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Abstract

We study the dynamic properties of growth rates in an overlapping generations economy with credit market imperfections. The analysis demonstrates that in early stages of financial development where credit constraints are severe, growth rates evolve monotonically. At the intermediate level of financial development, as the degree of credit market imperfections diminishes, growth rates exhibit endogenous fluctuations for some parameter values. However, as the financial sector matures, fluctuations disappear and the growth rates evolve once again monotonically.

Keywords: Credit market imperfections; Endogenous business fluctuations; Endogenous growth; Heterogeneous agents.

JEL Classification Numbers: O41

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1 Introduction

Endogenous business fluctuations have been studied with overlapping generations models for over twenty years. The dynamic properties of exchange economies have been investigated in the literature initiated by Benhabib and Day (1982) and Grandmont (1985). They have paid attention to deterministic cycles or chaotic behavior in equilibrium. Azariadis (1981) and Azariadis and Guesnerie (1986) study sunspot equilibria of exchange economies. For an overlapping generations economy with a production sector, Farmer (1986), Reichlin (1986), Benhabib and Laroque (1988), and Rochon and Polemarchakis (2006) derive competitive equilibrium cycles. Galor (1992) develops a model with two production sectors and makes clear the conditions for indeterminacy of equilibrium. None of these studies deal with an overlapping generations economy with credit market imperfections, although credit market imperfections are important in understanding macroeconomic phenomena.[1]

Indeed, overlapping generations economies with credit market imperfections have been studied in an extensive literature.[2] Among those, Matsuyama (2007) and Assenza, et al. (2009) derive endogenous fluctuations in overlapping generations economies with credit market imperfections. To the best of our knowledge, however, no studies demonstrate under what degree of credit market imperfections the growth rates of an overlapping generations economy are highly volatile. The objective of this paper is to answer this question.

In this paper, we employ the Schumpeterian endogenous growth model developed by Aghion and Howitt (1992, 1998). Many researchers have studied the relationship between economic growth and volatility induced by exogenous productivity shocks empirically and theoretically (e.g., Ramey and Ramey, 1995; Aghion and Banerjee, 2005; Aghion, et al., 2010). While in investigating the relationship, endogenous growth models have been useful tools, the AK approach cannot derive a negative correlation between economic growth and

[1] For finance and growth, see Levine, et al. (2000) and Aghion, et al (2005) and for business cycles and credit market imperfections, see, for instance, Kiyotaki and Moore (1997). For financial market globalization and credit market imperfections, see, for instance, Matsuyama (2004).

[2] See, for instance, Bernanke and Gertler (1989), Galor and Zeira (1993), and Boyd and Smith (1998).

volatility under a small inter-temporal elasticity of substitution of agents' utility (Aghion and Banerjee, 2005), which is consistent with empirical evidence of Ramey and Ramey (1995). In contrast, the Schumpeterian approach with credit market imperfections derives a negative correlation between economic growth and volatility under a small inter-temporal elasticity of substitution. Although we do not assume any exogenous shocks in our model, the existing literature motivates us to use the Schumpeterian growth model.

In our model, we assume heterogeneous agents within a generation in terms of their productivity in creating capital goods for a R&D sector. Usually, it is difficult to construct a tractable model with credit constraints in a closed economy where interest rates are determined endogenously. By incorporating heterogeneous agents into an economy, we make the model tractable in handling credit market imperfections. In our model, each agent can make a deposit in or borrow from an infinitely-lived financial intermediary. If the productivity of an agent is greater than a cutoff, he starts an investment project, borrowing from the financial intermediary up to the limit of a credit constraint. Meanwhile, if the productivity of an agent is lower than the cutoff, he only deposits a part of his income in the financial intermediary. Since all of the financial trades are executed via the financial market, each agent unconsciously makes financial transactions with other agents intra- or inter-generationally. The cutoff of the agents' productivity which divides agents into savers and investors is determined endogenously and fluctuates endogenously for some parameter values. The growth rate of the economy is a one-to-one function of the cutoff and thus the growth rates endogenously fluctuate as well.

The main finding is as follows. In early stages of the development of a financial sector where credit market imperfections are severe, growth rates evolve monotonically. At the intermediate level of financial development, growth rates exhibit endogenous fluctuations for some parameter values. However, as the financial sector matures and credit market imperfections are resolved, fluctuations disappear and the growth rates evolve once again monotonically.

This paper proceeds as follows. Section 2 provides a model. In section 3 we study equilibrium, deriving a function for the growth rate with respect to the cutoff of agents' productivity. In section 4, we investigate the dynamic properties of the economy. In section 5, we discuss the mechanism with which endogenous fluctuations arise in equilibrium and in section 6, we provide numerical examples and observe that it is when the degree of financial development is at the intermediate level that the growth rates endogenously fluctuate. We provide concluding remarks in section 7.

2 Model

The economy consists of overlapping generations: young and old agents. Time goes from 0 to ∞. Each agent lives for two periods. The rate of population growth is assumed to be $n > -1$, namely $L_{t+1} = (1+n)L_t$ where L_t is the population of young agents at time t. Young agents can borrow up to a certain limit from an infinitely-lived financial intermediary. In other words, the financial intermediary imposes credit constraints on young agents.

2.1 Debt and Credit

Young agents can deposit a part of their income in the financial intermediary. Those deposits become financial resources for the financial intermediary to loan. Meanwhile, each young agent can borrow against his future income by selling bonds to the financial intermediary. Young agents can consume, or invest in a project, more than their income in the first period by going into debt to the financial intermediary.

If the total stock of deposits made by young agents is greater than the total stock of loans lent to them, this is the case in which the financial intermediary is indebted to the private sector (the debt case). On the other hand, if the total stock of deposits made by young agents is less than the total stock of loans lent to them, this is the case in which the private sector is indebted to the financial intermediary. In this case, the financial intermediary owns a stock of loans to the private sector (the credit case).

A loan made by the financial intermediary is an exchange of real output in this period for real output in the next period. The debt of the financial intermediary at time t, B_t, is equal to the total deposit minus the total loan in the economy at time t. So in the debt case, it holds that $B_t > 0$, whereas in the credit case, it follows that $B_t < 0$. From a point of view of a balance sheet of the financial intermediary, $B_t < 0$ reflects the equity capital of the financial intermediary. By contrast, $B_t > 0$ reflects the national debt. The law of motion of the total debt of the financial intermediary is given by:

$$B_{t+1} = r_{t+1} B_t, \tag{1}$$

where r_{t+1} is the (real) interest rate at time $t + 1$.

As discussed in De La Croix and Michel (2002, Chapter 4, pp.212), B_0 is a predetermined variable since we do not incorporate nominal money into this economy. At time zero, the financial intermediary can be a net creditor or a net debtor to the private sector. The initial amount of credit or debt is determined historically.

2.1.1 Individuals

Each agent is born with one unit of endowments which we call labor. In the first period of his lifetime, he supplies labor inelastically and earns a wage income. An agent born at time t consumes c_{1t} when young and c_{2t+1} when old. There are two saving methods for each agent. One is depositing his income in the financial intermediary. If an agent deposits one unit of his income in the financial intermediary at time t, he will gain a claim to r_{t+1} units of consumption goods at time $t + 1$.

Alternatively, an agent can start an investment project, which will produce capital goods. The capital goods are thought of broadly as human capital or physical capital and they are used as input goods for an R&D sector.

In the first period, each agent consumes, starts an investment project, deposits his income in the financial intermediary, and/or borrows from it. In the second period, he consumes all his earnings from the investments and from the deposits, and he repays the financial intermediary if he borrowed in the first period.

