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Di Gaetano, Luigi

Department of Economics and Quantitative Methods, University of Catania

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Luigi Di Gaetano

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Abstract Several new auction formats are spreading over the Internet. They have usually the aim of raising revenues by increasing the number of participant, who will pay a participation fee, rather than selling the object at the highest possible price.

The aim of this paper is to study a format of descending price auction with hidden starting price and endogenous price decrease. In this format, usually known as price reveal auction, the price is hidden and players have to pay a fee to observe it. The price decreases only if a bidder observes it and not because of the time, like in the usual Dutch format.

In the following pages, we will analyse the effect of the concealment of the price in a standard Dutch auction. We will, then, define a model for price reveal auction, and analyse its most important aspects. We will, finally, derive players' best strategy and the Nash equilibrium of the game. Our result is that players use a threshold strategy to decide whether or not participate the auction (observe the price and pay the fixed fee). However, in our model there is not a separating equilibrium.

Moreover, we will find that there is a process of beliefs updating, which takes account of the time as a signal of the price. Therefore, if the game continues, players infer that the price is too high and update their beliefs accordingly.

We will finally compare our theoretical results with empirical data about 135 price reveal auctions held between December 2009 and April 2011 on the website Bidster.com.

Keywords Price reveal auction · Endogenous price · Descending auction

JEL Classification D44 · C72 · D82

Department of Economics and Quantitative Methods
University of Catania
E-mail: luigi.digaetano@unict.it

1 Introduction and Related literature

Auctions have always been employed as a method for selling objects since the earliest moment of the human history. During the last years, they were used to sell an increasing number of objects, and different new types of auction have been modelled to be adapted to various situations.

Several are the factors for these changes. In the first place, the greater deal of attention given by the literature – both theoretical and empirical – of the last decades, have contributed to understand mechanisms which regulate bidders' actions. Therefore, these improvements constituted also the basis to design new auctions, which had the aim of meeting certain requirements. The technological advantages, on the other side, had the effect of increasing the number of possible participant of auctions – for instance, by widening the geographical catchment area or increasing the number of bidders – and gave the possibility of organising more complex auction mechanisms.

Over the past few years, several new formats are spreading over televisions and the Internet, which represent new models of trade, and are different from traditional auctions because of differences in cost structure and in the organisation rules[1]. Among these, the most common new formats are Penny auction, Lowest unmatched bid auction and Price reveal auction (also known as scratch auction).

In a theoretical point of view, these auctions are interesting to study because of their structure, which differs from standard auction models, and for the mechanism by which revenues are raised by the auctioneer.

While in standard auction formats, the auctioneer tries to sell the object at the maximum possible price, in these new formats, objects are sold at a relatively low price. The auctioneer's profit derives, instead, from fixed costs which bidders have to sustain in order to enter the game or submit the bid.

This paper have the aim of analysing a relatively new auction format called price reveal auction. This format is a descending price auction, where the price is hidden for bidders – who have to pay a fee in order to observe the price and buy the object – and the price decrease mechanism is endogenous.

Because it is a descending price auction, it is apparently similar to a Dutch auction. However, in this format, the price is hidden and decreases not because of the time, but because of players' actions (endogenous price decrease mechanism). The two principal websites which offer this format are Bidster.com and Dealwonders.com.

To better understand how this auction mechanism works, we can quote the description of the auction rules of one of the websites which organise this auction format. Bidsters.com organises "scratch auction" since the 29th December 2009 (Gallice 2010), the auction is defined as follow:

> "On Bidster's Scratch Auctions the current price is hidden from the participant until the participant scratches the auction. Then the participant will see the current price. Every time someone scratches the auction the price is lowered. So the more people that scratch the auction the faster the price will drop. [. . .]
> Auction example (Scratch auction): The market price of the product is £ 100. Starting price for the auction is £ 95 (the start price is hidden for the participants). Every scratch lowers the price £ 2.5.
> When the auction starts Maria scratches the auction and gets the current price of £ 92.50. She sees the price in 10 seconds, but thinks it is a bit expensive and chooses not to buy the product for the current price. [. . .] Maria scratches the auction again and get the current price £ 85.00. She thinks it's a good price and choose to purchase the product.

[1] See, for example, Gupta and Bapna (2001) for a review of the principal online auction mechanisms and of the related literature.

Then the auction ends and Maria is the winner." (Bidster.com (2011), consulted on 28th June 2011).

As we said before, price reveal auction is part of a larger class of pay–per–bid auctions[2] which are spreading over the Internet.

Penny auction and least unmatched bid auction, were analysed by several authors.[3] Eichberger and Vinogradov (2008); Augenblick (2009), for instance, analyse the theoretical properties and empirical data of, respectively, lowest unique bid auction and penny auction. Eichberger and Vinogradov (2008) find that there is not a Nash equilibrium in pure strategies in their model of lowest unique bid auction, and they characterise the equilibrium in mixed strategies. However, they find also that data do not fit the predicted optimal strategy. Houba et al. (2008) focus, instead, mainly on theoretical issues as the endogenous entry of players, which creates uncertainty on the number of participants, and how players bid in least unmatched bid format. Augenblick (2009) analyses the equilibrium hazard rates of penny auction and the mixed–strategy equilibrium. He, then, compares his results with empirical data, finding that auctioneer's revenues are usually higher than the objects' market value and that the starting hazard rate is similar to that predicted, but it decreases in time.

A first prospective on price reveal auction is given by Gallice (2010), who sets up a theoretical model for this format. This is, so far, the only contribution to the literature with regard to this type of auctions. Thus, his paper represents the main background for our model.

Gallice (2010) develops a model where the price is hidden in each period. The starting price is set equal to the retail price of the object, and it is assumed that players' valuation are distributed between zero and the retail price. Players could observe the price or not. After observing it, they decide to buy the object or not. To observe the price they should pay a fee c. Every time someone observes the price, it is decreased by a value smaller than c. The price, in Gallice (2010), decreases only for this reason.

