Asset Price Bubbles in the Kiyotaki-Moore Model

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Abstract

We examine the effect of asset price bubbles in the Kiyotaki-Moore model. We show that the dynamic interactions between bubble-asset price, land price, and output generate powerful bubbly dynamics. The boom-bust cycles in bubble-asset price cause boom-crash cycles in the land market simultaneously, like a contagion by affecting the fundamentals of land. We also numerically analyze the welfare effects of bubbles in transitional dynamics.

Key words: Bubbly Dynamics, Contagion, Welfare Effects of Bubbles

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1 Introduction

Many countries have experienced bubble-like dynamics. The boom-bust in asset prices in equity markets tends to be associated with the boom-crash of land or housing markets, which, in turn, has large effects on real economic activity. Notable examples include the U.S. before and after the financial crisis of 2007–2008 and Japan from the late 1980s to the beginning of the 1990s.[1] The episodes in these countries suggest that financial markets are connected to each other, in the sense that the boom and collapse in one asset market has contagious effects on other asset markets.

In this paper, we theoretically investigate contagious effects of asset price bubbles on the other asset market. For this purpose, we incorporate a bubble-asset into the model of Kiyotaki and Moore (1997). Following traditional literature such as Tirole (1985), this asset produces no real dividend, i.e., the fundamental value of the asset is zero. One interpretation of the asset is that an equity with no dividends circulates. We first show that the dynamic interactions between bubble-asset price, land price, and output generate powerful bubbly dynamics. Second, the boom-bust cycles in bubble-asset price cause boom-crash cycles in the land market simultaneously, like a contagion, by affecting the fundamentals of land.

Figure 1 shows the mechanisms of the dynamic interactions in a bubbly episode. Output, land price, and bubble-asset price interact with each other not only within a period, but also between periods. This dynamic interaction between asset prices and aggregate quantity is similar to the Kiyotaki-Moore model. The key innovation of our paper is that the presence of bubbles enhances this interaction, increasing investment, output, consumption, and land price compared to the bubbleless economy. Once a bubble collapses, a reversal mechanism operates. Investment, output, consumption, and land price all fall eventually.

[Insert Figure 1]

A feature of bubbly equilibrium is that bubbles affect the land market by changing fundamentals of land itself, i.e., cash flows from land. In the bubbleless economy, land allocation is inefficient, in the sense that even unproductive entrepreneurs use land to produce goods as in Kiyotaki and Moore

[1]See Brunnermeier (2009) for the U.S. experiences and see Ueda (2011) for Japan's case.

(1997). As a result, output, investment, consumption, TFP, and land price are all low in the steady-state bubbleless economy. Bubbles not only increase the net worth of entrepreneurs facing the borrowing constraint, but also improve land allocation, i.e., more land is used in productive sectors. This improves cash flows from land and increases land price, which in turn relaxes credit limits, thereby generating expansionary effects.

Although bubbles improve resource allocation and produce expansionary effects, are they welfare-improving or welfare-reducing? We also investigate welfare effects of bubbles. Traditional wisdom such as Tirole (1985) suggests that bubbles are welfare-improving. In his framework, although bubbles increase consumption, they are contractionary in investment and output. On the other hand, Grossman and Yanagawa (1993) shows that bubbles are welfare-reducing. In their model, consumption, investment, and output all decrease by bubbles. This contractionary view has been criticized, because it seems to be inconsistent during bubbly episodes. We investigate welfare effects within a framework in which bubbles are expansionary in all three variables. Kocherlakota (2009) also examines welfare effects of bubble in the expansionary case. The difference between his analysis and ours is that we consider welfare effects including transitional dynamics, while he focuses on welfare in the steady state.

Our paper is related to a number of recent theoretical studies on rational asset price bubbles. Since a seminal paper by Farhi and Tirole (2009), the recent literature have provided a theoretical framework to analyze expansionary effects of asset price bubbles. Examples include Hirano and Yanagawa (2010a, 2010b), Martin and Ventura (2010, 2011), Aoki and Nikolov (2011), Ventura (2011), Miao and Wang (2012), and Sakuragawa (2012).[2] These studies only have one asset market, bubble-asset market.[3] Different from these studies, our model has two asset markets, bubble-asset market and land market. Thus, we can analyze interactions between two asset markets, and investigate contagious effect of changes in the bubble-asset market on the land market.

Kocherlakota (2009) and Miller and Stiglitz (2010) are closely related to our study, in the sense that they examine the effects of boom-bust cycles of bubbles in the Kiyotaki-Moore model. There are significant differences. In

[2]Kocherlakota (1992), Santos and Woodford (1997), and Hellwig and Lorenzoni (2009) analyze asset price bubbles in an endowment economy with an infinitely lived agent.

[3]The model of Miao and Wang (2012) only has equity markets. The equity prices include a bubble component.

Kocherlakota's model, land is not used as a factor of production and does not produce any output, i.e., the fundamental value of land is zero. He analyzes the case of positive land price as a bubbly economy. On the other hand, in our model, land is used as a factor of production. Miller and Stiglitz describe bubbles as corrective error of forecast on land price. In contrast, our model is based on a rational bubble model.

