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The studies of the International Monetary Fund offer a model for recommending sustainable budget policy to oil-exporting countries including Russia. The model does not contain any resource as a factor of production and assumes that Russian oil reserves will be exhausted by the middle of the 21st century. The current paper examines the sustainability of open and closed models, which are calibrated on Russia's data and include a resource as a factor of production. The open-model case shows that monotonic economic growth is impossible given the current state of the Russian economy. This paper offers an approach for estimating changes that improve long-term sustainability.

Keywords: **nonrenewable resource, weak sustainability, open imperfect economy, Russian oil extraction.**

JEL classification: O13, Q32, Q38

1. Introduction

The famous first report to the Club of Rome (Meadows et al. 1972) initiated the second – after the works of T. Malthus – wave of interest to the problems of the dependence of economic growth on natural resources. The report declared that the continuation of the exponential growth of population, resource extraction, and pollution of the environment can result in a global social-economic disaster during the current century.

Among the publications that followed the report, the works of Dasgupta, Heal (1974), Solow (1974) and Stiglitz (1974) take a special place. These works offer a model (DHSS), based on the Cobb-Douglas production function, that contains, in addition to labor and capital, a nonrenewable resource as a necessary[1] factor of production. As the authors of the DHSS model show, the Cobb-Douglas function with the unity elasticity of substitution between factors is the only function from the family of functions with the constant elasticity of substitution that reflects the uncertainty of the problem of extraction of a nonrenewable resource under the requirement of sustainable[2]

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[1] According to some authors (for example, van der Ploeg 2011), a resource is *necessary* for production if output is zero in the absence of the resource and, in the presence of the resource and other necessary factors, output is positive; a resource is *essential* if consumption goes to zero when the resource flow goes to zero. Hence, a resource can be *necessary* for production *but not essential* if output is zero in the absence of the resource, and there is a feasible economic program along which consumption is bounded away from zero while the flow of the resource goes to zero always remaining positive. Dasgupta and Heal (1974) defined a resource as essential for production if output is zero in the absence of the resource.

[2] According to the definition offered in WCED (1987), "sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs." Pezzey (1992) defined sustainable economic growth (development) as nondeclining consumption (utility). As a specific indicator of social progress, UN (2010) offers the Human Development Index, which is an alternative to the traditional measure of social well-being in terms of GDP per capita. The reviews on the evolution of the notion of SD are in Pezzey (1992), Appendix 1, and in Hammond (1993). The idea of SD is attractive due to its consistency with the theories of justice requiring nondeclining quality of life (see, for example, a review in Asheim 2010).

development (SD). Functions with the elasticity less then one predetermine a pessimistic outcome: regardless of investment or resource policy, production and consumption decline to zero with the exhaustion of the resource. Although, in the real economy, increasing efficiency in the use of resources and the development of renewable sources of energy give hope that this dismal result can be avoided. If the elasticity is greater than one, the model economy can grow without resources. This is also not obvious in the real world, because it is still unknown if it is possible to substitute completely and adequately all the production processes that use nonrenewable resources by the technologies that use only renewables; and, if that is possible, how long and costly this substitution will be.

According to some empirical studies, the elasticity of substitution between energy resources and capital exceeds unity, whereas other works show that energy and capital are complements rather than substitutes (the elasticity is less than one). There are publications that show that the elasticity is very close to unity. These results are discussed in more detail in, for example, the review of Neumayer (2000), Section 4. Hence, from the point of view of empirical estimates, the use of the Cobb-Douglas function with a resource as a factor of production is not implausible.

The DHSS model allows for non-decreasing consumption during the infinite period of time under the condition that the rates of extraction of a nonrenewable resource are decreasing in the long run remaining positive, and capital is growing, substituting for the shrinking resource. This substitution can practically mean that more expensive (per unit of energy) capital that uses a renewable resource, such as ethanol, substitutes for cheaper capital that uses a nonrenewable resource, such as oil. For simplicity, renewable resources, as a rule, do not enter this model, because the problem of limitedness of reserves for this kind of resource is not as acute as for nonrenewable ones.

The conception that assumes that natural capital can be substituted by a man-made one is called in the literature *the weak form of SD*. The followers of *the strong form of SD,* for example, N. Georgescu-Roegen and H. Daly, criticize the assumptions of the weak form for being too optimistic. The strong form of SD assumes that natural resources and capital can only complement each other in production; in other words, that the elasticity of substitution between them is zero. Intermediate conceptions of SD claim that a part of the reserve of a nonrenewable resource must be kept intact, and that the reserve of a renewable resource must be maintained at a constant level per capita. A dispute among the proponents of various forms of SD is published in *Ecological Economics*, 22(3), 1997. A detailed discussion of the forms of SD can be found, for example, in Neumayer (1999).

A large body of research in resource economics suffices to show the importance of the resource extraction policy for maintaining non-decreasing indicators of social well-being. However, many economic schools do not offer courses in environmental and natural resource economics, or these courses are not mandatory. Meanwhile, conventional theories of economic growth still do not consider natural resources as production factors.[3] As a result, some studies, connected with extraction of natural resources, assume an infinite elasticity of substitution between a resource and capital.[4] For example, Jafarov et al. (2006) offered recommendations for constructing a sustainable budget[5] policy in Russia on the assumption that Russian oil reserves will be exhausted by 2048.[6]

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[3] See, for example, Barro, Sala-i-Martin (2003).

[4] In other words, that the resource and capital are perfect substitutes.

[5] A sustainable budget in Jafarov et al. (2006) is the one that maintains nondeclining per capita consumption during a long period of time.

[6] The authors estimated the time of exhaustion of oil reserves given the total estimate of proven, probable, and possible reserves (149,3 trillion barrels in 2004) and an exogenous scenario, where the rates of oil extraction gradually increase in 2006-2011 and reach four percent a year in 2011-2012, which is followed by gradual deceleration of growth and then by a decline in the rates of extraction" (Jafarov et al. 2006, p. 43). The most quoted sources for scenarios of oil extraction

The recommendations are based on numerical estimates with the use of a neoclassical model of economic growth specified in Barnett, Ossowski (2003) for oil-exporting countries. The resource reserves in this model are considered a part of financial assets: if these assets are being consumed, the country's wealth is declining, but, if the resource rent is being invested, the wealth remains constant and only the portfolio structure is changing (Jafarov et al. 2006, p. 5).

The aggregate budget constraint in this model is

$$C(t) + \dot{K}(t) \leq F_I[A(t), K(t), L(t)] + p(t)R(t) + iK_W(t), \qquad (1)$$

where $C(t)$ – the aggregate government and private consumption at the time t;

$\dot{K}(t)$ – investment into government and private "non-oil" capital;

$R(t)$ – the rate of oil extraction (all oil is being exported);

$p(t)$ – the export price of oil;[7]

K_W – the government holdings in the world's financial assets;

i – the rate of return on K_W (constant);

$F_I[A(t), K(t), L(t)]$ – domestic "non-oil" production function, where

F_I – Cobb-Douglas function;

$A(t)$ – the level of technology, exogenously growing at a constant rate;

$K(t)$ – government and private "non-oil" capital;

$L(t)$ – labor.

Sustainable long-run consumption $C(t)$ is maintained by investment into non-oil capital $K(t)$ and foreign financial assets K_W, while oil revenue $p(t)R(t)$ declines to zero in final time. The scenario that provides the maximum level of constant consumption[8] after exhaustion of fossil fuels is chosen from four alternative budget policies.

