Deterministic and stochastic trends in the time series models: A guide for the applied economist

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Deterministic and Stochastic Trends in the Time Series Models : A Guide for the Applied Economist[1]

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Abstract

Applied economists working with time series data face a dilemma in selecting between models with deterministic and stochastic trends. While models with deterministic trends are widely used, models with stochastic trends are not so well known. In an influential paper Harvey (1997) strongly advocates a structural time series approach with stochastic trends in place of the widely used autoregressive models based on unit root tests and cointegration techniques. Therefore, it is important to understand their relative merits. This paper suggests that both methodologies are useful and they may perform differently in different models. This paper provides a few guidelines to the applied economists to understand these alternative methods.

Keywords: Stochastic and Deterministic Trends, Bai-Perron Tests, STAMP, Structural Time Series Models.

JEL Codes:

[1] I thank Gyaneshwar Rao, Rup Singh and Saten Kumar for some useful suggestions. All errors are my responsibility.

1. Introduction

There is a methodological controversy on how to estimate time series models. Like other such controversies, this is a difficult to resolve. By and large many applied economists, working with time series models, assume that the variables have strong deterministic trends. Therefore, they add a time trend with constant level and slope parameters to variables in the unit root tests and when estimating the models with the cointegration techniques. Models based on this methodology are well known as autoregressive models. In contrast Harvey (1997, p. 192), in an influential methodological paper, argued that "….unless the time period is fairly short, these trends cannot be adequately captured by straight lines. In other words, a deterministic linear time trend is too restrictive…." Harvey suggests that time series models should incorporate slowly evolving stochastic instead of deterministic trends. Such models are known as the unobserved components models or structural time series models.

In between these two approaches, there is another equally influential view in which the changing nature of the trend is examined so that unit root tests and cointegration can proceed with the appropriate shift dummy variables. Perron is a well known proponent of this approach; see Perron (1989) and Perron and Wada (2005). His approach can be justified with the following analogy. If a circle can be seen as a collection of an infinite number of straight lines, with varying slopes, time series technique with deterministic trends which allow for major changes in the level and slope of the trend should be also satisfactory. Perron goes further and says that in the decomposition of US GDP:

"Unobserved Components models, …. yield very different cycles which bears little resemblance to the NBER chronology, ascribes much movements to the trend leaving little to the cycle, and some imply a negative correlation between the noise to the cycle and the trend. We argue that these features are artifacts created by the neglect of the presence of a *change in the slope of the trend function* in real GDP in 1973. Once this is properly accounted for, the results show all methods to yield the same cycle with a trend that is non-stochastic except for a few periods around 1973." Perron and Wada (2005) with my italics.

The aforesaid view of Perron seems consistent with the following observation of Harvey (1997, pp.192-93):

"Since a deterministic time trend is too restrictive, the obvious thing to do is to make it more flexible by letting the level and slope parameters change over time. In a structural time series model, these parameters are essentially assumed to follow random walks. This leads to a stochastic trend in which the level and slope are allowed to evolve over time".

Even before this observation by Harvey, several attempts have been made to develop tests for unit roots and cointegration with known or unknown structural breaks. In fact it is well known that Perron (1989) in a path breaking work and has laid the foundation for this approach. The main difference between Harvey and Perron seems to be that Perron finds it satisfactory if the major breaks in the trends are adequately captured, but Harvey (1997, p.195) considers it desirable to let the trend evolve over time and expresses some reservations on the Perron approach. Consider the following:

"Some econometricians have advocated the acquiring of additional flexibility by introducing breaks, to give a piecewise linear trend. This has the advantage that it can be estimated by regression. The disadvantages are that the break points are assumed to be known and forecast mean square errors do not allow for the possibility of further breaks. The merit of the stochastic trend model is that it will adapt to a break whenever it occurs and the forecast mean square error will reflect the possibility of similar breaks in the future".

While this may correct, the Bai and Perron (2003) and Perron and Qu (2005) tests for endogenous multiple structural breaks seem to have filled these gaps. Therefore, we may say that each of these alternative methodologies may have merits and limitations. In other words they may be appropriate for some data sets and relationships but not for all. For example, it may be difficult to use the stochastic trend approach to the demand for money in a developing economy where monetization is a gradual process. A straight line trend, with a break or two, may explain the data better than a model with an evolving stochastic trend. On the other hand stochastic trends are found to be useful in several energy studies at the Surrey Energy Economics Centre; see for example Dimitropoulos, Hunt and Judge (2004). Therefore, we take the view that it is worth estimating time series models, in spite of some strong reservations by the proponents, with alternative assumptions on modeling the trend.[2] The main aim of this paper is to illustrate these alternative approaches and encourage applied economists to share experiences with both approaches.