Gallice (2010) derives a very clear prediction for the (unique) perfect Bayesian equilibrium. According to his model, every player decides not to observe the price and the auctioneer's revenues are zero. Since the starting price is known and equal to the maximum valuation and due to the presence of the cost c, it is not optimal for the player with valuation equal to the retail price[4] to observe the price. Consequently, the price does not decrease, and for players with lower valuations, it is not optimal to observe the price neither.

The result, as said before, is that no one observes the price. However, these results – as the same author observes – are not consistent with empirical data.

As we will see later on next chapters, due to the hidden price and the bidding fee, the entry of players is endogenous. Moreover, since the price decreases because of the action of players, the price decrease mechanism is also endogenous.

The problem of endogenous entry of players in standard auction setting was studied by Levin and Smith (1994), McAfee and McMillan (1987b), Menezes and Monteiro (2000) and Chakraborty and Kosmopoulou (2001). In particular, Menezes and Monteiro (2000) studied the case in which players known only their information and the potential number of participants (not the number of players who decide to participate). They introduced a threshold strategy according to which only bidders who have a valuation above a certain cut-off value submit a bid. They, consequently, derived the optimal bidding functions for first price and second price sealed bid auctions.

[2] Among which there are penny auctions and least unmatched/lowest unique bid auction

[3] For a list of these see Gallice (2010)

[4] That is the maximum possible valuation.

This threshold strategy will be used (with some modifications) to determine the entry of bidders. In each period, players will decide to observe the price only if their valuation is greater than a certain threshold.

The paper is organised as follow. In the next section, we will analyse what is the effect of a hidden starting price in the simplest setting of Dutch auction. So, we will study a Dutch auction with uniform descending price and hidden and random price. Then, we will introduce a theoretical background for the model of price reveal auction and underline the characteristics of the new auction format. Then, we will analyse the theoretical aspects of the model, derive the predicted best strategy for players and analyse the theoretical results. Finally, we will compare our results with some empirical data from Bidster.com and make some remarks about the model.

2 Dutch auction with hidden random starting price

Before analysing our model of price reveal auction, we shall address a first theoretical question, which could be useful to understand players' strategies in the next chapter.

In particular, we shall analyse what is the effect of introducing a hidden and random starting price in a standard Dutch auction.

2.1 Structure of the game

The (modified) Dutch auction consists in a sale of a unique and indivisible object with market value M known by all players and by the auctioneer. There are $I = \{1, \cdots, n\}$ risk neutral bidders. It is assumed that each player has an *i.i.d.* private valuation v_i distributed over the interval $[0, \bar{v}]$, according to a generic cumulative distribution function $F(\cdot)$[5]. This distribution is assumed to be monotone non decreasing, continuous and differentiable.

The valuation M, positive, is not necessary below $\bar{v}$. The hidden starting price is αM, where α is a random variable distributed over the interval $[\epsilon, 1]$ according to a cumulative distribution function $G(\cdot)$ and a *pdf* $g(\cdot)$. The value ϵ is known by all players, as well as its distribution $G(\cdot)$.

At $t = 0$ nature selects α and, consequently, the starting price αM. At each time $t \geq 1$ the price is hidden, and it decreases uniformly and exogenously according to the following rule: $p_t = \alpha M - \delta(t-1)$.

The game starts in $t = 1$ and finishes if a player stops the auction, or when $t = T$ (It is assumed the auctioneer is always willing to sell the object).

At each time a player could decide to stop the game (buy the object) or to wait. Therefore, $A_{i,t} = \{stop, wait\}$ is the set of actions each player ($\forall i = 1, \cdots, n$) has in each period t. If she stops the game, she will buy the object whatever the price is. This means that she has to buy it even if the price is above her valuation.

Player i's payoff is:

$$\pi_i = \begin{cases} v_i - p_{\tilde{t}} & \text{if she stops the game at time } \tilde{t} \\ 0 & \text{otherwise} \end{cases}$$

2.2 Bidding strategy

Suppose all players $j \neq i$ follow a strictly decreasing and continuous strategy $t(v_j) = obs$ which maps each v_j to the strategy of observing the price at time t. And suppose player i uses a strategy

[5] and probability density function (from now on *pdf*) $f(\cdot)$.

$t(x) = obs$, for an arbitrary x. Moreover, although the price in each period is hidden, there is a one–to–one relationship which maps every period with an expected price p_t^e.

Thus, the risk neutral player i is willing to maximise the following expected payoff:

$$\max_x \left(v_i - p_t^e\left(t(x)\right)\right) Pr\left(t(x) \leq t(v_j), \forall j \neq i\right)$$

Since $t(x)$ is a monotonic decreasing and continuous function of x (thus invertible)[6], the maximisation problem becomes:

$$\max_x \left(v_i - p_t^e\left(t(x)\right)\right) F\left(x\right)^{n-1}$$

Taking the first derivative with respect to x, we obtain the first order condition:

$$\left[v_i - p_t^e\left(t(x)\right)\right] F\left(x\right)^{n-2}(n-1)f(x) - p_t'\left(t(x)\right)t'(x)F\left(x\right)^{n-1} = 0$$

$$p_t'\left(t(x)\right)t'(x)F\left(x\right)^{n-1} + p_t^e\left(t(x)\right)F\left(x\right)^{n-2}(n-1)f(x) = v_iF\left(x\right)^{n-2}(n-1)f(x)$$

This condition should be optimal for $x = v_i$

$$p_t'\left(t(v_i)\right)t'(v_i)F\left(v_i\right)^{n-1} + p_t^e\left(t(v_i)\right)F\left(v_i\right)^{n-2}(n-1)f(v_i) = v_iF\left(v_i\right)^{n-2}(n-1)f(v_i)$$

Since[7] the left hand side is the derivative of $p_t^e(t(v))F(v)^{n-1}$ and because a player with valuation 0 would buy the object only if the expected price is zero, we have that $t(v_i) = obs$ when:

$$p_t^e\left(t(v_i)\right) = \frac{1}{F\left(v_i\right)^{n-1}}\int_0^{v_i} xF\left(x\right)^{n-2}(n-1)f(x)dx$$