The problems of long-term forecasting and planning, connected with the exhaustion of nonrenewable resources, are usually solved under uncertainty in the resource reserves and in the rate of technical progress. The latter can be expressed, in particular, in the value (behavior) of the elasticity of substitution between the resource and capital and in the behavior of total factor productivity (TFP). The errors in planning a budget or a resource policy caused by the uncertainty in future production possibilities can be divided into two main types:

1) future possibilities are overestimated; the resource is overextracted in the short run with possible collapse of the economy (for example, Brander, Taylor, 1998);

2) future possibilities are underestimated; the resource is underextracted in the short run, causing inefficiency in the economy (the level of utility is lower than it could be under precise forecasting).

For sustainable development, the second type of error is obviously preferable because the error of the first type is irreversible due to nonrenewability of the resource. For the second type, the policy can be corrected (the rates of extraction can be increased) after knowledge is updated. As a

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(IEA: www.iea.org, EIA: www.eia.gov, and OPEC: www.opec.org) consider also a scenario of low prices, which is connected with a possible fast development of renewable energy. This most favourable scenario for SD assumes that the world demand for oil will essentially decrease before the exhaustion of Russian reserves. The critics of scenarios of IEA and EIA can be found, for example, in Jakobsson et al. (2009).

[7] The export price of oil in 2006-2011 is based on the forecasts of the *World Economic Outlook* (www.imf.org). Starting from 2012, the price is constant in real terms with respect to long-term consumer price inflation (two percent a year) in the developed countries (Jafarov et al. 2006, p. 44).

[8] Constant-consumption criterion is a convenient instrument for quantitative comparison of scenarios. In a two-factor model, a scenario with a constant consumption can be transformed into a scenario of growth by increasing investment at the initial moment and (or) by redistribution of the resource in favour of future generations.

result, the economy can be sustainable and asymptotically efficient. In this connection, at least three assumptions of model (1) might prove to be too optimistic.

1. The elasticity of substitution between the resource and capital can be lower than assumed. Model (1) assumes that oil and financial assets are perfect substitutes. In fact, oil is a necessary factor of production; in other words, $F_I(t)$ depends on $R(t),$ and it is uncertain, so far, if renewable resources can completely replace oil so that the level of utility can be kept the same. Hence, the level of domestic production $F_I(t)$ can decline when the rates of extraction are decreasing.

2. The pace of technical progress, as known, is irregular, and undeniable advances of the past do not guarantee high rates of development in the future (for example, Brander 2010). Moreover, the development of science is not always followed by the growth of TFP. Sometimes, TFP can even decline (for example, Lipsey, Carlaw 2004). Therefore, $A(t)$ can grow more slowly than the exponential function, not compensating for the declining rates of extraction.

3. The rate of return on the world's financial assets can decline. There are known historical tendencies to declining interest rates (Homer, Sylla 1996). At present, the rates of return in developed countries (for example, in Japan) are very close to zero. The assumptions of neoclassical models in resource economics also result in the marginal productivity of capital and, correspondingly, the rate of return asymptotically declining to zero. This plausible scenario implies that, for maintaining constant utility (consumption) at the expense of the interest from assets abroad, it will be necessary to increase these assets, which might prove to be impossible due to decreasing rates of resource extraction and shrinking domestic production.

The current paper uses well-known models, which include a resource as a production factor, to examine the sustainability of the Russian economy with respect to oil-extracting policy. This paper offers the notions of potential sustainability and survivability of a resource-based model and finds necessary and sufficient conditions for the potential sustainability and survivability depending on the economy's initial state, using the example of the Cobb-Douglas production function with a resource as a production factor (Section 4). The condition of potential sustainability, obtained for an imperfect economy, can be used as an indicator (Definition 7), the positive value of which guarantees the existence of an economic program with non-decreasing utility during an infinite period of time. This indicator includes the Hartwick rule (Hartwick 1977) as a particular case, when the economy's initial state satisfies a perfection condition with respect to a constant-consumption criterion (Bazhanov 2010).

The theoretical results of the paper are illustrated by numerical examples where oil is considered as a resource and the models are calibrated on data from the Russian economy (Section 5). According to the conditions of potential sustainability, the closed model turns out to be potentially sustainable. However, the use of the open model, similar to model (1), reveals that there is no economic program with sustainable domestic production. A ban on oil exports, in this case, does not solve the problem, since the main cause of unsustainability is a relatively low growth rate (for smaller capital) for domestic output. Hence, a simpler closed model may be not adequate for a study of potential sustainability of an open economy.[9]

The numerical analysis shows also that potential sustainability of the open model of the Russian economy can be increased, in particular, by a more thrifty resource policy. In reality, the resource policy should be even more conservative due to some simplifications in the models. For example, following the assumptions of model (1), this paper ignores the damages from oil use to utility and production. As is known from the literature,[10] when a social planner takes into account

[9] In some cases, however, the use of a closed model can result in the same conclusions as the use of an open one (for example, for qualitative comparison of dynamic scenarios (Bazhanov, Belyaev 2009)).

[10] See, for example, a review in Bazhanov (2011).

these damages, the optimal rate of extraction is lower than is prescribed by a theory that neglects these effects.

2. Dasgupta-Heal-Solow-Stiglitz (DHSS) model

This paper deals only with the weak form of sustainability, which assumes that a nonrenewable resource (oil) can be (maybe in the remote future) replaced by capital that uses only renewable resources (wind, sunlight, etc.). The minimum requirement of the weak form of SD is at least constant per capita utility in the long run, meaning by the long run an infinite period of time.[11]

In the study of sustainability, it is natural to require that the model must allow both for sustainable and unsustainable outcomes. As mentioned above, the simplest model, satisfying this requirement, can be based on the Cobb-Douglas production function with a resource as a factor:

$$F(t) = A(t)K(t)^{\alpha} R(t)^{\beta} L(t)^{\gamma}, \qquad (2)$$

where $\alpha, \beta, \gamma \in (0,1); \alpha + \beta + \gamma = 1,$ F – GDP, A – the scale multiplier (TFP), K – the stock of capital, R – the rate of resource extraction, L – labor, which is constant and equal to population.[12]

The DHSS model has been widely used in the studies on sustainability of economies with a nonrenewable resource mostly for analysis of the role of investment in the long-term behavior of a social well-being indicator, for example, in the following works: Hartwick (1977), Dasgupta, Heal (1979), Pezzey, Withagen (1998), Stollery (1998), Asheim et al. (2003), Buchholz et al. (2005), Asheim (2005), Hamilton, Hartwick (2005), Hamilton, Withagen (2007), Bazhanov (2010) and (2011). The Cobb-Douglas production function was combined with various assumptions about population growth (exponential – Stiglitz (1974) and Takayama (1980); quasiarithmetic – Mitra (1983) and Asheim et al. (2007)) and about the form of technical progress (exogenous exponential growth of TFP – Stiglitz (1974), Suzuki (1976), and Solow (1986); endogenous resource augmenting – Takayama (1980); exogenous quasiarithmetic – Pezzey (2004a)[13] and Asheim et al. (2007); compensating for capital decay – Bazhanov (2010) and (2011)). Function (2) is also being used in applied studies, for example, in the Integrated Assessment Models of climate change (for example, Nordhaus, Boyer, 2000).

After dividing both sides of equation (2) by $L,$ the model takes the form:[14]

$$f = Ak^{\alpha} r^{\beta}, \qquad (3)$$

where the low-case variables denote the values of the correspondent upper-case variables in per capita units. Since the goal of the paper is not connected with the study of knowledge development, the assumption about the rate of technical progress takes a simple form of such a growth of TFP that compensates for capital decay.[15] The rate of investment is constant, namely, $\dot{k}(t) = wf(t),$ $w \in (0,1),$ $w = const,$ and the balance equation is $f = \dot{k} + c,$ where c is per capita consumption.

3. A technique for calibration

The parameters $A, \alpha,$ and β of family of functions (3) are estimated in this study with the

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[11] Solow (1974).

