A model of waste control and abatement capital: Permanent versus temporary environmental policies

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A model of waste control and abatement capital: Permanent versus temporary environmental policies

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Abstract

In this paper we investigate the effects of introducing explicitly abatement capital in a welfare function which depends on waste stock and consumption. Consumption is assumed to produce an undesiderable residue. Society can control waste accumulation using abatement capital. We focus on two issues: the intertemporal relationship between abatement investment and waste emission, and the effects of permanent and temporary environmental policies on the long-run equilibrium of the economic-ecological system. We get three main results. *First*, for a society the problems of waste control and abatement investment are very interrelated. Any change in investment affects waste emission and consumption, but not always in a predictable manner. *Second*, we show that the adoption of either temporary subsidies or taxes do not change the long-run properties of the economy. It is not just current subsidies or taxes, but their entire path over time that affects accumulation of waste and capital. *Third*, we get that environmental policies may have ambiguous effects: in response to subsidies or taxes a society might accumulate less abatement capital than desidered, allowing the stock of waste to rise in the long run.

Key words: Abatement investment; waste accumulation; dynamic optimization; environmental policy.

JEL classification codes: E22, L51, H23, Q28.

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1 Introduction

In *Polluted Golden Age* Brock (1977) contended that economists generally tend to overlook the process of waste accumulation associated with economic activities. He remarked that waste product accumulates over time and provides disutility to society. Abatement investment can be done to control waste emissions and productivity, but it takes a long time to be made (Plourde, 1972; Smulders and Gradus, 1996; Pindyck, 2000; Färe *et al.*, 2007). Finally, corrective taxes and subsidies can be used to control waste emissions, but different environmental policies may have different impacts on waste emissions, abatement investment and welfare over time (Perman *et al.,* 2003; Dinda, 2004; Xepapadeas, 2005).

In what follows, we will think over these issues. Our aim is to scrutinize the timing and persistence of environmental policies to assess their relative desirability in terms of environmental quality and social welfare. A special attention will be devoted to study the effects of environmental policies on abatement investment decisions.

As said above, the issue of waste control is not new in the economic debate. Seminal papers are Smith (1972), Plourde (1972) and Forster (1973) which employ optimal control techniques to study the intertemporal effects of waste stock externality. Later on, a number of studies (Forster, 1975; Dasgupta, 1982; Weitzman, 1976; Solow, 1986; Hartwick, 1990; Bovenberg and Smulders, 1995; Smulders and Gradus, 1996; Dasgupta and Maler, 2000; van der Ploeg and Withagen, 1991; Withagen, 1995; Brock and Starrett, 2003; Travaglini, 2012b) have investigated the effect of waste accumulation on growth to amend the Solow (1956) and Ramsey model (1928). The common denominator of these papers is that waste is a by-product of economic activity, so that growth and environmental protection could be compatible only under certain restrictive conditions (Xepapadeas 2005; Travaglini 2012a).

A further remarkable line of research is based on cost-benefit analysis (Pearce and Nash, 1981; Hanley and Spash, 1993; Schulze, 1994). In its most updated formulation, it is concerned with the optimal timing of a discrete policy that a society should adopt to reduce waste emissions (Pindyck, 2000, 2002). Uncertainty about future, externality of waste, and irreversibility of investments and policies are the central elements of this class of models. Real option is the more appropriate methodology to study the optimal timing problem of environmental policies with uncertainty and irreversibility

(Pindyck, 2002; Lin et al., 2007; Ansar and Spark, 2009; Lin and Huang, 2010, 2011). An interesting application is in Saltari and Travaglini (2011a, 2011b). They shift the attention from the net *social* benefits to the net *private* benefits gained from green firms which invest in abatement capital. Specifically, they show that the "private view" may explain why environmental policies, promoted to stimulate protective investment, can sometimes generate unexpected allocation of resource, reducing, instead of increasing, abatement capital stock over time.

The present paper improves on the current literature in several dimensions. As far as we know, this is the first attempt to introduce *both* abatement capital and waste stock in a social utility function. We study the relationship between abatement investment and waste control. Using this innovative view, we analyze the intertemporal choices of a society concerning the timing and impacts of taxes and subsidies on waste emissions and abatement investments. In addition, we will study the characteristics of environmental policies to be either permanent or temporary, focusing on their capabilities of affecting the long-run equilibrium of the economy.

Basically, we combine the two previous cited strands of research in one unique coherent framework. On the one hand, we have the society which needs to control waste emissions to improve welfare. On the other hand, we have the firms which invest in abatement capital, looking at costs and revenues. Here, we try to merge together these two approaches. Therefore, we will assume that abatement investment is an argument of the welfare function. Welfare is negatively affected by waste accumulation, while it is positively affected by abatement capital and consumption. But consumption generates an undesiderable residue.

For even the basic framework presented here, one is concerned not only with the optimal waste emission, but also with the optimal rate of abatement investment. The two planes are strictly interrelated and the combined intertemporal problem is complex. From a technical point of view, when there are two state variables – such as waste and capital stock – the maximum principle continues to provide a set of necessary and sufficient conditions for an optimum; and if all functional forms and other restrictions are fully specified we could provide an explicit solution to the problem. As we will show, the recursive structure of our problem provides, under specific conditions an explicit solution, and a general qualitative solution as well. This is a step forward with respect to the previous literature on waste accumulation where, in presence of two state variables and constraints, it is very difficult to find either an explicit or an implicit solution with waste stock and resource constraints (as it is in Smith, 1972).

The model has a basic structure. Society can control waste emission using abatement capital, corrective taxes and subsidies. In making investment society will, however, incur in an additional cost whose magnitude varies positively with the investment ratio. This additional cost is modelled by a convex quadratic function. Because of pressure on natural resources this increasing cost function explicitly takes into account the society's external obstacles to investment.

We get three main results. Firstly, for a society the problems of consumption control, waste control and optimal abatement investment are interrelated. Any change in protective investment affects waste emission and consumption, but not always in a predictable manner. Secondly, we show that the adoption of either temporary subsidies to stimulate investment, or temporary taxes to discipline polluters do not change the long-run equilibrium of the economy. Indeed, it is not just current subsidies or taxes, but their entire path over time that affect investment decisions and waste accumulation. Thus, expectations about how long the environmental policies would last have a special role in affecting investment decisions and waste accumulation. Finally, environmental policies may have ambiguous effects: in response to subsidies or taxes a society might accumulate *less* abatement capital than desidered, allowing the stock of waste to rise in the long run.

The organization of the paper is as follows. In the next section we present the model. We then study the dynamic properties of the model using phase diagrams. Next, we study the effects of permanent and temporary environmental policies on both abatement investment and waste accumulation. The last section concludes.

2 The model

Following Plourde (1972), Smith (1972) and Forster (1973, 1975) consider a society consuming some good and which, as a by product of this consumption, increases the stock of waste. Let's indicate the social utility function as

$$u(C_t, A_t, M_t) \tag{1}$$

where C_t is consumption (a "good"), A_t is stock of abatement capital (a "good"), M_t is stock of waste (a "bad"). We assume that $u_C, u_A > 0$ and

$u_M < 0$ and $u_{CC}, u_{AA} < 0$ and $u_{MM} > 0$.

