1}W_1 - \lambda_{L1}W_1 \sigma_1 - W_2^* \rho (1 - \varphi \rho) \right] \dot{W}_1 + \hat{\lambda}_{L1}W_1 \dot{X}_1 = W_2^* \rho^2 \varphi \left( \frac{W_2^*}{W_1} - 1 \right) \dot{\varphi} - \lambda_{L2}W_2^* \dot{K}_F
\]

(15)

\[
\sigma_1 \theta_{L1} \dot{W}_1 + \theta_{K1} \dot{X}_1 = 0
\]

(16)

Solving Equations (15) and (16) one gets,

\[
\frac{\dot{W}_1}{\dot{\varphi}} = \frac{\theta_{K2}W_2^* \rho^2 \varphi}{\Delta} \left( \frac{W_2^*}{W_1} - 1 \right) < 0
\]

(17)

\[
\frac{\dot{W}_1}{\dot{K}_F} = - \frac{\theta_{K1} \lambda_{L2}W_2^*}{\Delta} > 0
\]

(18)

\(^5\) See Appendix A.1 for detailed derivations.
\[
\begin{align*}
\left( \frac{\dot{X}_1}{\dot{\phi}} \right) &= -\frac{\sigma_1 \theta_{L_1} W_2^* \rho^2 \varphi \left( \frac{W_2^*}{W_1} - 1 \right)}{\Delta} > 0 \\
\left( \frac{\dot{X}_1}{\dot{K}_F} \right) &= -\frac{\sigma_1 \theta_{L_1} W_2^* \lambda L_2}{\Delta} < 0
\end{align*}
\] (19)

where \( \Delta = \lambda_{L_1} W_1 \left( \theta_{K_1} - \sigma_1 \right) - \theta_{K_1} W_2^* \rho (1 - \varphi \rho) < 0 \), if \( \sigma_1 \geq \theta_{K_1} \) (21)

It is important to note that if the production function of sector 1 is of Cobb-Douglas type we have \( \sigma_1 = 1 \). So \( \Delta < 0 \) without of any restrictions on the parameters.

3.1. Effects on \( U \):

The total differential of Equation (7) yields\(^6\):

\[
(\dot{U} / \dot{\phi}) = \frac{L \lambda_{L_1} \sigma_1 W_2^* \rho^2 \varphi \left( \frac{W_2^*}{W_1} - 1 \right)}{U \Delta} < 0
\] (22)

\[
(\dot{U} / \dot{K}_F) = -\left( \frac{L \lambda_{L_2}}{\Delta U} \right) \left[ W_1 \lambda_{L_1} \left( \theta_{K_1} - \sigma_1 \right) - \sigma_2^\* \left( \theta_{K_1} \rho (1 - \varphi \rho) \right) - \lambda_{L_1} \sigma_1 \right]
\] (23)

So, from Equation (23) we find that \( (\dot{U} / \dot{K}_F) < 0 \) if \( \rho (1 - \varphi \rho) > \lambda_{L_1} \).

Here, the unemployment rate is:

\[
\mu = \{U / (U + a_{L_2} X_2)\} = [1 / \{1 + (a_{L_2} X_2 / U)\}]
\] (24)

Result (24) shows that \( \mu \) also falls when \( \varphi \) or \( K_F \) rises.

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\(^6\) See Appendix A.2.
These lead to the following proposition:

**Proposition 1:** Both an improvement in job-search efficiency and the inflow of foreign capital lower the volume and the rate of urban unemployment if capital is specific to each sector.\(^7\)

