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Estimating Standard Error of Inflation in Pakistan: A Stochastic Approach

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Abstract

We estimate standard errors (S.Es.) of month on month and year on year inflation in Pakistan based on data for the period of July 2001 to June 2010 using stochastic approach as well as extended stochastic approach to index numbers. We develop a mechanism to estimate S.E. of period average headline inflation using these approaches. This mechanism is then applied to estimate S.Es. of 12-month average inflation in Pakistan for July 2003 to June 2010. The systematic changes in the relative prices of different groups in the CPI basket for Pakistan are also estimated. The highest (positive) relative price inflation occurred in ‘food, beverages and tobacco’ group and lowest (negative) for ‘recreation and entertainment’ group, during FY01 to FY10.

Key Words: Inflation, Relative Prices, Pakistan

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“The answer to the question what is the mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as many purposes; and we may almost say in the matter of prices as many purposes as many writers.” Edgeworth (1888)

1. Introduction

Stochastic approach to index numbers has recently attracted renewed attention of researchers as it provides the standard error of index number (and its growth). One of the most important uses of index number is in the case of measurement of the general price level in an economy (and then inflation, of course). Stochastic approach to index numbers has been applied to measure inflation in studies like Clement and Izan (1987), Selvanathan (1989), Crompton (2000), Selvanathan (2003), Selvanathan and Selvanathan (2004), and Clement and Selvanathan (2007).

Historically, there are two main approaches to measure the index number: the functional approach and the stochastic approach. In the functional approach, prices and quantities of various goods and services are looked upon as connected by certain typical observable relationship (Frisch, 1936). The stochastic approach is less well known although it has a long history dating back to Jevons (1865) and Edgeworth (1888). In the stochastic approach prices and quantities are considered as two sets of independent variables. It assumes that (ideally) individual prices ought to change in the same proportion from one point of time to other. This assumption is based upon quantity theory of money - as the quantity of money increases all prices should increase proportionally. Any deviation of individual prices from such proportionality is seen as ‘errors of observation’ and/or may be the result of non-monetary factors’ affect on prices. Thus the rate of inflation can be calculated by averaging over the proportionate changes in the prices of all individual goods and services. Keynes (1930) criticized the assumption that all prices must change equiproportionately so that that relative prices remain same by considering this as being
‘root and cause erroneous’. In functional approach the deviations from proportionality are taken as expressions for those economic relations that serve to give economic meaning to index numbers (Frisch, 1936).

The recent interest of researchers in the stochastic approach to index number theory is led by Balk (1980), Clements and Izan (1981, 1987), Bryan and Cecchetti (1993) and Selvanathan and Rao (1994). Clements and Izan (1987) recognized the Keynes (1930) criticism on the assumption of identical systematic changes in prices and viewed the underlying rate of inflation as an unknown parameter to be estimated from the individual price changes by linking index number theory to regression analysis.

Furthermore, using functional approach to index numbers we obtain an estimate of the inflation rate without knowing its distribution. Thus, we have no basis to make any statistical comment, say about ‘efficiency,’ of the estimated inflation rate. For this purpose we need to know the standard error of the estimated rate of inflation as well. The stochastic approach leads to familiar index number formulae such as Divisia and Laspeyres type index numbers. As uncertainty plays a vital role in this approach the foundations differ markedly from those of the functional approach, linking index number theory to regression analysis we not only get an estimate of the rate of inflation but also its sampling variance. With the relaxation of the assumption that prices of goods and services change equiproportionately, the individual prices in the basket of price index move disproportionately (which usually happens) and thus the overall rate of inflation may become less well defined (Selvanathan and Selvanathan 2006b). In such situations the ability of stochastic approach becomes important by allowing us to construct confidence interval around the estimated rate of inflation with the help of standard error (of inflation). Confidence interval
built around the estimated rate of inflation can be used for some practical purposes such as wage negotiations, wage indexation, inflation targeting (in interval), etc.

