Technology choice and endogenous productivity dispersion over the business cycles

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Abstract

Firm-level productivity dispersion is countercyclical. I incorporate firms' technology adoption decision into firm dynamics model with business cycle features to explain these empirical findings both qualitatively and quantitatively. The option of endogenous exiting and credit constraint jointly play an important role in motivating firms' risk taking behavior. The model predicts that relatively small sized firms are more likely to take risk, and that the dispersion measured as the variance/standard deviation of firm-level profitability is larger in recessions.

Keywords: Countercyclical Productivity Dispersion, Business Cycles, Firm Dynamics.

JEL Classification Codes: E32, L11, L25

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1 Introduction

Cross-sectional productivity dispersion rises in bad times. Recently, this phenomenon attracts growing attention of economists, with numerous new evidences from individual level data sets[1]. However, this significantly negative correlation between uncertainty and aggregate economic condition is often treated as a calibration discipline, while not many works have been done to explain it.

In this paper, I provide a possible mechanism through which the worsened aggregate economic condition leads to an increase in the measured dispersion in individual level productivity. The model at work stands close to the standard industry dynamics model with firm entry and exit built in the seminal work Hopenhayn (1992), with aggregate fluctuations in terms of "technology shocks" as the driving force of model dynamics, which is also a standard approach in real business cycles literature. Meanwhile, it differs from the standard in that in each period, after observing the aggregate "technology realization", a staying firm has the option to adopt a risky technology, in addition to the standard safe technology whose productivity realization is determined by the aggregate state. Given the same capital input, the output and productivity associated to the risky technology is a mean-preserving spread of the safe one's output and productivity. Although firms are risk neutral and the risky technology does not give higher flow payoff, there is a positive fraction of firms that strictly prefer to take the risk. This is because the option of exit provides a lower bound to a firm's continuation value as a function of working capital and creates a local convexity in it. Therefore, firms in this region have the incentive to randomize over their future values by choosing the risky technology, and when the uncertain productivity realizes, dispersion arises. This setup resembles Vereshchagina and Hopenhayn (2009) on occupational choice. In bad times, this risky region gets larger and the fraction of risky firms then gets larger. Consequently, the average or aggregate riskiness in firms' production increases, so does the realized productivity dispersion. Despite the model is only a standard one with a little twist, it is capable of generating productivity dispersion negatively correlated to aggregate state, with the correlation coefficient in line with data.

This model's mechanism is also strongly motivated by empirical findings. It has features and implications that mirror the following observations: (1) new firms are relatively small and small firms have low survival rate; (2) small and/or young firms tend to bear more risk and/or show larger productivity dispersion; (3) business cycles indicators lead the change in productivity dispersion; and (4) in recessions, more firms become risky and the exit rate is therefore countercyclical.

[1] Examples are Higson, Holly and Kattuman (2002), Higson, Holly, Kattuman and Platis (2004), Bloom (2009), Bloom, Floetotto and Jaimovich (2010), Bachmann and Bayer (2011), Arellano, Bai and Kehoe (2009), Bachmann, Elstner and Sims (2011), Chugh (2010), Kehrig (2011), to name a few.

The first two points are closely related, as the exit hazard is a special form of firm level risk. The relation between firm size and dynamics is well established and can be dated back to, for example, Dunne, Roberts, and Samuelson (1988). This is further discussed in Section 2. The findings on firm size and riskiness mainly come from two directions. Firstly, it is well established in the entrepreneurship literature that entrepreneurs, especially poorer ones, bear substantial amount of risk and tend to hold largely undiversified assets by investing heavily in their own firms, despite no or little premium in doing so. The risk here is interpreted as either the dispersion in small firm owners' personal income, or dispersion in return to private equity. At the same time, privately owned businesses are on average smaller in scale, measured in either capital stock, number of employees, or output[2]. The second stream of empirical findings, more relevant to my work, regards the productivity and firm size differential. Gertler and Gilchrist (1991), using the Quarterly Financial Report for Manufacturing Corporations, find that smaller firms exhibit higher standard deviation in sales growth rates than larger ones do. Dhawan (2001) looks at publicly traded firms in COMPUSTAT and find that small firms have higher failure rate and larger standard deviation in profit rate, while conditional on surviving, small firms show higher average profit rate. The superior profitability in small firms reduces if adjusted according to the failure rates. Here, Dhawan defines the profit rate as operating income per unit of capital, and he defines the firm-level riskiness or volatility as the variance in the random realizations of production. Using his definitions, my model generates the same pattern of profit rate and riskiness differential in size. There is also evidence from outside U.S.. For example, utilizing German data set USTAN, Bachmann and Bayer (2011) find decreasing productivity risk in firm size, where the risk is measured as average cross-sectional standard deviation in log-differences in firm-level Solow residuals.

The latter two points are on the cyclical change. Increase in measured cross-sectional dispersion lags the worsened business cycles indicator, for example, GDP growth rate, as shown in Bachmann, Elstner and Sims (2011) and Kehrig (2011) among others. Similar response is observed on the stock market. The last point relates to the key feature of the model. Although unfortunately I do not have direct observation from the data, there are indirect evidences that imply a larger fraction of risky firms in recessions, consisting of mainly small firms. Exit rate raises in bad times. The findings on the relation between firm size and exit rate show that small firms and establishments drive the negative correlation between exit rate and business cycles. This indicates that small firms are more sensitive to the cyclical change, as the model predicts. The increased exit rate in bad times is shown in papers such as Campbell (1998) and Jaimovich and Floetotto (2008), and

[2] Examples for works dedicated in this direction are Barton H. Hamilton (2000), Moskowitz and Vissing-Jorgensen (2002), and Herranz, Krasa and Villamil (2009). See Quadrini (2009) for a detailed review.

is discussed in Section 2. A maybe more direct evidence is on cyclical pattern of price dispersion recently documented in Bachmann and Moscarini (2011) and Berger and Vavra (2011). Cross-sectional dispersion in price changes is countercyclical, both within and across sectors. Meanwhile, the dispersion is positively correlated to the frequency of adjustments, which is also countercyclical. The higher adjustment frequency in bad times is interpreted as result of firms doing more frequent risky pricing experiments due to lower experimentation cost in Bachmann and Moscarini (2011).

The goal of this paper is to complement existing theories. It is true that, if measured uncertainty and aggregate economic condition are correlated, the causal direction can be either. The real option literature that aims at explaining such countercyclicality suggests the opposite direction of causal relationship, from increased uncertainty to decline in aggregate economic activity. An influential paper dedicated in this direction is Bloom (2009), which is later generalized by Bloom et. al. (2009). Bloom shows that increased uncertainty, through the channel of adjustment costs to capital and labor, leads to larger option value of waiting and a pause in investment and employment. A sizable drop in aggregate economic activity occurs because of this "wait-and-see" effect. The time varying uncertainty is twofold in his model: (1) time series standard deviation of productivity can be either high or low, evolving as a Markov process, and (2) the one-step-ahead conditional variance of this Markov process depends on current realization. However, Bachmann and Bayer (2011) and Bachmann, Elstner and Sims (2011) show that there is little evidence of sizeable "wait-and-see" effects in data. In addition, the process of entry and exit is neglected. Arellano, Bai and Kehoe (2009) do consider the entry and exit dynamics that interact with financial constraints, but, again, the causal direction is from time series uncertainty shock to a sizeable response in aggregate variable.

It is important to notice that the importance of uncertainty shock is not denied in this paper, and the inverted causality may still be true, but there is an issue regarding measuring uncertainty, which relates to the lead-lag relationship between uncertainty and cycles. Time series variances of major business condition indicators are often interpreted as uncertainty. In addition, a parallel family of uncertainty measures regards the realized cross-sectional dispersion in individual level performances, which include, among others, cross-sectional variance in measured firm-level total factor productivities, levels or growth rates, and sales growth rates. However, realized cross-sectional dispersion is only a proxy of uncertainty. Besides, increased micro-level cross-sectional dispersion tends to lag the recessions. This suggests a possible causality from aggregate economic state to measured uncertainty, in particular, cross-sectional dispersion in productivities. This paper then tries to look at this interesting issue from an alternative angle to the one adopted by the aforementioned literature.

The other paper that entertains the same causal direction as mine is Bachmann and Moscarini

(2011). They build a model in which firms need to run costly experimentation and hence learn about their own market powers. As a result of lower experimentation costs, the dispersion of productivity measured in sales is larger during recessions due to more experimentations conducted. My model shares a similar feature with theirs, in that the option of exiting promotes the risky performance of firms, while the rest of the mechanism is very different. At the same time, my model differs from theirs by predicting that smaller firms are the major contributor of productivity dispersion and entry/exit dynamics.

The rest of the paper is organized as follows. Section 2 describes the stylized facts on cyclical dispersion of productivity, firm size distribution and dynamics. Section 3 contains a simple three-period model that illustrates the mechanism and shows preliminary results. Section 4 takes the simple model into infinite horizon. Section 5 concludes.

