Investment and firm dynamics

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March 2006

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First Draft: March 1, 2006
This version: April 19, 2007

Abstract

In this paper I ask whether a model of firm capital accumulation with entry and exit calibrated to match the investment regularities of U.S. establishments is capable of generating the dependence of firm dynamics on size and age. Firms face uncertainty in the form of idiosyncratic productivity shocks and are subject to non-convex capital adjustment costs. I solve for the stationary equilibrium to show that the model can account for the simultaneous dependence of industry dynamics on size (once we condition on age) and on age (once we condition on size).
JEL D21, E22, G11.

1 Introduction

It is well documented that non-convexities and irreversibility play a central role in the investment process. The primary basis for this view, is plant level evidence of a non-linear relationship between investment and measures of fundamentals, including investment bursts as well as periods of inaction (see Cooper and Haltiwanger (2006) for example). Moreover, Doms and Dunne (1994) discover that smaller plants have higher maximum growth rate and larger maximum investment shares than the largest plants. That is, as plant size increases, investment expenditures become smoother. Empirically, the investment behavior of firms is characterized by the following facts:

(i) The investment rate distribution is non-normal having a considerable mass around zero.

(ii) The investment rate distribution has fat tails and is highly skewed to the right.

(iii) Long periods of inaction are complemented by rather intensive adjustment of the capital stock.

Furthermore, empirical studies have shown that firm size and growth are *not* independent for manufacturing firms in the U.S.. Evans (1987) and Hall(1987) show that the growth rate of employment of manufacturing firms, and the volatility of growth is negatively related to firm size and age. Dunne et al. (1988) study the U.S. manufacturing plants and show that the output of an entrant is considerably smaller than an average incumbent. These findings are important because Gribrat's Law, which states that firm size and growth are independent, was widely used in the firms' dynamics literature (see for example the influential model of the size distribution of firms by Lucas (1978)). Evans (1987) also finds that firm growth decreases with firm age and that this relation is valid after conditioning on firms' size. Second, it examines the relationship between firm growth and firm size and observes that firm growth decreases with firm size even when firms' age is held constant. Davis, Haltiwanger and Schuh (1996) show that the rates of job creation and job destruction in U.S. manufacturing plants are decreasing in age and size and that conditional on the initial size, small establishments grow faster than large firms. Thus, as pointed by Cooley and Quadrini (2001) the empirical regularities of firm dynamics are:[1]

(i) Firm growth decreases with firm age and size.

(ii) The variability of firm growth decreases with firm age and size.

(iii) Job creation and destruction decrease with firm age and size.

(iv) The probability of firm survival increases with firm age and size.

(v) **Size dependence and age dependence:**

[1]Some of these empirical facts are shown using establishment data while others correspond to firm-level data. However, many of the empirical facts based on firm data also hold for single-unit establishments (i.e. establishments that are firms) and small establishments (see Evans (1987)). Moreover, a recent study by Rossi-Hansberg and Wright (2006) showed that the firm and establishment size distributions are similar, reflecting the fact that only the very largest firms possess more than a single plant. This paper focuses on the thecnology of a single production unit and does not address questions of ownership or control.

- *Size dependence:* Conditional on age, the dynamics of firms (growth, volatility of growth, job creation, job destruction, and exit) are negatively related to the size of firms;
- *Age dependence:* Conditional on size, the dynamics of firms (growth, volatility of growth, job creation, job destruction, and exit) are negatively related to the age of firms.

After the regularities just presented, it seems natural to link patterns of firm growth with their capital accumulation decision. I ask whether a model of capital accumulation with entry and exit calibrated to match the investment characteristics of manufacturing firms is capable of generating also the size and the age dependence.

The results of the paper can be summarized as follows. I show that in the stationary equilibrium a model of firm dynamics with entry and exit can capture the features of the investment behavior cited above. I observe that the model investment rate distribution has a considerable mass around zero, that smaller firms invest more and that as plant size increases, investment expenditures become smoother. Furthermore, I show that the combination of a standard model of investment with adjustment costs and the introduction of entry and exit can generate the *simultaneous dependence* of industry dynamics on size (once we condition on age) and on age (once we condition on size). Hence, there is no conclusive evidence that financial frictions are a necessary condition to replicate the age and size dependence.

Cooley and Quadrini (2001) were the first to capture the size and age dependence, linking the patterns of firm growth with their financial decisions. They introduce financial frictions in a basic model of industry dynamics with idiosyncratic productivity shocks and instantly adjustable capital and labor. In the absence of financial frictions the exogenous productivity shock fully characterize the size and dynamics of the firm and the model does not reproduce all the stylized facts stated before. With financial frictions, the size of the firm also depends on its equity. They motivate the introduction of financial frictions by pointing to the relation of investment rates to Tobin's Q and cash flows. However, there are theoretical arguments and empirical evidence showing that investment-cash flows sensitivities are not good indicators of financing constraints or financial frictions (see for example, Gomes (2001), Cooper and Ejarque (2003) and Kaplan and Zingales (2000)). Moreover, the dynamics of the model are driven by the assumption that new entrants are of the highest productivity level contradicting and the fact that entrants begin less capital-intensive and less profitable than incumbents. Finally, in Cooley and Quadrini (2001), firms' capital dynamics are at odds with the investment behavior observed in the data, specially the lumpiness of investment rates.

In my model the main friction is the presence of capital adjustment costs. The literature on capital accumulation has found that the standard assumptions of the neoclassical model of the firm, such as strictly convex adjustment costs and reversibility, fail to adequately explain the investment behavior (see Abel and Eberly (1994, 1996), Caballero, Engel and Haltiwanger (1995), Caballero and Engel (1999), Cooper and Haltiwanger (2006), Cooper, Haltiwanger and Power (1999), Doms and Dunne (1994) for example). Motivated by the disappointing empirical evidence, other economists have argue in favor of the existence of non-convexities. The sources of the speculated non-convexities in the cost of capital adjustment include increasing returns, the cost of the equipment, costs associated with disruption and installation costs. The adjustment cost function in the model includes not only includes the traditional convex cost of investment but also a non-convex cost associated with the level of profitability in periods of adjustment. The model still reproduces the relation between cash flows and investment rates that is cited as evidence of financial constraints. However, explicit financial constraints are not necessary to obtain the main results.

Firms are characterized by its capital stock and productivity level. The optimal decision rules and the evolution of the idiosyncratic shocks generate an endogenous distribution of establishment across capital, productivity and age. The size dependence derives from the standard conditions of optimal investment and labor decisions in this environment, that is an abundance of capital leads to low rates of return and slower accumulation. On the contrary, a relatively small stock of capital leads to higher returns and lower variability of future profits deriving in higher investment rates. Hence, small firms will grow faster than large firms. The age dependence is driven by the technological composition of firms in each age class. The distribution of entrants and the persistence of the productivity level play an decisive role. As a cohort of entrants gets older the persistence parameter defines how fast the distribution of this firms across shocks becomes equal to the stationary distribution. Hence, an initial distribution that differs from the ergodic distribution and a low persistent parameter increases the chances of the model of getting the right age dependence. We calibrate the stochastic process so that the model can reproduce the main facts of U.S. firms.

Besides Cooley and Quadrini (2001), a number of authors have tried to explain the relation between size, age and firm dynamics arising from persistent idiosyncratic shocks to firms' production technology or from learning about the technology. This literature includes the models studied by Jovanovic (1982), Hopenhayn (1992), Campbell and Fisher (2000), Alburquerque and Hopenhayn (2002), Clementi and Hopenhayn (2006) and Rossi-Hansberg and Wright (2006). These models can generate an *unconditional* dependence of the firm dynamics on size and age. In other words, as Cooley and Quadrini (2001) indicated, without conditioning on age, the firm dynamics are negatively related to its size, and without conditioning on size, the firm dynamics are negatively related to the firm's age. However, they cannot account simultaneously for the *conditional* dependence on both size and age. My paper is also related to the earlier work of Castro, Clementi and Corbae (2005). Their paper tries to discriminate between two models of firm dynamics: (i) a learning model (symmetric and incomplete information) and (ii) a moral hazard model (asymmetric information). They assess whether informational frictions can successfully explain the conditional moments of firm dynamics in a model that also incorporates fixed and convex adjustment costs. Boyarchenko (2006) constructs a model of a competitive industry equilibrium refining the work of Dixit and Pindyck (1996) to study the implications of capital irreversibilities in continuous time where investment is made in several stages.