To produce consumption goods and abatement investments society uses, in any activity $i = C, E$ an homogeneous environmental resource X^i, such as land, water or energy, so that

$$X = X_t^C + X_t^E + \frac{\epsilon}{2}\left(X_t^E\right)^2 \tag{2}$$

is the total endowment of this environmental input. Equation (2) includes adjustment costs $\frac{\epsilon}{2}\left(X_t^E\right)^2$ as an additional term. In fact, natural resource should be thought of as gross input. Adjustment costs make a claim on this gross input, and the net-adjustment-cost input is the gross input X minus the adjustment cost, that is $X - \frac{\epsilon}{2}\left(X_t^E\right)^2$. The quadratic component $\frac{\epsilon}{2}\left(X_t^E\right)^2$ is the additional cost (convex) of abatement investment measured in units of X. This component captures the external and internal obstacles (such as pressure on natural resource X) the community faces in adjusting its stock of abatement capital in the short-run.

Waste M accumulates according to

$$\frac{dM}{dt} = I_t - vM_t \tag{3}$$

that is, the stock M is increased by the waste flow I_t and it is decreased by the natural biological decay vM. The undesiderable residue I_t arises from aggregate consumption C_t, but it can decrease by some abatement activities, e.g. a recycling process.

Society can perform any or all of two activies: the production of consumption goods and the production of abatement goods to process waste residue. Basically, we assume that society produces the amount of consumption goods C_t according to the production function

$$C_t = hX_t^C, h > 0 \tag{4}$$

Then, as in Clarke and Reed (1994), the gross stock of waste I_t accumulates at a rate

$$I_t = hX_t^C - (X - X_t^C), \tag{5}$$

and the net accumulated rate is

$$\frac{dM}{dt} = hX_t^C - (X - X_t^C) - vM_t \tag{6}$$

where X is the constant bulk of resource available at any time, and $X - X_t^C$ is the amount of the natural resource assigned to abatement activities.

It is convenient to write the dynamics of the abatement stock A_t, chosen by the society, as

$$\frac{dA}{dt} = X_t^E - \delta A_t \tag{7}$$

where X_t^E is the gross defensive investment per unit time, and δ is the depreciation rate of A_t.

As time periods are linked together through the stock-flow relationships, efficient abatement activities and waste targets must be derived from an intertemporal analysis. We proceed assuming that the society aims at maximize discounted social net utility over some suitable time horizon. The dynamic optimization problem can be stated as follows: select values for the control variables X_t^C and X_t^E for $t = 0, \ldots, \infty$ so as to maximize the functional

$$W = \int_0^\infty u(C, A_t, M_t) e^{-\rho t} dt \tag{8}$$

where ρ is the constant social discount rate subjects to

$$\begin{aligned}
\frac{dM}{dt} &\equiv \dot{M} = I_t - vM_t && (9)\\
\frac{dA}{dt} &\equiv \dot{A} = X_t^E - \delta A_t && (10)\\
I_t &= hX_t^C - (X - X^C) - vM_t && (11)\\
X &= X_t^C + X_t^E + \frac{\epsilon}{2}\left(X_t^E\right)^2 && (12)
\end{aligned}$$

Finally, associated with each state variables there is a shadow price λ_t for the waste stock M_t and ψ_t for the abatement stock A_t. The auxiliary variable μ is the Lagrangean multiplier of the constraint (12).

This problem is particularly simple to illustrate when the utility function is additively separable (Plourde, 1972; Smith, 1972). Therefore, to find a closed form solution we assume that

$$u(C_t, A_t, M_t) = u(C_t) + u(A_t) + u(M_t) \equiv a\left(X_t^C\right)^\alpha + bA^\beta - gM^\gamma$$

with $0 < \alpha, \beta, \gamma < 1$ to assure convergence.

2.1 Optimality

The current valued Hamiltonian takes the form

$$H = \left\{ \begin{array}{c} \left[a\left(X_t^C\right)^\alpha + bA^\beta - gM^\gamma\right] + \lambda_t\left(hX_t^C - \left(X - X^C\right) - vM_t\right) + \\ +\psi_t\left(X_t^E - \delta A_t\right) \end{array} \right\} e^{-\rho t} \tag{13}$$

and the Lagrangean is

$$L = \left\{ \begin{array}{c} \left[a\left(X_t^C\right)^\alpha + bA^\beta - gM^\gamma\right] + \lambda_t\left(hX_t^C - \left(X - X^C\right) - vM_t\right) + \\ +\psi_t\left(X_t^E - \delta A_t\right) + \mu\left[X - X_t^C - X_t^E - \frac{\epsilon}{2}\left(X_t^E\right)^2\right] \end{array} \right\} e^{-\rho t} \tag{14}$$

The transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t M_t = 0 \qquad \text{and} \qquad \lim_{t\to\infty} e^{-\rho t}\psi_t A_t = 0$$

Ignoring time subscripts, the optimal solution must satisfy the following conditions in terms of the Lagrangean

$$L_{X^C} = 0 : a\alpha\left(X^C\right)^{\alpha-1} + \lambda(1+h) - \mu = 0 \tag{15}$$

$$\Rightarrow \quad X^C = \left[\frac{\mu - \lambda(1+h)}{a\alpha}\right]^{\frac{1}{\alpha-1}} \tag{16}$$

$$L_{X^E} = 0 : \psi - \mu\left(1 + \epsilon X^E\right) = 0 \tag{17}$$

$$\Rightarrow \quad X^E = \frac{1}{\epsilon}\left[\frac{\psi}{\mu} - 1\right] \tag{18}$$

with

$$\mu \geqslant 0 \quad X - X_t^C - X_t^E - \frac{\epsilon}{2}\left(X_t^E\right)^2 \geqslant 0 \quad \text{and} \quad \mu\left[X - X_t^C - X_t^E - \frac{\epsilon}{2}\left(X_t^E\right)^2\right] = 0 \tag{19}$$

$$-L_A + \rho\psi = \dot{\psi} : -b\beta A^{\beta-1} + \delta\psi + \rho\psi = \dot{\psi} \tag{20}$$

$$\Longrightarrow \quad \psi = b\beta\frac{A^{\beta-1}}{\rho+\delta} + \frac{1}{\rho+\delta}\dot{\psi} \tag{21}$$

$$-L_M + \rho\lambda \quad = \quad \dot{\lambda} : g\gamma M^{\gamma-1} + v\lambda + \rho\lambda = \dot{\lambda} \tag{22}$$

$$\Longrightarrow \quad \lambda = -g\gamma \frac{M^{\gamma-1}}{\rho + v} + \frac{1}{\rho + v}\dot{\lambda} \tag{23}$$

Because X is constant by assumption one expects that $\mu > 0$ is indipendent of time.

2.2 Properties of the optimal conditions

Equation (15) states that in each period the efficient consumption will be one in which the marginal utility of consumption must be equal to the total price of consumption. μ is the implicit (fixed) rental price of the resources employed to produce consumption goods, while $\lambda(1+h)$ is the unit shadow social cost of the waste stock. Thus, condition (15) states that a marginal unit of consumption must equal its marginal social cost.

