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by

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Abstract

Stylized facts on financial time series data are the volatility of returns that follow non-normal conditions such as leverage effects and heavier tails leading returns to have heavier magnitudes of extreme losses. Value-at-risk is a standard method of forecasting possible future losses in investments. A procedure of estimating value-at-risk using time-varying conditional Johnson S_U distribution is introduced and assessed with econometric models. The Johnson distribution offers the ability to model higher parameters with time-varying structure using maximum likelihood estimation techniques. Two procedures of modeling with the Johnson distribution are introduced: joint estimation of the volatility and two-step procedure where estimation of the volatility is separate from the estimation of higher parameters. The procedures were demonstrated on Philippine-foreign exchange rates and the Philippine stock exchange index. They were assessed with forecast evaluation measures with comparison to different value-at-risk methodologies. The research opens up modeling procedures where manipulation of higher parameters can be integrated in the value-at-risk methodology.

Keywords: Time Varying Parameters, GARCH models, Nonnormal distributions, Risk Management

JEL Classification: C22, C58, G12, G32

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Introduction

Financial institutions engage in investment activities to expand their assets so that they can provide quality financial products and services to their clients. In these activities, financial institutions incur risks of loss from their investment which may cause bankruptcy to their firms. In the intricate web of the financial sector, the downfall of large institutions or many firms may lead to financial crisis.

Central banks as financial regulators require financial institutions to comply within levels of allowable incurred risk in financial activities. Risks in financial activities are categorized in three kinds: (1) credit risk, incurred by lending to other institutions, (2) market risk, incurred by keeping a portfolio of assets where prices are determined by market forces, e.g., stocks, commodities, and currencies, and (3) operational risks, incurred from internal operations of the institutions, such as electricity and office equipment failures. (BSP Memo. Cir. 538)

Financial regulators observed international guidelines on risk capital adequacy over financial institutions as stipulated by the Basel Committee on Banking Supervision (Basel, 2004). There are different multiple suggested methods in dealing with different risks, yet the paper focuses on market risks where time series analysis and econometric modeling are preferred. In managing market risks, one of the tools to comply with these specifications is the Value-at-Risk (VaR) methodology, the measure of minimum possible loss as returns of investment given a probability of extreme loss (Jorion, 2006).

Stylized facts in the distribution of returns are the time-varying nature in mean and variance, which brought time series models autoregressive-moving average (ARMA) models and generalized autoregressive conditional heteroscedasticity (GARCH) models in the fray of financial econometrics (Tsay, 2002). In higher moments, unequal leverage effects due to negative shifts and fat tails have been evident in many researches, which correspond left-side skewness and leptokurtosis which are changes in the shapes of distributions of returns (Tsay, 2002). In the existence of means and variances that change in time, the concept of time-varying densities in financial returns is gaining ground and questions arise as whether there is strong evidence for these behaviors in profits and losses (Jondeau, et. al. 2007).

The aim of the paper is to derive a VaR methodology that incorporates time-varying shape characteristics in the framework. The Johnson S_U distribution with time-varying parameters is assumed in the value-at-risk model as the underlying distribution of the returns (Yan, 2005). Two procedures are devised that incorporate density changes, (1) a joint estimation in which mean and variance models are incorporated in the likelihood function and (2) a two-step approach where the appropriate mean and variance models would generate residuals to be fitted with the Johnson S_U density. These procedures would be compared to econometric methodologies for risk management on Philippine financial time series data with the aid of different evaluation measures and statistical tests.

Returns from Asset Prices

Returns are relative capital gains from possessing financial assets and equities (Jorion, 2007; Tsay 2002). For an asset of price P_t at time *t*, an arithmetic return at time *t* is defined as (Jorion, 2007):

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

The arithmetic return describes the relative change of assets based on most recent previous asset price.

The geometric return, also known as log-return, of an asset at time *t* is defined as (Tsay, 2002):

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{2}$$

The logarithm is of base e, the Euler number. It is favorable to use the log-returns due to its additive property (Jorion, 2007). The logarithmic transformation of the data is more favored since it restricts prices as positive values compared to the arithmetic returns and reduces the magnitude of volatility in price changes (Chatterjee, et al., 2000). With the statistically favorable advantages, returns computed in the paper are log-returns.

The Definition of Value-at-Risk

Value-at-risk in simple description is a minimum threshold value for possible losses in a given probability of risk to extreme loss (Jorion, 2007). A more formal definition is given by Tsay (2002), which deals with probability. Suppose that at current time *t*, a VaR value is to be estimated for *k* periods ahead. Let r_t be the financial asset return series of interest to be evaluated with a distribution function $F_{r_t}(x)$, where a negative return means loss in the long position. Define $F_{r_t}^{-1}(q) = \inf\{x \mid F_{r_t}(x) \geq q\}$ to be the quantile function for a left-tail probability *q*. Let the risk probability for extreme loss be p, commonly used values are 0.01, 0.05, or 0.10. Then the $100(1-p)\%$ value-at-risk of possessing 1 unit of an asset *k* periods ahead is equal to (Tsay, 2002):

$$VaR: p = P\left(r_{t+k} \leq VaR\right) = F_{r_{t+k}}\left(VaR\right) \ or \ VaR = F_{r_{t+k}}^{-1}\left(p\right) \tag{3}$$

In estimating VaR for an asset, the following elements should be known: (1) the probability p, (2) the forecast horizon *k*, (3) the data frequency, e.g., daily or weekly, (4) the distribution of asset returns, and (5) the amount of position for the asset (Tsay, 2002). In VaR estimation, statistical modeling and time series analysis are used in describing the distribution of asset returns and research on deriving VaR from different methods and techniques and comparison of these different techniques are fruitful and numerous.

The Family of Value-at-Risk Methods

Historical Method

In evaluating the quantile of a distribution for VaR, a simple approach is to solve for sample quantiles based on historical data on asset returns. If $\{r_1, r_2, \ldots, r_t\}$ is a subset of data on consecutive periods of the return series of an asset with window length *t* and $r_{(i)}$ is the *i*th smallest return in the window, then the one-period ahead $100(1-p)\%$ VaR is equal to:

$$VaR_{Hist} = r_{([tp])} + \left(tp - [tp]\right)\left(r_{([tp]+1)} - r_{([tp])}\right) \tag{4}$$

where $[q]$ means the integer part of the real number q (Tsay, 2002; Fallon and Sarmiento-Sabogal, 2003). For example, in a window of an asset return series with 1500 data points, the long-position 99% VaR would be the first percentile of the data, which is 15th smallest return value.

In this method of estimation, the assumed distribution of the data is the empirical distribution of returns. It avoids the possible misspecification by not assuming mathematical probability distributions. It is a very easy method that deals no statistical complexity. Caveat of the method is that it assumes that the distribution is similar between past observed values and future unobserved values of the returns (Tsay, 2002). The static approach ignores the time-varying nature of asset returns especially in volatility. In addition, the use of sample quantiles in estimating true quantiles at the tails of a distribution is very unreliable with very high variation (Danielsson and de Vries, 1997).

Econometric Methods

Econometric methods use time series modeling and analysis methodologies in modeling the distribution of asset returns (Tsay, 2002). The methodology involves the specification of the following: (1) conditional mean structure μ_t as a function of time t, e.g. using autoregressive-moving average (ARMA) models (Box ,et al. ,

1994) or regression models with exogenous explanatory variables, (2) conditional variance equation h_t for volatility as a function of time, e.g., using autoregressive conditional heteroscedasticity (ARCH) models (Engle, 1982; Bollerslev, 1986; Nelson, 1991), and (3) specification of the standardized error distribution $\varepsilon_t \sim F_\varepsilon$, e.g., using the standard normal distribution (Engle, 1982; Longerstaey and Spencer, 1996), the standardized t distribution (Tsay, 2002), or the generalized error distribution (Nelson, 1991).

Given that the proper three elements of the econometric methods has been fully specified and all model parameters are estimated, the one-period ahead $100(1-p)\%$ VaR based on the econometric method is equal to:

$$VaR_{Econ} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}} F_\varepsilon^{-1}(p) \tag{5}$$

The hats over the mean and variance specification imply one-step ahead forecasts for the mean and variance of the return series. The function F_ε^{-1} is the quantile function of the standardized error distribution.

An example of popular models in the econometric method is the RiskMetrics model of J. P. Morgan (Longerstaey and Spencer, 1996). The model assumes that the conditional mean of the returns is zero always and the conditional variance follows a special IGARCH(1,1) model. The assumed error distribution of the data is the standard normal distribution. In equation form for the log-return series r_t:

$$\begin{aligned} r_t &= 0 + \sqrt{h_t}\varepsilon_t \\ \varepsilon_t &\overset{iid}{\sim} N(0,1) \\ h_t &= \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \end{aligned} \tag{6}$$

The parameter λ describes the variance process as an exponentially weighted moving average and is determined to be any number between 0.9 and 1 (Longerstaey and Spencer, 1996; Tsay, 2002). A problem of the RiskMetrics system is the assumption of normality. The distribution of financial returns tend to deviate from the normal distribution and are more likely to be heavy-tailed, meaning that the data has greater chances of tail values occurring in the changes of asset prices than compared to the normal distribution (Tsay, 2002).

