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22. June 2011

Online at http://mpra.ub.uni-muenchen.de/35667/
MPRA Paper No. 35667, posted 2. January 2012 04:50 UTC
Firms’ Organizational Modes with Productivity Heterogeneity, Demand Uncertainty and Production Capacity∗

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Abstract
This paper investigates how firms’ demand uncertainty with capacity constraints and their productivity heterogeneity affect their making-or-buying organizational choices in a general equilibrium framework with incomplete contracts. It shows that a final-good producer may adopt integrating a part of the production of its intermediate input in-house and outsource it at arm’s length domestically or abroad simultaneously. Moreover, five organizational modes, exiting the market, outsourcing in the North, outsourcing in the South, integrating and outsourcing in the North simultaneously, and integrating in the North and outsourcing in the South simultaneously, in turn occur with an increase of firm-level productivity, as well as its demand uncertainty. Influences of uncertainty and productivity on prevalence of various organizational modes are also explored.

Keywords: Uncertainty, organizational mode, productivity heterogeneity, incomplete contract, outsourcing, integration

JEL Subject classification:F12, D23

∗The authors appreciate the comments and suggestions from Professor Larry Qiu, Zhihao Yu, Dr. Hong Ma, Qing Liu, Huanlang He, Chia-Hui Lu, and other friends. More comments and suggestions are still welcome.
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1. Introduction

The phenomenon of the so-called "bi-sourcing", i.e., a final-good producer acquires the same set of inputs both by purchasing from external suppliers (outsourcing) and carrying out in-house production (insourcing), widely exists. For example, according to Johnson (2007), Mattel made most of its own die-casting molds at a facility in Malaysia, but also outsourced them to firms in Hong Kong. Carey and Frangos (2005) also reports that U.S. airlines’ heavy-over haul work is conducted half-to-half by in-house mechanics and outside vendors in the U.S. and overseas from less than a third in 1990. Various theories have been developed to explain this phenomenon. In the point of view in Du et al. (2006, 2009), a firm adopts strategic bi-sourcing because they want to apply the cross-threat effect between the internal and external suppliers, as well as the possible cost advantage brought forward by outsourcing. This idea is followed by Stenbacka and Tombak (2010), which adopts the same analytic framework except that the bargaining power of the external supplier increases with the amount of the intermediate outsourced to it. Not directly relevant but closely associated with bi-sourcing, Spencer and Raubitschek (1996) finds that joint venture will adopt in-house production of intermediate inputs with higher marginal cost and also import them from their abroad rivals, because competition in the market of intermediate input will reduce the price of importing intermediate inputs. Beladi and Mukherjee (2008) proposes another theoretic explanation for the occurrence of bi-sourcing. In their analysis, a firm is faced with a deterministic demand. It can produce a good himself at a constant cost $c$, or acquire it from an external supplier at a cost $w$. The firm must determine its product capacity before it observes the cost $w$, after which the supplier determines $w$, then it decides whether to outsource or not. They show that bi-sourcing occurs in this framework. The problem in this framework is that it’s not plausible that firms can not observe $w$ before they makes their outsourcing decision. Moreover, their analysis is limited in a closed economy, and characteristics of firms is ignored.

This paper tries to propose a new and more plausible explanation for bi-sourcing in a general equilibrium of international trade. We investigate the relationship among a final-good producer’s organizational choices, its productivity level and the uncertainty of its demand in a two-country (the North and the South), two-factor (labor and capital), $n + 1$ -industry (with one homogeneous industry and another $n$ differentiated industries) general equilibrium framework with monop-
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Another competition, increasing returns to scale, and free trade. In our model, a representative firm in a representative, monopolistically competitive, and differentiated industry must pay a fixed cost $f_E$ to enter into the market before it starts to produce its differentiated final good, whose production requires two intermediated inputs, $h$ and $m$, where $h$ can only be produced in the North by the final-good producer itself, while $m$ can be produced in both countries. After paying the fixed entry cost, the final-good producer gets aware of its productivity level, which is randomly drawn from a known cumulative distribution. Knowing its productivity level, it is faced with investment decisions, which are dividend into two stages. In the first stage, it shall determine the production capacity of $h$ (i.e., invest how many capitals and labors to produce $h$), which can not be expanded afterward. In the second stage, the final-good producer observes the demand of its variety and decides whether to integrate or outsource the production of $m$ and whether to do it domestically or abroad. A representative final-good producer has totally five potential organizational choices, integrating the production of $m$ in-house, outsourcing it to a supplier in the North or in the South, integrating and outsourcing it in the North simultaneously, and integrating it in the North and outsourcing it in the South simultaneously. Both Integration and outsourcing (no matter in the North or in the South) incur fixed organizational costs. The final-good producer has to pay a sum of the organizational costs if it adopts integration and outsourcing simultaneously. After it decides to outsource the production of $m$ partially or completely, the producer and (if partial outsourcing occurs) or (if complete outsourcing occurs) its supplier invest capitals and labors to produce $m$. Then bargaining on division of the surplus of the final sale between the two parties occurs before its realization. As the production capacity of $h$ must be determined prior to the outsourcing decisions of $m$ and it can not be expanded, the contract between the two parties is incomplete and the intermediate input $m$ is specific to its final good, under-investment problem may occur. Knowing this, the final-good producer has two ways to mitigate it—adjusting the production capacity ex ante or integrating a part of the production of $m$.

The main contributions of this article are multi-fold. First, it incorporates demand uncertainty, capacity constraints, and firms’ organizational choices into a general equilibrium framework with incomplete contracts and firms’ heterogeneous productivity to investigate how the uncertainty of firms’ demands and their productivity heterogeneity affect their making-or-buying organizational choices.
This setting is new according to the authors’ knowledge. Second, the paper shows that a final-good producer may adopt integrating a part of the production of its intermediate input in-house and outsource it at arm’s length domestically or abroad simultaneously, which does not occur in many other literatures on multinationals’ organization and trade. This paper also shows that the increase of a firm’s productivity level results in successively exiting the market, outsourcing in the North, outsourcing in the South, integrating and outsourcing in the North simultaneously, and integrating in the North and outsourcing in the South simultaneously. Third, the paper shows that the increase of the uncertainty of a firm’s demand has the same effects on its organizational choices as that of its productivity. The paper further investigates the influences of uncertainty and productivity on the prevalence of a firm’s various organizational modes.

Except for those literatures cited above for explanations of bi-sourcing, the framework proposed in this paper also connects to those literatures on multinationals’ organization under the general equilibrium framework of international trade, which include Antras (2003), Antras and Helpman (2004, 2006) and Acemoglu et al. (2007), Grossman and Helpman (2002, 2003, 2004, 2005), and Grossman et al. (2003, 2005). For detailed overviews of these work, we refer readers to Antras (2005), Helpman (2006) and Antras and Esteban (2008). Our paper more closely connects to Antras (2003), Acemoglu et al. (2007), Antras and Helpman (2004, 2006), in which they apply the same GHM framework and the Melitz model to investigate firms’ organizational choices. Similar to them, this paper also investigate how firms’ productivity level influences their organizational choices, but it is set under the uncertainty environment, in which firms’ demand is unknown before they make their organizational choices. Under uncertainty, Firm’s production capacities and their un-expandability are introduced. Firms must first determine their production capacities of one intermediate input, which are not expandable afterward, and then observe their demand uncertainty before making organizational choices. That firms must determine the production capacities of one input before the realization of their demand uncertainty incurs

1 which apply the productivity heterogeneity model proposed in Melitz (2003) and the Grossman-Hart-Moore (GHM) framework proposed in Grossman and Hart (1986) and Hart and John (1990); Hart and Moore (1999), to investigate how multinationals organize their global production in ownerships and locations

2 which use transaction costs method to study multinationals’ integration and outsourcing strategies

3 which investigates how multinationals determines their optimal organizational strategies.
possible risks of loss, while that they make their organizational decisions (integrating or outsourcing) after the demand uncertainty is realized makes them possible to diversify risks of loss to suppliers and supply investment if demand realized ex post is larger than expected ex ante. Hence there exists a tradeoff among ex ante production capacities of one input, ex post investment of another input, demand uncertainty, and firms’ productivity level. This setting thus results in more complicated but rich results. Moreover, the organizational choices investigated in this paper are somewhat different from those explored in Antras (2003), Acemoglu et al. (2007), Antras and Helpman (2004, 2006). We investigates the cases that firms adopt simultaneously integration and outsourcing, but do not investigate firms’ FDI. As a result, the relationship between firms’ organizational modes and their productivity level are different.

The sequel of this paper is organized as follows. Section 2 briefly introduces the model structure. Section 3 devotes to analyze the model, including both among a final-good producer’s optimal organizational choice, its productivity level, and the uncertainty of its demand. Section 4 investigates how demand uncertainty affects the prevalence of various organizational modes, as well as firms’ productivity. Section 5 outlines how a general equilibrium and thus the equilibrium prevalence of various organizational modes can be determined. Conclusions are drawn in section 6, with future extensions included.

2. The model

In the world we consider are there two countries, the North and the South, denoted respectively by $N$ and $S$, and two factors, the labor and the capital, denoted respectively by $L$ and $K$. Suppose that the labor and the capital in country $l \in \{N, S\}$ are $L^l$ and $K^l$, respectively. There are two sectors (industries) in both country, one producing homogeneous numeraire goods, whose price is then $p_0 = 1$, and the others producing differentiated final goods, which are indexed by $1, 2, \cdots, n$ and whose prices are respectively $p_1, \cdots, p_n$. We suppose that all consumers in the two countries have the same preferences, whose corresponding utility function is of the CES form, i.e.,

$$U = \nu \ln y_0 + \frac{1 - \nu}{\rho} \ln \left( \sum_{i=1}^{n} x_i^p \right), \quad 0 < \nu, \rho < 1,$$  
(1)
where $x_0$ is the consumption of a homogeneous good, and $x_i$ is the consumption of the $i^{th}$ differentiated good, where the number of the differentiated goods is endogenously determined. From a representative consumer’s utility maximization problem, one can find the demand function of each good as follows:

$$p_i = z y_i^{\alpha-1}. \tag{2}$$

In this paper, we assume that $z$ is uncertain because of the random change of the income of the economy or the random change of consumers’ preference. More specifically, we assume that $z$ follows a uniform distribution on $[\bar{z} - \frac{\sigma}{2}, \bar{z} + \frac{\sigma}{2}]$, where $0 < \sigma < 2\bar{z}$. We make this assumption because demand must be positive, and we want to keep a symmetric uncertainty so that we can analyze the effect of demand uncertainty on firms’ organizational choices. We will show that it’s a crux for firms to partially outsource the production of differentiated goods.