### 3.2. Effect on welfare:

The total differential of Equation (10.2) and then using (11) gives:\(^8\)

\[
\frac{\hat{Y}}{\hat{\phi}} = \left(\frac{V}{\Delta Y}\right) W_2^* \rho^2 \phi \left(\frac{W_2^*}{W_1} - 1\right) \left[ L \lambda L_1 W_1 \left(\theta_{K1} - \sigma_1\right) - \theta_{L1} R_1 K_D \right] > 0
\]

\[
(-)
\]

\[
\frac{\hat{Y}}{\hat{K}_F} = -\left(\frac{V}{\Delta Y}\right) \left\{ W_2^* \rho (1 - \varphi \rho) - W_1^* L_1 \right\} \left[ \frac{\theta_{K1} P_2^* X_2}{(1 + t)} \left( \theta_{L2} R_1 K_D - \theta_{K2} \right) \right] < 0
\]

\[
(-)
\]

---

\(^7\) Theoretically, the specific factor models have been developed with three factors and two goods cases (Jones 1971, Samuelson 1971). Imperfect factor mobility is also observed both in large and small economies. We find capital immobility in developing countries like India and China. In India, as much as 80% of investment capital in small and medium-sized firms is from informal sources and internal funds. We observe strong correlation between district wealth and investment in India (Sharma, S 2008).

Immobility of capital may occur due to transportation barriers, language and cultural barriers, information barriers, heavy reliance on specialized equipment and knowledge etc. So, there may exist some complex conditions in a small open economy that lead to imperfect capital mobility within the economy (Mavromatis and Verikios 2008).

\(^8\) See Appendix A.3.
From (26) it follows that \( \hat{Y} / \hat{K}_F < 0 \) if \( t \geq (\theta L_2 / \theta K_2) \).

These give us the following proposition:

**Proposition 2:** An improvement in job-search intensity raises social welfare, whereas an inflow of foreign capital lowers this.

We explain Proposition 1 and Proposition 2 as follows: As job search is more efficient for the rural workers, a demerit of search for a job in the urban sector while staying in the rural area is smaller. So, the value of being a rural worker is higher, which encourages workers to stay in the rural area, choosing the part time job searching strategy and therefore, \( L^1 \) rises. Given the labour constraint, \( L^2 \) falls. However, the urban production remains unchanged due to fixed capital and no change in factor intensity in this sector.\(^9\) So, the level of unemployment falls as the number of full time job seekers (ex-ante labour force in the urban sector) reduces and the ex-post level of urban employment remains fixed. Equation (24) shows that as \( U \) falls, \( \mu \) also falls, given \( a L_2 X_2 \). Again, as more and more workers stay in the rural area, rural wage rate falls and the rental rate rises to maintain price-unit cost equality at the competitive equilibrium. The falling rural wage lowers probability of getting jobs to maintain the expected income equality between the two job search strategies. Thus, at the initial equilibrium, the improved search intensity lowers wage income and raises rental income, keeping the tariff revenue unchanged. The rise in rental income outweighs the fall in wage income and so, the domestic factor income rises and so also the social welfare.

On the other hand, an inflow of foreign capital expands the urban sector and contracts the rural sector. At the same time, it also raises the probability of getting urban jobs. So, the number of people adopting the strategy of full time job searching rises. The magnification

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\(^9\) Here, factor intensity means the ratio of ex-post capital to ex-post labour. Here, ex-ante labour force in the urban sector changes but not the ex-post labour force. So, factor intensity remains the same even if \( L^2 \) falls, when capital is fixed.
effect implies that the level of unemployment in the urban sector falls if $\rho(1-\varphi \rho) > \lambda_{L1}$. Thus, the employment–unemployment ratio in the urban sector rises and this lowers the rate of unemployment. Further, the inflow of foreign capital raises wage income by upgrading rural wage and probability of getting urban jobs. At the same time, it lowers rental income on domestic capital. At the initial equilibrium, it also lowers tariff revenue. These two effects jointly outweigh the wage effect and so the domestic factor income falls if $t \geq \left(\frac{\theta_{L2}}{\theta_{K2}}\right)$ and so also the social welfare. Hence, an inflow of foreign capital in this job search model lowers both unemployment rate and social welfare.

4. An extension: mobile capital case:

In this section we relax the assumption of fixed capital and capital is assumed to be mobile between the two sectors. Thus, we have a common rate of return on capital. We assume that sector 1 is more labour-intensive (less capital-intensive) vis-à-vis sector 2 in value sense.