[12] This simplification does not change the results since the ratio of labor to population oscillates usually around a constant.

[13] Pezzey called it «hyperbolic», because he assumed that the growth rate of TFP is inversely proportional to time.

[14] Sometimes, for simplicity, the dependence of variables on time will be omitted.

[15] The assumption implies that TFP is $\tilde{A}(t) = A\left(1 + \mu k^{1-\alpha} r^{-\beta}\right),$ where μ is the rate of capital decay. This TFP is close to a linear function with a small slope (Bazhanov 2009). This assumption is neither extremely optimistic as in the models with exponentially growing TFP, nor extremely pessimistic as in the models with no technical progress.

use of the following system:[16]

$$\frac{\dot{f}_0}{f_0} = \alpha \frac{\dot{k}_0}{k_0} + \beta \frac{\dot{r}_0}{r_0}, \tag{4}$$

$$1-\gamma = \alpha + \beta, \tag{5}$$

where γ is the given share of labor in GDP. The unique solution of system (4), (5), which has the form

$$\alpha = \left[\frac{\dot{f}_0}{f_0} - (1-\gamma)\frac{\dot{r}_0}{r_0}\right] \Bigg/ \left[\frac{\dot{k}_0}{k_0} - \frac{\dot{r}_0}{r_0}\right], \tag{6}$$

$$\beta = \left[(1-\gamma)\frac{\dot{k}_0}{k_0} - \frac{\dot{f}_0}{f_0}\right] \Bigg/ \left[\frac{\dot{k}_0}{k_0} - \frac{\dot{r}_0}{r_0}\right], \tag{7}$$

exists for $\dot{k}_0/k_0 \neq \dot{r}_0/r_0$. Subscript 0 means that the corresponding value is given at the initial moment t_0. Knowing α and β, the parameter A can be found from equation (3).

The technique of calibration assumes that

- a real economy $\mathbf{E_0}$ is represented at t_0 by the following data: $\mathbf{E_0}=\{\gamma, f_0, \dot{f}_0/f_0, k_0, \dot{k}_0, r_0, \dot{r}_0, s_0\}$, where s_0 is the estimate of all economically valuable reserves of a nonrenewable resource per capita that can be extracted during the period under consideration. As noted before, it is preferable for SD that s_0 be a lower bound of the real reserve.
- $f_0, k_0, r_0, s_0 > 0,$ and the values of f_0 and $\dot{k}_0$ are such that the level of the initial utility is not below a subsistence minimum: $u_0 \geq u_{\min},$ where $u_0 = u(c_0)$ and $c_0 = f_0 - \dot{k}_0.$
- The use of the resource at the initial moment can be both statically and dynamically inefficient, and non-optimal. This assumption contrasts to many theoretical studies with model (3), where the initial state is determined by the stocks of k_0 and $s_0,$ which are used as initial data to define the initial rate of extraction $r_0,$ investment $\dot{k}_0,$ and consequently the rest of the data from $\mathbf{E_0}$ as the solutions of the problem of welfare maximization.

The last assumption allows for including into the model (implicitly) the influence of imperfect institutions, x-inefficiencies, and externalities, which brings the model closer to real life; although there are still some simplifications that are not always true in a real economy. For example, this study assumes that (a) the resource is productive[17] ($\partial f/\partial r > 0$) and (b) the economy is non-wasteful (all output is being used either in consumption or in investment: $f = c + \dot{k}$).[18]

Definition 1. A model is *calibrated on the economy* $\mathbf{E_0}$ *at the moment* t_0 or a model *has the initial state* $\mathbf{E_0}$ if the chosen for calibration values of this model coincide with the correspondent data of economy $\mathbf{E_0}$ at $t = t_0$.

Definition 2. A model, calibrated on the economy $\mathbf{E_0}$, is *feasible for the economy* $\mathbf{E_0}$ if the

[16] Sometimes, expert estimates are used for α and $\beta,$ for example, $\alpha = 0.3,$ $\beta = 0.05$ (Andreeva, Bazhanov 2007) or $\beta = 0.25$ (Bazhanov, Tyukhov 2008; Bazhanov, Belyaev 2009). In this case, the "model" values of capital and the rate of GDP growth can be found from equation (4). This paper does not use expert estimates because they do not allow the use of technique of evaluation of potential sustainability, offered below.

[17] A resource use is *not productive* (*anti-productive*) if, all other variables being fixed, the resource stock is decreasing and GDP is not increasing (decreasing), for example, during wildfires or oil spills.

[18] An economy is *wasteful* if $f > c + \dot{k}$.

parameters of this model exist and take feasible values.

Proposition 1. *Model (3) is feasible for the economy* $\mathbf{E_0}$ *at* $t = t_0$ *if and only if*

1) $f_0 > 0, k_0 > 0, r_0 > 0 \qquad (\Leftrightarrow \qquad A > 0);$ (8)

2) for $\dot{r}_0\left(\dot{k}_0/k_0 - \dot{r}_0/r_0\right) > 0$:

$$1 - \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0} < \gamma < \frac{\dot{k}_0 r_0}{k_0 \dot{r}_0} - \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0} \qquad (\Leftrightarrow \quad \alpha \in (0,1)); \tag{9}$$

for $\dot{r}_0\left(\dot{k}_0/k_0 - \dot{r}_0/r_0\right) < 0$:

$$\frac{\dot{k}_0 r_0}{k_0 \dot{r}_0} - \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0} < \gamma < 1 - \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0} \qquad (\Leftrightarrow \quad \alpha \in (0,1)); \tag{10}$$

3) for $\dot{k}_0\left(\dot{k}_0/k_0 - \dot{r}_0/r_0\right) > 0$:

$$\frac{k_0 \dot{r}_0}{\dot{k}_0 r_0} - \frac{\dot{f}_0 k_0}{f_0 \dot{k}_0} < \gamma < 1 - \frac{\dot{f}_0 k_0}{f_0 \dot{k}_0} \qquad (\Leftrightarrow \quad \beta \in (0,1)); \tag{11}$$

for $\dot{k}_0\left(\dot{k}_0/k_0 - \dot{r}_0/r_0\right) < 0$:

$$1 - \frac{\dot{f}_0 k_0}{f_0 \dot{k}_0} < \gamma < \frac{k_0 \dot{r}_0}{\dot{k}_0 r_0} - \frac{\dot{f}_0 k_0}{f_0 \dot{k}_0} \qquad (\Leftrightarrow \quad \beta \in (0,1)). \tag{12}$$

P r o o f follows directly from formulas (3), (6), (7), and the conditions of feasibility of the parameters. For example, for $\dot{k}_0/k_0 - \dot{r}_0/r_0 < 0$ and $\dot{r}_0 < 0$, the condition $\alpha > 0$ takes the form:

$$\left[\frac{\dot{f}_0}{f_0} - (1-\gamma)\frac{\dot{r}_0}{r_0}\right] \Big/ \left[\frac{\dot{k}_0}{k_0} - \frac{\dot{r}_0}{r_0}\right] > 0 \quad \Leftrightarrow \quad \frac{\dot{f}_0}{f_0} - (1-\gamma)\frac{\dot{r}_0}{r_0} < 0 \quad \Leftrightarrow \quad \frac{\dot{f}_0}{f_0} < (1-\gamma)\frac{\dot{r}_0}{r_0} \quad \Leftrightarrow$$

$$1 - \gamma < \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0} \quad \Leftrightarrow \quad \gamma > 1 - \frac{\dot{f}_0 r_0}{f_0 \dot{r}_0},$$

which is the left-hand side of inequality (9). The other cases can be shown in a similar way.■

Proposition 1 restricts the set of initial states for which model (3) makes economic sense. The restriction is natural, because model (3), intended for examining trends, can be inapplicable to the current state of economy $\mathbf{E_0}$ if this state resulted from a process that is not specified in the model. For example, in 2009, Russian rates of oil extraction were growing, the rate of growth of capital exceeded the rate of growth of oil extraction ($\dot{r}_0(\dot{k}_0/k_0 - \dot{r}_0/r_0) > 0$), and GDP was declining ($\dot{f}_0/f_0 < 0$). According to the left-hand side of inequality (9), the condition $\alpha > 0$ requires in this case $\gamma > 1$, which is infeasible.[19] Therefore, in order to reduce the influence of the short-run deviations, some of the data can be represented by their time averages.[20]

4. Potential sustainability

Definition 3. The set of paths $\Pi(t)$ is called *economic program* (program)[21] for a model of an economy if this set uniquely determines the dynamics of the model.