The production of the homogenous good requires both capitals and labors, whose production functions are identical in both countries and are supposed to be $y_0 = L^\alpha K^{1-\alpha}$, $\alpha \in (0, 1)$. Suppose the production technology of the homogenous good is of constant returns to scale, whose market competition is perfect, while that of each differentiated final good is monopolistic, so that each differentiated good is produced by only one producer, which we call a final-good producer. Each producer’s output is small enough relative to the total outputs of the market so that the price change of any differentiated good does not affect prices of other differentiated goods. Suppose differentiated good $i$’s production requires two inputs, a headquarter service $h_i$ and an intermediate component $m_i$, where $m_i$ can only be supplied by operators of manufacturing plants, which are called intermediate-component or intermediate-input suppliers in this paper.

The production of the headquarter service $h_i$ and that of the intermediate component $m_i$ for differentiated good $i$ require both capitals and labors, whose production functions are supposed to be $h_i = L^\gamma K^{1-\gamma}$ and $m_i = L^\beta K^{1-\beta}$, respectively. We suppose that the production functions of headquarter services and intermediate components are identical respectively for all differentiated-good producers. We assume that $\alpha > \beta > \gamma$, so that the headquarter service is the most capital-intensive while the homogeneous good is the most labor-intensive. Let $w_N$ and $w_S$ be the wage rates and $r_N$ and $r_S$ be the interest rates in the North and the South, respectively.

Now, different from the usual assumption of production, we assume that the
production of both homogeneous good and that of the intermediate components are instantaneous, i.e., producers of these goods invest capitals and hire labors instantaneously and then produce the goods, while that of the headquarter services is divided into two stages. In the first stage, a final-good producer must determine its production capacity, i.e., it must determine how many capitals to invest and how many labors to hire before its production. The principle to determine its production capacity includes cost minimization and expected profit maximization, i.e., the final-good producer shall choose appropriate capitals and labors to minimize its production cost, given its production capacity, while the capacity is determined according to expected profit maximization, with demand unknown when the production capacity is determined.

As we set a symmetric model for the economy, the sequel proceeds for a representative final producer. A final-good producer does not know its productivity level before it pays an indispensable fixed cost $F_E$ to enter the differentiated-good market. This cost $F_E$ is identical for all potential entrants. After that, the producer draws a productivity level $\theta$ from a random distribution $G(\theta)$, which is a common knowledge for all potential entrants. We suppose that the production function of each differentiated good is $y = \theta \left( \frac{h}{\eta} \right)^{\eta} \left( \frac{m}{1-\eta} \right)^{1-\eta}$ with $0 < \eta < 1$, where $h$ and $m$ are the quantities of headquarter service and intermediate component required to produce $y$ quantity of the representative producer’s differentiated good. Herein we assume that each final-good producer owns the same production technology.

After observing its productivity level, the final-good producer decides whether to exit the market or not, in the latter case it will observe its demand uncertainty $z$, and then it engages in production. In doing so, an additional fixed cost of organizing production is incurred, which is a function of the structure of ownership and the location of production. If the producer selects to start production, it has two choices, integrating the production of its headquarter service and intermediate component or outsourcing the latter completely or partially domestically or abroad. That is, the producer is faced with two kinds of decisions, ownership decision and location choice. In the former, it must determine whether to integrate or outsource the production of its intermediate components, and in the latter, it must determine where to do them. In this paper, integration and outsourcing do not just mean to keep the production of an intermediate input in-house or at arm’s length completely, but imply partially or completely. For example, the final-good producer may keep the production of an intermediate input $m$ in-house and
outsourcing its production at arm’s length partially simultaneously. In this case, we call the producer integrate and outsource the production of the input simultaneously. We denote the integration and outsourcing in the ownership choices by \( V \) and \( O \), and denote the North and the South in the location choices by \( N \) and \( S \), respectively. We don’t want to investigate foreign integration (usually called FDI) in this paper. Moreover, we do not consider the case that a final-good producer outsource the production of the same intermediate input \( m \) to suppliers in both countries at the same time, but require that the producer only outsource it to a supplier in one country, if it is willing to do so. Extensions can be made to cover these cases in future work. Thus, the final-good producer faces five potential choices, integrating in the North (denoted by \((V, N)\) ), outsourcing in the North (denoted by \((O, N)\) ), outsourcing in the South (denoted by \((O, S)\) ), (partially) integrating and outsourcing its intermediate component \( m \) in the North (denoted by \((V O, NN)\) ), (partially) integrating in the North and partially outsourcing its component in the South (denoted by \((VO, NS)\) ), respectively. We call \((V, N)\), \((O, N)\), \((O, S)\), \((VO, NN)\) and \((VO, NS)\) the potential organizational modes for a representative final-good producer.

For choice of each organizational mode \((k, l) \in \{(V, N), (O, N), (O, S), (VO, NN), (VO, NS)\}\), a representative final-good producer must pay a fixed organizational cost \( f_{k,l} \), with \((k,l) = (V,N)\), or \((k,l) = (O,l)\), \(l \in \{N, S\}\). We suppose that \( f_{V,N}^N > f_{S,O}^N > f_{O,N}^N \) to avoid a complex potential taxonomy of organizational choices. This ranking implies that the organizational costs of any firm integrating the production of \( m \) in the North is larger than those outsourcing it in each country, respectively. For its partial integration or outsourcing choice, it has to pay a sum of fixed costs of two corresponding independent organizational modes. For example, if the producer partially integrate in the North, then it has to pay a fixed cost of \( f_{V,N}^N + f_{O,N}^N \) as it is engaged into two different relationships with two different kinds of suppliers.

Free entry into each sector ensures zero expected profits for a potential entrant. To simplify the description of the industry equilibrium, we assume that upon entry the supplier makes a lump-sum transfer \( T_k(i) \) to the final-good producer, which can vary by industry \( k \) and variety \( i \). Ex-ante, there is a large number of identical, potential suppliers for each variety in each industry, so that competition among these suppliers will make \( T_k(i) \) adjust to zero so as to make them break even.
We apply the Grossman-Hart-Moore framework (first proposed in Grossman and Hart (1986), and later developed in Hart and John (1990); Hart and Moore (1999) to analyze the incomplete contracts signed ex-ante between final-good producers and their suppliers. According to this framework, the contracts signed between the two parties can not specify the purchase of specialized intermediate components for a certain price, the amount of capital and labor hired or the volume of sales revenues obtained after the final good is sold. That is, that the parties can not commit not to renegotiate an initial contract and that the precise nature of the required input is revealed only ex-post, and it's not verifiable by a third party. To divide the ex-post revenue from the final sale, the two parties have to bargain over the surplus from the relationship after the inputs have been produced. In this paper, we assume that it's a generalized Nash bargaining game between the two parties. Suppose a representative final-good producer obtains a fraction $\zeta \in [0, 1]$ of the ex-post gains from the relationship, where the parameter $\zeta$ measures the bargaining power of the final-good producer. Here $\zeta$ may be distinct for each producer as the producers are heterogenous in productivity.

Suppose that ex-post bargaining between a representative producer and its supplier takes place only under outsourcing. In integration, the producer organizes the total production processes itself and thus seizes the total surplus of the production. This assumption deviates a little from those of the classical papers, such as Grossman and Hart (1986), Hart and Moore (1999) and many successive applications of GHM framework in analysis of global organizational choices of multinationals, such as Antras and Helpman (2004, 2006), Acemoglu et al. (2007), etc. For outsourcing, we assume that the final-good producer obtains what it can get by using $h$ and $m$ it produces to produce its final good and selling it, while its supplier gets zero, if the contract between it and its supplier breaches. This assumption is also different from Antras and Helpman (2004, 2006), Acemoglu et al. (2007), etc. In fact, the distribution of surplus is related to the organizational modes. Specifically, if the producer does not own a manufacturing plant (i.e., complete outsourcing), then when the contract between the the producer and its supplier breaches, both sides obtain no income as the components are tailored specifically to the opposite party in the relationship; if the producer totally owns a manufacturing plant as its supplier does (i.e., outsourcing partially in the North or the South), then when the contract breaks down, the producer can fire the supplier, get the component $m$, with a loss of a fraction of $1 - \delta_l$ fraction of final-good production for $l \in \{N, S\}$, as the producer can ask its integrated workers producing intermediate components to modify the supplier's components to fit its own demand. We can assume that $\delta_N \geq \delta_S$, which implies that a contractual breach is more likely costly to the producer when the supplier is in the South than in the North, as the North have a better institutional environment. Our model can be extended to this setting without any difficulty, by noticing that $\zeta_l = \delta_l \zeta + \zeta (1 - \delta_l)$, $l = N, S$ for this setting, where $\zeta_l$ is the bargaining power of the final-good producer toward its supplier in country $l$. See
After the final-good producer makes its decisions on organizational modes when the demand uncertainty realizes, it and its supplies decide how much \( m \) to produce. If a final-good producer decides to integrate the production of \( m \) in the North, it must determine how much \( m \) to produce. If the producer decides to outsource, then it must choose the bargaining power with its supplier, how much \( m \) to produce, while its suppliers decides how much \( m \) to produce. When the final-good producer or both parties makes or make their decisions on how much \( m \) to produce, they choose the optimal capitals and labors to minimize the production cost of producing \( m \). Finally, when the final-good producer or both parties have produced \( h \) and \( m \), the final-good producer combines them to produce its final good and then sell it in the market, with the final sale and thus the surplus realized. Both parties divide the surplus according the contract signed ex ante.

### 3. Organizational decisions after the uncertainty is realized

In this section, we consider the case that a representative final-good producer makes its organizational decisions after the uncertain demand is realized. According to the modeling in section 2, the timing of the game is as follows.

**Time 0** A representative final-good producer (\( F \) in short) decides whether to pay \( F_E \) to enter the market, and it knows its productivity \( \theta \) afterward, which follows a Pareto distribution.

**Time 1** Knowing its productivity \( \theta \) and staying in the market, \( F \) decides its production capacity \( H \) of \( h \). After that, it observes \( z \). Its production capacity can not be expanded thereafter.

**Time 2** \( F \) decides whether to integrate or outsource the production of \( m \). If \( F \) adopts only integration, it needs to pay \( f^N_i \). If \( F \) adopts only outsourcing in country \( l \), it needs to pay \( f^O_i \), where \( l \in \{N, S\} \). If \( F \) adopts integration and outsourcing simultaneously, it has to pay \( f^O_i + f^N_i \).

**Time 3** Both the final-good producer and its intermediate-input supplier (\( S \) in short) decide how much \( m \) to produce. Suppose their outputs are \( m \) and \( m^l_O \), respectively, where \( l \in \{N, S\} \) is the country the supplier is in.

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**Time 4** After that, both parties bargain on division of the surplus.