The paper is organized as follows. In section 2, we present a basic model and then derive the existence condition of bubbles. We also compare the bubble economy to the bubbleless economy in the steady state. In section 3, we analyze bubbly dynamics and discuss the welfare implications of bubble.

2 The Model

We develop an infinitely lived agent model in which the financial market is imperfect. Our model is based on Kiyotaki and Moore (1997).

2.1 Entrepreneur's Problem

Consider a discrete-time economy with a continuum of entrepreneurs. A typical entrepreneur has the following expected discounted utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t^i \right], \tag{1}$$

where i is the index for each entrepreneur and c_t^i is his or her consumption in period t. $\beta \in (0,1)$ is the subjective discount factor and $E_0[\cdot]$ is the conditional expectation on information at the beginning of period 0.

In each period, each entrepreneur has high production opportunities to produce the homogeneous goods (hereinafter H-projects) with probability p, and low production ones (L-projects) with probability $1-p$.[4] Both high- and low-productivity entrepreneurs (hereafter, H-entrepreneurs and L-entrepreneurs) use land and intermediate goods as inputs to produce the homogeneous goods. The land has a fixed supply, which is normalized to be one. The intermediate goods fully depreciate one period after production.

[4] A similar setting is used in Woodford (1990), Kiyotaki (1998), Kiyotaki and Moore (2008), and Kocherlakota (2009).

The production technologies are as follows:

$$y_{t+1}^i = \alpha_t^i \left(\frac{k_t^i}{\sigma}\right)^{\sigma} \left(\frac{z_t^i}{1-\sigma}\right)^{1-\sigma}, \tag{2}$$

where $k_t^i(\geq 0)$ is land, $z_t^i(\geq 0)$ is intermediate goods in period t, and y_{t+1}^i is output in period $t+1$. α_t^i is productivity in period t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he or she has L-projects. We assume $\alpha^H > \alpha^L$. The probability p is exogenous and independent across entrepreneurs and over time. At the beginning of each period t, the entrepreneur knows his or her own type in period t, whether he or she has H-projects or L-projects. Assuming that the initial population measures of H-types and L-types are p and $1-p$, respectively, the population measures in subsequent periods are the same.

In this economy, we introduce bubbles. We define a bubble-asset as an asset that produces no real return, i.e., the fundamental value of the asset is zero. Let P_t be the per unit price of bubble-asset in period t in terms of consumption goods. Then, the entrepreneur's flow of funds constraint is given by

$$c_t^i + q_t(k_t^i - k_{t-1}^i) + z_t^i + P_t(x_t^i - x_{t-1}^i) + r_{t-1}b_{t-1}^i + q_t\gamma k_{t-1}^i = y_t^i + b_t^i, \tag{3}$$

where x_t^i is the amount of bubble-asset purchased in period t. The left hand side of (3) shows expenditure on consumption, net purchase of land, investment of intermediate goods, net purchase of bubble-asset, repayment, and maintenance costs. As in Lorenzoni (2008), we assume that in order to keep the land productive, each entrepreneur must pay maintenance costs for a proportion γ of his or her land holdings. The right-hand side shows the available funds in period t, which includes the return from investment in the previous period and new borrowing. We define the net worth of the entrepreneur in period t as $e_t^i \equiv y_t^i - r_{t-1}b_{t-1}^i + q_t k_{t-1}^i(1-\gamma) + P_t x_{t-1}^i$.

We assume that because of frictions in the financial market, the entrepreneurs are credit-constrained. Following Kiyotaki and Moore (1997), creditors limit credit so that debt repayment cannot exceed the value of collateral, i.e., the value of the land minus maintenance costs. That is, the borrowing constraint becomes

$$r_t b_t^i \leq q_{t+1}k_t^i - q_{t+1}\gamma k_t^i, \tag{4}$$

where q_{t+1} is the price of land in period $t+1$. r_t and b_t^i are the gross interest rate and the amount of borrowing in period t, respectively.

We also impose a short sale constraint on bubble-asset:[5]

$$x_t^i \geq 0. \tag{5}$$

2.2 Equilibrium

Let us denote the aggregate consumption of H- and L-entrepreneurs in period t as $\sum_{i \in H_t} c_t^i \equiv C_t^H$ and $\sum_{i \in L_t} c_t^i \equiv C_t^L$, respectively, where H_t and L_t are families of H- and L-entrepreneurs in period t. Similarly, let $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in L_t} z_t^i \equiv Z_t^L$, $\sum_{i \in H_t} b_t^i \equiv B_t^H$, $\sum_{i \in L_t} b_t^i \equiv B_t^L$, $\sum_{i \in H_t} k_t^i \equiv K_t^H$, $\sum_{i \in L_t} k_t^i \equiv K_t^L$, $\sum_{i \in H_t \cup L_t} x_t^i \equiv X_t$, and $\sum_{i \in H_t \cup L_t} y_t^i \equiv Y_t$ be aggregate investment, aggregate borrowing, aggregate land holdings, and aggregate demand for bubble-asset of each type, respectively. Assuming that the aggregate supply of bubble-asset is fixed over time X, then the market clearing conditions for goods, credit, land, and bubble-asset are, respectively,

