Social security and retirement across the OECD

Jorge Alonso-Ortiz

Instituto Tecnologico Autonomo de Mexico

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Jorge Alonso-Ortiz
Centro de Investigación Económica, ITAM

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Abstract

Employment to population ratios differ markedly across OECD countries, especially for people over 55. Social security features also differ markedly across the OECD, particularly with respect to replacement rates, entitlement ages and earnings tests. I conjecture that differences in social security features explain many differences in employment to population ratios at older ages. I assess my conjecture quantitatively with a life cycle general equilibrium model of retirement. At ages 60-64 the correlation between my model’s simulations and observed data is .67. Replacement rates and the earnings tests are key features.

Keywords: Social security, retirement, idiosyncratic labor income risk

JEL Codes: E24, H53, J14, J26
1 Introduction

Employment to population differ markedly across OECD countries. In 2006 the employment ratios ranged from 42% in Turkey to 66% in Norway. Differences are most significant for older people, as illustrated by ratios at ages 60-64; which ranged from 13% in Hungary to 60% in New Zealand. At the same time, there are large differences on many features of social security systems across the OECD. For example, the net replacement rate\(^1\) ranged from 38% in Mexico to 124% in Turkey, while the entitlement age varied from 55 in Australia to 67 in Norway. Some countries (such as Denmark) do not social security benefits to be collected by those who work, whereas others (such as Canada) permit workers to collect benefits. A third group of countries make social security to be means tested. In the US, this means test is called the “earnings test” and I will refer to means test as such from now on.

My paper seeks to answer two questions: Can differences in social security features account for large differences in employment to population at older ages? And what features of social security are the most important contributors to differences in employment to population? The answer to these two questions is crucial to assess the current policy debate on social security reform in aging societies. It is also a good exercise to validate whether a standard model of policy evaluation can deliver cross country differences in employment as we see in the data.

To answer to these questions I develop a life cycle general equilibrium model of retirement, with a discrete labor choice, idiosyncratic labor income risk and incomplete markets. I calibrate the model to match key statistics of the US economy and its social security system. A key feature of my model is that I am able to capture heterogeneity in employment by age found in the data. Idiosyncratic labor income risk makes that people in the model economy are ex-post heterogeneous and therefore make different retirement decisions. This

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\(^1\)The OECD supplies a variety of measures of the generosity and progressively of social security systems. In particular, the net replacement rate measures the promised entitlement relative to average individual earnings at the age of retirement when taxes on these entitlements are taken into account.
is consistent with data, as in the US more than 60% of the population is working at age 62 and 40% is still working at age 65. My model is able to capture this feature very well.

To evaluate the effects of cross country differences in social security, I solve for the stationary equilibrium calibrated to the US, but with social security systems of each OECD country. I ask what would happen if suddenly the US had a completely different social security system.

It turns out that differences in social security account for two thirds of the differences in employment to population at ages 60-64 and ages 65-69. This imply that variations in social security only make my model able to match the employment to population age profile of many countries in my sample.

Another way of summarizing this finding is using the coefficient of variation of employment to population across OECD countries by age. At ages 60-64 this statistic is .45 in the data and .42 in the model. At ages 65-69 it is .80 in the data and .70 in the model. As a matter of fact, when I run a regression between my model’s predictions and data at ages 60-64 I find that the $R^2$ equals .90. Using different assessments I consistently document the crucial importance of the incentives that social security systems provide to people older than 50.

When I ask what features of social security are salient, I find that the replacement rate and the “earnings test” are of utter importance, while differences in retirement age are not. This result is salient as many policy makers believe that increasing retirement age is the the way to increase employment to population at older age. Reduced form regression analyses find that there is a positive correlation between entitlement age and retirement age, however they do not take into account that savings decisions change dramatically under different social security systems. As a result, people’s planned retirement age does not change that much.

To assess the magnitude of each feature, I shut them down to US levels one at a time. I find that the coefficient of variation of employment to population at ages 60-64 in the model is .20 when there are difference only in the replacement rate and .22 when countries have differences only in “earnings tests.” In contrast, the coefficient of variation is .05 when there are solely differences in
the entitlement age. It follows that the replacement rate and the “earnings tests” account for roughly 50% of the variability in the model.

My paper is most related to two streams of literature. The first follows Prescott (2004,) who sought to explain large differences in hours of work per person through differences in the average tax rates for G-7 economies. However this raises the question about which features of tax are transfer programs are most important to his results. It also raises the question on the relative importance of the extensive versus the extensive margin. Most of the variability of hours per person is accounted for by the extensive margin, so models that explicitly incorporate this feature are worth exploring. Prescott et al. (2007) and Roger-son & Wallenius (2009) introduce an extensive-intensive margin to explore the effect of a simple tax and transfer program on hours of work. It turns out that their results are similar to those in Prescott (2004.) Wallenius (2009) extends this framework to include human capital accumulation to study differences in hours per capita in Belgium, France and Germany and she incorporates features of social security. She finds that social security plays a role on hours of work through the extensive margin. Guvenen et al. (2009) examine the role of progressive taxation in accounting for the evolution of ex-ante wage inequality in Continental Europe relative to the US. They find that different features of tax on income, in particular progressive taxation, are able to account for most differences in wage variance. Relative to Wallenius (2009) my paper has two novel features. My model incorporates heterogeneity, through idiosyncratic productivity and mortality risk. Idiosyncratic productivity is crucial to match the distribution of retirement that is found in the data, whereas everybody retires at the same age in her model. Mortality risk is critical to evaluate the true impact of social security on hours per person. My model is also applied to a large sample of countries, allowing to quantitatively measure the importance of social security to account for the variation in employment to population at older age. Having a large sample of countries helps to identify which features of social security are most salient.

