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The Single-Mindedness Theory: Micro-foundation and Applications to Social Security Systems

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Abstract

In this paper I will analyse the redistribution of income amongst two generations using the Single-mindedness Theory. I will introduce a new expression for the balanced-budget constraint, no longer based on lump-sum transfers as in the traditional literature, but rather on more realistic labour income taxation. Since the Government has to clear the budget, some generations obtain a benefit, whilst some other must pay the entire cost of social security systems. I will demonstrate that generations which are more single-minded on leisure are the most better off since they are more able to capture politicians in the political competition. Furthermore, it could be the case that candidates are not forced to undertake the same policies in equilibrium and I will demonstrate that this result holds only once an endogenous density function for individual preferences for politicians is considered.

We work in order to have leisure (Aristotle)

1 Introduction

The participation to the labour force of the older persons in the U.S. labour market has been steadily declining over the last century. If the labour force participation of men aged 65-69 was around 60% in the 50's, the same figure had fallen to 26% in the 90's [17]. In many OECD countries, workers withdraw from the labour market well before the official retirement age. Eventually this long-term decline, associated with an increase in life expectancy, has led to a considerable increase in retirement years. Otherwise, the Government expenditure for social security has been skyrocketing and so has been the percentage of workers covered by the system. This situation runs into risk to become financially unsustainable over the next years, unless governments undertake structural reforms as suggested by many economists (see Feldstein & Liebman [20] amongst the others).

Over the last few years, the economic literature has been trying to give plausible explanations to this strong change in the old workers' lifestyle. According to an OECD survey [42] financial incentives embedded into public pensions and other assistance schemes pull old workers into retirement. Nevertheless, the OECD makes a distinction between *pull factors of retirement* and *push factors of retirement*. The former include all of those financial benefits that incentive workers to anticipate their retirement, whilst the latter refer to negative perceptions by old workers about their ability or productivity and to socio-demographic characteristics.

In this paper I will take the distance from the OECD's view, which considers financial benefits as a *pull factor* which reduces the amount of work. I suggest that preferences of workers for leisure shape the characteristics of modern social security systems. Thus, generosity of governments' transfers is not exogenously given but it is rather the effect of a precise political mechanism; this is driven by old workers who use their political power to obtain what they need to finance their retirement years.

To explain the early retirement phenomenon, I will use an overlapping generation model (OLG) model which considers a society divided into two groups of workers: the old and the young. I will assume that there is a political competition between two candidates who must choose effective marginal tax rates on labour in order to maximize the probability of winning elections.

The core assumption of the model is based on the idea of "single-mindedness", introduced by Mulligan & Sala-i-Martin [40]. They assumed that the old prefer leisure more than the young; this structure of preferences would explain why the old require (and eventually obtain) more generous transfers from the government and why social security expenditures have been increased so much over the last decades. They adopted an OLG model where society is divided into old and young workers and showed that

retired elderly can concentrate on issue that relate only to their age
such as the pension or the health system

while the young have to choose amongst

age-related and occupation issues

Eventually, they concluded,

the elderly are politically powerful because they are more single-
minded and (...) more single-minded groups tend to vote for larger
social security programs that benefit them

According to this theory the group of old workers, because more single-minded, would have a greater power of influence over politicians and they are more able to drive the optimal taxation (a sort of tyranny of the elder or "Gerontocracy", to quote authors).

Indeed, neither Demographics nor the need for an assistance would explain the skyrocketing increase in the governments' expenditure for social security

systems and the broad reduction in retirement age over the last decades, but preferences of the old for leisure would provide a more suitable explanation to this upward trend. In a recent work, Diamond [17], attempting to describe the linkage between the social security system and the retirement in the U.S., wrote in his conclusions:

there is clear evidence from both previous work (...) that the broad structure of the SS program influences retirement timing. Evidence on the effects of variation in the benefits provided by this program is less clear, however.

In particular, I will assume that the Government has to decide how to divide the revenues generated by the taxation of the two groups. In doing this, it exploits a balanced budget constraint which is based on (distortionary) labour income taxation. Eventually, I will demonstrate that the older generations obtain a higher tax credit (or a reduction of the effective marginal tax rate) than the younger generations and that they get a higher level of leisure. A situation which is consistent with the old's needs, since their preferences are more oriented toward retirement than toward work. The work also explains the importance of single-mindedness of social groups and the role of preferences of individuals in political competition. The more single-minded a group, the higher is its political power, captured by a density function which is assumed to be monotonically increasing in the level of leisure. Since more single-minded groups are, other things being equal, more politically powerful, they are more able to obtain favourable policies by political candidates in equilibrium.

2 The basic model

I consider an OLG model, where each generation lives for two periods: the *youth* and *old* age. At any period of time, the generation of youths coexists with the generation of the elderly. At the beginning of the next period, the elderly die, the youths become old and a new generation of youths is born. As a consequence, there are two overlapping generations of people living at any one time. Generations are unlinked, meaning that for whatever reason, a generation does not leave any bequest to another generation. Individuals consume all the available income earned at a given period of time; thus, it is not possible neither to save nor to borrow money.

Then, at time $t = 0, \dots, +\infty$, let a continuum of voters of size one be partitioned into two generations of workers $I = t - 1, t$. The *old* represent the generation born at time $t - 1$ and it is denoted by $t - 1$ whilst the *young* represent the generation born at time t and it is denoted by t . The two generations have same size, which does not change over time¹. A single worker is denoted by $i \in [0, \frac{1}{2}]$.

Each worker has to decide how to divide his total endowment of time T between work, $L_t^i > 0$ and leisure, $l_t^i > 0$. If leisure is almost equal to the total endowment of time, I assume that the worker retires and gets a benefit (i.e. a pension).

The component of every voter's welfare depends on fiscal policies chosen by two political candidates $j = A, B$ which affect his consumption and which is known by both parties, whilst the other component of welfare, which derives from personal attributes of candidates, is only imperfectly observed by parties. Both candidates have an ideological label (i.e. they are seen as Democrats or Republicans), exogenously given. In other words, I assume that individuals' preferences for consumption are perfectly visible, whilst other political aspects such as ideology are not (Linbeck & Weibull's *stochastic heterogeneity*). The deterministic component of a worker's welfare is captured by a quasi-linear utility function in consumption and leisure, whilst the stochastic component is captured by the expression $D^A \cdot (\xi^{i,I} + \zeta)$, where $D^A = 1$ if candidate A wins elections and $D^A = 0$ if candidate B wins elections. The term $\zeta \lesseqgtr 0$ reflects candidate A's general popularity amongst the electorate and it is only realized between the announcement of parties' policy vector and elections. It is not idiosyncratic and it is uniformly distributed on the interval $(-\frac{1}{2h}, \frac{1}{2h})$ with mean zero and density h , known by the two candidates and normalized to one for simplicity. Otherwise, the term $\xi^{i,I} \lesseqgtr 0$ represents an individual component of preferences for candidate A. It is known by political candidates and uniformly distributed on $(-\frac{1}{2s^I}, \frac{1}{2s^I})$, again with mean zero and density s^I .

A representative old worker at time t has the following utility function:

$$U_t^{i,t-1} = c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + D^A \cdot (\xi^{i,t-1} + \zeta) \quad (1)$$

¹Note that this is a different with respect to Profeta who assumes that the two groups have different sizes.

where c_t^{t-1} is consumption and $\psi^{t-1} \in [0, 1]$ is a parameter representing the intrinsic preference of the old worker for leisure.

The old worker consumes all his income:

$$c_t^{t-1} = w^{t-1}(1 - \tau_t^{t-1})(T - l_t^{t-1}) \quad (2)$$

where w^{t-1} is the unitary wage per hour worked, $\tau_t^{t-1} := \tau(1 - a_t^{t-1})$ the effective tax rate on labour income equal to the nominal tax rate $\tau \in [\tau^{\min}, \tau^{\max}]$ net of the tax credit $a_t^{t-1} \in [a_t^{t-1 \min}, a_t^{t-1 \max}]$, with $a_t^{t-1 \min} < 1$ and $a_t^{t-1 \max} > 1$. I assume that τ is equal for every generation and steady over time. τ^{\min} and τ^{\max} denotes the minimum and maximum legal tax rates, whilst a^{\min} and a^{\max} the minimum and maximum tax credits, both written in the budget law.

Similarly, preferences of a representative young worker t are given by the following utility function:

$$U_t^{i,t} = c_t^t + \psi^t \log l_t^t + \beta(c_{t+1}^t + \psi^{t-1} \log l_{t+1}^t) + D^A \cdot (\xi^{i,t} + \zeta) \quad (3)$$

subject to

$$c_t^t = w^t(1 - \tau_t^t)(T - l_t^t) \quad (4)$$

$$c_{t+1}^t = w^t(1 - \tau_{t+1}^t)(T - l_{t+1}^t) \quad (5)$$

where β is a discount factor and $a_t^t \in [a_t^{t \min}, a_t^{t \max}]$ the tax credit, with $a_t^{t \min} < 1$ and $a_t^{t \max} > 1$.

Condition $a_t^{t \min} < 1$, $a_t^{t \max} > 1$ makes a redistribution program feasible since, as we see later in studying the budget constraint of the government, it allows a generation to obtain positive transfers paid by the other generations.

2.1 Different preferences for leisure

I assume that the old and the young are identical in every respect except one

Axiom 1 *the intrinsic value of the old workers for leisure is assumed to be greater than the young workers; that is, $\psi^{t-1} > \psi^t$.*

This axiom is supported by the empirical evidence. In fact, the economic science has produced many works which provide possible explanations to the existence of a difference in preferences. Moreover, over the last years, other social sciences like Sociology and Psychology have added some very useful contributions. I distinguish the *economic reasons* from the *non-economic reasons*.

The economic reasons are summarized in the work by Mulligan & Sala-i-Martin (1999).