Since the normal distribution is inadequate in modeling financial returns, another compromise is the t-distribution, which has a bell-shaped density curve but with fatter tails compared to the normal distribution. When an appropriate mean and variance model has been fitted for the standardized t distribution, the one-step ahead $100(1-p)\%$ VaR for the t distribution with v degrees of freedom is given by (Tsay, 2002):

$$VaR_{Econ,t} = \hat{\mu}_{t+1} + \frac{t_{p,v}}{\sqrt{\frac{v}{v-2}}}\sqrt{\hat{h}_{t+1}} \tag{7}$$

The $t_{p,v}$ is the pth lower quantile of the t distribution with *v* degrees of freedom. The parameter *v* and other model parameters are jointly estimated.

Extreme Value Theory Methods

In targeting tail values directly, techniques in extreme value theory are another set of tools for VaR estimation. Extreme value theory methods are large-sample procedures designed to describe and make inferences on the tail values of distributions of data (Coles, 2001). These techniques are described as semi-parametric because of the involvement of parameter estimation in the methodology but the parameters are not directly related to the distribution that generated the data (Jondeau, et al., 2007)

There are two basic techniques in extreme value theory: (1) the block maxima method (Berman, 1964; Longin, 2000) and (2) the peaks-over-thresholds model (Pickands, 1975; Davison and Smith, 1990; Smith, 1999).

The block maxima technique has the following steps for a return series $\{y_1, y_2, ..., y_T\}$ (Longin, 2000): (1) divide the data into exclusive *g* subsamples, each of equal subsample size of *m* periods (e.g., *m* = 20 for months as subsamples, 60 for quarterly, 250 for yearly) such that $T = mg$, thus the data is of the form $\{[y_1, y_2, ..., y_m], [y_{m+1}, y_{m+2}, ..., y_{2m}], ..., [y_{m(g-1)+1}, y_{m(g-1)+2}, ..., y_{mg}]\}$; if a remainder *r* exists, then remove the first *r* periods in the return series; (2) from each of the g subsamples, the subsample minimum value $y_j^{\min},\ j = 1, 2, ..., g$ is gathered for long-position VaR, creating a new data series $\{y_1^{\min}, y_2^{\min}, ..., y_g^{\min}\}$; (3) the minimum series is fitted to the generalized extreme value (GEV) distribution for the minimum (Berman, 1964; Coles, 2001; Longin, 2000) and its parameters are estimated; the cumulative distribution function of the GEV is shown below:

$$F_{Min}(u) = \begin{cases} 1 - \exp\left\{ -\left(1 + \tau \dfrac{u-\alpha}{\beta}\right)^{1/\tau} \right\} & if\ \tau \neq 0 \\ 1 - \exp\left\{ -\exp\left(\dfrac{u-\alpha}{\beta}\right) \right\} & if\ \tau = 0 \end{cases} \tag{8}$$

The domain of F_{Min} depends on the value of τ; if $\tau > 0$, then $u > \alpha - \beta/\tau$ (Weibull distribution), $u > \alpha - \beta/\tau$ when $\tau < 0$ (Frechet distribution), and $u \in$ □ when $\tau = 0$ (Gumbel distribution). From the estimation of the parameters and fit of the distribution, the one-step ahead $100(1-p)\%$ VaR is (Tsay, 2002):

$$VaR_{BM} = \begin{cases} \hat{\alpha} + \dfrac{\hat{\beta}}{\hat{\tau}}\left\{\left[-\ln(1-p)\right]^{\hat{\tau}} - 1\right\} & if\ \hat{\tau} \neq 0 \\ \hat{\alpha} + \hat{\beta}\ln\left[-\ln(1-p)\right] & if\ \hat{\tau} = 0 \end{cases} \tag{9}$$

The peaks-over-thresholds approach is conducted in this manner (Smith, 1999): (1) transform the data by changing the sign of the returns, i.e., let the transformed data $r_t^- = -r_t$; (2) choose a threshold value η so that when $r_i^- \geq \eta$, the data point r_i^- will be included in a new data set for model fitting; otherwise, the data point is excluded, creating a new data set $\left\{r_1^{-*}, r_2^{-*}, \ldots, r_{n*}^{-*}\right\}$, where $n*$ is the number of included data points; (3) the new dataset will be fitted to the Generalized Pareto distribution (GPD) (Pickands, 1975) with its cumulative distribution shown below:

$$\Pr(X \leq u \mid X > \eta) = \begin{cases} 1 - \left[1 + \xi\left(\dfrac{x-\eta}{\sigma}\right)\right]^{-1/\xi} & if\ \ \xi \neq 0 \\ 1 - \exp\left\{-\dfrac{x-\eta}{\sigma}\right\} & if\ \ \xi = 0 \end{cases} \tag{10}$$

When the parameters of the GPD are estimated, the one-step ahead $100(1-p)\%$ long position VaR is equal to (Tsay, 2002; Coles 2001), with n as the total number of data points in the original data set:

$$VaR_{POT} = \begin{cases} -\eta + \dfrac{\hat{\sigma}}{\hat{\xi}}\left\{1 - \left[\dfrac{n*}{n(1-p)}\right]\right\} & if\ \hat{\xi} \neq 0 \\ -\eta + \hat{\sigma}\ln\left[\dfrac{n(1-p)}{n*}\right] & if\ \hat{\xi} = 0 \end{cases} \tag{11}$$

The problem with these procedures is their static properties, such that the time dynamics are not included in the model (Tsay, 2002). These methods have been adjusted to included time dependence such as a mixture with econometric methods (McNeil and Frey, 2000; Suaiso, 2009) and the use of explanatory variables (Tsay, 2002; Cayton et al., 2010)

Conditional Density Methods

These methods deal with fitting distributions with parameters that are time-dependent (Jondeau, et al., 2007). The econometric methods and time-varying extreme value theory models are special cases of this family of procedures. Distributions used in these sets of procedure commonly involve more than two parameters, which would include shape parameters such as those that affect skewness and kurtosis. In this family, higher parameters are modeled with a time-varying structure to adapt to the concept dynamics in higher moments. Distributions that have been used in literature are such as the skewed Student's t distribution (Hansen, 1994; Harvey and Siddique, 1999), Pearson Type IV distribution (Yan, 2005), Johnson S_U distribution (Yan, 2005), Edgeworth series densities (Rockinger and Jondeau, 2001), and the Gram-Charlier densities (Jondeau and Rockinger, 2001).

These researches generally underpin that time-varying higher moments and parameters may exist on some financial assets, thus to fit returns with time-varying structures would be required in such cases. Each distribution has their caveats and advantages, such as those that directly influence the moments of skewness and kurtosis but are computationally intensive for computation of quantiles, e.g., the Gram-Charlier densities and Edgeworth series densities (Jondeau and Rockinger, 2001; Rockinger and Jondeau, 2001), and those which are computationally and analytically derivable quantiles yet do not directly influence the coefficients of skewness and kurtosis, such as the Pearson IV and Johnson S_U distributions (Yan, 2005). Due to the computational ease of estimation for parameters using maximum likelihood estimation and analytically derivable quantiles after estimation (Yan, 2005), the Johnson S_U distribution is used by the paper for time-varying conditional density.

The Johnson S_U Distribution

The Johnson S_U distribution was one of the distribution derived by Johnson (1949) based on translating the normal distribution to certain functions. Letting $Z \sim N(0,1)$, the standard normal distribution, the random variable Y has the Johnson system of frequency curves from this method of transformation:

$$Z = \gamma + \delta g\left(\frac{Y-\xi}{\lambda}\right) \tag{12}$$

The parameters γ and ξ may be any real number, while δ and λ should be positive numbers. The form of the distribution depends on the function g : (1) if $g(u) = u$, it results to the normal distribution; (2) $g(u) = \log\left[u/(1-u)\right]$, then the distribution is bounded , called the Johnson S_B distribution; (3) when

$g(u)=\ln(u)$, it is the log-normal distribution, and (4) when $g(u)=\sinh^{-1}(u)$, the distribution is unbounded, called the Johnson S_U (JS_U) distribution.

From the transformation of the normal distribution, the cumulative distribution function of the JS_U distribution is shown below. If $Y \sim JS_U(\xi,\lambda,\gamma,\delta)$:

$$F_Y(y)=\Phi\left[\gamma+\delta\sinh^{-1}\left(\frac{y-\xi}{\lambda}\right)\right] \tag{13}$$

The function $\Phi(u)$ is the cumulative distribution function of the standard normal distribution. From the equation above, the quantile function F_Y^{-1} can be directly derived as:

$$F_Y^{-1}(p)=\xi+\lambda\sinh\left[\frac{\Phi^{-1}(p)-\gamma}{\delta}\right] \tag{14}$$

The quantile function simply depends on the quantiles of the standard normal distribution $\Phi^{-1}(p)$, which are tractable due to the many functions available in computers and tables of standard normal quantiles in literature.