While I also find large effects brought about by social security, heterogeneity reduces the impact on employment to population. Why? Older people weight
less than younger people as my model takes into account mortality risk. To investigate the role of heterogeneity I cut the variance of the income risk by half relative to my benchmark economy and recalibrate my model. I find that idiosyncratic productivity risk increases the effect of every feature of social security, which is not surprising when individuals are risk averse. A country with a social security system that is twice as generous, on average, than the US will have an employment to population ratio 3 percentage points below the US, whereas it would be 6 percentage points in my benchmark economy. This means that it doubles the effect on the extensive margin. The employment to population at ages 60-64 will be 9 percentage points below the US, whereas in my benchmark economy it would be 25 percentage points. So including idiosyncratic labor income risk is critical to quantify the role of social security.

A second stream of literature uses reduced form econometric models, microsimulation and structural models. I will not survey this literature here as it is vast. They study both positive and normative aspects of social security. Relative to the reduced form and microsimulation literature my work allows people to change their behavior when social security rules change. This is crucial to find that increases in entitlement age would have meagre effects on employment to population at older ages. However, increasing entitlement may still be a good policy as it will ease the burden of social security finances. Most related to my work is French (2005 & 2007.) He develops a model with labor income and health risk to study the role of social security into accounting for retirement behavior in the US. I find similar results in a general equilibrium environment and applied to a large sample of countries: the interaction of risk, market incompleteness and social security matters to understand retirement decisions.

2 Employment and Social Security in the OECD

This section presents empirical evidence for OECD countries in 2006. I use labor force statistics by age and sex from the OECD on-line database, social security data from “Pensions at Glance 2009” and productivity data from the “Total Economy Database". To study the role of social security in accounting for cross country differences in retirement I collect from the OECD database on employment to population and employment to population at ages 20-75, 50-54, 55-59, 60-64, 65-69 and 70-74. Even though employment to population at ages 20-75 (which can be considered total employment to population) is not the main objective of this paper, it is useful as a benchmark to understand the magnitude of cross national differences in employment at older ages.

Figure 1 shows the magnitude of employment to population at ages 60-64 (a) compared to the total employment to population (b.) Turkey has the lowest employment rate at 42%, whereas Norway has the highest at 66%. Differences become even larger for older individuals. For example, if we look at the employment to population at ages 60-64, differences range from 13% in Norway to 60% in New Zealand. The US is at the upper end of the distribution with employment to population of 65% and 55% for ages 60-64.

![Graphs showing employment differences in OECD](image)

**Figure 1. Employment differences in OECD**

Social security systems are complex, and they differ in many dimensions. For

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3 The Conference Board and Groningen Growth and Development Center
example, consider three countries: Belgium, France and the US. In Belgium to qualify for full social security benefits one has to be at least 65 with no fewer than 45 years of employment, although one may be entitled to reduced benefits if she is at least 60 and has worked no fewer than 35 years. Social security depends also on marital status and the presence of dependents in the household. Belgian social security is means tested and includes different types of allowances (i.e. vacation allowances.) In France one needs to be at least 60 and have been employed at least 40 years but if one entered the labor force at ages 14-16 she may qualify for full benefits at ages 56-59. One may continue any gainful activity and collect social security but a worker has to wait 6 months out of employment when she claims entitlement to cash her first social security check. In practice that means that you must leave the labor force to collect social security benefits. One may also defer social security benefit collection subject to some conditions, and there are “solidarity pensions” that do not depend on earnings. Social security is based on the best 25 years, it is indexed to cost of living and marital status as well. In the US social security is not simpler than in Continental Europe countries. Individuals are entitled to full benefits at age 65, but they may collect reduced benefits at age 62. Benefit reductions can be compensated by suspending benefit claims later on; this compensation it is roughly actuarially fair. Any individual is required to be employed at least 10 years to qualify. Dependents are also entitled to benefits; these depend on family structure.

Given these complexities a good question to ask is what of these features are key to understand their effect on retirement decisions. The empirical literature usually compute accrual rates and other measures that are correlated to employment rates at the age of retirement, but those are just by-products of the primitive rules of the system. I rather focus on simple measures based on the primitives of social security rules. I focus on three simple features that could be retrieve by anybody from the OECD web: replacement rates, entitlement ages and “earnings tests.” The definition of replacement rate that I use is the ratio of social security net benefits at entitlement age to individual average net earnings at entitlement age for a single male whose earnings equals the average
earnings of the economy (AW hereafter) and entered employment at age 20 with no career breaks. The OECD provide alternative definitions social security benefits’ generosity but given that my model abstracts from other taxes and taxes on social security benefits, this is the closer measure of what an average individual gets out of his lifetime earnings to expend on consumption after retirement. The definition is also compatible with the structure of my model\(^4\). The entitlement age is defined by each country’s social security law. The entitlement ages sometimes depend on sex and occupation. I will choose the first age at which a male is entitled to claim social security benefits. More information can be found in the appendix. Finally, to simplify my computations I assume that the “earnings test” for each country is a 0-1 variable. I rely on two sources to determine whether a country allows a person to collect social security and work. Duval (2003) computes a tax on continuing to work based on social security rules of a sub-sample of OECD countries. The US Social Security Administration provides detailed information on social security rules around the world. For many countries they explicitly state if workers can collect social security benefits and work. I will combine both and in case of uncertainty about what did the social security rules established, I conduct sensitivity analysis. Neither quantitative results nor qualitative results change substantially.