This implies that $(\frac{\theta_{L1}}{\theta_{K1}} > \frac{\theta_{L2}}{\theta_{K2}}) \Leftrightarrow (\frac{W_{1}a_{L1}}{a_{K1}} > \frac{W_{2}a_{L2}}{a_{K2}}) \Rightarrow (\frac{\lambda_{L1}}{\lambda_{K1}} > \frac{\lambda_{L2}}{\lambda_{K2}})$. This means that if sector 1 is more labour-intensive (less capital-intensive) vis-à-vis sector 2 in value sense it is also more labour-intensive (less capital-intensive) than sector 2 in physical sense.

Equations (1) and (2) of the previous section become:

\begin{align*}
W_{1}a_{L1} + Ra_{K1} &= 1 \quad (1.1) \\
W_{2}^*a_{L2} + Ra_{K2} &= P_{2}^* = P_{2}(1+t) \quad (2.1)
\end{align*}

where $R$ stands for the common return to capital in both the sectors.

The two capital endowment equations of the previous section are changed into one:

\begin{align*}
a_{K1}X_{1} + a_{K2}X_{2} &= F + K_{D} = K \quad (8.1)
\end{align*}
Here $K = (K_d + K_f)$ is the aggregate capital stock of the economy (domestic plus foreign).

Equation (10) becomes:

$$Y = X_1 + P^*_2 X_2 + t P_2 M - RK_F$$

(10.3)

Equivalently,

$$Y = \rho W^*_2 L + RK_D + t P_2 \{ D_2 (P^*_2 Y - X_2) \}$$

(10.4)

All other equations are same as the previous section.

Here, $R$ is determined from (2.1) and then $W_1$ from (1.1). Thus, all $a_{ji}$'s are determined.

Now, (3.1) and (8.1) give $X_1$ and $X_2$. Equation (4) yields $\rho$. Then, we get $L^1$ from Equation (5) and $L^2$ from Equation (6). $U$ is obtained from (7). We get $Y$ from (10.3). Then, $D_2$ and $M$ are obtained from Equations (11) and (12).

Totally differentiating Equation (4) we get,

$$\dot{\rho} = \left( \frac{W^*_2}{W_1} - 1 \right) \varphi \rho \dot{\varphi}$$

(27)

Totally differentiating Equations (3.1) and (8.1) and using (27) we get,

$$\left\{ \frac{\dot{X}_1}{\dot{\varphi}} \right\} = \left( \frac{1}{\Delta'} \right) \lambda K_2 W^*_2 \rho^2 \varphi \left( \frac{W^*_2}{W_1} - 1 \right) > 0$$

(28)

10 See Appendix B.1.
\[
\left( \frac{\dot{X}_1}{K} \right) = -\left( \frac{1}{\Delta'} \right) \lambda_{L2} W_2^* < 0 
\] (29)

\[
\left( \frac{\dot{X}_2}{\phi} \right) = -\left( \frac{1}{\Delta'} \right) \lambda_{K1} W_2^* \rho^2 \phi \left( \frac{W_2^*}{W_1} - 1 \right) < 0 
\] (30)

\[
\left( \frac{\dot{X}_2}{K} \right) = \left( \frac{1}{\Delta'} \right) \lambda_{L1} W_1 > 0 
\] (31)

where \( \Delta' = \left( W_1 \lambda_{L1} \lambda_{K2} - W_2^* \lambda_{K1} \lambda_{L2} \right) > 0 \), (since sector 1 is assumed to be labour-intensive relative to sector 2 in value sense). (32)

The total differential of Equation (7) and then using (B.1.3) and (B.1.4) yields 11:

\[
(\dot{U} / \phi) = \left( \frac{\rho^2 W_2^*}{\Delta' U} \right) \left( \frac{W_2^*}{W_1} - 1 \right) \phi \left( \lambda_{L2} \lambda_{K1} - \lambda_{L1} \lambda_{K2} \right) < 0 
\] (33)

\[
(\dot{U} / K) = \left( \frac{\lambda_{L1} \lambda_{L2}}{\Delta' U} \right) (W_2^* - W_1) > 0 
\] (34)

and, These lead to the following proposition:

**Proposition 3:** An improvement in job-search efficiency lowers the volume of urban unemployment and raises the rate of unemployment, while the inflow of foreign capital gives the opposite results if capital is mobile between the two sectors.