The density of the JS_U distribution, which will be used for the estimation procedure, is equal to (Yan, 2005):

$$f_Y(y)=\frac{\delta}{\lambda\sqrt{1+\left(\frac{x-\xi}{\lambda}\right)^2}}\phi\left[\gamma+\delta\sinh^{-1}\left(\frac{x-\xi}{\lambda}\right)\right] \tag{15}$$

The function $\phi(u)$ is the probability density function of the standard normal disitribution. The parameters of the JS_U are $(\xi,\lambda,\gamma,\delta)'$ with each affecting the location, scale, skewness, and kurtosis of the distribution. The parameters are not the direct raw moments of the distribution. The first four moments, the mean, variance, third central moment, and fourth central moment, respectively, of the distribution are the following (Yan, 2005):

$$\mu=\xi+\lambda\omega^{1/2}\sinh\Omega \tag{16}$$

$$\sigma^2=\frac{\lambda^2}{2}(\omega-1)(\omega\cosh 2\Omega+1) \tag{17}$$

$$\mu_3=-\frac{1}{4}\omega^2\left(\omega^2-1\right)^2\left[\omega^2\left(\omega^2+2\right)\sinh 3\Omega+3\sinh\Omega\right] \tag{18}$$

$$\mu_4 = \frac{1}{8}\left(\omega^2 - 1\right)^2 \left[\omega^4\left(\omega^8 + 2\omega^6 + 3\omega^4 - 3\right)\cosh 4\Omega + 4\omega^4\left(\omega^2 + 2\right)\cosh 2\Omega + 3\left(2\omega^2 + 1\right)\right] \quad (19)$$

The quantities in the moment formulas are $\Omega = \gamma/\delta$ and $\omega = \exp\left(\delta^{-2}\right)$. The standard distribution for the JS_U exists when $\xi = 0$ and $\lambda = 1$, but the mean and the variance are not 0 and 1 respectively. To use the Johnson distribution as a standardized error distribution in econometric modeling (e.g., in ARMA-GARCH modeling), set the parameters in the following manner (Yan, 2005):

$$\xi_S = -\omega^{1/2}\sinh\Omega\left[\sqrt{\frac{1}{2}(\omega - 1)(\omega\cosh 2\Omega + 1)}\right]^{-1} \quad (20)$$

$$\lambda_S = \left[\sqrt{\frac{1}{2}(\omega - 1)(\omega\cosh 2\Omega + 1)}\right]^{-1} \quad (21)$$

Joint Estimation Procedure for JS_U Distribution

From the standardization of the distribution, mean-variance specifications can be introduced for econometric modeling with the JS_U distribution. For maximum likelihood estimation, the higher parameters can be modeled to have time-varying structure, ultimately introducing dynamic properties to the skewness and kurtosis. In modeling with the JS_U with joint estimation of parameters having time-varying structures, the following are sets of equations are defined:

Mean-Variance-Error Interaction: $y_t = \mu_t + \sqrt{h_t}\, z_t$ (22)

Mean Specification: $\mu_t = g_\mu(t)$ (23)

Variance Specification: $h_t = g_h(t)$ (24)

Error Specification: $E(z_t) = 0;\ \mathrm{var}(z_t) = 1;$ $z_t \sim JS_U\left(\xi_{S,t}, \lambda_{S,t}, \gamma_t, \delta_t\right)$ (25)

Third Parameter Specification: $\gamma_t = g_\gamma(t)$ (26)

Fourth Parameter Specification: $\delta_t = g_\delta(t)$ (27)

The functions $g_\mu, g_h, g_\gamma,$ and g_δ are time-dependent functions related to *t*, e.g., $g_\mu \equiv ARMA(p,q)$ process for the mean, $g_h \equiv GARCH(p_1, q_1)$ for the variance, $g_\gamma(t) = \beta_0 + \beta_1 x_{t-1}$ for the structure of the third parameter, and $g_\delta(t) = \delta_0$ a constant value for the fourth parameter. The location and scale parameters since they are function of time-varying third and fourth parameters due to standardization of the JS_U

distribution, i.e., $\xi_{S,t} = f_1(\gamma_t, \delta_t)$, $\lambda_{S,t} = f_2(\gamma_t, \delta_t)$. In this structure, it is implied that the skewness and kurtosis would have time-varying properties due to the structure of the third and fourth parameters.

The log-likelihood sum to be maximized for estimation is written below:

$$\ell(g_\mu, g_h, g_\gamma, g_\delta \mid y_1, \ldots, y_n) = \sum_{t=1}^{n} \left\{ \log g_\delta(t) - \log \lambda_{S,t} - \frac{1}{2}\log\left[1 + \left(\frac{\frac{y_t - g_\mu(t)}{\sqrt{g_h(t)}} - \xi_{S,t}}{\lambda_{S,t}}\right)^2\right] + \log\left[\phi\left(g_\gamma(t) + g_\delta(t)\sinh^{-1}\left(\frac{\frac{y_t - g_\mu(t)}{\sqrt{g_h(t)}} - \xi_{S,t}}{\lambda_{S,t}}\right)\right)\right] - \frac{1}{2}\log\left[g_h(t)\right] \right\} \tag{28}$$

The use of the functions $g_\mu, g_h, g_\gamma,$ and g_δ as arguments in the log-likelihood function implies the estimation of the parameters inside these functions. The location and scale parameters should be substituted to the appropriate functional form based on g_γ and g_δ. If lagged values of the time series data are being used, the addends of the summation are reduced to adapt to the use of lags.

Two-Step Procedure for JS_U Distribution

Another procedure that would introduce time-varying mean and variance specifications in the JS_U distribution is a two-step procedure, where first the return series r_t are fitted with the appropriate model for mean μ_t and variance h_t and estimation is carried out using quasi-maximum likelihood estimation [QMLE] (Bollerslev and Wooldridge, 1992). From the estimated model equations $\hat{\mu}_t$ and $\hat{h}_t$, the standardized residuals e_t of the model are computed:

$$e_t = \frac{r_t - \hat{\mu}_t}{\sqrt{\hat{h}_t}} \tag{29}$$

From these residuals, they are fitted with the JS_U distribution with structures in the third and fourth parameters. The log-likelihood to be minimized would be of the form below:

$$\ell\left(g_\gamma, g_\delta \mid e_1,\dots,e_n\right)$$
$$=\sum_{t=1}^{n}\left\{\log g_\delta(t)-\log\lambda_{S,t}-\frac{1}{2}\log\left[1+\left(\frac{e_t-\xi_{S,t}}{\lambda_{S,t}}\right)^2\right]+\log\left[\phi\left(g_\gamma(t)+g_\delta(t)\sinh^{-1}\left(\frac{e_t-\xi_{S,t}}{\lambda_{S,t}}\right)\right)\right]\right\} \quad (30)$$

After the proper estimation of the parameters of the model, either through the joint estimation or the two-step procedure, the one-step ahead $100(1-p)\%$ long position VaR is equal to:

$$VaR_{JS_U}=\hat{\mu}_{t+!}+\sqrt{\hat{h}_{t+1}}\left(\hat{\xi}_{S,t+1}+\hat{\lambda}_{S,t+1}\sinh\left[\frac{\Phi^{-1}(p)-\hat{\gamma}_{t+1}}{\hat{\delta}_{t+1}}\right]\right) \quad (31)$$

The paper would compare the performance of the VaR of the time-varying JS_U with the econometric VaR in Philippine financial time series datasets in terms of the number of exceptions and the magnitude of values given by the VaR of the different methods.

The Evaluation of Value-at-Risk Methods

Number of Exceptions: Basel Requirements

A perspective of the evaluation of VaR methods is through the number of exceptions (Basel, 1996). A VaR exception occurs when the actual loss exceeded the value of the anticipated VaR. Depending on the VaR probability level, a specific amount of VaR exceptions are allowed per year. For example, for the 99% VaR, it is expected and permitted that the number of exceptions be equal to 1% of the total number of periods in a year. If a year had 250 time periods, about two or three exceptions are allowed per year. . Based on the number of exceptions of the VaR model of a financial institution, it is classified into three zones: (1) the green zone, where when institutions are inside this zone, it implies that their VaR model are able to fulfill the 99% specification, (2) the yellow zone, where institutions in this zone imply that their VaR model may be able to meet the 99% requirement but with low confidence, (3) and the red zone, where it indicates that the VaR model are not able to meet the 99% specification. Depending on the number of exceptions received in a year, a penalty multiplier in introduced in the calculation of appropriate risk capital based on the VaR. The table below displays the multipliers for each zone and number of exceptions.