Each agent maximizes his lifetime utility:

$$u(c_{1t}, c_{2t+1}) \tag{2}$$

which is a function of consumption (c_{1t}, c_{2t+1}) in youth and old age, subject to:

$$c_{1t} + k_t + b_t \leq w_t \tag{3}$$

$$z_{t+1} = \phi k_t \tag{4}$$

$$c_{2t+1} \leq q_{t+1} z_{t+1} + r_{t+1} b_t, \tag{5}$$

$$b_t \geq -\mu k_t, \quad 0 \leq \mu < 1 \tag{6}$$

$$k_t \geq 0. \tag{7}$$

Specifically, we assume $u(c_{1t}, c_{2t+1}) = c_{1t}^{\gamma} c_{2t+1}^{1-\gamma}$ throughout the current model, where $0 < \gamma < 1$. By specifying the utility as a Cobb-Douglas function, we can highlight the effects of credit market imperfections on the dynamic properties of the economy. If the income effect of the return to savings is too strong relative to the substitution effect, equilibrium cycles will appear without credit market imperfections.[3] Since we use a Cobb-Douglas utility function, however, the income and substitution effects are cancelled by each other. Accordingly, equilibrium cycles will not arise without credit market imperfections. Inequality (3) is a budget constraint for the first period. k_t is investments in a project and b_t is a deposit if positive and is a debt if negative. w_t is a wage income at time t. Eq.(4) is a production function for capital goods. z_{t+1} is the capital goods produced by the agent, and $\phi > 0$ is the productivity of the production. Inequality (5) is a budget constraint for the second period. q_{t+1} is the (real) price of the capital goods in terms of the consumption goods at time $t + 1$. Again, r_{t+1} is the (real) interest rate at time $t + 1$. Inequality (6) is a credit constraint which the agent faces. This type of credit constraints is often imposed in the literature.[4]

[3] See for instance Grandmont (1985).

[4] See Aghion, et al. (1999), Aghion and Banerjee (2005) and Aghion et al. (2005). The credit constraint given by inequality (6) is equivalent to the wealth-backed lending by the financial intermediary. In particular, even if we replace inequality (6) with $b_t \geq -\nu w_t$, $\nu \in [0, \infty)$, the results of this paper do not change.

In appendix, we provide a microfoundation for the credit constraint following Aghion and Banerjee (2005). While the agent can deposit his income in the financial intermediary as much as he wants to, he can borrow from it only up to some proportion of the investments. μ measures the degree of financial development: if μ is large, the financial sector is fully developed, whereas if μ is small, it is poorly developed. Inequality (7) is a non-negativity constraint for the investment project.

2.1.2 Heterogeneous Agents

Now the heterogeneity of agents is introduced in terms of their productivity in producing capital goods. When an agent is born, he receives a shock for his productivity level ϕ from a time-invariant distribution $G(\phi)$ whose support is $[0, a]$, where $a > 0$. $G(\phi)$ has a continuous density $g(\phi)$ on $[0, a]$.

We assume that each agent knows his own productivity at his birth, while other agents do not know his productivity.[5] In the Diamond type overlapping generations model (Diamond 1965), agents are homogeneous, i.e., $\phi = 1$ for every agent. By contrast, in our model, the agents' saving technology depends upon their heterogeneous productivity. As we solve a consumer's problem in what follows, we shall find that the heterogeneity of agents is crucial when savers and investors are endogenously determined.

To solve the utility maximization problem for a consumer, we let $s_t := k_t + b_t$. Since $k_t \geq 0$ and $b_t \geq -\mu k_t$, it follows that $s_t \geq (1-\mu)k_t \geq 0$. Hence, we can rewrite an agent's maximization problem as follows:

$$\max_{0 \leq k_t \leq \frac{s_t}{1-\mu}} \quad u(w_t - s_t, r_{t+1}s_t + (q_{t+1}\phi - r_{t+1})k_t). \tag{8}$$

In this maximization problem, $s_t > 0$ holds because $\lim_{c_{2t+1} \to 0} u_2(.,.) = \infty$. Since $u_2(.,.) > 0$, if $r_{t+1} > q_{t+1}\phi$, then it is optimal for the agent to choose $k_t = 0$ and $s_t = b_t$. Since we have assumed $u(c_{1t}, c_{2t+1}) = c_{1t}^{\gamma} c_{2t+1}^{1-\gamma}$, the first-order condition with respect to s_t is given by: $-\gamma \frac{u(c_{1t}, c_{2t+1})}{c_{1t}} + (1-\gamma) r_{t+1} \frac{u(c_{1t}, c_{2t+1})}{c_{2t+1}} = 0 \iff c_{2t+1} = \frac{(1-\gamma) r_{t+1}}{\gamma} c_{1t}$. From this equation,

[5] Hence, less capable agents cannot ask more capable agents to produce capital goods. For this situation, one could imagine that less capable agents face prohibitively high costs to identify more capable agents.

inequality (3) and inequality (5), we obtain $b_t = (1-\gamma)w_t$. On the other hand, if $r_{t+1} < q_{t+1}\phi$, then the agent chooses $k_t = \frac{s_t}{1-\mu}$. The first-order condition is then given by: $-\gamma\frac{u(c_{1t},c_{2t+1})}{c_{1t}} + (1-\gamma)\frac{q_{t+1}\phi - r_{t+1}\mu}{1-\mu}\frac{u(c_{1t},c_{2t+1})}{c_{2t+1}} = 0 \iff c_{2t+1} = \frac{1-\gamma}{\gamma}\frac{q_{t+1}\phi - r_{t+1}\mu}{1-\mu}c_{1t}$. Now from this equation, inequality (3) and inequality (5), we obtain $k_t = \frac{(1-\gamma)w_t}{1-\mu}$ and $b_t = -\frac{\mu(1-\gamma)w_t}{1-\mu}$.

ϕ is used for the index of the heterogeneity of agents. Therefore, we henceforth put it on each variable as $c_{1t}(\phi)$, etc. Lemma 1 summarizes the above results.

Lemma 1 *Let $\phi_t := \frac{r_{t+1}}{q_{t+1}}$. Then, the following claims hold.*

- *If $\phi_t > \phi$, then $k_t(\phi) = 0$ and $b_t(\phi) = (1-\gamma)w_t$.*
- *If $\phi_t < \phi$, then $k_t(\phi) = \frac{(1-\gamma)w_t}{1-\mu}$ and $b_t(\phi) = -\frac{\mu(1-\gamma)w_t}{1-\mu}$.*

Proof: The claims have been proven in the above discussion. □

As seen in lemma 1, ϕ_t is a cutoff, i.e., if an agent's productivity is greater than ϕ_t, he starts an investment project, borrowing from the financial intermediary. In this case, the returns to his savings are subject to his productivity. He obtains more returns from the investment in a project than from a deposit in the financial intermediary. On the other hand, if an agent's productivity is less than ϕ_t, he only deposits a part of his income in the financial intermediary. He prefers to deposit in the financial intermediary rather than to invest in a project. We can ignore agents with $\phi = \phi_t$ because they have no impact on the economy. As the parameter for financial development μ gets greater, both investments and borrowings of the agents with $\phi > \phi_t$ become large.

2.2 Final Production Sector

The general goods are produced in a final production sector, which become consumption goods and investment goods. The general goods are produced from a continuum of intermediate goods, which is distributed uniformly in $[0,1]$. The production function is given by:

$$Y_t = \left[\int_{i\in[0,1]} (A_{it}x_{it})^{\alpha} di\right]^{\frac{1}{\alpha}}, \tag{9}$$

where A_{it} is the quality of the ith intermediate good, x_{it} is its quantity, and $\frac{1}{1-\alpha}$ is the elasticity of substitution between input goods, which is greater than one since $\alpha \in (0,1)$.

