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Endogenous Labour Market Imperfection and the HOS Model: Some Counterintuitive Trade-theoretic Results

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ABSTRACT: This paper introduces endogenous labour market imperfection in an otherwise Heckscher-Ohlin-Samuelson (HOS) model. It demonstrates that this framework satisfies the Stolper-Samuelson theorem and the magnification effect and that it is capable of producing certain trade-theoretic results which are contrary to the standard HOS and the Corden and Findlay (1975) results.

Keywords: Labour market imperfection, Heckscher-Ohlin-Samuelson model, Corden and Findlay model, foreign capital, trade liberalization, general equilibrium.

JEL Classification: F2, F21, O17.

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1. Introduction

The Heckscher-Ohlin-Samuelson (hereafter, HOS) model with Stolper-Samuelson (SS) theorem at its core is the foundation of the neoclassical theory of international trade. In the HOS model both commodity and factor markets are perfectly competitive that limits the application of such a model for the purpose of analyzing the problems of a developing economy where the existence of factor market imperfection is a salient feature. Attempts have been made to use derivatives of the HOS model with labour market imperfection to address the problems of such economies. One example of this is the Corden and Findlay (1975) (CF hereafter) model). They have extended the Harris-Todaro (1970) (HT) model by allowing perfect capital mobility between the rural and the urban sectors, thereby integrating the HT structure with the orthodox HOS trade model. The CF is a dual economy model with exogenous labour market distortion in the urban sector and does not satisfy the Stolper-Samuelson theorem completely. For example, if the price of the labour-intensive commodity changes the return to capital in nominal terms does not change.

Otherwise, as shown by Marjit and Beladi (2003), this structure more or less behaves like the standard HOS model and certain important trade theoretic results remain undisturbed. For example, the welfare effect of foreign capital with full repatriation of foreign capital earnings is immiserizing if the import-competing sector is capital-intensive[1] and is protected by an import tariff. This is the standard 'Brecher-Alejandro' (1977) (BA) proposition which is valid in the CF set-up despite the presence of labour market distortion (Khan 1982). This is due to the 'envelope

[1] The BA (1977) proposition has been proved in terms of the HOS framework. Here a necessary condition for the proposition to hold good requires that the tariff-protected import-competing sector must be capital-intensive in physical sense. However, when we consider an HT structure, as has done by Khan (1982), the import-competing sector has to be capital-intensive in value sense which automatically implies that the sector is capital-intensive in physical sense.

property' of the CF model where the average wage of the workers is equal to the rural sector wage. So long as the 'envelope property' holds the BA proposition will carry over in the HT framework under the modified assumption on factor intensity. There are a number of works that have examined trade policy and growth with domestic distortions. This includes works of Kemp and Negishi (1970), Ohyama (1972), Panagariya and Eaton (1979, 1982), Chaudhuri (2005) etc.

On the other hand, there exists a theoretical literature on endogenous wage determination in the unionized labour market in the formal sector of a two sector general equilibrium model and it consists of works of Calvo (1978), Quibria (1988), Chaudhuri and Mukhopadhyay (2009), etc. Wage determination in each of these models is based on the monopoly trade union framework as well as on the Nash bargaining framework. They have derived a unionized wage function where the unionized and non-unionized wages are positively correlated. However, little has been said about conditions under which inflows of foreign capital or a policy of trade liberalization would be necessarily harmful or beneficial. Batra (1973) has shown that in the presence of a tariff distortion in the product market how the presence of an imperfection in the labour market affects welfare in a small open economy. However, he has considered only exogenous distortion in the labour market and does not examine the validity of the properties of the HOS model and compare his results with the CF case.

The present paper introduces endogenous labour market distortion in an otherwise HOS structure. The analysis has found that the Stolper-Samuelson theorem and the magnification effect are valid despite introduction of endogenous labour market imperfections where the unionized wage is a positive function of the non-unionized wage rate, the bargaining power of the trade union and the commodity price. Besides, we have derived precise conditions for inflow of foreign capital to be welfare-improving in the presence or absence of any tariff distortion and also for tariff reforms to be welfare-deteriorating. These results cannot be obtained in the CF framework with exogenous distortion in the labour market.