Table 1. Classification Zones Based on Number of Exceptions and Appropriate Scaling Factors for Risk Capital

Zone	Number of Exceptions	Scaling Factors for the Market Risk Capital
Green Zone	0	3.00
	1	3.00
	2	3.00
	3	3.00
	4	3.00
Yellow Zone	5	3.40
	6	3.50
	7	3.65
	8	3.75
	9	3.85
Red Zone	10 or more	4.00

Source: Basel (1996)

Number of Exceptions: Likelihood Ratio Tests

Another method for assessing VaR models for their performance is based on their adherence to the desired risk probability. Christoffersen (1998) introduced a system of successive chi-square testing procedure for assessment of VaR methods based on their number of exceptions within the forecast evaluation period. Three tests are conducted on the frequency of exceptions, done successively: (1) unconditional coverage test, which tests whether the risk probability is fulfilled by the VaR model, (2) independence test, which test whether the probability of two successive exceptions is equal to the proportion of exceptions succeeded by non-exceptions, and (3) conditional convergence test, which tests whether the probabilities of successive and non-successive exceptions are equal to the coverage probability. When an initial tests leads to the acceptance of the null hypothesis, it is a favorable result for the VaR model and a succeeding test is conducted. If an initial test leads to rejection, then the succeeding test is not done and therefore the VaR procedure does not have a favorable property based on the test. The sequence of tests is listed in the table below.

Table 2. Likelihood Testing Procedures for VaR Assessment

Name of Test	Hypotheses	Test Statistic	Implication of Acceptance	Implication of Rejection
Unconditional Coverage	$H_0: \pi = p$ (exception proportion equals risk probability) $H_A: \pi > p$ (VaR model does not attain proper risk probability)	$LR_{uc} = 2\log\left[\left(\frac{1-\hat{\pi}}{1-p}\right)^{T-T_1}\left(\frac{\hat{\pi}}{p}\right)^{T_1}\right] \sim \chi^2_{(1)}$ T =number of data points in the forecast period T_1 =number of VaR exceptions in the forecast period $\hat{\pi} = T_1/T$ = estimated proportion of VaR exceptions	VaR method is appropriate in coverage of risk probability. Do next test.	VaR model is not appropriate in the given level of risk probability. Adjust VaR model.
Independence	$H_0: \pi_0 = \pi_1$ (Proportion of un-clustered VaR exceptions is equal to the proportion of clustered exceptions) $H_A: \pi_0 < \pi_1$ (proportion of clustered VaR exceptions is higher than un-clustered exceptions)	$LR_{ind} = 2\log\left[\frac{(1-\hat{\pi}_0)^{T_{00}}\hat{\pi}_0^{T_{01}}(1-\hat{\pi}_1)^{T_{10}}\hat{\pi}_1^{T_{11}}}{(1-\hat{\pi}_{pool})^{T_{00}+T_{10}}\hat{\pi}_{pool}^{T_{01}+T_{11}}}\right] \sim \chi^2_{(1)}$ T_{00} =number of two consecutive days with no exception T_{01} =number of periods with no exceptions followed by an exception T_{10} =number of periods with exceptions followed by no exceptions T_{11} = number of two consecutive days with exceptions $\hat{\pi}_0 = \frac{T_{01}}{T_{01}+T_{00}}; \hat{\pi}_1 = \frac{T_{11}}{T_{11}+T_{10}};$ $\hat{\pi}_{pool} = \frac{T_{01}+T_{11}}{T_{01}+T_{00}+T_{11}+T_{10}}$	Occurrences of exceptions are independent of each other. Exceptions clustering is rare or none. Do next test.	VaR model produces exception clustering. VaR model is not appropriate in mitigating risks in times of volatility clustering. Adjust VaR model.
Conditional Coverage	$H_0: \pi_0 = \pi_1 = p$ (the proportions exceptions are equal to the risk probability) $H_A: \pi_0 = \pi_1 > p$ (The proportions of exceptions are larger than the risk probability)	$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2_{(2)}$	Exception proportions are within prescribed risk probability levels. VaR model is appropriate.	Proportions are higher than desired risk probability. Adjust VaR model.

Source: Christoffersen (1998)

Magnitudes of Values

As risk capital is based on the value of VaR, the magnitude of the VaR methods are analyzed and compared. Three features should be possessed by an appropriate method: (1) conservatism, which indicates that it generally give a relatively higher VaR compared to other methods, (2) accuracy, in which the method is able to identify the level of loss with minimum error in the magnitude, and (3) efficiency, in which the method is able to compute the adequate level of risk capital such that risk is fully accounted yet not too high that opportunity loss for other financial activity are constrained (Engel and Gizycki, 1999). A statistical measure is selected for each quality as a measure of their compliance to each desired feature. The table below shows the statistics and their intended analysis. Long position VaR values, where negative means loss, are assumed as used in the following formulas.

Table 3. Statistical Measures for VaR Comparisons

Desired Quality	Statistical Measure	Formula	Analysis of Statistic
Conservatism	Mean Relative Bias [MRB] (Engel and Gizycki, 1999)	$MRB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{VaR_{it} - \overline{VaR}_t}{\overline{VaR}_t}$; $\overline{VaR}_t = \sum_{i=1}^{N} VaR_{it}$ VaR_{it} = the VaR on time t based on method i T = return series data length of evaluation period N = number of VaR methods being compared.	The higher the MRB of a VaR method, the more conservative it is relative to other models.
Accuracy	Average Quadratic Loss Function [AQLF] (Engel and Gizycki, 1999)	$AQLF = \frac{1}{T}\sum_{t=1}^{T} L(VaR_t, r_t)$ $L(VaR_t, r_t) = \begin{cases} 1 + (VaR_t - r_t)^2 & if\ VaR_t > r_t \\ 0 & otherwise \end{cases}$	The lower or the closer to zero the AQLF is, the more accurate the VaR method in forecasting and accounting for possible loss.
Efficiency	Average Market Risk Capital [AMRC] (Basel, 1996)	$AMRC = \frac{1}{T}\sum_{t=1}^{T} MRC_t$ $MRC_t = \max\left[-\frac{k}{60}\sum_{k=t-1}^{t-60} VaR_t, -VaR_{t-1}\right]$ k = the penalty multiplier based on the number VaR exceptions (see Table 1)	If the lower the AMRC, the lower the risk capital to be allocated on the average. The AMRC is jointly analyzed with results of the other statistics

From the established measures and procedures of assessment and evaluation, the paper wishes to evaluate the time-varying methods of estimating VaR with the JS_U distribution and compare it with other VaR methodologies in literature.

Research Methodology

Data Used

In the evaluation of VaR methodologies, the following financial time series data are used: (1) the Philippine Peso-US Dollar Exchange Rate (RUSD) from 4 January 1999 to 10 November 2011, (2) the Philippine Peso-Euro Exchange Rate (REUR) from 4 January 1999 to 18 November 2011, (3) the Philippine Peso-Singaporean Dollar (RSGD) from 3 January 2006 to 21 Novermber 2011, and (4) the Philippine Stock Exchange Index (PSEI) from 3 January 2000 to 18 November 2011.

In division of the datasets, the most recent 250 data points per series would be the forecast evaluation period and the rest of the peiords would be under the model estimation period.

Models Used

The data series would be evaluated with long position 99% one-step-ahead VaR values. The VaR based on the time-varying JS_U methods will be evaluated with the following econometric models: (1) GARCH(1,1)-normal distribution-QMLE, (2) GARCH(1,1)-t distribution,(3) TARCH(1,1,1)-normal distribution-QMLE, and (4) TARCH (1,1,1)-t-distribution. The GARCH(1,1) is based on the model by Bollerslev (1986) with the form for the variance given below:

$$h_t = \alpha_0 + \alpha_1 h_{t-1} z_{t-1}^2 + \beta h_{t-1} \tag{32}$$

The argument z_{t-1} is the standardized error of one period before, and the parameters $(\alpha_0, \alpha_1, \beta)'$ are estimated. The model assumes a symmetric effect of changes in the immediate past to the variance of current changes, i.e., it assumes no leverage effect. To account for asymmetric effect of past changes to current volatility, the TARCH(1,1,1) (Zakoian, 1994) adds a term on the volatility and models the conditional standard deviation. Thus in modeling the variance, the equation is modified as shown below:

$$h_t = \left(\alpha_0 + \alpha_1 \left|\sqrt{h_{t-1}} z_{t-1}\right| + \psi I_{(0,\infty)}\left(\sqrt{h_{t-1}} z_{t-1}\right)\left|\sqrt{h_{t-1}} z_{t-1}\right| + \beta \sqrt{h_{t-1}}\right)^2 \tag{33}$$

$$I_{(0,\infty)}(u) = \begin{cases} 0 & if \quad u \le 0 \\ 1 & if \quad u > 0 \end{cases} \tag{34}$$

For the joint estimation of parameters on the JS_U distribution, the following specification of the variance, third, and fourth parameters are shown below:

$$h_t = \exp\left\{\theta_0 + \theta_1 \left|\sqrt{h_{t-1}} z_{t-1}\right| + \theta_2 \sqrt{h_{t-1}} z_{t-1}\right\} \tag{35}$$

$$\gamma_t = \varphi_0 + \varphi_1 \left|\sqrt{h_{t-1}} z_{t-1}\right| + \varphi_2 \sqrt{h_{t-1}} z_{t-1} \tag{36}$$

$$\delta_t = \zeta_0 + \zeta_1 \left|\sqrt{h_{t-1}} z_{t-1}\right| + \zeta_2 \sqrt{h_{t-1}} z_{t-1} \tag{37}$$

For the two-step procedure, the variance is estimated with the GARCH(1,1) model and the residuals were modeled with the JS_U using equations (36) and (37) with $\sqrt{h_{t-1}} = 1$ since the residuals have unit variance. The mean specification for all models was set to zero, i.e., $\mu_t = 0$ and $r_t = \sqrt{h_t} z_t$.