One of the criticisms on this new stochastic approach of Clements and Izan (1987) was on the restriction of homoscedasticity on the variance of the error term in the OLS regression (Diewert, 1995). Crompton (2000) also pointed out this deficiency and extended the new stochastic approach to derive robust standard errors for the rate of inflation by relaxing the earlier restriction on the variance of the error term by considering an unknown form of heteroscedasticity. Selvanathan (2003) presented some comments and corrections on Crompton’s work. Selvanathan and Selvanathan (2004) showed how recent developments in stochastic approach to index number can be used to model the commodities prices in the OECD countries. Selvanathan and Selvanathan (2006) calculated annual rate of inflation for Australia, UK and US using stochastic approach. These studies provided mechanism for calculating standard error for inflation. Rather than targeting the headline (YoY) inflation, some countries track 12 month moving average inflation as goal of monetary policy. However, there is no work in the literature to estimate standard error of period average inflation. We contribute by developing a mechanism to estimate standard error of period average inflation.

In this study we estimate standard errors of month on month (MoM) and year on year (YoY) inflation in Pakistan using stochastic approach, following Selvanathan and Selvanathan (2006). Since State Bank of Pakistan (the central bank) targets 12-month average of YoY inflation, we contribute by applying our mechanism to estimate the standard error of 12-month average inflation in Pakistan.

1 Clements, Izan and Selvanathan (2006) presented a review on the stochastic approach to index number theory.
The criticism on the assumption while estimating the standard error of inflation, that when prices change they change equally proportionally, has been responded by Clements and Izan (1987) by extending the stochastic approach by considering the underlying rate of inflation separate from the changes in relative prices. We also estimate the standard errors of (MoM, and YoY) inflation in Pakistan using extended stochastic approach of Clements and Izan (1987). We contribute by developing a mechanism, and applying this to Pakistan, to estimate standard error of period average inflation using the extended stochastic approach as well. Furthermore, by applying extended stochastic approach we also estimate the systematic (MoM, and YoY, and 12-month average) change in relative prices based upon individual prices of 374 commodities in the CPI basket of Pakistan for period July 2001-June 2010. However, in this paper we present only the average systematic change in relative prices of different groups in CPI basket of Pakistan.

In the following section we provide details of existing mechanisms of stochastic approach and their applications to index number theory in the context of price index. We then further build upon this stochastic approach to estimate YoY inflation, period (12-month) average inflation and their standard errors. In section 3 we present results of the application of stochastic and the extended stochastic approach for estimating MoM inflation, YoY inflation, and period (12-month) average inflation along with their standard errors. In section 4 we present the estimated average systematic change in relative prices of different groups in CPI basket of Pakistan. Concluding remarks follow in the last Section.

2. Unfolding the Stochastic Approach to Index Numbers

Different ways to apply the stochastic approach to index numbers give various forms of index numbers like Divisia, Laspeyres etc. Since Federal Bureaus of Statistics (Pakistan’s official
statistical agency) uses Laspeyres index formula for measuring inflation in period \( t \) over the base period, we would like to confine following analysis to derive Laspeyres index.

2.1 Derivation of Laspeyres index number\(^2\)

Following conventional notations let \( p \) represents price and \( q \) represent the quantity. We subscript these notations by \( it \) where \( i(i = 1, 2, \ldots, n) \) represents commodity and \( t(t = 1, 2, \ldots, T) \) the time. Under stochastic approach any observed price change is a reading on the ‘underlying’ rate of inflation and a random component (\( \epsilon_{it} \)). If \( \gamma_t \) is the price index, relating expenditures in period \( t \) to expenditures in base period, then following stochastic approach we can write

\[
 p_{it}q_{i0} = \gamma_t p_{i0}q_{i0} + \epsilon_{it} \quad t(t = 1, 2, \ldots, T) 
\]

(1)

We assume

\[
 E(\epsilon_{it}) = 0, \quad \text{Cov}(\epsilon_{it}, \epsilon_{jt}) = \sigma_t^2 p_{i0}q_{i0} \delta_{ij} \quad (\delta_{ij} \text{ is the Kronecker delta}) 
\]

(2)