2. The Model

We consider the standard HOS model with labour market imperfection in sector 2. In sector 2 (formal sector) workers receive the unionized wage, W^*, while their counterparts in sector 1 (an informal sector) receive a low and competitive wage, W. All other standard assumptions of the HOS model are retained. Commodity prices are given by the small open economy assumption.

The unionized wage is determined as a solution to the Nash bargaining game between the representative firm and the representative labour union in the unionized formal sector and is written as follows.[2]

$$W^* = U\frac{PQ(L)}{L} + (1-U)W \qquad (1)$$

The unionized wage function in general form is written as follows.

$$W^* = W^*(P, W, U) \qquad (1.1)$$

Differentiating (1) it can be easily shown that[3]

$$\left.\begin{aligned}
&\left(\frac{\partial W^*}{\partial U}\right) = \left(\frac{1}{L}\right)(PQ(.) - WL) > 0 \\
&\left(\frac{\partial W^*}{\partial W}\right) = (1-U) + \frac{U(WL - PQ(.))}{L^2 PQ_{LL}} > 0\,; \\
&\left(\frac{\partial W^*}{\partial P}\right) = \left[\frac{UPQ(.)(Q_{LL}L + Q_L) - UWLQ_L}{PQ_{LL}L^2}\right] > 0 \text{ if } (Q_{LL}L + Q_L) \le 0 \text{ i.e } \xi_L \ge 1,
\end{aligned}\right\} \qquad (2)$$

where $\xi_L = -\left(\frac{LQ_{LL}}{Q_L}\right)$ is the elasticity of marginal product curve of labour.[4]

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[2] This function has been derived in Appendix I.

[3] See Appendix I.

This establishes the following proposition.

<u>Proposition 1</u>: The unionized wage is a positive function of both the informal wage rate and the bargaining strength of the labour union. It is also a positive function of the commodity price if $\xi_L \geq 1$.

From (1) it can also be checked that[5]

$$\left.\begin{aligned} &E_W = \left(\frac{\partial W^* W}{\partial W W^*}\right) = \left[\frac{\{(1-U)L^2 Q_{LL} + U(LQ_L - Q(.)\}W}{UPf(L^*) + WL^*(1-U)}\right] > 0; \\ &E_P = \left(\frac{\partial W^* P}{\partial P W^*}\right) = \left(\frac{PU}{LQ_{LL}}\right)\left[\frac{\{Q(.)LQ_{LL} - Q_L(LQ_L - Q(.))\}}{(UPQ(L) + WL(1-U))}\right] > 0 \text{ if } \xi_L \geq 1 \text{ and,} \\ &(E_W + E_P) = 1 \end{aligned}\right\} \quad (3)$$

2.1 The S-S theorem and magnification effect

The general equilibrium set-up is given by the following set of equations.

$$Wa_{L1} + ra_{K1} = P_1 \quad (4)$$

$$W^*(P_2, W, U)a_{L2} + ra_{K2} = P_2 \quad (5)$$

$$a_{K1}X_1 + a_{K2}X_2 = K \quad (6)$$

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (7)$$

where a_{ji}s is the requirement of the jth factor required to produce one unit of output of sector i for $j = L, K$; and, $i = 1, 2$.

Equations (4) and (5) are the two zero-profit conditions for the two sectors while equations (6) and (7) are the full-employment conditions for capital and labour, respectively. We assume that

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[4] This is only a sufficient condition. There can be another sufficient condition under which $\left(\frac{\partial W^*}{\partial P}\right) > 0$. See appendix I.