The final production sector is competitive and the representative firm solves the maximization problem given by:

$$\max_{x_{it}} \; Y_t - \int_{i\in[0,1]} \tilde{p}_{it} x_{it} di, \tag{10}$$

where $\tilde{p}_{it}$ is the price of the intermediate good i. From the first-order condition, we have an inverse demand function for the intermediate good i:

$$\tilde{p}_{it} = A_{it}^{\alpha} x_{it}^{\alpha-1} Y_t^{1-\alpha}. \tag{11}$$

2.3 Intermediate Sector

The intermediate sector consists of a continuum of firms, which is distributed uniformly in $[0,1]$. This distribution is time-invariant because for the intermediate sector, new innovators come out into the market at each period. Due to the newly invented technologies, new innovators make monopolistic profits. The newly invented technologies may be protected by patents or it may take time for the technologies to be imitated. The monopolistic profits, however, will disappear in one period since the next newly invented technologies are introduced into the market by other innovators after one period goes by.[6]

The intermediate goods are produced from labor with a one-for-one technology, i.e., an intermediate firm needs one unit of labor to produce one unit of x_{it}. The maximization problem for an intermediate firm is given by:

$$\begin{aligned} & \max_{x_{it}} \; \tilde{p}_{it} x_{it} - w_t x_{it} \\ = & \max_{x_{it}} \; A_{it}^{\alpha} x_{it}^{\alpha} Y_t^{1-\alpha} - w_t x_{it}, \end{aligned} \tag{12}$$

where w_t is the wage rate. From the first-order condition, we obtain:

$$w_t = \alpha A_{it}^{\alpha} x_{it}^{\alpha-1} Y_t^{1-\alpha}, \tag{13}$$

[6]The profits gained by the newly invented technologies are greater than those by the old technologies. Therefore, the newly invented technologies are always adopted rather than the old ones.

and

$$\pi_{it} = (1-\alpha)A_{it}^{\alpha}x_{it}^{\alpha}Y_t^{1-\alpha}, \tag{14}$$

where π_{it} is a profit. From Eq.(11) and Eq.(13), we note that $\tilde{p}_{it} = \frac{w_t}{\alpha}$. The prices of all the intermediate goods are the same in each period.

An intermediate firm supplies the intermediate good with up-to-date quality, which is developed in an R&D department within the firm.[7] The quality of the intermediate goods is improved by the R&D activities. Ha and Howitt (2007) empirically examine a production function for the quality improvement of intermediate goods. Using the data for the USA, the UK, France, Germany, and Japan, they study which model is most suitable to reality: the first generation endogenous models, the semi-endogenous models, or the fully-endogenous models with product proliferation.[8] They conclude that a functional form which is used in the fully-endogenous models with product proliferation such as Howitt's (1999) model is most plausible. Following them, we assume a functional form for the quality improvement as follows:

$$\frac{A_{it+1} - A_{it}}{A_{it}} = \eta\Big(\frac{z_{it+1}}{A_{it}L_t}\Big)^{\sigma}, \tag{15}$$

where we assume that $0 < \sigma \leq 1$.[9] η is a productivity parameter of the R&D department, z_{it+1} is the capital goods for the R&D activities, and L_t is the population of young agents at time t. Following Jones' (1995) critique, scale effects are adjusted in the right-hand side of Eq.(15). First, we obtain the quality-adjusted input goods to eliminate a scale effect associated with the current thechnology level, A_{it}, by dividing z_{it+1} by A_{it}. Second, we divide z_{it+1}/A_{it} by L_t in order to exclude a scale effect which comes from the increasing

[7]As discussed in Aghion and Howitt (1992), we may assume that separate firms are engaged in the R&D activities. In this case, the firms in an R&D sector sell their newly invented technologies to the firms in an intermediate sector. No results will change with this alternative assumption.

[8]For details, see Ha and Howitt (2007).

[9]Even though we alternatively assume a functional form for the quality improvement such as $A_{it+1}/A_{it} = \eta(z_{it+1}/A_{it}L_t)^{\sigma}$, our results are unchanged. Or we may allow technology transfers from abroad, by replacing Eq.(15) with an equation which expresses the rate of technology adoption as in Aghion, et al. (2005) and Howitt and Mayer-Foulkes (2005).

number of individuals. Finally, the quality improvement exhibits non-increasing returns to scale with respect to the input goods, i.e., $0 < \sigma \leq 1$.

Since each R&D department is in the patent race, the demands for capital goods are determined by the research-arbitrage condition:

$$\pi_{it+1} = q_{t+1} z_{it+1},$$

where again q_{t+1} is the (real) price of capital goods. Using Eq.(14), we rewrite this equation as:

$$(1-\alpha) A_{it+1}^{\alpha} x_{it+1}^{\alpha} Y_{t+1}^{1-\alpha} = q_{t+1} z_{it+1}. \tag{16}$$

3 Equilibrium

3.1 Growth Rate

The heterogeneity of the intermediate firms is assumed away: all the intermediate firms are symmetrical. From the labor market clearing condition, we have $\int_{i\in[0,1]} x_{it} di = L_t$. Since due to the symmetry, the same amount of labor is used in each intermediate firm, $x_t = L_t$ holds. Each R&D division uses the same amount of input goods as well. Let this amount be $\tilde{z}_t$. Then, $\tilde{z}_t := \int_{i\in[0,1]} z_{it} di$ holds. Hereafter, all variables are independent of i. From the assumption of symmetry and Eqs.(9), (13), and (16), we have the following equations:

$$\begin{aligned}
Y_t &= A_t L_t \\
w_t L_t &= \alpha A_t L_t = \alpha Y_t \\
q_t \tilde{z}_t &= (1-\alpha) A_t L_t = (1-\alpha) Y_t.
\end{aligned}$$

We note that $Y_t = w_t L_t + q_t \tilde{z}_t$ holds, namely the final output is distributed to the wages and the returns to the investments.

From lemma 1 and $w_t = \alpha A_t$, the market clearing condition (of the R&D sector) is given by:

$$
\begin{aligned}
\tilde{z}_{t+1} &= \int_{\phi_t}^{a} \frac{(1-\gamma)w_t}{1-\mu} L_t \phi dG(\phi) \\
\iff \quad \tilde{z}_{t+1} &= \frac{(1-\gamma)\alpha A_t L_t}{1-\mu} F(\phi_t),
\end{aligned}
\tag{17}
$$

where $F(\phi_t) = \int_{\phi_t}^{a} \phi dG(\phi)$. As seen in the right-hand side of Eq.(17), input goods for the R&D sector are supplied by the talented agents.

Substituting Eq.(17) into Eq.(15), we can derive a growth rate (which is defined by the growth rate of per capita output) as follows:[10]

$$
\begin{aligned}
\Gamma(\phi_t) &:= \Big(\frac{A_{t+1}}{A_t}\Big) - 1 \\
&= \eta\Big(\frac{(1-\gamma)\alpha}{1-\mu} F(\phi_t)\Big)^{\sigma}.
\end{aligned}
\tag{18}
$$

We note that the direct effect of μ on the growth rate is positive, i.e., for a given ϕ_t, as μ increases the growth rate goes up. We also note that the growth rate is a decreasing function with respect to ϕ_t. As ϕ_t increases, the number of investors decreases. Accordingly, for a given μ, the total investments go down and thus the growth rate decreases.

The above discussion about the effect of μ captures only the direct effect on the growth rate. However, a change in μ will affect the value of ϕ_t indirectly, which is thought of as a general equilibrium effect. In particular, as we will see, μ has a positive effect on ϕ_t in the steady states. Therefore, at this point, if an economy is in a steady state, we are uncertain whether μ has a positive or negative effect on the growth rate because the growth rate is decreasing with ϕ_t. In the analysis of the steady state below, we will discuss the effect of μ on the growth rate, taking into account the indirect effect.

[10]Again, due to symmetry, Eq.(15) is independent of i.