Differences in labour Productivity. Since the labour productivity is declining in age, the old are less productive than the young and, as a consequence, they earn a lower wage. This theory would explain the willingness by the old to retire: less productive workers in the labour market find profitable to devote relatively more of their time and effort to the political sector as to gain monetary transfers that they would not get if they relied on labour market. Nevertheless, for the theory to hold it is important to assume that leisure time devoted to political activities is a *normal good*. That is, an increase in the total leisure time provokes an increase in leisure time devoted to political activities, due to the income effect. Of course these assumptions are not entirely accepted in the literature. In particular, evidence about the effects of age on productivity and wages does not lead to clear-cut conclusions. For example, a work by Skierbekk ([47]) found that individual job performance decreases from around 50 years of age and that productivity reductions at older ages are particularly strong for work tasks where problem solving, learning and speed are needed, while in jobs where experience and verbal abilities are important, older individuals' maintain a relatively high productivity level.

Differences in Human Capital Accumulation. The young are more engaged in self-financed human capital accumulation while they work than the old. As a consequence, the value of time for the young may be higher than their average hourly wage (see Stafford and Duncan [48]).

Long-term employment contracts. The empirical evidence shows that due to the Lazear-type contracts, labour productivity for workers aged 60+ is significantly lower than wages.

As for the non-economic reasons, I refer to a work by Hershey, Henkens and Van Dalen [25]. In comparing the Dutch with the U.S. Social Security System, the authors discovered that “the Americans had significantly longer future time perspectives, higher level of retirement goal clarity and they tended to be more engaged in retirement planning activities”. Thus, these findings are able to explain the existence of socio-cultural differences in the preferences for retirement. They go on affirming that “American workers think, prepare and save more for retirement... beginning in early adulthood”, focalizing on the difference among societies, where there exists a major difference in financial responsibility, different level of uncertainty for future pension payouts and different psychological pressures. Finally, in concluding that the success of political initiatives depends in part on “changing the dimensions of the psyche that motivate individuals to adaptively prepare for old age”, they implicitly recognize that preferences of individuals for leisure may endogenously change over time, again due to cultural and psychological issues.

2.2 Definition of Single-Mindedness

I introduce now two important definitions:

Definition 2 *a generation A is said to be more single-minded than a generation B with respect to leisure if its preferences for leisure are higher than preferences*

of B . That is if $\psi^A > \psi^B$.

Definition 3 a generation A is said to be more politically powerful than a generation B if its density is higher than B 's. That is if $s^A > s^B$.

The political power of a generation is represented by its ability of influencing candidates' choices, when they have to take decisions about the optimal policy vector. In traditional probabilistic voting models this power is expressed by the density function which captures the distribution of the electorate.

Axiom 4 the density function of a generation is monotonically increasing in the level of leisure. That is $s^I = s(l)$, with $\frac{\partial s}{\partial l} > 0$.

Note, that this axiom brings something new with respect to previous probabilistic voting models, where the density function is only a constant and does not depend on anything.

In the resolution of the game it will be demonstrated that $l^I = l(\psi)$ and $\frac{\partial l}{\partial \psi} > 0$; that is, leisure is monotonically increasing in preferences for leisure. This result, jointly read with axiom 2, allows us to show that, other things being equal, *an increase in the single-mindedness of a generation entails an increase in its political power*. To demonstrate this, it is sufficient applying the chain rule

$$\text{to obtain } \frac{ds^I}{d\psi^I} = \overbrace{\frac{\partial s^I}{\partial l^I}}^{>0} \cdot \overbrace{\frac{\partial l^I}{\partial \psi^I}}^{>0} > 0.$$

This result says that the linkage between preferences of a generation and its political power passes through an increase in the level of leisure which the density depends upon. In other words, it must be the case where over leisure, different generations have different preferences for political parties. A greater level of single-mindedness entails higher values of the density function which tends to give to the distribution a ticker shape. Figure 1 shows an example of different distributions amongst cohorts.

[FIGURE 1 HERE]

The figure shows how distributions of the two generations depend on leisure and that the old generation (red) has a ticker distribution than the young generation (orange). The distribution is assumed to be uniform. The broadness of the interval $(-\frac{1}{2s^I}, \frac{1}{2s^I})$ is not fixed, because s is a monotonically increasing function of leisure, and higher levels of leisure increase s reducing the broadness of the interval. As a result, we obtain a higher concentration of swing voters around ζ .

Figure 2 shows the effects of an increase in ψ within a generation. A change in ψ (from ψ to ψ' , with $\psi' > \psi$) entails an increase both in l and s . Since s stands at the denominator of the expression representing the endpoints of the interval, the broadness of the interval reduces and the distribution becomes thicker.

[FIGURE 2 HERE]

2.3 The Government

I consider two self-interested candidates which choose an element $q_t^j = \{\tau_t'^{t-1}, \tau_t'^t\}$, encompassing the two effective tax rates $\tau_t'^{t-1}$ and $\tau_t'^t$, from the (common) strategy set $Q \subset \mathbb{R}^2$.

Furthermore, I introduce the budget constraints of the Government:

$$\Upsilon^j \equiv \frac{1}{2}\tau_t'^{t-1}(T - l_t^{t-1})w_t^{t-1} + \frac{1}{2}\tau_t'^t(T - l_t^t)w_t^t = 0 \quad (6)$$

where $\frac{1}{2}\tau_t'^{t-1}(T - l_t^{t-1})w_t^{t-1}$ represents total revenues generated by the taxation of the old and $\frac{1}{2}\tau_t'^t(T - l_t^t)w_t^t$ total revenues generated by the taxation of the young.²

Since revenues are proportional to the amount of labour, taxation entails inefficiencies, since it distorts workers' decisions on the amount of labour supplied.

As suggested by Lindbeck and Weibull, I assume the existence of a *balanced-budget redistribution* where the government cannot redistribute more resources than those available in the economy, and cannot use tax revenues for any other purpose than redistribution – so that the condition $\Upsilon^j = 0$ says that revenues obtained via labour taxation are only used to redistribute wealth amongst cohorts. To avoid the case in which a difference in wage levels is the solely responsible for the existence of retirement I impose that wages are equal for every generation: $w_t^{t-1} = w_t^t = w$. Furthermore, without loss of generality, I normalize the wage rate to the unity.

The advantage of adopting a budget constraint with distortionary taxation like that I use is realism. Economists like Profeta [43] and Mulligan & Sala-i-Martin [37] formalized models in an attempt to explain the linkage between intergenerational redistribution and early retirement; nevertheless, they seem to be affected by a fundamental problem due to the use of lump sum transfers; in Mulligan & Sala-i-Martin “an interest group may tax its members with a labour income tax and distribute the proceeds to them in a lump sum fashion”; Profeta used a lump-sum mechanism to transfer wealth both within the cohort and amongst different cohorts. Finally, also Linbeck and Weibull [35] study a redistributive model with political competition where gross incomes are fixed and known and, hence, “first-best (individual) lump-sum redistributions are in principle feasible”. A redistributive system such as that all of these models assume, with the presence of lump-sum taxation, does not exist in the real world. All the most recent studies on characteristics of social security systems around the world show that the income taxation is the only source which finances social expenditures. For instance, Diamond found out that “The Social

²Note that Υ^j is a strictly concave function in a_t^I . The first order condition gives $\frac{\partial \Upsilon^j}{\partial a_t^I} = -\frac{\tau(T+(a_t^I-1)\tau)^2 - \psi^{I2}}{2(1+(a_t^I-1)\tau)^2}$ and the second order condition gives $\frac{\partial^2 \Upsilon^j}{\partial a_t^{I2}} = -\frac{\tau^2 \psi^{I2}}{2(1+(a_t^I-1)\tau)^3} < 0$.

Security system in the U.S. today is financed by a payroll tax which is levied on workers and firms equally”, whilst Mulligan and Sala-i-Martin, adopting a cross-section analysis of 89 countries, recognized that the 96% of social security programs are financed with payroll taxes.

2.4 The political competition

2.4.1 The Lindbeck & Weibull framework

As said before voters’ welfare depends both on a deterministic and on a stochastic component. The presence of uncertainty, captured by variables related to preferences for political candidates, assures the existence of a NE in a multi-dimensional space (*see* Lindbeck & Weibull and Dixit & Londregan [18]). In the absence of that candidates would be perfectly able to observe how voters cast their ballots and then each voter would abruptly switch support toward the candidate which promises him the most favourable policy. In such a case the non-existence of an equilibrium is due to the fact that any chosen policy would be beaten by another policy. Therefore, traditional Downsian electoral competition models lead to a negative result where no Condorcet winner exists. Probabilistic voting models, instead, smooth out this discontinuity because a small change in the policy chosen by a candidate entails only a small change in the probability of support from voters and not a total loss of support. Smoothing out the discontinuity in the probability of winning opens up the possibility that an equilibrium returns to exist.

Each voter in generation I votes for candidate A if and only if candidate A’s policy vector provides him with a greater utility than that provided by candidate B’s policy vector. That is i votes for A if and only if:

$$V^I(q^A) + \zeta + \xi^{i,I} > V^I(q^B) \quad \forall i \quad (7)$$

where $V^I(\vec{q}^j)$ represents the indirect utility function which generation I obtains under the vector of policies chosen by candidate j .

2.4.2 The role of swing voters

In each generation there are some *swing voters*, represented by all of those individuals who are indifferent between voting for candidate A or B. For these voters the following condition holds:

$$\xi^I = V^I(q_t^B) - V^I(q_t^A) - \zeta \quad (8)$$

Otherwise, all voters with $\xi^{i,I} < \xi^I$ vote for candidate B and all voters with $\xi^{i,I} > \xi^I$ vote for candidate A.

Swing voters are pivotal, since even a little change in the policy vector may force them to vote one candidate rather than another. Suppose to start from a situation where A’s policy, q^A , is exactly equal to B’s policy, q_t^B ; a candidate

knows that, should it deviate from that policy, some swing voters would be better off (and vote for him) whilst some others would be worse off (and vote against him). Thus, in choosing a policy, a candidate should calculate the number of swing voters which he gains and compare it with the number of swing voters he loses; a change in policy should be made if and only if a candidate evaluates that the number of swing voters gained outweighs the number of swing voters lost.