Equation (18) is a variant of the familiar marginal condition for efficiency. It states that for a society the optimal abatement investment depends on the cost $c = 1$ of an additional unit of abatement resourse and its adjusted real marginal value $\frac{\psi}{\mu}$ of an additional unit of investment. Putting $q \equiv \frac{\psi}{\mu}$, equation (18) becomes

$$X^E = \frac{1}{\epsilon}\left[q - 1\right] \tag{24}$$

A society finds optimal to support abatement activities when $q > 1$. Note that for a society, in presence of waste, the q value captures the trade-off between abatement activities and polluting consumption. Indeed, the real q value of a unit of abatement goods is reduced by the shadow value μ of the resource X. It is worthwhile to remark that investing in protective capital is convenient for a society when the social shadow value q is bigger than the trigger value 1; disinvesting is optimal in the opposite case.

2.3 Properties of optimal paths of shadow values

2.3.1 Waste accumulation

To complete the analysis of waste accumulation, let's focus on equation (23). It describes how the shadow price λ of accumulated waste moves along the efficient path. λ equals the present value of the loss of welfare arising from the impact of a marginal unit of waste plus the present value of its variation. Dividing both sides by λ we get

$$\rho + v = -g\gamma \frac{M^{\gamma-1}}{\lambda} + \frac{\dot{\lambda}}{\lambda} \tag{25}$$

which states that the social cost of waste $-g\gamma \frac{M^{\gamma-1}}{\lambda} + \frac{\dot{\lambda}}{\lambda}$ must be equal to its user cost, $\rho + v$. In other words, since $\rho + v > 0$ the sign of the shadow price λ must be negative, $\lambda < 0$. This result reflects the disutility of M.

To further understand this result, consider a society that bears $M = 1$ units of waste. Recall that the shadow value of waste is λ, and consider the society's choice between decreasing the waste and continuing to endure it. The social real cost of waste is $-g\gamma M^{\gamma-1}\lambda$ per unit time. On the other hand, the stock of waste provides three distinct cash flows to the society. First, maintaining the stock of waste society gives up the gains it would receive if it reduces the waste and enjoy the social proceeds. This has a social cost of $\rho\lambda$ per unit time. Second, the waste is decreasing by natural decay so the society losses $v\lambda$ per unit time keeping its stock constant over time. And third, the shadow value of waste may be changing over time. It reduces the social proceeds if the social cost of waste is rising and it rises the social return if its value is decreasing. This variation has a cost of $-\dot{\lambda}$ per unit time. Putting these three components together, $-\gamma g M^{\gamma-1}\lambda = \rho\lambda + v\lambda - \dot{\lambda}$, yields the previous social user cost of waste. Note that if $\rho + v \lesseqgtr -g\gamma \frac{M^{\gamma-1}}{\lambda} + \frac{\dot{\lambda}}{\lambda}$ – that is if the social user cost of waste is smaller (greater) than the marginal disutility of waste plus its cost change – a society can find it optimal to increase (decrease) the waste stock. In steady state we have $\lambda = -g\gamma \frac{M^{\gamma-1}}{\rho+v}$, that is the shadow price of waste equals the discounted value of social disutility.

2.3.2 Abatement capital

Finally, equation (21) provides the motion of the marginal value of abatement capital A. Now, this equation can be re-expressed using the identity $q \equiv \frac{\psi}{\mu}$, which implies $\psi \equiv q\mu$ and $\dot{\psi} \equiv \dot{q}\mu$. Therefore, equation (21) can be rewritten as

$$q = \frac{b\beta A^{\beta-1}}{\mu\left(\rho + \delta\right)} + \frac{1}{\rho + \delta}\dot{q} \tag{26}$$

Intuitively, condition (26) states that for a society the internalization of the cost of waste affects directly the social q value of the abatement capital by the shadow price μ.

It is remarkable that the condition governing the optimal programme of abatement investment with waste emission should be so simple in form. In particular, condition (26) makes the role of q very clear. The marginal social value q of the abatement investment is equal to the discounted gross value of the marginal utility arising from an additional unit of investment, $b\beta A^{\beta-1}$, plus the discounted value of its capital gain $\dot{q}$. Thus, one important implication of our model is that even in a polluted environment the marginal q value captures all information relevant to the society's abatement investment decision.

As we have seen, a society will decide to increase its stock of abatement capital if q is higher than its social cost 1, and to reduce it if q is lower than 1. In other words, society does not need to know anything about the future other than the information that is summarized in q in order to make its decision. The long run equilibrium value of q can therefore be expressed as

$$q^* = \frac{1}{\mu}\int_0^\infty b\beta A_t^{\beta-1}e^{-(\rho+\delta)t}dt \tag{27}$$

which states that q is the present social value of an additional unit of abatement capital. In this perspective, the q ratio says that what is relevant for a society to invest in defensive capital is the net value of its marginal utility with respect to A.

3 Analyzing the model

Our solution provides two dynamic conditions, (23) and (26) describing how the shadow values q and λ move together over time. The dynamics of the system presents certain technical difficulties because there are two state variables (the abatement capital and the stock of waste). However, these equations have a recursive structure. The differential equation for q does not depend on M, but the differential equation for M in (31) depends indirectly on X^E and q. Hence, we employ a two steps procedure: we begin our analysis studying the intertemporal behavior of A and q; afterwards, we will proceed to solve the system for M and λ.

3.1 Dynamics and steady state

We start scrutinizing the dynamic properties of the system (26) and (10) using a phase diagram. We are looking at the following two differential equations

$$\begin{cases} \dot{q} = q\,(\rho+\delta) - \frac{1}{\mu} b\beta A_t^{\beta-1} \\ \dot{A} = \frac{1}{\epsilon}\,(q-1) - \delta A_t \end{cases} \tag{28}$$

The general solution for q of the system (28) is

$$q_t = C_1 e^{\varkappa_1 t} + C_2 e^{\varkappa_2 t} + \frac{1}{\mu}\int_0^{\infty} b\beta A_t^{\beta-1} e^{-(\rho+\delta)t} dt \tag{29}$$

where $\varkappa_1 > 0$ and $\varkappa_2 < 0$ are the roots of the characteristic equation obtained by (28). Therefore, we have to set $C_1 = 0$ in order to avoid speculative bubbles, and to reach the stable arm converging to the steady state

$$q^* = \frac{1}{\mu}\int_0^{\infty} b\beta A_t^{\beta-1} e^{-(\rho+\delta)t} dt = \frac{b\beta A^{\beta-1}}{\mu\,(\rho+\delta)} \tag{30}$$

This is of course only to be expected. In addition, since, the derivative of q with respect to A is negative $q_A < 0$ because $\beta < 1$, the demarcation curve $\dot{q} = 0$ has a negative slope. This curve delineates the subset of points in the space (A, q) where the variable in question are stationary, decreasing or increasing. Points off the demarcation curve are very much involved in dynamic motion. Accordingly, on the right of $\dot{q} = 0$ we find that $q > \frac{b\beta A^{\beta-1}}{\mu(\rho+\delta)}$, implying $\dot{q} > 0$ is positive. The positive sign of $\dot{q}$ means that the marginal shadow value of abatement capital rises. Similarly, on the left of the locus $\dot{q} = 0$ the sign of the variation is negative, $\dot{q} < 0$ and the marginal value of the abatement capital decreases. Hence, going from south to north in figure (1) the sign sequence of $\dot{q}$ is $(-, 0, +)$. The arrows in the phase diagram summarize this information.