The total differential of Equation (10.4) and then using (27), (32) gives 12:

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11 See Appendix B.2.

12 See Appendix B.3.
\[(\hat{Y} / \hat{\phi}) = \left( \frac{W_2^* \rho^2 \phi}{Y (1 - tP_2 D_{22})} \right) \left( \frac{W_2^*}{W_1} - 1 \right) \left[ L + \frac{tP_2 X_2 \lambda K_1}{\Delta'} \right] > 0; \text{ and,} \quad (35)\]

\[(\hat{Y} / \hat{K}) = \left( \frac{X_1 X_2 \theta L_1}{\Delta' L Y (1 - tP_2 D_{22})} \right) \left( P_2 - P_2^* \right) < 0, \quad \therefore \left( P_2 - P_2^* \right) < 0 \quad (36)\]

This gives us the following proposition:

**Proposition 4:** An improvement in job-search intensity is welfare improving, whereas an inflow of foreign capital is welfare reducing.

We may now give an intuitive explanation of Proposition 3 and Proposition 4: Here also as search intensity improves \( L^1 \) rises and \( L^2 \) falls. Again, as \( \phi \) rises the capital intensive sector contracts and the labour intensive sector expands. So, the ex-post level of employment in the urban sector falls. The level of unemployment also falls due to the specific factor intensity condition. Now, the ratio of unemployment to the ex-post labour force in the urban sector rises because the denominator reduces more substantially than the fall occurring in the numerator and so the rate of unemployment also rises. Again, as the job search efficiency rises, the wage income rises through the rise in the probability of getting urban jobs and tariff revenue also rises as \( X_2 \) falls. Thus, total domestic factor income rises and so also social welfare. On the other hand, the inflow of foreign capital expands the capital intensive sector and contracts the labour intensive sector\(^{13}\). Both the ex-post and the ex-ante level of employment fall in the rural sector. In the urban sector, both these levels rise. So, more labour is now available in the urban sector, but only a portion of it is absorbed. This accentuates the problem of unemployment. Here, the unemployment is the urban unemployment and the employment is the ex-post urban labour force. The ratio of urban unemployment to the ex-post labour force in the urban sector falls because the denominator rises more substantially than the rise in the numerator and so the unemployment rate falls. At

\(^{13}\) We assume that domestic capital and the foreign capital are perfect substitutes. So, the Rybczynsky effect works following an increase in the inflow of foreign capital.
the initial equilibrium, the inflow of foreign capital lowers the volume of import as $X_2$ rises. So, given $t, P^*_2$, the income from tariff revenues falls and the rental income rises, keeping wage income unchanged. Here, the fall in tariff revenue outweighs the rise in rental income. So, the domestic factor income falls and so also the social welfare.

5. Concluding remarks:

This paper aims to contribute to the literature on rural-urban migration and labour force allocation across sectors, in analyzing the dynamics of employment and unemployment. One implication of this analysis, in particular of the Harris-Todaro model (Harris and Todaro, 1970), is that it predicts an unemployment rate much higher than the ones observed in the data. The present paper extends the Fields’ (1989) model by introducing capital and international trade into the Fields’ framework and examines the effects of job-search efficiency and inflow of foreign capital on unemployment and social welfare. We consider two different cases: fixed capital case as well as mobile capital case. Our analysis shows that in both cases the improved job-search efficiency lowers the volume of unemployment. However, the rate of unemployment falls in the fixed capital case and rises in the mobile capital case if job-search efficiency improves. Thus, the result obtained in the mobile capital case is partly counterproductive to the Fields’ (1989) proposition: 1. Moreover, the increased job-search efficiency is welfare enhancing in both the fixed capital case and the mobile capital case.