For the goals of this paper, the pair of the paths $\{k(t), r(t)\}$ is sufficient to be considered as

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[19] In general, by the definition of production function, $\dot{k} > 0$ and $\dot{r} > 0$ yield $\dot{f} > 0$.

[20] The questions of quality of fit of a calibrated model with respect to historical data are discussed, for example, in Watson (1993).

[21] This notion was used, for example, in Dasgupta, Heal (1974 and 1979) and Arrow et al. (2003).

an economic program for $f(t)=f(k(t),r(t))$ since the investment rate is constant ($w(t)\equiv w_0$), the dynamics of the resource stock is $\dot{s}(t)=-r(t),$ and the production function formally does not depend on time.

Definition 4. A program $\Pi(t)$ is *feasible* for the model with the initial state $\mathbf{E_0}$ if $\Pi(t_0)$ coincides with the corresponding values from $\mathbf{E_0}$, $k(t),r(t)\geq 0$ for all $t\geq t_0,$ and $\int_{t_0}^{\infty} r dt \leq s_0.$

Definition 5. A model is *potentially survivable*,[22] if there exists at least one feasible program $\Pi(t)$ (*survivable program*) for which the level of utility is never below the subsistence minimum: $u(t)\geq u_{\min}$ for any $t\geq t_0.$ Otherwise, the model is *unsurvivable.*

A model may be unsurvivable, despite the existence of feasible programs, if, for example, the elasticity of substitution between the resource and capital is less than unity. In this case, an arbitrary high rate of capital growth is not able to compensate for the disappearing resource.

Proposition 2. *If model (3) with the initial state* $\mathbf{E_0}$ *is potentially survivable, then*

$$\gamma > 1-\frac{2\dot{f}_0/f_0}{\dot{k}_0/k_0+\dot{r}_0/r_0} \quad \textit{if } \left(\dot{k}_0/k_0\right)^2-\left(\dot{r}_0/r_0\right)^2>0, \textit{ or} \qquad (13)$$

$$\gamma < 1-\frac{2\dot{f}_0/f_0}{\dot{k}_0/k_0+\dot{r}_0/r_0} \quad \textit{if } \left(\dot{k}_0/k_0\right)^2-\left(\dot{r}_0/r_0\right)^2<0. \qquad (14)$$

Remark. Conditions (13) and (14) are necessary for potential survivability of model (3), because they make possible to stretch out the limited reserve s_0 over an infinite period of time, which is only necessary in this model for keeping the level of consumption above a positive value (which is necessary for $u\geq u_{\min}$) during all this period. In case (13), a program with the level of utility no less than $u_{\min}$ can exist due to the growth of capital complemented by labor. However, a survivable program may not exist, despite the convergence of the integral $\int_{t_0}^{\infty} r dt,$ for example, in case (14) with no investment ($\dot{k}_0=0$), decline in extraction ($\dot{r}_0<0$), and decline in labor force participation ($\gamma<1-2\beta$).

P r o o f of Proposition 2 follows from the necessary condition for potential survivability $\alpha>\beta$[23] and formulas (6) and (7). For example, for $\dot{k}_0/k_0-\dot{r}_0/r_0<0$ and $\dot{k}_0/k_0+\dot{r}_0/r_0<0,$ the condition $\alpha>\beta$ is

$$\frac{\dot{f}_0}{f_0}-(1-\gamma)\frac{\dot{r}_0}{r_0}<(1-\gamma)\frac{\dot{k}_0}{k_0}-\frac{\dot{f}_0}{f_0} \quad \Leftrightarrow \quad 1-\gamma<\frac{2\dot{f}_0/f_0}{\dot{k}_0/k_0+\dot{r}_0/r_0},$$

which yields inequality (13). The fulfillment of inequality (14) can be shown in a similar way.■

The value

$$\alpha-\beta=\frac{2\dot{f}_0/f_0-(1-\gamma)\left(\dot{k}_0/k_0+\dot{r}_0/r_0\right)}{\dot{k}_0/k_0-\dot{r}_0/r_0} \qquad (15)$$

can be used as a measure of potential survivability, implying the following result.

Corollary 1. *The potential survivability of model (3) with the initial state* $\mathbf{E_0}$ *can be*

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[22] The term *survivable* was used, for example, in Pezzey (1992). McKibben (2005) used the term *semisustainable* for the agriculture in Cuba, which managed to converge to an acceptable level after the collapse of the Soviet Union.

[23] This condition provides the convergence of the integral $\int_{t_0}^{\infty} r dt,$ (Solow 1974), which is necessary but not sufficient for non-zero consumption in the infinite period (see, for example, Bazhanov 2007 and 2008).

improved under the excess of the rate of investment over the rate of change in extraction ($\dot{k}_0/k_0 - \dot{r}_0/r_0 > 0$ *and* $\dot{k}_0/k_0 + \dot{r}_0/r_0 > 0$) *by*

- *increasing the rate of GDP growth* $\dot{f}_0/f_0$ *by increasing the marginal product of capital;*
- *increasing the share of labor* γ.

Remark. Corollary 1 considers only the cases when the convergence of the integral $\int_{t_0}^{\infty} r dt$ is accompanied by the growth of economy.

Definition 6. A model is *potentially sustainable* if there exists at least one feasible program $\Pi(t)$ (*sustainable program*) for which the level of utility is not declining: $\dot{u}(t) \geq 0$ for any $t \geq t_0$. Otherwise, the model is *unsustainable*.

This definition is partly equivalent to the following definition of Pezzey (2004b): a model (economy) is *sustainable at time* t_0, if $u(t_0) \leq u_{\max}$ (the economy is not overconsuming at t_0), where $u_{\max}$ is the maximum sustainable level of utility that can be maintained forever, given the stocks of man-made and natural capital at t_0. The equivalence is only partial because, as Pezzey noted, his definition works only for efficient economies. Otherwise, an overextracting economy may be underconsuming due to inefficient use of the resource. For such an economy, a sustainable program may not exist. Since real economies are, as a rule, inefficient, this paper uses Definition 6, which works for both efficient and inefficient economies. Pezzey (2004b) provides also a review of the literature relevant to this definition.

Definitions 5 and 6 imply that a model may be potentially survivable but unsustainable if there is at least one program for which utility is always not less than subsistence minimum, but for any feasible program there exists at least one moment of time when utility declines. If a model is unsurvivable, it is also unsustainable. If a model is potentially sustainable, it is also potentially survivable.

This paper, following IMF studies, assumes (too optimistically) that damages from resource use do not affect utility; therefore, potential sustainability and survivability can be measured in terms of per capita consumption since utility in this case monotonically depends on consumption. This simplification is used in the proof of the following proposition.