The outline of this paper is as follows. Section 2 discusses briefly deterministic and stochastic trends from the perspective of applied economists. Section 3 presents empirical results on two examples and finally in Section 4 the summary and limitations of this paper are stated.

2. Deterministic and Stochastic Trends

Developments in time series econometrics have implications for both economic interpretation and estimation. It is well known that many macroeconomic variables are non-stationary in their levels but stationary in first differences. Ignoring the ongoing controversy on the relative merits of various alternative unit root tests, application of the standard classical methods of estimation to models with non-stationary variables gives spurious summary statistics. Suppose, all the variables in a model are integrated of order one, that is they are stationary in their first differences. In such instances the model may be estimated in the first differences of the variables with the classical methods. However, this approach ignores information on the long run relationship between the levels of these variables, i.e., the long run equilibrium

[2] Harvey has other strong reservations on the mainstream time series work with autoregressive equations, unit root tests and cointegration techniques. His main reservation is that these tests and techniques are unreliable due to their lack power. Towards the end of his paper he says that "…..autoregressions, whether univariate or multivariate, are very limited in scope. The recent emphasis on unit roots, vector autoregressions and cointegration has focused too much attention on tackling uninteresting problems by flawed methods".

relationship implied by the economic theory.[3] In other words such estimates are not useful to test economic theories. In addition, there is a loss of efficiency in the estimated parameters due to the neglect of information on the equilibrium relationship. Cointegration techniques can be seen as methods of estimating time series models to overcome these two limitations.

Since most macro variables are highly trended, deterministic trends are used in unit root tests and in the estimation of the models with cointegration techniques. The implication of allowing for deterministic trend is that if the model is shocked, after some departures from the trend, the variables would return to their trend values. Cointegration techniques ensure this by estimating the model so that the residuals are stationary. Therefore, shocks have no permanent effects on the trend in the equilibrium relationships. This observation needs further explanation because large shocks like the oil shocks of the 1970s and shocks due to the current information and communications technology would have some permanent effects on these trends. Therefore, the assumption that shocks have no effects on the trend seems inappropriate at all times. But such large shocks are not frequent. The assumption that the parameters of trend remain unchanged can be justified only when these shocks are small and infrequent.

In contrast, shocks due to changes in the so called deep parameters of the system, such as changes in tastes, time preference rates and technology parameters etc., are perhaps likely to be small and frequent. These changes may or may not have significant effects on the trends over short spans of time but their cumulative effects may be significant and cannot be ignored. Therefore, there is also a strong

[3] Before time series econometrics made an impact, economists and econometricians at the London School of Economics (LSE) were aware that economic theory is silent on the dynamics of the relationships. Some of these academics were Phillips (Phillips curve fame), Sargan, Lipsey, Mizon and Hendry. They were unsatisfied with the then popular partial adjustment model of dynamics and the general to specific approach (GETS) was developed as an alternative. Negative feedbacks were introduced through Phillips's error correction mechanism (ECM) into GETS. GETS takes the view that dynamics is an empirical issue and through various procedures it is possible to reduce a very general and long dynamic structure into parsimonious specifications. Later developments in the cointegration techniques have used GETS approach to estimate the short run dynamic equations. Hendry and Mizon are the most well known contemporary proponents of GETS.

justification for using stochastic trends which generally show that the trend is an evolving processes over a period of time.

Models with stochastic trends i.e., structural time series models are useful in some instances. Firstly, it may be hard to identify multiple structural breakes in the deterministic trend when the sample size is small. Secondly, implementing endogenous multiple structural break tests is a demanding exercise. In contrast, estimation of structural time series models is relatively easy with a special software STAMP 7, developed by Koopman, Harvey, Doornik and Sheppard (2006). Thirdly, in structural time series models, standard classical methods of estimation can be used to estimate the effects of additional explanatory variables. Finally, Harvey points out that stochastic and deterministic trend hypotheses are nested within the structural time series approach and can be evaluated with the estimated values of the hyper parameters, although the power of these tests is not known. If the variances of the disturbances of the level and slope of trend are zero (known as hyper parameters), the structural time series model implies that a deterministic trend is preferable to a stochastic trend. In light of these observations, it hard to say which approach is better and our view is that both methods are worth using, especially to keep up with further refinements and developments in both approaches.