The rest of the paper is organized as follows. In Section (2) we describe the model and derive the conditions to find the stationary distribution. Section (3) presents the calibration and the computation of the model. In Section (4) I show the unconditional moments of firms' dynamics. Section (5) describes the main result of the paper, the size and age dependence. Finally, in Section(6) we conclude.

2 Model.

I present a model of firm entry, growth and survival in a monopolistic environment. This implies that while each firm has negligible impact on its rivals, it still maintains significant market power. The only source of uncertainty for firms currently in operation is the specific productivity shock. Incumbent firms maximize expected present value of discounted profits and in every period decide the optimal production plan. The framework described below is designed for the purpose of studying a competitive economy that is in a stationary or long-run equilibrium. In this equilibrium some firms will be undergoing change over time, with some of the expanding, others contracting, some exiting the market and others starting up. Despite all this change at the level of the individual firm, however, aggregate variables will be constant over time.

The firm[2] produces output y_t per time period with a production technology

$$y_t = f(s_t, k_t, n_t) = s_t k_t^{\alpha} n_t^{\gamma}, \tag{1}$$

with $\alpha \in (0,1)$, $\gamma \in (0,1)$, $\alpha + \gamma < 1$, where s_t is the idiosyncratic productivity shock, k_t is the stock of capital that the firm employs in period t and n_t is the labor input. Realizations of the idiosyncratic productivity shock s take values in the set $S \equiv \{s_1, \ldots, s_{ns}\}$ with ns finite. The process of s_t is assumed to follow a First Order Markov Process with transition matrix $\Pi(s'|s)$ and to be iid across firms. This implies that there is no uncertainty over the aggregate state of the economy even though there is uncertainty at the individual level. Denote $\pi_{s',s} = Pr(s_{t+1} = s'|s_t = s)$ the probability of receiving s' in period $t+1$ given that period t shock is equal to s. For each value of s, the vector $\Pi(\cdot|s)$ represents the distribution of future values of the shock, s'. It is assumed (as Cooley and Quadrini (2001)) that active firms face a probability of receiving a shock $s_t = 0$ denoted by π_x. Moreover, once s_t reaches 0 there is zero probability that s_t will receive a positive value in the future. Given these assumptions it is natural to identify a zero value for the productivity shock with the death of a firm.

2.1 Incumbent Firm's Problem.

The operative profits of an active plant are given by

$$P(s_t, k_t, n_t; w) = f(k_t, n_t, s_t) - w n_t \tag{2}$$

After observing the productivity shock and making the labor decision, every continuing plant decides the optimal level of investment

$$i_t = k_{t+1} - (1-\delta)k_t.$$

We normalize the price of new capital to 1 and denote the selling price of capital by p_s.

[2]Through the paper we consider single-unit firms, i.e. plants/establishments that are firms. See footnote 1.

Following the literature on plant dynamics, we assume that to modify the level of capital the plant must incur in adjustment costs. The function $g(k_t, k_{t+1})$ captures the presence of these costs and is defined as follows:

$$g(k_t, k_{t+1}; w) = \begin{cases} (1-\lambda)P(k_t, n_t, s_t; w) + \frac{\psi}{2}\left(\frac{i_t}{k_t}\right)^2 k_t, & \text{if } i_t \neq 0, \\ 0, & \text{if } i_t = 0; \end{cases}$$

For values of $i_t \neq 0$, the first term in $g(k_t, k_{t+1})$ captures the disruption costs associated with the installation of new capital. A fraction $\lambda \in (0,1)$ of the operative profits is lost in the period of adjustment. Empirical studies (see for example Power (1998) and Sakellaris (2001)) provide evidence that plant productivity is lower during periods of large investment. Note that ceteris paribus, investment rates are lower in periods of low productivity. The last term is the traditional convex adjustment cost (see Cooper and Haltiwanger (2006) for example).

The establishment's objective is to choose the optimal level of investment. The timing within period t for a plant that produced in period $t-1$ is as follows:

(i) The exit shock is realized. If the firm has to exit, it collects what it is left from the used capital and stops producing for ever.

(ii) If not, the idiosyncratic productivity level is realized.

(iii) Active plants, decide the optimal level of labor input. Later on the plant makes the investment decision and receives profits net of adjustment costs.

For any firm with $s \in S$ the optimal level of labor input solves the following problem:

$$R(k, s; w) = \max_n \left\{ s k^\alpha n^\gamma - wn \right\} \tag{3}$$

where $R(k, s; w)$ denotes the operative return of the firm. The solution implies that the optimal labor choice at state (s, k) is:

$$n(s, k; w) = \left[\frac{\gamma s k^\alpha}{w}\right]^{1/(1-\gamma)} \tag{4}$$

Thus, the return function of the plant, $R(k, s)$, after choosing the optimal level of labor is:

$$\begin{aligned} R(k, s; w) &\equiv P(k, n(k, s; w), s; w), \\ \Rightarrow R(k, s; w) &= s k^\alpha n(k, s; w)^\gamma - wn(k, s; w) \\ &= a k^\theta, \end{aligned} \tag{5}$$

where $a = (s/w^{\gamma})^{\frac{1}{1-\gamma}} \left[\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right]$ and $\theta = \frac{\alpha}{1-\gamma} < 1$.

Following Lucas (1978), I call θ the span of control parameter of a plants manager. Alternatively, we can assume that these properties derive from the monopolistic nature of the competitive environment where the firm faces a downward-sloping demand function. Now, for a given wage rate w, we can write the recursive problem of the active plant as follows:

$$V(k,s;w) = max\left\{V^b(k,s;w), V^s(k,s;w), V^i(k,s;w)\right\} \tag{6}$$

where $V^b(k,s;w)$ represents the value of "buying" more capital, $V^s(k,s;w)$ corresponds to the value of "selling" capital and finally $V^i(k,s;w)$, inaction, is the value of keeping the depreciated capital stock for the future period.

The value of buying is:

$$V^b(k,s;w) = \max_{k' \in (k(1-\delta),\bar{k}]} R(k,s;w) - i - g(k,k') + \frac{1}{1+r}\left[(1-\pi_x)\sum_{s'} V(k',s';w)\Pi(s,s') + \pi_x p_s k'\right]. \tag{7}$$

The value of selling is:

$$V^s(k,s;w) = \max_{k' \in [0,k(1-\delta))} R(k,s;w) - p_s i - g(k,k') + \frac{1}{1+r}\left[(1-\pi_x)\sum_{s'} V(k',s';w)\Pi(s,s') + \pi_x p_s k'\right]. \tag{8}$$

where $p_s \leq 1$ can be though as the selling price of capital.

Finally the value of inaction is given by:

$$V^i(k,s;w) = R(k,s;w) + \frac{1}{1+r}\left[(1-\pi_x)\sum_{s'} V(k(1-\delta),s';w)\Pi(s,s') + \pi_x p_s k(1-\delta)\right] \tag{9}$$

Note that in this last case the future value of capital is given by the depreciated capital stock after production in the current period.

2.2 Entry Decision.

We assume that there is a continuum of ex-ante identical potential entrants in each period. Entrants incur in a one time fixed cost κ_e, again denominated in units of output. Once this cost has been paid, the entrant is in the same position as the plant that was active in period $t-1$. Each potential entrant receives its initial shock from a continuous distribution $\nu(s^e)$. The size and the distribution of entrants will play an important role in the dynamics of firms. The determinants of this relation will be explained in more detail later.

It is assumed that in this economy there is free entry. The timing of events before entry is as follows:

(1) The potential entrant observes κ_e and then decides to enter or not.

