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2012

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MPRA Paper No. 36599, posted 13 Feb 2012 18:00 UTC

On the Optimality of Limit Cycles in Nature Based-Tourism

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Abstract

Virgin nature, as well as historical and cultural monuments located in National Parks, all form part of our national heritage. Tourism and recreation allow visitors to National Parks to enjoy nature, to reinstate, recover and broaden their personal outlook, to experience local history, culture, flora and fauna and to interact with the environment harmoniously. One of the objectives of the administration of a Governmental Institution 'National Park' is to maximize profits from tourism and recreation, where profit is defined as the difference between the revenues from visitors and the sum of expenditures on recreation investments and defensive expenditures for ensuring the preservation of natural and cultural heritage. This paper is an attempt to model some relevant aspects of these prey-predator relations. The model is formulated in terms of optimal control theory, and then is transformed into an 'augmented' dynamic system by meas of the optimal choice of control variables resulting form the application of Pontryagin's Maximum Principle. It turns out that, for reasonable parameter values, the optimal trajectory exhibits a cyclical behavior.

Keywords: bioeconomic model, tourism, optimal dynamic control model, optimal policy mix, financing and protected areas.

1 Introduction

Virgin nature, as well as historical and cultural monuments located in National Parks (hereinafter NPs) are part of our national heritage. Tourism and recreation allow visitors to National Parks to enjoy nature, to reinstate, to recover and broaden their personal outlook, to experience local history, culture, flora and fauna and to interact with the environment harmoniously. One of the objectives of the administration of a Governmental Institution 'National Park' (hereinafter GINP) is to regulate the access of tourists and visitors into a park (on their own as well as through encouraging private sector activities in this sphere), while ensuring the preservation of its natural and cultural heritage. There is one more reason to address the issue of regulated tourism and recreation in NPs. The term "regulated tourism and recreation" means the restriction by NPs authorities, through different regulatory and control mechanisms and activities, of the governments of tourists and operations of visitors' services within a NP and adjacent to its borders. The movements and services should agree with admissible anthropogenic pressures and should not inflict damage to the environment or to historical and cultural heritage. This means that NP develops and implements a system of management and economic measures that aim to involve tourists, tourist agencies and investors in the NP and create (especially around its borders) a highly efficient tourism infrastructure that promotes the integration of the NP into the social and economic structure of the region. The purpose of NPs and their recreation potential allows the development of various type of regulated tourism within both the NP and its transition areas (see [11], [13]). Such and approach facilitates the diversification of the tourist product, provides better employment opportunities and creates various income sources for local people.
The main result is that, for reasonable parameters values, the optimal solution exhibits a cyclical behavior which can be explained as follows. Assume that initially, a narrow number of visitors and one high natural resource. Since the GINP invests in order to increase the number of visitors, such increment leads to damage the natural resources, with a consequent decrease of the visitors and a consequent increment of the effort by the GINP in order to defend the stock of aggregated resources. The defensive action leads to an increment of the resource stock and resumes the increment of the visitors (for more works how the human activity may alter the natural evolution of biological species, see e.g. [3], [1], [2], [7], [12], [14], [15] [**?**]).
This paper is organized as follows. Section 2 illustrates the formulation of the problem. Section 3 defines the model. Sections 4 analyze the dynamics by means of the Pontryagin Maximum Principle and bifurcation theory. Section 5 outlines the conclusions.
This research is inspired by the article [9] that deals with an optimal development of a pollution generating tourism industry.

2 Formulation of the problem

The model problem is formulated with two state variables:

- $x(t)$ vulnerable natural resources
- $V(t)$ a number of persons visiting a NP for the period of 24 hours or one night for recreative, sports, religious, educational or other purpose with no gainful activities. Further in this paper, the synonym 'visitor' is used, because it is simpler and clearer. Besides, a tourist is always a visitor to a NP.