**Time 5** The sale of the final product and thus the total surplus is realized.

We solve the above problem by back induction. First, at **Time 5**, the total sale income is 
\[
 z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m + m_{O}^l}{1 - \eta} \right)^{\rho (1 - \eta)}.
\]
Knowing this, both the final-good producer and its intermediate-input supplier bargain on it at **Time 4**. We assume that the bargaining is of the form of Nash Bargaining. As the intermediate input \( m \) is totally specialized, it’s out of use for the supplier \( S \) if it fails to agree with \( F \), and hence its reserved payoff is 0. While for \( F \), it can perform the production of its final good with its own \( m \) even without the intermediate input \( m_{O}^l \) from \( S \), and hence its reserved payoff is 
\[
 z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m}{1 - \eta} \right)^{\rho (1 - \eta)}.
\]
Suppose the bargaining power of \( F \) is \( \zeta \). In this paper, we only consider the case that a final-good producer outsources the production of \( m \) in only one country if it does. Then there are three cases for the payoffs of both parties under given uncertainty \( z \) at **Time 4**. If \( F \) only adopts integration of the production of \( m \), then its payoff is
\[
 \pi_{V}^N = z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m}{1 - \eta} \right)^{\rho (1 - \eta)} - f_{m}^N m - f_{V}^{N}.
\]
If \( F \) only adopts outsourcing of the production of \( m \) in country \( l \in \{N, S\} \), then its payoff is
\[
 \pi_{O}^l = \zeta z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m_{O}^l}{1 - \eta} \right)^{\rho (1 - \eta)} - f_{O}^{l} + T_{O}^{l}.
\]
If \( F \) adopts integration in country \( N \) and outsourcing in country \( l \) simultaneously, then its payoff is
\[
 \pi_{VO}^l = \zeta z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m + m_{O}^l}{1 - \eta} \right)^{\rho (1 - \eta)} + (1 - \zeta) z^\theta \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m}{1 - \eta} \right)^{\rho (1 - \eta)} - f_{m}^N m - f_{O}^{l} + f_{V}^{N} + T_{O}^{l}.
\]
In the above expressions, \( f_{m}^{l} = \left( \frac{w_{l}}{\beta} \right)^{\beta} \left( \frac{r_{l}}{1 - \beta} \right)^{1 - \beta} \) is the minimal cost producing one unit of \( m \) in country \( l \in \{N, S\} \), and \( w_{l} \) and \( r_{l} \) are the prices of labor and capital in country \( l \), respectively.
On the other side, if outsourcing occurs, the payoff of the supplier in country \(l\) is

\[
\pi^l_S = (1 - \zeta)z\theta^\rho \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m + m^l_O}{1 - \eta} \right)^{\rho(1 - \eta)} - f^l_m m_O - T^l_O. \tag{6}
\]

In the above expressions, \(T^l_O\) is the lump-sum transfer made by \(S\) to the final-good producer. As there is a large number of identical potential suppliers for each variety in each industry, ex-ante, the competition among these suppliers adjust the lump-sum transfer \(T\) to zero so as to make them break even. This implies that \(F\) can seize all the positive profits from \(S\). Then the final-good producer chooses the organizational mode so as to maximize its ex-ante profits, which include the transfer.

Different from Antras (2003, 2005), Antras and Helpman (2004, 2006), we don’t add the so-called institutional parameter \(\delta\) in the above setting, which reflects a country’s quality of institutions.\(^5\) Our rationale is that we don’t think it’s appropriate for our setting. In their setting, when the contract breaches, the final-good producer can seize a proportion of the intermediate input produced by \(S\), but the intermediate-input supplier does not get anything. This implies that the contract is not fair. Besides, this setting does not tell us that the one breaches the contract shall be punished.

To find out whether a final-good producer will outsource or integrate the production of \(m\) at Time 3, we must compare its payoffs under different organizational choices, which is shown as follows.

### 3.1. Integrating in the North

If \(F\) only integrates the production in the North, then at Time 3, its optimal output of \(m\) is the solution \(m^N_V\) of the problem \(\max_m \pi^N_V\), from which one gets

\[
m^N_V = (1 - \eta) (\rho z \theta^\rho)^{-\frac{1}{\rho - \rho - (1 - \eta)}} (f^N_m)^{\frac{1}{\rho - \rho - (1 - \eta)}} \left( \frac{H}{\eta} \right)^{\frac{\rho \eta}{1 - \rho (1 - \eta)}}. \tag{7}
\]

\(^5\) In their setting, they assume that if the contract between the two parties breaches, then \(F\) can sell \(\delta\) part of final product \(y\), and thus its payoff is \(\delta z \theta^\rho \left( \frac{H}{\eta} \right)^{\rho \eta} \left( \frac{m + m_O}{1 - \eta} \right)^{\rho(1 - \eta)}\).
And thus its payoff at Time 3 is

$$\pi^N_3 = \frac{1 - \rho(1 - \eta)}{\rho} (\rho z \theta^\rho) \frac{1}{1 - \rho(1 - \eta)} (f^N_m)^{-\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \frac{H}{\eta} \right)^{-\frac{\rho}{1 - \rho(1 - \eta)}} - f^N_V. \quad (8)$$

At Time 2, the final-good producer's expected payoff from selecting the production capacity $H$ is then

$$\Pi^N_2 = \int \pi^N_V dz - f_h H - f_E$$

$$= \frac{1 - \rho(1 - \eta)}{\rho} (\rho \theta^\rho) \frac{1}{1 - \rho(1 - \eta)} f^N_m \frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)} \left( \frac{H}{\eta} \right)^{-\frac{\rho}{1 - \rho(1 - \eta)}} - \sigma f^N_V - f_h H - f_E, \quad (9)$$

where $f_h = (w_N^\gamma (r_N)^{1 - \gamma} is the minimal cost producing one unit $h$ in country $N$, $w_N$ and $r_N$ are the prices of labor and capital in the North, respectively, and

$$\varpi = \frac{1 - \rho(1 - \eta)}{2 \rho(1 - \eta)} \left[ (\bar{z} + \frac{\sigma}{2}) \frac{2 - \rho(1 - \eta)}{1 - \rho(1 - \eta)} - (\bar{z} - \frac{\sigma}{2}) \frac{2 - \rho(1 - \eta)}{1 - \rho(1 - \eta)} \right].$$

At Time 1, the final-good producer will choose the following optimal production capacity of $h$ to maximize its expected payoff

$$H^N_1 = \eta \varpi \left[ f^N_m \frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)} \left( \frac{H}{\eta} \right)^{-\frac{\rho}{1 - \rho(1 - \eta)}} - \frac{1}{\rho} \right], \quad (10)$$

and thus its expected payoff at Time 2 is

$$\Pi^N_2 = \left( \frac{1}{\rho} - 1 \right) (\rho \theta^\rho) \frac{1}{1 - \rho} \varpi \left[ f^N_m f^N_h \left( \frac{H}{\eta} \right)^{-\frac{\rho}{1 - \rho(1 - \eta)}} - \sigma f^N_V - f_E. \quad (11)$$

### 3.2. Outsourcing only in Country $l$

Now we consider the case that $F$ only outsources its production of $m$ in country $l \in \{N, S\}$. In this case, $S$'s optimal production plan of $m$ is

$$m^l_O = (1 - \eta) [(1 - \zeta) \rho z \theta^\rho] \frac{1}{1 - \rho(1 - \eta)} (f^l_m)^{-\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \frac{H}{\eta} \right)^{-\frac{\rho}{1 - \rho(1 - \eta)}} - \frac{1}{\rho}, \quad (12)$$

then its optimal payoff is

$$T^l_O = \left[ \frac{1}{\rho} - (1 - \eta) \right] [(1 - \zeta) \rho \theta^\rho] \frac{1}{1 - \rho(1 - \eta)} (f^l_m)^{\rho(1 - \eta)} \left( \frac{H}{\eta} \right)^{\rho(1 - \eta) - \frac{1}{\rho}}.$$


Thus $F$’s payoff at **Time 3** is

$$\pi^t_O = \left[ \frac{1}{\rho} - (1 - \zeta)(1 - \eta) \right] (1 - \zeta)^{\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \rho \theta \rho \right)^{\frac{1}{1 - \rho(1 - \eta)}} \left( f_m^l \right)^{-\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \frac{H}{\eta} \right)^{\frac{\rho m}{1 - \rho(1 - \eta)}} - f_O^l.$$  

Then its expected payoff at **Time 2** is

$$\Pi^t_O = \varpi \left[ \frac{1}{\rho} - (1 - \zeta)(1 - \eta) \right] (1 - \zeta)^{\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \rho \theta \rho \right)^{\frac{1}{1 - \rho(1 - \eta)}} \left( f_m^l \right)^{-\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \frac{H}{\eta} \right)^{\frac{\rho m}{1 - \rho(1 - \eta)}} - f_h H - \sigma f_O^l - f_E.$$  

Therefore, at **Time 1**, $F$ will select the following production capacity of $h$ to maximize its expected payoff

$$H^t_O = \kappa \varpi \left[ \frac{1}{\rho} - (1 - \zeta)(1 - \eta) \right] (1 - \zeta)^{\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \rho \theta \rho \right)^{\frac{1}{1 - \rho(1 - \eta)}} \left( f_m^l \right)^{-\frac{\rho(1 - \eta)}{1 - \rho(1 - \eta)}} \left( \frac{H}{\eta} \right)^{\frac{\rho m}{1 - \rho(1 - \eta)}},$$

where

$$\kappa = \left[ \frac{(1 - \rho(1 - \eta)(1 - \zeta))(1 - \zeta)^{\rho(1 - \eta)}}{1 - \rho(1 - \eta)} \right]^{\frac{1 - \rho(1 - \eta)}{1 - \rho(1 - \eta)}}.$$  

Substituting (14) into (13) yields

$$\Pi^t_O = \left( \frac{1}{\rho} - 1 \right) \kappa \left( \rho \theta \rho \right)^{\frac{1}{1 - \rho}} \varpi \left( f_m^l \right)^{\rho \theta \rho} \left[ f_h^m (1 - \eta) \right]^{-\frac{1}{1 - \rho}} - \sigma f_O^l - f_E, l = N, S.$$  

The final-producer choose an optimal bargaining power to maximize its expected payoff **Time 1**. Moreover, combining (11) and (15) and our previous assumption $f^N_V > f^S_O > f^N_O$ yields

$$\Pi^N_O - \Pi^N_V = \sigma (f^N_V - f^l_O) > 0,$$

which implies that comparing with only integration in the North, $F$ always prefers to outsourcing in the North.\(^6\)

Moreover, there is

$$\Pi^S_O - \Pi^N_O = \left( \frac{1}{\rho} - 1 \right) \left( \rho \theta \rho \right)^{\frac{1}{1 - \rho}} \varpi \left( f_m^l \right)^{\rho \theta \rho} \left[ (f_m^S - f_m^N) \right]^{-\frac{1}{1 - \rho}} - \sigma (f^S_O - f^N_O).$$

\(^6\)This result depends on the assumption $f^N_V > f^S_O > f^N_O$. The result would change had it be $f^N_V < f^S_O$ or $f^S_O < f^N_O$.  