For completeness, note, however, that, equation (30) does not provide an explicit solution for q^* because the term on the right hand side of this equation depends on future value of A. But, the future stock of abatement capital depends on the future q value, which is the unknown we are looking for. Thus, condition (30) provides a qualitative solution which captures the intertemporal relationship between q and A. Anyway, if $\beta = 1$ the marginal

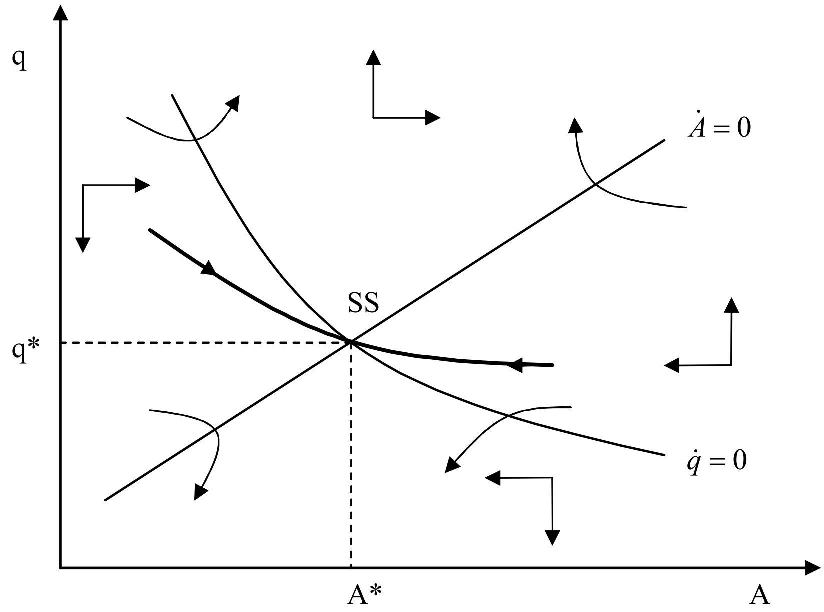

Figure 1: The saddle path of abatement investment

contribution of A would be constant over time, and condition (30) would provide the explicit solution for q.

A similar reasoning allows to draw the steady state for A and the demarcation line $\dot{A} = 0$. Along the locus $\dot{A} = 0$ the gross investment X^E is equal to the depreciation rate δA. The equilibrium value of the abatement capital is $A^* = \frac{1}{\delta \cdot \epsilon} [q^* - 1]$.

To determine the slope of the locus $\dot{A} = 0$ note the first addendum on the right hand side of the second equation in the system (28) is an increasing function of q, since an higher q implies an higher investment per unit of time. Nonetheless, the effect on $\dot{A}$ of an higher value of A is ambiguous. It is negative for the second addendum δA, but it is potentially positive for the first addendum. We assume that this positive effect dominates the first one, that is the locus $\dot{A} = 0$ has a positive inclination. Thus, when q exceeds the level that yields $\dot{A} = 0$, we find that $\frac{1}{\epsilon}(q-1) > \delta A_t$ and A is rising. On the other hand, when q is less than this level we find that $\frac{1}{\epsilon}(q-1) < \delta A_t$ and A is falling. Therefore, going from west to east the sign sequence of $\dot{A}$ is $(+, 0, -)$. The arrows in the phase diagram show the direction of motion of A.

3.2 Optimal abatement capital

Figure (1) combines the demarcation lines. The diagram shows how A and q must behave to satisfy the system (28). If $q > q^*_{|\dot{A}=0}$, the society tends to increase its abatement capital, thus $\dot{A} > 0$. But, since A is high and marginal utility is therefore low, q can be high only if it is expected to rise; thus $\dot{q}$ is also positive. Hence, A and q move up to the right in the diagram. Obviously, this is a divergent trajectory which drives the economy far away from the steady state SS. Specifically, for a given level of A there is a unique level of q that produces a stable path. If q starts below this level the economy eventually crosses into the region where K and q are both falling. Similarly, if q starts too high the economy eventually moves into the region where both K and q are rising indefinitely. Thus, the unique equilibrium is for q to equal the value that puts the economy on the saddle path and then moving along this saddle path to SS. This saddle path is shown in figure 1. The steady state is characterized by $\dot{A} = 0$ and $\dot{q} = 0$. Therefore, given the other parameters, society has not incentive to change its abatement capital stock.

3.3 Optimal waste accumulation

Equation (9), (11) and (23) determine the value of M and λ, and the motion of the system in the phase space (M, λ) is governed by the differential equations

$$\begin{cases} \dot{M} = I(q^*) - vM \\ \dot{\lambda} = (\rho + v)\,\lambda + g\gamma M^{\gamma-1} \end{cases} \tag{31}$$

Figure 2 illustrates the demarcation line of the points $M = \frac{I(q^*)}{v}$, such that $\dot{M} = 0$. Along this locus, the net emission of waste is just balanced by the rate at which waste is degraded by nature, so that accumulation is zero. In addition, since the demarcation line $\dot{M} = 0$ depend on λ, this locus will be increasing. At any point above of this locus, the stock of waste will increase since $Mv < I(q^*)$. But, at any point below this locus the decay of waste is higher than its net affluence $I(q^*)$, and the stock of waste will decrease. Therefore, going from west to east the sign sequence of $\dot{M}$ is $(+, 0, -)$. The arrows in the phase diagram show the direction of motion of M.

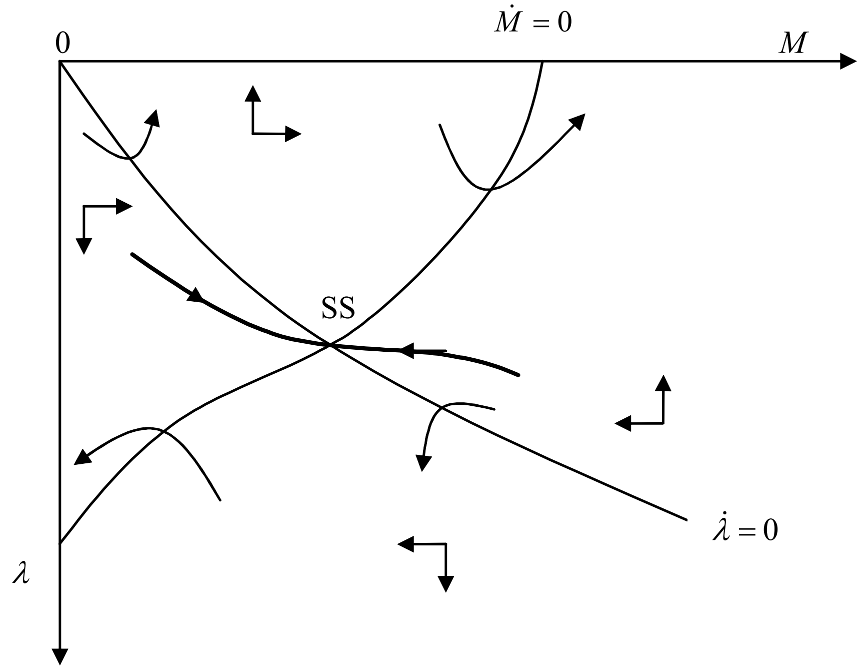

Figure 2: The saddle path of waste accumulation

In figure 2 it is illustrated the demarcation line $\lambda = -\frac{g\gamma M^{\gamma-1}}{(\rho+v)}$, defined by $\dot{\lambda} = 0$, where the shadow social cost of waste is stationary. At any point on the right of this locus the discounted marginal disutility of the stock of waste, $-\frac{g\gamma M^{\gamma-1}}{(\rho+v)}$, exceeds the shadow social cost $-\lambda$ associated with the stock of waste, and optimality requires that this cost will be decreasing, $-\dot{\lambda} < 0$. Then, at any point to the left of this locus, optimality necessitates increasing the social cost for waste emission. Hence, going from south to north in figure 2 the sign sequence of $-\dot{\lambda}$ is $(-, 0, +)$. The arrows in the phase diagram summarize this information. An optimal path, starting at some initial state (M, λ) and converging to the steady state SS in shown in figure 2.