And thus we have related the index number theory to regression analysis as now we can estimate the rate of inflation in period \( t \) by estimating the unknown parameter \( \gamma_t \) in (1). Rearranging (1) we get

\[
 \frac{p_{it}}{p_{i0}} = \gamma_t + \frac{\epsilon_{it}}{p_{i0}q_{i0}} \quad (3)
\]

\[
 \Rightarrow E\left(\frac{p_{it}}{p_{i0}}\right) = \gamma_t,
\]

\(^2\) This sub-section (2.1) is mostly based upon Selvanathan and Selvanathan (2006b)
To remove heteroscedasticity in the error term we transform equation (1) into new form which
gives homoscedastic variances in the error term across all the n commodities in any particular
time period t. For this purpose we divide equation (1) by \( \frac{p_{it}}{p_{i0} q_{i0}} \) and obtain

\[
y_{it} = \gamma_t x_{i0} + \eta_{it} \tag{4}
\]

Where \( y_{it} = \frac{p_{it} q_{i0}}{\sqrt{p_{i0} q_{i0}}} \); \( x_{i0} = \sqrt{p_{i0} q_{i0}} \) and \( \eta_{it} = \frac{\epsilon_{it}}{\sqrt{p_{i0} q_{i0}}} \)

Now assumptions in (2) after above transformation are

\[
E(\eta_{it}) = 0 \quad \text{and} \quad \text{Cov} \left( \eta_{it}, \eta_{jt} \right) = \sigma_t^2 \delta_{ij}
\]

Now we can apply, say, least squares to (4) to have an estimator for inflation as below:

\[
\hat{\gamma}_t = \frac{\sum_{i=1}^{n} x_{i0} y_{it}}{\sum_{i=1}^{n} x_{i0}^2}
\]

\[
= \frac{\sum_{i=1}^{n} \left( \frac{p_{it} q_{i0}}{\sqrt{p_{i0} q_{i0}}} \right) \left( \frac{p_{it} q_{i0}}{\sqrt{p_{i0} q_{i0}}} \right)}{\sum_{i=1}^{n} p_{i0} q_{i0}} = \frac{\sum_{i=1}^{n} p_{it} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} = \frac{\sum_{i=1}^{n} \left( \frac{p_{it} q_{i0}}{p_{i0}} \right) p_{i0}}{\sum_{i=1}^{n} \left( \frac{p_{it} q_{i0}}{p_{i0}} \right) p_{i0}} = \sum_{i=1}^{n} \left[ \frac{p_{it} q_{i0}}{\sum_{i=1}^{n} \left( \frac{p_{it} q_{i0}}{p_{i0}} \right) p_{i0}} \right]
\]

We know \( \frac{p_{i0} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \) is the budget share of commodity \( i \) in base period and if we write it as

\[ w_{i0} = \frac{p_{i0} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \text{ where } \sum_{i=1}^{n} w_{i0} = 1 \], we have

\[
\hat{\gamma}_t = \sum_{i=1}^{n} w_{i0} \frac{p_{it}}{p_{i0}} \tag{5}
\]

Which is weighted average of the n price ratios (with base-period budget shares being weights)
and is well-known Laspeyres price index. With the help of this price index we can have inflation
(MoM and/or YoY) by using simple formulae as below:

\[
\text{Inflation (MoM)} = \left( \frac{\hat{\gamma}_t}{\hat{\gamma}_{t-1}} - 1 \right) \times 100 \tag{6}
\]

\[
\text{Inflation (YoY)} = \left( \frac{\hat{\gamma}_t}{\hat{\gamma}_{t-12}} - 1 \right) \times 100 \tag{7}
\]

Variance of the estimator in (5) is given by
\[ \text{Var}(\hat{y}_t) = \frac{\sigma_t^2}{\sum_{i=1}^n x_{i0}^2} \]  
\hspace{1cm} (8)

The parameter \( \sigma_t^2 \) can be estimated as
\[ \hat{\sigma}_t^2 = \frac{1}{n-1} \sum_{i=1}^n (y_{it} - \hat{y}_t x_{i0})^2 \]  
\hspace{1cm} (9)