The paper also shows that the inflow of foreign capital lowers unemployment rate and social welfare in both cases. However, such inflow lowers the volume of unemployment in the fixed capital case and raises it when the capital is mobile.

Thus our results show that the nature of capital, whether it is fixed or mobile, plays important role to examine the impact of job-search efficiency and inflow of foreign capital on unemployment and social welfare. In the case of fixed capital, improvement in job search efficiency is better than the inflow of foreign capital because the former option allows an
increase in social welfare. If capital is mobile between the two sectors, increased job search efficiency raises both the unemployment rate and social welfare. However, the foreign capital inflow lowers both the unemployment rate and social welfare if capital is mobile. Thus, in the case of mobile capital which policy is better than other depends upon the objective of the economy: whether to reduce unemployment rate or to raise social welfare.

Appendices:

Appendix A.1:

The total differential of Equation (3.1) is,

\[
\lambda_{L1} W_1 LW_1 + \lambda_{L1} W_1 L\hat{x}_1 + \lambda_{L1} W_1 L\hat{\alpha}_1 = LW_2^* \rho \hat{\phi} - \lambda_{L2} W_2^* L\hat{x}_2 - \lambda_{L2} W_2^* L\hat{\alpha}_2
\]

(A.1.1)

Using Equation (16) and the definition of elasticity of factor substitution we get,

\[
\lambda_{L1} W_1 LW_1 + \lambda_{L1} W_1 L\hat{x}_1 - \lambda_{L1} W_1 L\sigma_1 \hat{W}_1 = LW_2^* \rho \left[ (1 - \varphi \rho) \hat{W}_1 + \varphi \rho \phi \frac{W_2^*}{W_1} \right] \\
- \lambda_{L2} W_2^* L\hat{x}_2
\]

or,

\[
\left[ \lambda_{L1} W_1 - \lambda_{L1} W_1 L\sigma_1 - W_2^* \rho (1 - \varphi \rho) \right] \hat{W}_1 + \lambda_{L1} W_1 \hat{\alpha}_1 = W_2^* \rho^2 \phi \frac{W_2^*}{W_1} \\
- \lambda_{L2} W_2^* L\hat{x}_2
\]

(A.1.2)

From Equation (9) we get,

\[
\hat{x}_2 = \hat{K}_F
\]

Putting this into (A.1.2) we get,
\[
\begin{align*}
\left[ \lambda_{L1} W_1 - \lambda_{L1} W_1 L \sigma_1 - W_2^* \rho (1 - \phi \rho) \right] \dot{W}_1 + \lambda_{L1} W_1 \ddot{W}_1 = & W_2^* \rho^2 \phi \left( \frac{W_2^*}{W_1} - 1 \right) \\
& - \lambda_{L2} W_2^* \dot{K}_F \quad \text{(15)}
\end{align*}
\]

Solving Equations (15) and (16) we get,
\[
\dot{W}_1 = \frac{1}{\Delta} \theta_{K1} \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) - W_2^* \lambda_{L2} \dot{K}_F \right] \quad \text{(A.1.3)}
\]

and,
\[
\dot{X}_1 = -\frac{1}{\Delta} \sigma_1 L \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) - W_2^* \lambda_{L2} \dot{K}_F \right] \quad \text{(A.1.4)}
\]

**Appendix A.2:**

The total differential of Equation (7) is given by:
\[
\lambda_{L1} \ddot{X}_1 + \lambda_{L2} \ddot{X}_2 + \left( \frac{U}{L} \right) \dot{U} = \lambda_{L1} \sigma_1 \ddot{W}_1 \quad \text{(A.2.1)}
\]