3.2 Dynamics

B_t is the net total asset held by young agents at time t, i.e., B_t is the debt at time t, which is equal to the total deposit minus the total loan. From lemma 1 and $w_t = \alpha A_t$, we obtain:

$$\begin{aligned} B_t &= \int_0^{\phi_t} (1-\gamma)\alpha A_t L_t dG(\phi) \\ &- \int_{\phi_t}^{a} \frac{\mu(1-\gamma)\alpha A_t L_t}{1-\mu} dG(\phi) \\ &= \frac{(1-\gamma)\alpha Y_t}{1-\mu}[G(\phi_t)-\mu]. \end{aligned} \tag{19}$$

We note that the total loan $\int_{\phi_t}^{a} \frac{\mu(1-\gamma)\alpha A_t L_t}{1-\mu} dG(\phi)$ is affected by the degree of financial development μ. Since $q_{t+1}\tilde{z}_{t+1} = (1-\alpha)Y_{t+1}$, substituting Eq.(17) into this yields $q_{t+1} = \frac{(1-\alpha)(1-\mu)Y_{t+1}}{(1-\gamma)\alpha F(\phi_t)Y_t}$. Therefore, $r_{t+1} = q_{t+1}\phi_t = \frac{(1-\alpha)(1-\mu)\phi_t Y_{t+1}}{(1-\gamma)\alpha F(\phi_t)Y_t}$. Substituting the last and Eq.(19) into Eq.(1), we have:

$$\underbrace{\frac{(1-\gamma)\alpha Y_{t+1}}{1-\mu}[G(\phi_{t+1})-\mu]}_{B_{t+1}} = \underbrace{\frac{(1-\alpha)(1-\mu)\phi_t}{(1-\gamma)\alpha F(\phi_t)}\frac{Y_{t+1}}{Y_t}}_{r_{t+1}}\underbrace{\frac{(1-\gamma)\alpha Y_t}{1-\mu}[G(\phi_t)-\mu]}_{B_t}, \tag{20}$$

which reduces to a difference equation of the cutoff, ϕ_t:

$$G(\phi_{t+1}) = \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi_t(G(\phi_t)-\mu)}{F(\phi_t)} + \mu. \tag{21}$$

Let us define a function as follows:

$$\Psi(\phi) := \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi(G(\phi)-\mu)}{F(\phi)} + \mu.$$

We note that the difference equation Eq.(21) is independent of η , σ, the population growth n, and the quality of input goods, A_t. The growth rate $\Gamma(\phi_t)$ is a one-to-one, continuous function of $\phi_t \in [0, a]$. Therefore, the dynamic properties of the growth rates are deduced directly from those of the cutoff. In what follows, we shift our focus to the study of the dynamic properties of the cutoff.

3.3 Steady-State Analysis

3.3.1 Two Steady-State Equilibria

A steady-state equilibrium $\phi_t = \bar{\phi}$ solves the following equation:

$$G(\bar{\phi}) = \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)} \frac{\bar{\phi}(G(\bar{\phi})-\mu)}{F(\bar{\phi})} + \mu. \tag{22}$$

We note that there exist two steady-state equilibria, $\bar{\phi} = \phi^*$ and $\bar{\phi} = \phi^{**}$, unless they are repeated values, such that:

$$G(\phi^*) = \mu \tag{23}$$

and

$$\frac{\phi^{**}}{F(\phi^{**})} = \frac{\alpha(1-\gamma)}{(1-\alpha)(1-\mu)}, \tag{24}$$

respectively.

In the steady-state equilibrium with $\bar{\phi} = \phi^*$, a generation's net credit position is always zero, whereas in the steady-state equilibrium with $\bar{\phi} = \phi^{**}$, the net credit position is positive or negative. We call equilibria with $\bar{\phi} = \phi^*$ and with $\bar{\phi} = \phi^{**}$ a non-trade steady-state equilibrium and a trade steady-state equilibrium, respectively. Whether a steady-state equilibrium is called a non-trade or a trade steady-state equilibrium, agents within a generation make financial trades via the financial intermediary. In the non-trade steady state, the credit market clears within a generation, whereas in the trade steady state, agents make financial trades inter-generationally and the credit market clears over two generations. In this sense, by "non-trade" we mean that agents in a generation do not trade with the agents in the other generation.

3.3.2 Comparative Statics with respect to μ

Now we investigate the effect of the change of μ on a growth rate, taking into account a general equilibrium effect.

Proposition 1 *As a financial sector is fully developed, i.e., as μ increases, the following hold:*

- *The growth rate goes up in the non-trade steady state, i.e.,* $\frac{\partial \Gamma(\phi^*)}{\partial \mu} > 0$.

- *The growth rate goes up in the trade steady state as well, i.e.,* $\frac{\partial \Gamma(\phi^{**})}{\partial \mu} > 0$.

Proof: See appendix.

King and Levine (1993a,b) and Levine, et al. (2000) give empirical evidence for the positive effect of financial development on economic growth. Our results for the steady states are consistent with their discoveries. From Eqs.(24) and (25), we note that both ϕ^* and ϕ^{**} increase as μ goes up. This means that the number of investors decreases, which negatively affects the growth rate. However, from lemma 1, although there are fewer investors, we note that each investor borrows and invests more now than before μ went up. This latter positive effect is stronger than the former negative effect in both steady states. Therefore, the growth rate always goes up in the steady states.

The model captures well the properties of financial development. As a financial sector is fully developed, the mis-allocation of production factors is corrected. As μ goes up, less capable investors turn into savers and the economic resources concentrate on more capable investors. As a result, efficiency in the economy is promoted. Less capable agents can utilize the ability of more capable agents. This is an essential characteristic of financial deepening.

4 Dynamic Properties

In this section, we investigate the dynamic properties of the economy. As seen in Eq.(18), $\Gamma(\phi_t)$ is a one-to-one, continuous function of $\phi_t \in [0, a]$. This means that if the equilibrium sequence, $\{\phi_t\}_{t=0}^{\infty}$, exhibits cyclical behavior, then so does the equilibrium sequence of the growth rates, $\{\Gamma(\phi_t)\}_{t=0}^{\infty}$.

We define a compact interval in $\Re$ as $X = [0, \max\{\phi^*, \phi^{**}\}]$. We restrict the domain of the dynamical system of Eq.(21) to X so as to obtain economically meaningful equilibria. If $\{\phi_t\}_{t=0}^{\infty}$ starts with $\phi_0 \in (\max\{\phi^*, \phi^{**}\}, a]$, then $G(\phi_t)$ becomes greater than one in finite time. Such a sequence does not become an equilibrium. We assume that the minimum of

$\Psi(\phi)$ (which is the right-hand side of Eq.(21)) is no less than zero. Having restricted the domain of the system to X, then the map, $\Psi : X \to X$, is continuous and maps X into itself. Henceforth, we use the pair (X, Ψ) to denote our dynamical system.

We linearize the difference equation Eq.(21) around a steady state:

$$\phi_{t+1} - \bar{\phi} = \Phi(\bar{\phi})(\phi_t - \bar{\phi}),$$

where $\Phi(\phi) = \frac{(1-\alpha)(1-\mu)\phi}{\alpha(1-\gamma)F(\phi)}\left[\left(\frac{1}{\phi g(\phi)} + \frac{\phi}{F(\phi)}\right)(G(\phi) - \mu) + 1\right]$.

Eq.(19) can be rewritten as $B_t + (1-\gamma)\alpha A_t L_t \mu(1 - G(\phi_t))/(1-\mu) = (1-\gamma)\alpha A_t L_t G(\phi_t)$, where the left-hand side is the debt plus the total loan and the right-hand side is the total deposit. This equation must hold at each point in time, expressing the credit market clearing condition, namely the left-hand side is the demand for the financial resources and the right-hand side is the supply of the financial resources. We should note that B_0, A_0, and L_0 are historically given. Therefore, the initial cutoff ϕ_0 is uniquely determined at time zero, implying that it is a predetermined variable. This means that the ratio of r_1 to q_1 is uniquely determined at time zero as well.

The local stability depends upon whether ϕ^{**} is greater than ϕ^* or not.

Proposition 2

- *If $\phi^{**} > \phi^*$, then the non-trade steady-state equilibrium ($\bar{\phi} = \phi^*$) is locally stable, whereas the trade steady-state equilibrium ($\bar{\phi} = \phi^{**}$) is locally unstable.*
- *If $\phi^{**} < \phi^*$, then the non-trade steady-state equilibrium ($\bar{\phi} = \phi^*$) is locally unstable, whereas the stability of the trade steady-state equilibrium ($\bar{\phi} = \phi^{**}$) is ambiguous.*

Proof: If $\phi^* < \phi^{**}$, then $|\Phi(\phi^*)| = |\frac{(1-\alpha)(1-\mu)\phi^*}{\alpha(1-\gamma)F(\phi^*)}| < |\frac{(1-\alpha)(1-\mu)\phi^{**}}{\alpha(1-\gamma)F(\phi^{**})}| = 1$ and $|\Phi(\phi^{**})| = \left|\left(\frac{1}{\phi^{**}g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})}\right)(G(\phi^{**}) - \mu) + 1\right| > 1$. If $\phi^* > \phi^{**}$, then $|\Phi(\phi^*)| = |\frac{(1-\alpha)(1-\mu)\phi^*}{\alpha(1-\gamma)F(\phi^*)}| > |\frac{(1-\alpha)(1-\mu)\phi^{**}}{\alpha(1-\gamma)F(\phi^{**})}| = 1$. However, we cannot know whether $|\Phi(\phi^{**})| = \left|\left(\frac{1}{\phi^{**}g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})}\right)(G(\phi^{**}) - \mu) + 1\right|$ is greater than one or not. $\square$

The phase diagrams for each case are given in figures 1-3. As seen in figure 3, if the trade equilibrium is locally unstable, it is possible for the dynamical system to exhibit cycles.