By substitution estimated parameter of \( \sigma_t^2 \) from (9) together with the values of \( x_{i0} \) and \( y_{it} \) in (8) and rearranging we get
\[ \text{Var}(\hat{y}_t) = \frac{1}{n-1} \sum_{i=1}^n w_{i0}(p_{it}/p_{i0} - \hat{y}_t)^2 \]  
\hspace{1cm} (10)

Thus, as the degree of relative price variability increases the variance of the estimated index increases. This agrees with the intuitive notion that when the individual prices move very disproportionately, the overall price index is less well-defined (Logue and Willet (1976)).

Now question is in (5) and (10) we can have the estimated price index and its estimated variance respectively. From (5) we can find the estimated rate of inflation but here we cannot find the variance of the estimated rate of inflation. For this purpose we have to proceed with inflation from the start rather than index.

2.2 **Application of Stochastic Approach to estimate headline inflation and its S.E.**

Following notations used above, if \( y_t \) is the price index, relating expenditures in current period to expenditures in base period, then, following stochastic approach, we can write
\[ p_{it}q_{i0} = y_t p_{i0}q_{i0} + \varepsilon_{it} \hspace{1cm} t(t = 1, 2, \ldots, T) \]  
\hspace{1cm} (11)

Base period can be somewhere in distant past (say five year back) and at any point in time we define headline (or, YoY) inflation as percentage change in price index over corresponding month last year then
\[ \pi_t^H = \frac{y_t - y_{t-12}}{y_{t-12}} \]
From (11) we can estimate of $\gamma_t$ only. For estimate of $\gamma_{t-12}$ we write (11) as

$$p_{it-12}q_{i0} = \gamma_{t-12} p_{i0} q_{i0} + \epsilon_{it-12} \quad t(t = 13, 14, 15 \ldots, T) \tag{12}$$

Here again $E\left(\frac{p_{it-12}}{p_{i0}}\right) = \gamma_{t-12}$, under the similar assumptions as in (2)

By subtracting (12) from (11) we have

$$p_{it}q_{i0} - p_{it-12}q_{i0} = \left(\gamma_t - \gamma_{t-12}\right)p_{i0}q_{i0} + \epsilon_{it} - \epsilon_{it-12} \tag{13}$$

Dividing (13) by $E\left(\frac{p_{it-12}}{p_{i0}}\right)$ and substituting $E\left(\frac{p_{it-12}}{p_{i0}}\right) = \gamma_{t-12}$ on right hand side, we get

$$\left[\frac{p_{it}q_{i0} - p_{it-12}q_{i0}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right] = \left(\frac{\gamma_t - \gamma_{t-12}}{\gamma_{t-12}}\right)p_{i0}q_{i0} + \frac{\epsilon_{it} - \epsilon_{it-12}}{p_{i0}q_{i0}} = \pi_t^H p_{i0}q_{i0} + \epsilon_{it} \tag{14}$$

Where $\epsilon_{it} = \frac{\epsilon_{it} - \epsilon_{it-12}}{\gamma_{t-12}}$. Again assuming that $E(\epsilon_{it}) = 0$ and $Cov(\epsilon_{it}, \epsilon_{jt}) = \rho_t^2 p_{i0}q_{i0} \delta_{ij}$

and proceeding as in subsection 2.1 we divide (14) by $\sqrt{p_{i0}q_{i0}}$ and get

$$\left[\frac{p_{it} - p_{it-12}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right] \sqrt{p_{i0}q_{i0}} = \pi_t^H \sqrt{p_{i0}q_{i0}} + \frac{\epsilon_{it}}{\sqrt{p_{i0}q_{i0}} \ \tag{16}$$

From (5) and (12) we can write $\gamma_{t-12} = E\left(\frac{p_{it-12}}{p_{i0}}\right) = \sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}$, Thus (16) becomes

$$\left[\frac{p_{it} - p_{it-12}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right] \sqrt{p_{i0}q_{i0}} = \pi_t^H \sqrt{p_{i0}q_{i0}} + \frac{\epsilon_{it}}{\sqrt{p_{i0}q_{i0}}}$$