Proposition 3. *Model (3) with the initial state* $\mathbf{E_0}$ *is potentially sustainable if and only if*

$$s_0 \geq \frac{r_0\left[1-\gamma-\dfrac{\dot{f}_0 k_0}{f_0 \dot{k}_0}\right]}{2\dot{f}_0/f_0 - (1-\gamma)(\dot{k}_0/k_0 + \dot{r}_0/r_0)} \tag{16}$$

or, in terms of α *and* β,

$$\frac{\dot{k}_0}{k_0}(\alpha-\beta) + \frac{\dot{s}_0}{s_0}\beta \geq 0. \tag{17}$$

P r o o f. For simplicity of notations, let $t_0 = 0$.

1) Necessity. Let model (3) with the initial state $\mathbf{E_0}$ be potentially sustainable. We show that the existence of such a program yields inequalities (16) and (17).

Denote $s_{\min}$ the reserve that is used along the "minimal" sustainable program that provides $c(t) \equiv c_0$. Since $w = const$, then $\dot{c}(t) = (1-w)\dot{f} \equiv 0$, which, owing to (3), yields the equation for $r_{\min}(t)$:

$$\frac{\dot{f}}{f} = \alpha\frac{\dot{k}}{k} + \beta\frac{\dot{r}}{r} = 0 \Leftrightarrow \beta\frac{\dot{r}}{r} = -\alpha w A k^{\alpha-1} r^{\beta} \Leftrightarrow r^{-1-\beta}\dot{r} = -\alpha w A k^{\alpha-1}/\beta.$$

Since $\dot{f} \equiv 0$, the investment rule $\dot{k} \equiv wf_0$ gives linear capital: $k(t) = k_0(1 + k_1 t)$, where $k_1 = wf_0 / k_0 = \dot{k}_0 / k_0$. Then the equation for $r_{\min}(t)$ takes the form

$$r^{-1-\beta} dr = -\frac{\alpha w A k_0^{\alpha-1}}{\beta}(1 + k_1 t)^{\alpha-1} dt,$$

which has the solution $r_{\min}(t) = r_0(1 + k_1 t)^{-\alpha/\beta}$. Integration of $r_{\min}(t)$ gives

$$s_{\min} = \int_0^\infty r_{\min}(t)dt = \frac{r_0 \beta}{k_1(\beta - \alpha)}(1 + k_1 t)^{1-\alpha/\beta}\Big|_0^\infty = \frac{k_0 r_0 \beta}{\dot{k}_0(\alpha - \beta)}.$$[24]

Feasibility of $\Pi(t)$, which provides $\dot{c}(t) \geq 0$ for any $t \geq 0$, implies the inequality $s_0 \geq \int_0^\infty r(t)dt \geq s_{\min} = k_0 r_0 \beta / [\dot{k}_0(\alpha - \beta)]$, resulting in condition (17). Substitution of expressions (6) and (7) into (17) yields inequality (16).

2) Sufficiency. Let inequalities (16) and (17) be satisfied for the economy $\mathbf{E_0}$ and calibrated model (3). We show that there exists a feasible program $\Pi(t)$, such that $\dot{c}(t) \geq 0$ for all $t \geq 0$. Consider a particular case of (17) when

$$s_0 = k_0 r_0 \beta / [\dot{k}_0(\alpha - \beta)] = k_0 r_0 \beta / [f_0 w(\alpha - \beta)] \qquad (18)$$

and show that per capita consumption is constant for a feasible program $\Pi(t) = \{k(t), r(t)\}$, where $k(t) = k_0(1 + r_1 t)$, and the path $r(t) = r_0(1 + r_1 t)^{-\alpha/\beta}$ is such that $\int_0^\infty r dt = s_0$. Integration of $r(t)$ gives $s_0 = r_0 \beta / [r_1(\alpha - \beta)]$. Then equality (18) is satisfied for $r_1 = f_0 w / k_0$. Consider

$$\dot{f} / f = \alpha \dot{k} / k + \beta \dot{r} / r = \frac{\alpha k_0 r_1}{k_0(1 + r_1 t)} - \frac{\beta \alpha r_0 r_1 (1 + r_1 t)^{-\alpha/\beta - 1}}{\beta r_0 (1 + r_1 t)^{-\alpha/\beta}} = \frac{\alpha r_1}{1 + r_1 t} - \frac{\alpha r_1}{1 + r_1 t}.$$

Hence, $\dot{f} \equiv 0$, which yields $\dot{c}(t) \equiv 0$ since $\dot{c} = (1 - w)\dot{f}$, where $w = const$. ■

Condition (17) for $\dot{k}_0 > 0$ and $\alpha - \beta > 0$ can be written as $\alpha - \beta \geq \frac{r_0 \beta k_0}{s_0 \dot{k}_0} > 0$, which, as one would expect, is a more strict requirement to the current state of economy than necessary conditions of potential survivability (13) and (14) based on the inequality $\alpha - \beta > 0$. Inequality (17) shows the ways for improving potential sustainability, which can be formulated as follows.

Corollary 2. *Potential sustainability of model (3) with the initial state* $\mathbf{E_0}$ *can be improved by*

- *increasing the resource reserve* s_0;
- *increasing the rate of capital growth* $\dot{k}_0 / k_0$;
- *decreasing the current rate of extraction* r_0;
- *increasing the share of capital* α *and decreasing the share of the resource* β *in GDP.*

In turn, according to formulas (6) and (7), the increase in α and decrease in β, under the fixed rate of capital growth, is equivalent to the increase in the rate of GDP and (or) to the decrease in the rate of change in the rate of extraction (decline in $\dot{r}_0 / r_0$). In other words, *if GDP is growing under the accelerating introduction of the technologies that do not use the nonrenewable resource, the potential sustainability is increasing.*

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[24] This equality can be written as $s_{\min} = k_0^{1-\alpha} r_0^{1-\beta} \beta / [(\alpha - \beta) A w]$, which is a more general form ($w \neq \beta$ and $A \neq 1$) of the "perfection condition" for the initial state of the DHSS economy with respect to the constant-consumption criterion (Bazhanov 2010).

Definition 7. For model (3) with the initial state $\mathbf{E_0}$ the value

$$LS_0 = s_0 - \frac{k_0 r_0 \beta}{\dot{k}_0(\alpha-\beta)} = s_0 - \frac{k_0^{1-\alpha} r_0^{1-\beta}\beta}{wA(\alpha-\beta)} = s_0 - \frac{r_0\left[1-\gamma-\dfrac{\dot{f}_0 k_0}{f_0 \dot{k}_0}\right]}{2\,\dot{f}_0/f_0 - (1-\gamma)(\dot{k}_0/k_0 + \dot{r}_0/r_0)},$$

where $\dot{k}_0 > 0$ and $\alpha - \beta > 0$, will be called *the level of potential sustainability (LPS)* of the model at the moment of time t_0. A positive value of LS_0 will be called *the reserve of potential sustainability*, a negative one – *the shortage of potential sustainability.*

Remark. (a) The second equality in Definition 7 shows that the growth rate of capital $\dot{k}_0/k_0$ and the indicator LS_0 are growing with the growth of the investment rate w and the level of TFP A. This growth is faster for the lesser values of k_0 due to the concavity of production function (3).[25]

(b) For an increase in the LPS, the TFP-augmenting technical progress is preferable to the increase in investment rate, because

- investment rate is limited ($w < 1$);
- consumption declines with the increase in w, whereas the growth of TFP provides both the growth of consumption ($c = (1-w)Ak^{\alpha}r^{\beta}$) and the increase in the level of sustainability.

(c) The indicator LPS coincides with the expression for genuine investment in accounting prices, derived in van der Ploeg (2011)[26] for $A = 1$, and, as noted above, with the expression for a perfection condition, derived in Bazhanov (2010)[27] for $A = 1$ and $w = \beta$. LPS includes also the Hartwick rule at t_0 ($\dot{k}_0 - f_r r_0 = 0$) as a special case, when the initial state of the economy satisfies the perfection condition ($r_0 = \left[As_0(\alpha-\beta)/k_0^{1-\alpha}\right]^{1/(1-\beta)}$) with respect to a constant-consumption criterion.