Particularly, a flip bifurcation occurs at $(\phi^{**}, \tilde{\mu})$ where $\left(\frac{1}{\phi^{**}g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})}\right)(G(\phi^{**}) - \tilde{\mu}) + 1 = -1$. Proposition 3 below shows that a (stable or unstable) period-two cycle exists when the trade steady-state equilibrium is locally unstable.

[Figures 1-4 around here]

Proposition 3 *Suppose that $\phi^{**} < \phi^*$. If the trade steady-state equilibrium is locally unstable, there exists a period-two cycle of $\{\phi_t\}_{t=0}^{\infty}$ in equilibrium.*

Proof: Let $\tilde{\Psi}(\phi) = G^{-1}(\Psi(\phi))$. Then Eq.(25) is written as $\phi_{t+1} = \tilde{\Psi}(\phi_t)$. We can take ϕ_0 close to ϕ^* so that $\tilde{\Psi}^2(\phi_0) < \phi_0 < \phi^*$ because $\Phi(\phi^*) > 1$. If the trade steady-state equilibrium is locally unstable, then $\Phi(\phi^{**}) < -1$. So we can take ϕ_0' close to ϕ^{**} so that $\tilde{\Psi}(\phi_0') < \phi^{**} < \phi_0' < \tilde{\Psi}^2(\phi_0') < \tilde{\Psi}^2(\phi_0)$. Therefore, by continuity, there exists $\bar{\phi}_0$ such that $\tilde{\Psi}(\bar{\phi}_0) < \phi^{**} < \bar{\phi}_0 = \tilde{\Psi}^2(\bar{\phi}_0) < \phi^*$, which means that there exists a period-two cycle. □

A graphical analysis gives a proof of proposition 2 as well. Since $\phi^{**} < \phi^*$, if the trade steady-state equilibrium is locally unstable, $\Phi(\phi^{**}) < -1$ holds. The configuration of $\phi_{t+1} = G^{-1}(\Psi(\phi_t))$ in this case is drawn in figure 4. Its mirror image relative to the 45 degree line is drawn as well. As seen in the figure, there exists at least one pair of $\bar{\phi}_0$ and $\bar{\phi}_1$, where $\bar{\phi}_1 \neq \bar{\phi}_0$, such that $\bar{\phi}_1 = G^{-1}(\Psi(\bar{\phi}_0))$ and $\bar{\phi}_0 = G^{-1}(\Psi(\bar{\phi}_1))$.

5 Financial Deepening and Cycles

In the previous section, we have examined the dynamical system analytically and we have seen the appearance of cycles in equilibrium. In this section, we investigate under what conditions are endogenous fluctuations more or less likely to arise.

Let us consider the case in which $\mu = 0$. If $\mu = 0$, then $\phi^* = 0$ holds from Eq.(23). Therefore, from the first part of proposition 2 and from its phase diagram, $\{\phi_t\}_{t=0}^{\infty}$ monotonically converges to $\bar{\phi} = \phi^* = 0$ whenever $\{\phi_t\}_{t=0}^{\infty}$ starts with $\phi_0 \in [0, \phi^{**})$. Therefore, cycles do not arise. By continuity, given other parameters and the distribution function for ϕ, there exists

μ_0 such that for $\mu \in [0, \mu_0]$ almost all of the sequences $\{\phi_t\}_{t=0}^{\infty}$ which start with an arbitrary value of $\phi_0 \in X$ monotonically converge to $\bar{\phi} = \phi^* = 0$. Meanwhile, if μ is sufficiently close to one, the first term of Eq.(21) degenerates and thus wherever $\{\phi_t\}_{t=0}^{\infty}$ starts in X, it converges to an asymptotically stable steady state.[11] In this case, no cycles appear either. Again by continuity, we can claim that given other parameters and the distribution function for ϕ, there exists μ_1 such that for $\mu \in [\mu_1, 1)$ almost all of the sequences $\{\phi_t\}_{t=0}^{\infty}$ with $\phi_0 \in X$ converge to an asymptotically stable steady state. From these discussions, we note that if a financial sector is fully developed or poorly developed, endogenous cycles do not appear. It is when financial development is at an intermediate level that endogenous fluctuations can arise.

From proposition 2, we note that it is in the credit case that the economy fluctuates. Essentially, the dynamic properties of $\{\phi_t\}_{t=0}^{\infty}$ depend upon $B_{t+1} = r_{t+1}B_t$ because Eq.(21) originates in this equation. In it, both r_{t+1} and B_t are functions of ϕ_t. Since $r_{t+1} = \phi_t q_{t+1} = \frac{(1-\alpha)(1-\mu)(1+n)\phi_t}{(1-\gamma)\alpha F(\phi_t)}(\Gamma(\phi_t) + 1) = \frac{(1-\alpha)(1-\mu)(1+n)\phi_t}{(1-\gamma)\alpha}\left[\frac{\eta}{F(\phi_t)^{1-\sigma}}\left(\frac{(1-\gamma)\alpha}{1-\mu}\right)^{\sigma} + \frac{1}{F(\phi_t)}\right]$, r_{t+1} is increasing with ϕ_t. From Eq.(19), B_t is increasing with ϕ_t as well. Nevertheless, an increase in ϕ_t at the beginning of period t has an ambiguous effect on ϕ_{t+1}. This is because B_t could be negative. As we have seen in the previous section, if the trade steady-state equilibrium $\bar{\phi} = \phi^{**}$ is locally unstable, then the economy exhibits endogenous fluctuations in equilibrium. In what follows, we will see that it is possible that the economy oscillates when μ is the intermediate value.

Suppose that the economy is in the trade steady-state equilibrium at the beginning of time t. In this case, since $\frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi^{**}}{F(\phi^{**})} = 1$, we obtain the steady-state interest rate as follows:

$$r^{**} = (1 + n)(\Gamma(\phi^{**}) + 1). \tag{25}$$

We note that if the growth rate is zero, i.e., $\Gamma(\phi^{**}) = 0$, then the steady-state interest rate r^{**} is equal to the biological interest rate of Samuelson (1958), where less capable agents

[11] However, in this case, we cannot specify to which steady state, $\bar{\phi} = \phi^*$ or $\bar{\phi} = \phi^{**}$, the economy converges.

receive a net interest rate, n, which is the population growth. On the other hand, from (19) the debt in the steady state is given by:

$$B^{**} = \Big[(1-\gamma)\alpha G(\phi^{**}) - \frac{\mu(1-\gamma)\alpha}{1-\mu}(1-G(\phi^{**}))\Big]Y_t, \tag{26}$$

where Y_t is given when we look at B^{**} at time t. Since we assume the credit case now, $B^{**} < 0$ holds.

Now suppose that the economy faces a shock so that the prospective interest rate at time $t+1$, r_{t+1}, goes up. In this case, agents whose productivity is slightly greater than ϕ^{**} turn from investors into savers. Therefore, the number of savers increases whereas the number of investors decreases relative to the case in which the economy is in the trade steady state. As a result, B_t goes up; however, it is still negative and thus the absolute value of B_t decreases. If μ is not close to one, i.e., μ is an intermediate value, then from Eq.(26) the increase in B_t is small relative to the increase in r_{t+1}. In this case, $r_{t+1}B_t$ decreases, which means that the financial resources of the financial intermediary at time $t+1$ go up. Accordingly, the prospective interest rate at time $t+2$, r_{t+2}, goes down. As a result, ϕ_{t+1} becomes smaller than ϕ^{**} and thus the economy oscillates. If the sequence $\{\phi_t\}_{t=0}^{\infty}$ does not converge, equilibrium cycles emerge.