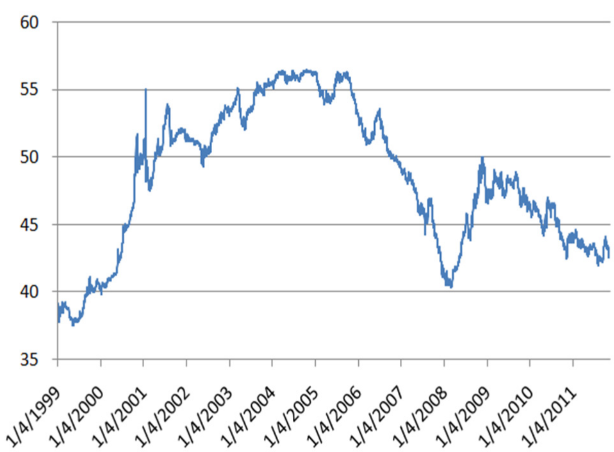
Descriptive Analysis of the Data

Graphical Analysis on Levels and Returns

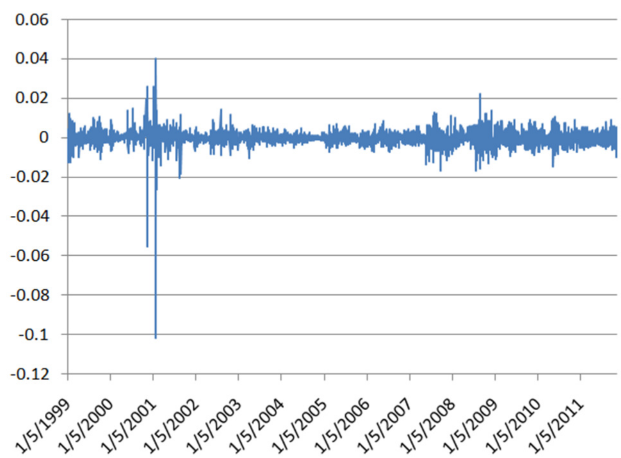
Figure 1 displays the graphs of levels and returns of the RUSD. With the data, a period of increase or depreciation of the Philippine peso occurred from before 1999 to 2001due to the effects of the Asian financial crisis and national political crisis of the Philippines. Stability in the level of the exchange rate was achieved from 2002 to 2004 and the appreciation of the peso started in 2005. In the offset of the 2008, agricultural price inflation, called 'agflation,' has occurred and depreciation began. The increase in the rate was augmented with the offset of the 2008 financial crisis but by 2009, the appreciation of the peso returned, continuing until the last point of the data. In the return series, the occurrence of very large changes were in end-of-2000 to start-of-2001, due to political uncertainty during the impeachment and ouster of the president of the country. After 2001 stability in the changes were observed, yet after 2008 the changes were gaining wider ranges compared to period in the years 2002-2007.

Figure 1. Time Plots of US Dollar Exchange Rate

US Dollar Exchange Rate Levels

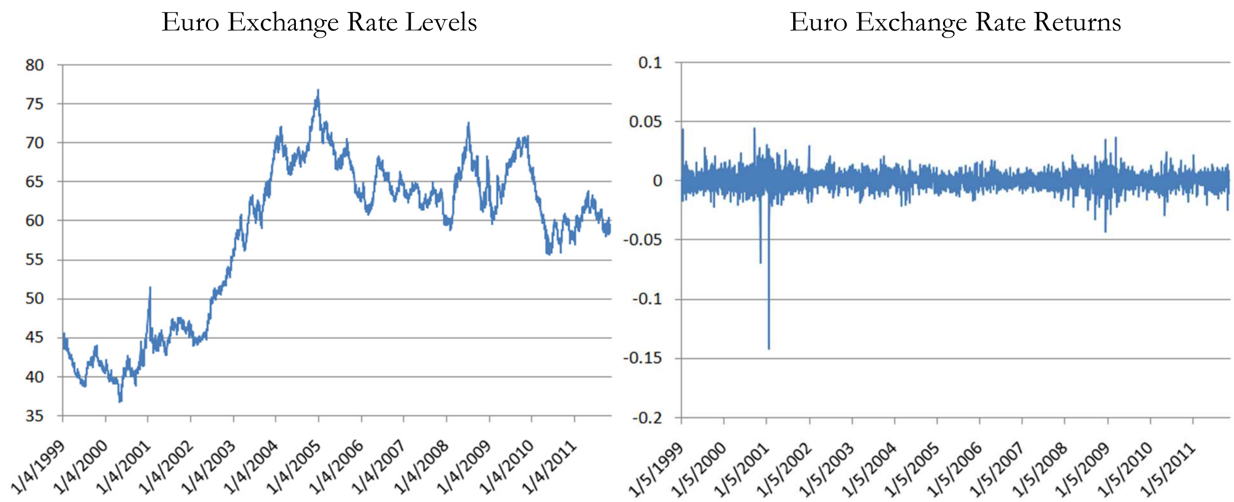


US Dollar Exchange Rate Returns



The graphs of the REUR are shown in figure 2. With the Euro, a surge of depreciation of the peso occurred from 2001 to the highest point of valuation of the euro in terms of the peso at 2005. From then the changes were relatively stable except in 2008-2010 where the euro had a wave of upturns and downturns of value in terms of the peso. In terms of volatility large changes have occurred at end-of-2000 to start-of-2001 with political unrest. A period of stability in changes was observed in 2002-2007 yet beginning 2008 to 2009 wider volatility occurred due to the global financial crisis.

Figure 2. Time Plots of Euro Exchange Rate



The RSGD levels and returns are graphed and displayed in Figure 3. The data is a short series yet it maintains a level with the ranges of Php 28 to Php 36 per dollar. Appreciation occurred pre-2008 followed by sharp depreciation in the first half of 2008. From then on a steady depreciation has occurred with the exchange rate. In terms of volatility, wider and more volatile changes occurred in 2008 and 2009. A stable 2010 was observed except for two abrupt changes in the early half of the year.