2 Empirical Facts

Cyclical Productivity Dispersion. Eisfeldt and Rampini (2006) use data from COMPUSTAT and find countercyclical movement of dispersion in Tobin's q. At the same time, they show a similar pattern for dispersion of total factor productivity growth rates at four digit SIC level, with correlation being -0.465. Bloom (2009) shows that the US stock market volatility measured as VXO index is positively correlated to the cross-sectional standard deviations of firm profit growth, firm stock return, and industrial TFP growth at four digit SIC level, but its correlation with industrial production is significantly negative. Moreover, Bloom, Floetotto and Jaimovich (2010) take an even closer look at this issue and examine the Census of Manufactures, and find that various measures of uncertainty are significantly countercyclical at all of establishment, firm, industry, and aggregate levels. Bachmann and Bayer (2011) take a long panel of German firm-level micro-data that covers all single digit industries, and show that the correlation between dispersion in growth rates of firm-level TFP, sale, and value added and economic performance is significantly negative. This pattern preserves in subsamples divided by sector and by size. Although a different economy, their USTAN data set has the clear advantage in coverage. Moreover, by looking at different size quantiles, they document that time series averaged productivity dispersion in smaller firms tend to be larger than bigger firms. Chugh (2010) explores the profitability series constructed by Cooper and Haltiwanger (2006) from Longitudinal Research Database and calculates the cyclical correlation between average productivity and the dispersion of profitability to be -0.97. However, the sample is of relatively short length as annual data and covers only 1977-1988, which exhibits unusually large degree of opposite movement. Kehrig (2011) focuses more on the dispersion of productivity levels rather than profit rates. He looks at the establishment-level data of

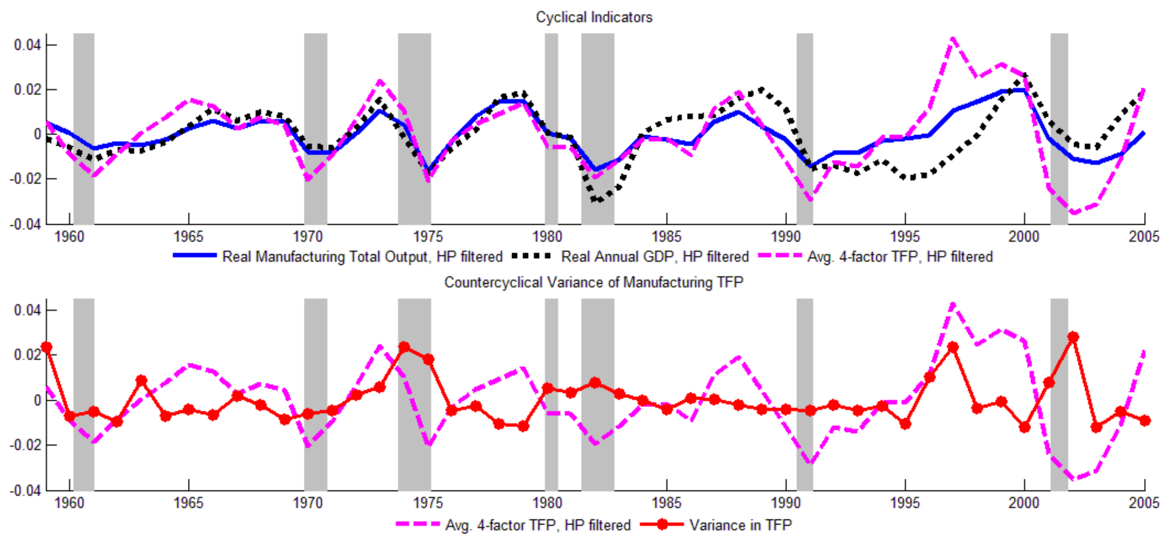

Figure 1: Cyclical Indicators and Variances in TFP. Upper panel plots different cyclical indicators, Real GDP (dotted line), Real total manufacturing output (solid line), Average TFP across industries at SIC 4 Digit level (dashed line). Lower panel shows cyclical behavior of TFP dispersion measured as variance (solid line with dots), together with Average TFP (dashed line). All series are HP-filtered. The shaded bars illustrate official NBER recessions. Real GDP data is from FRED; TFP series are from MIPD, and so is Manufacturing output measured as Real Total Shipment.

the US manufacturing sector that consists of the Annual Survey of Manufactures, Census of Manufactures, Plant Capacity Utilization Survey, and Longitudinal Business Database. Though the manufacturing sector as a whole shows countercyclical dispersion in establishment-level TFP, the durable industries show stronger cyclicality and it is the firms at bottom quantile of productivity distribution that drive the dispersion dynamics.

In this paper, I study how the aggregate economic state affects the dispersion in individual-level productivity. To link my model to data, ideally, the aggregate state is the average productivity measured as the cross-sectional average of plant-level TFP, and the dispersion is then the variance or inter-quantile range of plant-level TFP. Lacking the plant-level data, I use industry data at four-digit SIC level to approximate the desired measures. The paper is silent on the validity of this approximation, but Bloom et. al (2010) show that the countercyclical patterns of productivity dispersion are similar at the plant-, firm-, and industry-levels.

The upper panel of Figure (1) shows the co-movement of different business cycle indicators. In particular, I claim that the average TFP is a valid aggregate state indicator for the manufacturing sector. The correlation coefficient between average TFP (HP filtered) and sectoral output (HP filtered) is 0.86 with p-value of scale 10^{-9}. The average TFP corresponds to the cyclical indicator used through out the model, and the fluctuation in it represents technology or productivity shock, which drives the dynamics of model economy. Following Eisfeldt and Rampini (2006) and Bloom (2009), I use dispersion in cross-sectional TFP growth rate distribution at four digit SIC level to approximate that at the individual level, without arguing the validity of the approximation. Note that, the desired distribution is that of the *levels* of TFP instead of growth rates. The result is the lower panel of Figure (1), illustrating countercyclical movement of variance in TFP.[3] The precise correlation coefficients for the US manufacturing sector are documented in detail in both Bloom, Floetotto and Jaimovich (2010) and Kehrig (2011), and are summarized in Table 1 together with my own calculation.

Table 1. Correlations between Dispersion and Cyclical Indicator[4]

	For US Manufacturing Sector	GDP Growth	GDP HP Res.	Avg. ΔTFP
Kehrig (2011)				
(1)	Estab. TFP, Std. Dev. (Durables, HP Residual)	-0.420	-0.528	–
(2)	Estab. TFP, Std. Dev. (Non-durables, HP Residual)	-0.172	–	–
Bloom et. al. (2010)				
(3)	Estab. Output Growth, IQR	-0.364	–	–
(4)	Estab. TFP Growth, Std. Dev.	-0.273	–	–
(5)	Firm Sales Growth, IQR	-0.265	–	–
(6)	Firm Stock Returns, IQR	-0.339	–	–
Calculated from NBER-CES MIPD				
(7)	Ind. TFP Growth, IQR	-0.502 (0.000)	-0.298 (0.021)	-0.184 (0.108)
(8)	Ind. TFP Growth, Std. Dev.	-0.262 (0.038)	-0.241 (0.051)	-0.129 (0.194)
(9)	Ind. TFP Growth, Var.	-0.249 (0.046)	-0.245 (0.048)	-0.123 (0.206)

[3] I obtain data from the same sources as the aforementioned two papers, yet with more recent data up until 2005. I get the same signaficantly negative correlations as in these two papers if I only use the same range of data as they do. However, if I include the newly update data as shown in the figure, I can only a negative correlation that is not significant and is much smaller in absolue scale, which is less than 0.11.

[4] The first column of results show correlation coefficients (p-value) with Real GDP growth rate, the second with residuals of HP-filtered Real GDP, and the last with weighted average TFP growth rate in manufacturing sector.

Due to the limitation of data, I use dispersion measures at TFP growth rate instead of TFP level. The corresponding cyclical indicators are then GDP growth rate, sectoral output growth rate, and average TFP growth rate. To be comparable to other works, I only include GDP growth rate and GDP HP residuals in Table 1.

Firm Dynamics. One important cyclical feature of firm dynamics that motivates this paper is that exit rate moves countercyclically. This phenomenon is well documented in Campbell (1998) exploring ASM data between the second quarter of 1972 and the last quarter of 1988. In addition, Jaimovich and Floetotto (2008) assemble a new annual data set from 1956 to 1996 at the firm level across a broader range of industries and find that despite the difference in numbers, exit rates of all examined industries are countercyclical. To illustrate firm dynamics over time, I obtain annual data from 1977 to 2009 in Business Dynamics Statistics (BDS) at CES, a data set that recently became publicly accessible. To be consistent with micro-level evidence on countercyclical dispersion, I only look at the establishments in manufacturing sector. [5]

Table 2 summarizes the property of establishment entry and exit rates by firm size[6]. A firm is classified to be small if it has less than 50 registered employees. This is again not ideal, but

Row (1) and (2) are taken from Table 3 and 4 in Kehrig (2011), in which the microlevel data sources are mainly ASM/CM/LBD continuously covering period of 1972-2005 at annual frequency. Row (3) to (6) are from Table 1 in Bloom, Floetotto and Jaimovich (2010). Establishment-level data are also from ASM/CM/LBD, 1972-2006, while the firm-level infomation is from Compustat at quarterly frequency, 1967:II-2008:III for sales growth and 1969:I-2010:III for stock returns. Row (7) to (9) are TFP dispersions cross industries at four digit SIC level and NBER-CES Manufacturing Industry Productivity Database is the source, covering annually 1959-2005. Except for IQR, all other moments of industrial TFP growth are weighted by real value of total shipment. Numbers in parentheses are one-sided p-values under the null of non-negative correlation.

[5] A noteworthy issue here is how to define an entrant and an exiting establishment. According to the official overview of BDS dataset, "An establishment opening or entrant is an establishment with positive employment in the current year and zero employment in the prior year. An establishment closing or exit is an establishment with zero employment in the current year and positive employment in the prior year. The vast majority of establishment openings are true greenfield entrants. Similarly, the vast majority of establishment closings are true establishment exits (i.e., operations ceased at this physical location). However, there are a small number of establishments that temporarily shutdown (i.e., have a year with zero employment) and these are counted in the establishment openings and closings." Therefore, an inevitable caveat is that, although of relatively small number, an "idling" establishment can show up in the data as exit first, and then as entrant, for potentially many times. However, one clear advantage especially over firm-level data is that merging and acquisition are not reasons for disappearing units. Therefore, I can safely assume that exiting establishments suffer from low realizations of productivity.

[6] The entry and exit rates are indeed calculated utilizing the numbers of new born establishments, closed establishments, and existing establishments. However, the size is classified using the number of employees in a firm, instead of an establishment. One can only argue that large firms tend to own large establishments, and therefore large establishments exhibit similar dynamics to the ones owned by large firms. Otherwise, it is not clear whether this is a valid approximation.

subject to data availability. The preferred size classification is by capital stock. A more detailed illustration of entry and exit rates by year and by establishment size can be found in the Appendix.

Table 2. Entry and Exit Rates in Manufactures[7]

For US Manufacturing Sector 1977-2009				
		Total	Large	Small
(1)	Avg. Entry Rate (%)	9.36	5.18	31.18
(2)	Avg. Exit Rate (%)	9.28	6.00	30.06
(3)	Std. Dev. (Entry^{HP}) (%)	0.52	0.64	1.85
(4)	Std. Dev. (Exit^{HP}) (%)	0.67	0.90	1.56
(5)	Corr(Entry^{HP}, $(\text{Avg. TFP})^{HP}$)	0.20 (0.29)	0.19 (0.33)	0.21 (0.29)
(6)	Corr(Exit^{HP}, $(\text{Avg. TFP})^{HP}$)	-0.26 (0.17)	-0.17 (0.37)	-0.23 (0.24)
(5')	Corr(ΔEntry, Avg. ΔTFP)	0.22 (0.26)	0.13 (0.51)	0.31 (0.11)
(6')	Corr(ΔExit, Avg. ΔTFP)	-0.10 (0.62)	0.06 (0.76)	-0.06 (0.73)

Comparing establishment dynamics in small firms to that of large ones, they are of much larger scales, more volatile, and more cyclical. Therefore, in the quantitative model, I only focus on the dynamics in small firms, and treat the entry and exit of large firms mainly as exogenous, and they happen only with small probability.