4 Environmental policies

The model developed in the previous sections can be used to address many issues of environmental policies. This section examines its implications for the effects on changes in subsidies and taxes. In this setup, the subsidy s is a rebate which rises the marginal value of an additional unit of capital A. We suppose that the subsidy s is financed through the corrective taxes on waste whose value is λ. In fact, the costate variable λ can be seen as a Pigouvian tax on waste emission. This matter is examined deeply in the next section. It is, however, evident that there are several points at which either an environmental subsidy or a tax could be applied. This choice depends on waste targets. In what follows we assume that the subsidy s affects the marginal value of the abatement capital, while λ is a tax on emissions.

4.1 The effects of subsidies

By now, notice that when there is a subsidy s of this form, the (social) q value can be written as

$$q^* = \frac{b+s}{\mu}\frac{\beta A^{\beta-1}}{(\rho+\delta)}$$

Therefore, a rise of s will increase the social profit and stimulate society to enlarge the stock of abatement capital. The rise of s shifts the $\dot{q} = 0$ locus up, moving the economy to a new long run equilibrium. Since rents are higher, for a given capital stock A, smaller capital gains are needed to be willing to demand additional units of abatement capital. From our analysis of phase

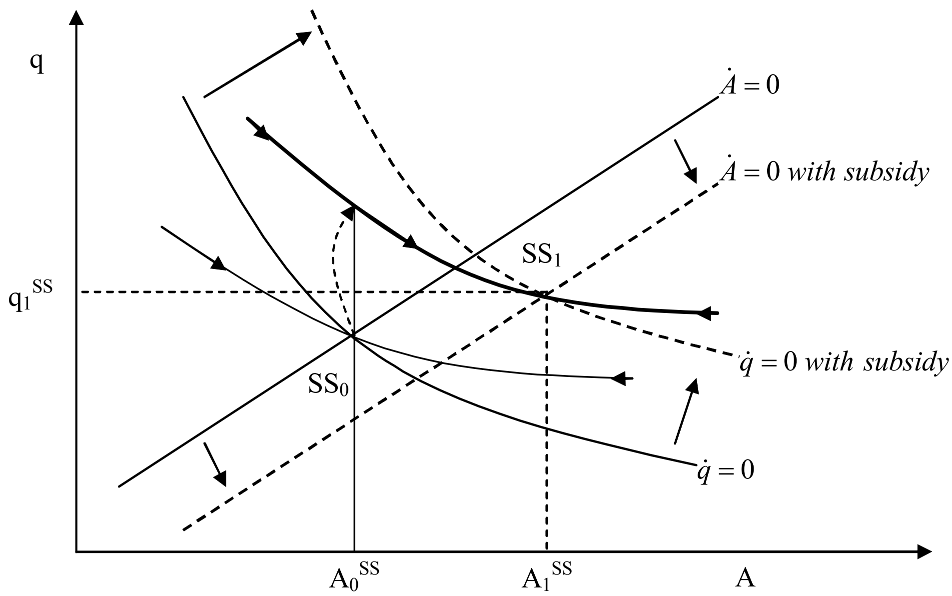

Figure 3: The effect of a permanent subsidy

diagrams, we know the effect of a higher subsidy: q jumps immediately on the new saddle path for a given capital stock. Then, A and q move down the path to the new steady state.

4.1.1 A permanent subsidy

Assume that the subsidy is *permanent.* Such a change shifts both the locus $\dot{q} = 0$ and $\dot{A} = 0$. In fact, along the $\dot{q} = 0$ locus we have that $q^* = \frac{b+s}{\mu}\frac{\beta A^{\beta-1}}{(\rho+\delta)}$. This implies that the derivative of q with respect to s is positive. Therefore, q rises when s rises. Similarly, along the locus $\dot{A} = 0$ the effect of an increase in s is given by $A_s = \frac{1}{\delta\epsilon}q_s > 0$, so that the effect of a change of s on A is positive. Let's focus on the case $q_s > A_s$. The phase diagram, therefore, looks as the one drawn in figure 3.

Figure 3 states that initially $(t < t_0)$ society is doing replacement investment just sufficient to maintain its abatement capital stock A_0^{ss}. Following the rise in s the marginal q value and the corresponding rate of investment rises instantaneously to a new higher level, causing the society's abatement capital stock to gradually rise to the new steady state level A_1^{ss}. But, the

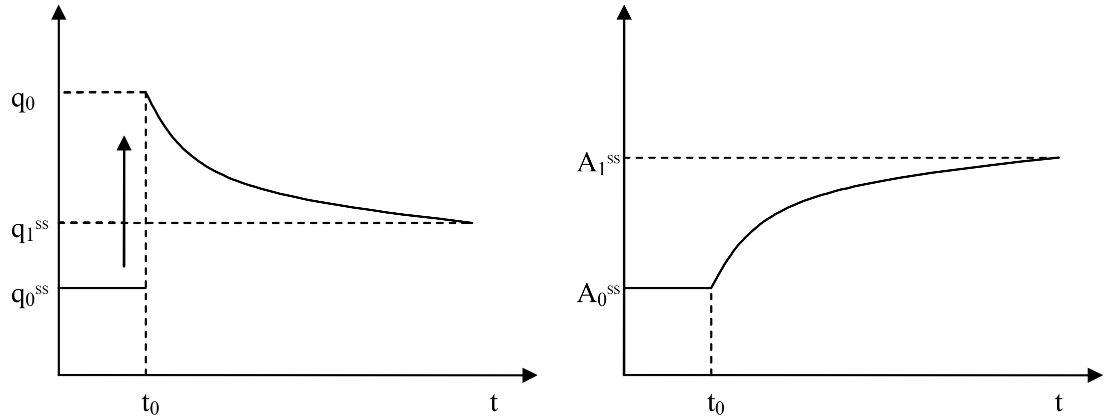

Figure 4: The effect of a subsidy on q and A

only path which leads to the new steady state, with no future jumps in q, is the new stable path. Therefore, q jumps instantaneously to q_0 and thereafter moves down the stable path. The time paths of q and abatement capital stock A therefore look like the ones drawn in figure (4).

The intuition behind this result is straightforward. A permanent subsidy s raises the abatement capital stock chosen by the society. However, in the short run since the abatement capital stock A cannot be adjusted instantaneously because the additional social cost c, existing capital earns rents and so its shadow value q rises strongly. The higher shadow value stimulates society to invest enlarging the abatement capital stock. But, given the assumption of decreasing marginal utility of A, the q value decreases over time until the shadow marginal value of the capital reaches the new long run equilibrium q_1^{SS}.