I denote the expected share of votes for candidate A in generation I with:

$$\pi^{A,I} = \frac{1}{2}s^I[\xi^I + \frac{1}{2s^I}] = \frac{1}{2}s^I\xi^I + \frac{1}{4} \quad (9)$$

and substituting (8) into (9) I obtain:

$$\pi^{A,I} = \frac{1}{2}s^I[V^I(q_t^B) - V^I(q_t^A) - \zeta] + \frac{1}{4} \quad (10)$$

The total number of expected votes candidate A gets must sum the expected number of votes of the two groups:

$$\pi^A = \frac{1}{2}s^{t-1}[V^{t-1}(q_t^B) - V^{t-1}(q_t^A) - \zeta] + \frac{1}{2}s^t[V^t(q_t^B) - V^t(q_t^A) - \zeta] + \frac{1}{2} \quad (11)$$

Notice that π^A is a random variable since it depends on ζ which is also random. Candidate A's probability of winning is simply the probability to obtain the simple majority of votes:

$$p^A = \Pr[\pi^A \geq \frac{1}{2}] = \Pr[\frac{1}{2} \sum_I s^I [V^I(q_t^B) - V^I(q_t^A) - \zeta] + \frac{1}{2} \geq \frac{1}{2}]$$

and rearranging terms we obtain:

$$p^A = \Pr[\pi^A \geq \frac{1}{2}] = \Pr[\sum_I s^I [V^I(q_t^B) - V^I(q_t^A)] \geq \zeta \sum_I s^I]$$

Denoting $\frac{1}{2} \sum_I s^I = s$ and $\frac{1}{2s} \sum_I s^I [V^I(q_t^B) - V^I(q_t^A)] = \widehat{\zeta}$ I obtain:

$$p^A = \Pr[\pi^A \geq \frac{1}{2}] = \Pr[\widehat{\zeta} \geq \zeta]$$

Finally, we also take into account the distribution of the other random variable ζ to write a final expression for the probability of winning:

$$p^A = \Pr[\pi^A \geq \frac{1}{2}] = [\widehat{\zeta} + \frac{1}{2}]$$

Similarly, candidate B wins with probability $p^B = 1 - p^A$.

Notice that p^j may be written as the sum of probability of winning with respect to single generations, weighted by the numerosity of the generation, equal to $\frac{1}{2}$; that is $p^j = \frac{1}{2}p_t^j + \frac{1}{2}p_{t-1}^j$, where p_I^j indicate the probability of winning for candidate j for generation I .

Each candidate maximizes *expected plurality*; that is a candidate wants either to maximize the expected margin of victory or to minimize the expected margin of loss, given the other candidate's policy vector.

Define

$$P\ell_I^j(q_t^A, q_t^B) = \frac{1}{2} \left[p_I^j(q_t^A, q_t^B) - p_I^{-j}(q_t^A, q_t^B) \right] \quad (12)$$

the expected plurality for candidate j at a particular (q_t^A, q_t^B) from a given generation I and

$$P\ell^j(q_t^A, q_t^B) = \frac{1}{2} \sum \left[P\ell_I^j(q_t^A, q_t^B) \right] \quad (13)$$

the expected plurality for candidate j .

We have now all the elements to define a two-person, *zero-sum* game Γ where the two candidates $j = A, B$ are players, the two policy vectors $q_t^j \in Q \subset \mathbb{R}^2$ the strategies and expected pluralities $P\ell^j(q_t^A, q_t^B) : Q \times Q \rightarrow \mathbb{R}$ the payoffs. Γ is written as $(Q, Q; P\ell^A, P\ell^B)$.

Definition 5 A Pair $(q_t^{A*}, q_t^{B*}) \in Q \times Q$ is called a (*pure strategy*) Nash equilibrium (NE) of Γ if and only if $P\ell^j(q_t^A, q_t^B) \leq P\ell^j(q_t^{A*}, q_t^{B*}) \leq P\ell^j(q_t^{A*}, q_t^B)$, $\forall q_t^A, q_t^B \in Q$ which satisfy the budget constraint.

It is also useful to remind that in a two-person, zero-sum game a pair of policies $(q_t^{A*}, q_t^{B*}) \in Q \times Q$ is an equilibrium if and only if it is a saddle point for the game

$$\Gamma = (Q, Q, P\ell^A(q_t^A, q_t^B), -P\ell^A(q_t^A, q_t^B))$$

2.5 Timing of the game

I consider a three-stage game where candidates aim to maximize the number of votes³.

In the first stage of the game, the two candidates, simultaneously and independently, announce (and commit to) their policy vectors.

In the second stage elections take place. A candidate wins elections if and only if obtains the majority of votes; in the case of a tie a coin is tossed to

³Lindbeck and Weibull 1987 and Dixit and Londregan 1996 demonstrated that the Nash equilibrium obtained if candidates maximize their vote share is identical to that obtained when candidates maximize their probability of winning

choose the winner. Furthermore, I assume that each party prefers to stay out from the competition than to enter and lose, that prefers to tie than stay out and it prefers to win than to tie.

Finally, in the third stage, workers choose their leisure, given the level of tax credits chosen by the government.

2.6 Calculate the equilibrium

I solve the game by backward induction, starting from the final stage.

A representative old worker solves the following optimization problem:

$$\begin{aligned} \max_l U_t^{i,t-1} &= c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + D^A \cdot (\xi^{i,t-1} + \zeta) \\ \text{s.t. } c_t^{t-1} &= (1 - \tau_t'^{t-1})(T - l_t^{t-1}) \end{aligned}$$

Solving with respect to l_t^{t-1} I obtain an expression for the optimal amount of leisure:

$$l_t^{t-1*} = \frac{\psi^{t-1}}{1 - \tau_t'^{t-1}} \quad (14)$$

and substituting (14) into (1) I obtain an expression for the Indirect Utility Function:

$$V_t^{t-1} = T(1 - \tau_t'^{t-1}) - \psi^{t-1} + \psi^{t-1} \log \psi^{t-1} - \psi^{t-1} \log(1 - \tau_t'^{t-1}) + D^A \cdot (\xi^{i,t-1} + \zeta) \quad (15)$$

with $1 - \tau(1 - a_t^{t-1}) > 0 \implies a_t^{t-1} > 1 - \frac{1}{\tau}$

I do the same for the representative young worker:

$$\begin{aligned} \max U_t^{i,t} &= c_t^t + \psi^t \log l_t^t + \beta(c_{t+1}^t + \psi^{t-1} \log l_{t+1}^t) + D^A \cdot (\xi^{i,t} + \zeta) \\ c_t^t &= (1 - \tau_t')(T - l_t^t) \\ c_{t+1}^t &= (1 - \tau_{t+1}') (T - l_{t+1}^t) \\ l_t^{t*} &= \frac{\psi^\tau}{1 - \tau_{t+1}'} \end{aligned} \quad (16)$$

$$l_{t+1}^{t*} = \frac{\psi^{\tau-1}}{1 - \tau_{t+1}'} \quad (17)$$

$$\begin{aligned} V_t^t &= T(1 - \tau_t') - \psi^t + \psi^t \log \psi^t - \psi^t \log(1 - \tau_t') \\ + \beta &\left(T(1 - \tau_{t+1}') - \psi^{t-1} + \psi^{t-1} \log \psi^{t-1} - \psi^{t-1} \log(1 - \tau_{t+1}') \right) + D^A \cdot (\xi^{i,t} + \zeta) \end{aligned} \quad (18)$$

Comparative statics shows that the optimal level of leisure is increasing in preferences of groups for leisure and decreasing in the amount of tax credits. That is $\frac{dl_t^{I*}}{d\psi^I} = \frac{1}{1-\tau_t^I} > 0$ and $\frac{dl_t^{I*}}{da_t^I} = -\left(\frac{\tau\psi^I}{1-\tau_t^I}\right) < 0$.

Analysing the indirect utility functions we may see that there are two effects coexisting together: a *tax effect*, $T(1-\tau_t^I)$, and a *leisure effect*, $-\psi^I \log(1-\tau_t^I)$.

What is the effect of an increase in the optimal tax credit on the wealth of an individual? At a first glance, one would be prone to answer that an increase in tax credits increases the individual's utility because the effective marginal tax rate reduces and the net-of-taxes labour income increases. But leisure effect says that an increase in tax credits reduces leisure, and eventually increases the utility. Therefore, the total effect on the welfare of an individual depends on which effect prevails.

In the second stage of the game

Proposition 6 *the political equilibrium is a tie.*

Proof. Candidates solve the following problem:

$$\begin{aligned} & \max_{\{a_t^{j-1}, a_t^j\}} \pi^j \\ & s.t. \Upsilon^j = 0 \end{aligned}$$

$j = A, B$

The set of First Order Conditions may be written as follows:

$$\left\{ \begin{aligned} \lambda^A &= \frac{\frac{\partial \pi^A}{\partial a_t^A}}{\frac{\partial \Upsilon^A}{\partial a_t^A}} \\ \lambda^A &= \frac{\frac{\partial \pi^A}{\partial a_t^{A-1}}}{\frac{\partial \Upsilon^A}{\partial a_t^{A-1}}} \end{aligned} \right. \quad (19)$$

$$\left\{ \begin{aligned} \lambda^B &= \frac{\frac{\partial \pi^B}{\partial a_t^B}}{\frac{\partial \Upsilon^B}{\partial a_t^B}} \\ \lambda^B &= \frac{\frac{\partial \pi^B}{\partial a_t^{B-1}}}{\frac{\partial \Upsilon^B}{\partial a_t^{B-1}}} \end{aligned} \right. \quad (20)$$

$$\Upsilon^j = 0$$

where λ^A, λ^B are the two Lagrange multipliers which may be interpreted as the per capita marginal gain in expected votes, with respect to a marginal shift in transfers. In equilibrium λ^A must be equal to λ^B , because the per capita marginal gain in expected votes should be equal for every candidate. Suppose it is not; then, the expected number of votes of a party could be improved without violation of the public budget constraint. As a consequence, it would mean that there exists an incentive for a candidate to increase transfers towards

those groups which promise a greater increase in the expected number of votes; as long as this incentive exists such a situation cannot be an equilibrium. ■