The model I build in the following sections tries to explain the negative correlation between average productivity and cross-sectional productivity dispersion. The main mechanism emphasizes the different behavior between small and large firms, which leads to observed difference in their entry and exit dynamics.

3 A Simple Model

To highlight the mechanism, I start from a simplified and tractable three period version of the full model. I remove some features of the working model that is not as crucial, and focus only on the incumbents' problem. The main idea is that the option to exit promotes risk taking of

[7] The data source is still BDS. The binary grouping rule in size can be found in caption of Figure (2). In Row (1) and (2), the numbers are simple time series averages. Row (3) and (4) are time series standard deviations for HP residuals. Row (5) to (6) are correlations for HP residuals, (7) and (8) are for changes. Numbers in parenthesis are p-values. I choose to compute correlation coefficient this way instead of using original entry/exit rates because there is a declining trend in both series. This is an interesting observation on its own sake, but this paper is silent on it.

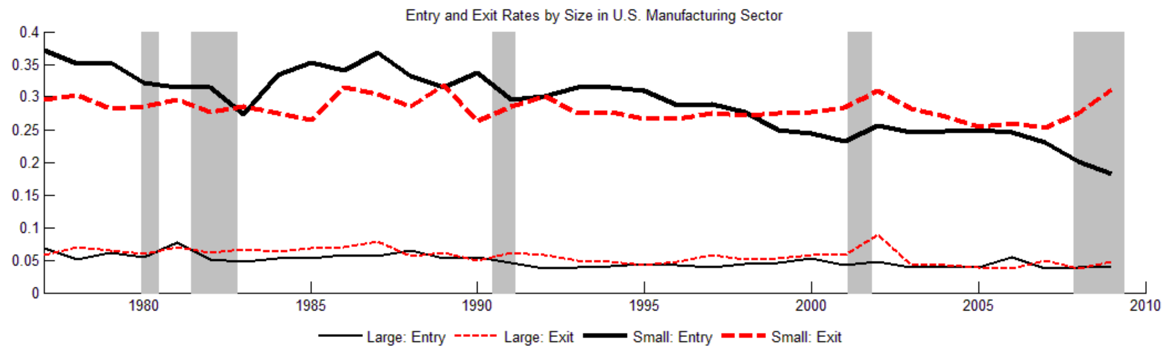

Figure 2: Cyclical behavior of entry and exit in manufacturing sector by size. A small firm is classified as one with less than 50 registered employees, and a large one with at least 50. This figure shows original series of entry (solid lines) and exit (dashed lines) rates by size. The two thinner lines at the bottom are for large firms, and the two thicker ones are for small firms. Data on entry and exit rates are from BDS of CES.

small firms by creating a local non-concavity in a firm's continuation value function, which in turn generates a non-degenerate dispersion in productivity. Moreover, as is shown in the comparative statics analysis, such dispersion becomes larger in bad time, due to a larger fraction of risk taking firms. The same mechanism drives the infinite horizon model as well.

3.1 Setup

There are 3 periods, $t = 0, 1, 2$. There are a continuum of risk neutral firm owners, each of whom owns a firm with different level of initial resource $w_0 \in [0, \bar{w}]$. Assume that each firm has only one plant or plant that produces one kind of product. The c.d.f. of owners' initial endowment of the single good is given as $G(w_0)$. At period 0, initial wealth w_0 can be divided into investment k_0 for future payoff and immediate consumption $w_0 - k_0$. If an owner decides to invest k_0, then she will get $w_1 = F(Z, k)$ as period 1 wealth, where

$$F(Z, k) = Zk^{\alpha}, 0 < \alpha < 1,$$

and Z represents the realized productivity of the technology the firm owner chooses after investment decision. A production project is associated with a technology. Assume that production requires full attention of the firm owner and utilizes the full capacity of the plant, hence a firm cannot undertake multiple production projects simultaneously. An owner can choose one and only one out of two available technologies: a safe one and a risky one, differing in the riskiness and realizations of productivity. For the safe technology, $Z = A$ for sure, while for a risky one, with

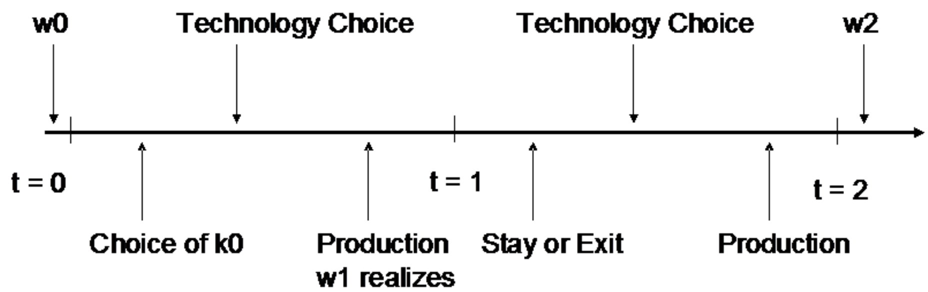

Figure 3: Timing of the Simple Model

probability $p \in (0,1)$, $Z = \bar{z} > A$, and with probability $1-p$, $Z = \underline{z} = 0$. Both technologies give the same expected value of Z, that is, $p\bar{z} + (1-p)\,0 = A$.[8] The risky technology has a variance in productivity as a function of p and $\bar{z}$, $\sigma^2(p,\bar{z}) = p(1-p)\,\bar{z}^2$. As a result of linearity of $F(Z,k)$ in Z, the expected flow output of the risky technology is the same as the safe one. Under this setup, A corresponds to the average establishment-level productivity measured as TFP in data, and plays the role of economic condition indicator (or cyclical indicator in the full model); the riskiness of the risky technology represents the risk at the establishment level, while its aggregated counterpart measures the dispersion in productivity.

3.2 Analysis

At period 1, after the uncertainty in Z realizes, the agent can decide whether to close her firm, exit the industry and get outside option value V^0, or stay. Conditional on staying, she makes the investment choice k_1 and technology adoption choice again based on period 1 wealth w_1. In the last period, she simply consumes her final wealth w_2. The objective of an agent with initial wealth w_0 is to maximize her discounted consumption, with discount factor β:

$$V_0(w_0) = \max_{0 \le k_0 \le w_0} \left\{ (w_0 - k_0) + \beta \max\left\{ V_1\left(Ak_0^\alpha\right), (1-p)\,V_1(0) + pV_1\left(\bar{z}k_0^\alpha\right) \right\} \right\}$$

where $V_t(w_t)$ is the time t value for an agent with wealth w_t.

It is convenient to work backwards. At time $t = 2$,

$$V_2(w_2) = w_2.$$

[8]For tractability, I assume only one type of risky technology and binary possible realization of it. In fact, a risky technology can be represented by a random variable Z with any distribution that is a mean preserving spread of A.

At time $t = 1$, an agent with $k_1 > 0$ will be indifferent between operating a safe project and a risky one. Assume that all agents will perform safely in this case, which is consistent with their choice if they were risk averse. For simplicity, I do not allow borrowing in the short model, and the period 1 value for a staying firm will be:

$$V_1^1(w_1) = \max_{0 \le k_1 \le w_1} \{(w_1 - k_1) + \beta A k_1^\alpha\}.$$

Let k^* be the optimal capital choice without borrowing constraint. The value of a firm with wealth level w_1 at the beginning of period 1 will be given by

$$V_1(w_1) = \max\left\{V^0, V_1^1(w_1)\right\}.$$

Let w_1^* be such that $V^0 = V_1^1(w_1^*)$. Note that there is a kink at w_1^* and $V_1(w_1)$ is convex in a neighborhood of w_1^*. This gives a firm with relatively low wealth level an incentive to take a risky project before it enters period 1. At $t = 0$, a firm makes the investment decision and chooses a technology:

$$\begin{aligned} V_0(w_0) &= \max_{0 \le k_0 \le w_0} \{(w_0 - k_0) + \beta \max\{V_1(Ak_0^\alpha), (1-p)V_1(0) + pV_1(\bar{z}k_0^\alpha)\}\} \\ &= \max_{0 \le k_0 \le w_0} \left\{(w_0 - k_0) + \beta \max\left\{V^0, V_1^1(Ak_0^\alpha), pV_1^1(\bar{z}k_0^\alpha) + (1-p)V^0\right\}\right\}. \end{aligned}$$

To explicitly characterize a firm's technology choice, it is useful to introduce the following condition on parameters.

Condition 1. $0 < V^0 < \alpha^{\frac{2\alpha^2}{1-\alpha^2}} \beta^{\frac{1+\alpha^2}{1-\alpha^2}} \bar{z}^{\frac{1}{1-\alpha}} p^{\frac{2\alpha^2}{1-\alpha^2}} \left(p^{1+\alpha} - p^2\right) / (1-p).$

The risky and safe continuation values intersect at most once in the region where they are both greater than V^0. This condition ensures the existence of intersection, and makes the analysis tractable as shown in Proposition 1. The intuition is that given $(\bar{z}, p)$, the option value V^0 of exiting cannot be too high, otherwise exit becomes very appealing, so does the risky technology. If it is violated, then all staying firms strictly prefer the risky technology. In particular, if V^0 is given, this happens when A is low enough.

