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28 November 2011

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November 28, 2011

Abstract

We present a microeconomic model of social stratification, which includes an endogenous fertility component. In the model, egalitarian and stratified societies coexist. The latter are divided into two hereditary classes: a warrior elite and a productive class. The model entails that the extra cost warriors must incur to train and equip their children for war determines the relative sizes of both classes and the degree of economic inequality. Higher costs of warrior children imply a greater economic advantage for warriors and a smaller ratio of warriors to producers. These results are consistent with the historical evidence. Finally, we explore conditions under which the social contributions of the warrior elite could discourage a revolution.

Keywords: Social stratification; income inequality; warfare; military participation ratio.

1 Introduction

Most anthropological and sociological theories of social stratification in premodern societies share three recurrent themes. First, premodern stratified societies divide labour between warriors, who fight wars, and peasants, who work the land. Second, stratified societies must be able to produce a sizable food surplus (i.e., more food than is needed to feed the peasants and their families) in order to support a non-food-producing warrior elite. Third, social positions in premodern stratified societies are, for the most part, hereditary (Summers 2005).

We integrate these three themes into a microeconomic model of social stratification, which includes an endogenous fertility component. Unlike previous studies, we base our model on individual

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rational choice, rather than treating social classes as organic units. We recognise that institutions, such as hereditary nobility, constrain individual behavior. But we also recognise that, within the constraints imposed by institutions, each individual will pursue his own interest by using the means at his disposal.

Using our model, we study the effects of production technology, military technology, and the costs of weapons and military training on the class structure of premodern stratified societies. We focus on two main issues: what determines the class composition of a stratified society and what determines the degree of economic privilege of its upper classes. While explaining the emergence of stratification is beyond the scope of our model, we explore conditions under which the military and organisational services provided by the elite could discourage a lower class revolution.

Below we present a brief sketch of the model and we summarise its results.

A group of rival societies divide up a region according to the balance of military power. These societies may be egalitarian or stratified.

In an egalitarian society there are no social classes. All of its members are food producers. They hunt and gather or cultivate their own plots of land. They also serve in the local militia. Military duties do not interfere with food production.

In a stratified society there are two hereditary social classes: a warrior elite and a productive class. The warriors own the land, while the producers cultivate the land to produce the food necessary to sustain both social classes. The classes divide up the crop in predetermined shares. There are diminishing returns to labour in food production. Hence, per capita incomes in units of food are decreasing in the size of the productive class.

The warrior elite performs two social functions. First, it provides public goods that enhance the productivity of the land (e.g., the warriors could organise the construction and maintenance of large-scale irrigation systems). Second, the warriors fight in wars, using specialised weapons that only they can handle. Due to the effectiveness of the specialised weapons, an army of professional warriors is more powerful than a militia of equal size.

Fertility is an individual choice: each adult chooses how many children to have, given his income and the cost of children. These are more costly for warriors than for producers. The cost difference between warrior and producer children can be read as the extra cost that warriors must incur to train and equip their children for war (O'Connell 2002, 42; Prestwich 1996; 1968, 58). Demographic equilibrium is reached when warriors and peasants decide to reproduce at their replacement rates, given their incomes and the costs of warrior and producer children.

The equilibrium results of the model are the following.

In egalitarian societies, the producers' per capita income is increasing in the cost of producer children, and is not affected by changes in total factor productivity (TFP). Any increase in TFP will be offset over time by population growth.

In stratified societies, warriors earn higher per capita incomes than producers. This result is entirely due to demographic pressures: the warriors earn higher incomes than peasants because warrior children are more costly than producer children.

The sharecropping agreement between the social classes does not affect the equilibrium per capita incomes. If the producers get a raise in their share, producer population will grow and warrior population will decline until the per capita incomes of both classes return to their original levels. In the model, the share of the product taken by the warrior elite does not have a permanent impact on income inequality.

The producers' per capita income is the same in egalitarian and in stratified societies. This means that any distributive gains that the producers may obtain from the abolition of social classes will inevitably vanish in the long term, as a consequence of population growth.

The warriors' per capita income is increasing in the cost of warrior children, while the producers' per capita income is increasing in the cost of producer children. Per capita incomes are not affected by changes in TFP. As in the case of egalitarian societies, any increase in TFP will be offset over time by population growth.

Following Andreski (1968) we define a society's military participation ratio (MPR) as follows:

$$\text{MPR} = \frac{\text{Number of warriors}}{\text{Number of peasants}}.$$

The model implies that the MPR is increasing in the warriors share of the crop. On the other hand, an increase in the relative cost of warrior children will reduce the MPR and at the same time will sharpen income inequality in favour of warriors. This result is consistent with the historical evidence surveyed by Andreski: increases in the cost of weapons and military training tend to reduce the MPR and to increase the economic advantage of warriors (Andreski 1968, 40–41, 73).

In the remainder of this introduction we survey the sociological, anthropological and economic theories of social stratification, and we present the main empirical regularities. We also cover the basics of Malthusian population theory.

The rest of the paper is organised as follows. In Section 2 we present our model in detail. In Section 3 we derive the demographic equilibrium of the model. We also discuss its implications, and compare them with the available historical evidence. Finally, in Section 4 we make some concluding remarks.

1.1 Agriculture and social stratification

Ethnographic studies reveal that most contemporary bands of hunter-gatherers, such as the Kung in the Kalahari or the Yolngu in Arnhem Land, are egalitarian and devoid of leadership (Boehm 1999; Knauft 1994; Winterhalder 2001). We do not know how prehistoric hunter-gatherers organised themselves, but the observation of their contemporary remnants and the archaeological evidence indicate that most prehistoric hunter-gatherers lacked social stratification. Many agrarian societies, on the contrary, were socially stratified (e.g., Sumer, Ancient Egypt and Mycenaean Greece). This fact suggests that agriculture and social stratification are related in some way (Price and Gebauer 1995).

The ability to produce a surplus of food lies at the core of many theories of social stratification. The conventional argument runs as follows. Surplus food is required to support a non-food-producing upper class. Agriculture has the potential to yield a surplus and hence the potential to support a class of non-food-producers. Hunter-gatherers, on the other hand, are always living on the edge of subsistence: chronically undernourished and constantly threatened by famine [evidence of this is surveyed by Kaplan (2000)]. They are thus unable to afford a class of non-food-producers. Gordon Childe (1942, 18; 1954) is the foremost surplus theorist of social stratification. Unsurprisingly, his ideas are very popular among Marxist thinkers (e.g., Beaucage 1976, 409–410; Mandel 1962, 26, 43).

Surplus theories of social stratification have several critics. Pearson (1957), for instance, argues that all societies have the potential to produce food in excess of biological necessity. According to Pearson, it is social organisation that generates a surplus and not the other way around. Sahlins (1972/1998) maintains that the key precondition for social stratification is not the ability to produce surplus food, but the feasibility of food storage [see Cashdan (1980) and Hayden (1995)]. Without storage, Sahlins argues, there is no accumulation of wealth, and without wealth, social inequalities cannot exist. Most hunter-gatherers are nomads. As they quickly deplete local resources, they have no alternative but to keep moving. Nomadism makes storing food, and thus stratification, impossible.