It is remarkable that initially the q_0 value and the associated investment jump *overshooting* the new long run equilibrium levels and then gradually fall to converge to the steady state. This happens because intertemporal optimality requires that at the time t_0 the dynamics of the system (28) must be on the saddle path in order to converge towards the steady state without discontinuity. Indeed, if the dynamics of q were discontinuous, it would be impossible to define $\dot{q}$, neither the investment equation or condition (21). Finally. it is worthwhile to note that, as the example of a permanent subsidy s shows, abatement investment is higher when s has recently rises than when it has been low for an extended period. This impact of the change in subsidy on the level of abatement investment can be labelled as *accelerator* mechanism.

4.1.2 A temporary subsidy

Now consider an increase in subsidy that is known to be *temporary*. This case is illustrated in 5 where s rises in the short run, but eventually will lead back to its initial level (say $s = 0$). Assume that the economy begins at time t_0 in the steady state SS_0. After the subsidy increase, there is an unexpected upward shift of the marginal q; but then the economy anticipates that q will return to its initial level at some later time T. The key insight needed to find the effect of this change is that there cannot be an anticipated jump in q. Therefore, at time T, both the variables A and q must be on the saddle path leading back to the initial long-run equilibrium: if they were not, q would have to jump to get back to its long-run equilibrium. But q is a continuos function and cannot jump between t_0 and T.

Together these facts tell us how the society responds to the temporary subsidy. At time t_0, q jumps immediately to the point B such that, given the initial capital, q and A reach the old saddle path at exactly time T. This is shown in 5 where q jumps from point SS_0 to point A at the time t_0 when the subsidy is adopted; then they move gradually to point C, arriving there at time T. Finally, they move to the old steady state SS_0.

This analysis of the subsidy has several implications. First, temporary subsidy rises abatement investment. Second, comparing figures 3 and 5 emerges that q rises less than it does if the increase in subsidy is permanent, so that investment responds less. Finally, figure 5 shows that the path of A and q crosses the locus $\dot{A} = 0$ before it reaches the old saddle path. Therefore, abatement capital stock begins to decline before temporary subsidy return to initial level (along the old saddle path). Indeed, when only a brief period of high subsidy remains, the society finds optimal to reduce investment immediately since it is too costly to adjust capital stock later.

Therefore, the main outcome of our analysis is that the temporary subsidy does not change the long-run equilibrium of the economy. In addition, it is not just current subsidy, but its entire path over time that affect abatement investment and capital (and respectively waste accumulation). Finally, the comparison of permanent and temporary subsidy shows that abatement investment is higher when subsidy is expected to be higher in the future than when it does not. Thus, expectations of high subsidy in the future raises

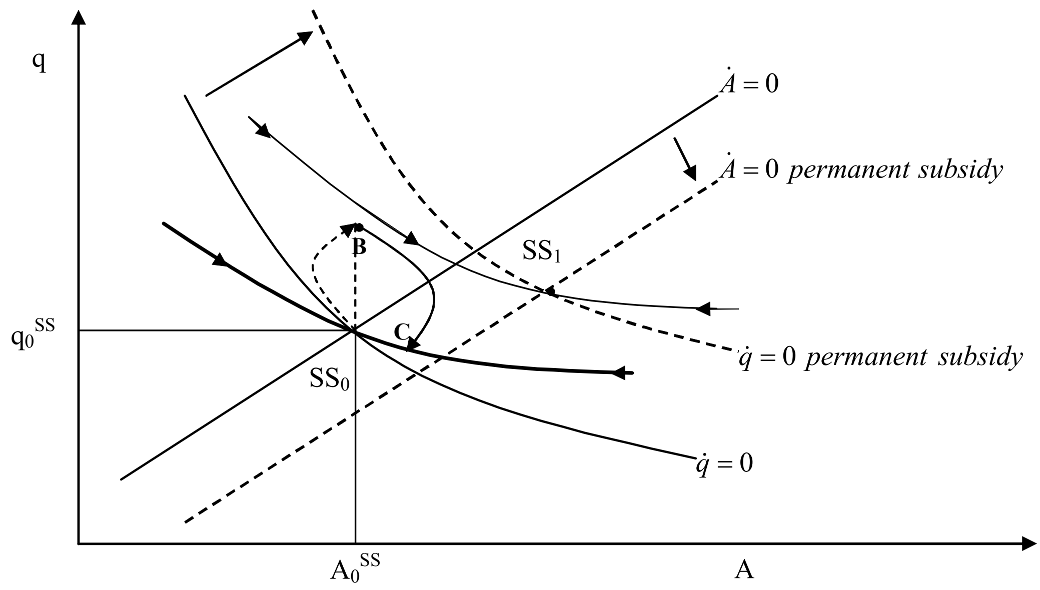

Figure 5: The effect of a temporary increase in subsidy

current abatement investment, reducing waste accumulation.

4.2 The effects of taxes

An environmental tax is often proposed as a way to attain waste targets. In most countries, the dominant method to control waste accumulation has been the use of direct controls over polluters, as consumers are in our setup. But, in the present model waste accumulation will also depend on the shadow value of abatement investment and the internalized *externality* caused by the (positive) accumulated effect of defensive capital goods.

4.2.1 Socially optimal tax

As said above, equations (15) and (17) give the shadow net price μ of the environmental resource X in terms of consumption and abatement investment. It is useful to rewrite these equations as

$$\mu = \lambda(1+h) + a\alpha\left(X^C\right)^{\alpha-1} \tag{32}$$

and

$$\mu = \frac{\psi}{1+\epsilon X^E} \tag{33}$$

They state that the gross price μ of the environmental resource X equals the social cost of waste plus the private marginal utility of consumption; and that the gross price μ must equal the net private value of abatement investment, including the adjustment cost, derived from an additional unit of capital goods. Or, more simply, the marginal social value of environmental resource X in its alternative forms must be equated so that

$$\frac{\psi}{1+\epsilon X^E} = \lambda(1+h) + a\alpha \left(X^C\right)^{\alpha-1} \tag{34}$$

It will be convenient to rearrange equation (34) to

$$\lambda = \frac{\psi - a\alpha \left(X^C\right)^{\alpha-1} \left(1+\epsilon X^E\right)}{(1+h)\left(1+\epsilon X^E\right)} < 0 \tag{35}$$

We can read this equation saying that in a perfectly functioning market, where social costs and benefits are internalized in market process, the gross price of waste λ (Pigovian tax) depends on several different components: the private shadow value ψ of abatement investments; the disutility of consumption $-a\alpha \left(X^C\right)^{\alpha-1}$; the environmental efficiency of the technology hX^C producing consumption goods; and the additional environmental cost $\left(1+\epsilon X^E\right)$ of producing the abatement capital. Therefore, we can interpret λ as the *socially optimal tax* which brings market price of waste into line with the optimal social allocation of resources. Note that if damages and benefits are not internalized, the price λ is not efficient and the intertemporal allocation of the resources is suboptimal.