Proposition 1. *At $t = 0$, if Condition 1 holds, then the continuation value functions associated with risky and safe technologies intersect only once, and $\exists k_0^I$ and k_0^{II} such that $0 < k_0^I < k_0^{II} < k^*$, and the decision rule of a firm owner with initial wealth w_0 will be one of the following:*

1. *If $0 < w_0 \le k_0^I$, she consumes all w_0 in period 0 and exits in period 1 for sure;*

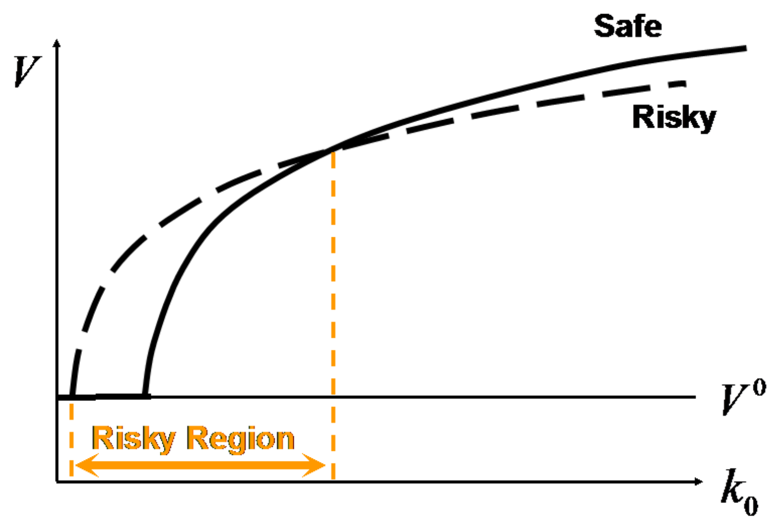

Figure 4: Continuation value as a functions of control variable, k_0. The horizontal axis is k_0, and the vertical axis is the continuation value for each level of k_0. The solid curve is the safe continuation value $V_1(Ak_0^\alpha)$, and the dashed curve is the risky continuation value $(1-p)V_1(0) + pV_1(\bar{z}k_0^\alpha)$. The horizontal line is V^0.

2. *If $k_0^I < w_0 < k_0^{II}$, she invests all w_0 in a risky project in period 0, then with probability p, $w_1 = \bar{z}k_0^\alpha$, she in turn invests all w_1 in period 1; with probability $1-p$, $w_1 = 0$, she exits in period 1;*

3. *If $k_0^{II} \leq w_0 \leq k_0^A$, she invests all w_0 in a safe project in period 0, then invests all $w_1 = Ak_0^\alpha$ in period 1;*

4. *If $k_0^A < w_0 \leq k^*$, she invests all w_0 in a safe project in period 0, then invests k^* and consumes the rest in period 1;*

5. *If $w_0 > k^*$, she invests k^* and consumes the rest in both periods.*

The interesting region, or the "risky region", is the interval $\left[k_0^I, k_0^{II}\right]$. The exiting option forms a lower bound in value function that is higher than in the case without exiting. This new lower bound alters the shape of continuation value function, in particular, the continuation value function has a local convexity if safe technology is chosen. This non-concavity region is roughly the same as the interval $\left[k_0^I, k_0^{II}\right]$, in which firms have limited amount of capital stock. Firms that fall into this region have incentive to smooth out such convexity by entering a lottery and randomizing over possible outcomes, which is exactly the role that risky technology plays in this model. The fraction of risk taking firms will then be determined given the initial distribution $G(w_0)$, and each of these firms bear the same risk in terms of the randomness of productivity[9]. As can be seen below, a change in A drives the changes in the risky region and the the fraction of risk taking firms, and leads to a different productivity dispersion.

Suppose that with probability p the risky technology realizes as high productivity. The cross-sectional variance in realized productivity in period 0, denoted as $\Gamma(p, \bar{z})$, is a function of p, the probability of good realization of risky technology, and $\bar{z}$, the good realization of productivity.

$$\begin{aligned}\Gamma(p, \bar{z}) &= E_{w_0,Z}\left(Z^2\right) - \left[E_{w_0,Z}(Z)\right]^2 \\ &= \sigma^2(p, \bar{z})\,\Lambda(p, \bar{z}),\end{aligned}$$

where Z represents the technology a firm chooses, and $\Lambda(p, \bar{z}) := \frac{G\left(k_0^{II}\right) - G\left(k_0^{I}\right)}{1 - G\left(k_0^{I}\right)}$ in which k_0^I and k_0^{II} are functions of p and $\bar{z}$ as well. $\sigma^2(p, \bar{z})$ is simply the variance of the Bernoulli distributed productivity of risky technology, while $\Lambda(p, \bar{z})$ represents the measure of firms in the risky region. $\Gamma(p, \bar{z})$ is ex ante variance, and coincides with realized dispersion in productivity, assumed a form of Law of Large Numbers. At the same time, the aggregate or average output in period 0, $O(p, \bar{z})$, is:

$$\begin{aligned}O(p, \bar{z}) &= E_{w_0,Z}\left(F(Z, k_0)\right) \\ &= p\bar{z}\int_{k_0^I}^{k^*} w_0^{\alpha}\, dG(w_0|k_0 > 0) + p\bar{z}(k^*)^{\alpha}\,\frac{1 - G(k^*)}{1 - G\left(k_0^I\right)}.\end{aligned}$$

3.3 Comparative Statics

The nature of the simple model does not permit cyclical features. Therefore, I will instead analyze the comparative statics mimicking different times of business cycles. In particular, I use A, the average productivity, as the economic condition indicator, which corresponds to the average TFP in data. In the model, a change in A can result from either a change in p, or in $\bar{z}$, or in both. Provided that the bad outcome of the risky technology is normalized to be zero, $\bar{z}$ then determines the range, the variance of the Bernoulli productivity $\sigma^2(p, \bar{z})$, and the measure of risky

[9] Once again, the same risk results from the assumption that only one way of randomization is permitted in the model for simplicity. To relax this restriction, one can assume that each firm can choose any distribution on productivity so long as the expection remains A, which results in a risky region larger than $\left[k_0^I, k_0^{III}\right]$. However, while making the model much more complicated, this will not alter the result qualitatively, neither will it provide more insight into the model.

region $\Lambda(p, \bar{z})$. At the same time, $\sigma^2(p, \bar{z})$ and $\Lambda(p, \bar{z})$ are also nontrivial functions of p. When A, p, and/or $\bar{z}$ changes, the resulting change in riskiness of a risky technology, that is, variance $\sigma^2(p, \bar{z})$ or range $\bar{z}$, is called the "riskiness effect", as such change directly affects the riskiness of available technology; and the change in the measure of firms in the risky region, $\Lambda(p, \bar{z})$, is the "mean effect", as the change in mean A determines the slope of continuation functions which in turn affects the width of risky region. The interesting one is the mean effect which highlights the novel mechanism of the model, therefore, I consider a particular change in A, such that $\bar{z}$ is held unchanged and p is also controlled to fully eliminate the riskiness effect, and examine the resulting mean effect.

Proposition 2. *Let V^0 and $\bar{z}$ remain unchanged and assume Condition 1 always holds. Let $A \in \{A^H, A^L\} = \{p^H\bar{z}, p^L\bar{z}\}$, p^H and p^L be such that $p^H > p^L > 0$. Suppose the distribution of initial wealth $G(\cdot)$ is Pareto or uniform and the lower bound of its support is below k_0^I when risky technology is p^H. Then:*

1. $O(p^H, \bar{z}) > O(p^L, \bar{z})$;
2. $\Lambda(p^H, \bar{z}) < \Lambda(p^L, \bar{z})$.

   To control the riskiness effect, assume $p^H + p^L = 1$, then:

3. $\sigma^2(p^H, \bar{z}) = \sigma^2(p^L, \bar{z}) = \bar{z}^2 p^H p^L$;
4. $\Gamma(p^H, \bar{z}) < \Gamma(p^L, \bar{z})$.

According to this proposition, given $\bar{z}$ fixed, A (or p) summarizes the aggregate state, higher A then means good times. When the aggregate state is good, the total output is high, and this is always the case whether the riskiness effect is controlled or not. Meanwhile, the risky region is smaller in good times, which in turn leads to smaller fraction of risk taking firms, regardless of the riskiness effect. The assumption on Pareto or uniform distribution is not very restrictive. In fact, it can be any distribution that results in the same pattern of change in fraction of risky firms. I choose Pareto distribution to mimic the actually observed size distribution of firms, which is only a sufficient but not necessary condition for the desired change in risky fraction. When the riskiness effect is controlled, the riskiness of a risky technology remains unchanged, therefore it is the change in fraction of risk taking firms that drives the change in resulting productivity dispersion, or average riskiness that firms choose to take, measured as variance in productivity.

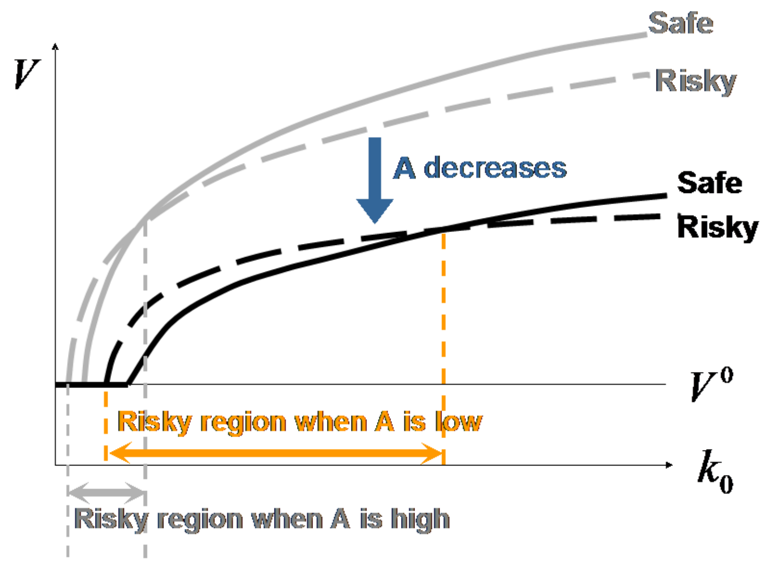

Figure 5: Comparative Statistics.

If $\bar{z}$ is not fixed or p is not controlled in such a way, then it is impossible to disentangle mean effect from riskiness effect, and these two effects jointly determine the resulting change in cross-sectional dispersion in productivity. In fact, in the calibrated quantitative model, it turns out that the riskiness effect is too small to generate significant difference in simulated results.

Figure (5) illustrates what happens to the model if A decreases, as described in Proposition 2. When A is low, the exiting threshold increases and more firms exit. At the same time, low A also leads to a larger risky region and a greater fraction of risk taking firms, so now there are more firms that strictly prefer to the risky technology. As a result, if the change in A is controlled as specified before, the average risk that firms choose to take is also larger, so is the realized productivity dispersion. To summarize, the key step for the model to generate countercyclical productivity dispersion is the change in the risky region as aggregate state changes. And it is mainly because of an enlarged fraction of risk taking firms that causes a larger productivity dispersion in bad times. This mechanism remains in the quantitative model with infinite horizon. In fact, if the aggregate state follows a Markov process with only two possible outcomes of A^H and A^L controlled in a similar way, then without introducing other features, the negative correlation between aggregate state and productivity dispersion is still almost perfect.