These three features of social security are sufficient to capture differences in social security programs around the world and it will be showed that they capture a substantial amount of retirement observed in the data. For example, a country that requires more years of employment to achieve full benefits will have a smaller net replacement rate (other things being equal) There are large differences in replacement rate across the OECD. Figure 2 shows the cross country distribution of net replacement rate (a) and entitlement ages (b)

There are large differences in replacement rates ranging from 38% in Mexico to 124% in Turkey. There are also large differences in entitlement age, which varies from 55 in Australia to 67 in Norway. Figure 3 shows Duval’s implicit

\(^4\)Other definitions of replacement rate are used in the computations and I get similar results
tax on continuing to work to illustrate the differences in rules that allow for collecting social security while the beneficiaries are also working.

It is clear from the figure that there is also large variability in “earnings tests.” Even though a 0-1 classification is arbitrary for some countries, I will use Duval’s numbers and results does not change substantially.

3 Model Economy

This section describes assumptions about demographics, preferences and endowments, technology, social security and market structure.
3.1 Demographics

I assume that the population is stable. Population \( N \) grow at a constant rate \( n \). People live a maximum number of \( A \) periods. Every period each individual face a probability of dying \( 1 - s_a \), which depends on age only. These assumptions induce a population structure where each age group is a constant fraction of the total population (of measure 1,) \( \mu_a \).

3.2 Preferences and Endowments

Every individual has identical preferences, in the US and abroad, over sequences of consumption \( \{c_a\} \) and leisure \( \{1 - h_a\} \). Consumption must be non-negative and hours of work are restricted to be either zero or \( \bar{h} \). Preferences over these stream are characterize with a standard utility function

\[
E_0 \left[ \sum_{a=1}^{A} \beta^a \left( \prod_{j=1}^{a} u(c_a, 1-h_a) \right) \right]
\]

3.3 Individual Productivity

People (indexed by \( i \)) when they are born to the economy are ex-ante identical. As they age their productivity \( (z_{i,a}) \) changes. There are two components that make productivity change. \( z^d_a \) is a deterministic component identical to every individual with the same age. \( z^w_{i,a} \) is a stochastic component that is idiosyncratic to each individual. Its distribution follows a Markov process that can be written in logs as:

\[
\log(z^w_{i,a+1}) = \rho \log(z^w_{i,a}) + \epsilon_{i,a+1}
\]

where \( \epsilon_{i,a} \to N(0, \sigma^2_{\epsilon}) \) is iid across individuals.

\[\text{footnote}{5}\]This number is obtained with the following recursion: \( \mu_{a+1} = \frac{s_{a+1}}{1+n} \mu_a \) with \( \sum_a \mu_a = 1 \)
3.4 Technology

There is a representative firm that operates a constant returns to scale technology. It transforms aggregate capital ($K$) and aggregate efficiency units of labor ($L$) into an homogeneous product ($Y$) which can be used in consumption or savings. Aggregate capital depreciates at a constant rate ($\delta$).

3.5 Markets

At each date there are markets for capital, labor and product. There are no insurance markets and no markets for borrowing and lending. As in Aiyagary (1994,) individuals accumulate precautionary savings.

3.6 Social Security

Social security is defined by two elements. The first is a payroll tax ($\tau$) that is levied on every worker. The second is a function $\phi(\bar{e}_a, h_a, a)$ that characterizes benefit payments and entitlement conditions. It is a function of average earnings, as the benefit amount replace different average earnings at a different rate. Progressive replacement rates are going to be held at US levels but average replacement rates will be scaled to each country. The benefit function also depends on labor choices as social security rules in some countries may restrict the possibility of accruing benefits while working. Finally, it depends on age as individuals are not entitled to receive social security until they reach a entitlement age ($\hat{a}$.) Further details about social security are given in the calibration section.

3.7 Accidental Bequests

As people have an idiosyncratic probability of dying every period they may leave some assets. I assume that the government collects all this capital and distributes it as a lump sum among those individuals alive. I will denote accidental bequests as $B$. 

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3.8 Individual Decision

I represent the individual decision recursively using dynamic programming. As I will focus on comparing steady states I will abstract from time subscript too.

Individual state variables are: assets \((k,\) the idiosyncratic component of productivity \((z^w,\) average earnings \((\bar{e},\) and age \((a,\) Each period, individuals decide how much to consume \((c,\) how much capital to hold \((k'),\) and employment \((h,\) In steady state, taking interest rates \((r,\) wages \((w,\) payroll tax \((\tau,\) social security benefit function \((\phi,\) and accidental bequests \((B,\) as given, each individual solve the following Bellman equation:

\[
V_a(k, z^w, \bar{e}) = \max_{c, k', h} u(c, 1 - h) + \beta s_{a+1} E_z w [V_{a+1}(k, z', \bar{e}')]
\]

s.t. \(c + k' = (1 + r)k + (1 - \tau) wz a h + \phi(\bar{e}, h, a) + B\)

3.9 Aggregate State Variable

The aggregate state variable of the economy is a list of measures over the individual states \(\{\Psi_a(k, z^w, \bar{e})\}\) In steady state, this list a function of the individuals’ policy functions and the idiosyncratic component of productivity.