[5] See Appendix I.

sector 1 is more (less) labour-intensive (capital-intensive) than sector 2 in value sense i.e. $\frac{Wa_{L1}}{a_{K1}} > \frac{W^* a_{L2}}{a_{K2}}$. As $W^* > W$ it automatically implies that sector 1 is more (less) labour-intensive (capital-intensive) than sector 2 in physical sense.

Differentiating equations (4) and (5) totally, applying the envelope conditions and arranging terms in a matrix notation we obtain

$$\begin{bmatrix} \theta_{L1} & \theta_{K1} \\ \theta_{L2}E_W & \theta_{K2} \end{bmatrix} \begin{bmatrix} \hat{W} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} \hat{P}_1 \\ \hat{P}_2(1 - E_{P2}\theta_{L2}) \end{bmatrix} \tag{8}$$

where the determinant to the coefficient-matrix is

$$|\theta| = (\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}E_W) = (\theta_{L1} - \theta_{L2}E_W) - \theta_{L1}\theta_{L2}(1 - E_W)$$
$$= (\theta_{K2} - \theta_{K1}E_W) - \theta_{K1}\theta_{K2}(1 - E_W) > 0 \tag{9}$$

θ_{ji} is the distributive share of the jth factor in the ith sector; and, "^" means proportional change.

We now state and prove the following proposition.

<u>Proposition 2:</u> If $\hat{P}_1 > \hat{P}_2$ the following ranking must hold: $\hat{W} > \hat{P}_1 > \hat{P}_2 > \hat{r}$.

Proof:

Solving (8) by Cramer's rule

$$\hat{W} = \frac{\theta_{K2}\hat{P}_1 - \theta_{K1}(1 - E_P\theta_{L2})\hat{P}_2}{|\theta|} \tag{10}$$

Now if $\hat{P}_1 > \hat{P}_2$ from (10) it follows that

$$\left(\frac{\hat{W}}{\hat{P}_1}\right) > \frac{(\theta_{K2} - \theta_{K1}) + \theta_{K1}\theta_{L2}E_P}{|\theta|} = B \text{ (say)}$$

Now $\left(\frac{\hat{W}}{\hat{P}_1}\right) > B \geq 1$ iff $(\theta_{K2} - \theta_{K1}) + \theta_{K1}\theta_{L2}E_P \geq |\theta|$. Using (9) and simplifying one gets

$\left(\frac{\hat{W}}{\hat{P}_1}\right) > B \geq 1$ iff $(E_{P2} + E_W) \geq 1$. Using (3) it follows that

$$\hat{W} > \hat{P}_1 \quad (11)$$

Similarly, solving (9) it is easy to show that

$$\hat{r} < \hat{P}_2 \quad (12)$$

Combining (11) and (12) one can write

$\hat{W} > \hat{P}_1 > \hat{P}_2 > \hat{r}$.

If $\hat{P}_1 > \hat{P}_2 = 0$, $\hat{W} > \hat{P}_1 > \hat{P}_2 = 0 > \hat{r}$.

This completes the proof.

It is important to note that the Rybczynski effect and the Rybczynski-type effect that occurs following an S-S effect if production technologies are of variable coefficient type also hold in this case.

2.2 Some counterintuitive trade-theoretic results

Now suppose that there a tariff on sector 2. Equation (5) is, therefore, modified to

$$W^*(P_2, W, U)a_{L2} + ra_{K2} = P_2^* \quad (5.1)$$

where $P_2^* = P_2(1+t)$ is the tariff-inclusive domestic price of commodity 2.

We also assume that the aggregate capital stock of the economy consists of both domestic capital (K_D) and foreign capital (K_F) and these are perfect substitutes. The capital endowment equation is now given by

$$a_{K1}X_1 + a_{K2}X_2 = K_D + K_F = K \quad (6.1)$$

The strictly quasi-concave social welfare function is given by

$$V = V(D_1, D_2) \quad (13)$$

where D_i denotes the demand for the ith commodity for $i = 1, 2$.