4 Quantitative Model

The simple three period model illustrates the main mechanism in a tractable setting. However, it is only feasible to look at the comparative statics in an essentially static model with three stages. Therefore, a richer model with infinite horizon is built in this section to include more realistic business cycle features and to examine the quantitative performance of the mechanism.

4.1 Setup

Time is discrete, with infinite horizon. The firms that have survived at least one period are called incumbents. There is a constant mass $M > 0$ of potential entrant firms every period, each of whom draws their initial capital k_0 from a distribution $G^0(k_0)$. $G^0(\cdot)$ determines the number and size distribution of newly born firms. Once entering, an entrant acts as an incumbent thereafter as long as this firm stays. The production function is the same as in the simple model, $F(Z,k) = Zk^\alpha$, with $0 < \alpha < 1$ and Z being the realized productivity depending on technology choice.[10] At the beginning of each period, all firms observe average productivity A. An incumbent firm owner makes the choice between staying and exiting, meanwhile, all firms also face an exogenous exiting probability $\eta > 0$. I allow additional exogenous exiting to generate the death of large firms, which always choose the safe technology, as in the simple model. If an incumbent exits, the owner closes her firm and sells all capital stock. Once exiting, the firm cannot come back to business again in the future. A staying firm then decides the amount of next period's working capital k' and whether to adopt the safe technology or the risky one. Again, assume full attention of a firm owner and complete utilization of plant capacity as a prerequisite of production. After production, capital

[10]In fact, $F(Z,k) = Zk^\alpha$ can be interpreted as a firm's profit function, that is, the revenue net of the cost for variable factors, for example labor and materials. Specifically, assume a plant faces an inverse demand function $P(y) = By^{-b}$, and therefore its revenue becomes $R(y) = By^{1-b}$. Suppose the actual production function is $y = \tilde{A}k^{\tilde{\alpha}}l^{\tilde{\phi}}$, and the price for other factors is ω, then after optimization of l, the revenue function becomes

$$R = \left(B\tilde{A}^{1-b}\right)^{1/\left(\tilde{\phi}(1-b)\right)} \left[\tilde{\phi}(1-b)/\omega\right]^{\tilde{\phi}(1-b)/\left(\tilde{\phi}(1-b)-1\right)} k^{\tilde{\alpha}(1-b)/\left(\tilde{\phi}(1-b)-1\right)},$$

and profit function

$$\pi = \left(1 - \tilde{\phi}(1-b)\right) R.$$

Redefining variables gives the form of Zk^α. Therefore, Z in the model is more appropriately interpreted as measured revenue total factor productivity that includes information from the demand side, instead of actual production technology. For the same reason, parameter A shown later in the model shall also be interpreted as aggregate state of the model economy, and change in A is more than just "technology shock". Under this specification, it is easier to link the model to data because only TFPR (TFP calculated using revenue data) is required for this model, but not TFPQ (actual TFP). Admittedly, TFPR is much easier to compute.

depreciates at rate δ.

Under these settings, firms in this economy are heterogeneous in realized productivity, capital stock, and depreciation rate in each period. Provided a realization of aggregate state, technology choice, investment, and depreciation jointly determine the incumbent's next period disposable resource.

The aggregate state for the model economy A evolves as a Markov chain with $A \in \mathbb{A} = \{A_1, ..., A_{N_A}\}$, and transition probability $\pi_{ij} = \Pr(A^j | A^i)$. In particular, this Markov chain is a discretized AR(1) process, such that $\ln A_t = \rho_A \ln A_{t-1} + \sigma_u u_t$, where $\rho_A \in (0, 1)$ is the serial correlation, and $u_t \sim N(0, 1)$ is white noise. Following conventional real business cycles models, I assume time invariant volatility in A, in terms of constant σ_u. This implies that the driving force of this modelled economy is the traditional "technology shocks", that is, the change in "first moment". This is different from Bloom (2009) and Bloom et. al. (2010), who use time varying higher moments as the pure source of aggregate fluctuation. Meanwhile, this also distinct from, for example, Bechmann and Bayer (2009a,b) and Chugh (2010), who allow time varying higher moments in addition to the usual first moment movement to account for the countercyclical dispersion observed in data. I do not allow σ_u to change over time is based on the following considerations that (1) σ_u is time series volatility, which is not the same as observed cross-sectional dispersion, (2) this model emphasizes a mechanism through which time varying A generates realized productivity dispersion, and it is of no need to introduce additional variation, and (3) fixed σ_u implies fixed unconditional mean of A.

Production is costly. In each period, a staying and active firm needs to pay a fixed operating cost, and, if the firm needs increase or decrease its capital stock, it pays a capital adjustment cost as well. Mainly following Cooper and Haltiwanger (2006) and Bloom (2009), I assume the capital adjustment cost consists of two parts: (1) a non-convex cost, and (2) a transaction cost. The non-convex cost represents the opportunity cost when a firm is under capital adjustment. Specifically, this firm foregoes a fraction c_k of its production if there is capital adjustment in a given period. The transaction cost represents the partial irreversibility. When a firm needs to increase capital, the price paid for every unit of new capital is normalized to be one, where the price is interpreted as how many units of output needed to get one unit of capital. However, if a firm wants to reduce capital, the selling price for each unit of capital is $\theta < 1$.

Each time period has several stages, which resembles period 1 in the simple three period model.

- Stage 1: Observation of state variables. Aggregate state A realizes, so does the capital depreciation. An incumbent firm observes A, and enters this period with remaining capital, $(1 - \delta) k$, and together with period's production $F(Z_{-1}, k)$, where Z_{-1} is the realization of last period's productivity of this firm. A potential entrant draws k_0 and observes A.

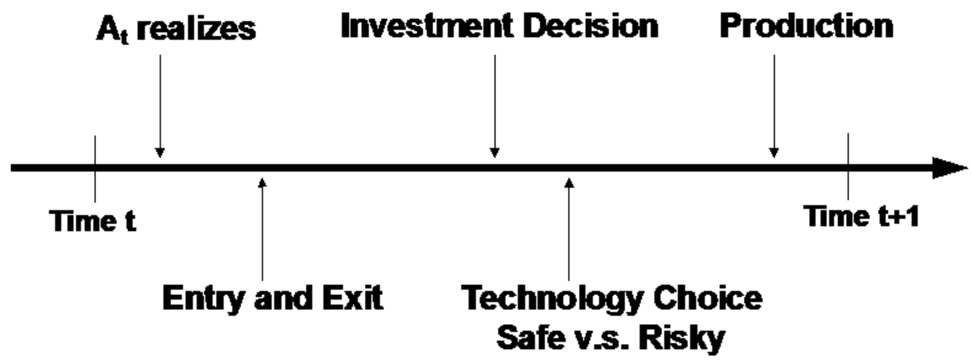

Figure 6: Timing of the Quantitative Model

- Stage 2: Entry and exit. An entrant with (k_0, A) enters if there is positive expected profit. An incumbent exits either voluntarily based on continuation values, or exogenously with probability η.

- Stage 3: Investment and technology decision. Both staying incumbents and new born firms decide how much to invest, and then choose between safe and risky technologies. At the same time, the operating cost and capital adjustment cost are paid.

- Stage 4: Production. Production takes place in the form $F(Z, k')$, where k' is the new working capital, and Z is the productivity. If a firm chooses safe technology, then the productivity is deterministic, $Z = A$. Otherwise, with probability $p(A)$, the risky technology turns out to be a success, $Z = \bar{z}$, and with probability $1 - p(A)$, it fails, and $Z = 0$.

4.2 Individual Decision

An Incumbent's Problem. At the beginning of each period, an incumbent firm is characterized by (Z_{-1}, k, A), where $Z_{-1} \in \{A_{-1}, 0, \bar{z}\}$ is the realized productivity of last period for a specific firm, which can be either of the safe productivity A_{-1}, the bad realization 0, or the good realization $\bar{z}$, k is the total amount of capital that was used in last period, and A represents the economic condition of current period.[11]

The first choice an incumbent firm owner makes is between keeping operating and closing the firm and leaving.

$$V(Z_{-1}, k, A) = \max (1 - \chi) V^1 (Z_{-1}, k, A) + \chi V^0 (Z_{-1}, k, A),$$

[11] The distribution of firms is not a state variable in this model, because it has an essentially partial equilibrium setup, and agents do not need to forecast future prices using information on distribution.

where $\chi \in \{\eta, 1\}$ is the exiting choice, and η is the exogenous exiting hazard. If a firm with (Z_{-1}, k, A) chooses to exit, the value is:

$$V^0 (Z_{-1}, k, A) = \theta (A) (Z_{-1} k^\alpha + (1 - \delta) k);$$

where $\theta (A) < 1$ is the fraction of resource a firm owner can take away when exiting, which is actually a resale price and is potentially a function of A. If this firm chooses to stay, the owner must then decide on investment, i, and technology choice, safe or risky. The capital stock evolves as follows

$$k' = (1 - \delta) k + i,$$

such that $k' \geq k_{\min} > 0$, where $k_{\min}$ is a very small positive number providing a lower bound of capital stock. The operating cost $C (i; Z_{-1}, k, A)$ of a firm consists of a fixed cost c_f and a capital adjustment cost:

$$C (i; Z_{-1}, k, A) = c_f + c_k F (Z_{-1}, k) 1_{\{i \neq 0\}} + (1 - \theta (A)) (-i) 1_{\{i<0\}}.$$