A positive value of LS_0, according to Proposition 3, shows that model (3) is potentially sustainable. Indicator LPS can be expressed in any units by multiplying LS_0 by a positive quantity like, for example, in inequality (17). In Definition 7, LS_0 is measured in the units of the resource reserve, which shows, for example, how much the resource reserve s_0 should be increased in order to eliminate the aggregate shortage of sustainability. Multiplication of LS_0 by $\dot{k}_0/(s_0 f_0)$ transforms this indicator into terms of genuine investment as a share of GDP.

Besides qualitative estimates, condition (17) allows to evaluate the comparative importance of investment policy for sustainability of an economy. For instance, let the rate of capital growth

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[25] The lesser value of capital corresponds to the faster growth in output, compensating more for losses due to the shrinking flow of the resource.

[26] As a social welfare function, van der Ploeg (2011) considered the maximum level of consumption $c_0(s_0, k_0)$ that a DHSS economy can maintain during an infinite period of time. Then, using the approach of Arrow et al. (2003), the accounting price of the resource is $p_G \equiv \dfrac{\partial c_0/\partial s_0}{\partial c_0/\partial k_0} = \left(\dfrac{\beta}{\alpha-\beta}\right)\dfrac{k_0}{s_0}$, and the genuine investment at t_0 can be defined as $\dot{k}_0 + p_G \dot{s}_0$, which, multiplied by $s_0/\dot{k}_0$, results in the first expression in Definition 7.

[27] Bazhanov (2010) derived a condition that allowed a DHSS economy to have a smooth continuation of its initial state by a constant-consumption path with $c(t) \equiv c_0$. The condition (for $A = 1$ and $w = \beta$) is $s_0 = r_0^{1-\beta}k_0^{1-\alpha}/(\alpha-\beta)$ $= r_0 k_0 \beta / \left[\dot{k}_0(\alpha-\beta)\right]$, which is equivalent to the requirement $LS_0 = 0$ or zero genuine investment.

$\dot{k}_0/k_0$ reduce to half the initial value. Then, for keeping the same level of potential sustainability, either the resource stock should grow twice the initial size, or the current rate of extraction should decline to half the initial value, or the share of the resource in GDP should be decreased to the value $\alpha\beta/(2\alpha-\beta)$[28] during the same period of time.

The growth of investments is limited by the current GDP and by the requirements to the minimum level of consumption. The opportunities for increase in the stock s_0 are also restricted due to the growing exploration cost, growing cost of extraction, and limitedness of the reserves. Therefore, *technical progress that increases TFP and reduces the share of the nonrenewable resource in GDP at the expense of increase in the use of renewable resources is the only reliable way to improve the potential sustainability of a resource-based economy*. The influence of this way on potential sustainability is limited only by technological opportunities, which are constantly improving.

5. Evaluation of potential sustainability of the Russian economy

5.1. A closed model

In this section, model (3) is calibrated on the data from Table 1. Inaccuracy of the model can be estimated, for example, by comparing the values of the marginal products of capital $f_k(t_0)$ and the resource $f_r(t_0)$[29] obtained both from the data directly and from the use of model (3). Rosstat (2010) gives

$$f_k(t_0) \equiv \frac{\partial f}{\partial k}(t_0) = \frac{\dot{f}_0/f_0}{\dot{k}_0/f_0} = \frac{\dot{f}_0/f_0}{w_0} = 0.2080.$$

Model (3) yields $f_k(t_0) = \alpha\, f_0/k_0 = 0.2074$. The estimate of the marginal product of oil for model (3) is $f_r(t_0) = \beta\, f_0/r_0 = 449$ [\$/t] or $f_r(t_0)$/7.3[bbl/t]=61.47[\$/bbl].[30]

It is easy to check that the necessary condition for potential survivability (Proposition 2) is satisfied for the data from Table 1, since, according to formulas (6) and (7), $\alpha = 0.37 > \beta = 0.16$,[31] which means that model (3) calibrated on Table 1, *is able* to avoid collapse. Moreover, this model, according to Proposition 3, is also potentially sustainable because

$$LS_0 = s_0 - k_0 r_0 \beta / \left[\dot{k}_0(\alpha - \beta)\right] = 129.05 > 0,$$[32]

which means that there are feasible programs that provide monotonically non-decreasing per capita consumption for any $t \geq t_0$. However, for model (3), these optimistic conclusions are true only when a positive amount of the resource is available at any moment $t \geq t_0$, which is not satisfied under the assumptions of model (1). Namely, the assumption about depletion of oil reserves by 2048 results in the collapse of production and consumption in the framework of model (3).

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[28] For example, if $\alpha = 0.3, \beta = 0.2$, then the new β should be 0.15 with the increase in the share of labor by 0.05.

[29] Under imperfect competition, these values, in general, do not coincide with the interest rate and the resource price correspondingly.

[30] According to EIA (http://www.eia.doe.gov), Urals oil price oscillated from \$34.2 in January 2009 to \$137.6 in July 2008 and to \$76.27 in October 2009.

[31] Then, from equation (3), $A = f_0 k_0^{-\alpha} r_0^{-\beta} = 2.47$ [(k\$/pers.)$^{1-\alpha}${t/(pers.× year)}$^{-\beta}$].

[32] In terms of genuine investments in accounting prices, expressed as a share of GDP, this indicator is $w_0 - k_0 r_0 \beta / [s_0 f_0 (\alpha - \beta)] = 0.222$. Genuine investment in marginal resource productivity (Hartwick rule) in this case is also positive: $w_0 - \beta = 0.097$.

5.2. An open model

Sections 2-5 dealt with the simplest model of a resource-based economy. In order to bring this model closer to model (1), the income from oil export and income from foreign assets will be considered as separate parts of production:[33]

$$y = c + \dot{k}_T = f_I(k_I, r_I) + pr_E + ik_W, \tag{19}$$

where y – GNP, $f_I = Ak_I^{\alpha_I} r_I^{\beta_I}$ – domestic production except oil export, r_I, r_E – the rates of domestic use and export of oil, p – the price of oil, k_W – holdings in the world's financial assets: $k_W = k_T - k_I$, k_T – total capital (domestic k_I plus the assets abroad), i – interest rate on k_W. The variables y, f_I, c, r_I, r_E, r_W, k_T, k_W, k_I are in per capita units. The main difference of model (19) from model (1) is that the flow of oil r_I, which is used in domestic production, is included into production function as a factor.

Unlike the closed case, formulas (6) and (7) for domestic production yield[34] $\alpha_I = 0.231 < \beta_I = 0.297$, which means that production $f_I(k_I, r_I)$ in model (19) is unsurvivable.[35] In other words, *given the shares α_I and β_I, production will eventually decline to zero, while oil is depleted, regardless of the path of extraction and of budget and investment policies.*

Remark. A mechanical transformation of the open economy to the closed one would require

(a) the ban on the export of capital and reinvestment of all the capital from abroad to the domestic production;

(b) the ban on the oil export.

Then, in order to make the *open* model as sustainable as *closed* model (3), the reinvested capital in combination with the oil, redirected from export into domestic production, must result in no less impact in GNP than it was in the open economy. Implausibility of this scenario suggests that potential sustainability can be improved mostly by qualitative transformations of domestic capital (development of renewable sources of energy) and growth of TFP.