(2) If he decides to enter, the entrepreneur pays the entry cost and makes the initial investment k^e, where k^e is the solution to:

$$\max_{k'} \left\{ \frac{1}{1+r} \sum_{s'} V(k', s'; w)\nu(s') - k' - \kappa_e \right\}. \tag{10}$$

For future reference we can define the value of creating a firm for a given wage rate w as

$$V^e(k^e; w) = \frac{1}{1+r} \sum_{s'} V(k^e, s'; w)\nu(s') - k^e - \kappa_e. \tag{11}$$

In equilibrium, new firms will enter and the wage rate will adjust until the expected discounted profits net of entry cost is at most zero, that is until:

$$V^e(k^e; w) \leq 0. \tag{12}$$

If this condition holds with equality an equilibrium with positive measure of entrants will exist.

For a given wage rate w, by the properties of the value function that solves problem (6), the solution to (11) exists and it is unique. The wage rate is endogenously determined in equilibrium to satisfy condition (12). The entry of new firms induce changes in prices and in the value of the firm until there are no gains from creating a new firm.

Problem (6), stated above, has a unique solution and can be solved numerically. It can be shown that the value functions are bounded, continuous and concave. We will focus our attention into the stationary distribution to study the long run properties of the model with adjustment costs. The stationary equilibrium implies a size and age distribution of firms. We provide conditions under which the empirical regularities hold.

2.3 Stationary Distribution and Aggregates

The only uncertainty in the model is generated by the idiosyncratic productivity shocks. At each point in time t the economy is characterized by a measure of firms

$\Gamma_t(k, s, j; w)$ for each level of capital stock $k \in \mathbf{K} = [0, \bar{k}]$, productivity shocks $s \in \mathbf{S} = \{s_1, \ldots, s_{ns}\}$ and age of the firms $j \in \boldsymbol{\Upsilon} = \{1, 2, 3, \ldots\}$. A discussion of the definition of the set $\mathbf{K}$ is in order. We will look for a stationary measure of firms, and this requires that firms never accumulate capital beyond some endogeneously determined level $\bar{k}$. Intuitively the value of $\bar{k}$ is where the decision rule $k'(k, s_{ns})$ crosses the 45° line, provided that the optimal capital accumulation rule is an increasing function of s. Conditions under which firms optimally decide to do this are given in the quantitative section.

With a positive probability of receiving $s = 0$ in any given period, the expected age of exit is finite. If we let the measure of firms at age j be given by μ_j, then $\mu_{j+1} = (1 - \pi_x)\mu_j$, where the measure μ_0 is given and corresponds to the mass of new entrants.

Let $\mathbf{B}(\mathbf{K})$ and $\mathbf{B}(\boldsymbol{\Upsilon})$ be the Borel $\sigma-$algebra of $\mathbf{K}$ and $\boldsymbol{\Upsilon}$ respectively, and $\mathbf{P}(\mathbf{S})$ the power set of $\mathbf{S}$. Define $\mathbf{X} = \mathbf{K} \times \mathbf{S} \times \boldsymbol{\Upsilon}$. Let $\mathcal{X} = \mathbf{B}(\mathbf{K}) \times \mathbf{P}(\mathbf{S}) \times \mathbf{B}(\boldsymbol{\Upsilon})$ and $\mathbf{M}$ be the set of all finite measures over the measurable space $(\mathbf{X}, \mathcal{X})$.

The law of motion[3] of $\Gamma_t(k, s, j)$ is given by:

$$\Gamma_{t+1} = H_t(\Gamma_t), \tag{13}$$

where the function H_t can be written explicitly as:

a. For all $\mathcal{T}$ such that $1 \notin \mathcal{T}$:

$$\Gamma_{t+1}(K \times S \times \mathcal{T}) = \int P_t((k, s, j); K \times S \times \mathcal{T}) \;\; \Gamma_t(dk \times ds \times dj), \tag{14}$$

where

$$P_t((k, s, j); K \times S \times \mathcal{T}) = \begin{cases} \pi(s', s)(1 - \pi_x) & \text{if } k'(k, s) \in K \\ 0 & \text{else} \end{cases}$$

b.

$$\Gamma_{t+1}(K \times S \times \{1\}) = \begin{cases} \nu(s)E & \text{if } k^e \in K \\ 0 & \text{else} \end{cases}$$

where E corresponds to the mass of entrants.

The explicit formulation of the law of motion for the distribution has to be divided in two parts in order to capture the assumption that entrant firms start their lives with capital value k^e.

[3] The dependence on the wage rate w is dropped for notational simplicity.

This paper focuses on the study of the invariant distribution of firms denoted by Γ^*. We find Γ^* as the fixed point of this mapping, that is, $\Gamma^* = H(\Gamma^*)$. We normalize the measure of firms to one. The mass of entrants, E, will coincide with the mass of firms that exit the market in equilibrium. In this way the total mass of firms is constant. Stockey and Lucas (1989) stated the necessary conditions for convergence of the measure Γ. The properties of the stochastic process and the decision rules give rise to a mapping from the current distribution to the next period measure of firms. An invariant measure of firms Γ^* exists. Moreover, Γ^* is unique, and the sequence of measures generated by the transition function, $\{H^n(\Gamma_0)\}_{n=0}^{\infty}$ converges weakly to Γ^* from any arbitrary Γ_0.

This result will allow me to calibrate the model using the stationary distribution and the moments from data on the U.S. manufacturing sector to then test the model against the conditional size and age dependence.

With the definition of the stationary distribution of active firms at hand it is straightforward to characterize the aggregate quantities in this economy. The aggregate supply of goods is given by

$$Y(\Gamma; w) = \sum_j \mu_j \int R(k, s; w)\Gamma(dk, ds, j), \tag{15}$$

total labor demand is

$$N(\Gamma; w) = \sum_j \mu_j \int n(k, s; w)\Gamma(dk, ds, j), \tag{16}$$

aggregate investment is

$$I(\Gamma; w) = \sum_j \mu_j \int [\iota_{\{i>0\}}(i) + p_s \iota_{\{i<0\}}(i)] i(k, s; w)\Gamma(dk, ds, j).$$

where $\iota_{\{y\}}(x)$ is the indicator function that takes value 1 if the condition y is true.

2.4 Stationary Equilibrium

Definition 1 (RSE)**.** *A Recursive Stationary Equilibrium (RSE) consists of a wage rate w^* a distribution of incumbent firms $\Gamma^*(k, s, j; w^*)$ and functions $V(k, s; w^*)$, $k'(k, s; w^*)$ and $n(k, s; w^*)$ such that:*

1. *Given w^*, $V(k, s; w^*), k'(k, s; w^*), n(k, s; w^*)$ solve the firm's problem.*

2. *The stationary distribution is such that $\Gamma^*(k, s, j; w^*) = H^*[\Gamma^*(k, s, j; w^*)]$*

3. The free entry condition is satisfied: $V^e(k^e; w^*) = 0$

4. The mass of entrants is $E^* = \pi_x \Gamma^*(k, s, j; w^*)$.

These are standard conditions for a stationary equilibrium. The set of prices and functions are such that they solve the firm's problem. Moreover, the evolution of the distribution, that reproduces itself in each period is consistent with decision rules and the evolution of the shocks. The mass of entrants equals the number of firms that exit in each period to keep the total measure of firms constant.

3 Calibration and Computation.

I parameterize the model assuming that a period is a year. To solve the firm's problem I approximate the value function using cubic splines. I assume that the firm's idiosyncratic shocks (defined as in equation (5)) follow an autoregressive form given by

$$\ln(a_{i,t}) = \rho_a \ln(a_{i,t-1}) + u_t \tag{17}$$

with $u_t \sim N(0, \sigma_u)$. Denote the standard deviation of $\log(a)$ by σ_a. To solve the model I will approximate the evolution of the idiosyncratic shocks using the method proposed by Tauchen and Hussey (1991) as explained in Adda and Cooper (2004, page 56). The parameters of the productivity process can be chosen to match the profile of US firms. They have implications for the degree of persistence and dispersion in the distribution of firms. The size of entrants is determined by the entry costs and the distribution of entry shocks. The study of Dunne et. al. (1988) reports that entrants that create a firm by building a new plant have market share[4] of 7.9% and their relative size[5] is 28.35% of the average firm. The general pattern observed is that entering firms are substantially smaller on average than existing or continuing firms. The distribution of entrants' productivity shocks is chosen to match these facts and the findings of some recent studies that show that organization learning appears to continue over a period of at least 10 years. We assume that $\ln(a^e)$ is distributed uniformly over the set $\Upsilon = [\ln(a_1), \chi]$ with

[4]Their study is based on Census data that is available every 5 years. For that reason the market share in the model corresponds to the ratio of total output produced by firms of age 1 through 5 over total output of older firms.