3.10 Steady State Recursive Competitive Equilibrium

To save notation I collect the individual state variables other than age in a vector \(x = (k, z^w, \bar{e})\). A stationary recursive competitive equilibrium is a list of functions and scalars: \(c_a(x), k'_a(x), h_a(x), V_a(x), \phi(\bar{e}, h, a), \Psi_a(x), w, r, \tau, K, L, B\), such that:

1. \(c_a(x), k'_a(x), h_a(x)\) and \(V_a(x)\) solve equation (3) for every \(a = 1, ..., A\)
2. \(K\) and \(L\) solve the representative firm profit maximization problem, so input prices are given by the first order conditions: \(r = F_K(K, L) - \delta\) and \(w = F_L(K, L)\)
3. Markets clear
(a) \( \sum_a \mu_a \int_X \left[ c_a(x) + k'_a(x) \right] d\Psi_a = F(K, L) + (1 - \delta)K \)

(b) \( \sum_a \mu_a \int_X k'_a(x) d\Psi_a = (1 + n)K \)

(c) \( \sum_a \mu_a \int_X z_a h_a(x) d\Psi_a = L \)

4. The aggregate state is consistent with individual behavior

5. Social security is balanced

\[ \tau L = \sum_{a \geq \bar{a}} \mu_a \int_X \phi(\bar{e}, h_a(x), a) d\Psi_a \]

6. Accidental bequest are distributed evenly among individuals alive

\[ \sum_a \mu_a (1 - s_{a+1}) \int_X (1 + r)k'_a(x) d\Psi_a = B(1 + n) \]

4 Calibration

I calibrate the model to key features of the US economy. Some parameters are selected independently, relying on various data sources and previous research. These are: demographics, productivity, fraction of time worked, labor share and social security system. The remaining parameters, depreciation rate, discount rate, inter-temporal elasticity of substitution and weight of leisure in the utility function are chose by solving the steady state equilibrium to match some key statistics of the US economy.

4.1 Parameters Calibrated Independently

I need to choose the growth rate of the population \((n,)\) the age when individuals enter the economy, length of life \((A,)\) probability of survival \((s_{a,})\) the individual productivity process \((z_{i,a,})\) labor share \((\alpha),\) fraction of time working \((\bar{h})\) and social security system.
4.1.1 Demographics

I set the population growth rate to be equal the US average of 1.2% over the period of 1960-2006. This number is taken from the US Census Bureau Statistical Abstract of 2009. Individuals enter the economy at age 20, and they die with probability 1 when they are 94, implying an $A = 75$. The probability of survival is taken from actuarial tables for males provided by the US Social Security Administration in 2004. Figure 4 shows survival rates for selected life spans as well as stationary population weights that are implied.

![Graphs showing survival and stationary weights.](image)

(a) Probability of Survival  
(b) Weight of the Population

**Figure 4.** Survival and Stationary Weights

4.1.2 Individual productivity process

Individual productivity ($z_{i,a}$) is characterized by two components: $z^d_{a}$, a deterministic component of age and $z^{wr}_{i,a}$, a stochastic component.

To characterize the deterministic component, I use annual earnings and annual hours worked for a sample of white non-disabled males with at least high-school from IPUMS-CPS\(^6\) over the period 1992-2006. The selection of the sample is driven by the objective of isolating the incentive effects on retirement of social security systems and not by life-cycle labor supply decisions driven by race, gender or education. I drop females as some of their choices are related to

\(^6\)http://cps.ipums.org
fertility. High school drop outs are also left out of the sample as they have remarkably different employment behavior and earnings dynamics that would not map that well under the OECD assumption that individuals have full careers beginning at 20. As my model abstracts from permanent heterogeneity, it is a reasonable first step to start without them. Finally, I abstract from disabled individuals as they face a rather different set of employment incentives due to disability insurance. Understanding the joint role of disability insurance and social security is an important research topic and should be addressed in the future, but at this stage it would make the model too complicated for the present purposes.

I express annual earnings in $US1982. The empirical literature usually decompose annual earnings in three different elements: age, time and cohort. A well known problem in this literature is that the time and the cohort effects can not be identified separately without making strong assumptions. Hugget et al. (2009) decompose earnings under three different hypotheses: they assume that either the time effect is zero, the cohort effect is zero or the time effect and the cohort effect are orthogonal. They find that none of the assumptions affect significantly the estimation of the age component of earnings. In the steady state, the time effect should be proportional to the time variable, so I assume that earnings grow at a 2% rate due to productivity gains. I construct hourly wages by dividing annual earnings and annual hours. Then I compute the ratio of mean hourly wage by age to mean hourly wage. This produces a hump-shaped profile. I fit a quadratic polynomial over ages 20-65 weighted by the sample importance of each observation to eliminate sample variability and noise related to selection around retirement age. Finally, I truncate the polynomial to zero when it goes below zero which happens at age 80. Estimating labor productivity is a difficult task at older ages as there are very

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7It is worth noting that the deterministic component of productivity for males and females look alike.

8Hugget et al. (2009) document a growth of wage per hour in the PSID of 1.5% for the period 1969-1992.

9Note that this does not deliver very different results than assuming that the deterministic component of the productivity is given by the average earnings by age relative to average earnings, as it is frequently done in the literature.
big selection effects. In the context of my model this does not seem to be crucial as it approximates relatively well the earnings profiles until age 75 and beyond this age there are few individuals working. Figure 5 shows the result of the calibration of the deterministic component (a) and compares CPS annual earnings with the earnings profile of the calibration (b). The simulated earnings profile is consistent with the earning profile from the CPS for most of the life cycle. It is worth saying that it is a very good fit as it was not a targeted moment in my calibration. As for how important this would be for the results, deviation from data tops 10 percentage points. We would likely find similar biases in other countries so it would not affect my cross country comparisons by much. The stochastic component of individual productivity is characterized by an AR(1)

\[ \log(z_{i,a+1}^{w}) = \rho \log(z_{i,a}^{w}) + \epsilon_{i,a+1} \]

with \( \epsilon_{i,a+1} \sim N(0, \sigma_{\epsilon}^{2}) \) The parameters \( \rho \) and \( \sigma_{\epsilon}^{2} \) are taken from French (2005) and equal .977 and .0141 respectively.