From the proof of the second part of proposition 2, it follows that if $\left(\frac{1}{\phi^{**}g(\phi^{**})}+\frac{\phi^{**}}{F(\phi^{**})}\right)(G(\phi^{**})-\mu) < -2$, then the trade steady-state equilibrium is locally unstable. In this case, we note from the phase diagram that endogenous fluctuations arise. From the assumption of the second part of proposition 2, we have $G(\phi^{**}) - \mu < 0$. Therefore, the condition for cycles must hold if $\frac{1}{\phi^{**}g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})}$ is very big. For instance, whenever the density at ϕ^{**} is very thin, i.e., $g(\phi^{**})$ is very small, this value becomes large. In other words, it is always possible that endogenous fluctuations arise when the distribution for ϕ has a thin density at ϕ^{**}.

6 Numerical Analysis

A growth rate has a one-to-one relationship with a cutoff and thus we have investigated the dynamic properties of the cutoff. While we have found that it is only when μ is the intermediate value that endogenous fluctuations appear, we have not made clear to what kinds of cycles the economy converges in equilibrium. In this section, in order to investigate the dynamic properties of the sequences $\{\phi_t\}_{t=0}^{\infty}$ and $\{\Gamma(\phi_t)\}_{t=0}^{\infty}$ numerically, we create bifurcation diagrams. These diagrams are helpful for us to understand the asymptotic dynamic properties of the economy concretely. We assume that $\phi \sim U(0,1)$. We would like to study the relationship between the degree of credit constraints and the dynamic properties of the growth rates. Therefore, we create bifurcation diagrams with respect to μ.

When ϕ has a uniform distribution $U(0,1)$, the dynamics of growth rates is given by the system of two equations:

$$\phi_{t+1} = \Psi(\phi_t) = \frac{2(1-\alpha)(1-\mu)}{\alpha(1-\gamma)} \frac{\phi_t(\phi_t - \mu)}{1-\phi_t^2} + \mu$$

and

$$\Gamma(\phi_t) = \eta\Big(\frac{(1-\gamma)\alpha}{1-\mu} F(\phi_t)\Big)^{\sigma}.$$

We create the bifurcation diagrams for the dynamical system iterating 10000 times. In this numerical analysis, we assume $\alpha = 0.65$, which implies that the labor share of output is 65%. We examine the various cases for γ: $\gamma = 0.9192$, $\gamma = 0.9210$, and $\gamma = 0.9228$. In either case, the saving rate of an agent is around 8%.[12] We choose the initial condition to be $\phi_0 = 0.01$.[13]

[Figures 5-7 around here]

Figures 5-7 give bifurcation diagrams for ϕ_t with respect to the degree of financial deepening, i.e., μ. As predicted in the previous section, for all cases, if μ is small or large, $\{\phi_t\}_{t=0}^{\infty}$

[12]Nowadays, there are many countries in which the household saving rate is less than 10%.

[13]We can verify that $\Psi : X \to X$ is a (upside-down) unimodal map: the critical point is $m := \frac{\mu}{1+\sqrt{1-\mu^2}}$. In this case, it is well known that for a given μ, if a Schwarzian derivative, $S(\Psi) := \frac{\Psi'''(\phi)}{\Psi'(\phi)} - \frac{3}{2}(\frac{\Psi''(\phi)}{\Psi'(\phi)})^2 < 0$ for all $\phi \in X - \{m\}$, then the asymptotic behavior of almost all sequences $\{\phi_t\}_{t=0}^{\infty}$ will be the same. Therefore, the choice of initial conditions does not matter. See for example Guckenheimer and Holmes (1983). Our Schwarzian derivative is too complicated to investigate its sign analytically. However, we plotted the values of our Schwarzian derivatives and numerically confirmed that the signs are negative. In addition, to make sure, we examined various initial values: we found that the asymptotic behavior of the dynamical system is invariant to the initial condition.

converges to an asymptotically stable steady state and thus the growth rates converge to a stationary growth rate as well. For each case, however, if the value of μ is in the intermediate region, endogenous fluctuations appear. When $\gamma = 0.9192$, as μ goes up from zero or goes down from one, the economy experiences period doubling bifurcations. At the intermediate values of μ, a globally stable period-eight cycle arises. When $\gamma = 0.9210$ and $\gamma = 0.9228$, the economy exhibits a complex dynamics in the intermediate region. For example, when $\gamma = 0.9228$, as μ increases from zero, we observe the first period doubling bifurcation around $\mu = 0.298$ and the second period doubling bifurcation around $\mu = 0.393$. These bifurcations are repeated over and over again and eventually the economy enters a complex region (shaded regions in the diagrams). As seen in figure 6, when $\gamma = 0.9210$ similar things happen.

A new finding from these bifurcation diagrams is that the amplitudes of cycles increase as γ goes up. We note from lemma 1 that as γ increases, agents put more weight on the first-period consumption and thus the total borrowing in the economy goes up. While the total borrowing goes up, the investments by each investor go down (lemma 1), which leads the price of capital goods to climb up. As a result, the cutoff goes down and the number of investors increases. This means that more agents are subject to the credit constraints when γ is large than when it is small. Hence, we may say that the enlarged amplitudes of cycles are due to the credit market imperfections agents are facing.

7 Concluding Remarks

We have investigated the relationship between credit market imperfections and the fluctuations of growth rates. Credit market imperfections have effects on macroeconomic phenomena such as economic growth and business cycles. Our main findings are as follows: (i) if the development of a financial sector is at an intermediate level, endogenous fluctuations of growth rates arise for some parameter values and (ii) if a financial sector is fully developed or poorly developed, the growth rates converge to an asymptotically stable steady state.

We make a final remark for future research. Investigating what would happen if our model is extended to a two-country model is future research. This investigation is important because a propagation mechanism of business cycles between two countries whose degrees of financial development are different is an open question. This is left for future research.

Appendices

i) Microfoundation for credit constraints

We provide a microfoundation for credit constraints. The idea is based on Aghion and Banerjee (2005).

Each borrower prepares his fund for investment s_t, where $s_t = (1-\gamma)w_t$ because the Cobb-Douglas utility function is assumed. Accordingly, his total investment resources are $k_t = s_t - b_t$. The return on one unit of investment is $q_{t+1}\phi$. If a borrower repays his obligations earnestly, then he earns a net income, $q_{t+1}\phi k_t + r_{t+1}b_t$. However, if the borrower does not repay his obligations, he incurs a cost, δk_t to hide his revenue. In this case, a financial intermediary monitors the borrower and it will capture him with probability p_{t+1}. Then, his expected income is given by $q_{t+1}\phi k_t - \delta k_t + p_{t+1}r_{t+1}b_t$.

Under this loan contract, the incentive compatibility constraint which incentivizes the borrower not to default is given by

$$q_{t+1}\phi k_t + r_{t+1}b_t \geq [q_{t+1}\phi - \delta]k_t + p_{t+1}r_{t+1}b_t, \tag{27}$$

which is rewritten as

$$b_t \geq -\frac{\delta}{r_{t+1}(1-p_{t+1})}k_t, \tag{28}$$

The left-hand side of Eq.(27) is the revenue which the borrower obtains when he invests in a project and repays his obligations, and the right-hand side is the gain when he defaults. Eq.(28) is independent of the return on one unit of investment.

The financial intermediary incurs a cost, $b_t C(p_{t+1})$, which is increasing and convex with respect to p_{t+1}, so that it attains the probability p_{t+1} to detect the borrower's deception. Following Aghion and Banerjee (2005), we assume $C(p_{t+1}) = \kappa \log(1-p_{t+1})$, where κ is strictly greater than δ so that all borrowers face credit constraints more severe than their natural debt limits. The financial intermediary will choose an optimal probability to solve a maximization problem such that

$$\max_{p_{t+1}} \; -p_{t+1}r_{t+1}b_t - \kappa \log(1-p_{t+1})b_t.$$

This maximization problem is rewritten as

$$\max_{p_{t+1}} \; p_{t+1}r_{t+1} + \kappa \log(1-p_{t+1}).$$

From the first-order condition, we have

$$r_{t+1} = \frac{\kappa}{1-p_{t+1}}. \tag{29}$$

From Eqs.(28) and (29), we obtain

$$b_t \geq -\frac{\delta}{\kappa}k_t. \tag{30}$$

As long as it imposes a credit constraint given by inequality (30) on all agents, no borrowers will default in equilibrium. Since $\delta < \kappa$, we can let $\mu := \frac{\delta}{\kappa} \in [0, 1)$, and thus

$$b_t \geq -\mu k_t,$$

which is a credit constraint in the main text. δ and κ are associated with a default cost and a monitoring cost, respectively. Therefore, μ can be regarded as a measure of financial sector development.

ii) Proof of proposition 1

We take four steps to show proposition 1.