As in [4], $x(t)$ is the total stock of species (wild resources) comprising the various populations or species found in a given area, aggregated in some appropriate way [1]. To activate a regulated tourism and recreation, GINP invests I, in NP tourism infrastructure, (i.e. the sum of tourism accommodation facilities, transport, feeding facilities, entertainment facilities and natural, historical and cultural monuments and objects located in NP), and it spends "to defend" the aggregated biomass of the park per time unit is denoted by E.
The manager of a National Park aims to maximize the cash flow resulting from the visitors.
Assuming an infinite planning period, denoting by r the time preference, which is exogenously given and constant over time, the GINP objective function is given by (see [8]):

$$\max_{I,E} \int_0^\infty (pV - C(I) - bE)e^{-rt}dt \tag{1}$$

where I and E are the control variables, p is the (constant) revenue visitors, b is the (constant) amount of money needed to activate one unit of enforcement and $C(I)$ are investment costs increasing and convex in I.
Define with $f(I, x)$ the function increasing in the number of visitors and with $g(x)$ the increasing of the biomass of the park, with $\psi(V)$ the function that defines the negative impact that visitors have on the natural resource and finally with $\phi(E)$ the function that defines necessary efforts in order to defend the ecological biomass of the park. Dynamics of interaction between the resource of the park and the number of the visitors can be defined as follows:

$$\begin{aligned} \dot{V} &= f(I, x) - aV \\ \dot{x} &= g(x) - \psi(V) + \phi(E) \end{aligned} \tag{2}$$

where the parameter $a > 0$ gives the decline in the number of tourists due to crowding effects. This means that a NP becomes less attractive when a lot of tourists visit that NP, leading to a decrease in the number of visitors.

Assumptions (Properties of f, g, ψ, ϕ)

[1] For example,the stock of species could be aggregated by number of individuals in each distinct population or species. Alternatively, the total 'biomass' of each population or species could be aggregated across all species. Some form of 'weighted average' combination of the two approaches could also be employed

a) $f_I > 0$, $f_{II} \leq 0$, $f_x > 0$, $f_{xx} \leq 0$, $f_{Ix} > 0$

b) $g_x > 0$, for $0 < x < \overline{x}$, $g_x < 0$, for $\overline{x} < x < k$, with $g_{xx} < 0$.

c) $\psi' \geq 0$, $\psi'' \geq 0$

d) $\phi' > 0$, $\phi'' < 0$

■

The Assumption *a*) states that both investment and the resource of the park influences positively the increase of the number of visitors and that the function f is not globally convex in the variables I and x.
The Assumption *b*) states that the function g that is chosen is a logistic function, where $\overline{x}$ is golden rule resource stock with $f' = 0$ and k the environmental carrying capacity i.e. $f(k) = 0$.The Assumption *c*) implies that the function ψ is convex.
The Assumption *d*) states that the function ϕ is concave, we can consider that the rate of growth of defensive expenses decrements with the increment of the resource.

3 Optimal Control Problem

Apply the Pontryagin's Maximum Principle to solve the model (1)-(2).
The current value Hamiltonian function is:

$$H = pV - C(I) - bE + \lambda(f(I,x) - aV) + \mu(g(x) - \psi(V) + \phi(E)) \tag{3}$$

where λ, μ are respectively the multipliers of V and x, and they have the usual interpretation of shadow "prices". Pontryagin's maximum Principle provides the necessary conditions of this problem. Let I^* and E^* be a solution of the problem defined above and let V^* and x^* be the optimal paths of associated state variables. First-order necessary conditions for optimality are given by following equations:

$$\dot{\lambda} = (r + a)\lambda - p + \mu\psi' \tag{4}$$

$$\dot{\mu} = (r - g_x)\mu - f_x(I,x)\lambda \tag{5}$$

Moreover, the Maximum Principles includes the condition that the control variables maximize the Hamiltonian. Setting the derivatives of (3) with respect to control variables I and E to zero, it yields:

$$\frac{\partial H}{\partial I} = -C'(I) + \lambda f_I(I,x) = 0 \tag{6}$$

$$\frac{\partial H}{\partial E} = -b + \mu\phi'(E) = 0 \tag{7}$$

The initial values of co-state variables are fixed according to the transversality conditions:

$$\lim_{t\to\infty} \lambda(t)V(t)e^{-rt} = \lim_{t\to\infty} \eta(t)x(t)e^{-rt} = 0 \tag{8}$$