The reported cases of stratified hunter-gatherer communities are located in exceptionally favourable ecological niches; for example, in rich marine and aquatic habitats (Erlandson 2001; Kennett 2005; Pálsson 1988; Soffer 1985; Soffer 1989; Testart 1982; Vanhaeren and d'Errico. 2005; Yesner 1980). In line with Sahlins' hypothesis, stratification among hunter-gatherers was particularly common in places where the abundance of food allows for permanent settlement and food storage. The Pomo people of Central California are a classic example of a stratified gathering society. Acorns, the staple of the Pomo diet before modernisation, were only available during one month in autumn. During that month the Pomo gathered the acorns and stored them for the rest of the year. The acorn stores were controlled by the chiefs (Kniffen 1939).

Whatever the preconditions for stratification may be (surplus, storage, or both), the emergence of an upper class of non-food-producers remains to be explained. Two opposing explanations have been proposed: a conflict-based explanation, advanced by Fried (1967; see also Hayden 1995), and a functionalist explanation, attributed to Service (1962; also found in Davis 1949, 367). Conflict theorists hold that 'aggrandisers' seized control of the means of production and then used the surplus to obtain a superior standard of living. This is the spirit of Acemoglu and Robinson's

(2001) economic theory of political transitions. The functionalists, on the other hand, believe that the upper classes provide goods that benefit society as a whole: they lead war parties and organise defence, build and maintain irrigation systems, store food as famine relief and manage intergroup trade. As a reward for their services, the lower classes allow the upper classes a greater share of society's wealth. Cioffi-Revilla (2005) presents a simulation model of functionalist inspiration.

Intermediate positions have emerged between these extremes. For instance, Johnson and Earle (2000) maintain that the intensification of agriculture and consequent population growth pose a number of problems that can only be solved through hierarchy and the centralisation of power: resource competition leading to raids and warfare, the risk of failure in food production, inefficient use of resources that call for major technology investments and resource deficiencies that can only be made up by foreign trade (pp. 29–32). Once power is acquired by an upper class, that group uses its power to establish privileges for itself (pp. 266–277, 301–303). At the same time, the lower classes face a trade-off between the benefits they derive from the public goods provided by the upper classes and the burden of inequality net of the cost of revolting (Boone 1992). Grossman (2002) and Leeson (2006) develop microeconomic models along these lines. Baker et al. (2010) complement the microeconomic analysis with Malthusian principles.

1.2 The rise of an hereditary warrior elite

According to Andreski (1968, 31–32), a class of warriors can emerge in two ways: either by gradual differentiation of warriors from the rest of the population, or by conquest and subjugation of another group. Gradual differentiation occurs when a group manages to monopolise arms-bearing in order to secure a privileged position in society or if the professionalisation of warriors is necessary for society to augment its military power. Andreski maintains that conquest was the most common mechanism of social stratification and provides a long list of historical cases to back up his claim: the subjugation of one city by another in Sumer (p. 42), the Dorian invasions in Greece (p. 43–44), and the Norse conquest of Russian Slavic tribes (p. 62), to mention just a few. Perhaps the chemically purest examples of stratification by conquest can be found in East Africa and Sudan. In those regions, ample kingdoms were founded through the conquest of agriculturalists by pastoralists (p. 32). In Ankole, for instance, the pastoralist Hima conquered the agricultural Iru sometime before the British colonisation. The Hima forced the Iru to pay tribute and allowed them no political rights. Only Iru men were allowed to bear arms and participate in war.

To explain the emergence of slavery, Lagerlöf (2009) proposes a microeconomic model with Malthusian foundations. In his model, an external elite forms a military alliance with an internal elite. Together they seize ownership of the land and enslave the commoners.

1.3 Social mobility

Betzig (1986) and Summers (2005) argue that the members of the upper classes use their privileged access to resources in order to further their own reproduction and that of their relatives. However, if this 'reproductive skew' is too extreme, it may eventually cause society to collapse. The lower classes may no longer be able to support the demands placed upon them by the mushrooming upper classes, and the latter may fragment as their members scrabble for an ever smaller share of the available food surplus.

The need to preserve a stable class composition puts limits on the extent to which the upper classes can outbreed the lower classes. There are various ways in which a stable class composition can be maintained. The members of the upper classes may voluntarily have fewer children, perhaps in response to their impoverishment as they become too numerous; or some upper-class individuals may become celibate and not reproduce at all; or enough of them may be killed in war before they get an opportunity to reproduce. All of these mechanisms help reduce the degree of reproductive skew. Some cultures have devised quite ingenious practices to restrain the growth of their upper classes. For instance, a newly appointed Ottoman sultan was obliged by law to kill all of his brothers. In the Central African kingdoms of Ankole and Kitara, the sons of a dead king had to fight for the throne until only one of them was left alive (Andreski 1968, 19).

The high reproduction rate of the upper classes can also be offset by inducing their excess members to leave. This may be achieved by some social rule, such as primogeniture, by virtue of which the eldest son inherits the whole family estate. The disinherited sons may sink down into the lower classes or be forced to seek their fortunes elsewhere. For example, the descendents of the king of Siam, except for his successors, were lowered in rank after each generation. After the fifth they became commoners (p. 19). In medieval Europe, noble scions roaming the country in search of fiefs were a common sight. This surplus of nobles was the principal source of knights for the Crusades (p. 137). The Austranesian stress on primogeniture forced the chiefs' younger sons to find and colonise uninhabited islands (Finney 1996).

Note that the permanent inflow of dispossessed aristocrats requires members of the lower classes to reproduce below the replacement rate (Eberhard 1962, 264–265; Lenski 1984, 190). Otherwise, the lower classes would grow beyond the point where they can provide for subsistence, and society would starve to death.

1.4 Malthusian principles of social stratification

To the extent that the adoption of agriculture involves the development of a more costly military technology and the emergence of a class of specialist warriors, not all of the extra output produced by agriculture can be translated into support for more producers and their families. A fraction of total production must be used to maintain and equip the warrior elite. This point was made by Sauvy (1969), who argued that achieving a 'power optimum' requires maximising the surplus available to support the military and the government.

The size and quality of the professional army that a society can support depends on the size of the surplus that the society can generate. In turn, the size of the surplus depends on three factors: the number of workers, the productivity of the average worker and the amount consumed by the average worker and his dependents. From Malthus onward, there has been a lively debate on the interplay between these factors (Coleman and Schofield 1986; Ashraf and Galor 2008). The classical tradition, exemplified by Malthus and Ricardo, assumed a perfectly elastic supply of labouring population at a constant subsistence wage rate together with diminishing returns to labour. If wages rise above subsistence, the labouring population will expand, leading to more employment. That will force down the marginal product of labour and thus wages. This process will only come to a halt when the marginal product of labour equals the subsistence wage, at which point the labouring population will stop growing. This is also the point at which the surplus product, in the form of rent, will be maximised.

How can population be regulated so as to generate a surplus in view of the limits set by technology and the environment? The growth of the population within a given territory is determined by a combination of fertility, mortality and migration, the relative importances of which have varied widely across time and geography. The role of migration is obvious and uncontroversial, so we shall focus on the other factors.