If we take $Y_{it} = \left(\frac{p_{it} - p_{it-12}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right) \sqrt{p_{i0}q_{i0}} \ , X_{i0} = \sqrt{p_{i0}q_{i0}} \quad \text{and} \ \varphi_{it} = \frac{\epsilon_{it}}{\sqrt{p_{i0}q_{i0}}}$. Under assumptions

that $E(\varphi_{it})=0$ and $Cov(\varphi_{it}, \varphi_{jt}) = \rho_t^2 \delta_{ij}$, for equation

$$Y_{it} = \pi_t^H X_{i0} + \varphi_{it} \tag{17}$$

Least square estimator of $\pi_t^H$ is
\[
\hat{\pi}_t^H = \frac{\sum_{i=1}^n X_{i0}^r Y_{it}}{\sum_{i=1}^n X_{i0}^2} = \frac{\sum_{i=1}^n \sqrt{P_{i0} q_{i0}} \left[ \frac{P_{it} - P_{it-12}}{P_{i0}} \right]}{\sum_{i=1}^n P_{i0} q_{i0}} = \frac{\sum_{i=1}^n w_{i0} \left[ \frac{P_{it} - P_{it-12}}{P_{i0}} \right]}{\sum_{i=1}^n w_{i0} P_{i0} q_{i0}} = \gamma_t - \gamma_{t-12} \tag{18}
\]

We knew this result from (7). Only benefit of the above process is that now we can have an estimate of the standard error of headline inflation as below:

\[
Var (\hat{\pi}_t^H) = \frac{\rho_t^2}{\Sigma_{i=1}^n X_{i0}^2} \tag{19}
\]

The parameter \(\rho_t^2\) can be estimated as

\[
\hat{\rho}_t^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_{it} - \hat{\pi}_t^H X_{i0})^2
\]

By substitution of the estimated parameter \(\rho_t^2\) in (19) and rearranging we get

\[
Var (\hat{\pi}_t^H) = \frac{1}{n-1} \sum_{i=1}^n w_{i0} \left[ \frac{P_{it} - P_{it-12}}{P_{i0}} \right]^2 \left( \hat{\pi}_t^H - \gamma_t - \gamma_{t-12} \right) \tag{20}
\]

Equation (20) shows that the variance of \(\hat{\pi}_t^H\) increases with the degree of relative inflation variability\(^3\). Now we move towards estimating the period average inflation and its standard error.

2.3 Application of Stochastic Approach to estimate period average inflation and its S.E.

We know that period average; (say 12 month average) inflation can be calculated either by averaging the last 12 YoY inflation numbers or by taking YoY inflation of the 12-month (moving) averaged index number. We would like to use result above in subsection 2.1 for estimating the 12-month average inflation, and those in subsection 2.2 for the standard error of period average inflation.

We have price index series as \(p_{it}\). If 12-month averaged price index series is denoted by \(p_{it}^A\) then following the results in subsection 2.1, estimate of YoY inflation of \(p_{it}^A\) series will be

\(^3\) Above procedure can be used to estimate the MoM inflation and its standard error.
\[ \hat{\gamma}^A_t = \sum_{i=1}^{n} w_{i0} \frac{p^A_{it}}{p_{i0}} \] (21)

And thus

\[ \hat{\pi}^A_t = \left( \frac{\hat{\gamma}^A_t}{\gamma^A_{t-12}} - 1 \right) * 100 \] (22)

Now for estimating the variance of the average inflation we use the result in above subsection 2.2 where we derived the standard error of YoY inflation. If we replace the index with the average index in (20) we will get the standard error of average YoY inflation, that is

\[ Var (\hat{\pi}^A_t) = \frac{1}{n-1} \sum_{i=1}^{n} w_{i0} \left[ \frac{p^A_{it}}{p_{i0}} - \frac{p^A_{i-12}}{p_{i0}} - \hat{\pi}^A_t \right]^2 \] (23)