Using Equations (A.1.3), (A1.4) and (9) into Equation (A.2.1) we get,
\[
\left( \frac{U}{L} \right) \dot{U} = \lambda_{L1} \sigma_1 \left( \frac{1}{\Delta} \right) \theta_{K1} \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) - W_2^* \lambda_{L2} \dot{K}_F \right] +
\]
\[
\lambda_{L1} \left( \frac{1}{\Delta} \right) \sigma_1 \theta_{L1} \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) - W_2^* \lambda_{L2} \dot{K}_F \right] - \lambda_{L2} \dot{K}_F
\]
or, $\hat{U} = \left( \frac{L}{U} \right) \lambda_L \sigma_1 (1/\Delta) W^*_2 \left[ \rho^2 \phi \left( \frac{W^*_2}{W_1} - 1 \right) - \frac{\Delta}{L} \lambda_L \hat{K}_F \right] - \left( \frac{L}{U} \right) \lambda_L \hat{K}_F$

or, $\hat{U} = \left( \frac{L}{U} \right) \lambda_L \sigma_1 (1/\Delta) W^*_2 \rho^2 \phi \left( \frac{W^*_2}{W_1} - 1 \right) - \left( \frac{L}{U} \right) \lambda_L \hat{K}_F \left( 1 + \frac{\lambda_L \sigma_1 W^*_2}{\Delta} \right)$ \hspace{1cm} (A.2.2)

Appendix A.3:

The total differential of Equation (10.4) gives:

$$Y \hat{Y} (1-tP_2 D_{22}) = W^*_2 L \rho \hat{\rho} + R_1 K_D \hat{R}_1 - tP_2 X_2 \hat{X}_2$$ \hspace{1cm} (A.3.1)

Suppose, $m_2$ stands for the marginal propensity to consume commodity 2. So,

$$m_2 = \frac{P_2}{(1+t)} \frac{\partial D_2}{\partial Y} = P_2 (1+t) (\partial D_2 / \partial Y), \hspace{0.5cm} (0 < m_2 < 1);$$

or, $m_2 = \frac{P_2 (\partial D_2 / \partial Y)}{(1+t)}$;

$$\therefore 1 - \frac{tm_2}{(1+t)} = \frac{1+t(1-m_2)}{(1+t)}$$

Substituting this into (A.3.1) we get,

$$\therefore \hat{Y} = \left( \frac{V}{Y} \right) \left[ W^*_2 L \rho \hat{\rho} + R_1 K_D \hat{R}_1 - tP_2 X_2 \hat{X}_2 \right]$$ \hspace{1cm} (A.3.2)

where $V = \frac{(1+t)}{1+t(1-m_2)} > 0$

Using Equations (11), (1) and (9) into (A.3.2) we get,

$$\hat{Y} = \left( \frac{V}{Y} \right) \left[ W^*_2 L \rho (1- \phi \rho) - R_1 K_{D} \frac{\theta L_1}{\theta K_1} \hat{W}_1 + W^*_2 L \rho^2 \phi \left( \frac{W^*_2}{W_1} - 1 \right) \phi \right]$$

$$- \left( \frac{V}{Y} \right) tP_2 X_2 \hat{K}_F$$ \hspace{1cm} (A.3.3)
Using Equation (A.1.3) we get,

\[
\dot{y} = \left( \frac{V}{Y} \right) \left[ W^*_2 L^* \rho (1 - \varphi \rho) - R^*_1 K^*_D \frac{\theta_{L^*_1}}{\theta_{K^*_1}} \right] \frac{\theta_{K^*_1}}{\Delta} \times \left[ \rho^2 W^*_2 \phi \frac{W^*_2}{W^*_1} \left( \frac{W^*_2}{W^*_1} - 1 \right) - W^*_2 \lambda_{L^*_2} \hat{K}_F \right] \\
+ W^*_2 L^* \rho \frac{W^*_2}{W^*_1} \phi - t^*_2 \rho X^*_2 \hat{K}_F 
\]