Thus, the optimal strategy for the Dutch auction with hidden random starting price is defined below:

Proposition 1 *In a Dutch auction with hidden and random starting price and with hidden price at every t, given player i's beliefs, the optimal strategy is:*

$$a_{i,t}^* = \begin{cases} stop\ \ if\ p_t^e \leq \frac{1}{F(v_i)^{n-1}}\int_0^{v_i} xF\left(x\right)^{n-2}(n-1)f(x)dx \\ wait\ otherwise \end{cases}$$

Where $p_t^e = E\left[\alpha\right]M - \delta\left(t-1\right)$ *is the expected price in period t.*

Note that we obtain the same result of a standard Dutch auction, $\frac{1}{F(v_i)^{n-1}}\int_0^{v_i} xF\left(x\right)^{n-2}(n-1)f(x)dx$ is the optimal bid for a first price auction[8]. Players act as in a (standard) Dutch auction but, instead of the known price, they use the expected price.

This result depends on two major assumption. First of all, bidders are risk neutral, and for this reason the only valuable information is the expected price.

If the hypothesis of risk neutrality does not hold, players should not follow the strategy outlined before. Risk aversion, for example, has the effect of having more aggressive bidders in a first price auction (Klemperer 1999). Moreover, without risk neutrality the revenue equivalence principle does not hold (Krishna 2002; Menezes and Monteiro 2005).

In our model the price is hidden. Therefore, depending on the form of bidders' utility function, we should consider not only the average, but also higher order moments of the distribution of α. The final effect on the best strategy – which is a sum of a more aggressive bidding strategy and

[6] Note that for the decreasing monotonicity and continuity, $t(x) \leq t(y) \iff x \geq y$

[7] Following the usual arguments for Dutch/first price auction with known price (Krishna 2002; Menezes and Monteiro 2005).

[8] Krishna (2002); Menezes and Monteiro (2005)

the characteristics of the expected price[9] – could be not trivial and it depends on the form of the utility function.

The second assumption is the randomness of the starting price, which has an important effect when calculating the expected price and the expected payoff. Players compute the expected value of α[10], and then update the expected price every period to account for the exogenous and uniform price decrease.

Results could be different, instead, if the auctioneer – and not the nature – is the one who chooses the (hidden) starting price.

Suppose there is a two-stage game where, in the first stage the auctioneer chooses the starting price (still hidden to bidders), and in the second stage the auction outlined before starts.

In this two-stage game, the auctioneer will anticipate that bidders use the expected price to decide whether to *stop* the auction or *wait*. Given players' beliefs about price, the auctioneer will always try to increase the starting price in order to enhance her profit.

Bidders, consequently, will understand that the auctioneer is trying to increase the hidden starting price.

Therefore, there are two possible cases. If the auctioneer could choose the price up to a certain known value $\dot{p}$, bidders will consider it as the starting price.

On the other hand, if the auctioneer's choice of the price has not an upper bound, players will anticipate it and, consequently, expect an infinitely large starting price. In the last case, they will never participate to the auction because, in each period, they expect an infinitely large price.

Bidders' best strategy will be the same of that outlined above. The effect of the removal of the assumption about the randomness of the starting price is on players' beliefs. Therefore, the expected price is $E\left[p_t\left|t\right.\right] = \dot{p} - \delta\left(t-1\right)$ in the first case and $E\left[p_t\left|t\right.\right] = \infty$ $(\forall t)$ in the second case.

3 A model of price reveal auctions

In the previous section we introduced a basic model of Dutch auction with random hidden starting price. In the next pages we will analyse a descending price auction with hidden price and endogenous price decrease. This format, as we said before, is known as price reveal auction and is usually organised on the Internet.

The remaining part of the paper is organised as follow. First, we will outline the characteristics of the model. Then, we will derive the optimal players' strategies in the same fashion as the previous model. Finally, we will analyse the theoretical results of the model and do some remarks about assumptions and findings.

3.1 The model

The model of price reveal auction consists in a sale of a unique and indivisible object with market value M, known by all players and by the auctioneer. There are $I = \{1, \cdots, n\}$ risk neutral bidders. It is assumed that each player has an *i.i.d.* private valuation v_i, distributed according to a generic cumulative distribution function $F(\cdot)$[11] and drawn from the interval $[0, \bar{v}]$. This distribution is assumed non decreasing, continuous and differentiable.

[9] For example, a big variance of α could lead to a less aggressive bidding behaviour when players are risk averse

[10] Where $1-\alpha$ is the discount of the price.

[11] and *pdf* $f(\cdot)$

In the same fashion of the previous model, the market value M is positive and could be greater than $\bar{v}$.

At time $t = 0$, nature selects the starting price and players' valuations. The game starts at $t = 1$ and it continues until T or until someone buys the object.

At each time $t \geq 1$, following Gallice (2010), players decide whether observing the price or not. After observing it, each buyer could buy the item or not. Therefore, in each period $t \geq 1$, a generic player i has a set of actions $A_{i,t} = \{nobs; (obs, b); (obs, nb)\}$, $(\forall i \in I)$.

To observe the price, bidders pay a fee $c > 0$ to the auctioneer. Each player could observe the price only once. When the price is observed by someone, it is decreased by a $\delta < c$. After observing the price, the player decides to buy or not the item (so also this option is proposed only once). If a player observes the price and does not buy the object, she will exit the game.

The initial price p_0 is hidden to all bidders and randomly chosen by nature. It is assumed that the auctioneer is always willing to trade whatever the price is.

The starting price is defined as $p_0 = \alpha M$, where α is a random variable, distributed over the interval $[\epsilon, 1]$ according to a cumulative distribution function (*cdf*) $G(\cdot)$ and a *pdf* $g(\cdot)$. The value ϵ is known by all players as well as the distribution $G(\cdot)$[12]. We should remark that the starting price could be above the highest valuation $\bar{v}$.