The original Malthusian theory assumed that population is automatically regulated through some kind of homeostatic mechanism. If the population gets too large relative to the amount of available resources, malnutrition, famine, disease and warfare will cause premature deaths. This formulation is based on the biological analogy that an animal species will blindly multiply up to the limits set by the carrying capacity of its habitat. Other formulations rely on conscious choice or social convention to limit the population. In his later writings, Malthus himself suggested prudential restraint involving late marriage or celibacy (Malthus 1820, 248–252). Abortion, infanticide and prolonged breast-feeding may also serve to space out births or get rid of unwanted children. All of these social practices were common among premodern societies (Douglas 1966; Cashdan 1985; Macfarlane 1997). Some practices were deliberately designed to limit the population, whereas others were followed without any such objective in mind. However, even non-deliberate social practices may have a homeostatic effect. Societies compete with each other and those with practices that most effectively regulate their populations may triumph over their rivals. Thus, group selection may lead to the emergence of population control practices that are well adapted to the prevailing environment (Wrigley 1978).

A controversial notion that needs to be clarified at this point is that of 'subsistence consumption.' Some versions of the Malthusian theory interpret this notion in biological terms, equating it to the minimum food intake that allows a human being to survive and produce an average of one offspring (e.g., Wolf 1966, 6). The later Malthus regarded such an idea as simplistic, and stressed the influence of socially conditioned preferences on reproductive behaviour (Malthus 1820, 248–252; Costabile and Rowthorn 1985). This was a common view among classical economists, such as Ricardo (1821, 91).

2 Model setup

2.1 Social structure

A region is populated by $M \geq 0$ competing societies. These are divided into $M_e \geq 0$ egalitarian societies and $M_s \geq 0$ stratified societies, where $M_e + M_s = M$ and $M > 0$. M_e and M_s are exogenous parameters.

The societies are indexed by $i \in \{1, 2, \ldots, M\}$. Society i is composed of $N_i > 0$ adults. The adults are divided into $N_{wi} \geq 0$ warriors and $N_{pi} > 0$ food producers, where $N_i = N_{wi} + N_{pi}$. N_{wi} and N_{pi} are endogenous variables.

If $N_{wi} = 0$, we say the society is egalitarian. If $N_{wi} > 0$, we say the society is stratified. In a stratified society, social positions are hereditary: the children of warriors become warriors, and the children of producers become producers.

2.2 Military power and land holding

If Society i is egalitarian, a non-professional militia defends the society and its military power is proportional to the number of producers. If Society i is stratified, only warriors carry arms and the military power of the society is proportional to the number of warriors. Let P_i represent Society i's military power. We define P_i as follows:

$$P_i = \begin{cases} N_{pi}^{\beta} & \text{if } N_{wi} = 0, \\ (\phi N_{wi})^{\beta} & \text{if } N_{wi} > 0. \end{cases} \tag{1}$$

The parameter $\phi > 1$ represents the relative effectiveness of a specialised military technology that only professional warriors can handle, and the parameter β measures the effectiveness of military power in acquiring or defending land. There are decreasing returns to military power, which means that $\beta \in (0, 1)$. To simplify the analysis, equation (1) assumes that in stratified societies the producers do not participate in war, or play an insignificant role. This was often the case in premodern times (Gat 2006, 298–299).

The total area of available land is given by the parameter $L_0 > 0$. The balance of military power determines the distribution of land among the M societies. Let L_i be the area controlled by

Society i. We define L_i as follows:

$$L_i = \frac{P_i L_0}{\sum_{k=1}^{M} P_k}. \tag{2}$$

This expression implies that the more powerful societies will get the larger shares of the land. Equation (2) can be rewritten in the following manner:

$$L_i = \frac{P_i L_0}{P_i + P_{-i}}, \tag{3}$$

where

$$P_{-i} = \sum_{i=1}^{M} P_i - P_i. \tag{4}$$

P_{-i} represents the combined military power of all societies with the exception of Society i.

2.3 Food production and allocation

The food production of Society i is given by

$$Y_i = A_i L_i^{\alpha} N_{pi}^{1-\alpha}, \tag{5}$$

where $\alpha \in (0,1)$ is a parameter that measures the intensiveness of land in production, and $A_i > 0$ is a variable that represents the society's total factor productivity (TFP).[1] We assume that stratified societies are at least as and possibly more technologically advanced than egalitarian societies:

$$A_i = \begin{cases} A_e & \text{if } N_{wi} = 0, \\ A_s & \text{if } N_{wi} > 0, \end{cases} \tag{6}$$

where A_e and A_s are constants, and $A_e \leq A_s$.

If Society i is egalitarian, the producers own the crop, which amounts to a total of Y_i. If Society i is stratified, the warriors take a predetermined share $\gamma \in (0,1)$ of the crop and the producers keep the rest. That is, the warrior elite gets a total of γY_i and the productive class gets a total of $(1-\gamma) Y_i$. Sharecropping agreements such as this were frequent among premodern agriculturalists (Raper and Reid 1941, 35–36). Often, the landlord's share was 50% of the crop, although shares between one third and to two thirds of the crop were frequent. Typically, the crop was divided into portions determined by tradition, rather than the market. The Spanish mediería, the Italian mezzadria, the French métayage and the Catalan masovería are notable examples of sharecropping

[1] More generally, Y_i represents the amalgamation of goods produced by society.

contracts. Nowadays, sharecropping is an infrequent but still relevant practice, especially in poor areas such as India and Pakistan.[2]

Not all people live long enough to reproduce. The parameters $\sigma_w, \sigma_p \in (0, 1]$ represent the probabilities of reaching reproductive age, for warriors and producers. There is some evidence that, before modern times, warriors had a lesser chance than producers of reaching reproductive age. Griffith (1970, 26) affirms that one third of Norwegian kings died in battle during the Viking era. According to Wrigley (1997, 206), the life expectancy at birth among the English aristocracy lagged behind that of the population as a whole until the 18th century, among other reasons, because the children of aristocrats were weaned earlier than the children of commoners. Hollingsworth (1957) reports that, during the 14th and 15th centuries, 46% of the sons of English dukes died violent deaths. The peasants, on the other hand, were free from the hazards of continual combat (although they were occasionally prey to marauding lords). Because these examples may not be typical, we make no assumption about the relative magnitudes of σ_w and σ_p.

Once the crop has been divided between the warrior elite and the productive class, each class distributes its share among its members. Since only $\sigma_w N_{wi}$ warriors and $\sigma_p N_{pi}$ producers survive, the per capita incomes of the surviving warriors and producers are given by

$$y_{wi} = \frac{\gamma Y_i}{\sigma_w N_{wi}}, \tag{7}$$

$$y_{pi} = \begin{cases} \dfrac{Y_i}{\sigma_p N_{pi}} & \text{if } N_{wi} = 0, \\ \dfrac{(1-\gamma) Y_i}{\sigma_p N_{pi}} & \text{if } N_{wi} > 0. \end{cases} \tag{8}$$

Combining equations (7) and (8) with equation (5) we can reformulate the per capita incomes as follows:

$$y_{wi} = \frac{\gamma A_i L_i^{\alpha} N_{pi}^{1-\alpha}}{\sigma_w N_{wi}}, \tag{9}$$

$$y_{pi} = \begin{cases} \dfrac{A_i L_i^{\alpha} N_{pi}^{1-\alpha}}{\sigma_p N_{pi}} & \text{if } N_{wi} = 0, \\ \dfrac{(1-\gamma) A_i L_i^{\alpha} N_{pi}^{1-\alpha}}{\sigma_p N_{pi}} & \text{if } N_{wi} > 0. \end{cases} \tag{10}$$

[2] Bardhan and colleagues (2000) analyse sharecropping agreements from the perspective of contract theory.