Apart from the fixed operating cost, there are two forms of capital adjustment costs: a non-convex adjustment cost and partial irreversibility. Actively adjusting capital stock and choosing $i \neq 0$, costs a firm c_k fraction of its revenue from last period's production. In addition, if a firm reduces its scale, it can only sell its current capital possession at price $\theta (A) < 1$. The fixed operating cost is to generate endogenous exiting behavior and therefore it creates a non-concave portion in the lower end of a firm's value function. The adjustment cost plays double role: one is to capture the observed inaction in investment and slow down the change in firm size, and the other is to dampen firms' reaction to change in aggregate states so that the correlation between productivity dispersion and aggregate state is not too close to -1. Combining these pieces gives the flow profit of this firm $D (k'; Z_{-1}, k, A)$, and

$$P (i; Z_{-1}, k, A) = F (Z_{-1}, k) - i - C (i; Z_{-1}, k, A) \geq 0.$$

I enforce non-negative profit as a constraint. The firm also has to choose between safe and risky technology. A safe technology produces $F (A, k')$ for sure; while a risky technology results in productivity at $\bar{z}$ with probability $p (A)$ and 0 with $1 - p (A)$. If the safe one is chosen, the firm gets:

$$V^1_{safe} (i; k, A) = \mathbb{E}_{A', \delta'} [V (A, k', A') | A],$$

and likewise,

$$V^1_{risky} (i; k, A) = p (A) \mathbb{E}_{A'} [V (\bar{z}, k', A') | A] + (1 - p (A)) \mathbb{E}_{A'} [V (0, k', A') | A].$$

Therefore, conditional on staying, an incumbent firm solves the following maximization problem:

$$V^1(Z_{-1}, k, A) = \max_i \left\{ P(i; Z_{-1}, k, A) + \beta \max \left\{ V^1_{safe}(k'; Z_{-1}, k, A), V^1_{risky}(k'; Z_{-1}, k, A) \right\} \right\}.$$

Denote the state variables of an incumbent as $\psi = (Z_{-1}, k, A) \in \Psi$, with Ψ being the set of all possible states. Solution to an incumbent's question with state ψ is a list of policy functions $\{\chi(\psi), \tau(\psi), \iota(\psi)\}$ such that (1) $\chi(\psi)$ is the exiting choice, $\chi : \Psi \to \{\eta, 1\}$; and conditional on surviving, (2) $\tau(\psi)$ is the technology choice, $\tau : \{\psi \in \Psi : \chi(\psi) = \eta\} \to \{0, 1\}$, where 0 represents the safe technology and 1 the risky one, and (3) $\iota(\psi)$ is the investment level, $\iota : \{\psi \in \Psi : \chi(\psi) = \eta\} \to \mathbb{R}$.

A Potential Entrant's Problem. A potential entrant draws initial capital holding k_0 from a invariant Pareto distribution $G^0(k_0)$ with parameter ξ. The value of staying outside the market is

$$V^0_0(k_0, A) = \theta(A) k_0.$$

To start up a business, one must pay a setup cost c_e from initial capital, and thereafter acts as an incumbent with state (Z_{-1}, k, A) being $\psi_0 = (0, (k_0 - c_e)/(1-\delta), A)$. Hence, the payoff of opening a firm will be:

$$V^1_0(k_0, A) = V^1(0, (k_0 - c_e)/(1-\delta), A).$$

A new firm will be born if

$$V^1_0(k_0, A) > V^0_0(k_0, A).$$

Solution to this problem is a binomial entry choice $\varepsilon : \Psi_0 \subset \Psi \to \{0, 1\}$, where Ψ_0 contains all possible ψ_0, and $\varepsilon(\psi_0) = 1$ if an entrant enters and 0 otherwise.

4.3 Aggregate Dynamics

Given the solutions to the individual problems described before, $\{\chi(\cdot), \tau(\cdot), \iota(\cdot); \varepsilon(\cdot)\}$, it is straightforward to write down the transition dynamics for the distribution over $\psi = (Z_{-1}, k, A)$.

For an arbitrary $\psi \in \Psi$, it is either $\psi \in \Psi_0$ or ψ can only be the state of an incumbent. I denote $\phi(\psi)$ as the measure or density of point $\psi = (Z_{-1}, k, A)$ at Stage 1 of a typical period with aggregate state A, before entry and exit takes place. If $\chi(\psi) = 1$, then a firm with this state exits for sure, and no other transition can happen. If $\chi(\psi) = \eta$, then with probability η this firm exogenously exits, and with a complementary probability, it stays. Conditional on staying, if the firm chooses the safe technology, $\tau(\psi) = 0$, then its individual state becomes $(A, (k + \iota(\psi)))$. On the other hand, if the firm chooses the risky technology, $\tau(\psi) = 1$, then with probability $p(A)$ its individual state becomes $(\bar{z}, (k + \iota(\psi)))$, and with probability $(1 - p(A))$ it becomes

$(0,(k+\iota(\psi)))$. Now turn to the new borns. Denote $g^0(\psi_0)$ the entrant's measure or density at point ψ_0 determined by $G^0(\cdot)$. A new born with ψ_0 enters if $\varepsilon(\psi_0)=1$. After entering, this firm acts exactly the same as a surviving incumbent with $\psi=\psi_0$. Finally, the aggregate states becomes A' with probability $\Pr(A'|A)$, $A'\in\mathbb{A}$. Formally, suppose the aggregate state at Stage 1 of a period is $A'=A_j$, and that of last period is $A=A_i$, meaning the realized productivity Z is one of $\{A_i,\bar{z},0\}$. Every state not on the realization path has zero measure, or

$$\phi'(A,k',A')=0 \text{ if } A\neq A_i \text{ or } A'\neq A_j,$$

where primed variables are ones realized at the same period as A'. The rest of the states can then be divided into three groups by realization of Z, all of which come from both incumbents and new borns. For $Z=A_i$,

$$\begin{aligned}\phi'(A_i,k',A_j) &= \int(1-\chi(\psi))(1-\tau(\psi))\,\mathbf{1}_{\{\psi:k'=(1-\delta)k+\iota(\psi)\}}\phi(d\psi)\\ &\quad +M\int\varepsilon(\psi_0)(1-\tau(\psi_0))\,\mathbf{1}_{\{\psi_0:k'=(1-\delta)k_0+\iota(\psi_0)\}}g^0(d\psi_0),\end{aligned}$$

where variables with no prime are ones observed one period back, with $\psi=(Z_{-1},k,A_i)$ and $\psi_0=(0,(k_0-c_e)/(1-\delta),A_i)$. For $Z=\bar{z}$ or 0,

$$\begin{aligned}\phi'(\{\bar{z},0\},k',A_j) &= \int(1-\chi(\psi))\tau(\psi)\,\mathbf{1}_{\{\psi:k'=(1-\delta)k+\iota(\psi)\}}\phi(d\psi)\\ &\quad +M\int\varepsilon(\psi_0)\tau(\psi_0)\,\mathbf{1}_{\{\psi_0:k'=(1-\delta)k_0+\iota(\psi_0)\}}g^0(d\psi_0).\end{aligned}$$

By independence, a fraction $p(A_i)$ has $Z=\bar{z}$, and the rest gets $Z=0$, that is,

$$\begin{aligned}\phi'(\bar{z},k',A_j) &= p(A_i)\,\phi'(\{\bar{z},0\},k',A_j),\\ \phi'(0,k',A_j) &= (1-p(A_i))\,\phi'(\{\bar{z},0\},k',A_j).\end{aligned}$$

Given the distribution measure ϕ and ϕ', the cross-sectional variance in productivity can be written as

$$\begin{aligned}\Gamma(A,\phi) &\propto \int\bar{z}^2\phi'(\bar{z},dk',dA')+\int A^2\phi'(A,dk',dA')-\left[\int\bar{z}\phi'(\bar{z},dk',dA')+\int A\phi'(A,dk',dA')\right]^2\\ &= \bar{z}^2p(A)(1-p(A))\int\phi'(\{\bar{z},0\},dk',dA')=\sigma^2(A)\Lambda(A,\phi).\end{aligned}$$

The expression of cross-sectional variance can be simplified in this way due to the linearity of productivity in production function.

4.4 Calibration

Before the description of calibration procedure, it is worth noticing that the mass of potential entrants M only affects the scale of the economy once other parameters are determined. Since the absolute scale is not of interest, the choice of M is irrelevant. For quantitative exercise, the number of potential entrants is fixed at 50000 each period. Furthermore, without aggregate fluctuation, starting from zero incumbents, the economy always converges to a stationary state in the sense that the exiting rate and entry rate are equal and the scale is neither expanding nor shrinking, as long as there is positive measure of entrants at the beginning, and this is the case with or without agents expecting the aggregate state to be varying over time. The reason is simple. Since there is no aggregate fluctuation, the measure of entrants (inflow) is fixed each period. The measure of exiting firms (outflow) is a fraction of remaining ones (stock). The outflow gradually increases to the same level as the inflow, and it is at this point that the scale of stock stops changing. Consequently, the entry and exit rates are the same. Because of this stationarity feature, the parameters that need to be internally determined are selected such that the statistics generated by the model at its stationary state match their empirical targets.

The setup of the model is very close to the standard, therefore some of the parameter values are directly taken from the literature. One period is chosen to be one year. The discount factor is set as $\beta = 0.938$ to match the long-run average for U.S. firm-level discount rate, as in Bloom (2009). According to the same paper, capital depreciates at rate $\delta = 0.1$. The production function, $F(Z, k) = Zk^{\alpha}$, is the same as the profit function in Cooper and Haltiwanger (2006), so I follow their estimation and set α to be 0.592. Taken from the same work, the standard deviation of aggregate process σ_A is 0.08, and the serial autocorrelation ρ_A is assumed to be 0.8 which is within the range between autocorrelation of common shock 0.76 and that of idiosyncratic shock 0.885 estimated in that paper.

The good productivity realization is predetermined as $\bar{z} = 2$ so that the probability of getting $\bar{z}$ is always around a half. This is to minimize the riskiness effect by controlling the uncertainty associated to the binary outcomed risky technology. The exogenous exiting hazard η that affects all firms alike is set to be 2%, which is in line with the exiting rate of large plants found by, for example, Lee and Mukoyama (2008). On the entrant side, it has been mentioned that the choice of M is not important. The distribution of initial endowment G^0 is Pareto such that, with slight abuse of notation, $G^0(k_0) = 1 - (k_{\min}/k_0)^{\xi}$ with $\xi > 0$. Clearly, ξ governs the shape of initial endowment distribution and it in turn determines the model generated firm size distribution. Ideally, this generated distribution shall also has a shape close to Pareto, however, the assumption on one common productivity shock and no idiosyncratic shocks makes this task infeasible. This can be corrected by introducing heterogeneous productivity, yet this practice will not provide more economic insight to this model. Therefore, for the numerical results, I set $\xi = 1$.