Note that such conditions are satisfied by every trajectory of system (2)-(6)-(7) approaching a fixed point or a limit cycle.
From (6), I depends from both λ e da x, that is $I = I(\lambda, x)$ then:

$$-C'(I(\lambda, x)) + \lambda f_I(I(\lambda, x), x) = 0$$

deriving further we obtain:

$$\begin{aligned} I_x &= \frac{\lambda f_{Ix}}{C'' - \lambda f_{II}} > 0 \\ I_\lambda &= \frac{\lambda f_I}{C'' - \lambda f_{II}} > 0 \end{aligned} \tag{9}$$

Remember that, f is no convex function Assumption a) and C is convex ($C'' > 0$),therefore the equations (9) are hold. The first equation of the (9) states that the investment for the services and infrastructures to the visitors grows with the increasing of the biomass of the park (or the conservation of the biomass), therefore the efficiency of unit additional of investment in terms of attraction for the visitors grows as grows the biomass (resource) of the park. While the second equation states that if the shadow price of the number of visitors is enough elevated the investment rate increases. From (7) follow that $E = E(\mu)$, deriving regarding μ follows that:

$$E_\mu = -\frac{\phi'}{\mu\phi''} > 0 \tag{10}$$

Remember that, ϕ is concave ($\phi' > 0, \phi'' < 0$), and (10) can be explained as it follows: when the price shadow of the resource park grows the visitors give greater value to the same resource and therefore the management of the park diminishes defensive expenses.

Note. From (9) and (10) follow the second-order necessary conditions, i.e. the Hamiltonian is concave in (I, E). The matrix of second derivatives of H with respect to I and E is negative defined since, for all I,E,V,x:

$$\begin{aligned} \frac{\partial^2 H}{\partial I^2} &= -C_{II} + \lambda f_{II} I_x < 0 \\ \frac{\partial^E H}{\partial E^2} &= \mu\phi_{EE} E_\mu < 0 \\ \frac{\partial^2 H}{\partial I \partial E} &= 0 \end{aligned}$$

The sign of the first two expressions above follows from convexity assumption. ■
Substituting the optimal choices of the control variables I and E in the equations (2)-(6)-(7) we obtain a dynamics system in the space (V, x, λ, μ):

$$\begin{aligned} \dot{V} &= f(I(\lambda, x), x) - aV \\ \dot{x} &= g(x) - \psi(V) + \phi(E(\mu)) \\ \dot{\lambda} &= (r + a)\lambda - p + \mu\phi' \\ \dot{\mu} &= (r - g_x)\mu - f_x(I(\lambda, x), x)\lambda \end{aligned} \tag{11}$$

Denote by $P = (V^*, x^*, \lambda^*, \mu^*)$, a fixed point, as solution of the nonlinear system $(\dot{V} = \dot{x} = \dot{\lambda} = \dot{\mu} = 0)$.

Proposition 1 *A necessary condition for* $(\dot{V} = \dot{x} = \dot{\lambda} = \dot{\mu} = 0)$ *is that* $r - g_x(x^*) > 0$

Proof. From Assumption (a), $\dot{\mu} = 0$ is met if and only if $r - g_x(x^*)$ is strictly positive ■

Proposition 2 *Let* $\underline{x}$ *such that* $g_x(\underline{x}) = r$*, if* $x^* > \underline{x}$*, then the optimal solution is met*

Proof. From the second of equations (11) follow that $g_x(x^*) = r - \dfrac{f_x(I(\lambda^*, x^*), x^*)\lambda^*}{\mu^*} < r$, but remembering that g_x is decreasing and $g_{xx} < 0$ implies that $g_x(x^*) < g_x(\underline{x}) = r$, therefore $x^* > \underline{x}$ ■

The Proposition 1 says that the discount rate must be greater than the rate of growth of the natural resource, besides a sufficient condition is that $x > \overline{x}$ $(g_x < 0)$. The Proposition (2) tells us that as the more we preserve the resource for the next generations (little r), the more the stock at equilibrium will be high. In other words a positive externality exists when (the GINP) attributes value to the resource.
Moreover from $\dot{\lambda} = 0$ follow that $\mu^* = \dfrac{p - (r + a)\lambda^*}{\phi'}$, then if p sufficiently large $\mu^* > 0$ is hold.