[5]The relative size is computed as the ratio of average output of entrants over average output of older firms.

$\chi \leq \ln(a_{ns})$. Note that if $\chi = \ln(a_{ns})$ the distribution of entrants will coincide with the ergodic distribution over shocks that can be derived from $\Pi(s'|s)$.

The set of parameters to calibrate in order to compute the model are the following:

$$\Theta = \{\delta, \alpha, \gamma, \rho_a, \sigma_a, \lambda, p_s, \phi, \kappa_e, \chi\}, \tag{18}$$

where δ is the depreciation rate, α is the capital share in the firms production technology, γ is the corresponding labor share, ρ_s and σ_s are the parameters that define the idiosyncratic shocks, λ is the parameter that captures the disruption costs associated with capital adjustment, p_s is the relative price of used capital to new capital, ϕ is the weight in the convex adjustment cost, κ_e is the entry cost and finally χ corresponds to the parameter of the entrant shock distribution.

We calibrate the parameters to match long-run moments from the U.S. economy and the investment dynamics reported before. We set the risk free rate to 4 percent, implying a value for the discount factor equal to 0.9615. I choose the depreciation rate to be 0.06 to match the NIPA value. Also, the labor-share parameter $\gamma = 0.64$ is in turn selected to replicate the labor share in the NIPA. Following Cooper and Haltiwanger (2005) the capital share α is set to 0.2186 that implies a value[6] of θ equal to 0.60 (Fuentes, Gilchrist and Rysman (2006), Gomes (2001) and Hennessy and Whited (2005) obtained similar estimates in related studies).

The parameters ρ_a and σ_a are taken from the study of Cooper and Haltiwanger (2005) for the calibration with disruption costs and after controlling for a time fixed effect[7]. The data come from the Longitudinal Research Database consisting of approximately 7000 large manufacturing plants that were in operation between 1972 and 1988. The number of grid points for s is chosen to be equal to twenty to

[6]This parameter value also produces an equilibrium capital-output ratio consistent with the US economy when the empirical counterpart for capital is identified with plant and equipment and is associated in NIPA with nonresidential investment.

[7]Recall that

$$a = (s/w^{\gamma})^{\frac{1}{1-\gamma}} \left[\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right].$$

Rearranging some terms we can write $\log(a) = (1-\gamma)^{-1}\log(s) + D$, where D is a constant that depends on the wage rate w and the labor share γ. Replacing in equation (17) we get

$$\log(s_{t+1}) = D(1-\gamma)(\rho_a - 1) + \rho_a \log(s_t) + e_t.$$

Hence, after this transformation we can recover the process of idiosyncratic shocks in the model where $e_t \sim N(0, \sigma_u(1-\gamma))$.

guarantee a good fit.

The exit probability is calibrated to $\pi_x = 0.045$ because in the sample analyzed by Evans (1987), the average probability of exit is about 4.5 percent.

The parameters associated with the adjustment cost function (λ and ϕ), the selling price of capital (p_s), the entrants' distribution (χ) and the entry cost (κ_e) are calibrated jointly so plants in the stationary distribution display the patterns that Cooper and Haltiwanger (2006) (investment facts) and Dunne, et. al. (1988) (entrants facts) documented.

The main features of Cooper and Haltiwanger (2006) findings could be summarized as follows: first, plants exhibit significant inaction in terms of capital adjustment (8.10 % of the total observations have investment rates of less than 1% in absolute value). Second, periods of inaction are complemented by periods of rather intensive adjustment of the capital stock. Cooper and Haltiwanger (2006) (and many others) define a spike as an investment episode in excess of 20 %. Negative spikes are found in 1.8 percent of the observations. The average investment rate in the data is 0.122.

The study of Dunne, et. al. (1988) shows that the entrant relative size of new firms entering through the construction of a new plant is 28.35% and that they account of around 8% of total production. This study summarizes the patterns of firm entry, growth and exit in the four-digit U.S. manufacturing industries over the period 1963-1982. Entrants are disaggregated into new firms, existing firms that diversify into an industry by opening new production facilities, and existing firms that enter by altering the mix of outputs they produce.

Thus, the remaining parameters of our model are calibrated to the following values: $\lambda = 0.84$, $p_s = 0.96$, $\phi = 0.25$, $\kappa_e = 55.91$ and $\chi = 0.58$. The selling price of capital is smaller than the price of buying new capital by approximately 4%. This wedge between the prices is in part responsible for the inaction that the model generates. The disruption cost is more than 10% of the current profits. Given the value of ϕ, firms with high enough levels of capital, will find it optimal to reduce the scale of production and do not wait until the depreciation process takes all the excess. Moreover, given the combination of adjustment costs present in the model, the firms will wait until the demand shocks are high enough to increase the capital stock up to the optimal level and we will observe the bursts of investment that are documented in the literature. To have a better sense of the magnitude of this parameters, the average adjustment cost paid relative to the capital stock was 4% in the stationary distribution. The value of κ_e implies that in equilibrium the total entry cost paid is around 3% of average capital of producing plants. The calibrated value of χ is around 20 % below the average productivity shock.

The full set of parameters values is reported in Table (3).

Parameter		Value	Moment Targeted
Discount Factor	$(1+r)^{-1}$	0.9615	interest rate 4 %
Depreciation Rate	δ	0.06	NIPA value
Labor Share	γ	0.64	Labor Share NIPA
Capital Share	α	0.2186	Cooper and Haltiwanger (2005)
Exit Probability	π_x	0.045	Evans (1987)
Autocorrelation	ρ_a	0.885	Cooper and Haltiwanger (2005)
Std. Dev. of u	σ_a	0.64	Cooper and Haltiwanger (2005)
Disruption Cost	λ	0.84	Inaction Rate
Selling Price	p_s	0.96	Negative Investment Spike
Convex Cost Func. Parameter	ϕ	0.25	Average Investment Rate
Entry Cost	κ_e	55.91	Entrant Relative Size
Entry Distribution	χ	0.58	Entrant Market Share

Table 1: Calibration values for model parameters.

4 Firm Dynamics and Stationary distribution

In this section we describe the firm dynamics generated by the calibrated model of adjustment costs at the stationary distribution. In figure (1) you can observe the value of the firm, the optimal investmet decision rule, the labor choice and Tobin's Q for different combinations of firm's capital size and idiosyncratic shocks.

The value of the firm is strictly increasing an concave on firm's size (capital). The marginal increase in firms value is decreasing in k, implying that for lower values of k the firm will prefer to invest in new capital, and for higher values it will prefer to sell some of its capital stock according to the current level of productivity. Furthermore, Figure (1.b) shows the optimal capital accumulation of the firm for different values of s. We observe that $k'(k,s)$ is strictly increasing in k and s. For low values of k, $k'(k,s) > k(1-\delta)$ that is, $i > 0$. There are middle range values of

k where $k'(k,s)$ coincides with $k(1-\delta)$. In this case the combination of k and s are such that the firm prefers not to invest. Finally, for high values of k, investment is negative, that is the firm is selling some portion of its capital.