$$C_t^H + C_t^L + Z_t^H + Z_t^L = Y_t - q_t \gamma, \tag{6}$$

$$B_t^H + B_t^L = 0, \tag{7}$$

$$K_t^H + K_t^L = 1, \tag{8}$$

$$X_t = X. \tag{9}$$

The competitive equilibrium is defined as a set of prices $\{r_t, q_t, P_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t^i, b_t^i, z_t^i, x_t^i, k_t^i, C_t^H, C_t^L, B_t^H, B_t^L, Z_t^H, Z_t^L, K_t^H, K_t^L, Y_t\right\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (6)-(9) are satisfied, and (ii) each entrepreneur chooses consumption, borrowing, land holdings, investment of intermediate goods, and bubble-asset to maximize his or her expected discounted utility (1) under the constraints (2), (3), (4), and (5). Because there is no aggregate uncertainty, the entrepreneurs have perfect foresight of future prices and aggregate quantities in equilibrium. We also rule out the explosion in land price:

$$\lim_{t \to \infty} \frac{q_t (1-\gamma)^t}{r_0 r_1 r_2 \cdots r_{t-1}} = 0. \tag{10}$$

[5] Kocherlakota (1992) shows that the short sale constraint plays an important role for the emergence asset price bubbles in an infinitely lived agent model.

As is well known, there are multiple equilibria in the rational bubble model. One is the equilibria with $P_t > 0$; the other is $P_t = 0$. In the rest of the analysis, we focus on $P_t > 0$ and analyze what happens in the bubble economy.

2.3 Entrepreneur's Behavior

We are now in a position to characterize the equilibrium behavior of entrepreneurs in the bubble economy. We consider the case

$$\frac{\alpha^L}{u_t^\sigma} < r_t < \frac{\alpha^H}{u_t^\sigma}, \tag{11}$$

where $u_t \equiv q_t - q_{t+1}(1-\gamma)/r_t$ is the opportunity cost, or user cost, of holding land from t to $t+1$. α^L/u_t^σ and α^H/u_t^σ are the rates of return of L-projects and H-projects, respectively.

In order that bubble-asset be held in equilibrium, the rate of return of bubble-asset must be equal to the interest rate:

$$r_t = \frac{P_{t+1}}{P_t}. \tag{12}$$

When (11) and (12) hold, both the borrowing constraint and the short sale constraint simultaneously become binding for entrepreneurs who have H-projects in period t, but not for entrepreneurs who have L-projects. Since the utility function is logarithmic, each entrepreneur consumes a fraction $1-\beta$ of the net worth in each period, that is,

$$c_t^i = (1-\beta)\left[y_t^i - r_{t-1}b_{t-1}^i + q_t k_{t-1}^i (1-\gamma) + P_t x_{t-1}^i\right]. \tag{13}$$

Then, by using (3), (4), and (5), we can derive demand functions of the type i agent of H-entrepreneurs for intermediate goods and land holdings in period t:

$$z_t^i = (1-\sigma)\beta\left[y_t^i - r_{t-1}b_{t-1}^i + q_t k_{t-1}^i (1-\gamma) + P_t x_{t-1}^i\right], \tag{14}$$

$$k_t^i = \frac{\sigma\beta\left[y_t^i - r_{t-1}b_{t-1}^i + q_t k_{t-1}^i (1-\gamma) + P_t x_{t-1}^i\right]}{q_t - \dfrac{q_{t+1}(1-\gamma)}{r_t}}. \tag{15}$$

Equation (14) indicates that H-entrepreneur spends a fraction $1 - \sigma$ of his or her savings on intermediate goods. Equation (15) indicates that H-entrepreneur spends a fraction σ of his or her savings to finance the difference between the land value q_t and the collateral value $q_{t+1}(1-\gamma)/r_t$.[6] The difference $q_t - q_{t+1}(1-\gamma)/r_t$ is the down payment to purchase one unit of land, and is the user cost u_t. The entrepreneurs buy bubble-asset when they have L-projects, and sell them when they have opportunities to invest in H-projects. As a result, their net worth increases (compared to bubbleless cases), which boosts their demand for intermediate goods and land as shown in (14) and (15).

L-entrepreneurs in period t prefer buying bubble-asset and lending to H-entrepreneurs instead of investing in their own L-projects when (11) and (12) hold because the rate of return of bubble-asset or lending is greater than the rate of return of L-projects.