2.4 Consumption, reproduction and utility

The utility of an adult who survives to reproduce is given by

$$u = \frac{c^{\theta} n^{1-\theta}}{\theta^{\theta} (1-\theta)^{1-\theta}}, \tag{11}$$

where $c \geq 0$ is his food consumption and $n \geq 0$ is the number of his children.[3] The adult chooses the values of c and n. The parameter $\theta \in (0, 1)$ represents the weight of consumption in utility.

We assume that adults who die before reproducing do not consume. Taking into account the risk of premature death, a person's utility at birth is

$$\bar{u} = \sigma^{\delta} u = \frac{\sigma^{\delta} c^{\theta} n^{1-\theta}}{\theta^{\theta} (1-\theta)^{1-\theta}}, \tag{12}$$

where $\sigma \in (0, 1]$ is the probability of surviving to reproduce, and $\delta > 1$ is a measure of death aversion.

Historical evidence suggests that fertility among premodern peoples depended on the income available to them. The methods they used to limit the number of their children included abstinence, celibacy, prolonged breast-feeding, abortion and infanticide (Douglas 1966; Cashdan 1985; Macfarlane 1997). Recourse to such methods was more frequent when times were hard than in times of abundance. To capture the link between consumption, fertility and income we will assume that each surviving adult solves

$$\begin{aligned} \max_{\{c,n\}} \quad & u = \frac{c^{\theta} n^{1-\theta}}{\theta^{\theta} (1-\theta)^{1-\theta}} \\ \text{s.t.} \quad & c + \kappa n = y, \\ & c, n > 0, \end{aligned}$$

where y is a variable that represents the surviving adult's income, and $\kappa > 0$ is the exogenous cost of a child in units of food.

[3] Consumption need not consist solely of food. Both warriors and producers may use part of their incomes to purchase other goods (e.g., clothes, weapons). We do not explore this issue and assume that all goods enter into the utility function in terms of their food equivalent.

The solution to the surviving adult's problem is given by

$$c = \theta y, \tag{13}$$

$$n = \frac{(1-\theta)\, y}{\kappa}, \tag{14}$$

$$u = \frac{y}{\kappa^{1-\theta}}. \tag{15}$$

Note that the expenditures on consumption and children are constant fractions of income, and indirect utility is increasing in food income and decreasing in the cost of children. This is the standard result of the consumer problem with a Cobb-Douglas utility function.

Children are more costly for warriors than for producers, which implies that κ_w is greater than κ_p. This difference can be read as the extra cost that warriors face in order to train and equip their children for war. As an example, consider the case of Spartans, who were taken away from their mothers to start their military training as young as seven years old (O'Connell 2002, 42). Another example is given by Prestwich (1996), who reports that a complete suit of armour in the Middle Ages would cost the equivalent of a 1939 light tank. According to Andreski (1968, 58), the total cost of equipping one knight amounted to the annual income of a whole village, making knighthood a heavy financial burden.

2.5 Population dynamics

There is no migration and no mobility between social classes. Therefore, population dynamics are governed by the following laws of motion:

$$N'_{wi} = \underbrace{\sigma_w n_{wi} N_{wi}}_{\substack{\text{Children of} \\ \text{surviving warriors}}}, \tag{16}$$

$$N'_{pi} = \underbrace{\sigma_p n_{pi} N_{pi}}_{\substack{\text{Children of} \\ \text{surviving producers}}}, \tag{17}$$

where N'_{wi} and N'_{pi} are the warrior and producer populations in the next generation. Recall that survival probabilities σ_w and σ_p are parameters. This implies that any changes in population must come about through endogenous variations in the birth rates, n_{wi} and n_{pi}.

3 Equilibrium and comparative statics

This section characterises the equilibrium of the model, and presents its comparative statics. We analyse separately the equilibrium in egalitarian and stratified societies. In the case of stratified societies, we investigate the determinants of income inequality, the relative sizes of the two social classes, social privileges and the potential benefits of social stratification.

3.1 Demographic equilibrium within egalitarian societies

Suppose that Society i is egalitarian. This means that $N_{wi} = 0$ and $A_i = A_e$. Also suppose that Society i has reached its demographic equilibrium; that is,

$$N'_{pi} = N_{pi}. \tag{18}$$

Replacing this condition into equation (17), we get the producers' replacement fertility rate:

$$n_{pi} = \sigma_p^{-1}. \tag{19}$$

Since $\sigma_p \in (0, 1]$, the producers' replacement fertility rate is at least one, for the following reason. In equilibrium, the producers must have one child on average. Otherwise, their population would not be stable. Since a fraction $1 - \sigma_p$ of the producers does not survive to reproduce, the surviving producers must compensate by having more than one child.

Combining equation (14) with identity (19), we can calculate the equilibrium income of surviving producers:

$$y_{pi} = \frac{\sigma_p^{-1} \kappa_p}{1 - \theta}. \tag{20}$$

Observe in equation (20) that the equilibrium income of surviving producers is increasing in the replacement fertility rate and in the cost of producer children. Intuitively, if the replacement rate or the cost of producer children increases, the typical producer will have to spend more to maintain his fertility at the replacement rate. He will not be willing to increase his spending unless his income also increases. This increase will be achieved through a temporary reduction in fertility and subsequent population decline, which will increase labour productivity and thus the producer's income.

Also observe that the equilibrium income of surviving producers is not affected by changes in TFP, for the following reason. In the equilibrium, the typical producer must choose to reproduce at the replacement rate. This choice depends exclusively on the producer's preferences, on his income and on the cost of producer children. Since the producer's preferences and the cost of producer children are fixed, his income must adjust to persuade him to reproduce at the replacement rate.

This is achieved through a standard Malthusian-Ricardian process: any increase in TFP will be offset over time by population growth.

Using equations (1), (3), (10) and (20) we obtain implicit solutions for the equilibrium producer population and for the land area of the egalitarian society:

$$N_{pi} = \frac{(1-\theta)\, A_i L_i^{\alpha} N_{pi}^{1-\alpha}}{\kappa_p}, \tag{21}$$

$$L_i = \frac{N_{pi}^{\beta} L_0}{N_{pi}^{\beta} + P_{-i}}, \tag{22}$$

where $A_i = A_e$.

It follows from equations (21) and (22) that TFP growth within an egalitarian society causes producer population to grow and also expands the land area of this society at the expense of other societies. This can be confirmed by log-differentiating equations (21) and (22) with respect to A_i, and then solving the resulting linear system for the elasticities:

$$\frac{\partial \ln N_{pi}}{\partial \ln A_i} = \frac{L_0}{\alpha\left[(1-\beta)\, L_0 + \beta L_i\right]}, \tag{23}$$

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \frac{\beta\,(L_0 - L_i)}{\alpha\left[(1-\beta)\, L_0 + \beta L_i\right]}. \tag{24}$$

Since $\alpha > 0$, $\beta \in (0,1)$, and $L_0 > L_i > 0$, both elasticities are positive. The mathematical derivations are presented in Appendix A.1.