Figure 3. Time Plots of Singaporean Dollar Exchange Rate

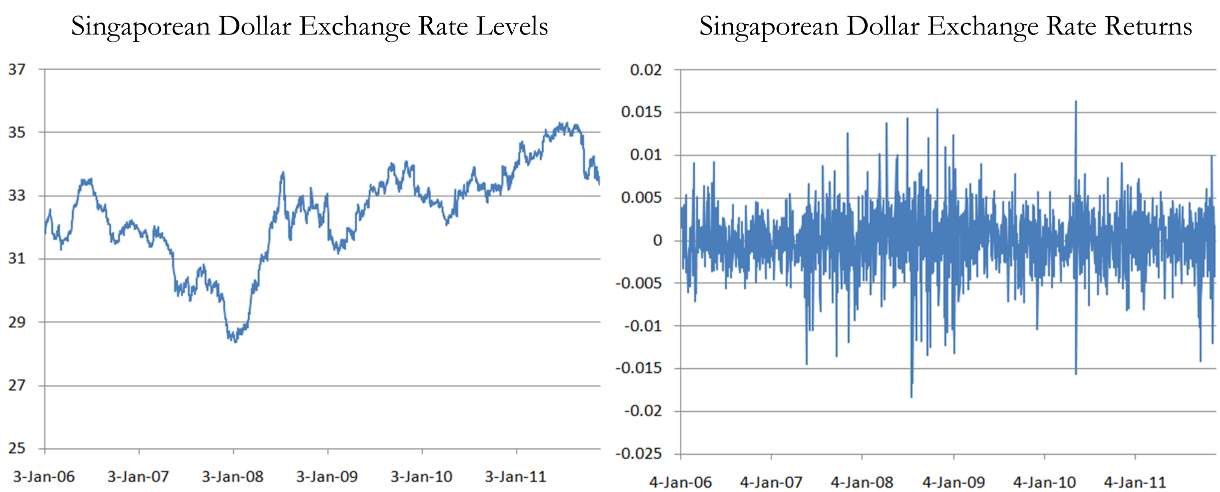
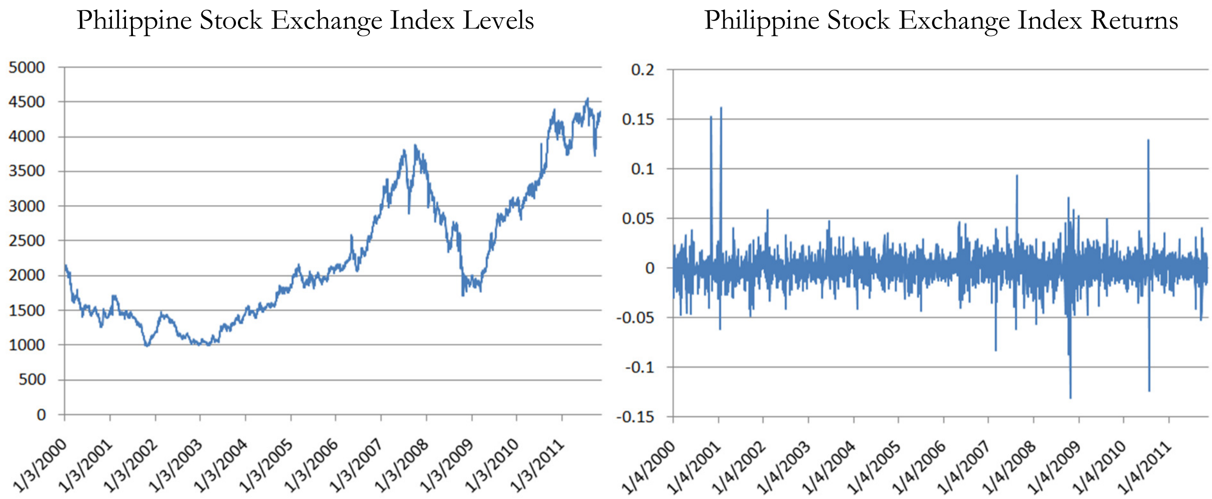


Figure 4 features the time plots of the PSEI. The index had experienced a surge in value from its bottom-of-the-bowl state in 2002 and the increase lasted from 2003 to 2007. In late 2007 the agflation crisis in the Philippines has started with rising food prices, followed by the global economic crisis of 2008-2009 which devalued the index. At the second quarter of 2009 the index had a continuing period of growth, escaping from the crisis. In the spectrum of volatility, two sharp increases were observed in end-of-2000 and start-of-

2001, and from then a stable variance has been observed. Around 2008-2009 was the widening of the range occurred, with sharp downward changes in the end of 2008. From then two abrupt changes in 2010 were observed and stability was observed.

Figure 4. Time Plots of Philippine Stock Exchange Index



Summary Statistics of Returns

Table 4 below shows the different statistics of financial time series returns. The four series have high kurtosis values for returns, evident of the non-normality of the time series returns. The three currencies had negative skewness, except for the PSEI which is skewed to the right. Reason behind the positive skewness of the PSEI is its long-term bearish performance in the observation period. The PSEI is also the most volatile of the four series with a wider range between the minimum and the maximum and high standard deviation. The most stable would be the RSGD yet its stability is within its short observation period.

Table 4. Summary Statistics of the Financial Returns

	RUSD	REUR	RSGD	PSEI
Obs	3192	3300	1477	2990
Mean	2.9499×10^{-5}	7.8269×10^{-5}	3.2838×10^{-5}	2.3321×10^{-4}
Std. Dev.	4.2902×10^{-3}	7.6022×10^{-3}	3.6339×10^{-3}	1.4409×10^{-2}
Skewness	-4.4820	-2.0704	-0.3035	0.4621
Kurtosis (unadjusted)	115.2209	43.3032	5.5069	20.6623
Minimum	-.10150	-.14193	-.01832	-.13089
Maximum	.04019	.04414	.01637	.16178

Results and Discussion

On the Number of Exceptions

Table 5 shows the results of statistics based on the number of exceptions and likelihood ratio tests for the different VaR methods in different return series. Generally, the econometric methods were better in terms of the number exceptions than the JS_U procedures. The econometric methods were generally placed in the green zone with 4 or less exception, except in the case when GARCH QMLE methods were used on RSGD. The JS_U methods were often on the yellow zone, with some exceptions when joint JS_U was used on REUR and two-step JS_U was used on the RUSD. This would mean that generally the JS_U methods can be able to make the prescribed 1% risk probability, but with low confidence.

Table 5. Table of Evaluation Measures Based on Exceptions

Model	Joint JS_U				Two-Step JS_U			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
Number of Exceptions	7	3	9	8	3	6	8	8
Likelihood Ratio Tests p-values								
Uncondtional Coverage	0.0190	0.7580	0.0014	0.0054	0.7580	0.0594	0.0054	0.0054
Independence	-	-	-	0.2389	-	-	-	0.2389
Conditional Coverage	-	-	-	0.0105	-	-	-	0.0105
Model	GARCH QMLE				GARCH t			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
Number of Exceptions	0	4	5	2	0	2	4	2
Likelihood Ratio Tests p-values								
Uncondtional Coverage	-	0.3805	0.1619	0.7419	-	0.7419	0.3805	0.7419
Independence	-	-	-	-	-	-	-	-
Conditional Coverage	-	-	-	-	-	-	-	-
Model	TARCH QMLE				TARCH t			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
Number of Exceptions	0	4	4	2	0	2	4	2
Likelihood Ratio Tests p-values								
Uncondtional Coverage	-	0.3805	0.3805	0.7419	-	0.7419	0.3805	0.7419
Independence	-	-	-	-	-	-	-	-
Conditional Coverage	-	-	-	-	-	-	-	-

With the likelihood ratio tests, the tests for independence and conditional coverage were not conducted in some cases since for the econometric methods in all series and in the JS_U methods in RUSD, REUR, and RSGD, no VaR exception clustering was observed. For the JS_U methods on the PSEI, though unconditional coverage would likely be rejected, independence and conditional coverage tests are continued and shown.

The JS_U methods on RSGD and PSEI were generally low performing with the formal test that these models on these series may not be able to cover the appropriate risk probability compared to econometric methods.

But when confidence is relaxed in the case of the PSEI, the JS_U methods were unlikely to have VaR exception clustering. The JS_U were less performing than the econometric methods in being able to predict points in time when high-loss scenarios would occur.

On the Magnitudes of Values

Table 6 listed the results of magnitude-based statistics for comparison between econometric VaR methods and JS_U VaR procedures.

With MRB, the JS_U methods were generally less conservative than the econometric methods in accounting for risk, having negative MRB values compared to econometric VaR methods.

The AQLF values were generally highest in JS_U methods compared to econometric methods except in the case of using joint JS_U in REUR. JS_U methods were less accurate in predicting the magnitude of risk compared to econometric methods.

Table 6. Table of Evaluation Measures Based on Magnitudes

Model	Joint JSU				Two-Step JSU			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
MRB	-0.1796	-0.0743	-0.0883	-0.2187	-0.1375	-0.1220	-0.1298	-0.2307
AMRC	0.0233	0.0509	0.0274	0.0845	0.0198	0.0477	0.0251	0.0831
AQLF	0.0492	0.0120	0.0572	0.0320	0.0332	0.0240	0.0532	0.0320
Model	GARCH QMLE				GARCH t			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
MRB	0.0124	0.0118	0.0072	0.0529	0.0955	0.0830	0.0925	0.1395
AMRC	0.0288	0.0440	0.0265	0.0907	0.0251	0.0472	0.0279	0.0986
AQLF	0.0000	0.0160	0.0412	0.0080	0.0000	0.0080	0.0372	0.0080
Model	TARCH QMLE				TARCH t			
Time Series	RUSD	REUR	RSGD	PSEI	RUSD	REUR	RSGD	PSEI
MRB	0.0567	0.0157	0.0172	0.0862	0.1525	0.0858	0.1012	0.1707
AMRC	0.0299	0.0440	0.0260	0.0933	0.0262	0.0472	0.0281	0.1006
AQLF	0.0000	0.0160	0.0372	0.0080	0.0000	0.0080	0.0372	0.0080

In assigning risk capital, JSU methods assigned lower capital in RUSD, where JS_U fared well in predicting when high loss periods would occur. With REUR, where JS_U methods were better in predicting periods of high risk, they placed higher risk capital, in due to their relatively poor performance in predicting the magnitude of risk. Lower risk capital were assigned on the average on the PSEI by JS_U methods compared to econometric methods yet JS_U methods were poor in predicting when high risk periods would occur. The risk capital assigned on RSGD by the JS_U and econometric methods are in range of each other, since the RSGD is a relatively stable series compared to other returns.