At each time, the price is $p_t = p_0 - \delta\eta_t$, where $0 < \delta < c$, and $\eta_t \in N$ is the number of times the price has been observed up to time t[13]. The price is unknown by players, because they do not know neither the starting price nor the overall number of times the price was observed (η_t). The payoff of a generic player with valuation v_i is:

$$\pi_i = \begin{cases} v_i - p_t - c & \text{if player } i \text{ observed and bought the object (at time } t) \\ -c & \text{if player } i \text{ observed the price and did not buy the object} \\ 0 & \text{otherwise} \end{cases}$$

This setup shares several characteristics with that of Gallice (2010). There are, however, some different assumptions.

The starting price is a random variable and is hidden to buyers. In Gallice (2010), the initial price was known and set equal to the retail price. The concealment of the starting price is a minor assumption, since it reflects the format run on the Internet[14]. The randomness of the starting price has the effect of stimulating the entry of players and, as we will see later, is a central assumption of our model. Moreover, the random price could be over the highest valuation $\bar{v}$. Finally, we assumed also that a buyer could observe the price only once.

3.2 Optimal strategy and second stage decision

The presence of randomness on the price of our auction, together with the fact that the player is allowed to refuse to buy the object, are critical points of the analysis of the optimal strategies.

Gallice (2010), in his model, introduced a threshold strategy for prices based on the fact that was not optimal for a bidder to observe the price and not buy the object. Due to this consideration, the strategy (obs, nb) was dominated and never chosen. Thus, he argued that the equivalence between first price auction and Dutch auction could be used as platform for building players' beliefs and strategies.

[12] The parameter ϵ is needed to avoid a price equal to zero.

[13] The price is lowered if someone observes the object's price (it is already decreased when players see it). Therefore, we have that $\eta_1 = 0$, since bidders start playing in period $t = 1$.

[14] In the first chapter we quoted, for instance, the rules of the format from the website Bidster.com, in which is stated that the starting price is hidden.

In our case, since the price could be above a bidder's valuation, we cannot exclude the strategy (obs, nb). However, we can separate players' strategies in two stages: in the first one, they choose whether observing the price or not, in the second – conditional on observing the price – they decide to buy or not the object.

The second stage decision is trivially dependent on the known price. Since players can enter the auction (observe the price) only once, they will decide to buy the object if the actual price is below their valuation.

Proposition 2 *Conditional on observing the price, player i will buy the object if the price is below her valuation, i.e. if $v_i \geq p_t$*

Proof Conditional on observing the price, player i has already sustained the entry fee c, thus it is a sunk cost[15]. If she buys the object, she receives a payoff $v_i - p_t$.

Since each player can observe the price (and buy the object) only once, there are not possibilities of re-entering the game[16], and hence the payoff is zero if the player decides not to buy the object .

Conditional on observing the price, the strategy obs, b dominates obs, nb if $v_i - p_t \geq 0$. Player i will, therefore, buy the object if $v_i - p_t \geq 0 \iff v_i \geq p_t$

3.3 First stage decision

In a standard Dutch auction, a generic player maximises her expected payoff, given that no one has stopped the game before[17].

In our model there is the possibility of observing the price and not buy the object[18]. Therefore, the game may continue also if someone already observed the price, and we should consider it when deriving the optimal strategy. Moreover, the price decreases only if someone observes it.

Suppose that each player $j \neq i$ follows a strictly decreasing and continuous strategy $t(v_j)$, which maps every valuation with a period t in which is optimal to observe the price. Suppose also that player i decides to follow a strategy $t(x)$, for an arbitrary value x.

According to these strategies, if a player k observes the price before i is because $v_k > x$[19]. Then, if a player k ($k \neq i$) observes the price before i and the game continues, we have that the price (already decreased[20]) is higher than $v_k > x$[21].

Consequently, if someone observes the price and refuses to buy the object, and if the optimal strategy for player i is when $x = v_i$, we can infer that the price is higher than v_i and it is not optimal for i (since the sunk cost to observe c is greater than δ) to observe it.

Hence, is not optimal to observe the price in the event that someone observed the price before $t(x)$ (for $x = v_i$). A generic player i is willing to maximise the expected utility:

$$\max\left[v_i - p_t\left(t(x)\right); 0\right] \cdot Pr\left(t(v_i) \leq t(v_j), \forall j \neq i\right) - c =$$

15 Therefore, players no more take into account it.

16 Without the restriction imposed, players could decide not to buy the object, once they observed the price, in order to buy it later and gain from new information acquired by observing the price.

17 Which happens with probability $Pr(t(v_i) \leq t(v_j), \forall j \neq i) = F(v_i)^{n-1}$.

18 Due to the second stage decision, this happens if the price is above player j's valuation

19 $t(\cdot)$ is a (strictly) decreasing and continuous function and we have that $t(v_k) < t(x) \iff v_k > x$.

20 Since the price decreases when someone observes it

21 This is trivial. The only reason for a player not to buy the object – after observing the price – is because the price (already decreased) is higher than his valuation (second stage decision). Then, considering players' strategies, if someone observes the price before i is because $t(v_k) < t(x) \iff v_k > x$. Using both considerations we have that the price is higher than v_k and, consequently, than x

$$\max\left[v_i - p_t\left(t(x)\right);0\right]\cdot F\left(v_i\right)^{n-1} - c$$

Note also that, since the price decrease mechanism is endogenous, the price does not decrease because of the time. And, for this reason, we have that $p_t' = 0$

Using the above considerations, we can derive the first order condition taking the first derivative, with respect to x, of the expected utility above, and imposing it equal to zero:

$$\max\left[v_i - p_t\left(t(x)\right);0\right]\cdot(n-1)f(x)F(x)^{n-2} - p_t'\left(t(x)\right)F\left(v_i\right)^{n-1} = 0$$

it should be optimal for $x = v_i$:

$$\max\left[v_i - p_t\left(t(v_i)\right);0\right]\cdot(n-1)f(v_i)F(v_i)^{n-2} - p_t'\left(t(v_i)\right)F\left(v_i\right)^{n-1} = 0$$

We find the usual condition for a Dutch auction, but we should consider also that $p_t' = 0$, therefore we have that:

$$\max\left[v_i - p_t\left(t(v_i)\right);0\right]\cdot(n-1)f(v_i)F(v_i)^{n-2} = 0$$

Since the price does not decrease because of the time, players do not find any gain in waiting (but only a possible loss due to the fact that someone could observe the price and buy the object), consequently they are willing to observe the price as long as $p_t^e = v_i$[22].