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\(\varpi\)
3.3. Integrating in the North and Outsourcing in country \( l \) simultaneously

This subsection considers the case that \( F \) simultaneously integrates the production of \( m \) in the North and outsources it in country \( l \).

Solving \( S \)'s profit maximization problem yields \( S \)'s optimal output of \( m_O^l \)

\[
m_O^l = (1 - \eta) \left\{ \left[ (1 - \zeta) \rho \theta^\rho \right]^{1-\rho/(1-\eta)} \left( f_m^l \right)^{\frac{1}{1-\rho}} \left( \frac{H}{\eta} \right)^{\frac{\rho}{1-\rho(1-\eta)}} - m_V^N \right\}. \quad (16)
\]

Substituting (16) into the first-order condition of \( F \)'s profit maximization problem and rearranging the resulted expression yields

\[
m_V^N = (1 - \eta) \left[ (1 - \zeta) \rho \theta^\rho \right]^{1-\rho/(1-\eta)} \left[ f_m^N - \frac{\zeta}{1 - \zeta} f_m^l \right]^{\frac{1}{1-\rho}} \left( \frac{H}{\eta} \right)^{\frac{\rho}{1-\rho(1-\eta)}}.
\]

Similarly, we know that \( F \) seizes \( S \)'s total positive profit from the lump-sum transfer \( T_O^l \), which can be calculated from substituting \( m_O^l \) and \( m_V^N \) back into the expression of \( S \)'s payoff. Plugging \( T_O^l \) back into the expression of \( F \)'s payoff yields

\[
\pi_{VO}^{NI} = \chi (1 - \zeta) \left[ (1 - \rho) \left( f_m^l \right)^{1-\rho/(1-\eta)} - f_m^l \right]^{\frac{1}{1-\rho}} \left( \frac{H}{\eta} \right)^{\frac{\rho}{1-\rho(1-\eta)}} - f_O^l - f_V^N. \quad (17)
\]

where

\[
\chi = \frac{1}{\rho} - (1 - \eta)(1 - \zeta) + \left[ \frac{1}{\rho} \left( (1 - \zeta) f_m^N - \zeta \right) - (1 - \eta)(1 - \zeta) \left( \frac{f_m^N}{f_m^l} - 1 \right) \right] \times \left( \frac{f_m^N}{f_m^l} - \frac{\zeta}{1 - \zeta} \right)^{-\frac{1}{1-\rho}}.
\]

We thus get \( F \)'s expected payoff at \textbf{Time 1} to be

\[
\Pi_{VO}^{NI} = \chi \omega (1 - \zeta) \left[ (1 - \rho) \left( f_m^l \right)^{1-\rho/(1-\eta)} - f_m^l \right]^{\frac{1}{1-\rho}} \left( \frac{H}{\eta} \right)^{\frac{\rho}{1-\rho(1-\eta)}} - \sigma (f_O^l + f_V^N) - f_h H - f_E. \quad (18)
\]

And then the optimal production capacity selected by \( F \) at \textbf{Time 1} should be

\[
H_V^N = \eta \tau \omega \left[ (1 - \rho) \left( f_m^l \right)^{1-\rho/(1-\eta)} \right]^{\frac{1}{1-\rho}} \left[ f_h (1 - \rho) \left( f_m^l \right)^{\rho/(1-\eta)} \right]^{\frac{1}{1-\rho}}.
\]

(19)
where
\[
\tau = \left[ \rho \chi \left( 1 - \zeta \right) \frac{\rho^1 \left(1 - \eta \right)}{1 - \rho (1 - \eta)} \right]^{\frac{1 - \rho (1 - \eta)}{1 - \rho}}.
\]

We then easily to get F’s maximal expected payoff to be
\[
\Pi_{N_{V'O}}^l = \left( \frac{1}{\rho} - 1 \right) \tau \omega^{\frac{1 - \rho (1 - \eta)}{1 - \rho}} \left( \rho \theta^\rho \right) \frac{1}{1 - \rho} \left[ f_h^p (f_m^l) \rho - \rho \eta \right]^{-\frac{1}{1 - \rho}} - \sigma (f_o^l + f_v^N) - f_E. \tag{20}
\]

\( F \) will choose an optimal bargaining power \( \zeta \) to maximize its payoff \( \Pi_{N_{V'O}}^l \), which exactly maximizes \( \tau \), a function of \( \zeta \). We still write its maximal value as \( \tau \). According to the appendix, we know that \( \tau > 1 \) for some \( \rho < 1 \).

Now we can compare \( F \)’s payoffs under only outsourcing in country \( l \) and under integration in the North and outsourcing in country \( l \). (20) minus (13) yields
\[
\Pi_{N_{V'O}}^l - \Pi_{V_O}^l = \left( \frac{1}{\rho} - 1 \right) \left( \tau - 1 \right) \omega^{\frac{1 - \rho (1 - \eta)}{1 - \rho}} \left( \rho \theta^\rho \right) \frac{1}{1 - \rho} \left[ f_h^p (f_m^l) \rho - \rho \eta \right]^{-\frac{1}{1 - \rho}} - \sigma f_V^N. \tag{21}
\]

### 3.4. Organizational choices and productivity

It’s easy to see that \( \Pi_{N_{V'O}}^l, \Pi_{V_O}^l, \Pi_{N_{V'O}}^N \) are all linear and increasing in \( \Theta = \theta \tau^\rho \). This implies that it suffices for us to analyze the relationship between \( F \) organizational choices and \( \Theta \) to analyze the relationship between \( F \)’s organizational choices and \( \theta \). We thus call \( \Theta \) the quasi-productivity level of \( F \).

In this subsection, we consider the relationship between \( F \)’s organizational choices and its productivity level (denoted by \( \Theta \)) with demand uncertainty level \( \sigma \) given. There are two cases: \( \tau \leq 1 \) and \( \tau > 1 \). Different values of \( \tau \) yield different organizational modes for \( F \).

We first consider the case \( \tau \leq 1 \). Let
\[
\Theta^l_O = \frac{\sigma f^l_O + f_E}{\left( \frac{1}{\rho} - 1 \right) \rho^{\frac{1 - \eta}{1 - \rho}} \omega^{\frac{1 - \rho (1 - \eta)}{1 - \rho}} \left[ f_h^p (f_m^l) \rho - \rho \eta \right]^{-\frac{1}{1 - \rho}}} \quad l = N, S, \tag{22}
\]
\[
\Theta^{NS}_O = \frac{\sigma (f_S^N - f_O^N)}{\left( \frac{1}{\rho} - 1 \right) \rho^{\frac{1 - \eta}{1 - \rho}} \omega^{\frac{1 - \rho (1 - \eta)}{1 - \rho}} \left[ f_h^p (f_m^l) \rho - \rho \eta \right]^{-\frac{1}{1 - \rho}}} \quad (f_S^N) - (f_N^N) - \sigma f_N^N. \tag{23}
\]

Then there are two potential organizational modes. If \( \Theta^{NS}_O < \Theta^N_O \), then for \( \Theta < \Theta^{NS}_O \), \( F \)’s net profit is less than 0, and thus the final-good producer will exit the market. The final-good producer’s net profit will stay in the market and its organizational...
mode is outsourcing in the South for the case $\Theta > \Theta_O^S$. This case is illustrated by Figure 1.

If $\Theta_O^S < \Theta_O^N$, then $F$’s net profit is less than 0 for $\Theta < \Theta_O^S$, and thus the final-good producer will exit the market. The final-good producer’s net profit of outsourcing in the South is positive and larger than that of outsourcing in the North for $\Theta > \Theta_O^{NS}$, and thus its organizational mode is outsourcing in the South. For $\Theta_O^S < \Theta < \Theta_O^{NS}$, the final-good producer’s net profit of outsourcing in the North is still positive and larger than that of outsourcing in the South, and thus its organizational mode is outsourcing in the North. This case is illustrated by Figure 2.

Note that $\Theta_O^S < \Theta_O^N$ if and only if $\frac{f_Nm + f_S}{f_O^N + f_E} > \left(\frac{\sigma f_S + f_E}{\sigma f_N + f_E}\right)^{\frac{1-\rho}{1-\rho(1-\eta)}}$, we thus have the following proposition synthesizing the above discussions.

**Proposition 1** If $\rho$, $\eta$ and $\frac{f_Nm + f_S}{f_O^N + f_E}$ are such that $\tau \leq 1$, then

1. the final-good producer outsources the production of $m$ in the South if $\Theta > \Theta_O^S$ and exits the market if $\Theta < \Theta_O^S$ in the case of $\frac{f_N}{f_m} \geq \left(\frac{\sigma f_S + f_E}{\sigma f_N + f_E}\right)^{\frac{1-\rho}{1-\rho(1-\eta)}}$, and

2. the final-good producer outsources the production of $m$ in the South if $\Theta > \Theta_O^{NS}$, outsources the production of $m$ in the North if $\Theta_O^{NS} < \Theta < \Theta_O^N$, and exits the market if $\Theta < \Theta_O^N$ in the case of $\frac{f_N}{f_m} < \left(\frac{\sigma f_S + f_E}{\sigma f_N + f_E}\right)^{\frac{1-\rho}{1-\rho(1-\eta)}}$,

where $\Theta_O^l$ and $\Theta_O^{NS}$ are defined by (22) and (23), respectively.

As $\frac{\sigma f_S + f_E}{\sigma f_N + f_E}$ is increasing in $\sigma$, it’s more possible that the second subcase occurs if the uncertainty increase under the situation $\tau \leq 1$ according to Proposition 1. Moreover, it’s more possible that final-good producers exits the market given the other parameters in this case.

If $\tau > 1$, then $F$’s organizational modes are more than complicated.

**Case 1.** $\Theta_O^N < \Theta_O^S$.

Let

$$\Theta_O^{NI} = \frac{\sigma (f_O^l + f_N^l) + f_E}{\left(\frac{1}{\rho} - 1\right)^{\frac{1}{1-\rho}} + \frac{1}{1-\rho} \left[ f_m^l (f_m^l)^{\rho - 1} - 1\right]^{\frac{1}{1-\rho}}}$$

where $l = N, S$.

Then $\Theta_O^{NS} < \Theta_O^{NN}$ if and only if $\frac{f_N}{f_m} > \left(\frac{\sigma f_S + f_E}{\sigma f_N + f_E}\right)^{\frac{1-\rho}{1-\rho(1-\eta)}}$.
We first consider the subcase $\Theta_{VOO}^{NS} < \Theta_{VOO}^{N}$. Let
\[
\Theta_{VOO}^{NS} = \frac{\sigma(f_{V}^{N} + f_{E}^{O} - f_{B}^{N})}{(1 - \rho) V^{O} \frac{1}{1-p} f_{h}^{O} \left( \tau(f_{m}^{S}) - \frac{\rho l}{1-p} \right) - (f_{m}^{l})^{\frac{\rho l}{1-p}}}, \quad l = N, S(24)
\]
then we have the following possibilities.