Step 1: If $\bar{\phi} \in [0, a)$, then $F(\bar{\phi}) > \bar{\phi}(1 - G(\bar{\phi}))$, since $F(\bar{\phi}) = \int_{\bar{\phi}}^{a} \phi dG(\phi) > \int_{\bar{\phi}}^{a} \bar{\phi} dG(\phi) = \bar{\phi}(1 - G(\bar{\phi}))$.

Step 2: $\frac{\partial \phi^*}{\partial \mu} = \frac{1}{g(\phi^*)}$, which follows from $G(\phi^*) = \mu$.

Step 3: $F(\phi^{**}) - (1-\mu)\phi^{**}g(\phi^{**})\frac{\partial\phi^{**}}{\partial\mu} = (1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial\phi^{**}}{\partial\mu}$. This is because from $\frac{\phi^{**}}{F(\phi^{**})} = \frac{\alpha(1-\gamma)}{(1-\alpha)(1-\mu)}$, we have:

$$\log\phi^{**} - \log F(\phi^{**}) = \log\Big[\frac{\alpha(1-\gamma)}{(1-\alpha)}\Big] - \log(1-\mu).$$

Therefore, we have:

$$\begin{aligned} & \Big[\frac{1}{\phi^{**}} + \frac{\phi^{**}g(\phi^{**})}{F(\phi^{**})}\Big]\frac{\partial\phi^{**}}{\partial\mu} = \frac{1}{1-\mu} \\ \iff \quad & F(\phi^{**}) - (1-\mu)\phi^{**}g(\phi^{**})\frac{\partial\phi^{**}}{\partial\mu} = (1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial\phi^{**}}{\partial\mu}. \end{aligned} \tag{31}$$

Step 4: Case 1: $\bar{\phi} = \phi^*$. From step 1, it holds that:

$$\frac{\phi^*}{F(\phi^*)} < \frac{1}{1-G(\phi^*)} = \frac{1}{1-\mu}. \tag{32}$$

From step 2 and Eq.(32), we have:

$$\begin{aligned} & \frac{-\phi^*(1-\mu) + F(\phi^*)}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{-\phi^* g(\phi^*)(1-\mu)\frac{1}{g(\phi^*)} + F(\phi^*)}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{-\phi^* g(\phi^*)(1-\mu)\frac{\partial\phi^*}{\partial\mu} + F(\phi^*)}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{\partial}{\partial\mu}\Big[\frac{F(\phi^*)}{1-\mu}\Big] > 0 \\ \iff \quad & \frac{\partial\Gamma(\phi^*)}{\partial\mu} > 0. \end{aligned}$$

Case 2: $\bar{\phi} = \phi^{**}$. From Eq.(31), $\frac{\partial\phi^{**}}{\partial\mu} > 0$ holds. Then, from step 3, we have:

$$\begin{aligned} & \frac{(1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial\phi^{**}}{\partial\mu}}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{-\phi^{**}g(\phi^{**})(1-\mu)\frac{\partial\phi^{**}}{\partial\mu} + F(\phi^{**})}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{\partial}{\partial\mu}\Big[\frac{F(\phi^{**})}{1-\mu}\Big] > 0 \\ \iff \quad & \frac{\partial\Gamma(\phi^{**})}{\partial\mu} > 0. \qquad \square \end{aligned}$$

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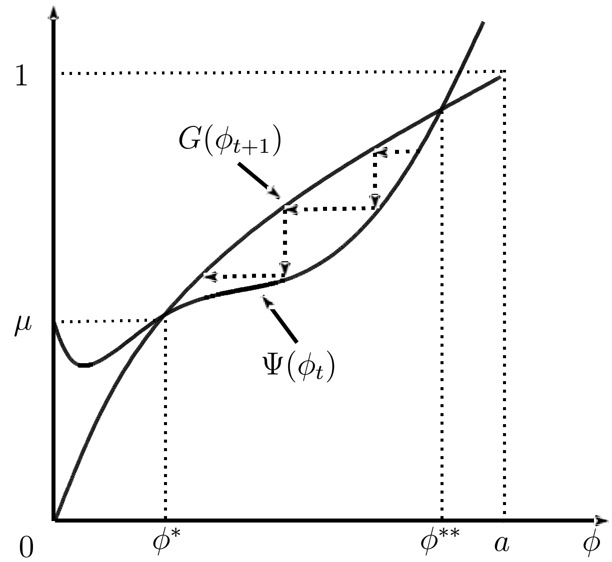