What is the reason for the worsening of sustainability for the open model? In the current case, the main cause is the cut of the stock of capital rather than the change in the resource flow, because the term $\dot{k}_0/k_0$ in formulas (6) and (7) is two orders higher than $\dot{r}_0/r_0$ both in the closed and in the open cases. Since production function is concave in both k and r, the rate of growth of production should be essentially higher in order that the domestic production f_I with the lower values of k and r be as sustainable as model (3). Namely, formula (6) implies that, for the data from Table 2, the share of domestic capital α_I would be equal to the value of α in model (3) if the rate of growth of production f_I would have been $\dot{f}_{I0}/f_{I0} = 0.084$, which is qualitatively consistent with the recommendations of Corollary 1. Therefore, *potential survivability can be increased by such an increase in capital that at least does not decrease the rate of growth of production.*

Of course, the estimate of potential sustainability may be data-biased. For example, in Table 2, it is assumed that $\dot{f}_{I0}/f_{I0} \approx \dot{f}_0/f_0$. This value can also be estimated from the equality

$$\frac{\dot{f}}{f} = \frac{\dot{f}_I + \dot{f}_W}{f} = \frac{f_I}{f}\frac{\dot{f}_I}{f_I} + \frac{f_W}{f}\frac{\dot{f}_W}{f_W},$$

which yields the following expression for $\dot{f}_{I0}/f_{I0}$:

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[33] This model was used, for example, in Dasgupta et al. (1978).

[34] Model (19) is calibrated on the data from Tables 1 and 2.

[35] Genuine investment in marginal resource productivity is negative: $w_{I0} - \beta_I = 0.284 - 0.297 = -0.013$.

$$\frac{\dot{f}_{I0}}{f_{I0}} = \frac{\dot{f}_0}{f_0}\frac{f_0}{f_{I0}} - \frac{\dot{f}_{W0}}{f_{W0}}\frac{f_{W0}}{f_{I0}}. \tag{20}$$

According to the data from Rosstat (2010), the change in the income "from the rest of the world" from the end of 2007 to the end of 2008 was

$$\frac{\dot{f}_{W0}}{f_{W0}} \approx \frac{1432.3486[\text{bln. rub.}]/29.38[\text{rub./\$}] - 1143.3291[\text{bln. rub.}]/24.55[\text{rub./\$}]}{1143.3291[\text{bln. rub.}]/24.55[\text{rub./\$}]} = 0.047,$$

which, according to formula (20), gives a higher estimate for $\dot{f}_{I0}/f_{I0}$:

$$\frac{\dot{f}_{I0}}{f_{I0}} = 0.052\frac{F_0}{F_{I0}} - 0.047\frac{F_{W0}}{F_{I0}} = 0.0592.$$

However, this estimate is also not enough for potential survivability, since it still yields $\alpha_I = 0.262 < \beta_I = 0.266.$ [36]

According to Corollary 1, potential survivability can also be improved by changing the share of labor. Since $\dot{k}_0/k_0 + \dot{r}_0/r_0 > 0,$ the share of labor γ_I should be increased. Indeed, if γ_I is increased by 1.15 times ($\gamma_I = 0.5428$), then $\alpha_I = 0.232 > \beta_I = 0.225.$ The same effect results from 1.15 times increase in the rate of production growth ($\dot{f}_{I0}/f_{I0} = 0.060$): $\alpha_I = 0.267 > \beta_I = 0.261.$ However, in both cases, according to Proposition 3, the model will be only potentially survivable but not potentially sustainable because the LPS in both cases is negative: $LS_0 = -215.2$ for $\gamma_I = 0.5428,$ and $LS_0 = -349.2$ for $\dot{f}_{I0}/f_{I0} = 0.060.$ [37]

The value of LS_0 shows how much additional reserve [t/pers.] is necessary in order to provide potential sustainability. According to ES-2030 (2010), the share of export in Russian oil extraction will decline, increasing the portion of the stock s_{I0} intended for domestic production. It is obvious, however, that the increase in s_{I0} will not solve the problem since the total stock $s_0 = 146.53$ is less than the shortage of potential sustainability. But if the rate of growth of production increases up to $\dot{f}_{I0}/f_{I0} = 0.060,$ and, at the same time, the share of labor grows to $\gamma_I = 0.5428,$ then these changes will already result in the reserve of potential sustainability $LS_0 = 59.21.$ [38]

According to Corollary 2, the level of potential sustainability can also be increased by decreasing the current rate of extraction.[39] For example, for $\gamma_I = 0.5428$ and $\dot{f}_{I0}/f_{I0} = 0.0524,$ the decrease in the use of domestic oil to $\tilde{r}_{I0} = 0.9 \times r_{I0}$=1.69 [t/(pers.$\times$year)] with the decreasing trend $\dot{r}_{I0} = -0.002$ [t/(pers.$\times$year^2)] and the reserve $s_{I0} = 2s_0/3 = 97.68$ [t/pers.] gives the decrease in the shortage of sustainability up to $LS_0 = -58.71.$ But if these changes are combined with the 1.1 times increase in the rate of production growth (up to $\dot{f}_{I0}/f_{I0} = 0.0577$), then the LPS will already be

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[36] The value $\dot{f}_{W0}/f_{W0}$, expressed in Euro, implies $\dot{f}_{I0}/f_{I0} = 0.058,$ which also yields $\alpha_I < \beta_I$.

[37] In terms of genuine investment (in accounting prices), these values are -0.7635 for $\gamma_I = 0.5428,$ and -1.239 for $\dot{f}_{I0}/f_{I0} = 0.060.$ Genuine investment in marginal resource productivity, however, is positive in both cases: 0.0582 and 0.0221.

[38] Genuine investment in marginal resource productivity is also positive: $w_{I0} - \beta_I = 0.093$.

[39] The structure of the model assumes in this case that oil is partly substituted by renewable sources of energy.

positive: $LS_0 = 74.13$.

According to the name of the indicator LS_0, all the aforementioned changes in the economy can improve only *potential* sustainability, which is being evaluated at a time t_0. Whether model (19) and the real economy, which was used for calibration, will follow a theoretically available sustainable program depends on the dynamics of investment and resource policies, and on the behavior of technical change (see, for example, Bazhanov, 2008).

As to the forecasts of the behavior of the factors that influence the LPS, according to ES-2030 (2010), the oil reserves will grow, and the share of the reserves meant for domestic production will also grow until 2030, which should increase the value of LS_0. However, these changes, being taken into account in the above calculations, did not help to solve the problem of unsustainability. Besides, ES-2030 (2010) does not consider any specific projects for the development of renewable energy, and instead, envisages the growing rates of extraction of fossil fuels, which should further decrease the level of potential sustainability.

6. Concluding remarks

This paper has introduced the notions of potential sustainability and potential survivability of a model. The notions are based on the possibility to keep a non-declining (non-declining below the subsistence minimum) level of utility during a long period of time. The approach to sustainability evaluation was illustrated by examples with the open and closed variants of neoclassical models with the Cobb-Douglas production function, which have been used in resource economics both in theoretical and applied studies since 1974. These models, with oil as a production factor, were calibrated on the data of the Russian economy.

The paper has derived the necessary conditions for potential survivability and the necessary and sufficient conditions for potential sustainability, depending on the economy's initial state. The conditions for potential sustainability can be used as an indicator of the level of potential sustainability (LPS). The indicator shows whether there exists an economic program that provides non-declining consumption during an infinite period of time. The paper assumes that the economy, at the moment of evaluation, may be non-optimal, and even statically and dynamically inefficient due to, for example, imperfection in knowledge or in institutions.

The opportunity to follow the program with non-declining consumption depends, of course, on the future dynamics of investment and resource policies; therefore, the offered conditions can be used as necessary for future sustainable growth or, as Pezzey (2004b) put it, as one-sided sustainability tests. Namely, if these conditions are not satisfied, the level of consumption will decline given the current structure of production. The conditions show the ways to improve potential survivability and sustainability (Corollaries 1 and 2).