Conditions (19), (20) state that candidates choose tax credits until the marginal political cost (MPC), which represents the reduction in expected votes, of raising an additional dollar is equalized across cohorts. Hence, the politically optimal structure is that one which minimizes total political costs and clear the balanced budget constraint. The optimal solution is depicted in figure 3. The shape of tax revenues reminds the famous "Laffer curve" or rate-revenue relationships, shown in panels *a* and *b*. With respect to the traditional Laffer curve, these ones have a negative tract; this is typical in a pure redistribution model, because if one generation gets a positive transfer the other one must pay it. Lambdas measure the intensity with which political tastes react to a change in full income by reducing expected support. Different preferences for leisure and different economic and political reactions to taxation solve in different tax rates. Panel *c* shows the political equilibrium. The marginal political benefit (MPB) equates the sum of single MPBs expressed per dollar of expenditure. The equilibrium is a point where $R^t + R^{t-1} = 0$ and $MPC^* = MPB^* < 0$. Lagrange multipliers are negative because $\frac{\partial \pi^j}{\partial a_t^I} < 0$ and $\frac{\partial \Upsilon^j}{\partial a_t^I} > 0$. Instead, nothing can be said about the shape of π^j , because second order conditions have an indeterminate sign.⁴

Corollary 7 *In equilibrium $\widehat{\zeta} = 0$.*

Proof. *By proposition 6 the electoral equilibrium is a tie; then the probability of winning must be equal to $\frac{1}{2}$ for every candidate. Since we have defined $p^j = \widehat{\zeta} + \frac{1}{2}$, then $\widehat{\zeta}$ must be equal to zero. ■*

In the first stage candidates choose optimal policy vectors which are obtained from the resolution of the maximization problem.

Proposition 8 *A tie in elections may be achieved (i) either if policies converge (ii) or if a policy chosen by one candidate favours one group and a policy chosen by the other candidate favours the other group.*

Proof. From Corollary 7 $\frac{1}{2} \sum_I s^I [V^I(q_t^{B*}) - V^I(q_t^{A*})]$ is equal to zero. This may be achieved only in two ways. Either (i) when policies are convergent, $q_t^{A*} = q_t^{B*}$, which entails that $V^I(q_t^{B*}) = V^I(q_t^{A*})$; or (ii) when policies are divergent, $q_t^{A*} \neq q_t^{B*}$, and in this case the following condition must hold:

$$\frac{1}{2} s^t [V^t(q_t^{B*}) - V^t(q_t^{A*})] + \frac{1}{2} s^{t-1} [V^{t-1}(q_t^{B*}) - V^{t-1}(q_t^{A*})] = 0$$

which may be also written as:

$$\frac{1}{2} s^t [V^t(q_t^{B*}) - V^t(q_t^{A*})] = \frac{1}{2} s^{t-1} [V^{t-1}(q_t^{A*}) - V^{t-1}(q_t^{B*})]$$

⁴Second order conditions give $\frac{\partial^2 \pi^j}{\partial a_t^I} = \underbrace{\frac{\partial V_t}{\partial a_t^I}}_{\leq} \underbrace{s^{-1}}_{>0} \underbrace{\frac{\partial s^I}{\partial a_t^I}}_{<0} \underbrace{(1 + s^I s^{-1})}_{>0} + \underbrace{s^I}_{>0} \underbrace{s^{-1}}_{>0} \frac{\partial^2 V_t}{\partial a_t^I}$

■

Notice that:

1. if an equilibrium is achieved via a policy convergence, then it must be true that $\Pr_t^A = \Pr_{t-1}^A = \Pr_t^B = \Pr_{t-1}^B = \frac{1}{2}$.
2. if an equilibrium is achieved via a policy divergence, one of the following statements must hold: (i) either $\Pr_t^A = 1, \Pr_{t-1}^A = 0, \Pr_t^B = 0, \Pr_{t-1}^B = 1$, (ii) or $\Pr_t^A = 0, \Pr_{t-1}^A = 1, \Pr_t^B = 1, \Pr_{t-1}^B = 0$.

Proposition 9 *if $q_t^A = q_t^B = q_t$ then $P\ell^j(q_t, q_t) = 0$.*

Proof. Notice that if $q_t^A = q_t^B$, $V^{t-1}(q_t^A) = V^{t-1}(q_t^B)$ and $V^t(q_t^A) = V^t(q_t^B)$ and thus the probability of winning for the two candidates for generations I is equal to $\frac{1}{2}$. Therefore, $P\ell^j(q_t, q_t) = \frac{1}{2} [\frac{1}{2} - \frac{1}{2}] = 0$. ■

The problem is now to evaluate whether the equilibrium of the model is achieved via a convergence or a divergence of policies. I will provide a sufficient (but not necessary) condition which assures that an equilibrium is achieved via policy convergence. Instead, note that the classical Lindbeck and Weibull's monotonicity condition to the policy convergence in probabilistic voting models is not applicable. Appendix 1 demonstrates the non-applicability of monotonicity condition.

Proposition 10 *In a zero-sum game $q_t^{A*} = q_t^{B*} = q_t^*$.*

Proof. First of all, we have defined Γ as a zero-sum game, since $P\ell^B(q_t^A, q_t^B) = -P\ell^A(q_t^A, q_t^B)$. Suppose now that the pair $(q_t^{A\circ}, q_t^{B\circ}) \in Q \times Q$ is the electoral equilibrium of the game. Suppose also that $q_t^{A\circ} \neq q_t^{B\circ}$. We know by (9) that $P\ell^A(q_t^{B\circ}, q_t^{B\circ}) = 0$. Therefore, by the definition of Nash Equilibrium it must be

$$P\ell^A(q_t^{A\circ}, q_t^{B\circ}) > P\ell^A(q_t^{B\circ}, q_t^{B\circ}) = 0 \quad (21)$$

By definition of a zero-sum game we also know that $P\ell^B(q_t^{A\circ}, q_t^{A\circ}) = -P\ell^A(q_t^{A\circ}, q_t^{A\circ}) = 0$ and again by definition of Nash Equilibrium, it must be

$$P\ell^B(q_t^{B\circ}, q_t^{A\circ}) > P\ell^B(q_t^{A\circ}, q_t^{A\circ}) = 0 \quad (22)$$

Since $P\ell^B(q_t^{B\circ}, q_t^{A\circ}) = -P\ell^A(q_t^{B\circ}, q_t^{A\circ})$, this implies that $P\ell^A(q_t^{B\circ}, q_t^{A\circ}) < 0$. By 21, this implies that $P\ell^A(q_t^{B\circ}, q_t^{A\circ}) > P\ell^A(q_t^{B\circ}, q_t^{A\circ})$, a contradiction. Therefore, $q_t^{B\circ} = q_t^{A\circ}$. ■

Hence, in this model an equilibrium is achievable via a convergence of policies but the Lindbeck & Weibull monotonicity condition cannot be applied. The NE of the game is $(q_t^*, q_t^*, 0, 0)$.

Proposition 11 *The optimal tax credits are a function of the density and numerosity of both groups, of the nominal marginal tax rate, of the total endowment of time and of preferences of groups for leisure. That is:*

$$a_t^{Ij} = a \left(s \left(l \left(\psi^I, \tau \right) \right), s \left(l \left(\psi^{-I}, \tau \right) \right) \tau, T, \psi^I, \psi^{-I} \right)$$

Proof. see the Mathematical Appendix 2. ■

Thus, the political economy framework suggests that tax rates should be differentiated. Indeed, if the traditional normative approach suggests that a benevolent governments *should tax* less the poorest social groups, this political economy approach suggests that in a real world vote-seeker governments *tax* groups according to their ability to threaten politicians in an electoral competition.

A complete analytical solution to the maximization problem of the first stage is difficult to find because it is an hard task to understand which shape the value function has. Nevertheless, since Q is a compact set, if the value function is continuous in $[a_t^{I \min}, a_t^{I \max}]$ by the meaning of the Weierstrass theorem we are sure that a maximum exist⁵. Then it only remains to understand whether the optimum is an interior solution or stands at the endpoints of the interval. If the maximum is an interior solution, it must come out from the resolution of the first order conditions (see Appendix 2) which finds all the stationary points.

Proposition 12 *If the maximum is not an internal solution, then the NE is either $(a_t^{t \min A}, a_t^{t-1 \max A}, a_t^{t \min B}, a_t^{t-1 \max B})$ or $(a_t^{t-1 \min A}, a_t^{t \max A}, a_t^{t-1 \min B}, a_t^{t \max B})$, or both.*

Proof. Note that in order to balance the budget constraint, if the marginal tax rate for a generation is greater than one, the marginal tax rate for the other generation must be lower than one; otherwise the sum of the two tax revenues can never be equal to zero. Since we know that $a_t^{I \min j} < 1$ and $a_t^{I \max j} > 1$, solutions such as $(a_t^{t \min A}, a_t^{t-1 \min A}, a_t^{t \min B}, a_t^{t-1 \min B})$ and $(a_t^{t \max A}, a_t^{t-1 \max A}, a_t^{t \max B}, a_t^{t-1 \max B})$ are not achievable. Therefore we must conclude that the only possible solution must be either $(a_t^{t \min A}, a_t^{t-1 \max A}, a_t^{t \min B}, a_t^{t-1 \max B})$ or $(a_t^{t-1 \min A}, a_t^{t \max A}, a_t^{t-1 \min B}, a_t^{t \max B})$, or both. ■

This proposition has an important meaning. It says that, if an internal solution is not achievable, candidates must favour a generation and penalize the other generation as much as it is possible, choosing the highest and the lower tax rates in the set of possible choices.