If $\Theta_{VOO}^{NS} < \Theta_{VOO}^{N}$, then $F$ will integrate in the North and outsource in the South simultaneously if $\Theta > \Theta_{VOO}^{NS}$, and it will exit the market if $\Theta < \Theta_{VOO}^{NS}$. This case is illustrated in Figure 3 as follows.

[Figure 3 is included here.]

If $\Theta_{VOO}^{O} < \Theta_{VOO}^{NS} < \Theta_{VOO}^{S}$, then $F$ will integrate in the North and outsource in the South simultaneously the production of $m$ if $\Theta > \Theta_{VOO}^{NS}$, it will exit the market if $\Theta < \Theta_{VOO}^{O}$, and it will outsource in the North if $\Theta_{VOO}^{O} < \Theta < \Theta_{VOO}^{NS}$. This case is illustrated by Figure 4 as follows.

[Figure 4 is included here.]

If $\Theta_{VOO}^{NS} > \Theta_{VOO}^{S}$, then $F$ will outsource in the North if $\Theta_{VOO}^{O} < \Theta < \Theta_{VOO}^{NS}$, it will outsource in the South if $\Theta_{VOO}^{NS} < \Theta < \Theta_{VOO}^{NS}$, it will integrate in the North and outsource in the South simultaneously if $\Theta > \Theta_{VOO}^{NS}$, and it will exit the market if $\Theta < \Theta_{VOO}^{O}$. The case is illustrated in Figure 5 as follows.

[Figure 5 is included here.]

Summarizing the above discussions, we have the following proposition.

**Proposition 2** Suppose that $\rho, \eta$ and $f_{m}^{N}$ are such that $\tau > 1$. Let $f_{m}^{N} \leq \left( \sigma\frac{f_{S}^{O} + f_{E}}{\sigma f_{O}^{S} + f_{E}} \right)^{\frac{1-\rho}{\rho (1-\eta)}}$. Then

1. the final-good producer integrates in the North and outsources in the South simultaneously the production of $m$ if $\Theta > \Theta_{VOO}^{NS}$ and exits the market if $\Theta < \Theta_{VOO}^{NS}$ in case of $\tau^{\frac{1-\rho}{\rho (1-\eta)}} f_{m}^{N} \geq \left( \frac{\sigma(f_{S}^{O} + f_{E})}{\sigma f_{O}^{S} + f_{E}} \right)^{\frac{1-\rho}{\rho (1-\eta)}}$;

2. the final-good producer integrates in the North and outsources in the South simultaneously the production of $m$ if $\Theta > \Theta_{VOO}^{NS}$, outsources in the North if $\Theta_{VOO}^{O} < \Theta < \Theta_{VOO}^{NS}$ and exits the market if $\Theta < \Theta_{VOO}^{O}$ in case of $\tau^{\frac{1-\rho}{\rho (1-\eta)}} f_{m}^{N} < \left( \frac{\sigma(f_{S}^{O} + f_{E})}{\sigma f_{O}^{S} + f_{E}} \right)^{\frac{1-\rho}{\rho (1-\eta)}}$ and $\tau \geq \frac{\sigma(f_{S}^{O} + f_{E})}{\sigma f_{O}^{S} + f_{E}}$,

3. the final-good producer outsources in the North if $\Theta_{VOO}^{O} < \Theta < \Theta_{VOO}^{NS}$, integrates in the North and outsources in the
South simultaneously if $\theta > \Theta^{SNS}_{VVO}$ and exits the market if $\theta < \Theta^S_O$ in case of $\tau < \frac{\sigma(f^S_O + f^V_N) + f_E}{\sigma f^S_O + f_E}$.

As $\frac{\sigma(f^S_O + f^V_N) + f_E}{\sigma f^S_O + f_E} = \frac{\sigma^2 (f^S_O - f^N_o)}{(\sigma f^S_O + f_E)(\sigma f^V_N + f_E)}$ is increasing in $\sigma$, the possibility that the case shown in Proposition 2 increases with the uncertainty $\sigma$, in which the subcase 3 is more possible to occur, the larger $\sigma$ is, wherein it's more possible that the final-good producer exits the market or outsources in the South as $\Theta^N_O$ and $\Theta^{SNS}_{VVO} - \Theta^{NS}_{VVO}$ increase with $\sigma$.

Now we consider the subcase $\Theta^{NS}_{VVO} > \Theta^{NN}_{VVO}$. Define

$$\Theta^{NNS}_{VVO} = \frac{\sigma(f^S_O - f^N_o)}{(1 - \frac{1}{\rho}) \tau \omega \rho^{1-\eta} f_h \frac{1}{1-\rho} \frac{1}{1-\rho} \rho^{\frac{1}{1-\rho} - (f^N_o) - \frac{\rho(1-\eta)}{1-\rho}}},$$

(25)

$$\Theta^{NNS}_{VVO} = \frac{\sigma(f^S_O + f^V_N - f^N_o)}{(1 - \frac{1}{\rho}) \tau \omega \rho^{1-\eta} f_h \frac{1}{1-\rho} \frac{1}{1-\rho} \rho^{\frac{1}{1-\rho} - (f^N_o) - \frac{\rho(1-\eta)}{1-\rho}}},$$

(26)

then there are the following several possible cases.

If $\Theta^{NN}_{VVO} < \Theta^N_O$, then $F$ will integrate and outsource in the North simultaneously the production of $m$ if $\Theta^{NN}_{VVO} < \theta < \Theta^{NNS}_{VVO}$, it will integrate in the North and outsource in the South simultaneously if $\theta > \Theta^{NNS}_{VVO}$ and it will exit the market if $\theta < \Theta^{NN}_{VVO}$, where $\Theta^{NNS}_{VVO}$ is the abscissa of the intersection of the lines $\Pi^{NS}_{VVO}$ and $\Pi^{NN}_{VVO}$. This case is illustrated by Figure 6 as follows.

[Figure 6 is included here.]

If $\Theta^N_O < \Theta^{NN}_{VVO} < \Theta^S_O$, then $F$ will outsource in the North the production of $m$ if $\Theta^N_O < \theta < \Theta^{NNS}_{VVO}$, it will integrate and outsource in the North simultaneously if $\Theta^{SNS}_{VVO} < \theta < \Theta^{NNS}_{VVO}$ and it will integrate in the North and outsource in the South simultaneously if $\theta > \Theta^{NNS}_{VVO}$, and it will exit the market if $\theta < \Theta^N_O$, where $\Theta^{NNS}_{VVO}$ is the abscissa of the intersection of the lines $\Pi^{SNS}_{VVO}$ and $\Pi^{NN}_{VVO}$, $\Theta^{NNS}_{VVO}$ is that of the intersection of the lines $\Pi^{NN}_{VVO}$ and $\Pi^{NS}_{VVO}$. This case is illustrated by Figure 7 as follows.

[Figure 7 is included here.]

If $\Theta^S_O < \Theta^{NN}_{VVO}$, then $F$ will outsource in the North if $\Theta^N_O < \theta < \Theta^{NS}_{VVO}$, it will outsource in the South if $\Theta^{NS}_{VVO} < \theta < \Theta^{SNS}_{VVO}$, it will integrate and outsource in the North simultaneously if $\Theta^{SNS}_{VVO} < \theta < \Theta^{NNS}_{VVO}$, it will integrate in the North and outsource in the South simultaneously if $\theta > \Theta^{NNS}_{VVO}$ and it will exit the market if $\theta < \Theta^S_O$, where $\Theta^{NS}_{VVO}$ is the abcissa of the intersection of the lines $\Pi^{NS}_{VVO}$ and $\Pi^{NS}_{VVO}$. This case is illustrated by Figure 8 as follows.
[Figure 8 is included here.]

Finally, summing up the above discussions yields the following proposition.

**Proposition 3** Suppose that $\rho$, $\eta$ and $\frac{f_N}{f_m}$ are such that $\tau > 1$. Let $\frac{f_N}{f_m} \leq \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$. Then

1. the final-good producer integrates and outsources in the North simultaneously the production of $m$ if $\Theta_{V O}^{N N} < \Theta < \Theta_{V O}^{N S}$, integrates in the North and outsources in the South simultaneously if $\Theta > \Theta_{V O}^{N N}$ and exits the market if $\Theta < \Theta_{V O}^{N N}$ in case of $\frac{1-\rho}{\rho(1-\eta)} \frac{f_N}{f_m} \geq \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$.

2. the final-good producer integrates in the North the production of $m$ if $\Theta_{O}^{N S} < \Theta < \Theta_{V O O}^{N S}$, integrates and outsources in the North simultaneously if $\Theta_{V O O}^{N S} < \Theta < \Theta_{V O}^{N S}$, integrates in the North and outsources in the South if $\Theta > \Theta_{V O}^{N S}$ and exits the market if $\Theta < \Theta_{O}^{N S}$ in case of $\frac{1-\rho}{\rho(1-\eta)} \frac{f_N}{f_m} < \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$ and $\frac{1-\rho}{\rho(1-\eta)} \frac{f_N}{f_m} > \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$.

3. the final-good producer outsources in the North the production of $m$ if $\Theta_{O}^{N S} < \Theta < \Theta_{O}^{N S}$, outsources in the South if $\Theta_{O}^{N S} < \Theta < \Theta_{V O O}^{N S}$, integrates and outsources in the North simultaneously if $\Theta_{V O O}^{N S} < \Theta < \Theta_{V O}^{N S}$, integrates in the North and outsources in the South simultaneously if $\Theta > \Theta_{V O}^{N S}$ and exits the market if $\Theta < \Theta_{O}^{N S}$ in case of $\frac{1-\rho}{\rho(1-\eta)} \frac{f_N}{f_m} \leq \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$.

As $\frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E}$ increases with $\sigma$, the possibility that the case shown in Proposition 3 is larger the larger $\sigma$ is given the other parameters. Moreover, in this case, the more possible the subcase 3 occurs the larger $\sigma$ is, wherein the final-good producer is more possible to exit the market or outsource in the South as $\Theta_{O}^{N}$ and $\Theta_{V O O}^{N S} - \Theta_{V O}^{N S}$ increase with $\sigma$.