Recall, however, that the system (31) provides the steady state of waste price, in terms of its social cost. Hence, our model implies that in the long run equilibrium it must be satisfied the condition

$$\lambda \equiv -\frac{g\gamma M^{\gamma-1}}{(r+v)} = \frac{\psi - a\alpha \left(X^C\right)^{\alpha-1} \left(1+\epsilon X^E\right)}{(1+h)\left(1+\epsilon X^E\right)} \tag{36}$$

This condition states that any variations of the parameters in condition (36) causes a reallocation of the environmental resource X between consumption and abatement goods, shifting the economy towards the new long-run equilibrium. Therefore, equation (36) is a necessary condition for the optimal solution of our waste problem. The value of λ depends on some parameters. Note that if $h = a = \epsilon = 0$, the previous formula reduces to $\lambda = \psi$. To understand the meaning of this equality recall that ψ is the shadow price of abatement capital. Thus, it quantifies the amount of social utility gains when a unit of environmental resource is diverted from polluting consumption to defensive expenditure. Finally, note that any changes of the magnitude of the parameters in equation (36) affects the optimal value of taxation λ. Our model can be used to investigate this matter.

4.2.2 A permanent rise in tax

For simplicity, assume that the tax λ rises *permanently.* The q value is not affected by a change of λ. But, equation (16) states that an higher λ increases the social charge for waste emissions, boosting society to reduce consumption and the use of the input X^C (recall $\lambda < 0$ and $(\alpha - 1)^{-1} < 0$). Obviously, from the constraint (12) X^E must increase. The most noteworthy feature of this change is the following: when the tax λ is high the level of abatement capital A will be high as well, but the waste stock M will be (relatively) low. Likewise, when the tax λ is low the abatement capital A will be low as well, and the waste stock M will be (relatively) high. This may seem puzzling: society holds a large amount of abatement capital when waste is low. We call this miss-allocation of the environmental resource X as *environmental dynamic inefficiency.* To see this possibility as simple as possible, look at the graph in figure (6). Note that the net effect of a rise of tax λ is to decrease the q value, rising the capital stock A in the long run.

4.2.3 A temporay tax

Now, consider a *temporary* rise of tax λ and its effect on waste. The initial effect of this change is to shift, in the phase space (M, λ), the demarcation line $\dot{\lambda} = 0$ on the left. In addition, from the previous section we know that an increase of λ rises X^E reducing the marginal value of the abatement investment, so changing the saddle path which converges to the equilibrium. But, when an increase in tax is known to be temporary, it is also known that the system will return to its initial path at some later time T. An example of this dynamics is illustrated in figure 7.

The temporary increase of a corrective tax causes waste stock to fall to a point where the dynamics of M and the tax λ bring them back to

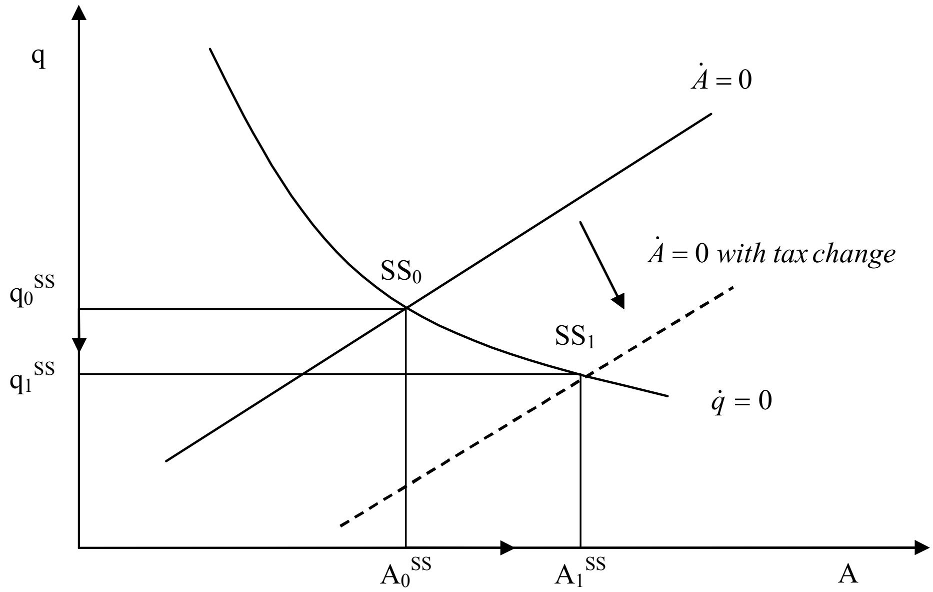

Figure 6: A permanent change in tax

the old saddle path just as the variation of tax expires. Then, they will move up that saddle path back to the initial steady state SS_0. As the figure 7 shows, λ does not fall all the way to its value on the new saddle path. Therefore, a temporary change of λ decreases the social cost of waste less than a comparative permanent tax does. The reason is that because the temporary tax does not lead to a permanent decrease in consumption, it causes a smaller reduction in the social cost associated to the existing waste stock.

Finally, note that the figure (7) shows that under the temporary tax, the social cost λ of waste is rising in the later part of the period that the increase of tax is in effect, even when waste is decreasing. Therefore, after a point, the corrective tax leads to a growing social cost and, eventually, to a growing waste accumulation as abatement capital stock begins to decline before corrective tax returns to normal. In other words, society anticipates that future tax will decrease, rising future consumption. As before, this result implies that it is not just current environmental tax, but its entire path over time that affects waste and investment accumulation.

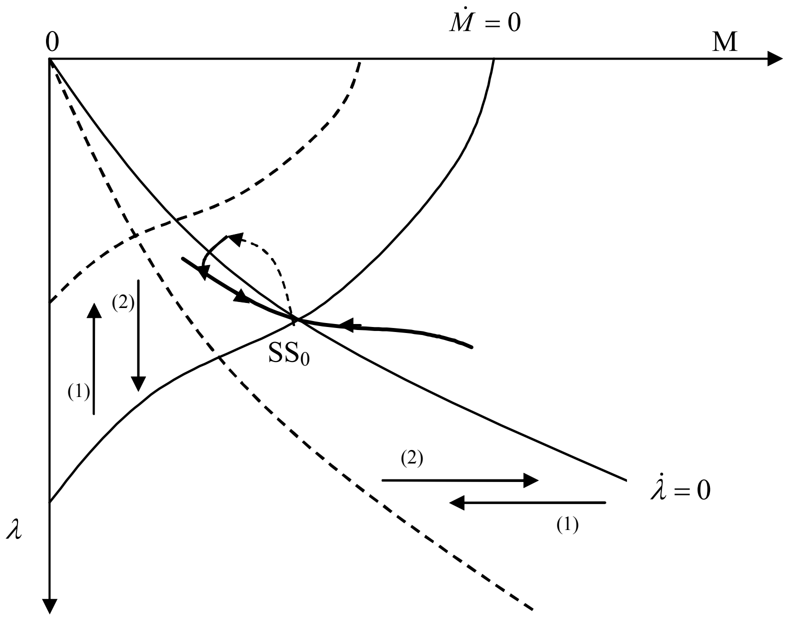

Figure 7: The effect of a temporary change in tax

5 Conclusions

In this paper we have studied a control problem of waste emission and abatement investment with fixed environmental resources. The innovative view of our model is that abatement capital is assumed to enter the welfare function as a public good, contrasting the disutility provided by the waste emissions (public bad). Therefore, the intertemporal maximization of welfare requires a social price to be associated with defensive capital, and a social cost to be associated with waste flow. The interrelation between the shadow prices, the social discount rate, the biological decay rate and the depreciation rate determine the optimal allocation of environmental resources over time.