Conjecture 13 *Tax credits are higher for the older generations.*

Proof. result obtained via numerical simulations. ■

Conjecture 14 *The older generations offer either a very low level of labour or retire at all, depending on the values which parameters assume, whilst the younger generations offer a greater amount of labour.*

⁵Weirstrass (or Extreme Value) theorem states that a continuous function on a compact set attains both a maximum and a minimum on the set. Note that the result gives only a sufficient condition for a function to have a maximum. If a function is continuous and is defined on a compact set then it definitely has a maximum and a minimum. The result does not rule out the possibility that a function has a maximum and/or minimum if it is not continuous or is not defined on a compact set

Proof. result obtained via numerical simulations. ■

Conjecture 15 *Tax revenues collected via the labour taxation of the younger generations are positive, whilst those of the older generations are negative.*

Proof. result obtained via numerical simulations. ■

Thus, a fiscal system where Leviathan governments take decisions helps older generations to the early retire. As a consequence, revenues collected from the taxation of the old are negative, whilst revenues collected with the taxation of the younger generations are positive and equal to the amount of pensions that the older receive. Thus, in this model there exists a net transfer from the younger to the older generations, suggesting that the former carry the burden of social security systems, whilst the latter gain a positive benefit.

3 A variant with altruism

The simple model described above is able to explain the very negative phenomenon of early retirement. It depicts an economic environment where politicians are captured by most single-minded groups. As long as candidates are self-interested and only aim to win elections, this political failure affects labour markets outcome. Of course this cannot be optimal for society, especially considering the effects on intergenerational equity: old generations are net receivers, whilst young generations carry the entire burden of social security systems. Is there any possibility to mitigate this persistent situation? As long as the old are selfish and only aim to maximise their welfare a solution which increases the young's welfare is not achievable. Otherwise, I think that altruism may represent a social solution to the early retirement. Altruism is seen as a change in preferences by the old which also pay attention to the young's needs. I argue that, if preferences are the real driver of political equilibrium, then a change in preferences must necessarily lead to another equilibrium.

In this chapter I consider a model where the old workers care of their offspring's wealth. A classical altruistic model considers that households can be represented by a dynasty who is willing to perpetuate forever. As a consequence, the old internalize the utility function of the young. The new utility function of the old may be written as:

$$U^{t-1} = c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + \sigma U^t \quad (23)$$

where $\sigma \in [0, 1]$ is a parameter which captures the degree of altruism of the old for the young; the higher σ the more the old attach a greater importance to the young's wealth. Under this new framework, we should expect that policies chosen by the government become less heavy for the young, since the old are now prone to share the burden of social security systems.

Conjecture 16 *with respect to the basic model, tax credits for the old (young) are lower (higher) and inter-generational transfers from the young to the old are reduced.*

4 Numerical Simulations

Numerical simulations were performed in order to assess the validity of conjectures 13-15, in the case where the maximum is an interior solution to the maximization problem. They jointly state that the old generation, because more single-minded, obtains more favourable policies by governments. That is, the old obtain higher tax credits (conjecture 14) and positive inter-generational transfers (conjecture 16). Furthermore, the combination of higher preferences for leisure and higher tax credits entails the old to reach higher levels of leisure (conjecture 15). As a consequence the young are the worse-off generation, since they get lower tax credits and have to pay the entire cost of social security systems. Unfortunately, the real problem is that we cannot say if the value function is concave, convex or neither concave nor convex, given the complexity of the expression. As a consequence, we cannot be sure if stationary points we found from first order conditions are maximum.

To perform simulations a suitable density function is required. As suggested by Profeta I will use one with a constant positive elasticity ε , $s^I = (l^I)^\varepsilon$, with $\varepsilon = 1$ for computational purposes. Table 1 shows results. The nominal marginal tax rate, τ , was set equal to 1 and the total endowment of time, T , equal to 0.9. Simulations were performed using different values of preferences of workers for leisure, always under the condition that the parameter of the old is higher than that of the young. Tax credits are always higher for the old but the difference between tax credits of the two generations reduces with respect to a reduction in the difference between preferences. Leisure is always higher for the old and the level of leisure increases both for the young and for the old from situation 1 to situation 9. Tax revenues are always positive for the generation of the young and negative for the generation of the old, meaning that the young borne the entire burden of social security systems; otherwise, the old get a transfer (i.e. a pension). Notice that the inter-generational redistribution effect is higher the higher is the difference between preference for leisure amongst cohorts. Finally, notice that, even though the sum of preferences for leisure of the old and the young is steadily equal to one, the total level of leisure is not constant. The worst situation for the total employment level is reached in situation 9, whilst the reverse is true for situation 1.

	ψ^{t-1}	ψ^t	τ	T	a^{t-1}	a^t	l^{t-1}	l^t	l	T^{t-1}	T^t
1	0.95	0.05	1	0.9	2.144	0.261	0.442	0.19	0.632	-0.261	0.261
2	0.9	0.1	1	0.9	1.915	0.385	0.469	0.259	0.728	-0.196	0.196
3	0.85	0.15	1	0.9	1.739	0.484	0.488	0.309	0.797	-0.152	0.152
4	0.8	0.2	1	0.9	1.592	0.571	0.502	0.35	0.852	-0.117	0.117
5	0.75	0.25	1	0.9	1.465	0.649	0.511	0.384	0.895	-0.09	0.09
6	0.7	0.3	1	0.9	1.352	0.722	0.517	0.415	0.932	-0.067	0.067
7	0.65	0.35	1	0.9	1.25	0.791	0.519	0.442	0.961	-0.047	0.047
8	0.6	0.4	1	0.9	1.159	0.859	0.517	0.465	0.982	-0.03	0.03
9	0.55	0.45	1	0.9	1.076	0.928	0.51	0.484	0.994	-0.014	0.014

Table 1 - Numerical simulation (basic model)

Notice that the result which states the old get lower effective marginal tax rates are utterly new with previous results of probabilistic voting models applied to social security systems. In Profeta, the old group is taxed heavier than the young group (Proposition 3.1, p. 345); the same result is achieved also by Mulligan and Sala-i-Martin (Proposition 8, p.31).

Table 2 shows results of simulations performed for the altruistic model in order to assess the validity of conjecture 16. The altruistic parameter was set equal to 0.3. With respect to previous results notice that in this case the old (young) obtain lower (higher) tax credits and that there are less redistributive effects since transfers from the young to the old reduce. Furthermore, notice that in situation 9 the young obtain a positive transfer, although this is rather small. Leisure increases for the old, suggesting that the higher effective marginal tax rate increases the incentive to withdraw from the labour force, whilst leisure of the young reduces. Total leisure reduces as well, but in situations 8, 9 where this is slightly higher than the previous situation.

	ψ^{t-1}	ψ^t	τ	T	σ	a^{t-1}	a^t	l^{t-1}	l^t	l	T^{t-1}	T^t
1	0.95	0.05	1	0.9	0.3	2.104	0.342	0.451	0.145	0.596	-0.247	0.247
2	0.9	0.1	1	0.9	0.3	1.845	0.503	0.487	0.198	0.685	-0.174	0.174
3	0.85	0.15	1	0.9	0.3	1.645	0.625	0.516	0.239	0.755	-0.123	0.123
4	0.8	0.2	1	0.9	0.3	1.479	0.723	0.540	0.276	0.816	-0.086	0.086
5	0.75	0.25	1	0.9	0.3	1.338	0.804	0.56	0.31	0.87	-0.057	0.057
6	0.7	0.3	1	0.9	0.3	1.217	0.873	0.575	0.343	0.918	-0.035	0.035
7	0.65	0.35	1	0.9	0.3	1.111	0.932	0.584	0.375	0.959	-0.017	0.017
8	0.6	0.4	1	0.9	0.3	1.019	0.987	0.588	0.405	0.993	-0.003	0.003
9	0.55	0.45	1	0.9	0.3	0.939	1.040	0.585	0.432	1.017	0.009	-0.009

Table 2 - Numerical simulation (altruistic model)

5 Conclusions

I introduced a very simple model where the generation of the old is more-single minded than the generation of the young, that is it has greater preferences for leisure. This enables this group to be more politically powerful in the political competition amongst two candidates which have to choose the effective marginal tax rate on labour. The equilibrium of the game is such that the old obtain more favourable policies; that is, higher tax credits, higher levels of leisure and positive intergenerational transfers. Eventually, I conclude, the young are the worse-off generation, which has to borne the entire burden of social security systems. Altruism may reduce this unfair redistribution scheme, lightening the excess pressure on younger generations. Intergenerational pacts could represent a possible solution to the early retirement problem, forcing the old generations to internalize the welfare of the young generations in order to make them share the entire burden of social security systems.

6 Mathematical Appendix

Proposition 17 (*Monotonicity condition*) Assume (i) V^i is concave in q_t^j (ii) for each group and candidate, $\frac{\partial \pi^j}{\partial a^j}$ is strictly monotonic. If (q_t^{A*}, q_t^{B*}) is a pure strategy electoral equilibrium, then $q_t^{A*} = q_t^{B*}$.

Proof. From Proposition 6 we know that λ^A and λ^B must be equal for every generation. This entails that the ratio between the two Lagrange multipliers of different candidates must be equal for every generation as well. I call this ratio $\rho^I = \frac{\lambda^{AI}}{\lambda^{BI}}$. The problem is to assess whether this condition may be achieved under a divergence or a convergence of policies. To prove this, I start assuming that $q_t^A \neq q_t^B$. Since candidates must clear the balanced-budget constraint, there must exist a generation which gets higher tax credits under candidate A (suppose it is t) and another generation which gets higher tax credits under candidate B ($t-1$). We have to assess whether the condition $\rho^t = \rho^{t-1}$ is achievable in such a situation. If it is, then an equilibrium is achieved under divergent policies; otherwise, policies are convergent. Notice that if both the numerator and the denominator are monotonic, the ratio is monotonic. If so, it means that (i) either $\rho^t > 1 > \rho^{t-1}$ or (ii) $\rho^{t-1} < 1 < \rho^t$; therefore, an equilibrium cannot be achieved via divergent policies. ■

In this model, and more in general in models where the direct utility function is quasi-linear in the consumption and leisure, the monotonicity condition cannot be applied to solve the candidate's problem. The failure of the monotonicity condition may have several implications. First of all, the possibility that the equilibrium is not achievable via a convergence of policies. Secondly, and more important, the convexity of V means that a maximum does not exist, since the value function is not concave.