**Case 2.** $\Theta_{O}^{N} > \Theta_{O}^{N S}$.

In this case, there are several possible organizational modes different from the case of $\Theta_{O}^{N} < \Theta_{O}^{N S}$. There are two subcases, $\Theta_{O}^{N S} < \Theta_{O}^{N N}$, which is equivalent to $\frac{f_N}{f_m} > \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$, and $\Theta_{O}^{N S} > \Theta_{V O}^{N N}$, which is equivalent to $\frac{f_N}{f_m} < \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$. Note that there is always $\left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}} > \left( \frac{\sigma(f_n^N + f_n^S) + f_E}{\sigma(f_O^N + f_O^S) + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}}$, hence $\Theta_{O}^{N} > \Theta_{O}^{N S}$ implies that $\Theta_{V O}^{N S} \geq \Theta_{V O}^{N N}$ will never be true, and thus it suffices for us to consider the former subcase.
For the former subcase, if $\Theta_{V0}^{NS} < \Theta_{O}^{S}$, then $F$ will integrate in the North and outsource in the South simultaneously if $\Theta < \Theta_{V0}^{NS}$ and it will exit the market if $\Theta < \Theta_{V0}^{NS}$. The case is illustrated in Figure 9 as follows.

If $\Theta_{S}^{O} < \Theta_{V0}^{NS} < \Theta_{O}^{N}$, then $F$ will outsource in the South if $\Theta < \Theta_{S}^{O}$, it will integrate in the North and outsource in the South simultaneously if $\Theta > \Theta_{NSS}^{S}$, and it will exit the market if $\Theta < \Theta_{O}^{S}$. The case is illustrated in Figure 10 as follows.

Summarizing the above discussion yields the following proposition.

**Proposition 4** Suppose that $\rho, \eta$ and $f_{N}^{m}f_{Sm}$ are such that $\tau > 1$ and $f_{N}^{m} > (\frac{\sigma f_{S}^{O} + f_{E}}{\sigma f_{O}^{N} + f_{E}}) \frac{1 - \rho}{\rho^{1-\eta}}$.

Then

1. the final-good producer will integrate in the North and outsource in the South simultaneously if $\Theta < \Theta_{V0}^{NS}$ and it will exit the market if $\Theta < \Theta_{V0}^{NS}$ in the case of $\tau > \frac{\sigma (f_{S}^{O} + f_{E}) + f_{E}}{\sigma f_{O}^{N} + f_{E}}$, and

2. the final-good producer will outsource in the South if $\Theta_{S}^{O} < \Theta < \Theta_{V0}^{NSS}$, it will integrate in the North and outsource in the South simultaneously if $\Theta > \Theta_{NSS}^{V00}$ and it will exit the market if $\Theta < \Theta_{O}^{S}$ in the case of $\tau < \frac{\sigma (f_{S}^{O} + f_{E}) + f_{E}}{\sigma f_{O}^{N} + f_{E}}$ and $f_{N}^{m} > (\frac{\sigma (f_{S}^{O} + f_{E}) + f_{E}}{\sigma f_{O}^{N} + f_{E}}) \frac{1 - \rho}{\rho^{1-\eta}}$.

As $\frac{\sigma f_{S}^{O} + f_{E}}{\sigma f_{O}^{N} + f_{E}}$ increases with $\sigma$, the possibility that the case shown in Proposition 4 decreases with $\sigma$.

### 3.5. Organizational choices and uncertainty

We see in the above subsection that the demand uncertainty does influence firms’ organizational modes. To investigate it in detail, we need to analyze the relationship between a final-good producer’s organizational choices and the uncertainty level $\sigma$. Let $\Sigma(\sigma) = \frac{1 - \rho}{\rho^{1-\eta}} / \sigma$. According to the discussion of the monotonicity of $\Sigma(\sigma)$ in $\sigma$, $\Sigma(\sigma)$ is increasing in $\sigma$, and hence $\Sigma = \Sigma(\sigma)$ has an inverse function $\sigma = \sigma(\Sigma)$. This implies that $\Sigma$ can be used to measure the “uncertainty” of demand of $F$’s final good.

We still denote $\Theta = \theta^{\frac{1}{1-\eta}}$. To investigate $F$’s organizational decisions, it suffices
for us to compare

\[ \tilde{\Pi}_O^l = \Pi_{O/l}^l = \left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta \Sigma \left[ f_h^{\rho_1} (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_E/\sigma), l = N, S \] (27)

with

\[ \tilde{\Pi}_{V/O}^{NI} = \Pi_{V/O}^{NI} = \left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta \Sigma \left[ f_h^{\rho_1} (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_V^l + f_E/\sigma) \] (28)

wherein we ignore the organizational mode of integrating in the North because it's dominated by outsourcing in the North.

It's easy to see that \( \tilde{\Pi}_O^l \) and \( \tilde{\Pi}_{V/O}^{NI} \) are increasing in \( \sigma \). Let \( \Sigma_{O/l}^l \) and \( \Sigma_{V/O}^{NI/l} \) be solutions of \( \hat{\Pi}_O^l = 0 \) and \( \hat{\Pi}_{V/O}^{NI} = 0 \), respectively. Then according to the Appendix, \( \Theta \Sigma_{O/l}^l \) and \( \Theta \Sigma_{V/O}^{NI/l} \) are increasing in \( \Theta \). Define

\[ \Sigma_{O/l}^{NS} = \frac{f_O^S - f_O^N}{\left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta f_h^{\rho_1} \left[ (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_E/\sigma)} \] (29)

\[ \Sigma_{V/O}^{NS} = \frac{f_O^V + f_O^N - f_O^S}{\left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta f_h^{\rho_1} \left[ (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_E/\sigma)} \], \( l = N, S \)

\[ \Sigma_{V/O}^{NS} = \frac{f_O^S - f_O^N}{\left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta f_h^{\rho_1} \left[ (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_E/\sigma)} \]

\[ \Sigma_{V/O}^{NS} = \frac{f_O^V + f_O^N - f_O^S}{\left( \frac{1}{\rho} - 1 \right) \rho^{1/\tau} \Theta f_h^{\rho_1} \left[ (f_m^{\rho_2})^{\rho_1} \right]^{1/\tau} - (f_O^l + f_E/\sigma)} \].

Obviously, they are all independent from \( \Sigma \). It suffices for us to compare \( \tilde{\Pi}_O^l - f_E/\sigma \) and \( \tilde{\Pi}_{V/O}^{NI} - f_E/\sigma \) instead of comparing \( \hat{\Pi}_O^l \) and \( \hat{\Pi}_{V/O} \) for \( l = N, S \). We conclude immediately that the four propositions formulated in the former subsections holds for the relationships between organizational modes and uncertainty \( \Sigma \), with those parameters of \( \Theta \) replaced by those of \( \Sigma \), shown above. Without proofs, we list them as follows. The shortcoming of these propositions is that their conditions are related to \( \Sigma \) itself. As a special case with \( f_E = 0 \), the demand uncertainty results in the same effects on a final-good producer's organizational choices.

**Proposition 5** If \( \rho, \eta \) and \( \frac{\sin}{\sin} \) are such that \( \tau \leq 1 \), then

1. the final-good producer outsources the production of \( m \) in the South if \( \Sigma > \)
\[ \Sigma^S_O \] and exits the market if \( \Sigma < \Sigma^S_O \) in the case of \( \frac{f^N_m}{f^m} \geq \left( \frac{\sigma f^N_m + f_E}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \), and

2. the final-good producer outsources the production of \( m \) in the South if \( \Sigma > \Sigma^N_S \), outsources the production of \( m \) in the North if \( \Sigma^N_S < \Sigma < \Sigma^N_O \) and exits the market if \( \Sigma < \Sigma^N_O \) in the case of \( \frac{f^N_m}{f^m} < \left( \frac{\sigma f^N_m + f_E}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \).

**Proposition 6** Suppose that \( \rho, \eta \) and \( \frac{f^N_m}{f^m} \) are such that \( \tau > 1 \). Let \( \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} < \frac{f^N_m}{f^m} \leq \left( \frac{\sigma f^N + f_E}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \). Then

1. the final-good producer integrates in the North and outsources in the South simultaneously the production of \( m \) if \( \Sigma > \Sigma^N_S \), and exits the market if \( \Sigma < \Sigma^N_O \) in case of \( \frac{f^N_m}{f^m} \geq \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \), \( \tau > 1 \).

2. the final-good producer integrates in the North and outsources in the South simultaneously the production of \( m \) in the North if \( \Sigma^N_O < \Sigma < \Sigma^N_S \) and exits the market if \( \Sigma < \Sigma^N_O \) in case of \( \frac{f^N_m}{f^m} < \left( \frac{\sigma f^N + f_E}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \) and \( \tau > 1 \).

3. the final-good producer outsources in the North if \( \Sigma^N_O < \Sigma < \Sigma^N_S \), outsources in the South if \( \Sigma^N_S < \Sigma < \Sigma^N_O \), integrate in the North and outsources in the South simultaneously if \( \Sigma > \Sigma^N_S \) and exits the market if \( \Sigma < \Sigma^N_O \) in case of \( \frac{f^N_m}{f^m} < \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \).

**Proposition 7** Suppose that \( \rho, \eta \) and \( \frac{f^N_m}{f^m} \) are such that \( \tau > 1 \). Let \( \frac{f^N_m}{f^m} \leq \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \). Then

1. the final-good producer integrates and outsources in the North simultaneously the production of \( m \) if \( \Sigma^N_O < \Sigma < \Sigma^N_S \), integrates in the North and outsources in the South simultaneously if \( \Sigma > \Sigma^N_S \) and exits the market if \( \Sigma < \Sigma^N_O \) in case of \( \tau \frac{f^N_m}{f^m} \geq \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \), \( \tau > 1 \).

2. the final-good producer outsources in the North the production of \( m \) if \( \Sigma^N_O < \Sigma < \Sigma^N_S \), integrates and outsources in the North simultaneously if \( \Sigma^N_S < \Sigma < \Sigma^N_O \), integrates in the North and outsources in the South if \( \Sigma > \Sigma^N_S \) and exits the market if \( \Sigma < \Sigma^N_O \) in case of \( \tau \frac{f^N_m}{f^m} < \left( \frac{\sigma (f^N_m + f^N)}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \) and \( \tau \frac{f^N_m}{f^m} > \left( \frac{\sigma f^N + f_E}{\sigma f^N + f_E} \right)^{\frac{1}{1-\eta}} \), \( \tau > 1 \).
3. the final-good producer outsources in the North the production of \( m \) if \( \Sigma^N < \Sigma < \Sigma^N_S \), outsources in the South if \( \Sigma^N_S < \Sigma < \Sigma^{NNS}_{VOO} \), integrates and outsources in the North simultaneously if \( \Sigma^{NNS}_{VOO} < \Sigma < \Sigma^{NNS} \), integrates in the North and outsources in the South simultaneously if \( \Sigma > \Sigma^{NNS} \) and exits the market if \( \Sigma < \Sigma^O \) in case of \( \tau < \rho \).

**Proposition 8** Suppose that \( \rho, \eta \) and \( \frac{f^N}{f^m} \) are such that \( \tau > 1 \) and \( \frac{f^N}{f^m} > \left( \frac{\sigma f^N + f_E}{\sigma f^O + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}} \).