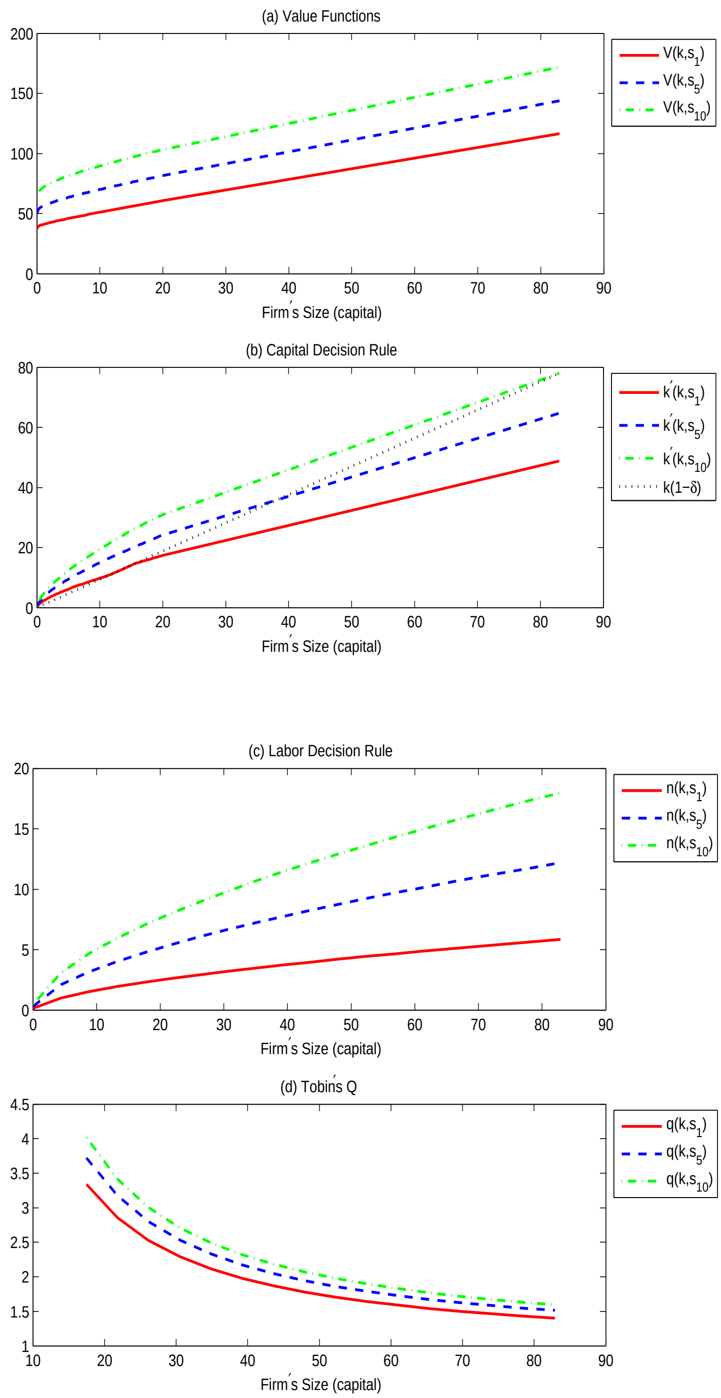

Figure 1: Firm Behavior

It is possible to note from the previous figure that at a given value of s there are two thresholds: one that defines when to stop investing and set $k' = (1-\delta)k$ and another that determines when to start selling capital. This thresholds are increasing in the productivity shock of the firm, that is firms will higher s will have higher investment rates conditional on current's capital k. Another feature that we can observe from this figure is the endogenous determination of $\bar{k}$. The labor decision rules is also depicted in Figure (1). This decision rule comes directly from equation (4). Firm's labor choice is strictly increasing in its capital and its productivity shock.

Now I turn to the properties of firms' dynamics in the stationary distribution. I calibrated the model mainly to match the moments reported by Cooper and Haltiwanger (2005) and the distribution of entrants presented in Dunne, Roberts and Samuelson (1988, page 504). The moments from the data and those from the model in the stationary distribution are reproduced in Table (2).

Moment	Data	Model
Spike Rate: Negative Investment (%)	1.80	2.33
Inaction Rate (%)	8.10	8.27
Average Investment Rate (%)	12.20	10.72
Entrant Relative Size (%)	28.35	28.22
Entrant Market Share (%)	7.90	7.31

Table 2: Data and Model Investment Moments.

The histogram of investment rates that emerges from this economy is reported in Figure(2). Clearly, there is a mass of firms around zero investment as we observe in the data (see Figure 1 in Cooper and Haltiwanger (2006)).

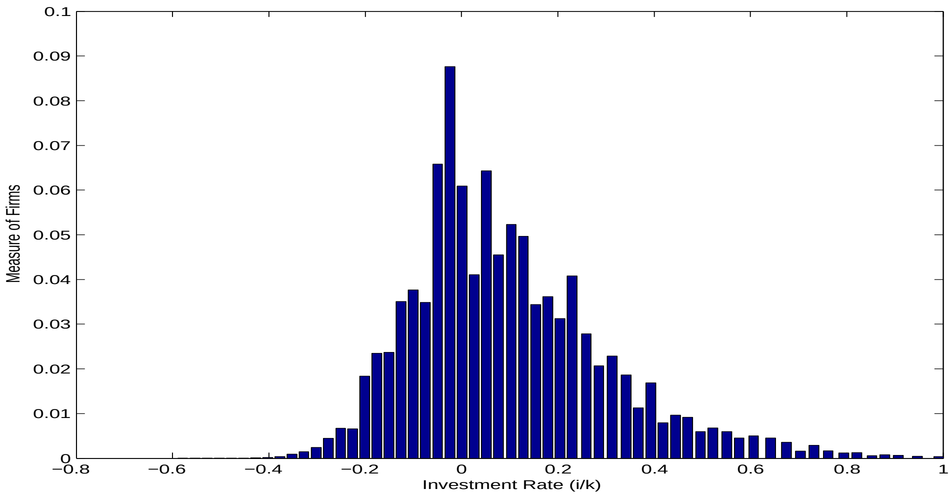

Figure 2: Model's Distribution of Investment Rates

As display in Figure (3), I also explore the properties of the growth rate of capital, profit's rate, the standard deviation of the growth rate and job creation rate dynamics to show that they are consistent with the observations in the U.S. economy. These unconditional moments are computed by averaging them according to the stationary distribution of each class of firms. At a given age, firms differ in two dimensions: their capital stock and their productivity shock. This heterogeneity is the driving force of all our results. The key properties of the behavior of firms that can be summarized as follows:

- Small and younger firms growth faster (Panels (a) and (b)).
- Profits rates are negatively correlated with size and age (Panels (c) and (d)).
- The variability of firm growth decreases with firm age and size (Panels (g) and (h)).
- Small and younger firms have higher job creation rates (Panels (e) and (f)).