Conclusions and Recommendations

The paper introduced and derived the VaR methodology in modeling returns using a time-varying conditional JS_U density. The performance of the model has been assessed and was compared with the econometric methods of estimating the VaR. The performance of the JS_U VaR in predicting periods of high risk was generally in the yellow zone of methods, since it had higher number of exceptions. It was a less conservative method of computing for VaR, with risks of underestimating the possible losses incurred.

The research paper contributes to the field of investigation into other nonnormal distributions in estimating the VaR of financial assets. It is possible to study the performance of the JS_U VaR in other returns data, especially in the assets of interest rates and commodities, especially when nonnormality is observed. The trials of other possible specification of models such as in the parameters or a new distribution different to the conducted study have a potential to produce better results, and such can be pursued in the future.

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Appendices

Model Results for RUSD

Joint JS_U distribution

Log likelihood = 15109.291

Number of obs = 2941
Wald chi2(2) = 197.67
Prob > chi2 = 0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnsigma_2						
r_usd_l1	59.32968	20.02176	2.96	0.003	20.08774	98.57162
ar_usd_l1	382.272	27.6263	13.84	0.000	328.1255	436.4186
_cons	-13.40537	.0661977	-202.50	0.000	-13.53511	-13.27562
gamma						
r_usd_l1	-26.61691	4.922242	-5.41	0.000	-36.26433	-16.9695
ar_usd_l1	-4.381315	6.168206	-0.71	0.478	-16.47078	7.708146
_cons	-.003158	.0109885	-0.29	0.774	-.024695	.018379
delta						
r_usd_l1	-36.91273	63.62302	-0.58	0.562	-161.6116	87.7861
ar_usd_l1	268.1492	82.89445	3.23	0.001	105.6791	430.6194
_cons	1.129331	.0713114	15.84	0.000	.9895633	1.269099

GARCH(1,1)-Normal-QMLE (also for Two-Step JS_U)

ARCH family regression

Sample: 1 - 2941
Distribution: Gaussian
Log likelihood = 15244.67

Number of obs = 2941
Wald chi2(.) = .
Prob > chi2 = .

r_usd	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.2238584	.0186823	11.98	0.000	.1872417	.260475
garch L1.	.7925047	.0149086	53.16	0.000	.7632843	.821725
_cons	3.64e-08	8.49e-09	4.28	0.000	1.97e-08	5.30e-08

Result of Model Parameters in Two-Step JS_U

Log likelihood = -4064.0317

Number of obs = 2940
Wald chi2(2) = 73.85
Prob > chi2 = 0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
gamma						
sres_j2_l1	-.0783668	.0091206	-8.59	0.000	-.096243	-.0604907
asres_j2_l1	-.0001061	.0139974	-0.01	0.994	-.0275405	.0273283
_cons	-.0098121	.0138945	-0.71	0.480	-.0370449	.0174206
delta						
sres_j2_l1	.2170272	.1516796	1.43	0.152	-.0802593	.5143137
asres_j2_l1	.1476059	.1845629	0.80	0.424	-.2141308	.5093426
_cons	1.82497	.1532824	11.91	0.000	1.52454	2.125398

GARCH (1,1)-t

ARCH family regression

Sample: 1 - 2941
Distribution: t
Log likelihood = 15314.2

Number of obs = 2941
Wald chi2(.) = .
Prob > chi2 = .

r_usd	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.2120506	.0232378	9.13	0.000	.1665053	.2575958
garch L1.	.7991809	.0188301	42.44	0.000	.7622747	.8360871
_cons	3.70e-08	1.03e-08	3.60	0.000	1.68e-08	5.71e-08
/lndfm2	1.617007	.1625315	9.95	0.000	1.298451	1.935563
df	7.037989	.8188318			5.663618	8.927942

TARCH(1,1,1)-QMLE

ARCH family regression

Sample: 1 - 2941
Distribution: Gaussian
Log likelihood = 15218.52

Number of obs = 2941
Wald chi2(.) = .
Prob > chi2 = .

r_usd	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.188798	.0083015	22.74	0.000	.1725274	.2050687
atarch L1.	.0390595	.0082874	4.71	0.000	.0228165	.0553024
sdgarch L1.	.8150362	.0066287	122.96	0.000	.8020442	.8280281
_cons	.000045	4.74e-06	9.49	0.000	.0000357	.0000543

TARCH(1,1,1)-t

ARCH family regression

Sample: 1 - 2941
Distribution: t
Log likelihood = 15311.65

Number of obs = 2941
Wald chi2(.) = .
Prob > chi2 = .

r_usd	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.1690324	.0162739	10.39	0.000	.1371362	.2009286
atarch L1.	.0257312	.015545	1.66	0.098	-.0047365	.0561989
sdgarch L1.	.839248	.0130014	64.55	0.000	.8137657	.8647304
_cons	.0000362	7.65e-06	4.74	0.000	.0000212	.0000512
/lndfm2	1.478495	.1430902	10.33	0.000	1.198043	1.758946
df	6.386338	.6276421			5.313626	7.806317

Model Results for REUR

Joint JS_U distribution

```
Log likelihood =  13390.366          Number of obs   =      3050
                                     Wald chi2(2)    =     51.27
                                     Prob > chi2     =    0.0000
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnsigma_2						
r_eur_l1	35.31211	11.37273	3.10	0.002	13.02196	57.60225
ar_eur_l1	107.2021	17.22417	6.22	0.000	73.44333	140.9608
_cons	-11.80824	.0528595	-223.39	0.000	-11.91184	-11.70464
gamma						
r_eur_l1	7.17846	2.709791	2.65	0.008	1.867367	12.48955
ar_eur_l1	.485567	3.916592	0.12	0.901	-7.190813	8.161947
_cons	-.0076292	.0127545	-0.60	0.550	-.0326275	.0173692
delta						
r_eur_l1	-15.06646	7.487115	-2.01	0.044	-29.74094	-.3919873
ar_eur_l1	-32.19464	8.674909	-3.71	0.000	-49.19715	-15.19214
_cons	1.795062	.11099	16.17	0.000	1.577526	2.012599

GARCH(1,1)-Normal-QMLE (also for Two-Step JS_U)

```
ARCH family regression

Sample: 1 - 3050                     Number of obs   =      3050
Distribution: Gaussian               Wald chi2(.)    =         .
Log likelihood =  13375.89           Prob > chi2     =         .
```

r_eur	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.0646709	.0200405	3.23	0.001	.0253923	.1039494
garch L1.	.9311778	.0223556	41.65	0.000	.8873617	.9749939
_cons	8.37e-08	5.25e-08	1.59	0.111	-1.92e-08	1.87e-07

Result of Model Parameters in Two-Step JS_U

```
Log likelihood = -4239.0844          Number of obs   =      3049
                                     Wald chi2(2)    =      3.79
                                     Prob > chi2     =    0.1506
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
gamma						
sres_j2_l1	.0173361	.0089698	1.93	0.053	-.0002444	.0349167
asres_j2_l1	.0025237	.0138495	0.18	0.855	-.0246207	.0296681
_cons	-.008187	.0137375	-0.60	0.551	-.0351121	.0187381
delta						
sres_j2_l1	-.0201295	.0550472	-0.37	0.715	-.1280201	.087761
asres_j2_l1	-.1178727	.0638471	-1.85	0.065	-.2430106	.0072653
_cons	2.075173	.1523107	13.62	0.000	1.77665	2.373697

GARCH (1,1)-t

```
ARCH family regression

Sample: 1 - 3050                     Number of obs   =      3050
Distribution: t                      Wald chi2(.)    =         .
Log likelihood =  13477.63           Prob > chi2     =         .
```

r_eur	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.0379317	.0082856	4.58	0.000	.0216921	.0541712
garch L1.	.9475774	.011043	85.81	0.000	.9259335	.9692213
_cons	1.29e-07	4.19e-08	3.09	0.002	4.74e-08	2.11e-07
/lndfm2	1.685577	.1600585	10.53	0.000	1.371868	1.999286
df	7.395561	.8636056			5.942708	9.383779

TARCH(1,1,1)-QMLE

```
ARCH family regression

Sample: 1 - 3050                     Number of obs   =      3050
Distribution: Gaussian               Wald chi2(.)    =         .
Log likelihood =     13396           Prob > chi2     =         .
```

r_eur	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.053323	.0068042	7.84	0.000	.039987	.066659
atarch L1.	.0460043	.0064571	7.12	0.000	.0333486	.0586601
sdgarch L1.	.927647	.0069683	133.12	0.000	.9139894	.9413046
_cons	.0000448	.00001	4.46	0.000	.0000251	.0000644

TARCH(1,1,1)-t

```
ARCH family regression

Sample: 1 - 3050                     Number of obs   =      3050
Distribution: t                      Wald chi2(.)    =         .
Log likelihood =   13484.8           Prob > chi2     =         .
```

r_eur	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.0402055	.0091205	4.41	0.000	.0223295	.0580814
atarch L1.	.0173125	.0092339	1.87	0.061	-.0007857	.0354107
sdgarch L1.	.9501725	.0089512	106.15	0.000	.9326285	.9677166
_cons	.0000369	.0000123	2.99	0.003	.0000127	.0000611
/lndfm2	1.700808	.134177	12.68	0.000	1.437826	1.96379
df	7.478372	.7350715			6.211529	9.126285