2.4 Aggregation

Since (14) and (15) are linear functions of net worth, we can the aggregate across H-entrepreneurs to derive the aggregate demand functions:

$$Z_t^H = (1-\sigma)\beta p \left[Y_t + q_t(1-\gamma) + P_t X\right], \tag{16}$$

$$K_t^H = \frac{\sigma\beta p}{q_t - \dfrac{q_{t+1}(1-\gamma)}{r_t}} \left[Y_t + q_t(1-\gamma) + P_t X\right], \tag{17}$$

where $Y_t + q_t(1-\gamma) + P_t X$ is the aggregate wealth of the entrepreneurs. Since every entrepreneur has the same chance to take on H-projects in each period, a proportion p of aggregate wealth is the wealth of H-entrepreneurs. Equation (16) and (17) indicate that the aggregate demand functions for intermediate goods and land depend upon cash flow from investments in the previous period Y_t as in Bernanke and Gertler (1989) and land price q_t as in Kiyotaki and Moore (1997). What is new in our framework is that the demand functions also depend upon bubble-asset price P_t, i.e., the presence

[6] This is a popular demand function under financial constraint problems such as Bernanke and Gertler (1989), Bernanke et al. (1999), Holmstrom and Tirole (1998), Kiyotaki and Moore (1997), and Matsuyama (2007). The difference between these literature and our paper is that the presence of asset bubbles affect the net worth of entrepreneurs.

of bubbles increases demands for land holdings and intermediate goods, does not crowds them out as in traditional literature (Tirole (1985)).

Since only H-entrepreneurs use land, from the land market clearing condition,

$$\frac{\sigma\beta p\left[Y_t + q_t(1-\gamma) + P_t X\right]}{q_t - \dfrac{q_{t+1}(1-\gamma)}{r_t}} = 1. \tag{18}$$

Note that q_t is an increasing function of P_t, Y_t, and q_{t+1}. When P_t increases, the net worth of H-entrepreneurs improves. As a result, they can buy more land with maximum leverage, which raises the current land price q_t. Moreover, when q_{t+1} is expected to increase, the borrowing constraint is relaxed, which increases the demand for land, as in the model of Kiyotaki and Moore (1997). In our model, q_{t+1} is affected by the bubble-asset price in period $t+1$, P_{t+1}.

From the aggregate flow of funds constraint of L-entrepreneurs and the market clearing condition for bubble-asset (9), we can derive the equilibrium the bubble-asset price in period t:

$$P_t X = \frac{\beta(1-p)\left[Y_t + q_t(1-\gamma)\right] - B_t^H}{1-\beta+p\beta} \tag{19}$$

We see that given B_t^H, P_t is an increasing function of Y_t and q_t. Intuitively, when cash flow or land price increases, the net worth of L-entrepreneurs improves and they can buy more bubble-asset, which raises the bubble-asset price.

Note that from (18) and (19), bubble-asset price P_t and land price q_t reinforce each other within a period. Moreover, as we will see, since both asset prices are forward-looking variables, they interact with each other between periods.

Since only H-entrepreneurs produce by using land and intermediate goods, output evolves over time as

$$Y_{t+1} = \frac{\alpha^H}{\sigma} u_t^{1-\sigma}. \tag{20}$$

From the definition of the user cost of land and equation (12), the land price should satisfy the dynamic equation

$$q_t = u_t + \frac{P_t}{P_{t+1}} q_{t+1}(1-\gamma). \tag{21}$$

From this, the current land price equals the discounted value of future user costs, which in turn depends on the discounted value of future output from (20). This means that when output is expected to increase in the future, the current land price rises. Additionally, future output is affected by the presence of bubbles.

Since both H- and L-entrepreneurs consume a fraction $1-\beta$ of the net worth, goods market clearing condition (6) can be written as

$$(1-\beta)A_t + \frac{1-\sigma}{\sigma}u_t = Y_t - q_t\gamma, \tag{22}$$

where $A_t \equiv Y_t + q_t(1-\gamma) + P_tX$ is aggregate wealth. The first term on the left hand side of (22) is aggregate consumption and the second is investment of intermediate goods.

Along the perfect foresight equilibrium path, aggregate wealth evolves over time as

$$A_{t+1} = \frac{\alpha^H}{u_t^\sigma}p\beta A_t + \frac{P_{t+1}}{P_t}(1-p)\beta A_t. \tag{23}$$

Equation (23) indicates that given the aggregate savings of the economy βA_t, the aggregate net worth of H-entrepreneurs earns a rate of return α^H/u_t^σ, while the net worth of L-entrepreneurs earns P_{t+1}/P_t.

2.5 Steady State Analysis

In order for bubbles to exist in the steady state, the following two conditions must be satisfied:

$$\frac{\alpha^L}{u^\sigma} \leq 1, \tag{24}$$

$$P > 0. \tag{25}$$

(24) indicates that the rate of return of one unit of bubble-asset must be greater than the rate of return of L-projects. Otherwise, no entrepreneur buys bubble-asset. (25) indicates that the bubble-asset price must be positive. This second condition is satisfied as long as L-entrepreneurs produce in the

bubbleless economy, which means $K_t'^L > 0$.[7] From these conditions, we obtain the following proposition.

Proposition 1 *Bubbles can exist if and only if the following condition is satisfied:*

$$\alpha^H \geq \frac{\alpha^L p\beta}{1-\beta+p\beta}, \tag{26}$$

Proof. *The proof is provided in the appendix A.* ■

In order for bubbles to exist in the steady-state equilibrium, productivity α^H must be sufficiently high. Intuitively, if productivity is high enough, the marginal productivity of production becomes lower, which means that the user cost becomes sufficiently large that the rate of return of bubble-asset becomes greater than the rate of return of L-projects.