Then

1. the final-good producer will integrate in the North and outsource in the South simultaneously if \( \Sigma > \Sigma^{NNS}_{VOO} \) and it will exit the market if \( \Sigma < \Sigma^{NNS}_{VOO} \) in case of \( \tau > 1 \) and \( \frac{f^N}{f^m} > \left( \frac{\sigma f^N + f_E}{\sigma f^O + f_E} \right)^{\frac{1-\rho}{\rho(1-\eta)}} \).

2. the final-good producer will outsource in the South if \( \Sigma^O < \Sigma < \Sigma^{NNS}_{VOO} \), it will integrate in the North and outsource in the South simultaneously if \( \Sigma > \Sigma^{NNS}_{VOO} \) and it will exit the market if \( \Sigma < \Sigma^O \) in case of \( \tau < \rho \). As \( \frac{\sigma f^N + f_E}{\sigma f^O + f_E} \) is increasing in \( \tau \), it suffices for us to investigate how the ratio \( \frac{\sigma f^N + f_E}{\sigma f^O + f_E} \) varies with the demand uncertainty \( \sigma \). If \( \theta_{uv} \) is decreasing in \( \sigma \), while \( \frac{\theta_{uw}}{\theta_{uw}} \) is decreasing in \( \sigma \), then we conclude immediately that the prevalence of \( v \) is decreasing in \( \sigma \).

4. **Prevalence of organizational modes**

It’s interesting to investigate how demand uncertainty (productivity) affects the prevalence of various organizational modes with productivity (demand uncertainty) of a final-good producer being fixed. According to Antras and Helpman (2004), the prevalence of an organizational mode is its occurrence probability. Suppose \( u, v, w \) are three two-two adjoint organizational modes occurring in a firm’s series of organizational modes, and the productivity cutoff from \( u \) to \( v \) is \( \theta_{uv} \), and that from \( v \) to \( w \) is \( \theta_{vw} \), where \( \theta_{uv} < \theta_{uv} \). Then the prevalence of organizational mode \( v \) is \( \Pr(v) = G(\theta_{uv}) - G(\theta_{uw}) \). As \( G(\theta) \) is increasing in \( \theta \), it suffices for us to investigate how the ratio \( \frac{\theta_{uv}}{\theta_{uw}} \) of \( \theta_{uv} \) and \( \theta_{uw} \) varies with the demand uncertainty \( \sigma \). If \( \theta_{uw} \) is decreasing in \( \sigma \), while \( \frac{\theta_{uv}}{\theta_{uw}} \) is decreasing in \( \sigma \), then we conclude immediately that the prevalence of \( v \) is decreasing in \( \sigma \).
Note that \( \frac{d\Theta}{d\sigma} < 0, \frac{d\Theta^{NS}}{d\sigma} < 0, \frac{d\Theta^{NI}}{d\sigma} < 0, \frac{d\Theta^{NIS}}{d\sigma} < 0, \frac{d\Theta^{NNIS}}{d\sigma} < 0 \) according to the increasing monotonicity of \( \Sigma(\sigma) \) in \( \sigma \). It’s also easy to see that \( \frac{d(\Theta^{NI}/\Theta^{NS})}{d\sigma} < 0, \frac{d(\Theta^{NIS}/\Theta^{NNIS})}{d\sigma} < 0 \). From these two facts and what asserted in Proposition 1 to Proposition 4, we conclude immediately the following result.

If a firm’s demand uncertainty \( \sigma \) is fixed, then we have immediately the following proposition according to (29).

**Proposition 9** If a final-good producer’s productivity is fixed, then with the increase of demand uncertainty, the prevalence of the organizational mode of

1. exiting the market decreases,
2. outsourcing in the North decreases,
3. outsourcing in the South increases,
4. integrating and outsourcing in the North simultaneously decreases,
5. and integrating in the North and outsourcing in the South simultaneously increases

if it occurs in the firm’s organizational modes.

Interestingly, though demand uncertainty and productivity of a final-good producer affects its organizational choices in a very uniform way, they have asymmetric influences on the prevalence of various organizational modes. In fact, applying the approach that we derive Proposition 9 and by the increasing Monotonicity of \( \Theta \Sigma^{i}_{O}(\Theta) \) and \( \Theta \Sigma^{NI}_{V,O}(\Theta) \), we conclude the following result.

**Proposition 10** If a final-good producer’s uncertainty is fixed, then with the increase of productivity, the prevalence of the organizational mode of

1. exiting the market decreases,
2. outsourcing in the North decreases,
3. integrating and outsourcing in the North simultaneously decreases,
4. integrating in the North and outsourcing in the South simultaneously increases,
if it occurs in the firm’s organizational modes. But the prevalence of outsourcing in the South does not necessarily decreases. If it occurs when the firm does not choose the mode of integrating in the North and outsourcing in the South simultaneously, its prevalence increases with the increase of the productivity, otherwise, the situation is reversed.

5. General equilibrium

In this section, we show how a general equilibrium is determined and how we can find it.

Let \( \Pi(\theta) = \max_{l=N,S}\{\Pi_O^l, \Pi_{VO}^N\} \). Then \( \Pi(\theta) \) is a function of \( \theta \) given \( \sigma, \rho \) and \( \eta \). Let \( H(\theta), m_V^N(\theta) \) and \( m_O^l(\theta) \) be respectively \( F’ \) production capacity of \( h \) determined at Time 1, its output of \( m \) produced in the North at Time 3, and \( S’ \)’s output of \( m \) produced in country \( l \) at Time 3, which maximize \( \Pi(\theta) \), given \( F’ \)’s productivity level \( \theta \) in an industry (if \( F \) does not outsource in country \( l \), then \( m_O^l(\theta) = 0 \); if \( F \) does not integrate in the North, then \( m_V^N(\theta) = 0 \)). Then \( F’ \)’s output of its final good is \( y(\theta) = \theta \left( \frac{H}{\eta} \right)^{\eta} \left( \frac{m_V^N + m_S^N + m_O^N}{1-\eta} \right)^{1-\eta} \). Let the output of the homogeneous good in country \( l \) be \( y_0^l \). Here \( m_V^N, m_O^l \) and \( H \) are all correlated with \( f_h \) and \( f_{m_l}^l \). Then the labor and the capital demanded for producing \( y_0^l \) units of homogeneous good in country \( l \) are \( \frac{\partial f_l}{\partial w} y_0^l \) and \( \frac{\partial f_l}{\partial r} y_0^l \), respectively, where \( f_0^l = \left( \frac{w_l}{\alpha} \right)^\alpha \left( \frac{r_l}{1-\alpha} \right)^{1-\alpha} \). Similarly, for each final-good producer with productivity level \( \theta \), the labor and capital demanded in the North to produce \( H(\theta) \) units of \( h \) are \( \frac{\partial f_{m_N}}{\partial w_N} H \) and \( \frac{\partial f_{m_N}}{\partial r_N} H \), and those demanded in the North under integration to produce \( m_{V_N}^N(\theta) \) units of \( m \) are \( \frac{\partial f_{m_N}}{\partial w_N} m_{V_N}^N \) and \( \frac{\partial f_{m_N}}{\partial r_N} m_{V_N}^N \), respectively. For its supplier, those demanded to produce \( m_O^l(\theta) \) units of \( m \) under outsourcing in country \( l \) are respectively \( \frac{\partial f_{m}}{\partial w_N} m_O^l \) and \( \frac{\partial f_{m}}{\partial r_N} m_O^l \). Then for each differentiated good, the expected labor and the expected capital demanded to produce it in country \( N \) are respectively

\[
\int \left[ \frac{\partial f_{m_N}}{\partial w_N} H(\theta) + \frac{\partial f_{m_N}}{\partial w_N} m_{V_N}^N(\theta) + \frac{\partial f_{m_N}}{\partial w_N} m_{O_N}^N(\theta) \right] dG(\theta)
\]

and

\[
\int \left[ \frac{\partial f_{m_N}}{\partial r_N} H(\theta) + \frac{\partial f_{m_N}}{\partial r_N} m_{V_N}^N(\theta) + \frac{\partial f_{m_N}}{\partial r_N} m_{O_N}^N(\theta) \right] dG(\theta),
\]

where \( G(\bullet) \) is the cumulative distribution of \( \theta \).
Those demanded to produce its intermediated-input in country $S$ are respectively $\int \frac{\partial f_N^S}{\partial w_N} y_0^N m^S_\theta (\theta) dG(\theta)$ and $\int \frac{\partial f_S^S}{\partial w_S} m^S_\theta (\theta) dG(\theta)$. Then the clearing conditions for the factor markets in country $N$ are

$$\int \left[ \frac{\partial f_N^N}{\partial w_N} y_0^N + n \int \left[ \frac{\partial f_h^N}{\partial w_N} H(\theta) + \frac{\partial f_m^N}{\partial w_N} m^N_\theta (\theta) + \frac{\partial f_m^N}{\partial r_N} m^S_\theta (\theta) \right] dG(\theta) \right] f(z) dz = L^N_N \quad (30)$$

$$\int \left[ \frac{\partial f_N^N}{\partial r_N} y_0^N + n \int \left[ \frac{\partial f_h^N}{\partial r_N} H(\theta) + \frac{\partial f_m^N}{\partial r_N} m^N_\theta (\theta) + \frac{\partial f_m^N}{\partial r_N} m^S_\theta (\theta) \right] dG(\theta) \right] f(z) dz = K^N_N \quad (31)$$

where $f(z)$ is the density function of $z$. Those for the factor markets in country $S$ are

$$\int \left[ \frac{\partial f_S^S}{\partial w_S} y_0^S + n \int \left[ \frac{\partial f_h^S}{\partial w_S} m^S_\theta (\theta) + \frac{\partial f_m^S}{\partial w_S} m^S_\theta (\theta) + \frac{\partial f_m^S}{\partial r_S} m^S_\theta (\theta) \right] dG(\theta) \right] f(z) dz = L^S \quad (32)$$

$$\int \left[ \frac{\partial f_S^S}{\partial r_S} y_0^S + n \int \left[ \frac{\partial f_h^S}{\partial r_S} m^S_\theta (\theta) + \frac{\partial f_m^S}{\partial r_S} m^S_\theta (\theta) + \frac{\partial f_m^S}{\partial r_S} m^S_\theta (\theta) \right] dG(\theta) \right] f(z) dz = K^S \quad (33)$$

where $n$ is the number of differentiated goods in the North, which can be found by solving the following zero-profit condition for each final-good producer

$$\int \left[ (1 - G(\theta)) \int_{\theta}^{+\infty} H(\theta) dG(\theta) \right] f(z) dz = \delta F_E \quad (34)$$

where $\delta$ is the probability that a firm survives to the next period, which is assumed to be constant over all the periods, and $\theta$ is the productivity cutoff, below which the firm will exit the market. Here, their prices (and quantities produced) at equilibrium are the same (denoted by $p$) according to symmetry of differentiated final goods. Solving (30)-(34) yields the equilibrium variables.