3. Measuring Standard Errors of Inflation in Pakistan

In this section we present an application of the results described and derived in the previous section by using monthly data of prices of 374 commodities covering the period July 2001- June 2010 for Pakistan\(^4\). We present the estimated MoM inflation, YoY inflation, along with their standard errors for the case of Pakistan. As discussed above there are different ways to apply the stochastic approach to index numbers and each culminates in different form of index numbers like Divisia, Laspeyres etc. Just to compare our estimated results of inflation with those from Federal Bureaus of Statistics we have used such application of stochastic approach which produces Laspeyres index formula for measuring inflation in current period over the base period. Since State Bank of Pakistan targets (12-month) average inflation, particular attention has been

\(^4\) Prices for consumer price index (CPI) are collected by Federal Bureau of Statistics (Government of Pakistan) on monthly basis for 374 commodities.
paid to estimate period (12-month) average inflation and its standard error which is first empirical application of its type.

Table A1 in the Appendix presents the official rate of (monthly, YoY and 12-month average) inflation and the estimated rate of (monthly, YoY and 12-month average) inflation based on stochastic approach along with standard error of the estimated of inflation for Pakistan economy based on the data for July 2001 to June 2010\(^5\).

Figures 1(a) to 1(c) present a scatter plot of the inflation versus the corresponding standard error for MoM, YoY and 12-month average inflation; the solid line is the linear trend line.

\(^5\) First 12 observations are lost in the YoY inflation calculation and next 12 are consumed in calculating the 12 month average. Thus, the results in the Table 1 start from July 2003 instead of July 2001.
From figures 1 (a) to 1(c) we can see that the standard error increases with increasing inflation as is depicted by the positive slope of trend line. An interpretation of this observation is that when inflation is higher it becomes difficult to predict it. Hence this agrees with the intuitive notion that when the individual prices move very disproportionately, the overall rate of inflation is less well defined. These observations are in line with the past literature on the rate and variability of inflation.

The inflation may become less predictable at higher inflation rate if government aims stabilizing prices rather than stabilizing expectations (Logue and Willet (1976))
Figure 2 (a) to figure 2(c) present graph of all the three types of inflation along with respective 95% confidence band. From figure 2 (a) it is clear that the time when there is a jump in inflation like in April 2005 and May 2008, there is an increase in the width of the confidence bands. Similarly we can note that in other figures where the inflation is high, the width of confidence band is also increased.
4. Extended Stochastic Approach and the Systematic Change in Relative Prices

As we discussed in section 2, different ways to apply the stochastic approach to index numbers give various forms of index numbers like the Laspeyres etc. However, as criticized by Keynes (1930), following this approach it is assumed that when prices change they change equipropotionately and thus relative prices remain same. Clements and Izan (1987) responded the Keynes criticism by considering common trend change in all prices as underlying rate of inflation separate from the systematic change in relative prices. Following Clements and Izan (1987), if we take $p_{it}$ as the price of commodity $i$ ($i = 1, 2, ..., n$) at time $t$ ($t = 1, 2, ..., T$) then price log change $Dp_{it} = \log p_{it} - \log p_{it-1}$ can be considered as

$$Dp_{it} = \alpha_t + \beta_i + \xi_{it} \quad i = 1, 2, ..., n; \text{ and } t = 1, 2, ...., T \quad (24)$$

Where $\alpha_t$ is the common trend change in all prices (the underlying rate of inflation) and $\beta_i$ is the change in relative prices of commodity $i$. Assuming the random component of change in prices, $\xi_{it}$, to be independent over commodities and time, and that the variances $[Var(\xi_{it})]$ are inversely proportional to corresponding arithmetic averages of budget shares; Clements and Izan (1987) showed that least squares estimates of $\alpha_t$ and $\beta_i$ subject to budget constraint$^7$ as given below:

$$\hat{\alpha}_t = \sum_{i=1}^{n} \bar{w}_i Dp_{it} \quad (25)$$

$$\hat{\beta}_i = \frac{1}{T} \sum_{t=1}^{T} \left[ \frac{1}{\bar{q}_i T} / \sum_{i=1}^{T} \frac{1}{\bar{q}_i T} \right] (Dp_{it} - \hat{\alpha}_t) \quad (26)$$