We should add a few more remarks on maintenance costs. As in traditional rational bubble literature such as Tirole (1985) and Farhi and Tirole (2011), in the steady-state bubble economy with no growth, the interest rate equals one, in which case (10) cannot be satisfied in our model because our model has a fixed asset, land. In order that (10) is satisfied, we must introduce costs associated with land holdings.

Proposition 2 *Output, investment, consumption, and aggregate TFP are all higher in the bubbly steady state than in the bubbleless steady state.*
Proof. *The proof is provided in the appendix B.* ■

In the steady state of the bubbleless economy,[8] production is inefficient in the sense that L-entrepreneurs as well as H-entrepreneurs produce by using land and intermediate goods, which means that resource allocation is inefficient. On the other hand, in the bubbly steady state, only H-entrepreneurs produce. This means that a bubble increases efficiency in production by eliminating inefficient L-projects, thus improving aggregate TFP. In other words,

[7] The condition that L-entrepreneurs produce in the bubbleless economy is the following:

$$1 - p(1-\sigma) - \sigma p \frac{\alpha^L}{\alpha^L - (1-\gamma)\left[\alpha^H p + \alpha^L (1-p)\right]\beta} > 0.$$

If this condition is satisfied, $P_t > 0$ holds true.

[8] The mathematical description in the bubbleless economy is in the proof of Proposition 2 in the appendix.

the presence of bubbles helps transfer resources from L-entrepreneurs to H-entrepreneurs. Since resource allocation is more efficient, all macroeconomic variables are higher than those in the bubbleless steady state.

With regard to the effects of a bubble on land price, since only H-entrepreneurs produce in the bubbly steady state, cash flow from land increases from the present to the future, which raises the steady-state land price. On the other hand, the interest rate is high in the bubbly steady state, which decreases land price.[9] In this sense, there are two competing effects of a bubble on land price. Which of these effects dominates depends on the value of γ.[10] In the following example, we focus on the case where a bubble increases land price in the steady state.

An interesting feature in our framework is that bubbles changes fundamentals itself, i.e., cash flow from land and interest rate. When the land price is higher in the bubble economy, it reflects an improvement in cash flow, and this improvement is supported by the presence of bubbles. This implies that boom-bust in bubble-asset price is associated with dramatic changes in fundamentals of land.

3 Macroeconomic Effects of Asset Bubbles

Given an initial condition Y_0, the perfect foresight equilibrium path is described by sequences $\{q_t, A_t, r_t, u_t, P_t, Y_t\}_{t=0}^{\infty}$, satisfying (10), (20), (21), (22), and (23). With regard to this dynamical system, we obtain the following proposition.

Proposition 3 *There is a saddle point path on which the economy converges to a steady-state bubble economy.*
Proof. *The proof is provided in the appendix C.* ■

[9] In Kiyotaki and Moore (1997), the interest rate equals the gatherers' time preference rate and becomes constant over time, even if productivity shocks occur, while in our model, the interest rate changes endogenously whether bubbles occur or not.

[10] There is a threshold value of $\gamma \equiv \hat{\gamma} < 1$, above which bubbles increase the steady-state land value and below which they decrease it. $\hat{\gamma}$ satisfies the following equation:

$$q = \frac{u}{\gamma} > q' = \frac{r'u'}{r' - (1-\gamma)},$$

where u' denotes the user cost of holding land in the bubbleless economy.

Once the economy gets on the saddle point path, the economy's output dynamics can be described as the following simple difference equation:

$$Y_{t+1} = \frac{\alpha^H}{\sigma}\left(\frac{\sigma p \beta}{1-\beta+p\beta}\right)^{1-\sigma}(Y_t)^{1-\sigma}, \tag{27}$$

and equilibrium land price, bubble-asset price, and user cost follow

$$q_t = \frac{\sigma p}{\gamma}\frac{\beta Y_t}{1-\beta+p\beta}, \tag{28}$$

$$P_t X = \frac{[1-(1-\sigma)p]\,\gamma - \sigma p}{\gamma}\frac{\beta Y_t}{1-\beta+p\beta}, \tag{29}$$

$$u_t = \frac{\sigma p \beta Y_t}{1-\beta+p\beta}. \tag{30}$$

What is happening during bubbly dynamics is as follows. Suppose that cash flow at date t increases. Together with this increase in cash flow, the net worth of H-and L-entrepreneurs improves, and their demand for land and their demand for bubbles also increase. As a result, both land price and bubble price at date t rise, reinforcing each other. Because of this, the net worth of H-entrepreneurs at date t improves furthermore, which produces further increase in output, the net worth, land price and bubble price at date $t+1$. These knock-on effects continue not only in period t, but also in periods $t+1$, $t+2, \cdots$. Moreover, this anticipated increase in land price in periods $t+1$, $t+2, \cdots$ is reflected by an increase in land price at date t, which affects bubble price at date t once again. In equilibrium, all these mechanisms occur simultanously, and the economy runs according to (27)-(30).