(i) Convexity of V

Write the worker problem where the direct utility function is quasi-linear in consumption and leisure

$$\max_l U_t = c_t + \psi \log l_t \text{ subject to the budget constraint } c_t = \tau(1 - a_t)(T - l_t), l_t >$$

0. The optimal leisure is $l_t^* = \frac{\psi}{1 - \tau(1 - a_t)}$. Obtain the indirect utility function $V_t =$

$T(1 - \tau(1 - a_t)) - \psi + \log\left(\frac{\psi}{1 - \tau(1 - a_t)}\right) = T(1 - t) - \psi + \psi \log \psi - \psi \log w - \psi \log(1 - \tau(1 - a_t))$. Define $A = T - 1 + \psi \log \psi$ substitute and obtain $V_t = A - \tau(1 - a_t)T - \psi \log(1 - \tau(1 - a_t))$. Write the first order condition

$$\frac{\partial V}{\partial t} = T\tau - \frac{\psi\tau}{1 - \tau + \tau a_t} = 0$$

Note that there exist only a stationary point, $a_t^o = 1 - \frac{\psi - T}{\tau\psi}$. Write the second order condition and

$$\frac{\partial^2 V}{\partial t^2} = \frac{\psi\tau^2}{(1 - \tau + \tau a_t)^2} > 0$$

That is, V is a convex function and a_t^o is a minimum.

(ii) *Non-monotonicity*

I impose the ratio $\rho^I = \frac{\lambda^{AI}}{\lambda^{BI}}$ equal to one, subtract the denominator from the numerator and verify whether the expression has a clear sign. Denoting $z = V^I(q_t^A) - V^I(q_t^B)$ we get the following:

$$\overbrace{\left(\frac{\partial}{\partial a_t^{At}}\left(\frac{1}{s}\right) - \frac{\partial}{\partial a_t^{Bt}}\left(\frac{1}{s}\right)\right)}^A \sum_I \frac{1}{2} s^t z + \frac{1}{s} \overbrace{\left(\frac{\partial s^t}{\partial a_t^{At}} z - \frac{\partial s^t}{\partial a_t^{Bt}} z + \frac{s^t \frac{\partial V^t}{\partial a_t^{At}}}{\frac{\partial \Upsilon}{\partial a_t^{At}}} - \frac{s^t \frac{\partial V^t}{\partial a_t^{Bt}}}{\frac{\partial \Upsilon}{\partial a_t^{Bt}}}\right)}^B \quad (24)$$

for t , and

$$-\overbrace{\left(\frac{\partial}{\partial a_t^{A^{t-1}}}\left(\frac{1}{s}\right) - \frac{\partial}{\partial a_t^{B^{t-1}}}\left(\frac{1}{s}\right)\right)}^C \sum_I \frac{1}{2} s^{t-1} z + \frac{1}{s} \overbrace{\left(\frac{\partial s^{t-1}}{\partial a_t^{A^{t-1}}}(-z) - \frac{\partial s^{t-1}}{\partial a_t^{B^{t-1}}}(-z) + \frac{s^{t-1} \frac{\partial V^{t-1}}{\partial a_t^{A^{t-1}}}}{\frac{\partial \Upsilon}{\partial a_t^{A^{t-1}}}} - \frac{s^{t-1} \frac{\partial V^{t-1}}{\partial a_t^{B^{t-1}}}}{\frac{\partial \Upsilon}{\partial a_t^{B^{t-1}}}}\right)}^D \quad (25)$$

for $t - 1$. By the meaning of Proposition 6 A and C are equal to zero. Thus we have only to verify that B and D are monotonic. Notice that by as demonstrated before $\frac{\partial V^t}{\partial a_t^{jt}}$ is not monotonic and that $\frac{\partial \Upsilon}{\partial a_t^{jt}}$ is not monotonic neither.

$sign(z)$ changes according to the interval where tax credits find. Denoting with $a_{V=0+}$ and $a_{V=0-}$ points where the IUF intersects the axis⁶ (respectively at the right and at the left hand side) representing the tax credit we may easily see that 6 cases to study arise:

$$a^{B+} < a^{A+} < a_{V=0} \implies z < 0$$

$$a^{B+} < a_{V=0} < a^{A+} < \widehat{a} \implies z < 0$$

$$a_{V=0} < a^{B+} < a^{A+} < \widehat{a} \implies z > 0$$

$$a_{V=0} < a^{B+} < \widehat{a} < a^{A+} < a_{V(a^A)} < a_{V(a^B)} \implies z > 0$$

⁶One may verify that indirect utility functions have two intersection points.

$$a_{V=0^+} < a^{B^+} < \widehat{a} < a_{V(a^A) < V(a^B)} < a^{A^+} < a_{V=0^-} \implies z > 0$$

$$a_{V=0^+} < a^{B^+} < \widehat{a} < a_{V(a^A) < V(a^B)} < a_{V=0^-} < a^{A^+} \implies z > 0$$

We study the sign of expression (24) and (25). Since the $sign(z)$ is discontinuous, $\frac{\partial V^t}{\partial a_t^{jt}}$ and $\frac{\partial \Upsilon}{\partial a_t^{jt}}$ are not monotonic, the sign of the expression is not clear and thus we cannot say *a priori* whether the monotonicity condition holds. As a consequence the Lindbeck & Weibull's monotonicity condition may not be exploited in this model to demonstrate that an equilibrium is only achievable via a convergence of policies.

7 Mathematical Appendix 2

In this Appendix I provide a complete resolution to candidates' problem. The two candidates face exactly the same optimization problem; they maximize their share of votes or, equivalently, the probability of winning.

$$\max_{\{a^t, a^{t-1}\}} \pi^j = \frac{1}{2} + \frac{1}{2s} \sum_I s^I [V^i(q_t^j) - V^i(q_t^{-j})]$$

$$\Upsilon^j \equiv \frac{\tau}{2} \sum_I (T - l_t^I) (1 - a_t^{Ij}) = 0$$

I write the Lagrangian function:

$$\mathcal{L} = \frac{1}{2} + \frac{1}{2s} \sum_I s^I [V^i(q_t^j) - V^i(q_t^{-j})] - \lambda (\Upsilon^j)$$

Deriving the Lagrangian I obtain first order conditions which may be seen as a modified version of the original Lindbeck and Weibull's first order conditions:

$$\left\{ \begin{array}{l} (1) \frac{\partial \mathcal{L}}{\partial a_t^{t-1j}} \equiv \frac{1}{2} \frac{\partial}{\partial a_t^{t-1j}} \left(\frac{1}{s} \right) \sum_I s^I [V^i(\bar{q}^j) - V^i(\bar{q}^{-j})] + \frac{1}{2s} \frac{\partial s^{t-1}}{\partial l_t^{t-1}} \frac{\partial l_t^{t-1}}{\partial a_t^{t-1j}} (V^{t-1j} - V^{t-1-j}) + \frac{1}{2s} s^{t-1} \left(\frac{\partial V^{t-1}}{\partial a_t^{t-1j}} \right) = \lambda^j \frac{\partial \Upsilon^j}{\partial a_t^{t-1j}} \\ (2) \frac{\partial \mathcal{L}}{\partial a_t^{tj}} \equiv \frac{1}{2} \frac{\partial}{\partial a_t^{tj}} \left(\frac{1}{s} \right) \sum_I s^I [V^i(\bar{q}^j) - V^i(\bar{q}^{-j})] + \frac{1}{2s} \frac{\partial s^t}{\partial l_t^t} \frac{\partial l_t^t}{\partial a_t^{tj}} (V^{tj} - V^{t-j}) + \frac{1}{2s} s^t \left(\frac{\partial V^t}{\partial a_t^{tj}} \right) = \lambda^j \frac{\partial \Upsilon^j}{\partial a_t^{tj}} \\ (3) \Upsilon^j = 0 \end{array} \right.$$

By Proposition 10 we know that $q_t^A = q_t^B$, such that first order conditions may be simplified as:

$$\left\{ \begin{array}{l} (1) \frac{\partial \mathcal{L}}{\partial a_t^{t-1A}} \equiv \frac{1}{2s} s^{t-1} \left(\frac{\partial V^{t-1}}{\partial a_t^{t-1A}} \right) = \lambda^A \frac{\partial \Upsilon^A}{\partial a_t^{t-1A}} \\ (2) \frac{\partial \mathcal{L}}{\partial a_t^{tA}} \equiv \frac{1}{2s} s^t \left(\frac{\partial V^t}{\partial a_t^{tA}} \right) = \lambda^A \frac{\partial \Upsilon^A}{\partial a_t^{tA}} \\ (3) \Upsilon^A = 0 \\ (4) \frac{\partial \mathcal{L}}{\partial a_t^{t-1B}} \equiv \frac{1}{2s} s^{t-1} \left(\frac{\partial V^{t-1}}{\partial a_t^{t-1B}} \right) = \lambda^B \frac{\partial \Upsilon^B}{\partial a_t^{t-1B}} \\ (5) \frac{\partial \mathcal{L}}{\partial a_t^{tB}} \equiv \frac{1}{2s} s^t \left(\frac{\partial V^t}{\partial a_t^{tB}} \right) = \lambda^B \frac{\partial \Upsilon^B}{\partial a_t^{tB}} \\ (6) \Upsilon^B = 0 \\ (7) \lambda^A = \lambda^B = \lambda \end{array} \right.$$

We then obtain the reaction functions:

$$r^A = \begin{cases} a^{tA} = r(a^{tB}, a^{t-1B}, s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ a^{t-1A} = r(a^{tB}, a^{t-1B}, s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \end{cases}$$

$$r^B = \begin{cases} a^{tB} = r(a^{tA}, a^{t-1A}, s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ a^{t-1B} = r(a^{tA}, a^{t-1A}, s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \end{cases}$$