Given that international trade occurs, trade balance requires that

$$(X_1 - D_1) = P_2(D_2 - X_2) + rK_F$$

or, $D_1 + P_2^* D_2 = X_1 + P_2^* X_2 + tP_2(D_2 - X_2) - rK_F$ (14)

where $(X_1 - D_1)$ is the amount of X_1 exported and $(D_2 - X_2)$ denotes the amount of X_2 that is imported.

Differentiation of (13) yields

$$(dV / V_1) = dD_1 + P_2^* dD_2 \qquad (15)$$

National income at domestic prices is given by

$$Y = X_1 + P_2^* X_2 + tP_2 M - rK_F \qquad (16)$$

where, M denotes the volume of import and is given by

$$M = D_2(P_2^*, Y) - X_2 \qquad (17)$$

2.3 Welfare consequences of foreign capital inflow and trade liberalization

Differentiating (13), (14), (16), (17), (6.1), (7) and the production functions and substituting into (15) the following expressions can be obtained.[6]

$$(\frac{1}{V_1}\frac{dV}{dK}) = (\underset{(+)}{\frac{v\lambda_{L1}X_2}{K|\lambda|}})[(W^* - W)a_{L2} - tP_2] \qquad (18)$$

$$(\frac{1}{V_1}\frac{dV}{dt}) = (\frac{v}{t})[(W^* - W)(\underset{(+)}{\frac{TL_2 S_{LK}^2}{|\theta|}})\underset{(+)}{\{E_W\theta_{K1} + \theta_{L1}(1 - E_P)\}}$$
$$+\{\underset{(+)}{\frac{(W^* - W)a_{L2} - tP_2}{|\lambda|}}\}X_2\underset{(+)}{(\lambda_{L1}A_2 + \lambda_{K1}A_1)} + \underset{(-)}{(tP_2)^2 H}] \qquad (19)$$

[6] See Appendix III for detailed derivations.

where: $v=[(1+t)/\{1+t(1-m)\}]>0; m=P_2^*(\partial D_2/\partial Y)$ is the marginal propensity to consume commodity 2 $(1>m>0)$; and, $H=[(\partial D_2/\partial P_2^*)+D_2(\partial D_2/\partial Y)]<0$ is the Slutsky's pure substitution term.

From (18) and (19) the following proposition can be established.

<u>Proposition 3</u>: An inflow of foreign capital improves social welfare iff $(W^*-W)a_{L2}>tP_2$. On the other hand, a policy of trade liberalization is welfare-worsening if $[\{\frac{(W^*-W)a_{L2}-tP_2}{|\lambda|}\}X_2(\lambda_{L1}A_2+\lambda_{K1}A_1)+(tP_2)^2H]\geq 0$.[7]

We explain proposition 3 as follows. An inflow of foreign capital leads to a Rybczynski effect. Sector 2 expands and sector 1 contracts, as the former sector is capital-intensive. As the higher wage-paying sector expands, both in terms of output and employment, the aggregate wage income of the workers increase. This we call the *labour reallocation effect* that works positively on social welfare and is captured by the first term in the right-hand side of (18). On the other hand, an expansion of sector 2 leads to further misallocation of economic resources, lowers volumes of trade thereby exerting a downward pressure on welfare. This is the cost of the tariff protection of the supply side[8] which is captured by the second term in the right-hand side of (18). We call it the output effect of the formal sector industry that affects welfare adversely. Welfare improves if the positive labour reallocation effect dominates over the increased cost of tariff protection.

A policy of trade liberalization, on the other hand, lessens, t. This lowers r and raises W and hence W^*. However, the higher wage-paying sector contracts both in terms of output and employment due to a Rybczynski type effect. Thus, there are two components of the labour

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[7] There can be other sufficient conditions under which a policy of trade liberalization might be welfare-deteriorating.