4 Analysis of the Model

4.1 Stability Analysis

Assuming that steady state point $P^\star$ exists, the local dynamics around that stationary state is determined by the eigenvalues of the Jacobian matrix at the the stationary point.
The Jacobian of the dynamic system (11) is derived as

$$J = \begin{pmatrix} -a & J_{12} & f_I I_\lambda & 0 \\ -\psi' & g_x & 0 & \phi' E_\mu \\ \mu\psi'' & 0 & r + a & \psi' \\ 0 & J_{42} & -J_{12} & r - g_x \end{pmatrix}$$

with $J_{42} := -\mu g_{xx} - \lambda f_{xx} I_x = \mu|g_{xx}| + \lambda|f_{xx}|I_x > 0$ and $J_{12} := f_x + f_I I_\lambda > 0$.
The eigenvalues of the Jacobian, are given by

$$\lambda_{1,2,3,4} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{K}{2} \pm \sqrt{\left(\frac{K}{2}\right)^2 - detJ}}$$

with K defined as (see [6])

$$K = \begin{pmatrix} -a & f_I I_\lambda \\ \mu\psi'' & r+a \end{pmatrix} + \begin{pmatrix} g_x & \phi' E_\mu \\ J_{42} & r-g_x \end{pmatrix} + 2\begin{pmatrix} J_{12} & 0 \\ 0 & \psi' \end{pmatrix}$$
$$= -a(r+a) - \mu\psi'' f_I I_\lambda + (r+g_x)g_x - \phi' E_\mu J_{42} + 2J_{12}\psi'$$

and with $detJ$ determinant of matrix J.
From analysis of K and $detJ$, arise the following proposition

Proposition 3 *If the effect of visitors on the environment is constant (ψ = constant), then the fixed point P^* is a saddle point with monotonous approach of the equilibrium on one or two-dimensional stable manifold*
moreover

a) if the NP manager carry out a defensive technology ($\phi = 0$) , the eigenvalues of the system (11) *are:* $\lambda_1 = -a$, $\lambda_2 = r - a$, $\lambda_3 = r + a$, $\lambda_3 = r - g_x$

b) if the NP manager no carry out a defensive technology ($\phi \neq 0$) , the eigenvalues of the system (11) *are:* $\lambda_1 = -a$, $\lambda_2 = r - a$, $\lambda_{3,4} = \frac{r}{2} \pm \sqrt{(r-g_x)^2 + 4J_{42}\phi' E_\mu}$

The proposition above states that, in the case a), if the intertemporal discount rate$r$ is low enough, the fixed point $P^\star$ is a saddle with a one-dimensional stable manifold (only one eigenvalue with negative real part), the fixed point cannot be (generically) reached, while in the case b) if the defensive technology ($\phi' E_\mu$) is sufficiently high at the fixed point, the fixed point is a saddle with two-dimensional stable manifold (two eigenvalue with negative real part). If case (b) holds, given the initial values of V and x, there exists (at least locally) a single initial value of λ and μ (determined by the representative agent) from which the economy approaches the fixed point.

Furthermore, looking at the formula for eigenvalues, one immediately realized that the eigenvalues are symmetric around $\frac{r}{2}$. Since $r > 0$ holds, this implies that the system is never completely stable (in the sense that all eigenvalues have negative real parts), it can be a saddle-point stable. From an economic point of view, saddle-point stability means that all variables are constant in the long run. That is a constant level of investment and level of defensive expenditures which are chosen in a way that the number of visitors in the NP also remains. Further, more the stock of environmental resource (or environmental quality) is also constant because the deprecation caused by visitors equals the absorptive capacity of nature. In this case, we may speak of a sustainable development since the environmental quality remains constant in the long run. The transitional behavior of the variables in case of saddle -point stability is characterized by unimodal time paths if the eigenvalues are real. If the eigenvalues are complex conjugate, however, the variables dispose cyclical oscillations until the stationary point is reached. This means that there are

periods with high investment I and defensive expenditure E followed by periods with low investment and defensive expenditures. But, in the long run, both I and E are constant and as high enough that the number of visitors is kept at a constant level. Besides convergence to the stationary state in long run, the system shows persistent endogenous cycles. In this work,in particular we are interested in the question of whether endogenous persistent cycles may occur for certain values of discount rate (r) and of technology parameter (ϵ) that measures the effectiveness of defensive expenditures.