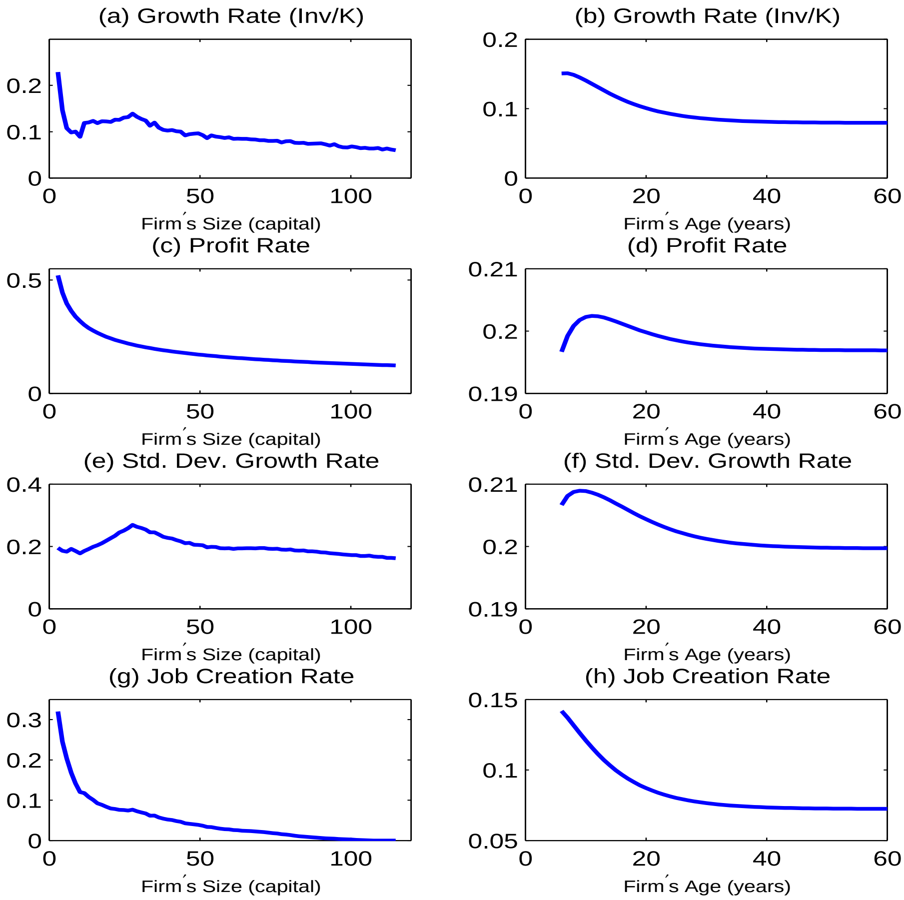

Figure 3: Firm's Moments and Invariant Distribution

The growth rate of capital is plotted in Panels (a) and (b). To understand why investment rates are a decreasing function of capital, we need to understand the trade-off that firms face when deciding the optimal level of capital for the future. On one hand, more capital allows them to increase the production scale and increase their expected profits; on the other hand the expansion of the production scale implies a higher volatility of profits. The investment behavior will be important in explaining the relation between growth rates and cash flows.

Panels (c) and (d) plot the profit rates as a function of firm's size and firm's age. This property derives from the decreasing return to scale production function and the optimal capital accumulation rule mentioned above. The higher profitability of smaller firms implies that they have a greater incentive to reinvest profits that relates to the ivestment rates observed in Panel (a).

The standard deviation of growth is also a decreasing function of size and age (see Panels (e) and (f)). This is because there is a lower bound for firm size (different than zero) for older firms. The optimal capital accumulation rules provides condition such that the capital level will never be lower than the value of k where $k'(k, s_1)$ crosses the $k(1-\delta)$ line. When productivity increases from s_1 to any higher s, they will accumulate more capital.

Panels (g) and (h) displays the job creation rate defined as the rate of employment gains summed over all plants that expand at age or size category (see Davis, Haltiwanger and Schuh (1996)). As pointed before, the labor decision is increasing in capital, so the dynamics of firms' growth stated above have a direct effect over the job creation rate of labor.

Finally, Figure (4) plots the joint distribution of firms over size (capital stock) and age. New entrants are of the same size; however they make different investment decisions according to their productivity shocks in their first period of life. In the model studied by Cooley and Quadrini (2001) entrants are always of the highest productivity shock. We observe a concentration of small and young firms. That is another feature of the model that matches the data. In the U.S. manufacturing sector, more than $\frac{4}{5}$ of new plants exit within 20 years. Furthermore, we observe that younger firms are smaller in average.

In summary, once calibrated to match the investment features of U.S. establishments, the model is capable to generate the *unconditional* size and age dependence of firms' dynamics. Moreover, the model is consistent with the evidence that relates investment rates and cash flows. Now is time to see if the model with adjustment costs and idiosyncratic productivity shocks is also able to generate simultaneously the conditional size and age dependence.

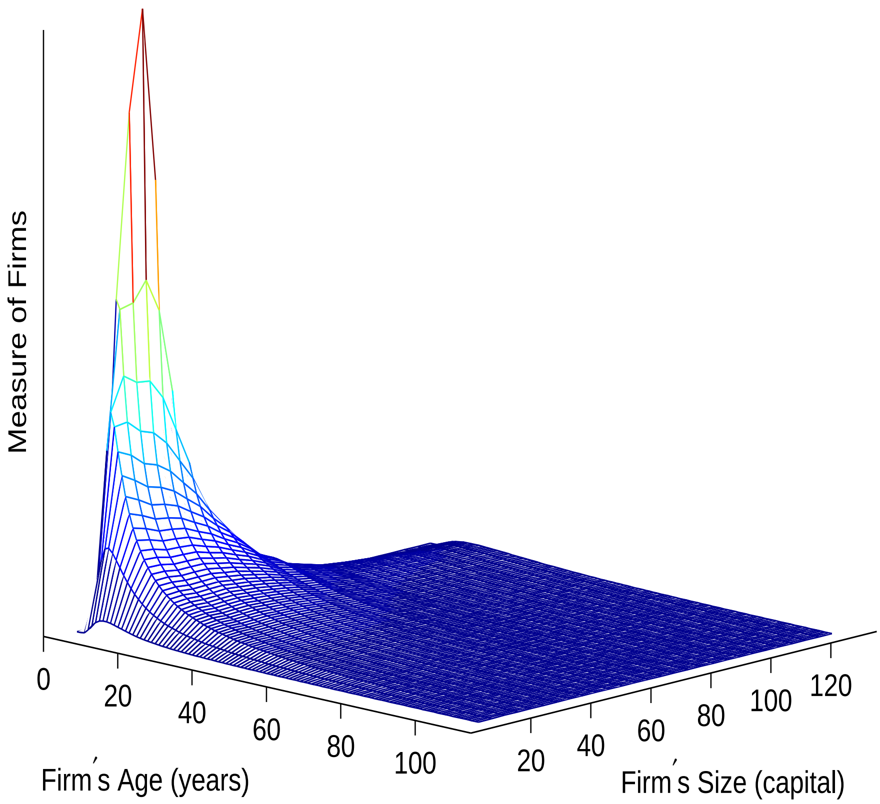

Figure 4: Stationary Distribution of Firms over Size and Age.

5 Size and Age Dependence.

The analysis conducted in the previous section showed that a model driven by productivity shocks, some level of capital irreverisibility with entry and exit captures many of the salient qualitative features of industry dynamics. In particular, we observe that higher investment rates for smaller firms, lower survival rates of small firms and an important degree of lumpiness in the investment evolution. However, the main point of the paper is to demonstrate that this model is also able to account for the conditional age and size dependence pointed by previous studies like Evans (1987), Hall (1987) and Davis et al. (1996) that Cooley and Quadrini (2001) describe as arising from financial frictions.