[8] As the output of the tariff-protected sector rises the deadweight loss to the society due to further misallocation of economic resources goes up thereby lowering social welfare.

reallocation effect. As the two wage rates increase the aggregate wage income increases. On the contrary, as the higher (lower) wage-paying sector contracts (expands) the aggregate wage income goes down. The net effect is, however, ambiguous. Finally, the consumers would be consuming more of commodity 2 leading to a decrease in cost of tariff protection of the demand side that works positively on welfare. Besides, as the protected sector contracts the efficiency of allocation of economic resources also improves. The cost of protection of the supply side decreases which also works favourably on welfare. If the net labour reallocation effect is negative and is stronger than the combined effect of the last two effects, welfare decreases due to trade liberalization. See equation (19).

In the absence of any labour market imperfection, we have $W^* = W$. Then the model boils down to the standard HOS model. There is no labour reallocation effect. The first terms in the right-hand side of equations (18) and (19) vanish and we get the following standard results: an inflow of foreign capital definitely worsens welfare. On the contrary, a policy of trade liberalization unambiguously improves social welfare.

It is worthwhile to mention that in the CF model we get different results due to the 'envelope property'. In the CF model there is no labour reallocation effect as the rural sector wage does not change following an inflow of foreign capital. Welfare unambiguously worsens due to negative tariff revenue effect. On the other hand, a reduction in import tariff lowers the return to capital and raises the rural sector wage. The aggregate wage income increases which works favourably on welfare. Besides, the tariff-protected import-competing sector contracts which also improves social welfare due to reduction in the cost of tariff protection. Welfare, therefore, unambiguously improves.

3. Concluding remarks

In this note, we have shown that the HOS model with endogenous labour market imperfection not only satisfies all the standard properties like S-S theorem, magnification and Rybczynski effects but also is capable of producing certain counterintuitive trade-theoretic results. A two-sector mobile capital version of the HT model does not satisfy these properties. Hence, the present framework may be useful in explaining as to why the developing countries are yearning for foreign capital despite the standard immiserizing result and why these countries are not reducing the tariff rates beyond certain levels.

References:

Batra, R.N. (1973): *Studies in the Pure Theory of International Trade*, Macmillan.

Brecher, R.A. and C.F. Diaz Alejandro (1977): 'Tariffs, foreign capital and immiserizing growth', *Journal of International Economics* 7, 317-22.

Calvo, G. (1978): 'Urban unemployment and wage determination in LDCs: trade unions in the Harris-Todaro model', *International Economic Review* 19(1), 65-81.

Chaudhuri, S. (2005): Labour market distortion, technology transfer and gainful effects of foreign capital', The Manchester School 73(2), 214-227.

Chaudhuri, S. and Mukhopadhyay, U. (2009): *Revisiting the Informal Sector: A General Equilibrium Approach*, Springer. New York.

Corden, W. M. and Findlay, R. (1975): 'Urban unemployment, intersectoral capital mobility, and development policy in a dual economy', *Economica* 42, 59-78.

Eaton, J. and Panagariya, A. (1979): 'Gains from trade under variable returns to scale, commodity taxation, tariffs and factor market distortions', *Journal of International Economics* 9, 481–501.

Eaton, J. and Panagariya, A. (1982): 'Growth and welfare in a small, open economy', *Economica* 49, 409–419.

Harris, J. R. and Todaro, M. (1970): 'Migration, unemployment, and development: a two-sector analysis', *American Economic Review* 60, 126-142.

Kemp, M.C. and Negishi, T. (1970): 'Variable returns to scale, commodity taxes, factor market distortions and their implications for trade gains',. *Swedish Journal of Economics* 72, 1–11.

Khan, M. A. (1982): 'Tariffs, foreign capital and immiserizing growth with urban unemployment and specific factors of production', *Journal of Development Economics* 10, 245-56.

Marjit, S. and Beladi, H. (2003).'Possibility or impossibility of paradoxes in the small country Harris-Todaro framework: a unifying analysis',*Journal of Development Economicsvol.* 72(1), 379-385.

Ohyama, M. (1972): 'Trade and welfare in general equilibrium', *Keio Economic Studies*, 9, 37–73.