Finally, the fraction of time spent working (\( \bar{h} \)) is set to .45 of available time in a year. To calculate the available time I assume that individuals can use 12 hours a day working which delivers 4380 hours in a year and 1971 hours spent at work.

Figure 5. Deterministic productivity (\( z_{a}^{d} \)) and earnings
4.1.3 Social security

The social security system is calibrated to that of the US. In my model individuals start collecting benefits at age 62 which is the early entitlement age in the US. There are computational reasons and economic reasons why I make this choice. Ideally I should also include the normal retirement age and the entitlement choice to benefits, but that would make the computations more time consuming than they already are. This modeling choice also makes economic sense for the calibration, as in the US individuals are not allowed to borrow against social security income. Therefore the asset poor individuals would like to get their benefits as soon available. On the other hand, the timing of benefits does not matter that much for the rich. Thus, setting the entitlement age to 65 would make my model to overestimate the employment rate of asset poor individuals.

I assume that the US has no restrictions on collecting social security while working. If we use the implicit tax on continuing to work obtained by Duval (2003) as a proxy for this restriction, it is one of the smallest across the OECD (12%). One strong penalty on collecting social security while working in the US was the "earnings test", which consisted on a tax on social security benefits for individuals that claimed entitlement before age 67 while still working. The test established two income thresholds: after the first threshold, $1 of social security benefits was taxed away for every $2 of labor earnings above this first threshold; and after the second threshold, $1 of social security benefits was taxed away for every $3 of labor earnings above this second threshold. On top of this arrangement, the US system included an actuarial compensation factor that allowed the individuals to compensate for some of the benefit loss later on. In 2000, the "earnings test" was reformed. Before the reform the test applied to people that continued to work and were younger than 67 and the actuarial compensation between ages 65 to 67 was not actuarially fair. After the reform, the test is only applied to individuals younger than 65 and the compensation is actuarially fair. Therefore as a first approximation it is

\[^{10}\text{I am not aware that in other countries they are allowed to.}\]
reasonable to abstract from it and assume that the US has zero restrictions on collecting social security while working. The social security benefit formula is taken from the US Social Security Administration. It is a piece-wise linear function of average individual earnings ($\bar{e}$) as in Hugget & Ventura (1999), French (2005) or Nishiyama & Smetters (2007). The bend-points are multiples of AW so they can be directly taken to the model economy. The US social security replaces 90% of the first $761 monthly, 32% from $761 and through $4,586, and 15% above $4,586. This is equivalent to .2,1.24 and 2.47 in multiples of annualized average earnings (AW). Therefore it written as

$$
\phi(\bar{e}, h, a) = \begin{cases} 
0 & \text{if } a < 62 \\
\varphi(\bar{e}) & \text{otherwise}
\end{cases}
$$

Note that as I assume that there are no restrictions on collecting social security while working $h_a$ does not play any role. I have made the following additional simplifications: the social security takes into account the 35 best years of earnings, while I just take a simple average over the lifetime, capped for individual earnings higher than 247% of AW. Again, this does not seem to be very important as the specification of the individual productivity process make that those highest earnings accrue early in life. I characterize individual

\textbf{FIGURE 6.} Social security benefits and earnings
average earnings by the following formula:

\[
\bar{e}' = \begin{cases} 
\frac{\bar{e} \cdot (a-1) + \min(w_{z_i,a}h,2.47\cdot AW)}{a} & \text{if } a < \hat{a} \\
\bar{e} & \text{otherwise}
\end{cases}
\]

I abstract from the fact that US social security requires the individuals to be employed for at least 10 years. This is not an issue in my model as everybody work more than 10 years whatsoever. I also assume that there are no earnings limits on the payroll tax, while in the US earnings above $100,000 are exempt (roughly 3AW). Still this seems a harmless assumption as the mass of individuals that earn more than 3AW is relatively small.

### 4.1.4 Labor Share

I assume that production technology is Cobb-Douglas, \( Y = K^\alpha L^{1-\alpha} \). Labor share \((1-\alpha)\) is set to .64 of production as it is customary found using NIPA. This number implies the same value in simulations by definition.

### 4.2 Parameters Calibrated Together

I calibrate preferences and the depreciation rate to match some key moments of the US economy. I assume that the utility function is separable in consumption and leisure and it takes the following form

\[
u(c, 1-h) = \frac{c^{1-\sigma}}{1-\sigma} + \lambda \cdot (1-h)
\]

this function is characterized by relative risk aversion \((\sigma)\) and the weight of leisure \((\lambda)\).

**Objective.** I choose \((\sigma, \lambda, \beta, \delta)\) to match the following key statistics in the US: a capital-output ratio of 3.0, an investment-output ratio of .20, a labor share of .64 and the employment to population ratio profile from ages 50 to 80. I calculate the ratio from the same sample of the CPS that I used to calculate hourly wages. I have 33 moments and 4 parameters so I choose the parameters to minimize the square deviation of the moments from the data.
and the analogous moments simulated by my model. I use the Nelder-Meade algorithm to find the minimum. Even though every parameter may impact any moment, the discount factor $\beta$ is related to the capital output ratio mostly. Once the algorithm finds a value of the discount factor that makes the capital-output equal 3.00, the value of $\alpha$ chosen to be .36 delivers a labor share of .64 and a value of $\delta$ of .066 delivers an investment output ratio of .20. The deterministic component of productivity ($\{z^d_n\}$), the weight of leisure in the utility ($\lambda$) and relative risk aversion ($\sigma$) interact to deliver an employment profile. At first sight, it is not obvious why does the relative risk aversion play a role to determine the shape of the employment profile and deserves a brief comment. For a high value of $\sigma$ (which implies a low elasticity of substitution), the drop in employment when individuals receive the social security benefits will be smaller than if $\sigma$ is small, thus the employment profile will be steeper for smaller values of the relative risk aversion coefficient.