3. Estimation Techniques and Software

For unit root tests and to estimate cointegrating equations we have used Microfit 4.1. For estimating the structural time series model we have used STAMP 7 of Koopman et. al. (2006). The data used for illustration in both approaches are from an example of STAMP 7 on the consumption of spirits in the Great Britain from 1870 to 1938. We shall also estimate the production function for Singapore because deterministic trend is frequently used to measure the rate of technical progress. The variables in the demand for spirits are the logarithms of consumption of spirits, income and the relative price of spirits. For unit roots tests the simple ADF test is used, but the results are not reported to conserve space. The GETS procedure of Hendry with the non-linear least squares is used for estimating the cointegrating relation.[4]

We have allowed for shifts in the deterministic trend based on the plots of the coefficients from the rolling least squares estimates in Microfit. We have also used the more demanding Bai and Perron (2003) tests to identify multiple endogenous structural breaks in the trend. We first report the results based the traditional unit roots and cointegration approach and then the estimates with the structural time series technique.

3. 1 Estimates with the Deterministic Trend

Instead of estimating demand for spirits with a constant deterministic trend, we first estimated the following equation, with the rolling least squares, to examine the behaviour of the trend.

$$\ln S_t = a + b\,t \qquad (1)$$

where S is demand for spirits and t is time. The window size is 25 and the period of estimation is from 1870 to 1938. Definitions of the variables are in the appendix. The plot of the intercept did not show much variation and the plot of the estimates of the slope is in Figure 1. The slope reached a maximum around 1890-1891 and since then declined until 1935. For the remaining three years it showed a small increase. Thus the recursive least square plot indicates one strong break in slope around 1890-1891 and then a mild break from 1935. However, inspection of Figure 1 suggests that a nonlinear cubic trend may also be appropriate. Therefore, when we estimate the cointegrating equations we shall use both linear and cubic trends without any breaks. Next we allow for one break during 1890-1891. A second break during 1935-1936 is not estimated because the post break regime would have only 3 observations.

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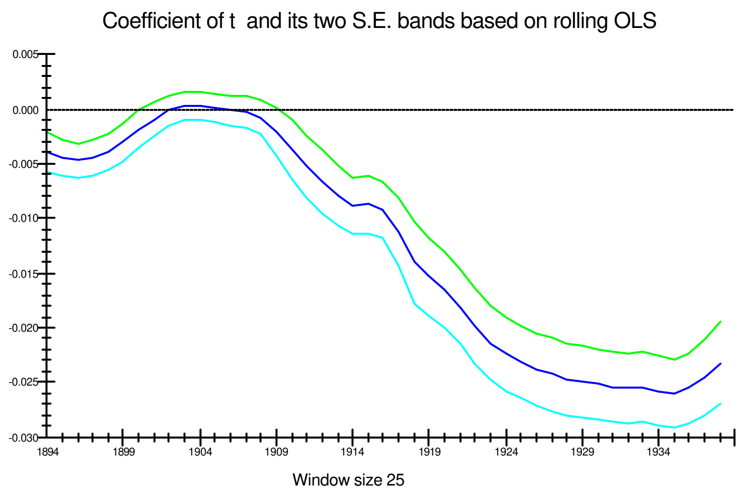
[4] Estimation of cointegrating equations with GETS is explained in Rao (2007). For the cointegration the Ericsson and MacKinnon (2002) response surface test is used.

Figure 1



We have also used the Bai and Perron (2003) structural break tests first by allowing for only one dominant break in both the level and slope of trend. It is found that this break date is 1916. Next we allowed for two break dates and these are found to be in 1898 and 1916. These estimates of breaks in the level and slope parameters with the Bai-Perron routine are summarised in Table 1 below.

We tested for unit roots in the variables viz., consumption of spirits, income and relative price of spirits and found that they are I(1) in levels and I(0) in first differences.[5] Therefore, we have used GETS to estimate the short and long run relationship between the logs of consumption of spirits (S), income (Y) and relative prices of spirits (P). Estimates are given in Table 2. The assumed long run equilibrium relation is:

$$\ln S = a + bt + \alpha \ln Y + \beta \ln P \qquad (2)$$

Equation (I) in Table 2 is an estimate with a deterministic linear trend and equation (II) is with the cubic trend. Equation (III) allows for a break in the slope of trend in

[5] To conserve space these test results are not reported and may be obtained from the author.