We can derive the same result with a different argument, which will be exposed below. We should check if is profitable to observe at $t(v_i) + dt$ instead of $t(v_i)$.

At $t(v_i)$, player i would receive a payoff of $\max\left[v_i - p_t;0\right]$. At $t(v_i)+dt$, if no one has observed the price, it does not change. And this happens with probability $1-(n-1)f(v_i)F(v_i)^{n-2}\frac{dt}{t'(v_i)}$[23]. So with this probability the price does not change.

If another player, instead, observes the price, there are two possibilities. If she buys the item, then player i payoff would be zero. If she does not buy the item, it is the case that the price $p_{t(v_i)+dt}$ is greater than her valuation and, consequently, $p_{t(v_i)+dt} > v_i$. Therefore the max operator gives zero as result in both sides.

So the optimal condition is:

- When the price is greater than v_i, the optimal first order condition is $0 = 0$ (always satisfied).
- When the price is smaller of v_i, between $t(v_i)$ and $t(v_i) + dt$, with probability $\left[1-(n-1)f(v_i)F(v_i)^{n-2}\frac{dt}{t'(v_i)}\right]$ no one observes the price and it does not change. With the complementary probability someone (with valuation equal to v_i) would observe the price and buy the object[24]. And the optimal condition is:

  $$\max\left[v_i - p_t;0\right] = \max\left[v_i - p_t;0\right]\left[1-(n-1)f(v_i)F(v_i)^{n-2}\frac{dt}{t'(v_i)}\right]$$

  which becomes[25]:

  $$\max\left[v_i - p_t;0\right](n-1)f(v_i)F(v_i)^{n-2}\frac{dt}{t'(v_i)} = 0$$

  The left hand side represents the marginal cost of waiting until $t(v_i) + dt$, while 0 represents the marginal gain from waiting. It is optimal to stop at $t(v_i)$ if the marginal cost of waiting (not stopping) is greater or equal to the marginal gain of not stopping (the LHS is greater or equal to 0).

[22] Note that this is only the first order condition but we should also consider the presence of the fee c.

[23] That is the complementary probability of the event in which a player with a valuation v_i observes the price between $t(v_i)$ and $t(v_i) + dt$.

[24] Since we supposed that the actual price is below v_i

[25] We can remove the max operator because we supposed that $p_t \leq v_i$

This is the same result as above. Players are willing to observe the price as soon as $p_t \leq v_i$[26] (but we should also consider the cost c, which is not present in the first order condition).

In the usual Dutch auction format, there is a trade off between the expected marginal gain of waiting the price to decrease (represented by the lower price in the future) and the marginal expected loss due to the lower probability of winning the auction.

In our model that marginal gain is zero. Since the price decreases because players observe the price, there are only two possibilities: either the price is too high or the object is bought by another player.

In this setting, consequently, there is always a gain in undercutting other players until the point in which the expected price is equal to player i's valuation.

Moreover, we should also consider the fee c, paid to participate the auction (observe the price). Given the expected price at time t, the probability of buying the object, which makes a generic player indifferent between the actions *obs* and *nobs*, is:

$$[v_i - p_t^e]\, Pr\,(p_t \leq v_i) - c = 0 \iff Pr\,(p_t \leq v_i) = \frac{c}{v_i - p_t^e}$$

In the left hand side, we have the expected utility of the action *obs*, that is the expected payoff times the probability of buying the object[27] and the sunk fee c. While in the right hand side, zero represents the payoff of the action *nobs*.

Proposition 3 *The optimal (first stage) strategy for a generic player i is to observe the price if $p_t^e \leq v_i - c$ and if $Pr\,(p_t \leq v_i) \geq \frac{c}{v_i - p_t^e}$*

Proof Before we demonstrated that players are willing to undercut the others players because there are not gains from waiting[28], due to the endogenous price decrease mechanism. Players are willing to observe the price as soon as the expected price is below or equal to their valuation.

Because the presence of the fee c (to observe the price), is not optimal to observe it when $p_t^e = v_i$. A necessary condition to make a player prefer to observe the price is that $Pr\,(p_t \leq v_i) \geq \frac{c}{v_i - p_t^e}$, which corresponds to the case where the expected utility of *obs* is greater or equal to that of *nobs*, given player's beliefs.

Moreover, since a probability is always included between 0 and 1, we have also that $0 \leq \frac{c}{v_i - p_t^e} \leq 1$. Which means that $v_i - p_t^e > 0$[29] and $v_i - p_t^e \geq c \iff p_t^e \leq v_i - c$.

Therefore, players are willing to undercut their rivals until they reach the upper bound $v_i - c$, which makes them indifferent between observing the price or not. Above this value of the expected price players prefer not to enter the auction (observe the price).

3.4 Qualification of Perfect Bayesian equilibrium

In the previous sections we derived the optimal players' strategies. We divided it in first and second stage strategies. Given those strategies we can define the perfect Bayesian equilibrium for the game.