Solving the system we obtain the optimal vector of policies from the set of intersection points Λ :

$$\begin{aligned} a_t^{t-1A} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ a_t^{tA} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ a_t^{t-1B} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ a_t^{tB} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t) \\ &\text{with } a_t^{t-1A} = a_t^{t-1B} \text{ and } a_t^{tA} = a_t^{tB} \end{aligned}$$

with altruism the first order conditions are modified as follows:

$$\left\{ \begin{array}{l} (1) \frac{\partial \mathcal{L}}{\partial a_t^{t-1A}} \equiv \frac{1}{2s} s^{t-1} \left(\frac{\partial V^{t-1}}{\partial a_t^{t-1A}} \right) = \lambda^A \frac{\partial \Upsilon^A}{\partial a_t^{t-1A}} \\ (2) \frac{\partial \mathcal{L}}{\partial a_t^{tA}} \equiv \frac{1}{2s} s^t \left(\frac{\partial V^t}{\partial a_t^{tA}} \right) + \frac{\sigma}{2s} s^{t-1} \left(\frac{\partial V^t}{\partial a_t^{t-1A}} \right) = \lambda^A \frac{\partial \Upsilon^A}{\partial a_t^{tA}} \\ (3) \Upsilon^A = 0 \\ (4) \frac{\partial \mathcal{L}}{\partial a_t^{t-1B}} \equiv \frac{1}{2s} s^{t-1} \left(\frac{\partial V^{t-1}}{\partial a_t^{t-1B}} \right) = \lambda^B \frac{\partial \Upsilon^B}{\partial a_t^{t-1B}} \\ (5) \frac{\partial \mathcal{L}}{\partial a_t^{tB}} \equiv \frac{1}{2s} s^t \left(\frac{\partial V^t}{\partial a_t^{tB}} \right) + \frac{\sigma}{2s} s^{t-1} \left(\frac{\partial V^t}{\partial a_t^{t-1B}} \right) = \lambda^B \frac{\partial \Upsilon^B}{\partial a_t^{tB}} \\ (6) \Upsilon^B = 0 \\ (7) \lambda^A = \lambda^B = \lambda \end{array} \right.$$

which gives a new set of intersection points Λ :

$$\begin{aligned} a_t^{t-1A} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t, \sigma) \\ a_t^{tA} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t, \sigma) \\ a_t^{t-1B} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t, \sigma) \\ a_t^{tB} &= a(s(l(\psi^{t-1}, \tau)), s(l(\psi^t, \tau)), \tau, T, \psi^{t-1}, \psi^t, \sigma) \\ &\text{with } a_t^{t-1A} = a_t^{t-1B} \text{ and } a_t^{tA} = a_t^{tB} \end{aligned}$$

References

- [1] Aguiar, M & Hurst, E.: *Measuring Trends in Leisure: the Allocation of Time Over Five Decades* (2006), NBER Working Paper 12082
- [2] Ambrosiano M. F., Bordignon M., Galmarini U. & Panteghini P.: *Lezioni di Teoria delle imposte*, (1997), ETAS Libri

- [3] Barr, N.: *Reforming pensions: Tales from China, Chile and elsewhere* (2007), Barclay Memorial Lecture, London School of Economics
- [4] Barr, N.: *Pensions: Overview of the Issues* (2006), Oxford Review of Economic Policy, Vol. 22. No. 1, pp. 1-14
- [5] Barr, N. & Diamond, P.: *The Economics of Pensions* (2006), Oxford Review of Economic Policy, Vol. 22. No. 1, pp. 15-39
- [6] Becker, G.: *A Theory of the Allocation of Time* (1965), Economic Journal, Vol. 75, pp.493-517
- [7] : Boeri, T., Borsch-Supan, A., & Tabellini, G.: *Would you like to shrink the welfare state?* (2000), Economic Policy (32), pp. 7-50
- [8] Brennan G. & Buchanan J.M: *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, (1980) Cambridge: Cambridge University Press
- [9] Canegrati, E.: *A Theory on the Allocation of Political Time* (2007) mimeo
- [10] Coile, C. & Gruber, J.: *Social Security and Retirement*, (2000) NBER Working Paper 7830
- [11] Coughlin, P.: *Probabilistic Voting Theory* (1992) Cambridge University Press
- [12] Costa, D: *Less of a Luxury: the Rise of Recreation Since 1888* (1997), NBER Working Paper 6054
- [13] Diamond P. & Mirrlees J. *Optimal Taxation and Public Provision 1: Production Efficiency*, (1971) American Economic Review, Vol.61, pp.8-27
- [14] Dorn, D. & Sousa-Poza, A.: *The Determinants of Early Retirement in Switzerland* (2005), Swiss Journal of Economics and Statistics, Vol.141(2), pp.247-283
- [15] Dorn, D. & Sousa-Poza, A.: *Motives for Early Retirement: Switzerland in an International Comparison* (2004) Diskussionspapiere St. Gallen
- [16] Diamond P. & Gruber J. *Social Security and Retirement in the U.S.*, (1997) NBER Working Paper 6097
- [17] Diamond P. *Pensions for an Aging Population*, (2005) NBER Working Papers 11877
- [18] Dixit, A. & Londregan J. *Redistributive Politics and Economic Efficiency*, (1994) NBER Working Papers 1056
- [19] Ferrera, M.: *EC Citizens and Social Protection: Main Results from a Eurobarometer Survey*, (1993), Brussels: European Commission, Division V/E/2.

- [20] Feldstein M. & Liebman J.: *Social Security*, (2001) NBER Working Paper 8451
- [21] Fuest C. & Huber B.: *Tax Coordination and Unemployment*, (1999) International Tax and Public Finance, Vol.6, pp.7-26
- [22] Fenge, R. & Pestieau, P.: *Social Security and Early Retirement*, (2005), MIT Press
- [23] Greenwood, J. & Vandenbroucke, G.: *Hours Worked: Long-run Trends*, (2005), NBER Working Paper 11629
- [24] Gruber, J. & Wise, D.A.: *Social Security and Retirement Around the World* (1999), Chicago University Press
- [25] Hershey D., Henkens K. & Van Dalen H.: *Mapping the Minds of Retirement Planners: A Cross-cultural Perspective*, (2006), mimeo
- [26] Hinich M.J.: *Equilibrium in Spatial Voting: The Median Voter Theorem is an Artifact*, (1977) Journal of Economic Theory, Vol. 16, pp. 208-219
- [27] Huovinen, P. & Piekola, H.: *Time is right? Early Retirement and Time use of older Finns* (2001) ETLA B189
- [28] Holzmann, R. & Palmer, E.: *Pension Reform. Issue and Prospects for Non-Financial Defined Contribution (NDC) Schemes* (2006) The World Bank
- [29] Hettich, W. & Winer, S.: *Democratic Choice and Taxation* (1999) Cambridge University Press
- [30] Jacobs, L. & Shapiro, R.: *Myths and Misunderstandings about Public Opinion Toward Social Security*. In Ardnold, R. D., Graetz, M., and Munnell, A., eds., *Framing the Social Security Debate: Values, Politics and Economics*, (pp. 355-388). (1998), Washington, DC: National Academy of Social Insurance
- [31] Koskela E. & Schob R.: *Optimal Factor Income Taxation in the Presence of Unemployment*, (2002) Journal of Public Economic Theory, Vol. 4(3), pp.387-404
- [32] Kotlikoff, L. & Rapson, D.: *Comparing Average and Marginal Tax Rates under the Fair Tax and the Current System of Federal Taxation*, (2006) Boston University, mimeo
- [33] Ihori, T.: *Public Finance in an Overlapping Generations Economy*, (1996) MacMillan Press
- [34] Ihori, T. & Tachibanaki, T.: *Social Security Reforms in Advanced Countries*, (2002) Routledge

- [35] Lindbeck A. & Weibull J. W.: *Balanced-Budget Redistribution as the Outcome of Political Competition*, (1987), Public Choice 52, pp. 273-297
- [36] McGrattan, R. & Rogerson, R.: *Changes in Hours Worked Since 1950* (1998), Federal Reserve Bank of Minnesota, Vol. 22(1), pp. 2-19
- [37] Mulligan C. B. & Sala-i-Martin: *Gerontocracy, Retirement and Social Security*, (1999) NBER Working Paper 7117
- [38] Mulligan C. B. & Sala-i-Martin: *Social Security in Theory and Practice (I): Facts and Political Theories*, (1999) NBER Working Paper 7118
- [39] Mulligan C. B. & Sala-i-Martin: *Social Security in Theory and Practice (II): Efficiency Theories, Narrative Theories and Implications for Reforms*, (1999) NBER Working Paper 7119
- [40] Mulligan C. B. & Sala-i-Martin: *Social Security, Retirement and the Single-Mindedness of the Electorate*, (1999) NBER Working Paper 9691
- [41] Ngai, R. & Pissarides, C.: *Trends in Hours and Economic Growth* (2006), LSE mimeo
- [42] OECD: *Live Longer Work Longer*, OECD Report, (2005)
- [43] Profeta, P.: *Retirement and Social Security in a Probabilistic Voting Model*, (2002) International Tax and Public Finance, 9, pp 331-348
- [44] Persson T. & Tabellini G.: *Political Economics: Explaining the Economic Policy*, (2000) MIT Press
- [45] Ramsey F.P.: *A Contribution to the Theory of Taxation*, (1927) Economic Journal, Vol. 37, No. 1, pp. 47-61
- [46] Ruzik, A.: *Retirement Behaviour in Poland, Hungary and Lithuania* (2006), mimeo
- [47] Skirbekk, V.: *Age and Individual Productivity: A Literature Survey* (2003), MPIDR WORKING PAPER WP 2003-028
- [48] Stafford, Frank P. and Greg J. Duncan: *The Use of Time and Technology by Households in the United States*, (1985) In F. Thomas Juster and Frank P. Stafford, eds., Time, Goods, and Well-Being. ANI Arbor, MI: Survey Research Center, University of Michigan, pp. 24588.