With respective variances of these estimators as below:

$$Var(\hat{\alpha}_t) = \frac{\sigma^2}{(n-1)} \quad (27)$$

$^7$ Budget share weighted average of the systematic component of relative price change is zero
\[ Var(\hat{\beta}_i) = \frac{1}{(n-1)\sum_{t=1}^{T} \frac{1}{w_i}} \left( \frac{1}{w_i} - 1 \right) \]  \hspace{1cm} (28)

Where \( \theta^2 \) is the sum (over commodities) of squares of estimated random component of price changes, that is

\[ \theta^2_t = \sum_{i=1}^{n} (\xi_{it})^2 \]

\[ = \sum_{i=1}^{n} w_i (\bar{Dp}_{it} - \bar{\alpha}_t)^2 + \sum_{i=1}^{n} w_i (\bar{Dp}_t - \bar{\alpha})^2 - 2 \sum_{i=1}^{n} w_i (\bar{Dp}_t - \bar{\alpha}_t) \bar{w}_i (\bar{Dp}_t - \bar{\alpha}) \]  \hspace{1cm} (29)

While it is obvious that \( \bar{Dp}_t = \frac{1}{T} \sum_{t=1}^{T} Dp_{it} \) and \( \bar{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \alpha_t \)

Our contribution in this section of the study is an application of the Clements and Izan (1987) extended stochastic approach to index numbers to Pakistan’s monthly data of prices of 374 commodities covering the period July 2001- June 2010. Extended stochastic approach to index number is closer to Divisia price index. As in above section 3, this approach also gives us the (trend) inflation rate and its standard errors which are presented in Table A1 of the Appendix. We can see that the estimated standard errors of the inflation rate based upon extended stochastic approach to inflation are lower than those based upon the stochastic approach for the MoM and YoY inflation. It needs not be true in the case of period average inflation (because of averaging effects).

In addition to inflation and its standard error, the Clements and Izan (1987) extended stochastic approach also gives us the systematic change in relative prices of each commodity in the basket. We have applied this approach upon prices of 374 commodities in the CPI basket of Pakistan for the period of FY01 to FY 2010 to investigate the systematic relative price changes. It may be difficult to extract any meaningful result from the detailed presentation of systematic (MoM, and
YoY, and 12-month average) change in relative prices of each of the 374 commodities. However, it will be useful if we present the systematic (MoM, and YoY, and 12-month average) change in relative prices for various groups in the CPI basket as in Table A2 of the Appendix. There are ten groups in the CPI basket of Pakistan as shown in the Table A2. From the Table A2 it is clear that coefficients of relative prices of all groups are significantly different from zero.

For comparison purpose we have also given the observed relative price changes as measured from FBS price data for all the three cases: MoM, YoY and 12-month moving average. The estimated relative prices of ‘Food Beverages & Tobacco’ group are increased by highest percentage point for MoM changes (0.18%). In case of YoY changes, we find ‘Food Beverages & Tobacco (FBT)’ and ‘Fuel and Lighting’ groups to exhibit increase in relative prices by 1.72 percent and 0.37 percent respectively. In the case of 12-month moving averages, we find ‘FBT’ and ‘House Rent’ groups to depict increase in relative prices by 1.72 percent and 0.04 percent respectively. In all the three cases, since ‘FBT’ group turned out to have the highest change in relative prices, we can say that inflation during most of the FY01 to FY10 was FBT prices changes driven.

Interestingly, for each of the three cases of MoM, YoY and 12-month moving average, the change in relative prices are found to be highest (positive) for ‘FBT’ group and lowest (negative) for ‘Recreation and Entertainment (RE)’ group during FY01 to FY10. Supply side factor(s) and/or elasticities of demand may be behind this observed phenomenon as commodities in the FBT group are more prone to supply shocks and tend to be less price elastic compared to those in RE group.