4.2 Specification of the model

In order to illustrate the possibility of persistent cycles we choose the following functions

$$\begin{aligned} &f(I,x) = f_1(I)f_2(x) = I\frac{x}{1+hx} \\ &g(x) = \alpha_1(k-x)x \\ &\psi(V) = \gamma V^2 \\ &\phi(E) = \epsilon\frac{E^\alpha}{\alpha} \\ &C(I) = \frac{1}{2}\beta I^2 \end{aligned} \tag{12}$$

The function $f(I,x)$ depends upon recreation investments ($f_1(I)$) and the culture of the visitors and, in particular, upon their sensitivity to the quality of the natural resources ($f_2(x)$), their ability to detect it. The first component is also modeled through linear function, while the attractiveness of the environment can be modeled as an increasing and saturating function of x, that is described as a Monod function [5].

The function $g(x)$ represents the quality of the environment x, in the absence of visitors and investment it is described by the well-know *logistic equation*, where the parameter α_1 is a measure of the maximum possible growth rate, while $k > 0$ represents the carrying capacity of the natural resource (i.e. the value that x reaches as $t \to \infty$).

Note that the quadratic form of the function $\psi(V) = \gamma V^2$ ensures, in contrast to Proposition 3, that $\psi'' = \gamma > 0$.

Finally, both the functions $\phi(E)$ and $C(I)$ are chosen so as to satisfy the Assumptions in Section 2.

From (12) the system (11) become:

$$\begin{aligned} &\dot{V} = \frac{\lambda x^2}{\beta(1+hx)^2} - aV \\ &\dot{x} = \alpha_1(k-x)x - \gamma V^2 + \frac{\epsilon}{\alpha}\left(\frac{\epsilon\mu}{b}\right)^{\frac{1}{1-\alpha}} \\ &\dot{\lambda} = (r+a)\lambda - p + 2\mu\gamma V \\ &\dot{\mu} = (r - \alpha_1(k-2x))\mu - \frac{\lambda x}{\beta(1+hx)^3} \end{aligned} \tag{13}$$

The system (13) is too complicated to generate analytical results, therefore, we have to rely on numerical method.

4.3 A Numerical examples

Let me assume that the values of parameters of the dynamics (13) are as follows: $\alpha = 0.5$, $a = 0.01$, $p = 0.008$, $\beta = 1.4$, $k = 1.5$, $h = 5$, $\gamma = 5$, $\alpha_1 = 0.05$, $b = 0.5$; The discount rate r and technology parameter ϵ are the bifurcation parameters.
Figure 1 show the equilibrium curve in the plane (r,x), fixed $\epsilon = 0$ (absence of defensive expenditures). We find two Hopf bifurcation, H^S, with coordinates $P^\star(H^s) = (0.056017, 0.251297, 0.06323, 0.00642)$ and $r_{H^s} = 0.059592$ and H^f, with coordinates $P^\star(H^f) = (0.074306, 0.648171, 0.044532, 0.00076)$ and $r_{H^f} = 0.168374$. The continuous line represents the equilibrium characterized by reachable fixed points (two eigenvalues with negative real part and two eigenvalues with positive real part), while the dashed line represent not reachable fixed points (only one eigenvalue negative positive real part).
Now we consider codimension-2 bifurcation of the system (13) as two parameters r and ϵ are varying (see Figure 2).
Figure 2 shows a Hopf curve, starting the H^S point and ending in H^f point. At $(r, \epsilon) = (0.16436, 1.37881)$ a generalized Hopf bifurcation was found, where the first Lyapunov coefficient vanishes (label GH)[2]. The GH has coordinates $P^\star(GH) = (0.074905, 0.629766, 0.045512, 0.000086)$. This codimension-2 bifurcation is nondegenerate, since the second Lyapunov coefficient $l2 = -0.1131215$ nonzero.
The continuous line corresponds to the supercritical Hopf bifurcation generating *stable* limit cycles, while the dashed line corresponds the subcritical Hopf bifurcation generating *unstable* limit cycles.
Moreover, Figure 2 shows that the plane (r, ϵ) is divided in two areas: the shaded area separates the unreachable fixed points to reachable fixed points.
Also Figure 2 shows two supercritical Hopf bifurcations H_1 and H_2 and one subcritical Hopf bifurcation H_3. Figure 3 shows a family of limit cycles (varying r) starting from Hopf bifurcation H_2 ,generating attractive limit cycles and starting from H_3 generating repulsive limit cycles.
Figure 4 illustrates how, fixed $\epsilon = 1$, a family of limit cycles which starts from supercritical Hopf bifurcation (H_1), varying r, approaches, the supercritical Hopf bifurcation H_2 "through" the area of the plan consists of fixed points can not be reached. Once you reach the point H_2, the attractive limit cycles return back to the point H_1. This means that a trajectory that arises inside the "ellipsoid" (see Figure 4) remains within it.