The numerical estimate of potential sustainability of the closed model, calibrated on the Russian data, showed that this model is potentially sustainable. However, this evaluation was not supported by the study of the open model, where GNP was considered as a sum of the oil-export income, income from capital shares abroad, and the rest of domestic production, which was modelled by the Cobb-Douglas function with oil as a production factor. The model of the rest of internal production turned out to be unsustainable even under a hypothetical assumption that all Russian oil reserves are used only inside of the country. According to Corollaries 1 and 2, potential sustainability in this case can be improved by

- increasing the marginal product of capital;
- decreasing the share of the resource and increasing the shares of labor and capital in GNP;

- increasing the growth rate of capital (increasing the investment rate, increasing TFP);[40]
- decreasing the rate of resource extraction;
- increasing resource reserve.

These recommendations imply, in particular, that LPS increases with the introduction of renewable sources of energy that reduce the use of fossil fuels. However, ES-2030 (2010) assumes that the development of the Russian economy will be based on the growing rates of extraction of nonrenewable resources, which, according to the results of this paper, will further decrease the level of potential sustainability. Polterovich et al. (2007) offer an analysis of the reasons for resource dependence of the Russian economy, including the role of institutions, and possible practical approaches to improving the possibilities for long-term economic growth.

Unlike the models that were used in the IMF studies (Barnett, Ossowski 2003; Jafarov et al. 2006), the models of resource economics, which were used in the current paper, were based on more cautious assumptions, in particular, regarding the opportunity of adequate substitution of a nonrenewable resource by financial assets or other forms of capital, as well as about the behavior of TFP. These assumptions prescribe, as expected, a more conservative resource policy and more efforts in the development of domestic capital. Moreover, the real sustainability policies should be even more resource-conservative and more growth-promoting. This follows, first, from considering only oil in this paper for evaluating sustainability regardless of other resources; and secondly, from the too optimistic assumptions that: a) the resource is always productive, b) the economy is always non-wasteful, and c) oil use does not cause damages to utility and production.[41]

The models used in this paper reflect the real-world uncertainty of dependence of economic sustainability on investment and resource policy. More comprehensive models with this important property are based on the production functions with the variable elasticity of factor substitution, which makes the analysis essentially more complicated.

Appendix

Table 1. The data for calibration of model (3)

Notation	Name	Estimate on 01.01.09; units of measure	Source of information
L_0	Population	141.9 [mln. pers.]	Rosstat (2010)
F_0	GDP	41428.56 [bln. rub.]	Rosstat (2010)
$\dot{F}_0/F_0 = \dot{f}_0/f_0$	Rate of GDP growth	0.0524 [per year]	Rosstat (2010)
D_0	$ USA	29.38 [rub./$]	Rosstat (2010)
f_0	GDP per capita	9.9371 [k$/(pers.×year)][42]	$= F_0 / D_0 / L_0$
w_0	The rate of	0.252	Rosstat (2010) – the share of gross

[40] These recommendations coincide with the conclusions of van der Ploeg, Venables (2011), who examined an extended variant of model (1) and showed that the augmenting of domestic capital and infrastructure is preferable for developing countries than the investment of resource rents into foreign assets.

[41] The optimal level of extraction is lower when a social planner takes into account the damages associated with the resource use (see, for example, Bazhanov 2011).

[42] This paper uses GDP estimate at official exchange rate, rather than at purchasing power parity since the oil-export income enters model (19) exactly in this unit.

	investment		savings in GDP
$\dot{k}_0$	Investment	2.5 [k\$/(pers.$\times$year)]	$= w_0 f_0$ [43]
K_0	Fixed assets	74471.182 [bln. rub.]	Rosstat (2010)
k_0	Capital per capita	17.86 [k\$/pers.]	$= K_0 / D_0 / L_0$
R_0	Rate of oil extraction	487.6 [mln. t/year]	ES-2030 (2010)
r_0	Id., per capita	3.4362 [t/(pers.$\times$year)]	$= R_0 / L_0$
$\dot{r}_0$	The rate of change in the rate of extraction	0.0034 [t/(pers.$\times$year^2)]	$\approx (3.4567 - 3.4362)/6$ (ES-2030 (2010); the assumption about linearity of r in each phase)[44]
s_0	Oil reserves per capita	146.53 [t/pers.]	=20792 [mln. t][45]/ L_0
γ	The share of labor in GDP	0.472	=Compensation of employees/GDP (Rosstat, 2010)

Table 2. The data for calibration of model (19)

Notation	Name	Estimate on 01.01.09; units of measure	Source of information
K_{W0}	The holdings in the world's financial assets	1011.377 [bln. \$]	CB RF, (2009)
k_{W0}	Id., per capita	7.1274 [k\$/pers.]	$= K_{W0} / L_0$
k_{T0}	Total national capital per capita	17.86 [k\$/pers.]	$\approx k_0$
k_{I0}	Domestic national capital per capita	10.74 [k\$/pers.]	$= k_{T0} - k_{W0}$
R_{E0}	Oil export	221.6365 [mln. t/year]	FTS (2009)
r_{E0}	Id., per capita	1.56192 [t/(pers.$\times$year)]	$= R_{E0} / L_0$
ε	The share of export in oil extraction	0.4545	$= r_{E0} / r_0$
$p_0 R_{E0}$	Income from oil exports	151.6686 [bln. \$]	FTS (2009)
$p_0 r_{E0}$	Id., per capita	1.0688 [k\$/(pers.$\times$year)]	$= p_0 R_{E0} / L_0$
r_{I0}	Domestic use of oil per capita	1.8743 [t/(pers.$\times$year)]	$= r_0 - r_{E0}$
s_{I0}	Oil reserves for domestic use per capita	79.92 [t/pers.]	$= (1-\varepsilon) s_0$

[43] For simplicity, investments are equal to savings.

[44] The average $\dot{r}_0$ for the period 2001-2009, estimated by the technique offered in Bazhanov (2006), is 0.018.

[45] The value 20792 mln. t includes the reserve estimate on 01.01.09, which equals 8219 mln. t (OGJ 2009) and the estimate of the reserve growth until 2030, which equals 12573 mln. t (ES-2030 2010).

F_{W0}	The income from the world's assets	1432.3486 [bln. rub.]	Rosstat (2010)
ik_{W0}	Id., per capita	0.3436 [k$/(pers.×year)]	$= F_{W0} / D_0 / L_0$
y_0	GNP per capita	9.9371 [k$/(pers.×year)]	$\approx f_0$
F_{I0}	Domestic "non-oil" production	35779.57 [bln. rub.]	$= F_0 - p_0 R_{E0} \cdot D_0 - F_{W0}$
f_{I0}	Id., per capita	8.525 [k$/(pers.×year)]	$= F_{I0} / D_0 / L_0$
$\dot{f}_{I0}/f_{I0}$	Growth rate of f_I	0.0524 [per year]	$\approx \dot{f}_0/f_0$
w_{W0}	The rate of investment into the world's assets	0.252	$\approx w_0$
$\dot{k}_{W0}$	Investment into the world's assets	0.087 [k$/(pers.×year)]	$= w_{W0} ik_{W0}$
$\dot{k}_{I0}$	Investment into domestic capital	2.4176 [k$/(pers.×year)]	$= \dot{k}_0 - \dot{k}_{W0}$
w_{I0}	The rate of investment into domestic capital	0.284	$= \dot{k}_{I0}/f_{I0}$
$\dot{r}_{I0}$	The rate of change in the rate of extraction for domestic use	0.00186 [t/(pers.×year^2)]	$\approx (1-\varepsilon)\dot{r}_0$
γ_I	The share of labor in F_{I0}	0.472	$\approx \gamma$

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