Pravin K. and Panagariya, A. (2000): 'A unification of second best results in international trade', *Journal of International Economics 52*, 235–257.

Appendix I: Determination of unionized wage

We consider a competitive formal sector industry. Labour (L) is the only variable input of production. The labour market facing the industry is unionized. Each firm in the industry has a separate trade union. The unionized wage is determined as a solution to the Nash bargaining game between the representative firm and the representative labour union.

The representative firm's profit function is given by:

$$\Pi = PQ(L) - W^{*}L \tag{A.1}$$

where P is the exogenously given price of the formal sector's product.

The representative labour union maximizes the aggregate wage income of its members net of their opportunity wage income i.e.

$$\Omega = (W^{*} - W)L \tag{A.2}$$

The informal sector wage, W, is the opportunity wage to the workers in the industry. This is because any worker failing to get employment in the formal will surely be getting a job in the informal sector.

We consider a cooperative game between the firm and the labour union that leads to determination of the unionized wage, W^{*} and the employment level, L. If the two parties fail to reach an agreement no production will take place and the workers have to accept jobs in the informal sector. So given the objective functions of the two parties, represented by equations (A.1) and (A.2), the disagreement pay-off is: $[0\ ,\ 0]$.

The Nash bargaining solution is obtained from the following optimization exercise.

$$\underset{W^*,L}{\text{Max}}\ J = [PQ(L) - W^*L]^{(1-U)} \times [(W^* - W)L]^U \tag{A.3}$$

where U is the bargaining strength of the labour unions.

The first-order conditions for maximization are

$$(1-U)[(W^* - W)L] = U[PQ(.) - W^*L] \tag{A.4}$$

and,

$$(1-U)(PQ_L - W^*)L = -U[PQ(.) - W^*L] \tag{A.5}$$

Using (A.4) and (A.5) one obtains

$$PQ_L = W \tag{A.6}$$

Differentiation of (A.6) leads to

$$\left(\frac{\partial L}{\partial W}\right) = \frac{1}{PQ_{LL}} < 0; \left(\frac{\partial L}{\partial P}\right) = -\frac{Q_L}{PQ_{LL}} > 0 \tag{A.7}$$

Simplification from (A.4) yields

$$W^* = U\frac{PQ(L)}{L} + (1-U)W \tag{1}$$

Equation (1) is the unionized wage function which has been presented in the text.

Differentiating (1) and using (A.7) we find the following results.

$$\left.\begin{array}{l}
(\frac{\partial W^*}{\partial U}) = (\frac{1}{L})(PQ(.) - WL) > 0 \\
(\frac{\partial W^*}{\partial W}) = (1-U) + \frac{U(WL - PQ(.))}{L^2 PQ_{LL}} > 0\,; \\
(\frac{\partial W^*}{\partial P}) = [\frac{UPQ(.)(Q_{LL}L + Q_L) - UWLQ_L}{PQ_{LL}L^2}] > 0 \text{ if } (Q_{LL}L + Q_L) \leq 0 \text{ i.e } \xi_L \geq 1, \\
\text{where } \xi_L = -(\frac{LQ_{LL}}{Q_L}) \text{ is the elasticity of marginal product curve of labour.}
\end{array}\right\} \qquad (2)$$

Appendix II: Some useful results

Differentiating equations (4) and (5.1) and arranging in a matrix notation one obtains

$$\begin{bmatrix} \theta_{L1} & \theta_{K1} \\ \theta_{L2}E_W & \theta_{K2} \end{bmatrix} \begin{bmatrix} \hat{W} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} 0 \\ (1 - E_P\theta_{L2})T\hat{t} \end{bmatrix} \qquad \text{(A.8)}$$

where $T = (t/1+t) > 0$.