[2]The generalized Hopf (GH) is a bifurcation of an equilibrium in a two-parameter family of autonomous ODEs at which the critical equilibrium has a pair of purely imaginary eigenvalues and the first Lyapunov coefficient for the *Andronov-Hopf bifurcation* vanishes. This phenomenon is also called the *Bautin* bifurcation.

4.3.1 Discussion of the limit cycle

Figure 6 shows the cyclical time path of visitors $V(t)$, of the 'stock' $x(t)$ of the environmental resources, of investments $I(t)$ and of environmental defensive expenditures $E(t)$. Notice that $I(t)$ and $V(t)$ follow very similar time paths; that is, investments do not counter the cyclic behavior of visitors. On the contrary, the evolution of the other control variable, $E(t)$, is inversely correlated with the evolution of the stock $x(t)$; that is, defensive expenditures tend to dampen the oscillatory behavior of $x(t)$. The link between the two state variables $V(t)$ and $x(t)$ is obvious; in particular, the path of $V(t)$ follows that of $x(t)$ with a time delay; the cyclic behavior is generated by an interaction of the prey-predator type, where $V(t)$ represents the size of the 'predators' while $x(t)$ that of the 'preys'. It is worth to stress that these cyclic paths are optimal, according to the objective function used in the model; consequently, the policy maker has no incentive to modify them.

5 Conclusions

The main issue of this paper was to establish the fact that periodic investments and defensive expenditure by in order to defend the natural resource may be optimal under certain parameters. An intertemporal approach was used to study the optimal design of the tourist infrastructure in the National Park as well as efficient defensive expenditures policies. Large investments call many visitors, this generates large revenues. This implies a damage for the natural resource; in order to preserve the investments it is necessary to defend the natural resource by defensive expenditure. The presence of cyclical dynamics, such as environmental resource needs to rest. The continuous cyclical pattern, in fact, allows the National Park to regenerate.

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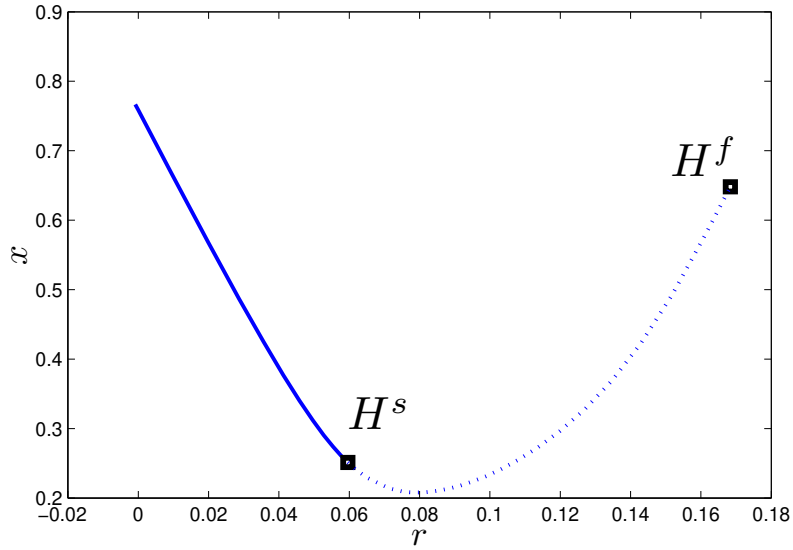