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8 Detailed results can be obtained from the authors, if desired.
9 We can see from the Table A2 in the Appendix that the weighted average of the relative prices is zero in each of the observed and estimated case, which should be.
Table A1 in the Appendix presents the official rate of (monthly, YoY and 12-month average) inflation and the estimated rate of (monthly, YoY and 12-month average) inflation based on stochastic as well as extended stochastic approaches along with standard error of the estimated of inflation for Pakistan economy based on the data for July 2001 to June 2010. Numerically, the official and estimated inflation rates seem different. But when we apply t-test we could not find official inflation rate to be statistically different from any of the estimated inflation rate based on stochastic as well as extended stochastic approaches. Which approach for measuring inflation is better? Obviously the stochastic approach has an advantage of estimating the standard errors along with the inflation rate and is thus preferable. Furthermore, in the case of using extended stochastic approach we also get estimates of systematic change in relative prices and their standard errors. Confidence interval can be built around the estimated rate of inflation for different useful purposes like wage bargaining.

5. Conclusion

In this study we estimate standard errors of month on month and year on year inflation using stochastic approach of Selvanathan and Selvanathan (2006) and extended stochastic approach of Clements and Izan (1987) based on individual prices of 374 commodities in CPI basket of Pakistan for the period of July 2001 to June 2010. We contribute to the literature on stochastic approach to index numbers by developing a mechanism to estimate the inflation and its standard error for period average inflation. Based on this mechanism, we estimate standard error of 12-month moving average YoY inflation for Pakistan for the period of July 2003 to June 2010. We find that standard error of inflation increases with inflation in Pakistan. Notwithstanding higher

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10 The results of t-test are not reported in the paper to save the space. However, those can be obtained from the authors, if required.
the standard error higher the inflation rate, the estimated standard errors of the inflation rate based upon extended stochastic approach to inflation are lower than those based upon the stochastic approach. Furthermore, for each of the three cases of MoM, YoY and 12-month average, the change in relative prices are found to be highest for ‘Food Beverages & Tobacco’ group and lowest for ‘Recreation and Entertainment’ group during FY01 to FY10.
References


## Appendix

### Table A1: Official rate of inflation, stochastic approach estimates of inflation and extended stochastic approach estimates of inflation

<table>
<thead>
<tr>
<th>Month</th>
<th>Official rate of inflation</th>
<th>Month on Month</th>
<th>Headline (Year on Year)</th>
<th>12-month moving average</th>
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</tr>
<tr>
<td></td>
<td>Official rate of inflation S.E.</td>
<td>Stochastic approach estimates of inflation S.E.</td>
<td>Extended stochastic approach estimates of inflation S.E.</td>
<td>Official rate of inflation S.E.</td>
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<td>0.89</td>
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Table A2: Group-wise change in relative price (July 01-June 10)

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<tr>
<th>Group</th>
<th>Weight in CPI basket</th>
<th>Month on Month Inflation</th>
<th>Headline (Year on Year)</th>
<th>12-month moving average</th>
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<tr>
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<td>Observed change in relative price (%)</td>
<td>Estimated change in relative price (%)</td>
<td>S.E.</td>
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<tr>
<td>Food Beverages &amp; Tobacco</td>
<td>0.403</td>
<td>0.14</td>
<td>0.18</td>
<td>0.001</td>
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<td>Apparel, Textile &amp; Footwear</td>
<td>0.061</td>
<td>-0.26</td>
<td>-0.21</td>
<td>0.004</td>
</tr>
<tr>
<td>House Rent</td>
<td>0.234</td>
<td>-0.06</td>
<td>-0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>Fuel &amp; Lighting</td>
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<td>0.03</td>
<td>-0.18</td>
<td>0.003</td>
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<td>Household Furniture &amp; Equipment</td>
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<td>-0.24</td>
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<td>0.008</td>
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<td>-0.14</td>
<td>0.003</td>
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<tr>
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<td>0.035</td>
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<td>0.008</td>
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