3.2 Demographic equilibrium within stratified societies

Suppose that Society j is stratified. This means that $N_{wj} > 0$ and $A_j = A_s$. Also suppose that Society j has reached its demographic equilibrium; that is,

$$N'_{wj} = N_{wj}, \tag{25}$$

$$N'_{pj} = N_{pj}. \tag{26}$$

Replacing these condition into equations (16) and (17), we get the warriors' and the producers' replacement fertility rates:

$$n_{wj} = \sigma_w^{-1}, \tag{27}$$

$$n_{pj} = \sigma_p^{-1}. \tag{28}$$

Since $\sigma_w, \sigma_p \in (0,1]$, both replacement rates are equal or larger than one.

Combining equation (14) with identities (27) and (28), we can calculate the equilibrium incomes of surviving warriors and producers:

$$y_{wj} = \frac{\sigma_w^{-1}\kappa_w}{1-\theta}, \tag{29}$$

$$y_{pj} = \frac{\sigma_p^{-1}\kappa_p}{1-\theta}. \tag{30}$$

Observe that the equilibrium income of surviving warriors is increasing in their replacement fertility rate and in the cost of warrior children. Analogously, the equilibrium income of surviving producers is increasing in their replacement fertility rate and in the cost of producer children. As in the case of egalitarian societies, these results arise from the combination of individual fertility choices and the Malthusian population dynamics.

Also observe that the equilibrium incomes are not affected by TFP. Any increase in TFP will be offset over time by population growth.

Finally, note that the equilibrium income of producers is the same in egalitarian and in stratified societies [compare equations (20) and (30)]. This is necessary for the producers of both types of societies to reproduce at the replacement rate.

Using equations (1), (3), (9), (10), (29) and (30) we obtain implicit solutions for the equilibrium populations of warriors and producers, and for the land area of the stratified society:

$$N_{wj} = \frac{(1-\theta)\,\gamma A_j L_j^{\alpha} N_{pj}^{1-\alpha}}{\kappa_w}, \tag{31}$$

$$N_{pj} = \frac{(1-\theta)\,(1-\gamma)\,A_j L_j^{\alpha} N_{pj}^{1-\alpha}}{\kappa_p}, \tag{32}$$

$$L_j = \frac{(\phi N_{wj})^{\beta} L_0}{(\phi N_{wj})^{\beta} + P_{-j}}. \tag{33}$$

where $A_j = A_s$.

It follows from equations (31)–(33) that TFP growth within a stratified society causes producer and warrior populations to grow and also expands the land area of this society at the expense of other societies. This can be confirmed by log-differentiating equations (31)–(33) with respect to

A_j, and then solving the resulting linear system for the elasticities:

$$\frac{\partial \ln N_{wj}}{\partial \ln A_j} = \frac{L_0}{\alpha\left[(1-\beta)L_0 + \beta L_j\right]}, \quad (34)$$

$$\frac{\partial \ln N_{pj}}{\partial \ln A_j} = \frac{L_0}{\alpha\left[(1-\beta)L_0 + \beta L_j\right]} \quad (35)$$

$$\frac{\partial \ln L_j}{\partial \ln A_j} = \frac{\beta\left(L_0 - L_j\right)}{\alpha\left[(1-\beta)L_0 + \beta L_j\right]}. \quad (36)$$

Since $\alpha > 0$, $\beta \in (0,1)$, and $L_0 > L_j > 0$, the three elasticities are positive. The mathematical derivations are presented in Appendix A.2.

The military power of a representative stratified society is given by ϕN_{wj}, while the military power of a representative egalitarian society is simply N_{pj}. If the following condition is met, stratified societies will be more powerful than egalitarian societies:

$$\phi^{\alpha}\left(1-\gamma\right)^{1-\alpha}\gamma^{\alpha}\frac{A_s}{A_e} > \left(\frac{\kappa_w}{\kappa_p}\right)^{\alpha}. \quad (37)$$

Therefore, stratified societies will be more powerful than egalitarian societies if specialised weapons are very effective, or if warriors are relatively cheap, or if the warriors' share of the crop is close to α. A full derivation of condition (37) is provided in Appendix A.3.

3.2.1 Income inequality and military participation ratio

The expected incomes of warriors and producers are given by the following expressions:

$$\bar{y}_{wj} = \sigma_w y_{wj} + (1-\sigma_w)\cdot 0, \quad (38)$$

$$\bar{y}_{pj} = \sigma_p y_{pj} + (1-\sigma_p)\cdot 0, \quad (39)$$

Replacing the equilibrium values of y_{wj} and y_{pj}, given equations (29) and (30), into equations (38) and (39) we get

$$\bar{y}_{wj} = \frac{\kappa_w}{1-\theta}, \quad (40)$$

$$\bar{y}_{pj} = \frac{\kappa_p}{1-\theta}. \quad (41)$$

Since $\kappa_w > \kappa_p$, it is direct that $\bar{y}_{ws} > \bar{y}_{ps}$. That is, in the equilibrium, warriors earn higher expected incomes than producers.

In the short run, population and hence production are fixed. Therefore, an increase in the warriors' share of the crop will immediately increase the warriors' expected income, and will reduce the producers' expected income. In the long run, however, population growth will inevitably bring the expected incomes back to their original level. The agreed division of the crop, captured by parameter γ, cannot affect incomes in the long run.

The ratio $\bar{y}_{ws}/\bar{y}_{ps}$ gives us a measure of the equilibrium degree of income inequality:

$$\frac{\bar{y}_{wj}}{\bar{y}_{pj}} = \frac{\kappa_w}{\kappa_p}. \tag{42}$$

Observe that the equilibrium degree of income inequality is increasing in the relative cost of warrior children, while the agreed division of the crop has no effect. This means that the share of the product taken by the warrior elite has no permanent impact on income inequality. In the model, the source of income inequality is purely demographic.

From equations (31) and (32) we obtain the society's equilibrium class composition:

$$\text{MPR} = \frac{N_{wj}}{N_{pj}} = \frac{\gamma}{1-\gamma}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1}. \tag{43}$$

Following Andreski (1968, 33), we will refer to ratio N_{wj}/N_{pj} as the society's military participation ratio, or MPR. As it is natural, the MPR is increasing in the warriors' share of the crop:

$$\frac{\partial \ln \text{MPR}}{\partial \ln \gamma} = \frac{1}{1-\gamma}. \tag{44}$$

Since $\gamma \in (0,1)$, this elasticity is greater than one.

An increase in the relative cost of warrior children will reduce the MPR and sharpen income inequality in favour of warriors:

$$\frac{\partial \ln \text{MPR}}{\partial \ln (\kappa_w/\kappa_p)} = -1, \tag{45}$$

$$\frac{\partial \ln(\bar{y}_{wj}/\bar{y}_{pj})}{\partial \ln (\kappa_w/\kappa_p)} = 1. \tag{46}$$

This result is consistent with the main stylised fact detected by Andreski: increases in the cost of weapons and military training tend to reduce the MPR and to increase the economic advantage of warriors (Andreski 1968, 40–41, 73). Andreski provides historical examples from a wide array of civilisations (pp. 39–72). We reproduce three of his examples to give the reader a taste of the historical evidence.

Persia— In the times of the Achaemenid Empire, the Persian army consisted of nobles and freemen. The MPR was high since the freemen were very numerous. Most of the army battled on foot, supported by a minimal cavalry. The main weapons in use were the bow and the long spear. Protective armour was scanty and uncommon. When the Sassanid dynasty rose to power in the third century A.D., it introduced a series of effective but very expensive military innovations; most importantly, the stirrup and heavy protective armour. As a result, the freemen disappeared, the warrior nobility shrank while its privileges expanded, and the peasants were reduced to harsh servitude (pp. 46–47).