Figure 1: Equilibrium manifold in the (r, x) plane: H^s and H^f are two Hopf point. The parameter value: $\alpha = 0.5$, $a = 0.01$, $p = 0.008$, $\beta = 1.4$, $k = 1.5$, $h = 5$, $\gamma = 5$, $\alpha_1 = 0.05$, $b = 0.5$, $\epsilon = 0$

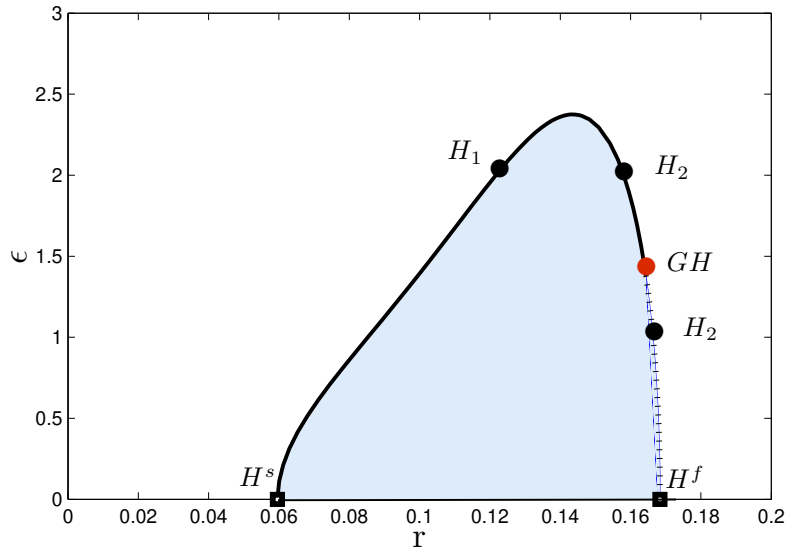

Figure 2: The Hopf bifurcation curve, varying r and ϵ: GH - Generalized Hopf point. The parameter values are those used in the simulation showed in Figure 1

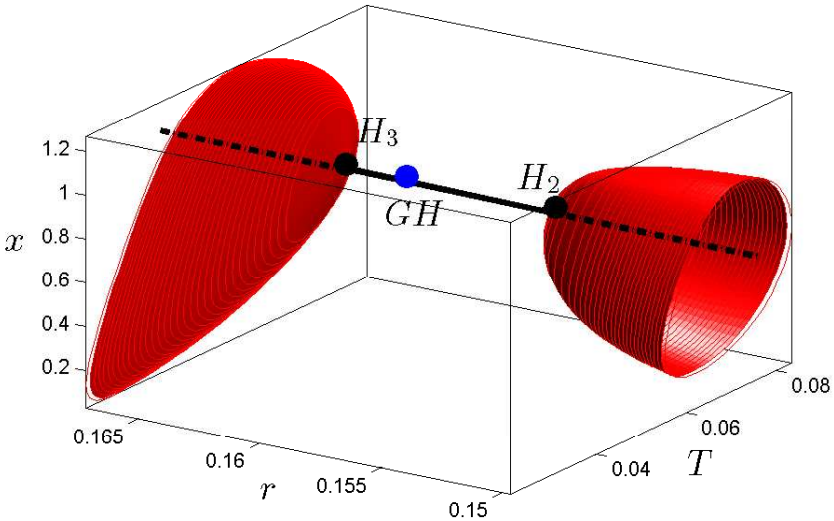

Figure 3: Limit cycle started subcritical Hopf point (H_3) and supercritical Hopf point (H_2).