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[26] Since the price is hidden, players will use the expected price at time t denoted as p_t^e

[27] That is the probability that the price is below player's valuation, because of the second stage decision

[28] gains that exist in a standard Dutch auction

[29] Because c is greater than zero

Proposition 4 *A perfect Bayesian equilibrium of the game consists in n sequences of actions $\left\{a^*_{i,t}\right\}_{t=1}^{T}$ (for every $i \in I$) where $a^*_{i,t}$ is the optimal action of player i at time t defined below:*

$$a^*_{i,t} = \begin{cases} (obs, b) & \text{if } \mu_{i,t} = Pr\left(p_t \leq v_i\right) \geq \frac{c}{v_i - p^e_t} \text{ and } v_i - p_t \geq 0 \\ (obs, nb) & \text{if } \mu_{i,t} = Pr\left(p_t \leq v_i\right) \geq \frac{c}{v_i - p^e_t} \text{ and } v_i - p_t \leq 0 \\ (nobs) & \text{if } \mu_{i,t} = Pr\left(p_t \leq v_i\right) < \frac{c}{v_i - p^e_t} \end{cases}$$

Where $Pr\left(p_t \leq v_i\right)$ represents player i's beliefs about the current price[30] *and p^e_t is the expected price at time t.*

Proof The decision of observing the price is taken according to the threshold rule depicted in the previous section (first stage decision). The action *nobs* dominates *obs* if $Pr\left(p_t \leq v_i\right) < \frac{c}{v_i - p^e_t}$, and is dominated otherwise. Conditional on observing the price, the decision of buying the object depends on v_i and p_t. According to the second stage decision, players are willing to buy the object if the known price is below their valuation: if $v_i - p_t \geq 0$ then (obs, b) weakly dominates (obs, nb) (or strictly dominates when the inequality is strict) and vice versa.

There is not a separating equilibrium. Given expectations of the initial price (and for next periods' prices if the game continues), there will be either no one or a bunch of players who are willing to observe price.

In particular – when the game starts – if $p^e_1 = E[\alpha]M \geq \bar{v}$, no one will enter the auction (since the expected price is higher or equal to the highest valuation and players should sustain c). The same happens if $E[\alpha]M < \bar{v}$ and $Pr(p_1 \leq \bar{v}) < \frac{c}{\bar{v} - E[\alpha]M}$.

Suppose instead that there is a value $\dot{v}$ such that $0 < \dot{v} < \bar{v}$ and $Pr(p_1 \leq \dot{v}) = \frac{c}{\dot{v} - E[\alpha]M}$ (and for smaller values the left hand side is strictly lower than the right hand side). Then, since $\frac{\partial Pr(p_1 \leq v)}{\partial v} \geq 0$[31] and $\partial \frac{c}{v - E[\alpha]M} / \partial v < 0$[32], all players with valuations between $\dot{v}$ and $\bar{v}$ will be willing to observe the price.

3.5 Characterisation of beliefs

There are different considerations to do when analysing players' beliefs. In period $t = 0$, players receives their own signals (their valuations) and have no other information at their disposal.

In period $t = 1$ the game starts. Therefore, no one has observed the price before – i.e. $\eta_1 = 0$ – and the expected price at $t = 1$ is $p^e_1 = E[\alpha]M$[33].

Due to the randomness of the initial price they formulate their beliefs – for the first period – as follow:

$$\begin{aligned} Pr\left(p_t \leq v_i \left| t = 1\right.\right) &= Pr\left(\alpha M \leq v_i\right) \\ &= Pr\left(\alpha \leq \frac{v_i}{M}\right) \\ &= G\left(\frac{v_i}{M}\right) \end{aligned}$$

[30] When $Pr\left(p_t \leq v_i\right) \geq \frac{c}{v_i - p^e_t}$ and $v_i - p_t = 0$ player i is indifferent between (obs, b) and (obs, nb).

[31] The probability that the price is less or equal to a certain valuation v does not decrease if v increases, given expectations about the price (See next section for a characterisation of beliefs).

[32] This ratio becomes smaller if v increases.

[33] Where $E[\alpha]$ is the expected value of α and is equal to $E[\alpha] = \int_{\epsilon}^{1} x g(x) dx$ (with $g(\cdot)$ *pdf* of α).

Where $G(\cdot)$ is the *cdf* of α. We can interpret $\frac{v_i}{M}$ as the maximum percentage value of α, above which the price is higher than player's valuation. Therefore, beliefs at time $t=1$ are represented by the probability that α is below that critical value. Note that if $\frac{v_i}{M} < \epsilon$ (the lowest value of α) the expression above is negative, and in this case the probability is simply zero.

Definition 1 For a certain player with valuation v_i the beliefs at time $t=1$ are:

$$\mu_{i,1} = \begin{cases} G\left(\frac{v_i}{M}\right) & \text{if it is positive} \\ 0 & \text{otherwise} \end{cases}$$

For every $t>1$, and until the game is finished, beliefs are updated using new information. Players consider the information that the game is still continuing as a signal of the current price.

There are two possible reasons why the game is still continuing. It could happen either that no one observed the price, or that someone observed it, but the object was not bought (because its price was too high).

Consequently, provided that someone is expected to observe the price and if the game is continuing, players will think that the hidden price is above the expected maximum valuation.

For this reason we should introduce the expected number of observing players and the expected maximum valuation.

Definition 2 Define $\xi_t = E(\eta_t)$ as the expected number of players who decide to observe the price up to period t. Note that, since players are rational, $\xi_1 = \eta_1 = 0$.

As anticipated before, players' expectation are updated each periods, provided that the game is still continuing.

If someone is expected to observe the price in the previous period and the game is continuing, players will update their beliefs thinking that the price is higher than the previous period expected maximum valuation.

Definition 3 We can define the value $v_{max}^{e|t}$. as the expected maximum valuation conditional on $v_i \geq \dot{v}_t$. That is the expected value of the highest order statistic, conditional on having values above $\dot{v}_t$, computed as $v_{max}^{e|t} = \frac{1}{1-F(\dot{v}_t)^{n-1}} \int_{\dot{v}_t}^{\bar{v}} x \cdot nF(x)^{n-1} f(x)\, dx$. Provided that $\dot{v}_t$ is smaller than $\bar{v}$.