Poland— The original Polish kingdom was despotic. Freemen and the king's personal guard, the Druzhina, comprised the army. Both groups were armed with primitive weapons and did not wear body armour. Gradually, the army incorporated more advanced equipment. Heavily armed horsemen were the mainstay of the Polish forces that repelled the Teutonic Knights in Grünewald (1410 AD). The modernisation of the army was accompanied by a reduction of the MPR and an increase in social inequalities: peasants were reduced to the status of serfs, and military service was restricted to the nobility (pp. 59–60).

England— The Norman conquest of England, which introduced heavy cavalry to the country, sharpened social inequalities relative to the preceding Anglo-Saxon period. This process began to be reversed during the wars against the Welshmen, when English warriors learned how to use the long bow. An inexpensive yet formidable weapon, the long bow was far superior to any other type of bow. In combination with cavalry, it was able to inflict enormous damage on enemy forces. The adoption of the long bow forced profound changes in military tactics and organisation. As a consequence of these changes, serfdom virtually disappeared from England, yeomen thrived, the MPR increased and social inequality became much less pronounced (pp. 64–65).

3.2.2 Privileges

In the equilibrium, the warriors' and producers' utilities at birth are given by

$$\bar{u}_{wj} = \frac{\sigma_w^{\delta-1}\kappa_w^{\theta}}{(1-\theta)}, \tag{47}$$

$$\bar{u}_{pj} = \frac{\sigma_p^{\delta-1}\kappa_p^{\theta}}{(1-\theta)}, \tag{48}$$

which follows from equations (12), (15), (29), and (30).

Using the equilibrium levels of utility at birth we can build a measure of the warriors' degree of privilege:

$$\frac{\bar{u}_{wj}}{\bar{u}_{pj}} = \underbrace{\left(\frac{\sigma_w}{\sigma_p}\right)^{\delta-1}}_{\gtrless 1} \cdot \underbrace{\left(\frac{\kappa_w}{\kappa_p}\right)^{\theta}}_{>1} \gtrless 1, \tag{49}$$

since $\delta > 1$, $\theta > 0$, $\kappa_w > \kappa_p > 0$, and $\sigma_w, \sigma_p > 0$.

If warriors face a higher risk of death than producers (i.e., if $\sigma_w < \sigma_p$), the warrior's degree of privilege could be lower than one, which means that the producers would be the privileged class. This is possible, but unlikely, as warrior children would probably shun the military career. Some form of compulsion or indoctrination would thus be required to keep the warrior elite from disbanding. If, on the contrary, warriors enjoy more utility than producers, coercion may be needed to enforce property rights or, in extremis, to prevent a revolution. This is probably why many warrior nobilities kept the commoners disarmed. Before the westernisation of Japan, for example, the bearing of arms was a strict prerogative of the nobles (with the very brief exception of the Taikwa reforms period during the 7th century). No peasant rebellion ever succeeded in Japan (Andreski 1958, 50).

Some authors suggest mechanisms that would allow a privileged upper class to subsist without coercing the lower classes. For example, if groups are segregated and investments in human capital generate positive externalities within groups, then individual choices may lead to self-perpetuating economic differences between groups (Lundberg and Startz 1998). In addition, upper-class propaganda could deceive the lower classes into believing that economic inequalities are in their best interest (Cronk 1994; DeMarrais et al. 1996). If people tend to be influenced by members of their own social classes, lower-class people could just learn to play their disadvantaged role in society without questioning (Henrich and Boyd 2008). Yet another possibility is that the network structure of a society may impede the formation of lower-class coalitions that could force a redistribution of wealth (Kets et al. 2011).

3.2.3 Benefits and costs of stratification

In our model, the existence of a warrior elite impinges in various ways on the productive class. On the one hand, the elite performs useful social functions: it expands and defends the frontiers of society, and it provides public goods that enhance the productivity of the land. On the other hand, the elite creams off a substantial share of food production for its own use.

Getting rid of the elite will produce redistributive gains for the productive class. However, the lack of warriors could reduce the military power of the society, exposing it to predation from other societies. Productivity may also fall as the organisational and intellectual skills of the elite are lost. The net impact on the producers' living standards will depend on the relative magnitude of these various effects.

As an illustration, consider the following thought experiment. A stratified society has reached its long-run equilibrium. One day, the producers revolt against the elite and kill all the warriors. For simplicity, assume that the revolution does not impose any direct costs on producers, and no producer is killed during the revolution.

Before the revolution, the income of the average producer was

$$\bar{y}_{pj} \quad = \quad \frac{(1-\gamma)\, A_s N_{pj}^{1-\alpha}}{N_{pj}} \left(\frac{(\phi N_{wj})^{\beta}\, L_0}{(\phi N_{wj})^{\beta} + P_{-j}} \right)^{\alpha}. \tag{50}$$

After the revolution, the income of the average producer is

$$\bar{y}'_{pj} \quad = \quad \frac{A_e N_{pj}^{1-\alpha}}{N_{pj}} \left(\frac{N_{pj}^{\beta} L_0}{N_{pj}^{\beta} + P_{-j}} \right)^{\alpha}. \tag{51}$$

To facilitate the math, assume that Society j is small in comparison to the aggregate of the other societies. This means that

$$\bar{y}_{pj} \quad \approx \quad \frac{(1-\gamma)\, A_s N_{pj}^{1-\alpha}}{N_{pj}} \left(\frac{(\phi N_{wj})^{\beta}\, L_0}{P_{-j}} \right)^{\alpha}. \tag{52}$$

$$\bar{y}'_{pj} \quad \approx \quad \frac{A_e N_{pj}^{1-\alpha}}{N_{pj}} \left(\frac{N_{pj}^{\beta} L_0}{P_{-j}} \right)^{\alpha}. \tag{53}$$

Note the differences between $\bar{y}_{pj}$ and $\bar{y}'_{pj}$. First, TFP falls from A_s to A_e, because the public goods provided by the warriors are no longer available after the revolution. Second, the military power of the society changes from $(\phi N_{wj})^{\beta}$ to N_{pj}^{β}, since the non-professional militia does not use specialised weapons. Third, the term $1-\gamma$ disappears from the expression, because the warriors no longer take a fraction γ of the total food production.

The ratio $\bar{y}_{pj}/\bar{y}'_{pj}$ gives us a measure of the net social contribution of the warrior elite:

$$\mathrm{SCW} \quad = \quad \frac{\bar{y}_{pj}}{\bar{y}'_{pj}} \quad = \quad (1-\gamma)\, \phi^{\alpha\beta} \frac{A_s}{A_e} \left(\frac{N_{wj}}{N_{pj}} \right)^{\alpha\beta}. \tag{54}$$

Combining equations (43) and (54), we obtain the equilibrium value of SCW:

$$\mathrm{SCW} \quad = \quad \gamma^{\alpha\beta} (1-\gamma)^{1-\alpha\beta}\, \phi^{\alpha\beta} \frac{A_s}{A_e} \left(\frac{\kappa_w}{\kappa_p} \right)^{-\alpha\beta}. \tag{55}$$

If SCW is greater than unity, social stratification provides a net benefit for producers. However, if SCW is less than unity, social stratification harms the producers. It is reasonable to think that, all else being equal, the probability of a revolt will be lower the larger the net benefits of stratification, or the smaller its net harm.