Figure 1: $\phi^{**} > \phi^*$

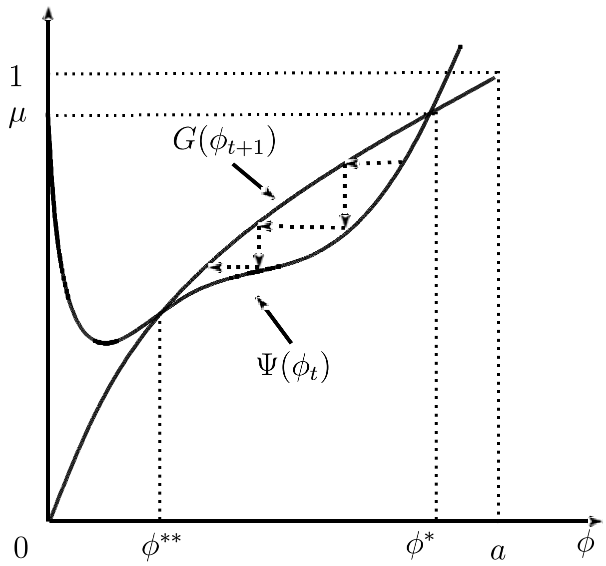

Figure 2: $\phi^{**} < \phi^*$, No Cycles

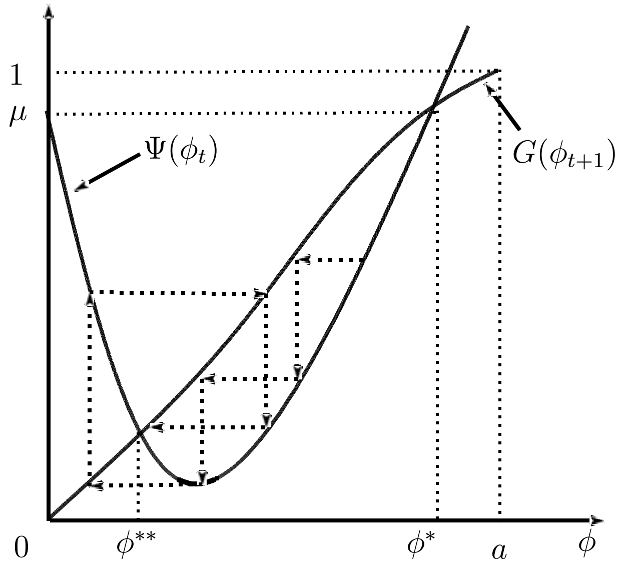

Figure 3: $\phi^{**} < \phi^{*}$, Cycles

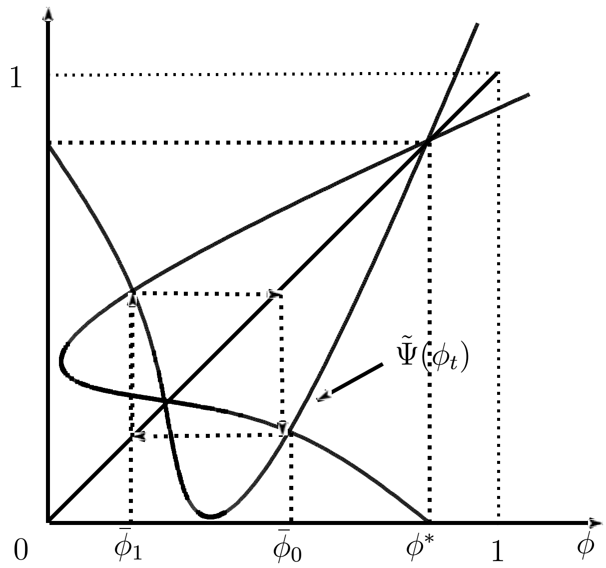

Figure 4: Period-Two Cycle

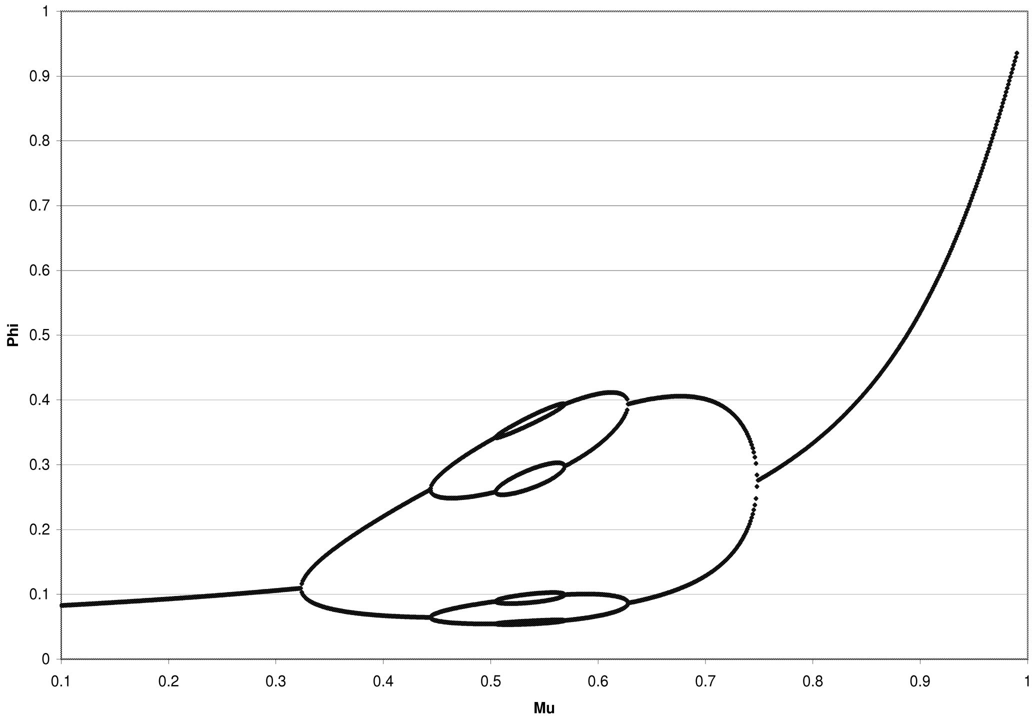

Figure 5: Bifurcation Diagram for phi, Gamma=0.9192

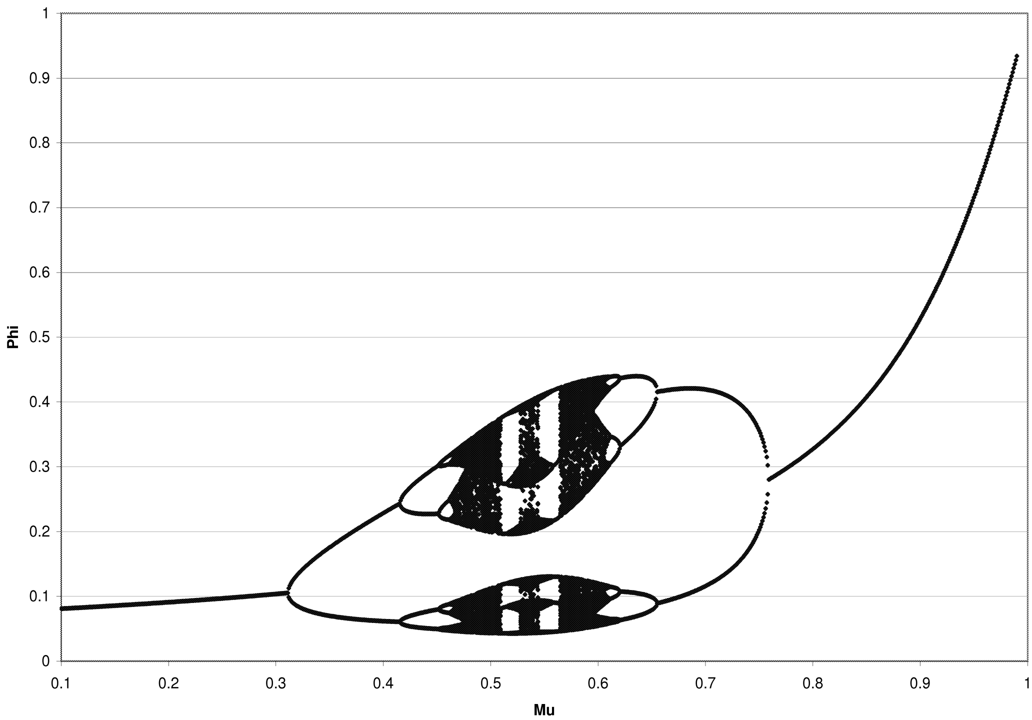

Figure 6: Bifurcation Diagram for phi, Gamma=0.9210

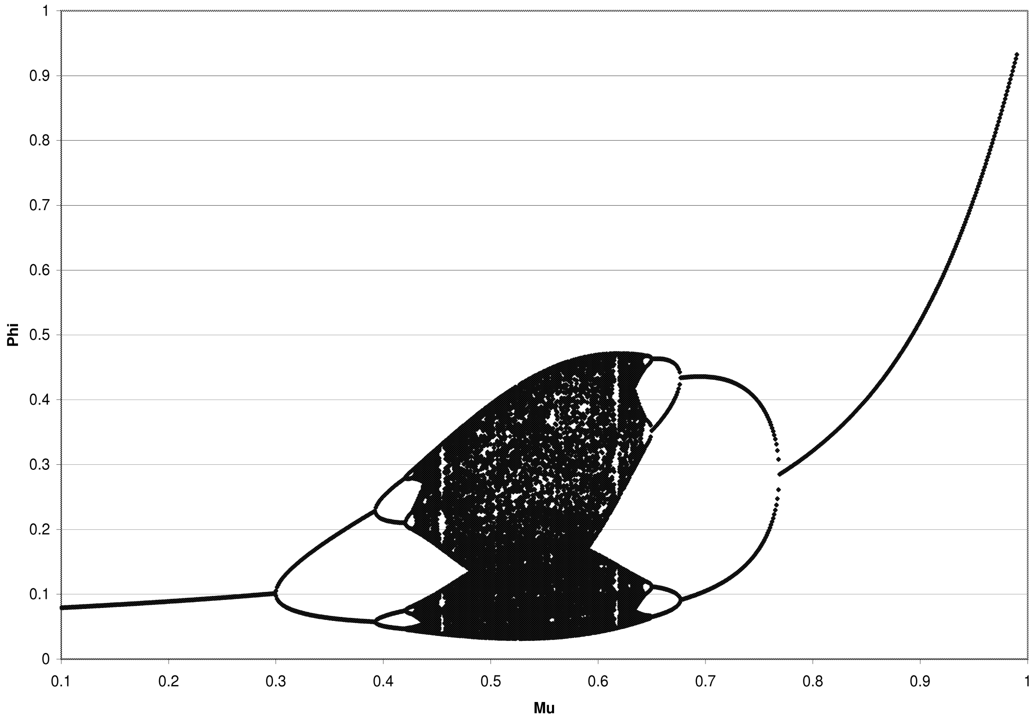

Figure 7: Bifurcation Diagram for phi, Gamma=0.9228