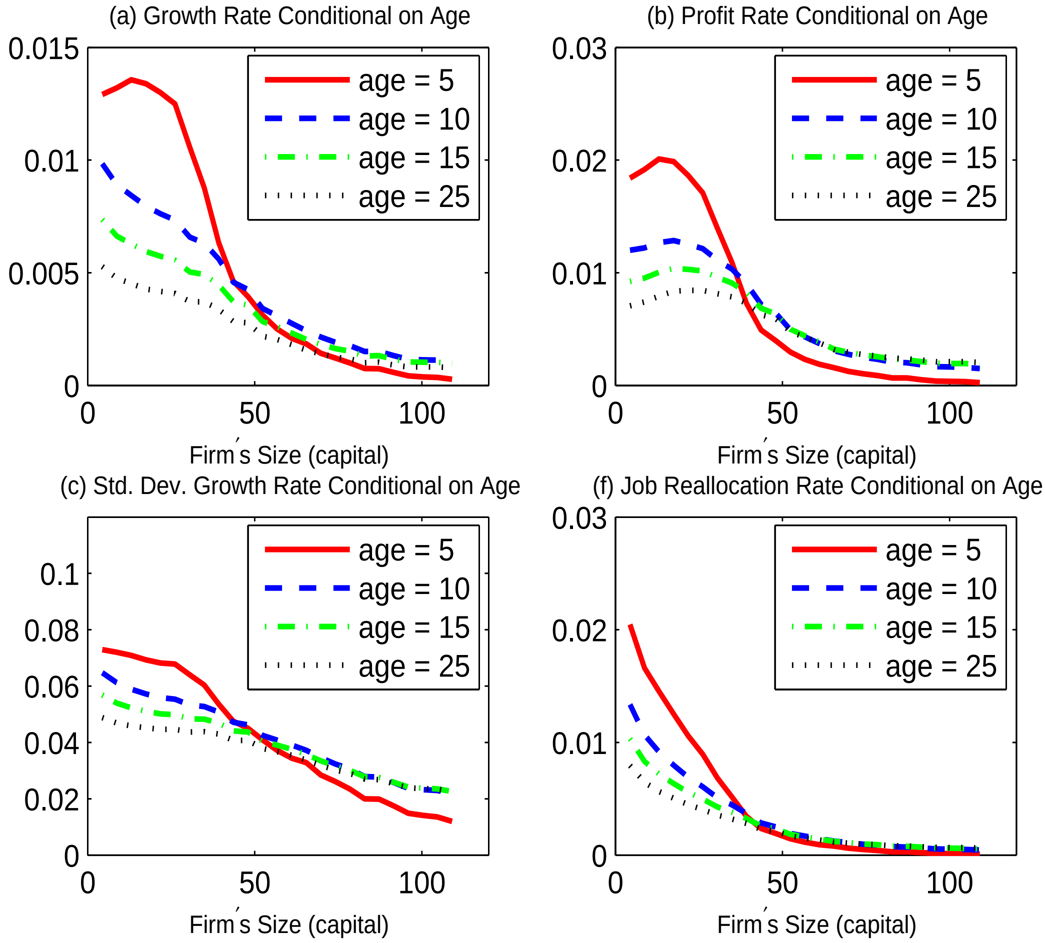

Figure 5: Size Dependence (Firms' Dynamics Conditional on Age).

Previous models of industry dynamics that consider investment decisions were not able to generate the age and size dependence because, once you control for the size of the firm, age becomes irrelevant in differentiating the dynamics of small and large establishments: the dependence on age derives only from the fact that young firms are in average smaller. In those models there exists only one dimension of heterogeneity, and thus once you fix age or size, firms are all alike independently of their history. In our model, there exists two dimensions of heterogeneity, because once you condition on size (capital stock depends on the previous history), firms could also be different in the productivity dimension. Furthermore, once you condition in the level of the idiosyncratic shock, firms differ in their size and this generates different patterns for the capital stock.

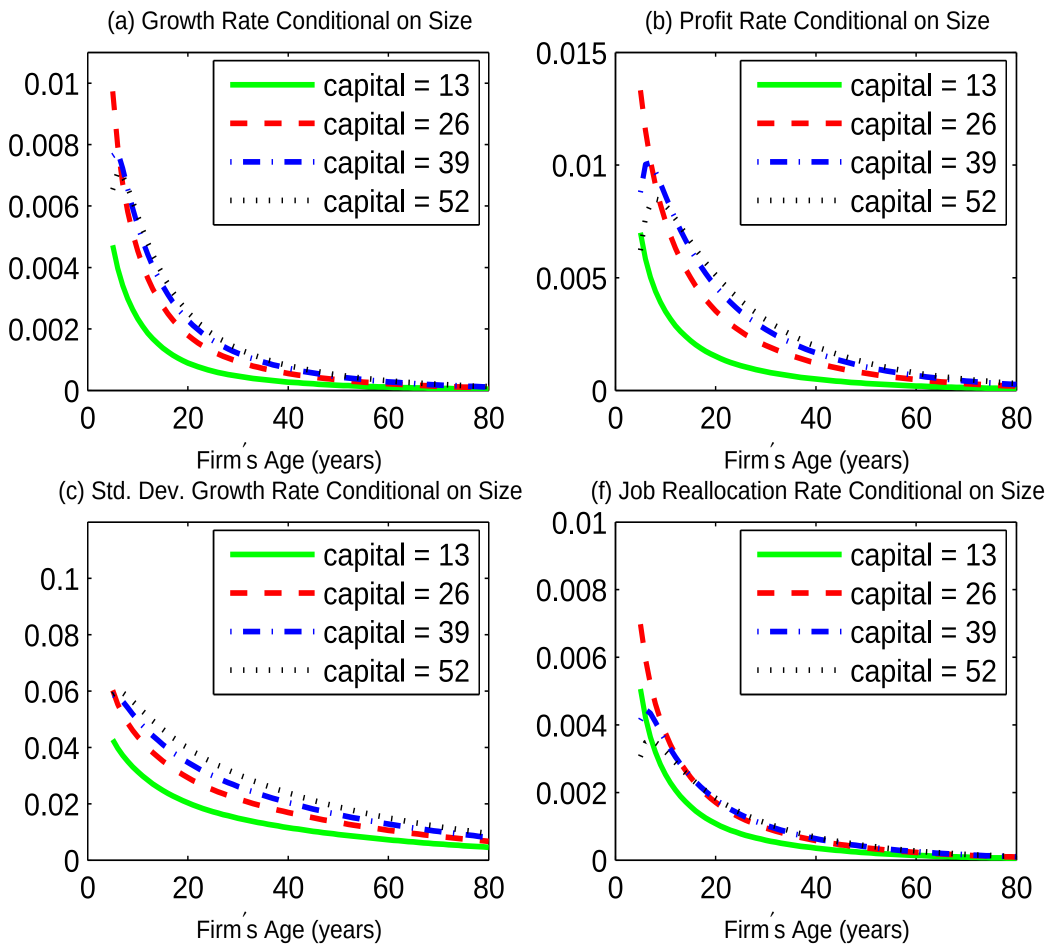

Figure 6: Age Dependence (Firms' Dynamics Conditional on Size).

As it was pointed before every decision of the firm depends on its level of capital stock as well as its productivity shock. Two firms with the same productivity shock will decide to invest, disinvest or continue with the same scale of production according to their level of k. Different values of k reflects the different histories. Similarly, two firms with the same scale of production will invest different amounts of capital according to their current value of s.

The heterogeneous behavior of firms in the stationary distribution, introduces the age and size dependence. To find the conditional dependence we need to separate the size effect from the age effect; for that reason we compute the measures of firms' dynamics for different age classes and different size classes.

Figures (5) and (6) plot the growth rate of firms, their profit rate, the standard deviation of the growth rate as well as the reallocation rate[8] as functions of the firms' size (conditional on age) and as a function of age (conditional on size) respectively.

We observe that most variables are decreasing in the size and in the age of the firm, even after controlling, respectively for age and size. The only exception is for job destruction of very small firms. Notice also, that the age and size dependence is more important for smaller and younger firms. The conditional size dependence derives from the same factors that affected the unconditional relation between firms' evolution and size. The higher growth rates of small firms is related with their higher profit rates and higher value of Tobin's q. This also introduces a negative relation with the rates of job reallocation (creation and destruction).

The age dependence is driven by the heterogenous behavior of firms of different ages classes conditional on their size. Conditional on their size, firms with higher productivity shocks experience higher rates of profits than firms with lower values of s. Conditional on the size of the firms, firms with different ages experience different productivity shocks, and thus, younger firms grow faster and face higher rates of job creation and failure than older firms.