Table 1

Break Dates and Implied Trend Parameters

	Number of Breaks	Break date	Estimate of Level	Estimate of Slope
Recursive Lest Squares	1	1890-1891	--	--
Bai-Perron Tests	1	1916	Level-1: 2.052 [0.00] Level-2: 2.494 [0.00]	Slope-1: -0.005 [0.00] Slope-2: -0.019 [0.00]
Bai-Perron Tests	2	1898 1916	Level-1: 2.026 [0.00] Level-2: 2.427 [0.00] Level-2: 2.494 [0.00]	Slope-1: -0.003 [0.00] Slope-2: -0.014 [0.00] Slope-3: -0.019 [0.00]
STAMP	1	1909	--	--

Notes: Figures in square brackets are *p*-values. Level-1 etc stands for the first regime etc., implied by the break dates. Level and slope are the intercept and slope of trend.

1891, based on the plot in Figure 1and equation (IV) allows for a breake in the level and slope of the trend based on the Bai-Perron test with the assumption that there is only one major break in 1916. We are not reporting the estimates with two break dates in 1898 and 1916 found by the Bai-Perron test because the results were poor. The level parameters for the three implied regimes (1873-1897, 1898-1915 and 1916-1938) by these two breaks were highly insignificant. The slope parameters for the first and third regimes were also insignificant. Therefore, we have searched again with the Bai-Perron tests only for changes in the slope parameter and found that the break dates are 1882 and 1908. When the equation is estimated by increasing and decreasing the break dates, around these two dates by one year, breaks in 1882 and 1909 gave good results. However, the slope of trend was insignificant in the second regime (1982-1908), but the estimated income and price elasticities were very close at 0.89043 and -0. 87384 respectively. To improve efficiency we reestimated this equation with the constraint that these two elasticities are equal but opposite in sign and the results are in equation (V) of Table 2. This is an important equation in that, unlike in the earlier equations, the income and price elasticities are close whereas in the previous estimates income elasticity is significantly higher tha n the absolute value of price elasticity. Furthermore, the Wald test showed that the slopes of trend, in each

Table 2

Estimates with Deterministic Trend

1870-1938

	I	II	III One Break 1916 (shift in slope)	IV One Break 1916 (shift in level & slope)	V Two Breaks 1882 & 1909 (shifts in slope)
C_1	0.160 [0.87]	-0.695 [0.49]	0.272 [0.821]	0.078 [0.910]	2.176 [0.00]
C_2	–	–	--	0.517 [0.466]	
C_3	–	–	--	--	
T_1	-0.016 [0.00]	-0.0330 [0.01]	--	-0.015 [0.00]	-0.002 [0.00]
T^2	–	$0.512E^{-3}$ [0.01]	--		
T^3	–	$-0.492E^{-5}$ [0.00]	--		
T_2	–	–	-0.015 [0.01]	-0.023 [0.00]	-0.006 [0.00]
T_3	–	–	-0.015 [0.00]	--	-0.011 [0.00]
λ	-0.226 (4.881)	-0.255 (4.551)	-0.227 (4.871)	-0.308 (4.567)	.32296 (4.20)
ln y	1.8137 [0.00]	2.168 [0.00]	1.762 [0.00]	1.689 [0.00]	0.876 [0.00]
ln P	-.674 [0.00]	-0.500 [0.00]	-.688 [0.00]	-0.516 [0.00]	-0.876 (constrained)
Ardls?					YES
$\bar{R}^2$	0.762	0.772	0.756	0.767	0.789
SEE	0.019	0.019	0.020	0.019	0.018
DW	2.246	2.329	2.246	2.381	2.2517
χ_{SC}^2	1.659 [0.198]	2.8172 [0.093]	1.679 [0.195]	4.593 [0.032]*	2.0509 [.152]*
χ_n^2					7.4198 [.024]

Notes: Figures in square brackets are *p*-values and figures in the parentheses are t-values.

regime, are significantly different. Finally, the χ^2 test for normality in the residuals in this equation, although significant at 5% level, is insignificant at the 1% level. In the other equations it is highly significant even at the 1% level.