Where $\dot{v}_t$ is the minimum valuation such that a player with $\dot{v}_t$ observes the price at time t, and a player with a smaller valuation does not. It is defined by the following equality[34]:

$$\dot{v}_t : Pr(p_t \leq \dot{v}_t) = \frac{c}{\dot{v}_t - p_t^e}$$

Therefore, if the game is still continuing and we expect that someone observed the price in the previous period, we can say that the game is still continuing because the price of the precedent period was higher than $v_{max}^{e|t-1}$.

Thus, if $\xi_t > \xi_{t-1}$ beliefs are updated as follow[35]:

$$Pr\left(p_t \leq v_i \,\middle|\, t, v_i, p_{t-1} \geq v_{max}^{e|t-1}\right) =$$

$$Pr\left(\alpha M - \delta\xi_t \leq v_i \,\middle|\, t, \alpha M - \delta\xi_{t-1} \geq v_{max}^{e|t-1}\right) =$$

[34] Note that – since the probability that the price is less or equal to a certain valuation v is not decreasing with respect to v, given expectations about the price (i.e. $\frac{\partial Pr(p_1 \leq v)}{\partial v} \geq 0$) and $\partial \frac{c}{v - p_t^e} / \partial v < 0$ – all players with valuations between $\dot{v}_t$ and $\bar{v}$ will be willing to observe the price.

[35] Note that we are applying Bayes rule: $P(A\,|B\,) = \frac{P(A \cap B)}{P(B)}$

$$\left[G\left(\frac{\delta\xi_t + v_i}{M}\right) - G\left(\frac{\delta\xi_{t-1} + v_{max}^{e|t-1}}{M}\right)\right] / \left[1 - G\left(\frac{\delta\xi_{t-1} + v_{max}^{e|t-1}}{M}\right)\right]$$

Bidders use the information "*no one has bought the object*" as a signal of the possible value of α. Note that the formula above could be negative for certain values of v_i. Since it is a probability, it is well defined only if it is positive. In case the formula above is negative, the probability is simply zero.

If, instead, players expect that no one has observed the price in the previous period (i.e. $\xi_t = \xi_{t-1}$), beliefs about the price are the same of the previous period.

Definition 4 In a generic period $t \geq 2$ beliefs are:

$$\mu_{i,t} = \begin{cases} \mu_{i,t-1} & \text{if } \xi_t = \xi_{t-1} \\ \frac{\left[G\left(\frac{\delta\xi_t + v_i}{M}\right) - G\left(\frac{\delta\xi_{t-1} + v_{max}^{e|t-1}}{M}\right)\right]}{\left[1 - G\left(\frac{\delta\xi_{t-1} + v_{max}^{e|t-1}}{M}\right)\right]} & \text{if this number is positive and } \xi_t > \xi_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

Where ξ_t is the expected number of times the price was observed up to time t (with $\xi_1 = 0$) and $v_{max}^{e|t}$ is the expected maximum valuation at time t as defined above. If $Pr\left(p_t \leq \bar{v}\right) < \frac{c}{\bar{v} - p_t^e}$ no one is expected to observe the price and we have that $\xi_{t+1} = \xi_t$.

4 Theoretical findings and other remarks

In our model of price reveal auction, we found that players are willing to undercut their rivals and observe the price as soon as it is expected to be below or equal to $v_i - c$ (and if $Pr\left(p_t \leq v_i\right) \geq \frac{c}{v_i - p_t^e}$[36]). This behaviour is a consequence of the endogenous price decrease mechanism. Since players do not expect the price to fall from time to time, because they want to observe the price only to buy it, they do not have any incentive in waiting.

In a standard Dutch auction, this incentive is represented by the price decrease, which creates a trade–off between lower probability of winning the auction and bigger (positive) difference between players' valuation and price.

Moreover, entry in the game is stimulated by the randomness of the starting price, together with its concealment. The incentive in entering the game, however, disappear during the time because players update their beliefs and expect the price to be higher if someone has not still bought it.

The fact that the game is still continuing is a bad new for players. Because it means that (if initial beliefs are such that someone is expected to observe the price) the price is higher than the valuation of players who observed it.

The time is, therefore, a signal about the price. And – similarly to the information *"winning the game"* in the winner's curse for the common value auctions – the information *"the game continues"* is a *bad new* for players, because they are informed about the (high) level of the price. Beliefs are, consequently, updated accordingly.

In Gallice (2010), in equilibrium, players decide not to observe the price in each period. This result, although theoretically strong, does not explain the empirical data. In his paper, Gallice (2010) has hypothesised that this is due to the presence of bounded rational players.

In our model, however, is theoretically possible to sustain an equilibrium where some players are willing to observe the price at least in the first period. There is not a separating equilibrium.

[36] That is the case where the strategy *obs* dominates *nobs*

Due to beliefs conformation and the randomness of the price, there is always either no one or a group of buyers who are willing to observe the price. These characteristics should be considered carefully, especially when analysing the efficiency properties of this descending auction format.

Another important remark, which should be done, concerns models' capability of explaining empirical data. Table n. 1 shows some summary statistics about all price reveal auctions held

Variable		Obs	Mean	Std. Dev.	Min	Max
marketp	Market price	135	441.3407	373.2719	25	1700
disc	Discount	135	.3495556	.1776331	.05	.999
c	Participation fee	135	1.062963	1.07278	.5	10
parti	N. of participants	13	140	86.93197	7	325
totdur_d	Tot. length (days)	135	50.24215	46.48102	.17	307.83

Table 1 *Descriptive statistics of all paying price reveal auctions ran in Bidster.com and ended between the 4th January 2010 and the 26th April 2011*

	marketp	disc	c	parti	totdur_d
marketp	1.00 (135)				
disc	-0.28 (135)	1.00 (135)			
c	0.11 (135)	0.07 (135)	1.00 (135)		
parti	0.51 (13)	-0.22 (13)	-0.03 (13)	1.00 (13)	
totdur_d	0.28 (135)	0.07 (135)	0.07 (135)	0.47 (135)	1.00 (135)

Table 2 *Pairwise correlation matrix for all paying price reveal auctions ran in Bidster.com and ended between the 4th January 2010 and the 26th April 2011* (N. of observations in brackets)

in Bidster.com between the 4th January 2010 2009 and the 26th April 2011. According to these data, the average discount is 34.95%, and auctions last, in average, for approximatively 50 days (with a maximum value of about 307 days).