Model Results for RSGD

Joint JS_U distribution

Log likelihood = 6223.477

Number of obs = 1226
Wald chi2(2) = 25.83
Prob > chi2 = 0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnsigma_2						
r_sgd_l1	4.770532	37.17653	0.13	0.898	-68.09412	77.63519
ar_sgd_l1	280.2138	55.16515	5.08	0.000	172.0921	388.3355
_cons	-13.26659	.0805503	-164.70	0.000	-13.42447	-13.10872
gamma						
r_sgd_l1	.3008584	8.660477	0.03	0.972	-16.67336	17.27508
ar_sgd_l1	37.63579	13.01215	2.89	0.004	12.13246	63.13913
_cons	-.0592509	.0209741	-2.82	0.005	-.1003593	-.0181425
delta						
r_sgd_l1	27.36721	41.33077	0.66	0.508	-53.63961	108.374
ar_sgd_l1	-149.5892	56.71475	-2.64	0.008	-260.7481	-38.43038
_cons	1.996604	.2178565	9.16	0.000	1.569613	2.423595

GARCH(1,1)-Normal-QMLE (also for Two-Step JS_U)

ARCH family regression

Sample: 1 - 1226
Distribution: Gaussian
Log likelihood = 6221.988

Number of obs = 1226
Wald chi2(.) = .
Prob > chi2 = .

r_sgd	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch						
L1.	.0828679	.0209009	3.96	0.000	.0419029	.1238328
garch						
L1.	.8964025	.0267768	33.48	0.000	.843921	.9488839
_cons	6.12e-08	2.76e-08	2.21	0.027	7.03e-09	1.15e-07

Result of Model Parameters in Two-Step JS_U

Log likelihood = -1708.7662

Number of obs = 1225
Wald chi2(2) = 4.59
Prob > chi2 = 0.1009

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
gamma						
sres_j2_l1	-.0018062	.0142355	-0.13	0.899	-.0297073	.026095
asres_j2_l1	.0470796	.0220011	2.14	0.032	.0039583	.090201
_cons	-.0440459	.0218843	-2.01	0.044	-.0869383	-.0011534
delta						
sres_j2_l1	.1254927	.1155307	1.09	0.277	-.1009434	.3519288
asres_j2_l1	-.210815	.1763266	-1.20	0.232	-.5564087	.1347787
_cons	2.180827	.298213	7.31	0.000	1.59634	2.765314

GARCH (1,1)-t

ARCH family regression

Sample: 1 - 1226
Distribution: t
Log likelihood = 6248.583

Number of obs = 1226
Wald chi2(.) = .
Prob > chi2 = .

r_sgd	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch						
L1.	.0738502	.0331151	2.23	0.026	.0089458	.1387546
garch						
L1.	.9021806	.0475548	18.97	0.000	.808975	.9953863
_cons	6.39e-08	4.65e-08	1.37	0.170	-2.74e-08	1.55e-07
/lndfm2	1.65679	.2568767	6.45	0.000	1.153321	2.160259
df	7.242457	1.346665			5.168699	10.67339

TARCH(1,1,1)-QMLE

ARCH family regression

Sample: 1 - 1226
Distribution: Gaussian
Log likelihood = 6222.323

Number of obs = 1226
Wald chi2(.) = .
Prob > chi2 = .

r_sgd	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch						
L1.	.0933566	.012129	7.70	0.000	.0695841	.117129
atarch						
L1.	.0305413	.0137091	2.23	0.026	.003672	.0574105
sdgarch						
L1.	.8871466	.011863	74.78	0.000	.8638956	.9103976
_cons	.0000475	.0000111	4.29	0.000	.0000258	.0000692

TARCH(1,1,1)-t

ARCH family regression

Sample: 1 - 1226
Distribution: t
Log likelihood = 6247.953

Number of obs = 1226
Wald chi2(.) = .
Prob > chi2 = .

r_sgd	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch						
L1.	.0872491	.0197714	4.41	0.000	.048498	.1260002
atarch						
L1.	.0146545	.0207853	0.71	0.481	-.0260839	.0553929
sdgarch						
L1.	.8983876	.0218249	41.16	0.000	.8556117	.9411636
_cons	.0000458	.00002	2.28	0.022	6.49e-06	.0000851
/lndfm2	1.669968	.2509502	6.65	0.000	1.178115	2.161821
df	7.311998	1.333047			5.248245	10.68695

Model Results for PSEI

Joint JS_U distribution

Log likelihood = 10389.917
Number of obs = 2740
Wald chi2(2) = 58.39
Prob > chi2 = 0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnsigma_2						
r_psei_l1	-16.97485	6.615072	-2.57	0.010	-29.94015	-4.009549
ar_psei_l1	65.77364	8.621792	7.63	0.000	48.87524	82.67204
_cons	-10.537	.0604384	-174.34	0.000	-10.65546	-10.41854
gamma						
r_psei_l1	-7.138833	1.419198	-5.03	0.000	-9.92041	-4.357256
ar_psei_l1	-1.149297	1.920681	-0.60	0.550	-4.913762	2.615168
_cons	-.0073575	.0121145	-0.61	0.544	-.0311014	.0163865
delta						
r_psei_l1	12.84385	6.882676	1.87	0.062	-.6459499	26.33364
ar_psei_l1	-3.486078	7.147872	-0.49	0.626	-17.49565	10.52349
_cons	1.344409	.0653743	20.56	0.000	1.216278	1.47254

GARCH(1,1)-Normal-QMLE (also for Two-Step JS_U)

ARCH family regression

Sample: 1 - 2740
Distribution: Gaussian
Log likelihood = 10154.39
Number of obs = 2740
Wald chi2(.) = .
Prob > chi2 = .

r_psei	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.1153213	.0174944	6.59	0.000	.0810329	.1496098
garch L1.	.7425473	.0428603	17.32	0.000	.6585427	.826552
_cons	5.64e-06	1.16e-06	4.87	0.000	3.37e-06	7.91e-06

Result of Model Parameters in Two-Step JS_U

Log likelihood = -3599.7307
Number of obs = 2739
Wald chi2(2) = 34.94
Prob > chi2 = 0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
gamma						
sres_j2_l1	-.054455	.0093085	-5.85	0.000	-.0726993	-.0362108
asres_j2_l1	-.0047896	.0132116	-0.36	0.717	-.0306838	.0211047
_cons	-.0065264	.0127573	-0.51	0.609	-.0315304	.0184775
delta						
sres_j2_l1	-.0778221	.0606603	-1.28	0.200	-.1967142	.0410699
asres_j2_l1	.0162679	.0627436	0.26	0.795	-.1067072	.139243
_cons	1.352966	.056759	23.84	0.000	1.24172	1.464212

GARCH (1,1)-t

ARCH family regression

Sample: 1 - 2740
Distribution: t
Log likelihood = 10429.78
Number of obs = 2740
Wald chi2(.) = .
Prob > chi2 = .

r_psei	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
arch L1.	.1647159	.0270476	6.09	0.000	.1117035	.2177283
garch L1.	.7163982	.0389079	18.41	0.000	.6401402	.7926562
_cons	4.72e-06	9.60e-07	4.92	0.000	2.84e-06	6.61e-06
/lndfm2	.8669395	.1583476	5.47	0.000	.556584	1.177295
df	4.379617	.3768066			3.744702	5.245583

TARCH(1,1,1)-QMLE

ARCH family regression

Sample: 1 - 2740
Distribution: Gaussian
Log likelihood = 10134.73
Number of obs = 2740
Wald chi2(.) = .
Prob > chi2 = .

r_psei	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.1261264	.0062795	20.09	0.000	.1138187	.138434
atarch L1.	-.0686956	.0075525	-9.10	0.000	-.0834982	-.0538929
sdgarch L1.	.8237644	.0111394	73.95	0.000	.8019317	.8455971
_cons	.0006865	.0000582	11.80	0.000	.0005724	.0008005

TARCH(1,1,1)-t

ARCH family regression

Sample: 1 - 2740
Distribution: t
Log likelihood = 10438.23
Number of obs = 2740
Wald chi2(.) = .
Prob > chi2 = .

r_psei	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
abarch L1.	.1749928	.024425	7.16	0.000	.1271207	.2228649
atarch L1.	-.0638546	.0215707	-2.96	0.003	-.1061324	-.0215767
sdgarch L1.	.7972977	.0301787	26.42	0.000	.7381486	.8564469
_cons	.000593	.0001265	4.69	0.000	.0003451	.0008409
/lndfm2	.8477833	.1352007	6.27	0.000	.5827947	1.112772
df	4.334466	.3156215			3.791037	5.042781