The remaining parameters to be internally calibrated are capital resale price θ, capital adjustment cost as a fraction of profit c_k, fixed operating cost c_f, and entry cost c_e. The model suggests that I shall look at the statistics of firm dynamics and investment rate distribution, and the remaining parameters (θ, c_k, c_f, c_e) are selected via simulated method of moments. The targets regarding firm dynamics are taken from Lee and Mukoyama (2008), and those on investment rate distribution are from Cooper and Haltiwanger (2006). I also compute from the model average five-year transition rates between different size classes, and compare the generated numbers to the actual rates found by Acemoglu, Akcigit, Bloom, and Kerr (2011) using census data. The parameters are calibrated without aggregate fluctuation, and the aggregate state sequence, $\{A_t\}$, is set to be constant at its mean, but the firms still expect the future states to be changing according to the transition probability of A, π_{ij}.

Table 3. Parameter Values and Rationale[12]

Parameters	Description	Notes
Aggregate Fluctuation		
$\bar{z} = 2$	Good productivity realization.	Predetermined. Normalization.
$\rho_A = 0.8$	Autocorrelation.	Cooper and Haltiwanger (2006)
$\sigma_u = 0.048$	Var. of innovation s.t. $\sigma_A = 0.08$.	Cooper and Haltiwanger (2006)
Production		
$\alpha = 0.592$	Production function parameter.	Cooper and Haltiwanger (2006)
$\beta = 0.938$	Discount factor.	Bloom (2009)
$\delta = 0.1$	Capital depreciation rate.	Bloom (2009)
$\eta = 0.02$	Exogenous exiting probability.	Lee and Mukoyama (2008)
$\theta = 0.84$	Capital resale price.	Internally determined.
$c_f = 1.62$	Fixed operating cost.	Internally determined.
$c_k = 0.165$	Capital adjustment cost.	Internally determined.
Entrants		
$c_e = 0.1$	Entry cost.	Internally determined.
$\xi = 1$	Shape of G^0.	Predetermined.

[12] I also tried several other sets of parameters. The negative sign of the correlations between aggregate state and dispersion measures is robust, which is not surprising because the mechanism works under mild restrictions of parameter space. However, it is true that the fraction of risky firms is sensitive to the shape of value function. In

Calibrated parameter values are summarized in Table 3, and simulated moments are compared with their empirical counterparts in Table 4. Cooper and Haltiwanger (2006) compute a thorough set of investment moments using a balanced panel from the LRD from 1972 to 1988. The model generated moments are close to their target with expected exceptions. The standard deviation in investment rates is much lower than data, because when the aggregate fluctuations are shut down, there is no idiosyncratic uncertainty other than the riskiness a firm chooses to take. With constant aggregate state and no growth, the model generated mean level of investment rate, together with fraction of large and positive investment rates, is below the target as well. The other set of targets regards the entry and exit dynamics of firms, which are taken from Lee and Mukoyama (2008). They use the ASM portion of the LRD from 1972 to 1997 to analyze the behavior of plants. At the same time, I look at the five-year transition rates between different size classes obtained by Acemoglu et. al. (2011) using the CM portion. Firms are divided into two size classes, small and large, by median shipments, and the third class is "not-in-business". For example, the transition rate from class small to class large is computed as the fraction of originally small firms that became large ones in the next census. Since the census data is only available every five years, I let the model produce the same transition rates for every five periods. Due to different sources of data, I choose to hit a number within the range of empirically computed entry and exit rates. The model failed to reproduce the eight transition rates, although it manages to capture the fact that small firms have higher exiting rates than large ones. Without assuming idiosyncratic shocks, the model cannot generate a highly right skewed size distribution with a relatively small median, therefore the simulated exiting rate is lower. At the same time, no further heterogeneity causes the large transition rates between large and small classes.

particular, when β is high, future profit flows are important, and the risky fraction declines, so does the exit rate. The realizations of δ are set to be $\{0.05, 0.1, 0.2, 0.5, 1\}$ with probabilities $\{0.69, 0.155, 0.1, 0.05, 0.005\}$ respectively.

Table 4. Moments Generated from Model and Targets

	Model	Data
Investment		
Mean of investment rate	0.097	0.122
Std. Dev. of investment rate	0.157	0.337
Fraction of inaction	0.059	0.081
Fraction w. positive investment	0.889	0.815
Fraction w. positive investment burst	0.064	0.186
Fraction w. negative investment burst	0.033	0.018
Data Source: Cooper and Haltiwanger (2006)		
Entry and Exit		
Mean entry rate	0.070	0.062
Mean exit rate	0.070	0.055
Relative size, entering	0.75	0.60
Relative size, exiting	0.58	0.49
Data Source: Lee and Mukoyama (2008)[13]		

4.5 Quantitative Results

The mechanism explained in the illustrative three-period model remains at work in the quantitative model with infinite horizon. The option to exit forms a lower bound for an incumbent's continuation value function, and in a conventional model without the additional choice of risky technology, this lower bound in turn creates a non-concave portion on the continuation value at the lower end with low capital levels. When the choice of risky technology is allowed as in this model, firms with capital levels in this portion have incentive to smooth out the non-concavity by taking the risk. Of course, anticipating future option of the risky technology, the continuation value function associated to the safe one becomes less convex compared to the conventional case.

The business cycle features can now be introduced in a more realistic fashion than comparative statics. Without recalibrating, I add the aggregate fluctuation by simulating a sequence of realizations of productivity level A, and let the model evolves accordingly. As the aggregate state changes, the reaction of firms is still very similar to the comparative statics in the simple model. If A drops, the slopes of both risky and safe continuation value functions decrease, which

[13] Lee and Mukoyama (2008) calculate the relative sizes of entering and exiting firms based on number of employees.

forms a larger portion where risky technology is strictly better. Consequently, a larger fraction of firms opt to take the risk, which results in a larger cross-sectional standard deviation in productivity. Therefore, the productivity dispersion is larger. The opposite happens when A increases. Nonetheless, provided the frictions and the law of motion of aggregate state, the magnitude of the changes in fraction of risk taking firms and in resulting standard deviation in productivity is history dependent.

The main goal of this numerical exercise is to show that changes in the level of A_t alone can generate countercyclical firm-level productivity dispersion as a result of firm's risk taking behavior, without introducing any time varying volatility in the driving force, A_t. The fluctuation in productivity A follows the Markov process specified in the Table 3, and not surprisingly, it is positively correlated with the total output with correlation coefficient 0.4030 (p-value = 0.0000). Therefore, the cross-sectionally averaged productivity can serve as an alternative cyclical indicator. The measures for productivity dispersion are chosen to be (1) standard deviation of cross-sectional distribution of realized Z, productivity, (2) fraction of firms that prefer risky technology, and (3) the 95% to 5% interpercentile range of realized Z, which is the value of Z at 95% percentile minus the value of Z at 5% percentile.

Table 5. Generated Cyclicality

Cyclicality: Correlations (p-value) with Cyclical Indicators

Variables of Interests		Cyclical Indicators: Avg. Productivity, A	Cyclical Indicators: Total Output, O
Productivity Dispersion	$std.dev.(Z)$	-0.4450 (0.0000)	-0.6969 (0.0000)
Frac. of Risky Firms	Λ	-0.4544 (0.0000)	-0.6063 (0.0000)
Interpercentile Range 95%-5%	IPR_5^{95}	-0.2089 (0.0000)	-0.6860 (0.0000)
Entry Rate	r^{EN}	0.0314 (0.4830)	-0.7679 (0.0000)
Exit Rate	r^{EX}	-0.4774 (0.0000)	-0.5649 (0.0000)

Table 5 shows that the correlation coefficients between productivity dispersion and cyclical indicators are significantly negative, and the absolute values are in line with the data counterparts. In fact, the correlations between productivity dispersion and total output is even larger in scale. Moreover, the cyclicality of productivity dispersion measured is in comparable scale to that of the fraction of firms that choose risky technology, and the movements are in very similar patterns as can be read off from Figure (8). This illustrate the mechanism that it is the change in fraction of risk taking firms that drives the cyclical movement of productivity dispersion. In bad times, more

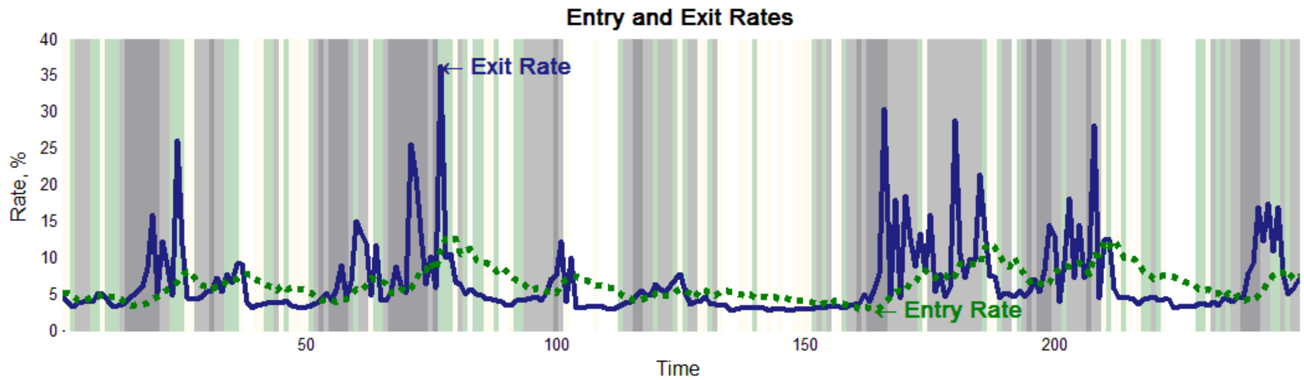

Figure 7: Simulated sequences of entry and exit rates. The solid line represents the exit rates, and the dashed line records entry rates. Grey bars indicate the value of A as in previous figure.

firms are willing to take the risk and randomize their future values. Consequently, the resulting dispersion measured as standard deviation of cross-sectional productivity distribution is larger, so is the interpercentile range.[14] The assumed binomial outcome of a risky technology has the potential to impact the behavior of the dispersion, however, such impact is controlled at a much smaller scale by the choice of μ_A and $\bar{z}$ and does not alter the main pattern. A somewhat unusual result is the significantly negative correlation between total output and entry rates. This is a result of modelling technique. The entry decision of potential entrants depends largely on discounted and expected future payoff, so the impact of current aggregate state is minimal. At the same time, entry rates increase when number of existing firms is smaller. However, the total output is not only a function of current state A, but it also positively depends on the number of existing firms. These two forces drive the entry rate series to move in the opposite direction to total output.