4.3 Calibration results

Table 1 shows the results of the calibration. Relative risk aversion ($\sigma$) is within the range of the values found in the literature which vary from 1 to 8, $\beta$ is in the low range for life cycle models but I still get a hump-shaped consumption profile as it is shown in Figure 7.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$n$</th>
<th>$\sigma$</th>
<th>$\lambda$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\rho$</th>
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The model matches the ratios of capital and investment to output and labor share perfectly. It is also successful matching the employment to population ratio by age. Figure 8 shows the match of employment to population for ages 50-80. After 80 almost nobody is working and in my model nobody is working. This is a key feature of my model as it is a prerequisite to analyze cross country differences in employment by age. French (2005) also matches the employment
profile but he seems to over-estimate the employment to population above age 62 more than I do. That he attempts to match the wealth distribution at the same time is the most likely reason of his results. To match the accumulation of wealth in the top wealth quintile you need a high $\beta$ but this would also induce individuals to retire early. In French (2007) he partially solves this issue introducing heterogeneous preferences.

My model performs well also along other dimensions that were not intended to match. In particular, the replacement rate in my model is 40% and the measure I use in the data for the US is 45% (the gross replacement on the other hand is 39%. The equilibrium payroll tax in my model is 10.38% which is similar to the US payroll tax when the contributions made to medicaid are discounted (9.5%) I take these as evidence that my model is capturing the
main features of US social security.
A reasonable question to ask is whether my model is able to capture the correlation that the empirical literature documents between replacement rates and employment to population of older people. Figure 9 shows that my model does a very good job. Panel (a) compares the replacement rate of OECD countries relative to that in US for ages 60-64. The solid line represents the unconditional correlation that a reduced form exercise would deliver. The solid line with squares is the result of simulating the model with replacement rates ranging from 70% of the US to 250% of the US. The slope is almost identical but the simulation is shifted up. Why? Because the model is conditional on US entitlement age and “earnings test.” Panel (b) does the same exercise for ages 65-69. Not only it shows that the relation between replacement rate and employment rate is non-linear but it delivers a better fit that the linear regression.

5 Policy Experiments
In this section I asses the importance of three key features of social security to explain cross national differences in employment to population rates at older ages using benchmark calibration. I also investigate the most salient features of social security, if any are salient. Note that the experiments should
be understood as exercises in determining circumstances of Americans that awaken one day under a different social security system that they believed if was always there. I don’t do transitions because I want to focus on explaining cross national variation in employment rates of older people and not on welfare analysis nor policy reform. These topics are a natural follow up of this work but they are computationally complex problems.

5.1 Description of the Experiments

Section 2 documented large differences in employment to population rates at older ages across the OECD. It also documented large differences in three key features of social security: replacement rates, entitlement ages and “earnings tests.” I use US simulation as a benchmark because I get a very good fit in my calibration exercise. Scaling things relative to the US also helps to prevent that the fit of the employment profile is not perfect and that it is performed on a sub-sample of US population. The implicit assumption made is that the relative effect of changes in social security rules are equal across every sub-group of any country. If I used raw employment to population ratios I would deliver an over-estimated employment to population profile when women, racial minorities, disabled and high school drop outs were included. In the Appendix I recalibrate the model including females and my results do not change. I abstract from different ability to redistribute lifetime earnings at older age. I believe that it is also an important feature of social security but it makes the comparative statics exercise more involved.

To account for differences in employment through differences in social security I solve for the stationary equilibrium of the model with different parameters for social security to mimic differences in the replacement rate, entitlement age and “earnings test.” Then I compare the results of simulations with OECD employment data in 2006.

I begin with employment to population at 60-64 because it illustrates a common age of retirement and gives a cross country picture of the performance

\[11\]

I believe it will have interesting implications both for the extensive and intensive margin and human capital accumulation.
of the model. Still, there are some countries that have entitlement ages below 60 or above 64, so I group countries by their entitlement ages and compute employment to population around entitlement ages. This means that if I compare a country like Italy, which has entitlement age of 57, to the US, I use employment to population for ages 55-59. I will also include some employment profiles of selected countries to show that a very parsimonious model is able to fit other countries outcomes, even though it was calibrated to the US only. Finally, I pin down the features of social security that are key to generating the large differences in employment to population found in the data by setting to US levels some features of social security while leaving others as they are in the OECD. The measure of variability that I will use is the coefficient of variation.

5.2 Results

5.2.1 Retirement relative to the US

The main idea of this work is that cross national differences in social security account for large cross national differences in retirement behavior. The main result is that it actually does. This is surprising as there are many things going on as people age. Their health worsens and there are obvious differences in health systems across the OECD, even though most European countries have universal health care. Countries also differ in the combination of social security and private pensions\textsuperscript{12}. To give an example roughly half of benefits of older people in Australia comes from their super-annuitization system. New Zealand has the “Kiwi-Saver” and Austria does have a growing private pension system. Acknowledging that there may be many interactions between these institutional features as people age, differences in social security account for two thirds of the differences in employment to population at ages 60-64. Figure 10 illustrates this result. Given the wide variation of institutional features in different countries, it is not surprising that we find some outliers such as Austria or Italy. If I dropped those countries, I would find an even better

\textsuperscript{12}The definition of net replacement includes other private defined benefit benefits
Does my model depend on whether I focus on males rather than males and females altogether? It does not. Comparing panel (a) with (b) shows that the fit is similar. Of course there are interactions between gender, family structure and social security rules but they do not seem to matter much for the aggregates. The correlation between benchmark model simulations and data is .67 at ages 60-64 and .65 at ages 65-69. Selecting males only, these numbers come to .71 and .64 respectively.