The heterogeneity just described plays the most important role in generating the age dependence in the economy. Moreover, the persistence of the shocks, produces that the heterogenous composition is maintained for different ages. If this parameter is close to one, the distribution of entrants $\nu(s)$ will shape the distribution of active firms for a long time. In the limit, if $\rho_a \to 1$ the distribution over shocks of active firms will be similar to $\nu(s)$. On the contrary, as $\rho_a \to 0$ only the distribution of very young firms will remain close to their initial distribution. To make it clearer consider the following example: assume that there are no disruption costs associated with adjusting the capital stock and $\pi_x = 0$. Then, the Euler equation of an active firm after substituting the envelope condition is

$$
\begin{aligned}
&-1 - g_{k'}(k,k') + \frac{1}{(1+r)}\Big[\sum_{s'} \Pi(s'|s) R_{k'}(s',k') + (1-\delta k') + g_k(k',k'')\Big] = 0 \\
&\Rightarrow 1 + g_{k'}(k,k') = \frac{1}{(1+r)}\Big[\sum_{s'} \Pi(s'|s) R_{k'}(s',k') + (1-\delta k') + g_k(k',k'')\Big] \quad (19)
\end{aligned}
$$

This is the usual capital accumulation equation of a firm where the left hand side

[8]Following Davis et. at. (1996) job reallocation is the sum of job creation plus job destruction where job creation is defined as the sum of employment gains of expanding firms and job destruction is the sum of employment losses of contracting firms.

represent the marginal costs and the left hand side represents the marginal benefits of investment. Consider the extreme case where shocks are iid and distributed according to the stationary distribution corresponding with Π. Denote this distribution with Π^*. In this world only the shocks of firms that are 1 year old, i.e. entrants, will depend on $\nu(s)$. For any firm with age greater or equal to 2, the shocks will be drawn from Π^*. Then the capital accumulation equation becomes

$$1 + g_{k'}(k, k') = \frac{1}{(1+r)}\Big[\sum_{s'} \Pi^*(s')R_{k'}(s', k') + (1 - \delta k') + g_k(k', k'')\Big] \tag{20}$$

that is independent of s. Thus, once you condition on size, the capital accumulation of the firm is independent of the shock and then firm with different ages will behave as identical firms conditional on the capital stock. The distribution of firms for active plants that are not entrants is the same across shocks.

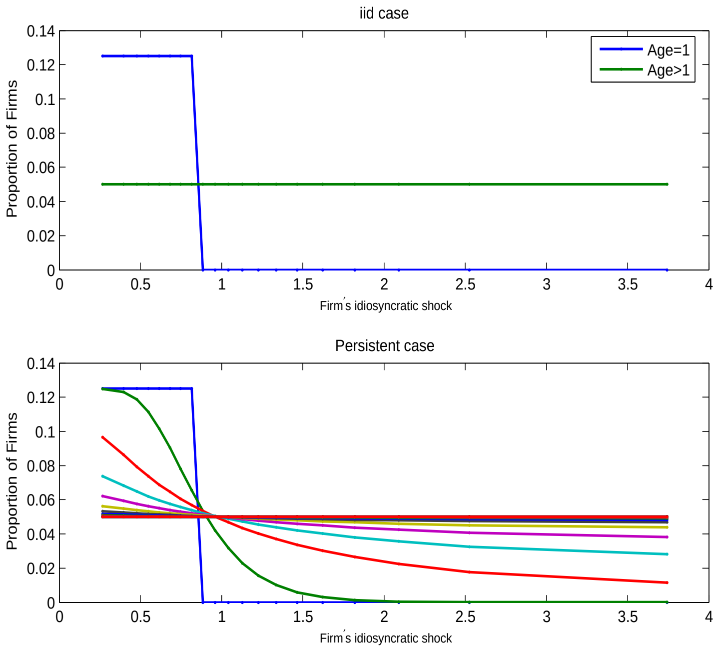

Figure 7: Distribution of firms over shocks for different ages

If you add some level of persistence, not only the decision of the firm will depend on the idiosyncratic shock, but also the distribution of firms across shocks will differ for firms with different ages. The optimal capital investment is the solution to problem (6). The measure of active firms with a particular shock will approach $\Pi^*(s)$ as you consider older firms. Figure (7) displays the proportion of firms with different shocks for different ages for the iid case and for the calibrated economy (persistent case). Each line correspond to a different age. In the iid case only two lines can be distinguished. After the first year of production, the distribution of firms over s is the same across firms with different ages. For the case where the shocks are persistent, the distribution of firms over shocks approaches $\Pi^*(s)$ only as firms get older. Each line correspond to firms with a different age. Older firms are distributed according to $\Pi^*(s)$. This shows the importance of the calibration of the entrant distribution and the relation with the results derived previously. We calibrate the initial distribution to facts of US manufacturing plants that is also consistent with the view of Atkeson and Kehoe (2006) of an increase in plant's productivity as they get older coming from a learning process. An important point is worth to mention here. Cooley and Quadrini (2001) needed to assume that the entrants were of a particular technology type to generate the right sign in the age dependence (see Cooley and Quadrini (2001) page 1303). In our model, the size of the entrants as well as the entry barrier are calibrated to match the facts observed in the U.S. manufacturing sector.

To close this section, I also conduct an econometric test of the relation between firm growth, size and age. Specifically, I simulate the model economy and run the following regression on the simulated data

$$\frac{i_{j,t}}{k_{j,t}} = a_0 + a_1 \ln(size_{j,t}) + a_2 \ln(age_{j,t}) + \epsilon_t \tag{21}$$

where the subscript j denotes firm j and t corresponds to the time period. Firm's size measure is the stock of capital. The results are displayed in Table (3).

Coefficient	Value	Std. Error
a_1	-0.018	6.56e-05
a_2	-0.11	6.26e-05

Table 3: Model Predictions.

We can therefore reject the hypothesis that growth is independent of either size or age. The elasticities of growth with respect to size and age are both negative. Therefore, firm growth decreases with firm size when firm age is held constant and decreases with firm age when size is held constant.

5.1 Evidence on Financial Constraints

Cooley and Quadrini (2001) motivated the introduction of financial constraints in a standard model of firm dynamics pointing to the the relation between investment rates and Tobin's Q and cash flows. However, there are theoretical arguments and empirical evidence showing that investment-cash flows sensitivities are not good indicators of financing constraints or financial frictions. Cooper and Ejarque (2001) find that the sensitivity of investment rates to cash flows does necessarily comes from a model with financial constraints. They estimate different models of capital investment to match the "Q-theory" regressions and obtain a better fit with a model with no financial frictions. Moreover, Erickson and Whited (2000) and Gomes (2001) argue that the relation between investment rates and Tobin's q comes from measurement error. In my model, the monotonicity of the investment function imply that investment of firms is sensitive to cash flows generating the significant relation obtain in the data. To show this I simulate my model economy and apply the same econometric procedures that previous studies pointing to financial constraints used. The estimated model takes the following form

$$\frac{i_{j,t}}{k_t} = a_0 + a_1 E[q_{j,t+1}] + a_2 \frac{\pi_{j,t}}{k_{j,t}} + \epsilon_t \tag{22}$$

where the subscript j denotes firm j and t corresponds to the time period. A significant coefficient a_2 in this type of regression motivated the inclusion of financial constraints. The results from my model are displayed in Table (4).

Coefficient	Value	Std. Error
a_1	0.087	1.92e-06
a_2	2.95	5.71e-04

Table 4: Model Predictions.

The values obtained are in line with the estimated coefficients reported by Gomes (2001) and Cooper and Ejarque (2001). The goal of this exercise was to understand the "cash-flow effect" and the relation with financial constraints. We observe that a model with no financial frictions and only some level of capital irreversibility also generates a significant "cash flow" coefficient. This is not an argument against financial frictions. I do not question the existence or importance of these constraints for investment decisions. Nevertheless, this result cast serious doubt on the common interpretation of cash-flow effects as evidence in favor of financing constraints.

Hence, the integration of a basic model of industry dynamics with non-convex adjustment costs and entry and exit is able to capture most of the stylized facts about the investment behavior and the growth of firms. In particular, we are able to reproduce the conditional age and size dependence that the empirical literature pointed before and that previous models of investment and firms' dynamics were not able to obtain. In contrast with previous models were financial frictions were necessary to address this question we developed a model were the friction present is the adjustment costs of capital accumulation.

6 Conclusion

Models of firm and industry dynamics that consider entry and exit did not include any type of adjustment costs and were unable to account simultaneously for the conditional dependence of the firm dynamics on size and age. At the same time, Cooley and Quadrini (2001) pointed that one possible explanation could be the introduction of financial frictions in an otherwise standard model of industry dynamics. They show that the integration of persistent shocks and financial-market frictions allows the model to generate the desire firm dynamics. In this paper, we find that a model of investment with adjustment costs with entry and exit can account also for all the firm dynamics. More importantly, the integration of these features, allows us to reconcile the characteristics of the firms with the investment moments that empirical studies describe, such as the periods of investment inaction and capital accumulation bursts.

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