The two-way feedback between asset prices and aggregate quantities operates as in Kiyotaki and Moore (1997). The key innovation of our paper is that the presence of bubbles enhances this two-way interaction, thus increasing output and land price compared to the bubbleless economy.

Since we obtain the analytical solutions of the equilibrium path, we can analyze the output dynamics. In Figure 2, we show the equilibrium dynamics of output in bubble and bubbleless economies. Suppose that the economy is initially in a bubbleless steady state, $Y^*_{bubbleless}$, and suddenly people believe that a bubble has emerged and will exist with a positive price forever. The output gets larger and converges to the steady state in the bubble economy Y^*_{bubble}.

[Insert Figure 2]

3.1 Numerical Example

We show a numerical experiment on the effects of boom-bust in the bubble-asset price on macroeconomic variables.[11] For $t \leq 0$, the economy is in the steady state in the bubbleless economy, in which all entrepreneurs believe that the bubble-asset price will be zero in the future: $P_t = 0$ for all t. We suppose that at the beginning of period $t = 1$, the bubble-asset price unexpectedly becomes positive $P_t > 0$, and all entrepreneurs expect that the bubble will last forever. At the beginning of period $t = 51$, the bubble collapses unexpectedly $P_t = 0$, and once the bubble bursts, all entrepreneurs expect that the bubble will not emerge at all in the future. Note that there is no aggregate productivity shock for all t. The model is solved by the shooting method. The parameter values are set as follows: $\alpha_H = 1$; $\alpha_L = .8$; $\beta = .99$; $\gamma = .2$; $p = .05$; $\sigma = .3$; $X = 1$. Most of these values appear standard.

Figure 3 plots the dynamics of the macroeconomic variables when a bubble occurs. Consumption rises immediately after the emergence of the bubble in period $t = 1$ and continues to increase over time because of the wealth effect. Recall that consumption is a fraction $1-\beta$ of net worth and net worth improves over time. Output remains unchanged in period $t = 1$ because it is predetermined. However, in period $t = 2$, it expands because of the reallocation of land, i.e., land is used only by H-entrepreneurs in the bubbly economy ($t \geq 1$). After $t = 3$, since the bubble continues to improve the net worth of H-entrepreneurs. This improvement enhances the crowd-in effect, output, expenditure of intermediate goods, and land price over time. We emphasize that the increase in land price reflects an improvement in fundamentals (or cash flow from land). Both land price and expenditure for intermediate goods in period $t = 1$ decline immediately after the emergence of a bubble. This is because when the bubble arises, the entrepreneurs buy bubble-asset, which crowds savings away from the purchase of land and intermediate goods.[12] This means that when the bubble first appears, the traditional crowd-out effect of the bubble dominates the crowd-in effect in the land market and intermediate goods market.

When the bubble bursts in period $t = 51$, a reverse mechanism operates.

[11] It is not necessary to show the numerical experiment in order to show the dynamics of output. However, to show the welfare analysis later, the simulation of a numerical example in the transition path is needed.

[12] Immediately after bubbles arise, the interest rate rises substantially, reflecting the tightness of the credit market. This decreases the land price.

The net worth of entrepreneurs decreases over time and land is used inefficiently, which means that L-entrepreneurs as well as H-entrepreneurs produce. As a result, output, consumption, expenditure of intermediate goods, and land price all fall and converge to the lower, steady-state level.[13] Our simulation result suggests that the boom-bust cycles in bubble-asset price cause boom-crash cycles in the land price simultaneously, like a contagion.

[Insert Figure 3]

Here we add a few remarks on the response of land price and expenditure for intermediate goods. In this numerical example, we investigate the pure effects of bubbles; the birth and burst of a bubble is caused by a sudden change in entrepreneurs' expectations. If the emergence and collapse of a bubble is triggered by a change in productivity α^H, land price and expenditure for intermediate goods will increase immediately after the emergence of the bubble, then will decline immediately after the bubble crashes.

3.2 Welfare Effects of Bubbles

A bubble's appearance has positive effects on macroeconomic variables. In this subsection, we discuss the welfare effects of asset price bubbles. We compare the ex-ante welfare of entrepreneurs under two situations. As in the previous numerical example, until period $t \leq 0$, the economy is in the steady-state of the bubbleless economy. In one situation, at the beginning of period $t = 1$, a bubble arises and after period $t \geq 1$, the economy runs on the bubbly economy's path. In the other situation, a bubble never arises for all $t \geq 0$ and the economy continues to stay in the steady state of the bubbleless economy.

When we compute the ex-ante welfare in period $t = 1$,

$$V_1^i = E_1 \left[\sum_{t=1}^{\infty} \beta^{t-1} \log c_t^i \right] = E_1 \left[\sum_{t=1}^{\infty} \beta^{t-1} \log(1-\beta) e_t^i \right]$$

where V_1^i is ex-ante welfare of the type i entrepreneur in period $t = 1$. Since

[13] Note that land price and expenditure for intermediate goods increase immediately after the bubble bursts because all the savings of entrepreneurs flow to the purchase of land and intermediate goods, causing the interest rate to fall substantially.