Figure (7) plots the truncated series of entry and exit rates from the model simulation. The sequence of exit rates remains mostly in a reasonably scale between 3 to 12 percent. On the contrary, there are quite a few episodes in which exit rates are really high. Extraordinarily high exit rates happen after a succession of bad realizations of aggregate state A, when the number of remaining firms is small. This is not surprising under the model assumptions that (1) all firms

[14] Due to the model assumption, cross-sectional IPR in productivity can only be either $\bar{z}$, $\bar{z} - A_t$, or A_t, and does not have very interesting dynamics, although it is still countercyclical. This can be overcome by allowing a richer set of productivity lotteries and keeping the expected productivity to be A. For example, in addition to $(p(A), \bar{z})$, firms can also choose any $(p, \bar{z}_A)$ pair with binary outcomes such that $p\bar{z}_A = A$. Intuitively, the IPR measure in this case will again be negatively correlated to A_t because smaller firms have incentive to take even more risk in bad times than in the original case. Therefore, the range of realized productivities is wider, and potentially the IPR is larger and has more possible values.

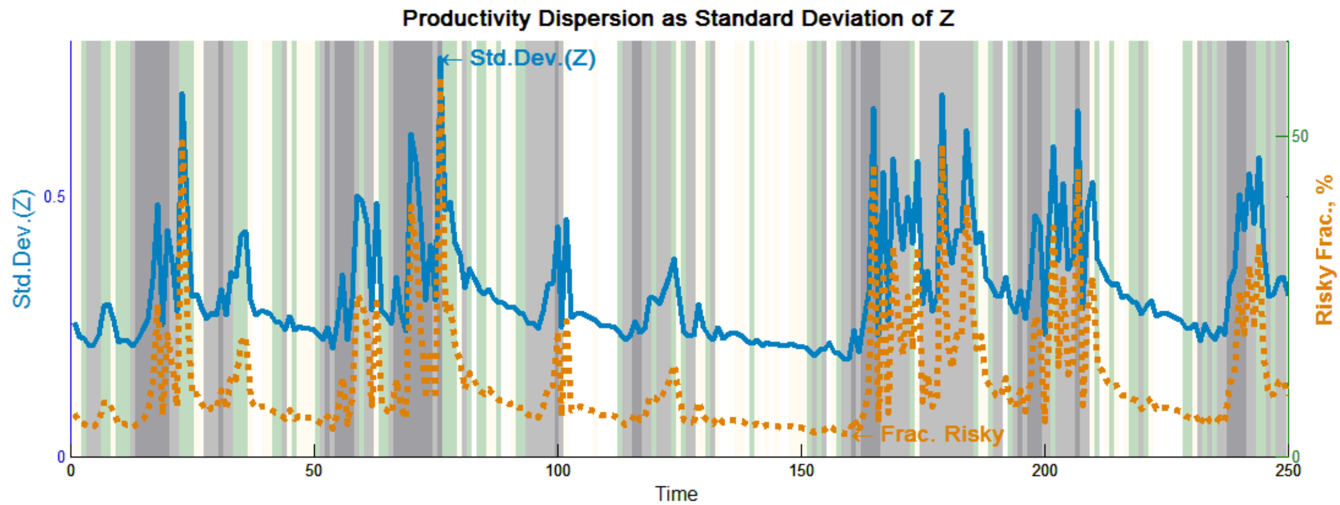

Figure 8: Simulated sequences of (1) cross sectional productivity dispersion measured as standard deviation in realized productivity Z (solid line, left axis), and (2) fraction of firms that choose risky technology (dotted line, right axis, in %). The grey bars indicate the economic condition as value of A. In particular, darker bars represent lower values of A.

share the same serially correlated A with no idiosyncratic shocks, and (2) given each realization of A, there is only one alternative risky technology allowed. Figure (8) shows the truncated sequences of countercyclical cross-sectional standard deviation in productivity and fraction of risk taking firms in each period. The realized standard deviation in productivity is mostly ranging between 0.25 and 0.65, and the fraction of firms choosing risky technology is mostly between 10% and 55%. Peaks of productivity dispersion and risky fraction are associated to excessive exiting rates, as the mechanism suggests.

Figure (9) shows how the productivity dispersion and fraction of risk-taking firms will react to a drop in A from its mean level. Originally, the model is simulated in the same way as it is for calibration: the aggregate fluctuation is shut down by fixing A at its mean level μ_A, while the firms behave under the belief that A evolves according to π_{ij}. Then, the value of A suddenly and permanently switches to one standard deviation lower, $\mu_A - \sigma_A$, and the firms' belief remains unchanged. The risky fraction and productivity dispersion increases immediately upon impulse, then oscillate with an ascending trend, and eventually settles at a higher level. The two paths may seem unusual at the first glance, but it is the joint work of (1) technology choice and (2) capital adjustment costs. Upon the bad shock, as the result of higher entering threshold, the number of entrants immediately drops to a lower level and then remains constant, and the scale of the economy measured as the total number of remaining firms decreases gradually to a new

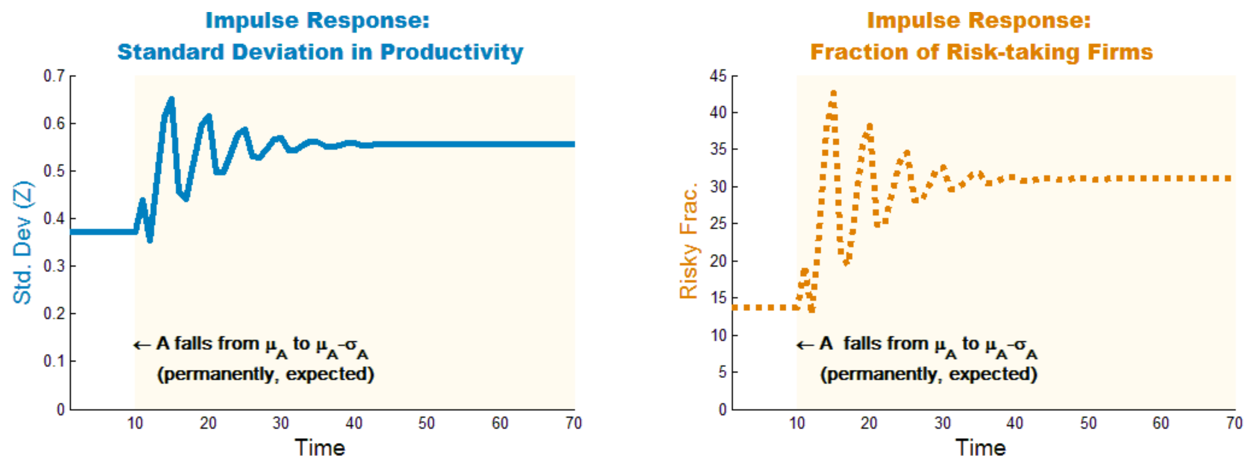

Figure 9: Impulse responses to a permanent (and expected) one standard deviation drop in aggregate state. The left panel is the response of cross-sectional productivity dispersion measured as standard deviation in realized productivity. The right one plots that of fraction of firms choosing risky technology.

stable level. If capital adjustment costs are shut down, then both the absolute number and the fraction of risk taking firms jump up upon impulse and drop in the following period. The reason of this sudden jump and plump is that the risky technology becomes more appealing to firms with a wider range of capital stock when the shock hits, even though there is a higher probability of bad outcome. Consequently, a large number of firms exit due to the choice of risky technology, which leaves less firms remaining in the risky region and this causes the following plump. The absolute number of risky firms then gradually decreases while the fraction increases to a higher level because of decreasing scale. This up-and-down trend is in line with what is shown in Figure (9), which is driven by the technology choice. On the other hand, the oscillation is due to the capital adjustment costs, which creates firms' inaction in investment and prevents firms from freely changing their capital stocks. Therefore, firms which should be in the risky region in the free adjustment case may now be outside, and vice versa. Note that, the fraction of risky firms is around 14% when A is kept at its mean, corresponding to the standard deviation in productivity at about 0.37. Cooper and Haltiwanger (2006) find that the plant-specific idiosyncratic shock has a standard deviation of 0.64. Without assuming idiosyncratic risk, the calibrated stationary version of this model is capable of reproducing at least half of the micro-level standard deviation.

5 Concluding Remarks

Productivity dispersion tends to be larger during recessions. The prevailing view is that increased uncertainty causes decline in aggregate economic activities. However, although uncertainty matters, this story seems to contradict the observation that recessions lead increase in productivity dispersion. To complement existing theories, I explore a simple mechanism through which aggregate fluctuations due to standard "technology shocks" can endogenously generate countercyclical dispersion in individual productivity. I alter the standard industry dynamics model with business cycle features by incorporating technology choice as part of the individual decision problem. By this feature, a firm in this model can then decide the riskiness of its production. The resulting productivity distribution is non-degenerate even if no other heterogeneity is modeled. The model provides the following predictions: small firms are more likely to take risk and have lower survival rates, but conditional on surviving, they exhibit higher productivity; a larger fraction of firms become risky in bad times, which also leads to higher exit rate; and realized individual-level productivity dispersion is larger in recessions.

Table 6. Additional Moments: Transition Rates

5-Year Transition Rates	Model	Data
Small → Exit	0.3491	0.5032
Small → Small	0.2900	0.4203
Small → Large	0.3609	0.0764
Large → Exit	0.2755	0.1803
Large → Small	0.3228	0.0564
Large → Large	0.4017	0.7633
Entry → Small	0.5070	0.7483
Entry → Large	0.4930	0.2517

Data Source: Acemoglu et. al. (2011)

The mechanism that I suggest in this paper is clearly complementary to the literature. The comparison between the model generated moments and empirical counterparts suggests that this is not the whole story and that there are some possible extensions for future work. The additional moments in Table 6 indicate that the shape of firm size distribution generated from the model is considerably different from the true one. Without altering the mechanism, introducing further heterogeneity in productivity can at least partly overcome this issue. In addition to that, adding more shocks, such as micro-level idiosyncratic shocks, and allowing for a richer set of risky technology, can improve the fit of calibration targets, especially the standard deviation in investment rates. This can also help reduce the extraordinarily high exit rate under aggregate fluctuation. Again, these extensions will not alter the mechanism at work. A potentially more interesting extension that I am currently working on is to generalize the model into a general equilibrium framework. One way to do so is endogenize the capital market in which exiting firms and shrinking firm can sell their capital holdings to growing ones. This may also help correct the high exit rate during really bad times.

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