Our model has a special property: the use of subsidies to promote abatement investment, and the introduction of corrective taxes to reduce waste emissions may have ambiguous effect on the allocation of natural resources. Indeed, in response to subsidies or taxes a society might accumulate *less* abatement capital than desidered, allowing the stock of waste to rise in the long run. In addition, as we have shown, the adoption of either temporary subsidies or temporary corrective taxes do not change the long-run equilibrium of the economic–biological system. Indeed, it is not just current subsidies or taxes, but their entire path over time that affect abatement investment decisions and waste accumulation. Thus, expectations about how long the subsidies and taxes would last have a special role in affecting defensive investment decisions and waste accumulation.

Traditionally, the control of waste and the accumulation of abatement capital goods use as device a system of Pigouvian prices. Our problem provide "user" charges which should redirect natural resources and its uses towards the alternative activities chosen optimally by the society. A socially optimal tax is derived as the solution of the intertemporal problem. As it is shown, the gross price of waste λ depends on the private shadow value of abatement investment, the disutility of consumption, the efficiency of the technology to produce consumption goods, and the non-linear factor component which strengthens the ability of abatement investments to absorb waste emissions. However, as it is clear from our previous discussion, changes of the "user" charge influence not only the accumulation of waste, but also the dynamic of the abatement investment through the marginal $q(\lambda)$ value of capital which is directly affected by the cost of waste. Hence, one important implication of our model is that waste accumulation and its social cost affect the optimal pattern of the abatement investment, altering its marginal value through the attributed social taxes.

Based on these considerations, the public bad problem of waste accumulation is actually much more complex than the way it is usually presented. Therefore, we believe that the issues raised in this paper need to be taken into account to provide a comprehensive analysis of the intertemporal problems related to waste emission and alternative resource uses. In fact, from our analysis it is apparent that the difficulties in designing environmental policies to defend environment and welfare are immense.

The present model is not conclusive, but it opens new perspectives concerning the efficient and optimal use of natural resources. The next step would be to introduce irreversibility and uncertainty. Further, abatement capital stock may be heterogeneous or located in such a way that costs may differ for different kind of stocks. Finally, the adjustment cost may be a logistic curve with an upper ceiling. Each of these issues can contribute to generalize the basic framework.

References

Ansar, J. and R. Spark (2009). The experience curve, option value and the energy paradox. *Energy Policy*, 37, 1012-1020.

Bovenberg A.L. and S. Smulders (1995). Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. *Journal of Public Economics,* 57, 369-391.

Brock W.A. (1977). A polluted Golden Age. In V. Smith *Economics of Natural and Environmental Resources*, 441-4661, ed. Gordon and Breach, New York.

Brock W.A. and D. Starrett (2003). Nonconvexities in ecological management problems". *Environmental and Resource Economics*, 26, 575-602.

Clarke H. and W. Reed (1994). "Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse", *Journal of Economic Dynamics and Control,* 18, 991-1010.

Dasgupta P. (1982). The control of Resources. Basil Blackwell, Oxford.

Dasgupta, P.S. and K. G. Mäler (2000). Net national product, wealth and social well-being. *Environment and Development Economics* 5, 69-93.

Dinda, S. (2004). Environmental Kuznets Curve Hypothesis: A Survey. *Ecological Economics*, 49, 4, 431-455.

Färe R., S. Grosskopf, and C. A. Pasurka Jr. (2007). Pollution abatement activities and traditional productivity. *Ecological Economics*, 62, 3–4, 1673-682.

Forster, B. A. (1973). Optimal capital accumulation in polluted environment. *Southern Economic Journal, 39, 544-557.*

Forster, B. A. (1975). Optimal pollution control with non.constant exponential decay rate. *Journal of environmental Economics and Management,* 2, 1-6.

Hanley, N. and C. Spash (1993). *Cost-Benefit analysis and the environment.* Edward Elgar, Aldershot.

Hartwick, J.M. (1990). Natural resources, national accounting and economic depreciation. *Journal of Public Economics,* 43, 291-304.

Lin, T. T., C. Ko and H. Yeh (2007). Applying real options in investment decisions relating to environmental pollution. *Energy Policy,* 35, 2426-2432.

Lin, T. T. and S. Huang (2010). An entry and exit model on the energy-saving investment strategy with real options. *Energy Policy,* 38, 794-802.

Lin, T. T. and S. Huang (2011). Application of the modified Tobin's q to an uncertain energy-saving project with the real options concept. *Energy*

Policy, 39, 408-420.

Pearce D.W., and C.A. Nash (1981). *The social appraisal of projects: A text in cost benefit analisys.* Harvester Wheatsheaf, Hemel Hempstead.

Perman R., Ma Y., McGilvray J. and Common M. (2003). *Natural Resources and Environmental Economics,* ed. Perman Education Limited, Essex, thrird edition.

Pindyck, R. S. (2000). Irreversibilities and the timing of environmental policy. *Resource and Energy Economics,* 22, 233-259.

Pindyck, R. S. (2002). Optimal timing problems in environmental economics. *Journal of Economic Dynamics & Control,* 26, 1677-1697.

Plourde C.G. (1972). A model of waste accumulation and disposal. *Canadian Journal of Economics*, 5, 119-125.

Ramsey F. (1928). A mathematical theory of saving. *Economic Journal*, 38.

Saltari E. and G. Travaglini (2011*a*). The effects of environmental policies on the abatement investment decisions of a green firm. *Resource and Energy Economics*, Elsevier, vol. 33, 3, 666-685.

Saltari E. and G. Travaglini (2011*b*). Optimal abatement investment and environmental policies under pollution, Chap. 20 in *Frontiers of Economic Growth and Development,* ed. O. de La Grandville, Emerald.

Schulze, P.C. (1994). Cost-benefit analyses and environmental policy. *Ecological Economics*, 9, 3, 197-199.

Smith V.L. (1972). Dynamics of waste accumulation: Disposal versus recycling. *Quarterly Journal of Economics*, 86, 600-616.

Smulders, S. and R. Gradus (1996). Pollution abatement and long-term growth. *European Journal of Political Economy,* 12, 505-532.

Solow R. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics,* 70, 65-94.

Solow, R. (1986). On the intergenerational allocation of natural resources. *Scandinavian Journal of Economics,* 88, 141-149.

Travaglini G. (2012a). Pollution control: targets and dynamics. *Working Papers 1201*, University of Urbino Carlo Bo, Department of Economics, Society & Politics.

Travaglini G. (2012b). Trade-off between labor productivity and capital accumulation in Italian energy sector. *Journal of Policy Modeling*, 34, 1, 35-48.

Van der Ploeg, F. and C. Withagen (1991). Pollution control and the Ramsey problem. *Environmental and Resource Economics,* 1, 215-236.

Withagen, C. (1995). Pollution, abatement and balanced growth. *Environmental and Resource Economics,* 5, 1-8.

Weitzman M.L. (1976). On the welfare significance of the national product in a dynamic context. *Quarterly Journal of Economics,* 90, 156-162.

Xepapadeas, A. (2005). Economic growth and the environment. in *Handbook of Environmental Economics,* edited by K.G. Maler and J. Vincent, ed. North-Holland, Amsterdam.