or,

\[
\dot{y} = \left( \frac{V}{\Delta Y} \right) \left[ \rho^2 W^*_2 \phi \frac{W^*_2}{W^*_1} - 1 \right] \{ W^*_2 L^* \rho (1 - \varphi \rho) \theta_{K^*_1} - R^*_1 K^*_D \theta_{L^*_1} + \Delta \} \phi^{-} \\
\left[ \{ W^*_2 L^* \rho (1 - \varphi \rho) \theta_{K^*_1} - R^*_1 K^*_D \theta_{L^*_1} \} W^*_2 \lambda_{L^*_2} + \Delta t^*_2 \rho X^*_2 \right] \hat{K}_F 
\]

or,

\[
\dot{y} = \left( \frac{V}{\Delta Y} \right) \left[ \rho^2 W^*_2 \phi \left( \frac{W^*_2}{W^*_1} - 1 \right) \{ L \lambda_{L^*_1} W^*_1 \left( \theta_{K^*_1} - \sigma_{1} \right) - R^*_1 K^*_D \theta_{L^*_1} \} \phi^{-} \right] \\
\left[ \{ W^*_2 \rho (1 - \varphi \rho) - \lambda_{L^*_1} W^*_1 \theta_{K^*_1} \} \left( L W^*_2 \lambda_{L^*_2} - t^*_2 \rho X^*_2 \right) \right] \hat{K}_F \\
- t^*_2 \rho X^*_2 \lambda_{L^*_1} W^*_1 \sigma_{1} 
\]

or,

\[
\dot{y} = \left( \frac{V}{\Delta Y} \right) \left[ \rho^2 W^*_2 \phi \left( \frac{W^*_2}{W^*_1} - 1 \right) \{ L \lambda_{L^*_1} W^*_1 \left( \theta_{K^*_1} - \sigma_{1} \right) - R^*_1 K^*_D \theta_{L^*_1} \} \phi^{-} \right] \\
\left[ \{ W^*_2 \rho (1 - \varphi \rho) - \lambda_{L^*_1} W^*_1 \theta_{K^*_1} \} \left( L W^*_2 \lambda_{L^*_2} - t^*_2 \rho X^*_2 \right) \right] \hat{K}_F \\
- t^*_2 \rho X^*_2 \lambda_{L^*_1} W^*_1 \sigma_{1} 
\]

or,

\[
\dot{y} = \left( \frac{V}{\Delta Y} \right) \left[ \rho^2 W^*_2 \phi \left( \frac{W^*_2}{W^*_1} - 1 \right) \{ L \lambda_{L^*_1} W^*_1 \left( \theta_{K^*_1} - \sigma_{1} \right) - R^*_1 K^*_D \theta_{L^*_1} \} \phi^{-} \right] \\
\left[ \{ W^*_2 \rho (1 - \varphi \rho) - \lambda_{L^*_1} W^*_1 \theta_{K^*_1} \} \left( L W^*_2 \lambda_{L^*_2} - t^*_2 \rho X^*_2 \right) \right] \hat{K}_F \\
- t^*_2 \rho X^*_2 \lambda_{L^*_1} W^*_1 \sigma_{1} 
\]

(A.3.4)
Appendix B.1:

Totally differentiating Equation (3.1) and using Equation (27) we get,

$$W^*_2 \lambda^*_2 \ddot{X}^*_2 + W^*_1 \lambda^*_1 \ddot{X}^*_1 = \rho^2 W^*_2 \{(W^*_2/W^*_1)-1\} \phi \dot{\phi}$$

(App B.1.1)

Totally differentiating Equation (8.1) we get,

$$\lambda^*_K \ddot{X}^*_2 + \lambda^*_L \ddot{X}^*_1 = \dot{\hat{K}}_F$$

(App B.1.2)

Solving (B.1.1) and (B.1.2) we get,

$$\ddot{X}_1 = (1/\Delta') \left[ \lambda^*_K \rho^2 W^*_2 \left( \frac{W^*_2}{W^*_1} - 1 \right) \phi \dot{\phi} - W^*_2 \lambda^*_L \ddot{\hat{K}} \right]$$