Observe that the social benefits of stratification are increasing in the effectiveness of the specialised weapons, and in the technological advantage of stratified societies, which is given by A_s/A_e. Also, the benefits are decreasing in the relative cost of warrior children, since a fall in the cost of warrior children allows a society to finance a larger army.

The warriors' share of the crop, denoted by γ, has an ambiguous effect on the social benefits of stratification. Log-differentiating SCW with respect to γ we get

$$\frac{\partial \ln \mathrm{SCW}}{\partial \ln \gamma} = \frac{\alpha\beta - \gamma}{1-\gamma}, \tag{56}$$

which will be positive if and only if $\gamma < \alpha\beta$.

The source of this ambiguity is the following. On the one hand, a high γ implies that getting rid of the warriors will liberate large quantities of food to be distributed among the current generation of producers. On the other hand, getting rid of the warriors will alter the military power of the society. If γ is large, the society's MPR will be large before the revolution. Thus, the militia formed after the revolution will be less powerful than the army which it replaces. The society's neighbours will take advantage of this weakness, capturing land from the emancipated producers. Food production will suffer from the loss of land. In the end, the producers' income may be even lower than before the revolution.

The previous analysis assumes that social structures come in two extreme types: egalitarian societies armed with primitive weapons, and stratified societies armed with advanced weapons. In practice, a variety of hybrid social structures is conceivable. In many premodern societies, a core of elite warriors was supplemented by cheaply equipped professional soldiers or a citizen militia. This practice reached its zenith in Ancient Greece, whose city-states relied heavily on part-time armies of citizen farmers (Gat 2006, 310). Note, however, that Greek city-states were not very egalitarian by modern standards, since they typically included a slave population who performed many of the menial and manual tasks.

The possibility of hybrid social structures, such as that of the Greek city-states, implies that SCW only sets up an upper bound for the social contribution of warriors. Nevertheless, this fact does not invalidate the comparative statics that follows from the previous analysis. Even if the social contribution of warriors turns out to be negative, increases in SCW should make a revolution less likely.

There is also the possibility that the emancipated producers are able to perform the functions of the displaced warrior elite. The producers could in theory raise taxes to train and equip some members of their own class to serve as professional soldiers and administrators. In this case, the newly formed egalitarian society would retain the use of specialized weapons and the public goods provided by the warrior elite before the revolution. However, taking over the functions of the warrior elite also imposes costs on society. Reallocating some producers to military and administrative tasks will reduce production. In addition, the taxes will further reduce the producers' per capita income. The taxes will be higher the higher the cost of training and equipping soldiers and administrators. Moreover, the possibility exists that the appointed soldiers mount a coup and subjugate the rest of the population. To reduce the likelihood of a coup, the egalitarian society may have to restrict the size of its army, which will reduce the effectiveness of the army in its offensive and defensive functions. In sum, the producers will have to consider all these costs when they decide whether or not to rebel against the warrior elite.

In our model, the role of the warrior elite is similar to that of the 'king' in Grossman (2002). In his model, a privileged elite provides a public service whose benefits to the rest of the population outweigh the cost of supporting the elite. There are, however, two differences between our model and Grossman's. In Grossman's model, the elite defends producers against internal predators who would otherwise steal part of their output. In our model, the elite defends producers against external predators and increases the productivity of the land. A second difference concerns the duration of the benefits. In Grossman's model, the policing activities of the elite ensure a permanently higher standard of living for producers, whereas in our model endogenous population growth will eventually bring the producers' standard of living back to the subsistence level.

In a related paper, Acemoglu and Robinson (2001) model the transition from a non-democratic society controlled by a rich elite to a democracy. They find that the poor will threaten to revolt when the cost of revolting is low; for example, during recessions. This threat may force the elite to democratise. On the other hand, the redistributive nature of democratic regimes may encourage the elite to mount a coup. The more unequal the distribution of resources, the more likely it is that society will end up oscillating between democratic and non-democratic regimes. Unlike our model, Acemoglu and Robinson's model does not account for the effects of demographic forces.

4 Concluding remarks

In our model, equilibrium requires that the number of warriors and the number of producers remain constant through time. Thus, per capita incomes must be just enough to ensure that the average member of each social class (taking into account individuals who die before reproducing) has exactly one child. Since warrior children are more costly than producer children, the per capita income of warriors must be higher than the per capita income of producers. Otherwise, warriors would choose to have too few children, and the ratio of warriors to producers would fall. Warriors use part of their higher income to train and equip their children for war, and part to finance their own superior level of consumption. If the relative cost of warrior children increases, the warriors will require even higher incomes to induce them to reproduce at the same rate as producers. As a result, income inequality will increase. The new equilibrium will be achieved through changes in warrior and producer populations. The warrior population will decline to increase their per capita income. Conversely, the producer population will grow to reduce their per capita income. It follows that the military participation ratio (MPR) must fall if the relative cost of warrior children increases.

In order to increase its realism, our model could be extended in various ways.

The assumption of strict social immobility could be relaxed, allowing warriors to father children who move down the social scale in later life. This extension is important because differential reproduction is a form of social inequality (Betzig 1986).

The ecology of stratified and egalitarian societies could also be analysed. A natural way would be to integrate our model with some existing ecological model. The Ideal Despotic Distribution Model would be a natural choice (Bell and Winterhalder 2011; Kennett and Winterhalder 2008; Kennett et al. 2009; Winterhalder et al. 2010).

In our model, we examine how the given land area is divided up between egalitarian and stratified societies, but in doing so the number of each type of society is taken as given. An extended model with an ecological component could be used to explore the processes by which societies rise and fall, and the mechanisms by which one type of society replaces another. For example, as stratified societies grow larger, they use their military power to seize the most productive lands, pushing egalitarian societies into increasingly marginal ecological niches (Scott 2010, 3–9). Our model could be extended so that stratified societies deliberately try to expand into their neighbours' most productive lands. In the extended model, a minor improvement in weapons technology might eventually transform into a big military advantage. Initially, a stratified society would use its improved weaponry to capture from its egalitarian neighbours a small area of their best land. Production from this extra land would used by the stratified society to sustain a bigger army. Meanwhile, the defeated egalitarian societies would lose some of their population, no longer able to produce enough food to sustain it. Consequently, the military superiority of the stratified society would increase. The stratified society would then take advantage of its increased superiority to capture its neighbours second-best land, igniting a chain reaction in which the stratified society becomes ever larger and more powerful, and egalitarian societies ever smaller and weaker. The chain reaction will come to an end only when a hypertrophied stratified society is surrounded by many small egalitarian societies, each confined to an area of low fertility soil.

Finally, the model could be extended to endogenise the social norms and institutions that provide structure to stratified societies, beginning with the class system itself. Militarism, hereditary nobility and primogeniture could also be endogenised. A plausible mechanism for the emergence of these norms and institution is group selection. According to some authors, group selection can explain the evolution of many individual behaviours that improve the fitness of the society, even if those behaviours are costly at the personal level (see, for instance, Bowles et al. 2003, Henrich

2004, and van den Bergh and Gowdy 2009). The spread of social stratification, despite its social injustices, may have been an endogenous evolutionary response to intersocietal competition.