Are my results due to the chosen definition of replacement rate? Table 4 in the Appendix shows it does not. OECD provides net replacement rates but also gross replacement rates, public replacement rates and net wealth. I feed into my model these definitions of replacement and results do not change. Worst fit (.48) is found when using gross replacement rate which is still a big number. Another important assumption was regarding Duval’s (2004) definition of implicit taxes on continuing to work. To address this issue I feed into my model his numbers. Results are shown on Table 5 in the Appendix. Correlation between model simulations and data is still .59 at ages 60-64 and .64 at ages 65-69. These high correlations remain regardless of the definition of replacement rate.

Differences in social security are also able to account for the employment profiles from ages 50-54 to ages 70-74 of many countries. I will show four countries: Australia, France, Japan and Finland; which are also remarkably different to
the US\textsuperscript{13}.

Figure 11. Employment Rate at Older Age: Selected Countries

It is remarkable how much of the employment profiles at older age are accounted for by differences in social security only. There may be other factors that are important such as health insurance and how people in different countries perceive survival rates. The importance of health insurance has been already documented by Rust & Pheland (1997) and French (2005) but in their framework social security is the most important feature. Differences in health insurance may matter to explain employment differences at older age between US and Europe but not that much within European countries as most have universal health insurance, so it is unlikely that health systems are a major source

\textsuperscript{13}I produced graphs for every country in my sample and they are available upon request.
of cross national variation in employment. The effect of different survival rates is unlikely to be important for cross national variation in employment rates at older age too as there are many countries alike. However, the link between mortality and retirement is worth considering in future research.

5.3 Employment to Population

Differences in social security also account for substantial differences in cross national employment. These differences are accounted for through retirement behavior, as there is not much action in the employment decisions before age 50. My model is not intended to capture total employment rates but this result substantiates the idea that cross national tax and transfer programs are responsible for a substantial amount of the differences in labor services. A model that incorporated the role of transfers in education to explain spells into employment of the young would have the potential to explain much of the variation in the extensive margin and the trade-offs between investing collected taxes in education or social security benefits. Figure 12 shows the fit of the employment to population ratio relative to the US.

![Graph of Employment to Population relative to the US](image)

(a) Males and Females

![Graph of Employment to Population, Males](image)

(b) Females

Figure 12. Employment to population

Overall the extensive margin accounts for over a half of the differences in hours per person. Roughly two thirds of the differences in the extensive margin is accounted for by differences in employment to population at older
ages. My model accounts for two thirds of these differences through social security. Therefore it accounts for 22% of the differences in employment to population across the OECD. This number is big given that social security is a tax and transfer program that targets older people.

5.4 What features of social security are most important

To pin down what the most important features of social security are, I run some counter-factual simulations. First, I will focus on the role of each feature of social security: the replacement rate, entitlement age and “earnings test.” I let one feature remain active at a time and set other features to US levels. My measure of variability is the coefficient of variation of data and model simulations. I compute standard deviation and mean of employment relative to the US for different ages. The ratio of standard deviation to mean gives a unit-less measure measure of variability. The results of the counterfactuals are shown in Figure 13.

![Figure 13. Features that account for variability](image)

From panel (a) we can learn that the important features individually are replacement rates and “earnings test”, whereas entitlement ages are not important in accounting for the measure of variability that I use. When I run simulations allowing for all the potential interactions between each feature of social security I find that still the entitlement age is unable to account for
the large differences in employment to population at older ages. Many policy makers argue about strategies to encourage their citizens to work later in life. One of the most popular policy reforms is increasing entitlement age. However, according to my model that would barely have effects on retirement age. People would change their saving behavior and retire at the age they planned in the steady state. This result does not mean that increasing retirement age is a bad policy. In the steady state social security will be cheaper and out of steady state it will have effects on employment to population of older people. If we had to take my model seriously, policy makers should worry about the distributional consequences at older age, as asset poor people will be forced to work longer.

6 Concluding Remarks

There are large cross national differences in employment to population. These differences are even larger at older ages (55+.) There are many factors that may affect retirement behavior but social security is a natural candidate. First, social security is a tax and transfer program present in mostly every country. It accounts for a large fraction of their GDP and it accounts for an even larger fraction of their tax proceeds. There are also large cross national differences in social security features. A question that we must ask is how much of these differences in employment to population at older ages can be accounted for through these different features. My paper conjectures that these differences in employment at older ages are accounted for through the differences in three features of social security only: replacement rates, entitlement ages and if a country allows people to work and collect social security benefits at the same time.

This question is very important as it hits the epicenter of a current cross national policy debate: how should we reform social security systems? The objective of many policy makers is to increase employment to population at older ages to reduce the dependency ratio (the ratio of current workers to retirees.) They usually propose either to cut benefits (reduce its replacement
rate or raise payroll taxes) or increase the entitlement age\textsuperscript{14}. Many researches use a life cycle model of labor supply with idiosyncratic labor income risk and incomplete markets to evaluate these reforms, but is it a good model? To answer to all these questions I build a life cycle model of labor supply that features idiosyncratic labor income risk and incomplete markets. With this model calibrated to the US I can give answer to all these questions: does social security account for retirement behavior? Which features are most important? and, can we take this model seriously when applied to a single country? I find a yes for an answer. My model accounts for two thirds of the differences in employment to population at ages 60-64 and it also accounts for a substantial amount of the differences in employment to population at older ages. These results are achieved by changing three features of social security only: replacement rates, entitlement ages and if social security rules allow people to work and collect social security benefits at the same time. Idiosyncratic labor income risk seems to play an important role to generate such big differences in retirement behavior. When I cut the variance of wages by half and re-calibrate the model, differences in social security have a substantially smaller effect on employment to population at older ages. Therefore using this kind of model to evaluate policy reforms may not be a bad idea at all.