$e^i_{t+1} = R^i_t \beta e^i_t$, the above equation can be rewritten as

$$V^i_1 = \sum_{t=1}^{\infty} \beta^{t-1} \log(1-\beta)\beta^{t-1} + \frac{1}{1-\beta} \log e^i_1 + \frac{\beta}{1-\beta} \log R^i_1$$
$$+E_1 \left[\sum_{t=2}^{\infty} \frac{\beta^t}{1-\beta} \log R^i_t \right], \qquad (31)$$

where, in the bubble economy,

$$R^i_t = \begin{cases} \frac{\alpha^H}{u^\sigma_t} & \text{if } i = H, \\ \frac{P_{t+1}}{P_t} & \text{if } i = L, \end{cases}$$

and where, in the bubbleless economy,

$$R^i_t = \begin{cases} \frac{\alpha^H}{u'^\sigma_t} & \text{if } i = H, \\ \frac{\alpha^L}{u'^\sigma_t} & \text{if } i = L. \end{cases}$$

u' denotes the user cost of holding land in the bubbleless economy. When we compute the ex-ante welfare welfare, how much net worth the entrepreneur has at date e^i_1, and how much he/she earns marginally in each period, $\{R^i_t\}^{\infty}_{t=1}$, have persistent effects on consumption in the future through affecting the net worth. The second term in equation (31) captures the discounted sum of the effect of e^i_1 on each period utility. The third term is the discounted sum of the effect of R^i_1, and the fourth term is the discounted sum of the effect of $\{R^i_t\}^{\infty}_{t=2}$ on each period expected utility.

We make two assumptions to compute ex-ante welfare in period $t = 1$. The first assumption is that at the beginning of period $t = 1$, each entrepreneur is endowed with X units of the bubble-asset. The second assumption is that we use the third term and the fourth term in (31) for computing the ex-ante welfare. When a bubble occurs at the beginning of period $t = 1$, the bubble-asset price jumps up, but land price jumps down as shown in the numerical example. Hence, the effect of a bubble's emergence on the net worth in period $t = 1$ (the second term in (31)) is theoretically ambiguous. However, in our numerical example, every individual's net worth in period $t = 1$ improves. Thus, it is not surprising that the effect of a bubble's appearance raises the welfare if we include the second term in (31). Since the first term is the same for all entrepreneurs, it is unnecessary to compare the levels of welfare. Therefore, we exclude the first two terms.

When computing the bubble economy, we take into account transitional dynamics from the steady state of the bubbleless economy to the steady state of the bubble economy. The parameter values are the same as before.

[Insert Figure 4]

Figure 4 plots the values of ex-ante welfare for H-entrepreneur in period 1 and L-entrepreneur in period 1, respectively. We compute their ex-ante welfare with respect to p. The first row shows the ex-ante welfare in the bubble economy, while the second row shows the ex-ante welfare in the bubbleless economy.

[Insert Figure 5]

Figure 5 shows the difference of ex-ante welfare for them between bubble and bubbleless economies and indicates the appearance of a bubble is welfare-improving for both types of entrepreneurs. Intuitively, this comes from the difference in the rates of return. Without bubbles, L-entrepreneurs end up with accumulating their wealth through a low return savings vehicle with a rate of return of $\alpha^L/u_t'^{\sigma}$. On the other hand, a bubble provides a high rate of return vehicle for them, P_{t+1}/P_t. This increase in the rate of return contributes to improving the net worth of entrepreneurs and their welfare.

4 Conclusion

We examined the effect of asset price bubbles in the Kiyotaki-Moore model. We have shown that the dynamic interactions between bubble-asset price, land price, and output generate powerful bubbly dynamics. The boom-bust cycles in bubble-asset price cause boom-crash cycles in the land market simultaneously, like a contagion by affecting the fundamentals of land. We also numerically analyzed the welfare effects of bubbles in transitional dynamics. We have found that bubbles tend to be welfare-improving.

Our framework can be extended in several directions. Let us discuss one of them here. In the above model, we have analyzed the interactions between bubble-asset price and other asset prices in a closed economy. It would be promising to include a two-country or a multi-country model. This would enable analysis of how the boom-bust cycles of bubbles in one country have contagious effects on asset prices in other countries. This analysis will be important to understand the recent global financial and debt crises across countries.

References

[1] Aoki, Kosuke, and Kalin Nikolov. 2011. "Bubbles, Banks and Financial Stability," Working Paper, CARF-F-253, The University of Tokyo.

[2] Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1): 14–31.

[3] Bernanke, Ben, Mark Gerlter, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. Taylor and M. Woodford eds, *the Handbook of Macroeconomics*, 1341-1393. Amsterdam: North-Holland.

[4] Brunnermeier, Markus. 2009. "Deciphering the Liquidity and Credit Crunch 2007-2008.," *Journal of Economic Perspectives,* 23(1): 77-100.

[5] Farhi, Emmanuel, and Jean Tirole. 2011. "Bubbly Liquidity," forthcoming in *Review of Economic Studies*.

[6] Grossman, Gene, and Noriyuki Yanagawa. 1993. "Asset Bubbles and Endogenous growth," *Journal of Monetary Economics*, 31(1): 3-19.