Note that it’s difficult to solve Equations (30)-(34) because any final-good producer’s organizational modes in the markets are influenced by $\frac{\partial f_S^S}{\partial f_m^S}$, while the latter can only be determined after the organizational modes have been determined. To solve the above equations and finally determine the equilibrium prevalence of various organizational modes in the markets, we shall apply numerically computational methods, which deserves another paper to investigate the sophistication under various factor endowments for the two countries. We neglect it here.
6. Conclusion

Though there are many literatures having investigated the relationship between integration, outsourcing and uncertainty, this paper proposes a new framework to investigate firm’s organizational decisions of ownership and location with heterogeneous firms under uncertainty. The key point in our model is that Demand uncertainty occurs after the unexpandable production capacity of $h$ is determined. However, this may cause profit losses for the final-good producer. Therefore, it causes the final-good producer to adjust its organizational decisions so as to mitigate it. We show that demand uncertainty results in partial outsourcing, i.e., a final-good producer may integrate part of the production of $m$ and outsource part of it domestically or abroad, which depends on the producer’s productivity level, the uncertainty degree of demand, the ratio between the variable costs producing $m$ in the North and in the South, the organizational costs, the entry costs, etc. Summing up, with other parameters given, a final-good producer never chooses only integrating the production of $m$ in the North, and its organizational choices are outsourcing in the North, outsourcing in the South, integrating and outsourcing simultaneously in the North, and integrating in the North and outsourcing in the South simultaneously, in turn with increasing of its productivity level $\theta$. A final-good producer does not only choose integration or outsourcing of the production of $m$. These new organizational modes never occur in other literatures with heterogeneous productivity, incomplete contracts, fixed entry and organizational costs, without incorporating uncertainty and production capacity. such as Antras and Helpman (2004), Antras and Helpman (2006), Acemoglu et al. (2007), etc. For the relationship among organizational modes and uncertainty, we also show that the increase of the uncertainty of a final-good producer’s demand results in turn in outsourcing in the North, outsourcing in the South, integrating and outsourcing simultaneously in the North, and integrating in the North and outsourcing in the South simultaneously.

The framework proposed in this paper can be extended in multiple directions. First, it can be extended to the case that the organizational choices are made before the demand uncertainty is realized. In this case, the investment in the intermediated input $m$ shall be made before the demand uncertainty and it can not be expanded afterward. We are sure that the order a final-good producer gets aware of its demand may influence its organizational choices. Second, one may consider the case that a final-good producer can simultaneously outsource the
production of $m$ in both countries. This consideration yields more organizational modes and thus the situation is more complicated. Moreover, as suppliers’ investment in the production of $m$ in the two countries may influence that of the opposite, so that their strategic interaction finally affects their bargaining powers toward the final-good producer. Hence, to analyze this situation, one has to model bargaining process among multiple parties, which is our main research work in the future. Finally, one can further investigate how factor endowments of countries influence the trade patterns under the setting of demand uncertainty, investment irreversibility and unexpandability and firms’ choices of various organizational modes.
References


Appendix

The maximal value of $\tau(\zeta)$

To see whether the optimal value of $\tau(\zeta)$ is possible to be larger than 1, it suffices to find the maximal value of the function $g(\zeta) = \chi(1 - \zeta)^{1/\rho(1 - \eta)}$. To simplify deduction, we take the following transformation:

$$x = \frac{\zeta}{1 - \zeta},$$

then

$$\zeta = \frac{x}{1 + x}, \quad 1 - \zeta = \frac{1}{1 + x}.$$

Let

$$a = -\frac{1}{\rho}, \quad c = \frac{f_N}{f_m}, \quad b = \left(\frac{1}{\rho} - (1 - \eta)\right)c + 1 - \eta, \quad d = -\frac{1}{1 - \rho(1 - \eta)},$$

then

$$g(x) = \left[-a(1 + x) - (1 - \eta) + (ax + b)(c - x)^d\right](1 + x)^d.$$

The first-order condition of the maximization problem of $g(x)$ reduces to

$$d(ax + b)(c - x)^{d-1}(c - 1 - 2x) + a(1 + x)(c - x)^d - a(1 + d)(1 + x) = d(1 - \eta).$$

The above equation has no analytic solutions. So we can not analytically know whether $g(x)$’s maximal value is larger than 1 or not. To see, we need to only consider the case of $x = -\frac{b}{a} = \rho(1 - \eta) + [1 - \rho(1 - \eta)]\frac{f_N}{f_m}$. Then $c - x = \left[\frac{f_N}{f_m} - 1\right]\rho(1 - \eta), 1 + x = 1 + \rho(1 - \eta) + [1 - \rho(1 - \eta)]\frac{f_N}{f_m}$. Thus

$$g\left(\frac{b}{a}\right) = \frac{1}{\rho} \frac{1 + (1 - \rho(1 - \eta))\frac{f_N}{f_m}}{\left[1 + (1 - \rho(1 - \eta))\frac{f_N}{f_m} + \rho(1 - \eta)\right]^{\frac{1}{1 - \rho(1 - \eta)}}}.$$

It’s easy to see that $\lim_{\rho \to 0} g\left(\frac{b}{a}\right) = +\infty$ for given $\eta \in (0, 1)$. Hence there exists indeed $\rho$ (for example, sufficiently small $\rho$, according to the continuity of $g\left(\frac{b}{a}\right)$ with respect to $(\rho, \eta)$), such that $g\left(\frac{b}{a}\right) > 1$. This implies that the maximal value of
g(x) hence the optimal value of $\tau(\zeta)$ is larger than 1 for some $\rho < 1$.

The monotonicity of $\Sigma(\sigma)$ in $\sigma$

To explore the monotonicity of $\Sigma(\sigma)$ in $\sigma$, it suffices for us to investigate that of the function

$$h(\sigma) = \frac{1 - \rho(1 - \eta)}{1 - \rho} \ln \left( \frac{\bar{z} + \frac{\sigma}{\sqrt{2}}}{\bar{z} - \frac{\sigma}{\sqrt{2}}} \right) - \ln \sigma. \quad (35)$$

It’s easy to get the derivative of $h$ with respect to $\sigma$ to be

$$h'(\sigma) = \frac{2 - \rho(1 - \eta)}{2(1 - \rho)} \left( \bar{z} + \frac{\sigma}{\sqrt{2}} \right)^{1 - \rho(1 - \eta)} + \left( \bar{z} - \frac{\sigma}{\sqrt{2}} \right)^{1 - \rho(1 - \eta)} - \frac{1}{\sigma}. \quad (35)$$

Let $a = \frac{1}{1 - \rho(1 - \eta)}$, $b = \frac{2 - \rho(1 - \eta)}{2(1 - \rho)}$, $x = \frac{\bar{z} - \sigma}{\bar{z} + \frac{\sigma}{\sqrt{2}}}$, then $x \in (0, 1)$, and $a > 1, b > 1$. Under these notations, we can rewrite $h'(\sigma)$ as

$$h'(\sigma) = \frac{1}{\bar{z} + \frac{\sigma}{\sqrt{2}}} \left( \frac{b(1 + x^a)}{1 - x^{a+1}} - \frac{1}{1 - x} \right).$$

To judge the sign of $h'(\sigma)$, it suffices for us to judge the sign of

$$t(x) = \frac{b(1 + x^a)}{1 - x^{a+1}} - \frac{1}{1 - x}.$$

Note that

$$t'(x) = \frac{abx^{a-1} + (a + 1)bx^a + bx^{2a}}{(1 - x^{a+1})^2} + \frac{1}{(1 - x)^2} > 0,$$

which implies that $t(x)$ is increasing in $x$. As $t(0) = b - 1 > 0$, we conclude that $t(x) > 0$ for all $x \in (0, 1)$. This implies that $h'(\sigma) > 0$ for all $\sigma \in (0, 2\bar{z})$. Hence $\Sigma(\sigma)$ is increasing in $\sigma$ for all $\sigma \in (0, 2\bar{z})$.

Increasing Monotonicity of $\Theta \Sigma^l_\Theta(\Theta)$ and $\Theta \Sigma^N_\Theta(\Theta)$

To see the increasing Monotonicity of $\Theta \Sigma^l_\Theta(\Theta)$ and $\Theta \Sigma^N_\Theta(\Theta)$, it suffices for us to consider the monotonicity of $g(\Theta) = \Theta \Sigma^\Theta_\Theta(\Theta)$, where $\Theta$ is the solution of the
following equation

\[ a \Theta \Sigma - \frac{f_E}{\sigma} - b = 0, \]  

(36)

where \( a, b > 0 \).

First, as \( \Sigma \) is increasing in \( \sigma \), we have \( \frac{d\Sigma}{d\sigma} < 0 \) and \( \frac{d\sigma}{d\Theta} > 0 \) according to Equation (36). Second, we have \( h(\Theta) = \frac{1}{a} \left( b + \frac{f_E}{\sigma(\Theta)} \right) \). And thus

\[ \frac{dh}{d\Theta} = -\frac{f_E}{a} \frac{1}{\sigma^2} \frac{d\sigma}{d\Theta} > 0, \]

which implies that \( h \) is increasing in \( \Theta \).

Specifying \( a = \left( \frac{1}{\rho} - 1 \right) \rho^{1-\tau} \left[ f^{m_1}(f_{m_1}^m)^{\rho - m} \right]^{-\frac{1}{1-\tau}} b = f_{O}^1 \) for \( \Theta \Sigma_{O}^1 \), and specifying \( a = \Pi_{V_{O}^{N}}/\sigma = \left( \frac{1}{\rho} - 1 \right) \rho^{1-\tau} \tau \sigma \left[ f^{m_1}(f_{m_1}^m)^{\rho - m} \right]^{-\frac{1}{1-\tau}} \) and \( b = f_{O}^1 + f_{V}^N \) for \( \Theta \Sigma_{V_{O}^{N}} \), we conclude their increasing monotonicity.

\textbf{Figures}
Figure 2: \( F \) outsources in the North first, and then outsources in the South

Figure 3: \( F \) integrates in the North and outsources in the South simultaneously
Figure 4: $F$ outsources in the North first, and then integrates in the North and outsources in the South simultaneously.

Figure 5: $F$ outsources in the North first, and then outsources in the South, then integrates in the North and outsources in the South simultaneously.
Figure 6: $F$ integrates and outsources in the North simultaneously first, and then integrates in the North and outsources in the South simultaneously.

Figure 7: $F$ outsources in the North first, and then integrates and outsources in the North simultaneously, and then integrates in the North and outsources in the South simultaneously.
Figure 8: $F$ outsources in the North first, and then outsources in the South, then integrates and outsources in the North simultaneously, then integrates in the North and outsources in the South simultaneously.

Figure 9: $F$ only integrates in the North and outsources in the South simultaneously.
Figure 10: F outsources in the South first, and then integrates in the North and outsources in the South simultaneously