Solving (A.8) by Cramer's rule we obtain the following expressions.

$$\left.\begin{array}{l}
\hat{W} = -(1 - E_P\theta_{L2})(\frac{\theta_{K1}}{|\theta|})T\hat{t} \\
\hat{r} = \theta_{L1}(1 - E_P\theta_{L2})(\frac{T\hat{t}}{|\theta|}) \\
(\hat{W} - \hat{r}) = -(\frac{1 - E_P\theta_{L2}}{|\theta|})T\hat{t}\,; \\
(\hat{W}^* - \hat{r}) = -(\frac{E_W\theta_{K1} + \theta_{L1}(1 - E_P)}{|\theta|})T\hat{t}\,; \text{ and,} \\
|\theta| = \theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}E_W > 0 \text{ (as sector 2 is more capital-intensive} \\
\text{than sector 1 in value sense; and, } (E_P + E_W = 1)).
\end{array}\right\} \qquad \text{(A.9)}$$

[Note that $\hat{W}^* = E_P T\hat{t} + E_W\hat{W} + E_U\hat{U}$. We do not intend to do comparative statics with respect to U. So we write $\hat{U} = 0$]

Differentiating equations (6.1) and (7), using (A.9), simplifying and writing in a matrix notation one gets

$$\begin{bmatrix} \lambda_{L1} & \lambda_{L2} \\ \lambda_{K1} & \lambda_{K2} \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = \begin{bmatrix} -A_1\hat{t} \\ \hat{K} + A_2\hat{t} \end{bmatrix} \tag{A.10}$$

$$\left.\begin{aligned} &\text{where: } A_1 = (\frac{T}{|\theta|})[\lambda_{L1}S_{LK}^1(1-E_P\theta_{L2}) + \lambda_{L2}S_{LK}^2(E_W\theta_{K1} + \theta_{L1}(1-E_P))] > 0 \\ &A_2 = (\frac{T}{|\theta|})[\lambda_{K1}S_{KL}^1(1-E_P\theta_{L2}) + \lambda_{K2}S_{KL}^2(E_W\theta_{K1} + \theta_{L1}(1-E_P))] > 0; \end{aligned}\right\} \tag{A.11}$$

S_{ji}^k = the degree of substitution between factors j and i in the k th sector, $j, i = L, K$; and, $k =$ 1,2. For example, $S_{LK}^1 \equiv (r / a_{L1})(\partial a_{L1} / \partial r)$, $S_{LL}^1 \equiv (W / a_{L1})(\partial a_{L1} / \partial W)$ etc. $S_{ji}^k > 0$ for $j \neq i$; and, $S_{jj}^k < 0$; and, '$\wedge$' = proportional change.

Solving (A.10) by using the Cramer's rule we get the following expressions.

$$\left.\begin{aligned} &\hat{X}_1 = -(\frac{1}{|\lambda|})[(\lambda_{K2}A_1 + \lambda_{L2}A_2)\hat{t} - \lambda_{L2}\hat{K}] \\ &\hat{X}_2 = (\frac{1}{|\lambda|})[\lambda_{L1}\hat{K} + (\lambda_{L1}A_2 + \lambda_{K1}A_1)\hat{t}] \end{aligned}\right\} \tag{A.12}$$

Appendix III: Expressions for welfare change

Differentiating (14) one gets

$$dD_1 + P_2^* dD_2 = dX_1 + P_2^* dX_2 + tP_2 dM - rdK_F \tag{A.13}$$

Using (15) and (A.13) we write

$$(dV / V_1) = dX_1 + P_2^* dX_2 + tP_2 dM - rdK_F \tag{A.14}$$

From differentiation of (16) one gets

$$dY = [dX_1 + P_2^* dX_2 + X_2 dP_2^* + tP_2 dM + P_2 M dt - rdK_F] \tag{A.15}$$

Differentiation of (17) and use of (A.15) yield

$$dM = (\partial D_2 / \partial P_2^*)dP_2^* + (\partial D_2 / \partial Y)[dX_1 + P_2^* dX_2 + X_2 dP_2^* + tP_2 dM + P_2 Mdt - rdK_F] - dX_2 \tag{A.16}$$