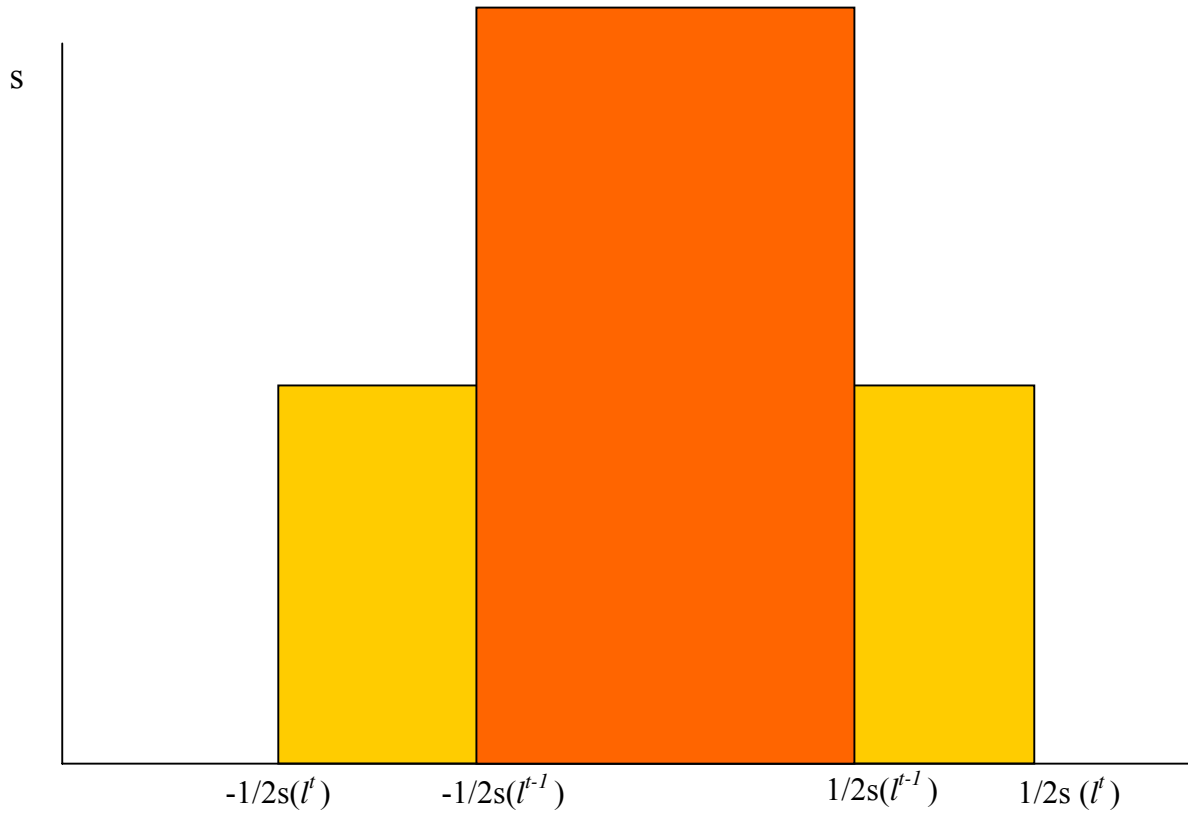


FIGURE 1
DISTRIBUTION FUNCTIONS WITH SINGLE-MINDED GENERATIONS

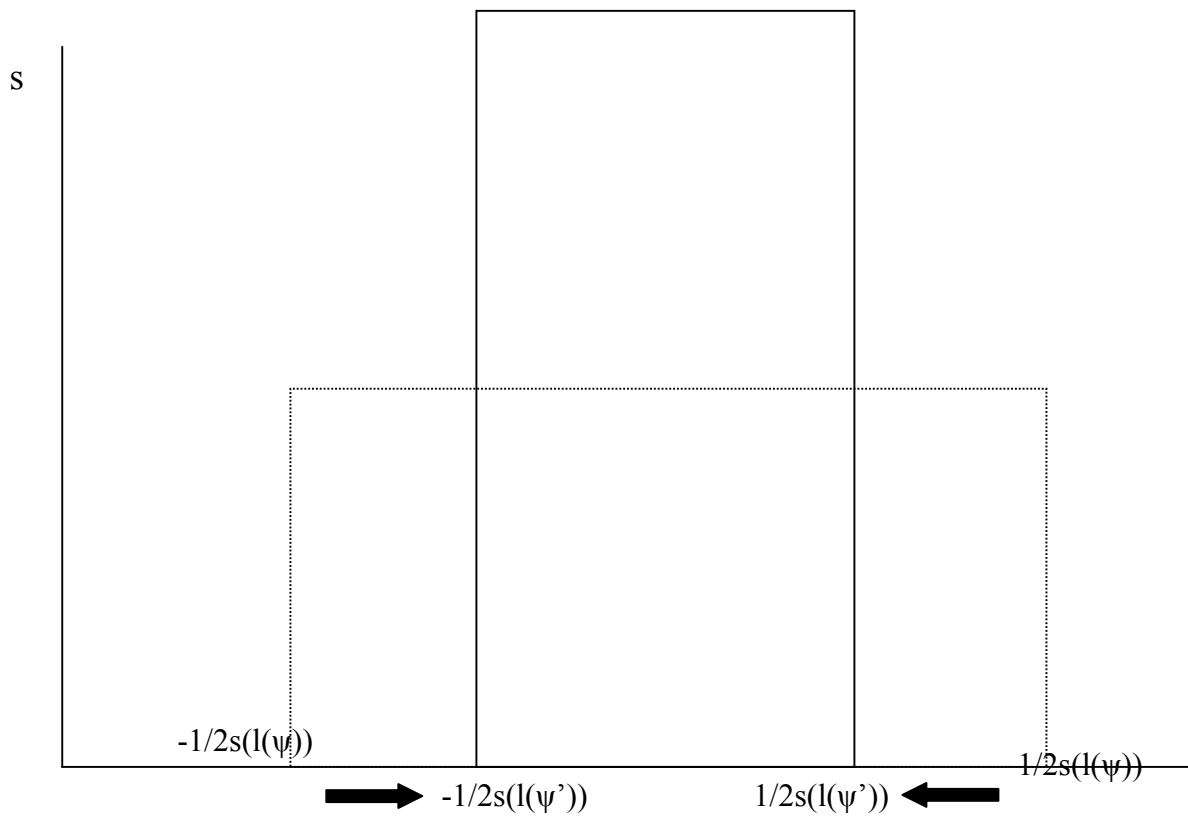


FIGURE 2
EFFECTS OF A CHANGE IN GENERATIONS' PREFERENCES ON DISTRIBUTION

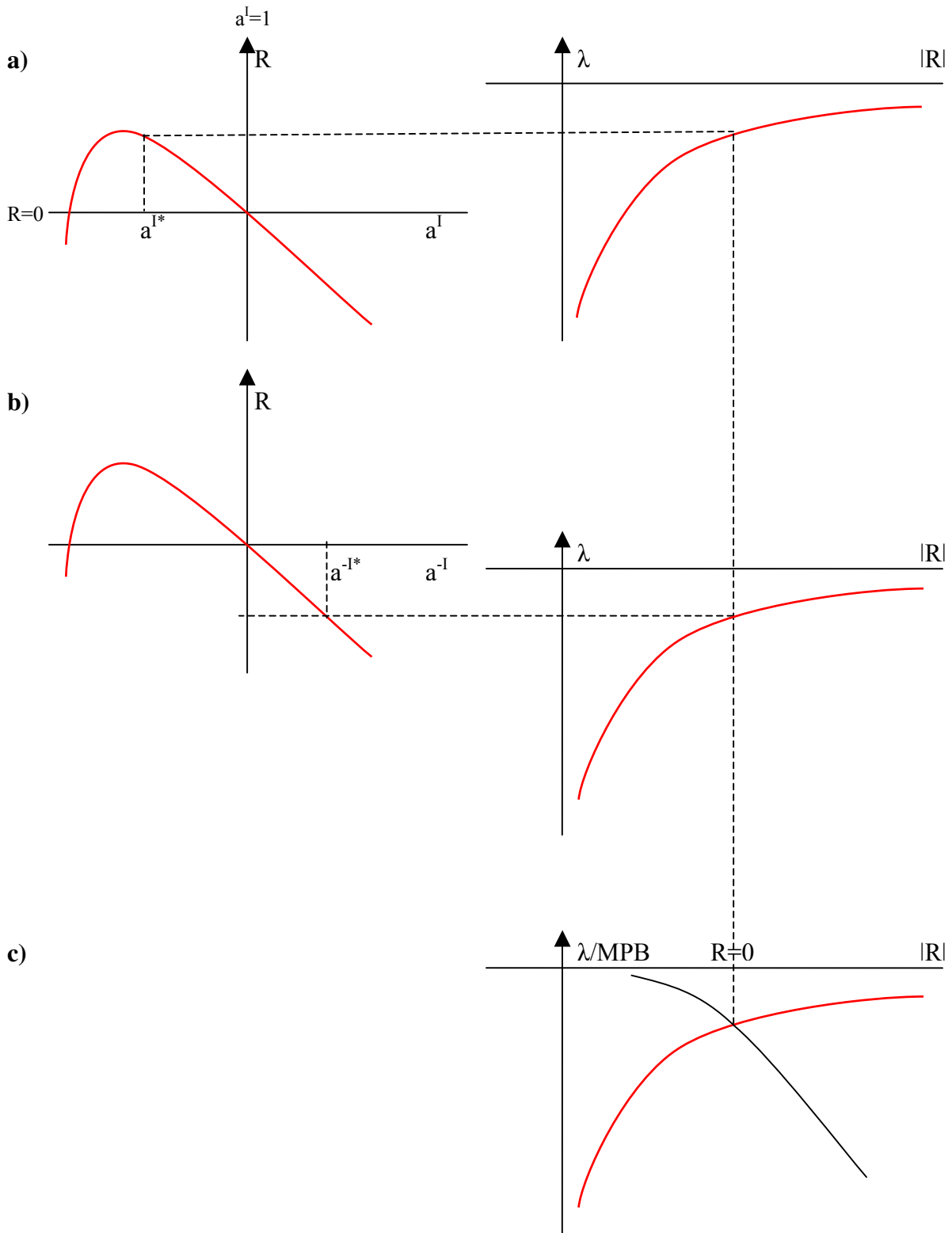


FIGURE 3
TAX STRUCTURE IN A POLITICAL EQUILIBRIUM