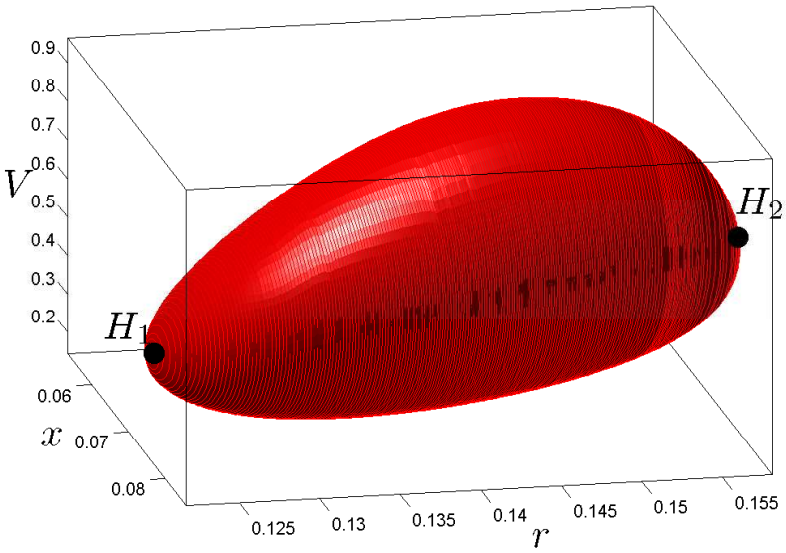

Figure 4: Limit cycles started from a Hopf point H_1.

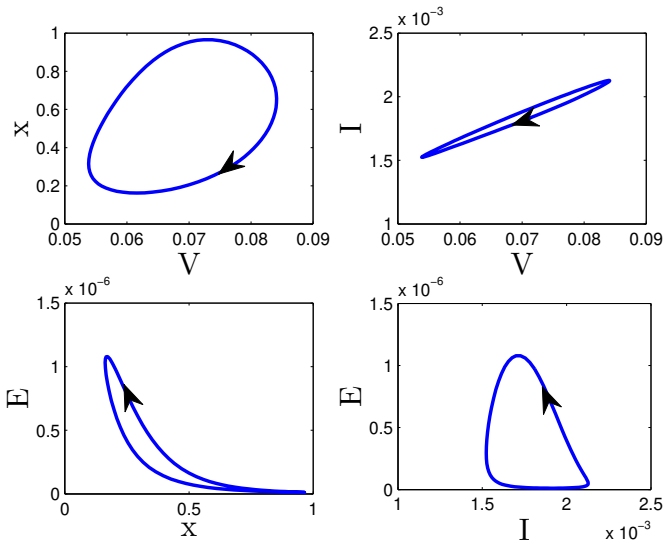

Figure 5: Phase portrait of the limit cycle in the plane, with $r = 0.14678$, and $\epsilon = 2$.

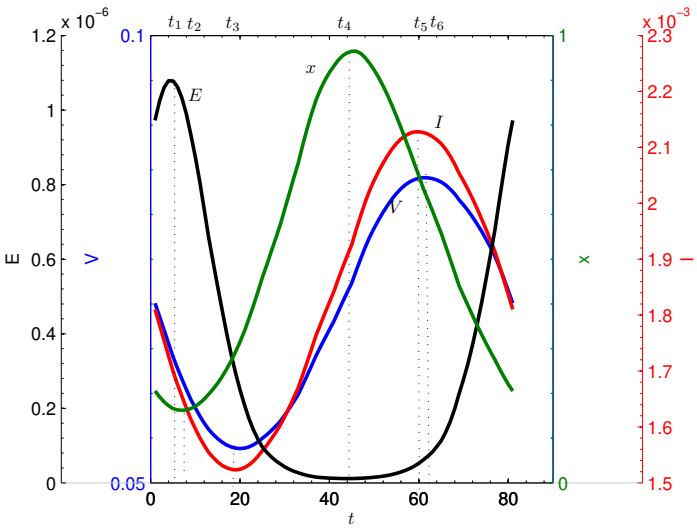

Figure 6: Time paths of the states and controls of the persistent cycle during one period.