Here note that $X_1 = F^1(L_1, K_1)$ and $X_2 = F^2(L_2, K_2)$ are the two production functions while the full-employment conditions for the two inputs are:

$L_1 + L_2 = L$ and $K_1 + K_2 = K_D + K_F = K$

Therefore, equation (A.15) may be expressed as

or, $dY = [F_L{}^1 dL_1 + F_K{}^1 dK_1 + P_2^* F_L{}^2 dL_2 + P_2^* F_K{}^2 dK_2 + P_2 X_2 dt + tP_2 dM + P_2 Mdt - rdK_F]$

$= [WdL_1 + rdK_1 + W^* dL_2 + rdK_2 - rdK_F + P_2 D_2 dt + tP_2 dM]$

$= [(W^* - W)dL_2 + r(dK_1 + dK_2) - rdK_F + P_2 D_2 dt + tP_2 dM]$

or, $dY = (W^* - W)dL_2 + P_2(D_2 dt + tdM)$ (A.17)

Using (A.17), equation (A.16) may be expressed as

$dM = (\partial D_2 / \partial P_2^*)dP_2^* + (\partial D_2 / \partial Y)[(W^* - W)dL_2 + P_2(D_2 dt + tdM)] - dX_2$

or, $dM[1 - tP_2(\partial D_2 / \partial Y)] = (\partial D_2 / \partial Y)(W^* - W)dL_2 + P_2 dt\{(\partial D_2 / \partial P_2^*) + D_2(\partial D_2 / \partial Y)\} - dX_2$

or, $dM = v[(mv / P_2^*)(W^* - W)dL_2 + HP_2 dt - dX_2]$ (A.18)

where: $v = [(1+t)/\{1 + t(1-m)\}] > 0; m = P_2(1+t)(\partial D_2 / \partial Y)$ is the marginal propensity to consume commodity 2 ($1 > m > 0$); and, $H = [(\partial D_2 / \partial P_2(1+t)) + D_2(\partial D_2 / \partial Y)] < 0$ is the Slutsky's pure substitution term.

Using (A.14) and (A.18) one gets

$(dV / V_1) = v[(W^* - W)dL_2 + tP_2(HP_2 dt - dX_2)]$

or, $(dV / V_1) = v[(W^* - W)L_2 \hat{L}_2 + (tP_2)^2 H\hat{t} - tP_2 X_2 \hat{X}_2]$ (A.19)

Now $L_2 = a_{L2} X_2$ (A.20)

Differentiating (A.20) and using (A.9) and (A.12) one can derive the following expression.

$$\hat{L}_2 = (\frac{\lambda_{L1}}{|\lambda|})\hat{K} + [\frac{TS_{LK}^2(E_W \theta_{K1} + \theta_{L1}(1 - E_P))}{|\theta|} + \frac{(\lambda_{L1} A_2 + \lambda_{K1} A_1)}{|\lambda|}]\hat{t} \tag{A.21}$$

Using (A.19) and (A.21) the following expressions can be written.

$$(\frac{1}{V_1}\frac{dV}{dK}) = (\underset{(+)}{\frac{v\lambda_{L1}X_2}{K|\lambda|}})[(W*-W)a_{L2} - tP_2] \tag{18}$$

$$(\frac{1}{V_1}\frac{dV}{dt}) = (\frac{v}{t})[(W*-W)(\underset{(+)}{\frac{TL_2S_{LK}^2}{|\theta|}})\underset{(+)}{\{E_W\theta_{K1} + \theta_{L1}(1-E_P)\}}$$

$$+\{\underset{(+)}{\frac{(W*-W)a_{L2} - tP_2}{|\lambda|}}\}\underset{(+)}{X_2(\lambda_{L1}A_2 + \lambda_{K1}A_1)} + \underset{(-)}{(tP_2)^2H}] \tag{19}$$