When I investigate which the most important features of social security are, I find that the most important features are replacement rates and whether a country allows people to collect social security and work. Differences in entitlement ages are not able to account for a substantial amount of the differences in employment to population at older age. When we compare steady state to steady state, people change their saving behavior rather than work longer. This does not mean that it is a bad policy as it will certainly make social security cheaper. However it will not increase employment to population at older ages by much.

There are many elements I have excluded from my analysis to make the numerical exercise as clean as possible. My model features social security only,\textsuperscript{14}Many countries including the US have already introduced reforms to increase entitlement age.
but there are other tax and transfer programs that may interact with social security. Unemployment benefits and disability insurance may be used by people in different countries as a bridge to retirement. Private healthcare linked to employment versus universal health care may encourage individuals to retire later. I have abstracted from how social security is financed. Different countries have different caps on payroll taxes’ proceeds. In this work I wanted to focus on the transfers side. There are two other factors that should be considered in future research. Almost every social security program has progressive replacement rates. In my analysis I have set them to US structure. A model that featured human capital accumulation would be a better device to study how people allocate their labor supply over the life cycle under different progressive replacement rates. Such a model would be a useful tool to understand the evolution of productivity as people age. In my model productivity is exogenous and set to US parameters. A model with human capital formation will shed some light over the goodness of this assumption.

References


# Appendices

## Appendix A: OECD Social Security Data

### Table 2

OECD Social Security Relative to the US

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<thead>
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Source: “Pensions at Glance”, OECD 2009 and Duval, 2004

ERA: Early Retirement Age

WSS: Working and Collecting Social Security
7.2 Appendix B: Alternative Calibration

In the benchmark calibration of the model I have used data for white males from the CPS and then run simulations under different configurations of social security. However, the simulations are compared to data from the OECD, which abstracts from the selection I choose. I recalibrate the model to the US employment to population at ages 50-54, 55-59, 60-64, 65-69 and 70-74 and find that the parameters that match the aggregate moments and the employment distribution are roughly similar but with a higher value of leisure in the utility function ($\lambda$) to compensate for an employment distribution that is shifted down when women and other ethnic groups are included. I keep the same relative wages by age as there are not big differences by sex.

**Table 3**

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7.3 Appendix C: Numerical Methods

The algorithm used to compute the equilibrium of the model is similar to Hugget & Ventura (1999) The following steps describe the salient features of the computation:

1. Choose an initial value of aggregate capital \((K_0)\), aggregate labor in efficiency units \((L_0)\), accidental bequests \((B_0)\) and payroll tax \((\tau_0)\)

2. For these values I solve iterating backwards, starting from \(V(x, A) = 0\), the Bellman’s equation of the individual at each point of the individual state space \((k, z^w, e)\). As a result I get the policy functions \(c(x, a), k'(x, a), h(x, a)\) for every \(a = 1, \ldots, A\)

3. I compute the distributions over the individual’s state space \((\Psi(a))\) using Montecarlo’s simulations. I start assuming that individuals start with a capital equal to accidental bequests, average earnings of zero and an initial draw of productivity belonging to the stationary distribution of \(z^w\)

4. I update \(K_0, L_0, B_0\) and \(\tau_0\) aggregating over the simulated distributions to \(K_1, L_1, B_1\) and \(\tau_1\)

5. If aggregate variables in the previous point are close enough and product markets clear, I stop iterations. Otherwise I continue until convergence.

I choose 90 points for the individual capital, 30 points for the idiosyncratic shock and 4 points for average earnings. I have to be careful in the computations as the problem is non-standard as there is a non-convexity on the labor choice. This probably is not a problem in theory, as I am integrating the value function over a continuous distribution with no mass points. Nevertheless, in the numerical computations I am on a grid and this can be a problem. As I do not attempt to prove that the objective function is concave and differentiable, I use golden section search at each point of the individual state for each employment status \((0 \text{ or } \bar{h})\) and then choose the maximum between these two numbers. Note that golden section search just require that the objective
is single peaked on an interval that you choose and do not use any derivative at all. There is a trade off between reliability and computational efficiency that makes this type of problems time consuming. For example, solving for the stationary equilibrium of the model may take between 30 min to 3 hours. Calibration may take from a few days to weeks.
### 7.4 Appendix D: Sensitivity analysis

**Table 4**

Correlation between data and simulations (full sample)

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<td>0.35</td>
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</tbody>
</table>

Full sample includes all the OECD

Benchmark simulations are under the assumption of a 0-1 earnings tests.

**Table 5**

Correlation between data and simulations (full sample)

Using Duval’s (2004) definition of implicit taxes

<table>
<thead>
<tr>
<th>Age</th>
<th>Net Replacement</th>
<th>Gross Replacement</th>
<th>Public Replacement</th>
<th>Net Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
<td>All Males</td>
</tr>
<tr>
<td>20-75</td>
<td>0.64</td>
<td>0.64</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>50-54</td>
<td>0.31</td>
<td>0.28</td>
<td>0.14</td>
<td>0.01</td>
</tr>
<tr>
<td>55-59</td>
<td>0.20</td>
<td>0.22</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>60-64</td>
<td>0.59</td>
<td>0.60</td>
<td>0.56</td>
<td>0.57</td>
</tr>
<tr>
<td>65-69</td>
<td>0.64</td>
<td>0.63</td>
<td>0.58</td>
<td>0.56</td>
</tr>
<tr>
<td>70-74</td>
<td>0.62</td>
<td>0.60</td>
<td>0.56</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Full sample includes all the OECD