The Ericsson and MacKinnon (2002) cointegration test (based on the surface response function) indicated that in all the equations of Table 2, the *t*-ratios of the adjustment parameter

λ are higher than the critical value $\kappa_{cT}(3) =$ -3.992 at the 5% level implying that the 3 variables in the ECM are cointegrated and all the five equations.[6] A choice between these five equation is necessary because the first four equations imply that income elasticity is two to three times higher that the absolute value of price elasticity where as equation (V) implies they are about equal. Although the $\bar{R}^2$ and SEEs of these five equations are close, equation (V) is preferred because the residuals in the other equations badly fail the normality test. Furthermore, the level coefficient of trend, which is important here, is significant only in equation (V). Therefore, we prefer equation (V). Among other things this equation has an important implication and supports the reservations of both Harvey and Perron on the limitations of the bulk of the time series empirical work which often ignore structural breaks in the trend. Equation (V) implies that ignoring structural breaks may yield biased estimates of the coefficients of the long run relationships. Consequently, it may be said that main methodological difference between Perron and Harvey now is whether it is adequate to allow for changes in the level and/or slope of a deterministic trend or one should use evolving stochastic trends. Therefore, it is necessary now to know how structural time series estimates perform. Nevertheless, it may also be said that Perron is partly justified in saying that determinist trends with structural breaks seem to be adequate.[7]

3. 2 Estimates with Stochastic Trend

In models with stochastic trends the following specifications are commonly used. To illustrate this we shall use the stochastic trend variant of equation (2) where the trend was deterministic. We need here time subscripts, an error term and a stochastic trend μ and its evolution. With these modifications the demand for spirits can be expressed as:

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[6] Strictly speaking estimating cointegrating equations with structural breaks and shift dummies is not satisfactory procedure. However, there is a gap in the literature here. There are only two known procedures to estimate cointegrating equations with structural breaks. Firstly, Gregory and Hansen (1992) developed a method where the two step Engle-Granger equation can be estimated with one endogenous structural break. Secondly, Juselius (1996) developed a procedure to estimate cointegrating equations with a known break date with the Johansen method.

[7] Needless to say this conclusion based only on a sample of one experience has many caveats and further experiments are necessary to draw conclusions with more confidence.

$$\ln S_t = \mu_t + \alpha \ \ln Y_t + \beta \ \ln P_t + \varepsilon_t \qquad (3)$$
$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \nu_t \qquad \nu_t \sim N(0, \sigma_\nu^2) \qquad (4)$$
$$\beta_t = \beta_{t-1} + \xi_t \qquad \xi_t \sim N(0, \sigma_\xi^2) \qquad (5)$$

Equation (4) says that the stochastic trend level (μ) and slope (β) parameters and the evolution of the trend parameters are expressed as first order difference equations augmented with two stochastic error terms. The solution to equation (5), for example, is

$$\beta_t = \beta_0 + \sum_{i=1}^{t} \xi_i \qquad (6)$$

where β_0 is the initial value of this parameter.

The important point to note is that random shocks have a permanent effects on the slope parameter. A similar interpretation can be given to equation (4) where random shocks have permanent effects on the level of trend. Note that both parameters evolve over time and capture the cumulative effects of the two random shocks ν and ξ. If the variances of these error terms are zero i.e., $\sigma_\nu = 0$ and $\sigma_\xi = 0$ they are no more stochastic shocks and the trend becomes deterministic.[8]

Models with variables, detrended in this way, can said to be structural time series models. STAMP is a specialized software to estimate them with an option to estimate conventional models with a deterministic trend. Equation (VI) in Table 3 is estimated with the deterministic trend and the maximum likelihood method in STAMP. The estimates of income and price elasticities are qualitatively similar to those in Table 2. However, the low DW statistic makes the other summary statistics unreliable. Equation (VII) is estimated with the stochastic trend and shows significant improvement over equation (VI). It is comparable to the estimate of equation (I) with the deterministic trend in Table 2. Particularly noteworthy features are significant improvements in $\bar{R}^2$ and DW statistics. Estimates of the income and price elasticities

[8] A proof of this is beyond the scope of this paper.

are also different now and the absolute value of the latter is higher than the former. However, the χ^2 statistic for normality of the residuals is too high and just insignificant only at the 0.5% level.