Therefore, even if our model theoretically allows player to observe the price, it seems that these data are not completely compatible with our theoretical findings.

The average discount and average duration of the game seems indeed too high for our model. The duration, in particular, is counterfactual with respect to the beliefs updating process. Because beliefs about the current price are updated each period (price are expected to be higher), the game is expected to finish early in time.

These differences could be explained by the presence of players' bounded rationality. Suppose, for instance, that a fraction of players randomly observe the price, because they do not properly understood the game.

If in each period is expected to have h observing not rational players, then we will have that the price is falling of δh in each period (δht is one of the two price decreasing rules). Therefore, we introduced an exogenous price decrease mechanism which could lead to delay the decision to observe the price[37].

Another explanation about this data regards the possibility that players do not start the auction at the same time. In our model, players start the auction together at $t = 1$. In the real format there is not a similar restriction. Therefore – even though this hypothesis could be difficult to address theoretically – this could explain empirical observations. Since is theoretically

[37] First of all, we no more have that $p'_t = 0$. Thus, players have an incentive in waiting.

possible to observe the price, allowing players to start in different times could explain the data pattern observed empirically.

A central assumption is the randomness of the starting price. It is important to characterise players' beliefs and to stimulate entry in the game. As we saw in the modified Dutch auction, if the hidden starting price is chosen by the auctioneer, she would choose the greatest possible price. Thus players' beliefs are calculated accordingly. This assumption seems too restrictive. But, even though the auctioneer does not choose the starting price, she could choose the distribution of α and the value of ϵ.

Several developments could be done starting from this model. First of all, allowing players to observe the price more than once could lead to different strategies (the second stage decision should consider the expected value of participating again the auction as a benchmark to look if is optimal to buy the object or not).

In the second place, as we discussed above, allowing the presence of bounded rationality could change the theoretical results and could explain the differences between theoretical and empirical findings.

The similarities of this auction format with lotteries (similarities which are common in all the new auctions format spreading on the Internet) are a clue about the possibility that the behaviour of players could be different from a risk neutral agent. For this reason, it could be interesting to analyse what will be the results with risk averse and risk seeking bidders.

Finally, we only analysed the auction considering bidders' point of view. Future developments can go in the direction of analysing the expected revenues and on the design of an optimal auction for the auctioneer. Questions to be addressed, for instance, concern what is the optimal distribution $G(\cdot)$ and value ϵ which has the effect of optimising the revenues and if this auction is an efficient allocation rule.

5 Conclusions

In the previous pages, we developed a model of descending auction with hidden price and endogenous price decrease. In this auction format, known as price reveal auction, the price is hidden in each period and decrease only if players observe the price. Our model is theoretically based on the model of Gallice (2010). In that model, the starting price is known by all bidders and is equal to the market value of the object (which is also equal to the highest value of players' valuations' distribution $\bar{v}$). However, we introduced a hidden random starting price, and we allowed the starting price to be either above or below $\bar{v}$ (the upper bound of valuations' support).

We divided players' decision in two stages (decision of observing the price or not, and, conditional on observing the price, decision to buy or to exit the game). Players could not observe the price (and buy the object) more than once.

We, finally, derived the optimal strategy given players' valuation and beliefs and we characterised the beliefs' structure.

Our results have some interesting characteristics. First of all, we found that, due to the endogenous price decrease mechanism, players do not have any incentive in waiting. They, therefore, are willing to observe the price as long as it is expected to be below their valuations (and, for this reason, we introduced a threshold rule based on the probability that the price is below v_i).

This is a huge difference between the Dutch auction (and first-price sealed bid auction) and our format. Buyers do not base their decision on the first price auction optimal bid[38].

[38] In a standard Dutch auction, players are willing to buy the object if the price is smaller or equal to the optimal first price auction bid. We saw in the second chapter that, this is also the case in a Dutch auction with hidden starting price.

Moreover, we found that is possible to have an equilibrium with some players who observe the price, at least in the first period. This results is different from that of Gallice (2010), who found that, in the unique perfect Bayesian Nash equilibrium, players do not observe the price because their beliefs. Our result could explain in part the empirical findings.

However, in our model there is not a separating equilibrium. Given beliefs and players' characteristics, in each period there will be either no one or a group of players who are willing to observe the price.

Finally, we characterised beliefs' structure. Beliefs are updated using different information. Time itself is a signal of the possible price. Because players are willing to buy the object as soon as possible, and due to the concealment of the price, the time is a signal about the price. This updating process has the effect of decreasing the incentive in observing the price from time to time. Therefore, players (in average) are less willing to observe the price when t increases. We observed, however, that this process does not explain why, in empirical observations, auctions last (in average) for several weeks.

The average number of days and number of participants seems to be too high to match with the results of this model. One explanation could be the presence of bounded rationality. Allowing for bounded rationality – as we suggested, for instance, in the previous chapter – could have the effect of introducing an endogenous price decrease mechanism and, consequently, delaying the decision of observing the price. We also argued that another explanation could be found on the fact that, in online auctions, players do not start the auction at the same time[39].

Future developments of the model could be represented by the possibility of allowing players to observe the price for more than once and by allowing for the presence of boundedly rational players. It will also be interesting to analyse the effect of a fee c smaller than the price decreasing factor δ. Further analysis could be done to find the expected auctioneer's revenues and the auction's efficiency in term of allocation[40].

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[39] They do not log–in the website and participate the auction contemporaneously.

[40] It is expected not to be efficient due to the absence of a separating equilibrium and the presence of randomness.