A Derivation of the mathematical results

A.1 Derivation of equations (23) and (24)

Log-derivating both sides of equation (21) we get

$$\frac{\partial \ln N_{pi}}{\partial \ln A_i} = 1 + \alpha \frac{\partial \ln L_i}{\partial \ln A_i} + (1-\alpha) \frac{\partial \ln N_{pi}}{\partial \ln A_i}. \tag{57}$$

Analogously, log-derivating both sides of equation (22) we get

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \beta \frac{\partial \ln N_{pi}}{\partial \ln A_i} - \frac{\partial \ln \left(N_{pi}^{\beta} + P_{-i} \right)}{\partial \ln A_i}. \tag{58}$$

But, from the chain rule we know that

$$\frac{\partial \ln [f(x)]}{\partial \ln x} = \frac{x}{f(x)} \frac{\partial f(x)}{x}. \tag{59}$$

Applying this identity in equation (58) we obtain

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \beta \frac{\partial \ln N_{pi}}{\partial \ln A_i} - \frac{\beta N_{pi}^{\beta}}{N_{pi}^{\beta} + P_{-i}} \frac{\partial \ln N_{pi}}{\partial \ln A_i}. \tag{60}$$

From this result and definitions (1) and (3), it follows that

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \beta \frac{\partial \ln N_{pi}}{\partial \ln A_i} - \beta \frac{L_i}{L_0} \frac{\partial \ln N_{pi}}{\partial \ln A_i}. \tag{61}$$

Equations (57) and (61) form a linear system for the elasticities of N_{pi} and L_i with respect to A_i. The solutions of this system are

$$\frac{\partial \ln N_{pi}}{\partial \ln A_i} = \frac{L_0}{\alpha \left[(1-\beta) L_0 + \beta L_i \right]}, \tag{62}$$

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \frac{\beta (L_0 - L_i)}{\alpha \left[(1-\beta) L_0 + \beta L_i \right]}, \tag{63}$$

as we set out to prove.

A.2 Derivation of equations (34), (35) and (36)

Log-differentiating both sides of equations (31), (32) and (33) with respect to A_j we get

$$\frac{\partial \ln N_{wj}}{\partial \ln A_j} = 1 + \alpha \frac{\partial \ln L_j}{\partial \ln A_j} + (1-\alpha) \frac{\partial \ln N_{pj}}{\partial \ln A_j}, \tag{64}$$

$$\frac{\partial \ln N_{pj}}{\partial \ln A_j} = 1 + \alpha \frac{\partial \ln L_j}{\partial \ln A_j} + (1-\alpha) \frac{\partial \ln N_{pj}}{\partial \ln A_j}, \tag{65}$$

$$\frac{\partial \ln L_j}{\partial \ln A_j} = \beta \frac{\partial \ln N_{wj}}{\partial \ln A_j} - \frac{\partial \ln \left[(\phi N_{wj})^{\beta} + P_{-j} \right]}{\partial \ln A_j}. \tag{66}$$

But, from the chain rule we know that

$$\frac{\partial \ln [f\ (x)]}{\partial \ln\ x} = \frac{x}{f\ (x)} \frac{\partial f\ (x)}{\partial x}. \tag{67}$$

Applying this identity in equation (66) we obtain

$$\frac{\partial \ln L_j}{\partial \ln A_j} = \beta \frac{\partial \ln N_{wj}}{\partial \ln A_j} - \frac{\beta\, (\phi N_{wj})^{\beta} L_0}{(\phi N_{wj})^{\beta} + P_{-j}} \frac{\partial \ln N_{wj}}{\partial \ln A_j}. \tag{68}$$

From this result and definitions (1) and (3), it follows that

$$\frac{\partial \ln L_j}{\partial \ln A_j} = \beta \frac{\partial \ln N_{wj}}{\partial \ln A_j} - \beta \frac{L_j}{L_0} \frac{\partial \ln N_{wj}}{\partial \ln A_j}, \tag{69}$$

Equations (64), (65) and (69) form a linear system for the elasticities of N_{wj}, N_{pj} and L_j with respect to A_j. The solutions of this system are

$$\frac{\partial \ln N_{wj}}{\partial \ln A_j} = \frac{L_0}{\alpha \left[(1-\beta) L_0 + \beta L_j \right]}, \tag{70}$$

$$\frac{\partial \ln N_{pj}}{\partial \ln A_j} = \frac{L_0}{\alpha \left[(1-\beta) L_0 + \beta L_j \right]} \tag{71}$$

$$\frac{\partial \ln L_j}{\partial \ln A_j} = \frac{\beta\, (L_0 - L_j)}{\alpha \left[(1-\beta) L_0 + \beta L_j \right]}. \tag{72}$$

as we set out to prove.

A.3 Derivation of condition (37)

For an arbitrary egalitarian Society i, equations (2), (21) and (22) imply that

$$N_{pi} = \frac{(1-\theta) A_e L_i^{\alpha} N_{pi}^{1-\alpha}}{\kappa_p}, \tag{73}$$

$$L_i = \frac{N_{pi}^{\beta} L_0}{\sum_{k=1}^{M} P_k}. \tag{74}$$

Together, these equations yield

$$N_{pi}^{\alpha(1-\beta)} = \frac{(1-\theta) A_e}{\kappa_p} \left(\frac{L_0}{\sum_{k=1}^{M} P_k} \right)^{\alpha}. \tag{75}$$

For an arbitrary stratified Society j, equations (31), (75) and (33) imply that

$$N_{wj} = \frac{(1-\theta) \gamma A_s L_j^{\alpha} N_{pj}^{1-\alpha}}{\kappa_w}, \tag{76}$$

$$N_{pj} = \frac{(1-\theta)(1-\gamma) A_s L_j^{\alpha} N_{pj}^{1-\alpha}}{\kappa_p}, \tag{77}$$

$$L_j = \frac{(\phi N_{wj})^{\beta} L_0}{\sum_{k=1}^{M} P_k}. \tag{78}$$

Together, these equations yield

$$(\phi N_{wj})^{\alpha(1-\beta)} = \frac{\phi^{\alpha}(1-\theta)\gamma A_s}{\kappa_w} \left(\frac{\gamma}{1-\gamma} \left(\frac{\kappa_w}{\kappa_p} \right)^{-1} \right)^{-(1-\alpha)} \left(\frac{L_0}{\sum_{k=1}^{M} P_k} \right)^{\alpha}. \tag{79}$$

From (75) and (79) it follows that

$$\left(\frac{\phi N_{wj}}{N_{pi}} \right)^{\alpha(1-\beta)} = \phi^{\alpha} (1-\gamma)^{1-\alpha} \gamma^{\alpha} \frac{A_s}{A_e} \left(\frac{\kappa_w}{\kappa_p} \right)^{-\alpha} \tag{80}$$

The stratified society will be more powerful that the egalitarian society if $\phi N_{wj} > N_{pi}$, and hence if the right-hand side of the above equation is greater than unity. This will be the case if

$$\phi^{\alpha} (1-\gamma)^{1-\alpha} \gamma^{\alpha} \frac{A_s}{A_e} > \left(\frac{\kappa_w}{\kappa_p} \right)^{\alpha},$$

as we set out to prove.

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