Table 3

Estimates with STAMP Period: 1870-1938

Dependent Variable *ln S*

	VI FIML Estimates Deterministic Trend (No AR Variables)	VII Stochastic Trend	VIII* One Break 1909	IX Two Breaks 1882 & 1909	
C	1.199 (2.69)	8.882 (2.91) $\sigma^2 = 0.6E^{-4}$	10.1367 (3.42) $\sigma^2 = 0.6E^{-4}$	9.8244 (3.30) $\sigma^2 = 0.6E^{-4}$	
T	-0.009 (7.88)	-0.012 (1.25)* $\sigma^2 = 0.3E^{-4}$	0.030 (3.31) $\sigma^2 = 0.3E^{-4}$	-1.19862 (1.34)* $\sigma^2 = 0.3E^{-4}$	
DUM 1882	–	–	--	-0.006 [0.65]	
DUM 1909	–	–	-0.064 [0.00]	-0.064 [0.00]	
ln y	1.062 [0.00]	0.695 [0.00]	0.678 [0.00]	0.678 [0.00]	
ln P	-0.860 [0.00]	-0.950 [0.00]	-0.994 [0.00]	-0.995 [0.00]	
ARDLs	NO	NO	NO	NO	
$\bar{R}^2$	-0.135 (First Differences)	0.706 (First Differences)	0.788 (First Differences)	0.789 (First Differences)	
SEE	0.042	0.021	0.018	0.018	
DW	0.263	2.076	2.101	2.088	
$\chi^2_{SC(1)}$	0.816	-0.046	-0.061	-0.055	
$\chi^2_{n(2)}$	0.266	10.461	1.602	1.490	

Notes: Figures in square brackets are *p*-values and figures in the parentheses are t-values.

In equations (VIII) and (IX) intervention (impulse) dummies are introduced for breaks in the stochastic trend. Equation (VIII) is estimated with a one time break in 1909 and equation (IX) is estimated with two breaks in 1882 and 1909. There is support for these break dates from the Bai-Perron tests. In the STAMP manual example, 1909 was used as a possible break date. Introduction of these two sets of break dates did not have any significant effect on the estimated elasticities and the normality test static for the distribution of residuals is insignificant now at the 5% level. However, the dummy for 1882 shift is insignificant in equation (IX). Consequently, equation (VIII) with a shift in trend in 1909 is the preferred equation.

Some other noteworthy features of these estimates are as follows: both the hyper parameters are non zero and the Wald test did not reject the null that the estimated income elasticity in equations (V) and (VIII) are equal. Although the Wald test rejected the null that the price elasticities are equal, the null was not rejected when the price elasticity in equation (VIII) was lowered by half of its standard error. From these observations it may be said that both alternative methodologies of Perron and Harvey have yielded very close results. Therefore, on this basis it can also be said that when properly used it is hard to say one method is better than the other.

In another experiment, we estimated a Cobb-Douglas (CD) production function for Singapore. CD production functions, with constant returns, the Hicks neutral technical progress and a deterministic trend are widely used in the growth models to capture the rate of growth of technical progress with constant level and slope parameters for the trend. The results with GETS are in Table 4 as equations (X) and (XI). Equation (X) is estimated without any structural breaks in the level and slope of trend. The summary statistics of this equation indicate that there is some first order sereal correlation at the 5% but not at the 1% level. Estimates of the profit share at 0.24 is plausible and the Ericsson and MacKinnon indicates that there is cointegration. The t-ratio of λ exceeds the critical value at the 5% level of $\kappa_{CT}(2) = 3.891$. We have used the Bai-Perron structural break tests to search for a major structural break in the level of slope of trend and found 1994 as a break date. The level parameter decreased marginally and the slope parameter significantly. Therefore, we estimated equation (X) allowing for the structural breaks in 1994 and a year before and after this break date. Good results are obtained with 1993 as the break date and the estimates are in Table 4 as equation (XI). This equation is a significant improvement over (X) and shows the need for structural breaks tests in even equations with a deterministic trend. Its $\bar{\bar{R}}^2$ has increased significantly and the first order serial correlation is insignificant. The rate of growth of technical progress has

Table 4

Production Function for Singapore (1970-2005)