(App B.1.3)

$$\ddot{X}_2 = (1/\Delta') \left[ W^*_1 \lambda^*_L \dot{\hat{K}} - \lambda^*_K W^*_2 \rho^2 \left( \frac{W^*_2}{W^*_1} - 1 \right) \phi \dot{\phi} \right]$$

(App B.1.4)

Appendix B.2:

The total differential of Equation (7) is:

$$\lambda^*_L \ddot{X}_1 + \lambda^*_L \ddot{X}_2 + U \dot{U} = 0$$

(App B.2.1)

Using Equations (B.1.3) and (B.1.4) into (B.2.1) we get,

$$U \dot{U} = -\lambda^*_L \left( \frac{1}{\Delta} \right) \left[ \lambda^*_K \rho^2 W^*_2 \left( \frac{W^*_2}{W^*_1} - 1 \right) \phi \dot{\phi} - W^*_2 \lambda^*_L \ddot{\hat{K}} \right]$$

$$- \lambda^*_L \left( \frac{1}{\Delta} \right) \left[ W^*_1 \lambda^*_L \dot{\hat{K}} - \lambda^*_K W^*_2 \rho^2 \left( \frac{W^*_2}{W^*_1} - 1 \right) \phi \dot{\phi} \right]$$

$$\therefore \dot{U} = \rho^2 W^*_2 \frac{W^*_2}{\Delta U} - 1 \phi \left( \lambda^*_L \lambda^*_K \dot{\hat{K}} - \lambda^*_L \lambda^*_K \ddot{\hat{K}} \right) + \frac{\lambda^*_L \lambda^*_K}{\Delta U} \left( W^*_2 - W^*_1 \right) \dot{\hat{K}}$$

(App B.2.2)
Appendix B.3:

The total differential of Eq. (10.4) is:

\[ \hat{Y} = \left( \frac{V}{Y} \right)[\rho \hat{\phi} W_2^* L + R \hat{K} - tP_2 X_2 \hat{X}_2] \quad (B.3.1) \]

Using Equations (27) and (B.1.4) we get,

\[ \hat{Y} = \left( \frac{V}{Y} \right) \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) \left\{ L + \frac{tP_2 X_2 \lambda K_1}{\Delta'} \right\} \hat{\phi} + \left\{ \frac{tP_2 X_2 W_1 \lambda L_1}{\Delta'} \right\} \hat{K} \right] \quad (B.3.2) \]

Using Equation (32) into Equation (B.3.2) we get,

\[ \hat{Y} = \left( \frac{V}{Y} \right) \left[ \rho^2 W_2^* \left( \frac{W_2^*}{W_1} - 1 \right) \left\{ L + \frac{tP_2 X_2 \lambda K_1}{\Delta'} \right\} \hat{\phi} \right. \\
\quad \left. + \frac{1}{\Delta'} \left\{ \frac{X_1 X_2 P_2^* (\theta_1 - \theta_2)}{L} - \frac{tP_2 X_2 W_1 a L_1 X_1}{L} \right\} \hat{K} \right] \]

or,

\[ \hat{Y} = \left( \frac{V}{Y} \right) \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) \left\{ L + \frac{tP_2 X_2 \lambda K_1}{\Delta'} \right\} \hat{\phi} \]

\[ + \frac{X_1 X_2}{\Delta' L} \left\{ P_2^* (\theta_1 - \theta_2) - tP_2 \theta L_1 \right\} \hat{K} \]

or,
\[ \hat{Y} = \left( \frac{V}{Y} \right) \left[ \rho^2 W_2^* \phi \left( \frac{W_2^*}{W_1} - 1 \right) \left\{ L + \frac{tP_1 \lambda_1 K_1}{\Delta} \right\} \phi + \left\{ \frac{X_1 X_2}{\Delta L} \theta L \left( P_2 - P_2^* \right) \right\} \bar{K} \right] \]

References:


