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Fiscal Multipliers Are Not Necessarily That Large: A Comment On Eggertsson (2010)

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Abstract

This paper comments on Eggertsson (2010a) who argued that some fiscal policy measures like an increase in government spending or sales tax cut can be very effective in the recent peculiar environment of zero Federal Funds rate in the US. In particular, we show that the size of multipliers depends on the type of factor market structure (economy-wide or specific) we assume. Regarding the robustness of the results of Eggertsson (2010a) we argue that multipliers under zero nominal interest rate are a magnitude higher than those with positive interest only if the fiscal stimulus is sufficiently long (around ten quarters under specific labor market).

JEL classification: E52, E62

Keywords: fiscal policy, multipliers, homogenous factor market, heterogenous factor market, zero nominal interest

1 Introduction

After the introduction of the American Recovery and Reinvestment Act of 2009 there has been a renewed interest on the effectiveness of fiscal policy in the recent environment of virtually zero Federal Funds Rate. The debate in the literature on the value of fiscal multipliers[1] was sparked by the report of Romer and Bernstein (2009) who provided very optimistic estimates on the impact of the American Recovery and Reinvestment Act of 2009.

Several academic papers appeared in the last two years discussing the magnitude of fiscal multipliers (see, e.g., Christiano et al., 2010; Eggertsson, 2010a; Uhlig, 2010; Woodford, 2010; Cogan et al. 2009; Hall, 2009). Most of these papers employ Neo Classical or New Keynesian type of

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[1] This is the change in output due to a change in government spending, dY_{t+k}/dG_t. For $k = 0$ we get back the impact multiplier.

models to investigate under what conditions the fiscal multiplier is large. It turns out that under *positive* nominal interest rate the multipliers in a New Keynesian model differ mainly due to the type of preferences used[2], the assumption on the conduct of monetary policy[3] and whether the nominal interest rate is positive or zero. This paper argues that the large multipliers of Christiano et al. (2010), Eggertsson (2010a) and Woodford (2010) in case of zero nominal interest rate are not necessarily large if we change some trivial features of the underlying model like the assumed factor market or the length of fiscal spending.

Surprisingly, none of the above papers discussed the role of assumptions on factor market—homogenous (economy-wide) or heterogenous (specific/industry-specific)[4][5]—on the size of fiscal multipliers. Using the basic New Keynesian model of Eggertsson (2010a) augmented for positive steady-state government spending (instead of his assumption of zero) and decreasing returns (instead of his assumption of constant-returns) we show that fiscal multipliers derived under economy-wide labor market are higher than the ones under specific labor market.

Even more importantly, we show that the size of multipliers in Eggertsson (2010a) are extremely sensitive to the duration of the shock that makes the zero bound binding. In particular, under specific factor market and zero nominal interest the shock has to last for at least ten quarters[6]—with a sustained increase in spending during this period—in order for the multiplier to spectacularly exceed the same multipliers derived under positive nominal interest rate with a Taylor rule in action. The two conditions that must be satisfied in order to have a meaningful gap between the size of multipliers under economy-wide and specific labor markets are the assumption of zero nominal interest and a sufficiently long deflationary shock that makes zero bound on nominal interest binding.

Also importantly, using the same simple New Keynesian model with only price staggering and specific factors, we confirm the result of Christiano (2010) who, employing a New Keynesian model with both price and wage staggering, presented in a deterministic experiment that the labor tax

[2] It matters whether we use separable or non-separable preferences as the latter implies complementarity between consumption and hours worked, a reduced (negative) wealth effect on consumption and, hence, a large multiplier.

[3] In a frictionless New Keynesian model with homogenous factor market, Calvo pricing and separable preferences Woodford (2010) shows that the value of the multiplier is one when monetary policy maintains a constant real interest rate implying—through the intertemporal Euler equation—a constant consumption path. Instead, when monetary policy follows a Taylor rule the multiplier is slightly under one.

[4] Factor market means labor market in this paper. However, instead of assuming firm-specific labor market we can arrive at similar results under the alternative assumption of firm-specific (fixed) capital market as well. That is, our results are robust to slightly different settings too. Woodford (2003) shows that specific labor market and firm-specific capital market leads to rather similar outcomes. These assumptions about factor markets are discussed below in detail.

[5] Specific factor market means that there is no instantenous factor price equalisation among firms after reallocation of capital or labor reflecting the fact that sectoral movements of inputs across firms is costly and takes time. Thus, it means that capital and labor may be priced differently across firms.

[6] Eggertsson (2010a) estimated his model using inflation and output data from the 'trough' of the Great Depression (i.e. one observation for inflation and another for output) by Bayesian methods and obtained mode of a 10 quarters shock.

hike multiplier of Eggertsson (2010a) is quantitatively negligible[7]. In addition, we extend the discussion of Christiano (2010) and show in our model[8] that the government spending and sales tax cut multipliers under nominal interest rate is similar in magnitude to those under positive interest contrary to the findings of Eggertsson (2010a).

It is also shown in this paper that the extension of the model of Eggertsson (2010a) for non-zero government spending (instead of his assumption of zero) and decreasing returns (instead of constant returns) implies a significant drop in the value of spending multiplier. In particular, using the calibration of Eggertsson (2010a) we show that a generalisation of his model for positive long-run government spending and decreasing returns in technology reduces the (absolute) size of the government spending and wage tax cut multipliers non-trivially. In particular, the former drops to 1.63 from 2.28 while the latter rises to -0.4 from -1.02.

In a related paper, Christiano et al. (2010) obtain large multipliers in a similar model under zero nominal interest rate with *non-separable* preferences, assuming homogenous factor market and a different calibration. In the simple model in section two and three of their paper they present a spending multiplier for zero interest rate that is obtained for empirically implausible value of Calvo parameter (0.85 implying that firms hold their price fixed, on average, for longer than a year). However, we show that in case of specific factor market instead of the homogenous one multipliers are large even for plausible lengths of price inertia.

Factor market assumption matters a lot. To highlight this fact, we make a comment on Woodford (2010) who derives large multipliers in a model similar to Eggertsson (2010a) with *homogenous*[9] factor markets using parameter values from Eggertsson (2010a) who have estimated his model parameters under the assumption of *heterogenous* labor market. The problem is that using the calibration of Eggertsson (2010a) in a model with homogenous labor market does *not* result in determinacy when nominal interest is zero. Thus, Woodford (2010) could not arrive at the results with his formulas unless he used the ones of Eggertsson (2010a).

The paper is organised as follows. In section 2 we describe the setup of the model. In section 3 we provide the log-linear optimality conditions of the model. Section 4 characterises how we solve for equilibrium as a function of fiscal policy under positive and zero nominal interest rate. Section 5 contains calibration. Section 6 presents results. Finally, section 7 concludes.

[7] The landmark contribution of Eggertsson (2010a) is the presentation of the seemingly counterintuitive negative labor tax cut multiplier in a baseline New Keynesian setting. Then, Christiano (2010) asked what is the relevance of the labor tax hike multiplier i.e. how big it is.

[8] Our paper—similar to Eggertsson (2010a) and unlike Christiano (2010)—considers a stochastic experiment.

[9] In particular, using a concave production function, Woodford (2010) assumed instantenous factor prize equalisation in both capital and labor markets. This implies an outcome that coincides with the one under under homogenous labor market.

Table 1: Extensions in this paper

	Separable prefs.		Non-separable prefs.	
	Eggertsson (2010)	This paper	Christiano et al. (2010)	This paper
Factor market	Heterogenous	Homog./Heterog.	Homog.	Homog./Heterog.
Spending/GDP ($\frac{\bar{G}}{\bar{Y}}$)	zero	positive	positive	positive
Technology	CRS	CRS/DRS	CRS	CRS/DRS
Mark-up (μ)	no effect	no effect	$1^{\S}$	$1.085^{\$}$

Remarks to Table 1: CRS=constant returns; DRS=decreasing returns; $\frac{\bar{G}}{\bar{Y}}$=steady-state government spending-GDP ratio. ¶: The non-separable preferences setup *with DRS* is not discussed below because of the lack of analytical solution in that case. However, it is discussed in detail in the Technical Appendix. §: The steady-state mark-up is eliminated by means of an employment subsidy. \$: this value is based on the estimated demand elasticity ($\theta = 12.78$) by Eggertsson (2010a) in Table (2).

2 The setup of the model

Before we show the elements of the model used in this paper we summarise our extensions in Table (1). The description of the model with separable preferences follows Eggertsson (2010a) who uses a standard RBC model enriched with monopolistic competition and staggered price setting. The setup of the model with non-separable preferences of the form in Christiano et al. (2010)—whose loglinear equilibrium conditions is shortly presented below—is left to the Technical Appendix.

Households. There is a continuum of households of measure one. The representative household maximise

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \xi^T \left[u(C_T) + f(G_T) - \int_0^1 v(l_T(j))dj \right],$$

where β is the discount factor, ξ^T is a preference shock and C_T is a Dixit-Stiglitz aggregate of continuum of differentiated goods, $C_t \equiv \left[\int_0^1 \{c_t(i)\}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ with the elasticity of substitution $\theta > 1$. There is a corresponding price index, $P_t \equiv \left[\int_0^1 \{p_t(i)\}^{1-\theta} di \right]^{\frac{1}{1-\theta}}$. When labor is homogenous, there is a single, economy-wide nominal wage, W_t. However, in case of heterogenous labor market each household provides a specific type of labor j which is used in industry i and renumerated with a specific nominal wage, $W_t(j)$. The standard assumptions apply for preferences: u', $f' > 0$, u'', $f'' < 0$, $v' > 0$ and $v'' < 0$ and G is perfectly substitutable for private consumption. The representative household budget constraint is:

$$(1+\tau_t^S)P_t C_t + B_t = (1-\tau_{t-1}^A)(1+i_{t-1})B_{t-1} + \int_0^1 Z_t(i)di + (1-\tau_t^W)\int_0^1 W_t(j)l_t(j)dj - T_t,$$

where Z_t is the profit distributed lump-sum to households. There are three types of taxes in the model: a tax on financial assets, τ_t^A, tax on sales, τ_t^S, and a tax on labor, τ_t^W. Thus, representative household maximise utility—taking wages and prices as given—with respect to the budget constraint choosing $c_t(i)$, $l_t(j)$, B_t for all j and i at all time t. The utility-maximisation problem yields the intratemporal and intertemporal Euler equations together with a transversality condition for bonds to eliminate Ponzi schemes:

$$\frac{W_t(j)}{P_t} = \frac{v_l(l_t(j))}{u_C(C_t)} \frac{1+\tau_t^S}{1-\tau_t^W},$$

$$E_t \left\{ \frac{u_C(Y_{t+1}-G_{t+1})}{u_C(Y_t-G_t)} (1-\tau_{t+1}^A) R_{t+1} \right\} = \beta^{-1} E_t \left\{ \frac{\xi_t}{\xi_{t+1}} \frac{(1+\tau_{t+1}^S)}{(1+\tau_t^S)} \frac{P_{t+1}}{P_t} \right\},$$

and

$$\lim_{T\to\infty} E_t \left\{ \frac{B_T}{P_T(1+\tau_T^S)} \right\} u_C(C_T) = 0.$$

Firms. There is a perfectly competitive firm that bundles intermediary goods, $y_t(i)$ into a single final good. The cost-minimisation problem of the perfectly competitive firm yields the demand curve for intermediate good i of the form: $y_t(i) = [p(i)/P_t]^{-\theta} Y_t$. There is a continuum of intermediate goods producer firms—in measure one—that hires specific[10] type of labor j from each household. Intermediary firm i that operates in industry j with a total cost function, $TC_t(j)$ maximises its profit, $Z_t(i) = p_t(i)y_t(i) - TC_t(j)$, taking the demand curve of good i as given. Following Edge (2002) and Woodford (2003) we assume specific-labor market i.e. household j can sell its labor to firm i only, and, thus, $i = j$. Under the assumption of staggered price-setting a' la Calvo (1983) the profit-maximisation problem of the intermediary is:

$$\max_{p_t^*} \sum_{T=t}^{\infty} (\xi\beta)^{T-t} Q_{t,T} \left[p_t^* \left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T - TC\left(\left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T\right)\right],$$

where $Q_{t,T} \equiv \frac{U_C(C_T)}{U_C(C_t)} \frac{P_t}{P_T}$ is the discount factor and we substituted for $y_t(i)$ the demand curve of intermediary good i given above. Eggertsson (2010a) assumes constant-returns-to-scale (CRS) technology. However, we derive multipliers under decreasing-returns-to-scale (DRS) as well using $y_t(i) = [l_t(j)]^{1/\phi}$, $\phi > 1$[11]. When $\phi = 1$ we are back to CRS. The first-order condition (FOC) associated with this problem is:

$$\sum_{T=t}^{\infty} (\xi\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \left[\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} mc_{t,T}(i)\right] = 0,$$

[10] Or, it hires homogenous labor and index j can be dropped.

[11] Below we discuss how sensitive our results are to the fact whether we use specific labor or specific capital market. In case of specific capital market we use a constant-returns Cobb-Douglas formulation: $Y_t(i) = \bar{K}^{1-a} N(i)^a$ where $\bar{K}$ expresses the fact that capital stocks are firm-specific and not variable input (i.e. $\bar{K}$ is a constant). In the latter case the Cobb-Douglas formulation is practically a decreasing-returns production function in variable input $N_t(i)$.

where $\frac{p_t^*}{P_T}$ is the optimal relative price, $\frac{\theta}{\theta-1}$ is the markup due to monopolistic competition and $mc_{t,T}(i)$ is the time-T real marginal cost of firm i which last set its price at time t. Note that assuming DRS instead of CRS implies that the marginal cost of an individual firm becomes dependent on its own production (for both types of factor markets) and also affecting the slope of the New Keynesian Phillips Curve (NKPC).

The aggregate price index is composed of a fraction $(1-\alpha)$ of firms' who set their prices optimally at p_t^* and the remaining fraction (α) of those who keep them fixed at P_{t-1}:

$$P_t = \left[(1-\alpha)\left(p_t^*\right)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

The model is closed with the aggregate resource constraint, $Y_t = C_t + G_t$, and a restriction for the nominal interest rate: $i_t \geq 0$. The monetary policy is conducted through a Taylor rule:

$$i_t = \max\left\{0,\ f\left(\frac{P_t}{P_{t-1}}, Y_t, \xi_t\right)\right\},$$

where f is a function that is specified in detail below. Fiscal policy is a sequence of variables, $\{G_t, \tau_t^W, \tau_t^S, \tau_t^A\}$, specified below.

3 The log-linear equilibrium conditions

We loglinearise the model around its non-stochastic zero inflation steady-state. The New-Keynesian IS curve—which is the loglinear Euler equation—together with the loglinear aggregate resource constraint, $\hat{Y}_t = (1-g)\hat{C}_t + \hat{G}_t$, yields what we can call Aggregate Demand (AD) curve

$$\Gamma[\hat{Y}_t - E_t\hat{Y}_{t+1}] = (\Upsilon+1)[\hat{G}_t - E_t\hat{G}_{t+1}] - \Theta\left(i_t - \vartheta E_t\pi_{t+1} - r_t^e\right) + \zeta\chi^S\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \zeta\chi^A\hat{\tau}_t^A \quad (1)$$

where for **separable preferences** coefficients are: $\Theta = \zeta \equiv \check{\sigma}$, $\Upsilon = 0$ and $\vartheta = \Gamma = 1$. Further, $\sigma \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}\bar{C}}$, $\check{\sigma} \equiv \sigma(1-g)$, $\chi^S \equiv \frac{1}{1+\bar{\tau}^S}$, $\chi^A \equiv \frac{1-\beta}{1-\bar{\tau}^A}$ and $g \equiv 1-\bar{C}/\bar{Y} = \bar{G}/\bar{Y} > 0$ are the intertemporal elasticity of subsitution (σ), σ re-scaled by the level of government spending ($\check{\sigma}$), constants scaling the sales and capital taxes (χ^S and χ^A) and the definition of the steady-state government spending ($\bar{G}$)-GDP ($\bar{Y}$) ratio, respectively. Variables with a hat are defined as: $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, $\hat{C}_t \equiv \log(C_t/\bar{C})$, $\hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, $\hat{\tau}_t^i \equiv \tau_t^i - \bar{\tau}^i$, $i \in \{A, S, W\}$ and $r_t^e \equiv \log\beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$ where $\hat{\xi}_t \equiv \log(\xi_t/\bar{\xi})$.[12]

[12] Note that, here, in the loglinearised model i_t refers to $\log(1+i_t)$ and not defined as log deviation from steady-state. Further, $\hat{\tau}_t^A$ is defined such that a one percent increase in capital income per year is comparable with the tax on labor income.

The New Keynesian Phillips curve (or Aggregate Supply—AS curve) is given by:

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}_t^W + \chi^S\hat{\tau}_t^S - \vartheta\hat{G}_t) + \beta E_t\pi_{t+1} \tag{2}$$

with

$$\begin{aligned} \kappa &\equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi\omega+\vartheta+(\phi-1)}{1+\mathcal{I}_{\text{het}}\omega_y\theta+\mathcal{I}_{\text{hom}}(\phi-1)\theta}; \qquad \psi \equiv \frac{1}{\phi\omega+\vartheta+(\phi-1)}; \\ \omega_y &\equiv \phi(1+\omega)-1; \qquad \omega \equiv \frac{\bar{v}_{ll}\bar{l}}{\bar{v}_l}; \qquad \vartheta \equiv \check{\sigma}^{-1}; \qquad \chi^W \equiv \frac{1}{1-\bar{\tau}^W}, \end{aligned}$$

where $\mathcal{I}_{\text{hom}}$ ($\mathcal{I}_{\text{het}}$) is an indicator variable which takes on the value of one when we assume homogenous (heterogenous) labor market. For $\phi = 1$, $g = 0$, $\mathcal{I}_{\text{hom}} = 0$ and $\mathcal{I}_{\text{het}} = 1$ we are back to the setup of Eggertsson (2010a). Note that only the content of parameters $\check{\sigma}$, κ and ψ change when we generalise Eggertsson (2010a) for positive long run government spending, DRS and two types of labor markets.

For **non-separable preferences** of the kind in Christiano et al. (2010)[13] the AD curve has parameters different from the separable case[14]: $\Gamma \equiv [\Upsilon(1-b)+1]$, $\Upsilon \equiv (\sigma-1)\gamma$, $\Theta \equiv \beta(1-g)$, $\varrho \equiv \beta^{-1}$, $\zeta \equiv \check{\sigma}$, $\phi = 1$, $b \equiv \frac{1-\bar{\tau}^W}{1+\bar{\tau}^S}$.[15] Here, besides loglinear Euler and loglinear market clearing equations we also made use of loglinear production function, $\hat{Y}_t = (1/\phi)\hat{N}_t$, to derive the AD equation.

The NKPC in case of non-separable preferences can be written with small change, i.e. only the content of some parameters change (these are denoted with a tilde):

$$\begin{aligned} \tilde{\kappa} &\equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi\tilde{\omega}+\tilde{\vartheta}+(\phi-1)}{1+\mathcal{I}_{\text{het}}\tilde{\omega}_y\theta+\mathcal{I}_{\text{hom}}(\phi-1)\theta}; \qquad \tilde{\psi} \equiv \frac{1}{\phi\tilde{\omega}+\tilde{\vartheta}+(\phi-1)}; \\ \tilde{\omega}_y &\equiv \phi(1+\tilde{\omega})-1; \qquad \tilde{\omega} \equiv \frac{\bar{N}}{1-\bar{N}}; \qquad \tilde{\vartheta} \equiv (1-g)^{-1}. \end{aligned}$$

Our setup investigates $\phi = 1$, $g > 0$, $\mathcal{I}_{\text{hom}} = 0$ and $\mathcal{I}_{\text{het}} = 1$. However, for $\phi = 1$, $g > 0$, $\mathcal{I}_{\text{hom}} = 1$ and $\mathcal{I}_{\text{het}} = 0$ we are back to the setup of Christiano et al. (2010). Below we argue that the choice

[13] Here and for the rest of the paper we refer to the frictionless model in their Section 2 and 3 of Christiano et al. (2010) and not their model with capital in Section 3 or the medium-sized DSGE model in Section 4.

[14] Observe that we use the non-separable preferences employed by Christiano et al. (2010), $u(C_t, N_t) = \frac{[C_t^\gamma(1-N_t)^{1-\gamma}]^{1-\sigma}-1}{1-\sigma}$. Note that for non-separable preferences the intertemporal elasticity of substitution $(\gamma(1-\sigma)-1)$ is different from the separable case (σ). Thus, the definition of σ in the separable case does no coincide with the σ in the non-separable case.

[15] The parameters here are derived under the assumption of CRS. Note that they are different under DRS. The only caveat under DRS is that steady-state hours—that is needed for derivation of the multipliers—cannot be calculated analytically (i.e. we cannot simply substitute for $\frac{\bar{N}}{1-\bar{N}}$ the steady-state of the intratemporal condition). However, $\frac{\bar{N}}{1-\bar{N}}$ can be calculated numerically using e.g. Matlab fsolve algorithm. The AD curve for non-separable preferences in case of DRS is shown in the Technical Appendix.

of making either $\mathcal{I}_{\text{hom}}$ or $\mathcal{I}_{\text{het}}$ equal to one matters a lot. The monetary policy respects the Taylor rule:

$$i_t = \max\{0, r_t^e + \phi_\pi \pi_t + \phi_Y \hat{Y}_t\}, \tag{3}$$

where coefficients must satisfy: $\phi_\pi > 1$ and $\phi_y > 0$ (for the restrictions on ϕ_π and ϕ_Y see Woodford (2003)).

The equilibrium can be characterised as collection of stochastic sequences $\{\pi_t, \hat{Y}_t, i_t, r_t^e\}$ that satisfy equilibrium conditions (1)-(3) given path for policy variables $\{\hat{G}_t, \hat{\tau}_t^W, \hat{\tau}_t^S, \hat{\tau}_t^A\}$ which are financed by lump-sum taxes either in period t or in future periods, $t+j$.

4 Brief description of equilibrium for positive and zero nominal interest rates

Even if this is an infinite horizon problem Eggertsson (2010a) shows that it is enough to analyse a short-run and a long-run equilibrium. Initially we are in steady-state ($t = 0$). Then from time $t = 1$, for some interval, $0 < t < T$, which we can call *short run*, a shock hits the economy. That is, when $t < T$ the shock is described by an exogenous decrease in $r_t^e = r_S^e < 0$ with T denoting the stochastic date at which the shock vanishes. Christiano et al. (2010) interprets this shock as a rise in people's propensity towards savings[16]. Short-run allocations are denoted with subscript S. Further, we assume that in period t the shock persists with probability μ or dies out with $1-\mu$ for all $t < T$. In the short-run zero lower bound on nominal interest is binding ($i_t = i_S = 0$) or not binding ($i_t = i_S > 0$). In the non-binding case the nominal interest is governed by the Taylor rule. For time, $t \geq T$, variables take on their *long-run* steady-state values.

Positive Interest rate. When nominal interest rate is positive the system can be solved by the method of undetermined coefficients. That is, we assume that inflation and output is a linear function of the fiscal variable, $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$:

$$\pi_S = A_\pi \hat{F}_S, \tag{4}$$

$$\hat{Y}_S = A_Y \hat{F}_S. \tag{5}$$

We have an exogenous AR(1) process for government spending (and the same could be written for labour, sales and capital tax as well):

$$F_{t+1} = F_t^\rho \exp(\varepsilon_{t+1}) \tag{6}$$

[16] In the explanation of Eggertsson (2010a), an exogenous decrease in r_t^e can be translated into an increase in the probability of default of borrowers creating a spread between risk-free rate and the rate paid on risky loans.

where ρ measures persistence of government spending process and ε is an *i.i.d.* shock with zero mean and constant variance. That is, for expectational variables we have $E_t\hat{F}_{t+1} = \rho\hat{F}_S$. We assume that the government spending, the labour tax cut, the sales tax cut and the employment subsidy to restore efficiency in steady-state is financed through *lump-sum* taxes. That is, the Ricardian equivalence holds under our assumptions and the exact timing of taxes is irrelevant and we don't have to take into consideration the government budget constraint.

We compute fiscal multipliers separately. That is, e.g. a sales tax cut implies no change in other fiscal instruments (i.e. there is no change in labor, capital tax and government spending). Furthermore, we assume that changes in spending (or taxes) are offset by present or future lump-sum taxes/transfers, i.e. Ricardian evidence holds[17].

Zero nominal interest rate. In period t and t+1 variable $\hat{X}_i = \{\hat{F}_i, \hat{Y}_i, \pi_i\}$ with $\hat{F}_i = \{\hat{G}_i, \hat{\tau}_i^W, \hat{\tau}_i^S, \hat{\tau}_i^A\}$ for $i \in \{t, t+1\}$ are taking, respectively, the following values:

$$\hat{X}_t = \begin{cases} \hat{X}_t = \hat{X}_S, \quad 0 < t < T, \text{ zero bound binding,} \\ \hat{X}_t = 0, \quad t \geq T, \text{ zero bound not binding,} \end{cases}$$

and

$$\hat{X}_{t+1} = \begin{cases} (1-\mu)\hat{X}_S = 0, \quad \text{with probability } 1-\mu \text{ variable } \hat{X}_{t+1} \text{ reverts back to steady-state,} \\ \mu\hat{X}_S, \quad \text{with probability } \mu \text{ zero bound continues to bind.} \end{cases}$$

Next we formulate two conditions under which the zero bound binds. Condition *C1* ensures that the shock in r_S is large enough to make the zero bound bind and imposes a constraint on the magnitude of fiscal action[18]:

$$\begin{aligned} r_t^e \quad > \quad & \frac{(\Upsilon+1)(1-\rho)[\kappa\phi_\pi + (1-\beta\mu)\phi_Y] + \kappa\psi[(\phi_\pi - \varrho\rho)\phi_Y - \vartheta(\Gamma - \Gamma\rho + \Theta\phi_Y)\phi_\pi]}{\kappa\Theta(\phi_\pi - \varrho\rho) + [\Gamma + \Theta\phi_2 - \Gamma\rho](1-\beta\mu)}\hat{G}_t \\ & + \frac{\kappa\psi\chi^W[(\Gamma\rho - \Gamma - \Theta\phi_2)\phi_\pi - \Theta(\phi_\pi - \varrho\rho)\phi_Y]}{\kappa\Theta(\phi_\pi - \varrho\rho) + [\Gamma + \Theta\phi_2 - \Gamma\rho](1-\beta\mu)}\hat{\tau}_t^W \\ & \frac{-\kappa\psi\chi^S[\phi_\pi(\Gamma - \Gamma\rho + \Theta\phi_2) + (\phi_\pi - \varrho\rho)\phi_Y] - \zeta\chi^S(1-\rho)[\kappa\phi_\pi + (1-\beta\rho)\phi_Y]}{\kappa\Theta(\phi_\pi - \varrho\rho) + [\Gamma + \Theta\phi_2 - \Gamma\rho](1-\beta\mu)}\hat{\tau}_t^S \\ & + \frac{\zeta\chi^A[(1-\beta\rho)\phi_Y + \phi_\pi] - \kappa\psi\chi^A[\Gamma - \Gamma\rho + \Theta\phi_2]\phi_\pi}{\kappa\Theta(\phi_\pi - \varrho\rho) + [\Gamma + \Theta\phi_2 - \Gamma\rho](1-\beta\mu)}\hat{\tau}_t^A \end{aligned}$$

[17] In a simple RBC model with capital, constant Frisch elasticity preferences and a fiscal rule that connects changes in spending to changes in income tax Uhlig (2010) shows that output turn to negative after around two years of the rise in spending.

[18] This condition can be derived by substituting equations (8) and (9) into the Taylor rule, equation (3).

while condition *C2* makes sure that the crises do not last for too long[19]:

$$\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa > 0. \tag{7}$$

Thus, in the short run when $i_S > 0$ and *C1* does not hold the equilibrium π_S, $\hat{Y}_S$ and i_S are described, respectively, by (this is the generalised version of Proposition 3 in Eggertsson (2010a)):[20]

$$\pi_S = \mathcal{A}\hat{G}_S + \mathcal{B}\hat{\tau}_S^S + \mathcal{C}\hat{\tau}_S^W + \mathcal{D}\hat{\tau}_S^A, \qquad \mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D} > 0 \text{ (constants)}, \tag{8}$$

$$\begin{aligned}
\hat{Y}_S \quad = \quad & \frac{\kappa\psi[\phi_\pi - \varrho\rho] + (\Upsilon+1)(1-\rho)(1-\beta\mu)}{(1-\beta\rho)(\Gamma+\Theta\phi_2-\Gamma\rho) + \kappa\Theta[\phi_\pi - \varrho\rho]}\hat{G}_S \\
& - \frac{\kappa\Theta\psi\chi^S(\phi_\pi - \varrho\rho) + \zeta\chi^S(\rho-1)(1-\beta\rho)}{(\Gamma+\Theta\phi_2)(1-\beta\rho) - \Gamma\rho(1-\beta\rho) + \kappa\Theta(\phi_\pi - \varrho\rho)}\hat{\tau}_S^S \\
& - \frac{\kappa\psi\chi^W\Theta(\phi_\pi - \varrho\rho)}{(1-\beta\rho)(\Gamma-\Gamma\rho+\Theta\phi_2) + \kappa\Theta(\phi_\pi - \varrho\rho)}\hat{\tau}_S^W \\
& + \frac{\zeta\chi^A(1-\beta\rho)}{(\Gamma+\Theta\phi_2)(1-\beta\rho) - \Gamma\rho(1-\beta\rho) + \Theta\kappa(\phi_\pi - \varrho\rho)}\hat{\tau}_S^A
\end{aligned} \tag{9}$$

and

$$i_S = i_S^e + \phi_\pi\pi_S + \phi_Y\hat{Y}_S.$$

Similarly, in the short run when $i = 0$, *C1* and *C2* hold, the equilibrium is as follows:[13]

$$\pi_S = \check{\mathcal{A}}\hat{G}_S + \check{\mathcal{B}}\hat{\tau}_S^S + \check{\mathcal{C}}\hat{\tau}_S^W + \check{\mathcal{D}}\hat{\tau}_S^A + \check{\mathcal{E}}r_S^e, \quad \check{\mathcal{A}},\check{\mathcal{B}},\check{\mathcal{C}},\check{\mathcal{D}},\check{\mathcal{E}} > 0 \text{ (constants)},$$

$$\begin{aligned}
\hat{Y}_S \quad = \quad & \frac{(\Upsilon+1)(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa\psi\vartheta}{\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa}\hat{G}_S + \frac{\Theta\mu\varrho\kappa\psi\chi^W}{\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa}\hat{\tau}_S^W \\
& + \frac{\Theta\mu\varrho\kappa\psi\chi^S - \vartheta\chi^S(1-\mu)(1-\beta\mu)}{\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa}\hat{\tau}_S^S + \frac{\vartheta\chi^A(1-\beta\mu)}{\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa}\hat{\tau}_S^A \\
& + \frac{\Theta(1-\beta\mu)}{\Gamma(1-\mu)(1-\beta\mu) - \Theta\mu\varrho\kappa}r_S^e
\end{aligned} \tag{10}$$

and

$$i_S = 0.$$

Note that the above expressions for $\hat{Y}_S$ contain the fiscal multipliers (the constants multiplying

[19]Condition *C2* also makes sure i) to avoid deflationary black hole—analysed in Eggertsson (2010a)—that would arise at $\bar{\mu}$ that satisfies $L(\bar{\mu}) = 0$ and ii) ensures that the coefficient on r_t^e in equation (10) is positive so that $r_t^e < 0$ remains to be satisfied.

[20]Constants $\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}$ ($\check{\mathcal{A}},\check{\mathcal{B}},\check{\mathcal{C}},\check{\mathcal{D}},\check{\mathcal{E}}$) are available in the Technical Appendix.

$\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S$ and $\hat{\tau}_S^A$) for $i > 0$ and $i = 0$ cases. In line with Eggertsson (2010a) we assume that $\rho = \mu$. An approximate equilibrium that is correct up to the first order is a collection of stochastic processes for $\{\hat{Y}_t, \pi_t, i_t, r_t^e\}$ that solves equations (1)-(3) given paths for fiscal policy, $\{\hat{G}_t, \hat{\tau}_t^W, \hat{\tau}_t^S, \hat{\tau}_t^A\}$.

5 Calibration

5.1 Separable preferences

We use the estimated parameters of Eggertsson (2010a) who calibrated his model to data prevailing under the Great Depression. The values are summasrised in Table (2):

Table 2: Parametrisation of the model with separable preferences

β	σ	ω	ρ	ϕ_π	ϕ_Y	$1/\phi$	θ	g	$\bar{\tau}^S$	$\bar{\tau}^A$	$\bar{\tau}^W$
0.9970	0.86	1.5692	0.9030	1.5	0.5/4	2/3	12.7721	0.2	0.05	0	0.2
α	μ^*										
0.7747	0.9030										

Remarks to Table 2: g is from Christiano et al. (2010). ϕ is taken from Woodford (2003).
*For this value multipliers are not defined for economy-wide labor market when $i = 0$.
Hence, we use a lower value, $\mu = .8$, that is employed by Christiano et al. (2010).

5.2 Non-separable preferences

We adopt the calibration in section 2 of Christiano et al. (2010) and reproduce them in Table (3). For parameters not found there we use the estimated (and for the steady-state tax rates calibrated) values of Eggertsson (2010a).

Table 3: Parametrisation of the model with non-separable preferences

β	σ	γ	ρ	ρ_R	ϕ_π	ϕ_Y	$1/\phi$	g	$\bar{\tau}^S$	$\bar{\tau}^A$	$\bar{\tau}^W$
0.9970	2	0.29	0.8	0	1.5	0	2/3	0.2	0.05	0	0.2
θ	α	μ									
12.7721	0.85	0.8									

Remarks to Table 3: $1/\phi$ is taken from Woodford (2003).
Values for θ, $\bar{\tau}^S$, $\bar{\tau}^A$ and $\bar{\tau}^W$ are borrowed from Eggertsson (2010a).

6 Discussion of Results

6.1 The role of labor market structure

Proposition 1 *Fiscal multipliers in case of specific factor market are (in absolute value) lower than those under economy-wide factor market when nominal interest is zero.*

The proof is in the Appendix we provide only intuition here. Factor market assumption influences the model through κ which is the multiplier on marginal cost in NKPC (see equations (2) above). When factor markets are homogenous ($\kappa > 1$), monopolisitically competitive firms pricing decisions are strategic substitutes—i.e. an individual firm which experiences a rise in the prices of goods of the other firms will decrease the price of its own good. Whereas the specific factor markets assumption ($\kappa < 1$) implies complementarity in pricing decisions among firms[21]. To illustrate the degree of strategic complementary (or strategic substitutability) we compiled Table (added soon!!!!) which shows how our key parameter, κ, changes along the different factor market assumptions. The lower is the higher is strategic complementarity in pricing. Woodford (2003) includes features inducing strategic complementarity as one of the baseline elements of the baseline New Keynesian model.

Under positive nominal interest ($i_t > 0$), specific and non-specific factor market implies multipliers similar in magnitude. (e.g., in Table (4) we can compare 0.61 and 0.67 for an increase in spending or 0.40 and 0.44 for a cut in sales tax). Also, multipliers under specific factor market are the slightly bigger than their economy-wide counterparts as those firms who employ specific labor (and, thus, having strategic complementarity in pricing decisons) that have less opportunity to change prices and thus, will increase their output a bit more in case of a positive fiscal shock.

However, the latter is not true anymore when the zero lower bound on nominal interest becomes binding. The slope of the labor demand curve is influenced by the value of κ. The lower κ is the steeper is the labor demand which, as shown by Eggertsson (2010a and 2010b), positively-sloped in the $i = 0$ case. Consequently, labor demand with homogenous input is flatter than with heterogenous input. On the left panel Figure (1) we can track a rise in spending that shifts out both LD and LS to the right (denoted by LD' and LS') resulting in lower wage and higher hours worked. Note that the shift in LD for heterogenous market (dashed line) is larger than the shift in LD for homogenous labor market (dashed-dotted line): the difference comes from the value of κ multiplying $\hat{G}_S$ in the labor demand curve. Further, under homogenous labor market the rise in labor is bigger (point B2) than under heterogenous market (point B1)[22]. On the right panel of Figure (1) we observe a rightward shift in LS after a decrease in labor tax. It is only the LS that

[21] Woodford (2003) Chapter 3 gives an overview on the importance of the assumptions regarding the factor market for the propagation of nominal income shocks.

[22] Note that a decrease in sales tax is analogous to a rise in spending. However, the sales tax cut has a smaller effect because it has a coefficient, $\breve{\sigma}$, in the AD curve in equation (1) that is smaller than one.

moves as LD does not contain the wage tax. One can further say that these expansionary fiscal policies have a deflationary aspect in the sense that the fall in wages reduce marginal costs exerting a downward pressure on prices (Eggertsson (2010a)). Also, Table (4) shows that spending (1.74), labor tax cut (-0.32) and sales tax cut (1.14) multipliers for homogenous labor market are higher than those for specific labor market (1.08, -0.03 and 0.71, respectively)[23]. Also important to note here that the labor tax multiplier under specific factor market is now close to zero[24].

For completeness, we have to add that the presence of DRS in production can itself imply strategic complementary without assuming specific labor market as explained by Woodford (2003). Therefore, it is fair to ask whether multipliers between economy-wide labor market with CRS and the same with DRS differ as strikingly as they do for the economy wide versus specific markets. The answer is that they still differ but the comparison can be made only for low persistence values because in case of homogenous labor market and CRS μ can take the maximum of 0.69 in order for *C2* to be satisfied. It remains to be true that the difference is significant for only sufficiently high level of persistence (i.e. for $\mu > 0.5$).

In the next, we plot multipliers for both types of labor markets and find that multipliers do not differ sharply under $i > 0$ and $i = 0$ if the persistence of the fiscal shock is sufficiently but not implausibly low. Woodford (2010) also argues that for low values of the persistence parameter ($\mu < 0.903$) the spending multiplier do not exceed one. Here, we make an even stronger claim. That is, multipliers practically coincide if the deflationary shock—and the accompanying fiscal stance—last for three (six) quarters or shorter in case of economy-wide (specific) labor market. On Figure (2), (3), (4) and (5) we can observe spending, labor tax cut, sales tax cut and capital tax cut multipliers, respectively[25], for both types of labor markets. The upper panels refer to calculations under the Taylor rule in equation (3). Figures on the lower panels corresponds to the same Taylor rule but with no reaction to output-gap (i.e. $\phi_2 = 0$ as assumed by Christiano et al. (2010) in their model in section 2). We can see in the upper panels that multipliers are very similar in magnitude under $i > 0$ and $i = 0$ for modest level of persistence (e.g., $\mu = 0.7$ (0.8) implies a $3\frac{1}{3}$ (6) quarters shock). The latter finding is even more spectacular in the lower panels when there is no reaction to output gap in the Taylor rule. It is shown in Woodford (2010) that even for low value of r^e_S, the shock can be quite large—and leading to huge deflation and output collapse—if the persistence

[23]Unfortunately, multipliers derived under the assumption of homogenous and heterogenous factor markets are not directly comparable when $i_t = 0$ as the estimated value of $\mu = 0.903$—which Eggertsson (2010) obtained using specific labor market—under homogenous factor markets no longer satisfies *C2* as κ differs depending on type of the factor market we assume. In case of homogenous factor market, $g > 0$ and DRS the maximal value of μ that satisfies the previous condition is 0.85. For $\mu = 0.85$ the multiplier is unplausibly large. Hence, we use the somewhat lower but empirically still plausible value of $\mu = 0.8$ of Christiano et al. (2010) for comparison.

[24]Therefore we can easily generate a case for which the negative and large (in absolute value) labor tax multiplier of Eggertsson (2010) is still negative but very close to zero.

[25]In constructing the graphs we excluded values of $\mu > 0.92$ for which the multiplier is implausibly large (it is 16 for $\mu = 0.93$) so that we can avoid distortion of the graphs for values of $\mu < 0.92$ that convey multipliers between 1 and 2.

parameter is big (e.g. the estimated value of $\mu = 0.903$ by Eggertsson (2010a) implies 10 quarters mean duration of the shock)[26]. However, one can assume a shock that lasts for somewhat shorter time (e.g. $\mu = 0.8$ in Christiano et al. 2010). Then, in the latter case, multipliers for $i = 0$ are not significantly larger than those for $i > 0$. Hence, the multipliers of Eggertsson (2010a) for $i = 0$ are large under relatively special circumstances (i.e. a sufficiently long period of shock *with* an expansionary fiscal policy at the same time). The latter also reflects the peculiar features of the two-state Markovian structure used to model the zero nominal interest rate.

6.2 The effect of positive government spending-output ratio and decreasing returns

Let us further discuss the numbers in Table (4). We realise that multipliers in case of $i > 0$ do not differ too much for constant returns (CRS) and decreasing returns (DRS) irrespectively whether g is zero or positive. However, for $i = 0$ we can see that the government spending multiplier with positive $g > 0$ and DRS is less (the wage tax cut multiplier is also smaller in absolute value) than those obtained by Eggertsson (2010a) who assumed $g = 0$ and CRS. Why does the spending multiplier increase in case of $i > 0$ if we allow for $g > 0$ (and DRS)? When $g > 0$ the value of intertemporal elasticity of substitution falls (downscaled)—this is apparent from the definition of coefficient $\breve{\sigma} = \sigma(1 - g)$—and people are less willing to substitute present consumption for future consumption after the positive government spending shock even if the negative wealth effect[27] forces consumers to do so. Thus, lower $\breve{\sigma}$ results in smaller consumption loss and a higher multiplier. In contrast when $i = 0$ multipliers diminish under positive long-run spending. When $i = 0$, expansionary fiscal policy leads to a sharp rise in inflation which—due to the lack of Taylor rule—implies a fall in the real rate. The latter serves as an incentive for households to consume more in the present and, thereby, increasing the multiplier. However, this incentive is less strong under lower substitution ($\breve{\sigma} < \sigma$) which is why $g > 0$ downsize the multiplier under $i = 0$.

Also introducing diminishing returns to technology implies that a unit of labor produces less than one unit of output. That is, when monopolistically competitive firms increase their labor demand due to the rise in demand for their products they can produce less under DRS than under CRS. Hence, under DRS multipliers (for wage tax cut we mean in absolute value) are lower than those for CRS irrespectively whether $i > 0$ or $i = 0$. The only exception is the wage tax cut that delivers multipliers seemingly different in size under CRS and DRS when $i = 0$.

[26] We can confirm this claim by looking at the coefficient on r_t^e in equation (10) that is increasing in μ.

[27] As Ricardian evidence holds and government spending is financed through present and future lump sum taxes the consumer is willing to delay current consumption and work more hours.

6.3 Some remarks on Christiano et al. (2010)

The assumption of homogenous labor market and constant returns in Christiano et al. (2010 section 2 and 3) constrain the value of Calvo parameter to a minimum of 0.82 through condition *C2* in equation (7). However, we argue that the generalisation of the model in Christiano et al. (2010) for heterogenous factor market (instead of homogenous) conveys high multipliers even for low values of the Calvo parameter. In particular, the lowest possible value is $\alpha = 0.61$ which—implying less than three quarters of price stickiness—results in a multiplier of around 58. If we allow for decreasing returns (instead of constant returns) as well then α can be even smaller. For an empirically plausible value of the Calvo parameter (e.g. $\alpha = 0.75$ instead of $\alpha = 0.85$)[28] and using configuration for the other parameters in Table (3)[29] we found a spending multiplier of 1.60 under specific labor market. Instead, using the full calibration in Table (3) the spending multiplier for heterogenous labor market is 'only' 1.36 while for homogenous factor—which is the one used by Christiano et al. (2010)—it is huge (3.7). Even if multipliers with separable and non-separable preferences are not directly comparable we can observe that they are very similar in magnitude if we allow for the same persistence, Calvo parameter and heterogenous labor market under $i = 0$.

The reason why multiplier can be very high even for low α lies in the value of κ[30]: for heterogenous labor market the value of κ can be very small without a high α. Ohanian (2010) argues when discussing Eggertsson (2010a) that condition *C2* is hard to satisfy if prices change frequently ($\alpha \to 0$). However, we can claim that condition *C2* is less stringent if we allow for some straightforward real friction like factor specificity (and/or features like DRS instead of CRS). For non-separable preferences, κ in condition *C2* are influenced by even more parameters as, now, the labor supply elasticity—which is ω for separable preferences—corresponds to ratio, $\bar{N}/(1-\bar{N})$, consisting steady-state hours which is a function of the weighting parameter, γ, the steady state consumption tax, labor tax and the steady-state markup ($\theta/(\theta-1)$). Surprisingly, Christiano et al. (2010) eliminated steady-state markup distortion by imposing a lump-sum employment subsidy. However, we have no obvious reason to do alike. The consequence of eliminating the steady-state distortion due to monopolistic competition is, interestingly, a fall in the multipliers. It is because the presence of steady-state wage (the inverse of the markup), $(\theta-1)/\theta$, which multiplies ω and makes denominator of multipliers larger. Hence, multipliers gets smaller (3.38 instead of 3.7 for the baseline calibration with non-separable preferences). However, the firm-specific setup slightly

[28] Including sales prices Nakamura and Steinsson (2008) found an average price stickiness of one year on average ($\alpha = 0.75$). The same value is used by Christiano et al. (2011) in their middle-sized DSGE model in section 5. However, some author assume shorter period of price stickiness: e.g. Edge (2002) assumes six months price inertia while Bils and Klenow (2004) excluding sales prices estimated a mean duration of seven months.

[29] Christiano et al. (2010) uses a Taylor rule which allows for the possibility of accomodative monetary policy as well (or, to put it differently, interest rate smoothing that is governed by coefficient ρ_R). However, they calibrate ρ_R to equal zero so the Taylor rule here in equation (3) remains valid. Otherwise, for $\rho_R > 0$, it is not possible to solve for the multipliers analytically.

[30] Note that the value of κ is inversely related to α.

boost multipliers because the non-zero markup raises κ which reduces the denominator by more than the amount it reduces the nominator in the formulas of multipliers (for proof see Technical Appendix). Thus, e.g. for $\alpha = 0.67$ the spending multiplier with non-zero markup is 2.56 instead of 2.43 with zero markup. Also, for smaller α, the difference between the size of multipliers with zero and non-zero markup is even more pronounced.

7 Conclusion

This paper showed that the baseline New Keynesian model employed by Eggertsson (2010a) delivers large multipliers in case of zero nominal interest *only if* the deflationary shock—that makes the zero bound binding on the nominal interest—and the accompanying transitory fiscal measure is very persistent. In particular, multipliers are large when the fiscal shock lasts for at least ten (six) quarters in case of specific (economy-wide) labor market. When the zero bound is not binding a Taylor rule is operative which implies multipliers that is slightly lower than one as shown, for example, by Woodford (2010). We also demonstrated that even straightforward extensions of Eggertsson (2010a) like positive government spending (instead of zero) in steady-state and decreasing returns (instead of constant returns) in technology sizeably decrease the size of government spending multiplier when nominal interest is zero. Christiano (2010) expressed his concern about the quantitative relevance of the labor tax hike multiplier of Eggertsson (2010a). Here, we also demonstrated that even small departures from the assumptions of Eggertsson (2010a) can cast doubt on the quantitative nature of the other multipliers like spending increase or sales tax cut as well. Based on our calculations we are skeptic about relevance of the policy conclusions of Eggertsson (2010a) and Christiano et al. (2010).

8 Appendix

Proof. Following Eggertsson (2010a) let us employ the labor market equilibrium. Combining the loglinear Euler, NKPC and market clearing equations we obtain the inverse labor demand curve:

$$\hat{W}_S = \Lambda\mu\phi^{-1}\hat{N}_S - \Lambda(1-\mu)^{-1}r_S^e - \Lambda\check{\sigma}^{-1}\hat{G}_S + \Lambda\chi^S\hat{\tau}_S^S - \Lambda\chi^A(1-\mu)^{-1}\hat{\tau}_S^A \tag{11}$$

where $\Lambda \equiv \frac{1-\beta(1-\mu)}{\kappa\psi}$. Similarly, let us substitute the loglinear market clearing for consumption into the loglinear intratemporal condition to arrive at the inverse labor supply:

$$\hat{W}_S = \left[\frac{\omega\phi + \check{\sigma}^{-1}}{\phi}\right]\hat{N}_S + \chi^W\hat{\tau}_S^W + \chi^S\hat{\tau}_S^S - \check{\sigma}^{-1}\hat{G}_S. \tag{12}$$

The value of κ does not influence labor supply. However, it enters labor demand through Λ. First, we show that the value of κ in case of homogenous labor market (denoted as κ^{hom}) is higher than the κ under heterogenous labor market (κ^{het}):

$$\kappa^{\text{het}} < \kappa^{\text{hom}}. \tag{13}$$

Using the definition of κ^{het} and κ^{hom} we can write:

$$\kappa^{\text{het}} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi\omega+\vartheta+(\phi-1)}{1+\omega_y\theta}; \quad \kappa^{\text{hom}} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi\omega+\vartheta+(\phi-1)}{1+(\phi-1)\theta},$$

where the difference between κ^{het} and κ^{hom} lies in their denominator. Further, we observe that

$$\omega_y \equiv \phi(1+\omega)-1 > (\phi-1)$$

is always true as $\omega > 0$. Then, it follows that inequality (13) holds and the slope of the labor demand under heterogenous factor market is higher than the one under homogenous labor market

$$\left(\frac{\partial \hat{W}_S}{\partial \hat{N}_S}\right)_{\text{hom}} < \left(\frac{\partial \hat{W}_S}{\partial \hat{N}_S}\right)_{\text{het}}$$

as

$$\Lambda^{\text{hom}} < \Lambda^{\text{het}}.$$

A demand shock that shifts labor demand (and also labor supply), is less effective if the labor demand is steep. As shown above the specific labor market assumption delivers a labor demand which is steeper than its economy-wide counterpart.

In the next we argue that the rightward shift in labor demand under specific labor market is larger than the same under economy-wide one. To do so, it is necessary to realise that a fiscal shock (a rise in spending or a cut in sales tax) that directly affects demand (i.e. it enters the demand) shift out labor demand more under specific and less under economy-wide labour market structure by studying the coefficients on government spending (or sales tax) in the labor demand (LD) and supply (LS) equations (11) and (12) we find:

$$(\breve{\sigma}^{-1})^{LS} < (\Lambda\breve{\sigma}^{-1})^{LD-\text{hom}} < (\Lambda\breve{\sigma}^{-1})^{LD-\text{het}}.$$

The latter inequality shows that it is the labor supply (LS) which moves the least and labor demand with heterogenous factor market ($LD-\text{het}$) which moves the most to the right on figure (1) with the shift in labor demand under homogenous factor market ($LD-\text{hom}$) in between after an expansionary fiscal shock. Thus, the rise in labor demand due to a fiscal stimulus leads to higher output produced under economy-wide labour market with a correspondingly higher multiplier than the one of specific factor market. ■

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Table 4: Summary of Multipliers—Separable Preferences

Multipliers	Constant Returns (CRS)		Decreasing Returns (DRS)	
	$i_t > 0$	$i_t = 0$	$i_t > 0$	$i_t = 0$
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g = 0$	0.46	2.28	0.44	1.94
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5208	1.81	0.5	**1.63**
			[0.61|0.67]	[1.74|1.08]
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0815	-1.02	0.04	-0.41
			[0.07|0.04]	[-0.32|-0.03]
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.38	1.87	0.37	1.6
			[0.40|0.44]	[1.14|0.71]
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0104	-0.10	-0.01	-0.06

Remarks to Table 4: Gray cells indicate the extensions by this paper.
White cells contain the values calculated by Eggertsson (2010a). g is the steady-state government speding-GDP ratio, i is the nominal interest rate. Un-bracketed values are derived under the assumption of heterogenous labor market. Values # and ♮ in brackets, [#|♮], stand for homogenous (#) and heterogenous (♮) labor market with a persistence parameter $\mu = .8$ chosen so that multipliers in $i = 0$ case exist under both types of labor market. Comparison not reported for capital tax as no striking difference. Also omitted for CRS and for $g = 0$ where determinacy for $i = 0$ is limited to relatively low values of μ that may not be empirically relevant.

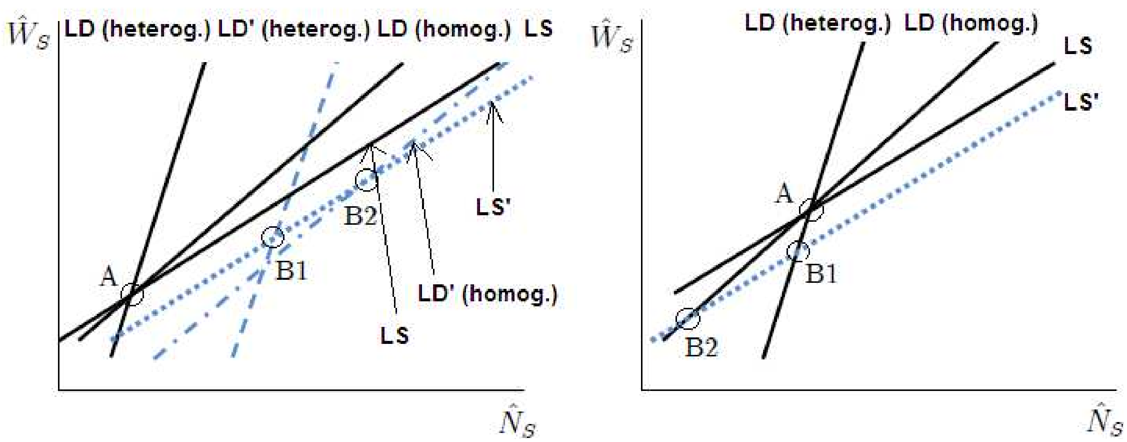

Figure 1: Left panel: an increase in government spending. Right panel: a decrease in labor tax.

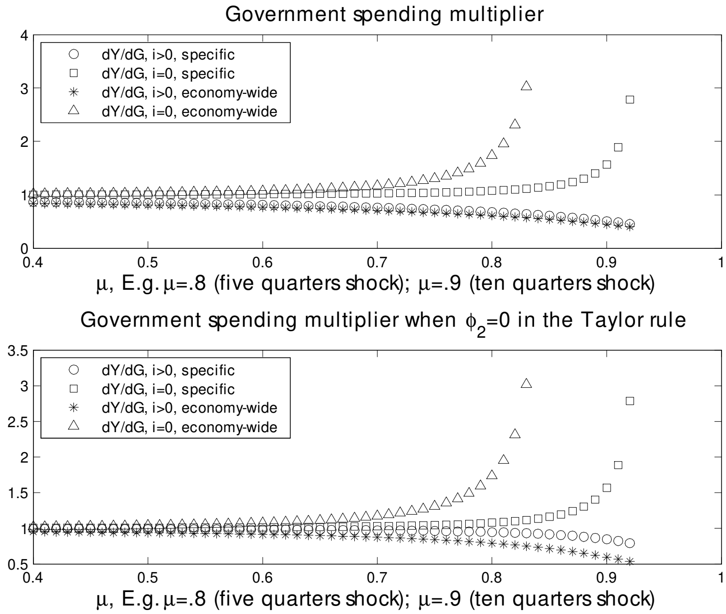

Figure 2: Government Spending Multipliers

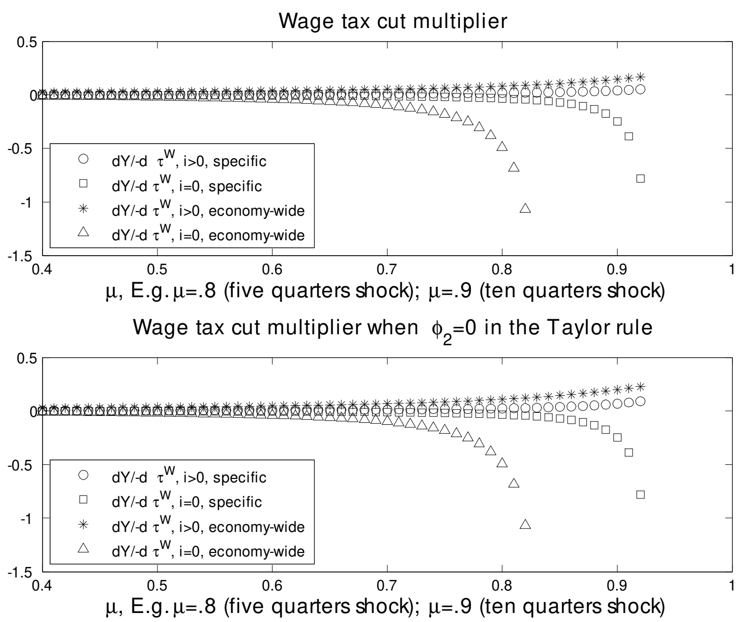

Figure 3: Wage tax cut multiplier

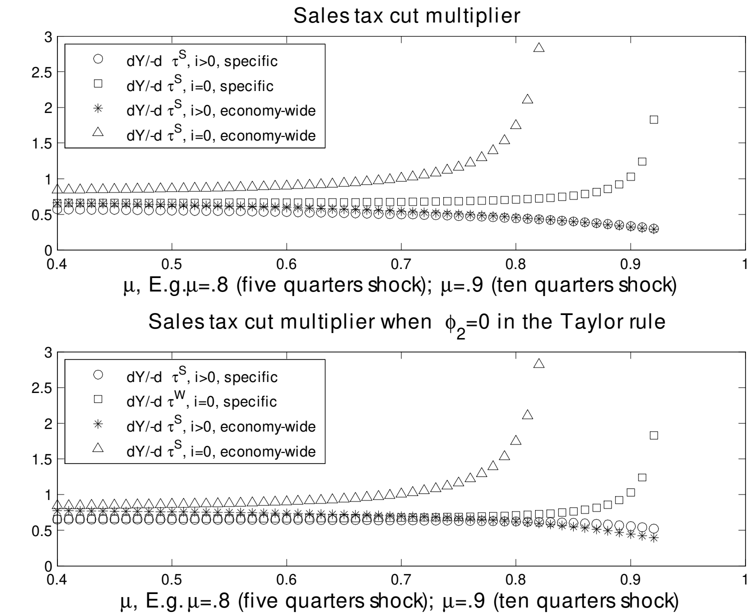

Figure 4: Sales tax cut multipliers

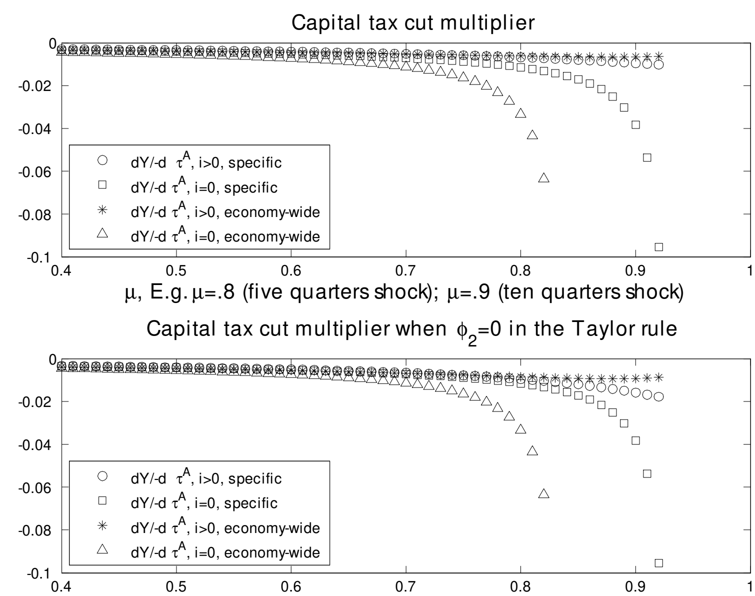

Figure 5: Capital tax cut multiplier