	Deterministic Trend (X)	Deterministic Trend with break in 1993 (XI)	StochasticTrend (XII)	Stochastic Trend (XIII)
C_1	7.188 [0.00]	8.776 [0.00]	8.260 [0.00] $\sigma^2 = 0.454^{-3}$	7.793 [0.00] $\sigma^2 = 0.440^{-3}$
C_2	–	8.3879 [0.00]	--	
T_1	0.031 [0.00]	0.029975 [0.00]	0.027 [0.00] $\sigma^2 = 0.284\text{e-}005$	0.026 [0.00] $\sigma^2 = 0.275\text{e-}5$
T_2	–	0.018 [0.01]	--	
λ	-0.294 (4.05)	-0.496 (3.96)	--	
k	0.243 [0.12]	0.178 [0.01]	0.236 [0.03]	0.23468 [0.03]
DUM1998	–	--	--	-0.040 [0.17]
ARDLs	YES	YES	NO	NO
$\bar{R}^2$	.29553	.50978	-0.044 (First Differences)	0.20901 (First Differences)
SEE	.028604	.023861	0.035	0.030565
DW	1.4939	1.8778	1.240	1.6805
$\chi^2_{SC(1)}$	4.7919 [0.029]*	.067402 [0.795]	0.36973	0.13288
$\chi^2_{n(2)}$	.19810 [.906]	36075 [.835]	0.67223	0.70107

Notes: Figures in square brackets are *p*-values and figures in the parentheses are t-values. Subscripts for intercept and trend correspond to the regimes.

declined from nearly 3% before 1993 to 1.8% afterwards.[9] The share of profits has also declined from 24 % to 18%, perhaps due to the slowdown of the economy due to the decline in the rate of growth of productivity.

Equations (XII) and (XIII) are estimated with stochastic trend. In equation (XII) there is no intervention dummy. Although the chi-square statistics for first order serial correlation and normality of residuals are insignificant, the $\bar{R}^2$ is poor. The estimated share of profits at 0.23 is close to that of equation (X) with the deterministic trend and without structural breaks. When the stochastic trend specification is estimated with an intervention dummy for 1993and 1994, the results were poor. When intervention dummies for the East Asian financial crisis in 1997 and 1998 were used, only the 1998 dummy had an expected negative coefficient but it is significant only at the 17% level of confidence. A noteworthy feature of equation (XIII) is that the share of profits remained almost the same as in equation (XII). On the whole the stochastic trend specification does not seem to have performed well in this example compared to Perron's method of allowing for structural breaks with deterministic trend. This can be explained as follows. While technological inventions might be stochastic , firms are likely to adopt improved technologies only gradually over a period. Therefore, there is some justification for deterministic trends in production functions and growth models.

4. Summary and Conclusions

This paper considered two major alternative methodologies of modeling with time series data. The mainstream approach, based on the unit root tests and estimating cointegrating equations uses deterministic trends. In contrast, the alternative approach of Harvey suggests that trend should be treated as a stochastic variable because of the effects of several unobservable shocks to the economy. In between these two

[9] The decline in the growth rate seems to have started earlier than the 1997 East Asian financial crisis. The Hodrick-Prescott filtered growth rate also showed 1993 as the beginning of the downturn in the growth rate of Singapore. When 1997 or 1998 was selected as plausible break dates the estimated equations were unsatisfactory in that the share of profits was close to zero and insignificant. The reasons why Singaporean economy started slowing down from 1993 are beyond the scope of this paper.

methodologies there is another view suggested by Perron that it is desirable to test for major breaks in the parameters of the deterministic trends.

This paper has recommended that it is desirable to use all the three approaches because they may perform differently in different models and data sets. However, this conclusion is based on a limited experience of two applications. In the demand for spirits both Perron and Harvey's approaches performed well but the Perron approach performed better in the production function. This conclusion is also consistent with Harvey's observation that there should be adequate justification for deterministic trend with breaks. We suggested that while inventions may be stochastic, firms are likely to use new technologies gradually.

The Harvey stochastic trend approach has a few advantages. It is relatively easy to use because it saves time in models based on the conventional approach to search for the autoregressive distributed lag terms. It is also useful for forecasting with univariate models and most importantly it helps to decide whether the Perron structural break tests need to be used in models with deterministic trends. We hope that this paper would encourage others to experiment with other examples and share their experiences.

Data Appendix

STAMP Example

S = is demand for spirits

Y = real income

P the ratio of prices of spirits to CPI.

Source: STAMP examples.

Production Function for Singapore

y = Real GDP per worker

k = capital per worker

Source: Real GDP in national currency is downloaded from the UN database available at http://unstats.un.org/unsd/snaama/selectionbasicFast.asp; Capital stock is estimated with the perpetual inventory method from real investment downloaded from the UN database. Depreciation rate was assumed to be 5% and the initial capital stock was estimated with the assumption that the capital-output ratio is 1.5. Employment data was downloaded from the IMF CD Rom 2006.

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