[7] Hellwig, Christian, and Guido Lorenzoni. 2009. "Bubbles and Self-Enforcing Debt," *Econometrica*, 77(4): 1137-1164.

[8] Hirano, Tomohiro, and Noriyuki Yanagawa. 2010a. "Asset Bubbles, Endogenous Growth, and Financial Frictions," Working Paper, CARF-F-223, The University of Tokyo.

[9] Hirano, Tomohiro, and Noriyuki Yanagawa. 2010b. "Financial Institution, Asset Bubbles and Economic Performance," Working Paper, CARF-F-234, The University of Tokyo.

[10] Holmstrom, Bengt, and Jean Tirole. 1998. "Private and Public Supply of Liquidity," *Journal of Political Economy,* 106(1): 1-40.

[11] Kiyotaki, Nobuhiro, 1998. "Credit and Business Cycles," *The Japanese Economic Review*, 49(1): 18–35.

[12] Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles," *Journal of Political Economy*, 105(2): 211-248.

[13] Kocherlakota, R. Narayana. 1992. "Bubbles and Constraints on Debt Accumulation," *Journal of Economic Theory*, 57(1): 245-256.

[14] Kocherlakota, R. Narayana. 2009. "Bursting Bubbles: Consequences and Cures," University of Minnesota.

[15] Martin, Alberto, and Jaume Ventura. 2010. "Economic Growth with Bubbles," Universitat Pompeu Fabra.

[16] Martin, Albert, and Jaume Ventura. 2011. "Theoretical Notes on Bubbles and the Current Crisis," *IMF Economic Review*, 59(11): 6-40.

[17] Matsuyama, Kiminori. 2007. "Credit Traps and Credit Cycles," *American Economic Review*, 97(1): 503-516.

[18] Miao, Jianjun, and Pengfei Wang. 2011. "Bubbles and Credit Constraints," mimeo, Boston University.

[19] Miller, Marcus, and Joseph Stiglitz. 2010. "Leverage and Asset Bubbles: Averting Armageddon with Chapter 11?," The Economic Journal, 120(544): 500-518.

[20] Santos, S. Manuel, and Michael Woodford. 1997. "Rational Asset Pricing Bubbles," *Econometrica*, 65(1): 19-58.

[21] Sakuragawa, Masaya. 2012. "Savings and Bubbles," mimeo, Keio University.

[22] Tirole, Jean. 1982. "On the Possibility of Speculation under Rational Expectations," *Econometrica*, 50(5): 1163-1182.

[23] Tirole, Jean. 1985. "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6): 1499-1528.

[24] Ueda, Kazuo. 2011. "Japan's Deleveraging since the 1990s and the Bank of Japan's Monetary Policy: Some Comparisons with the U.S. Experience Since 2007," Working Paper No. CARF-F-259, The University of Tokyo.

[25] Ventura, Jaume. 2011. "Bubbles and Capital Flows," CREI, Forthcoming in *Journal of Economic Theory*.

[26] Woodford, Michael. 1990. “Public Debt as Private Liquidity,” *American Economic Review*, 80(2): 382-388.

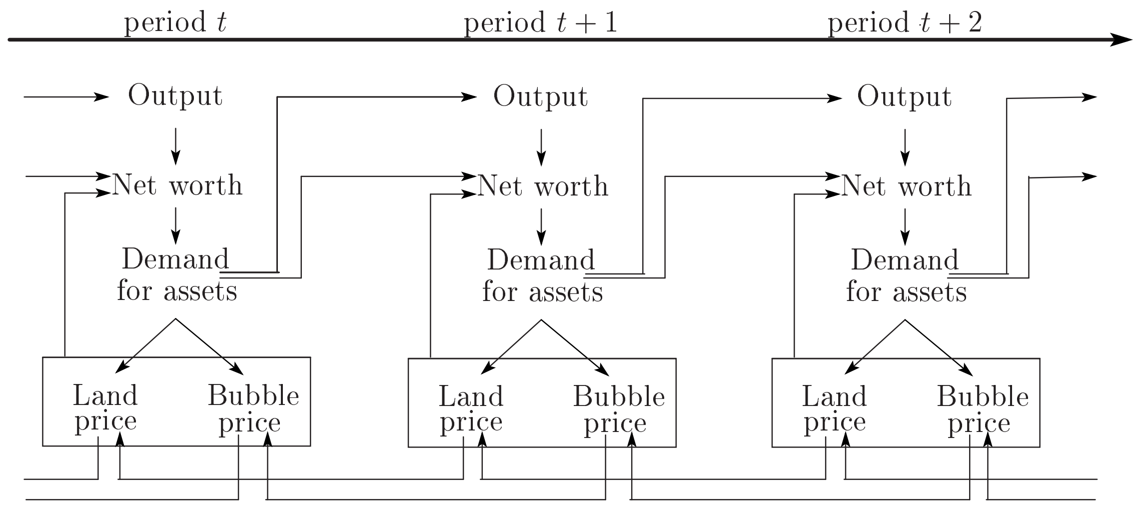

Figure 1: Mechanism of Bubbly Dynamics

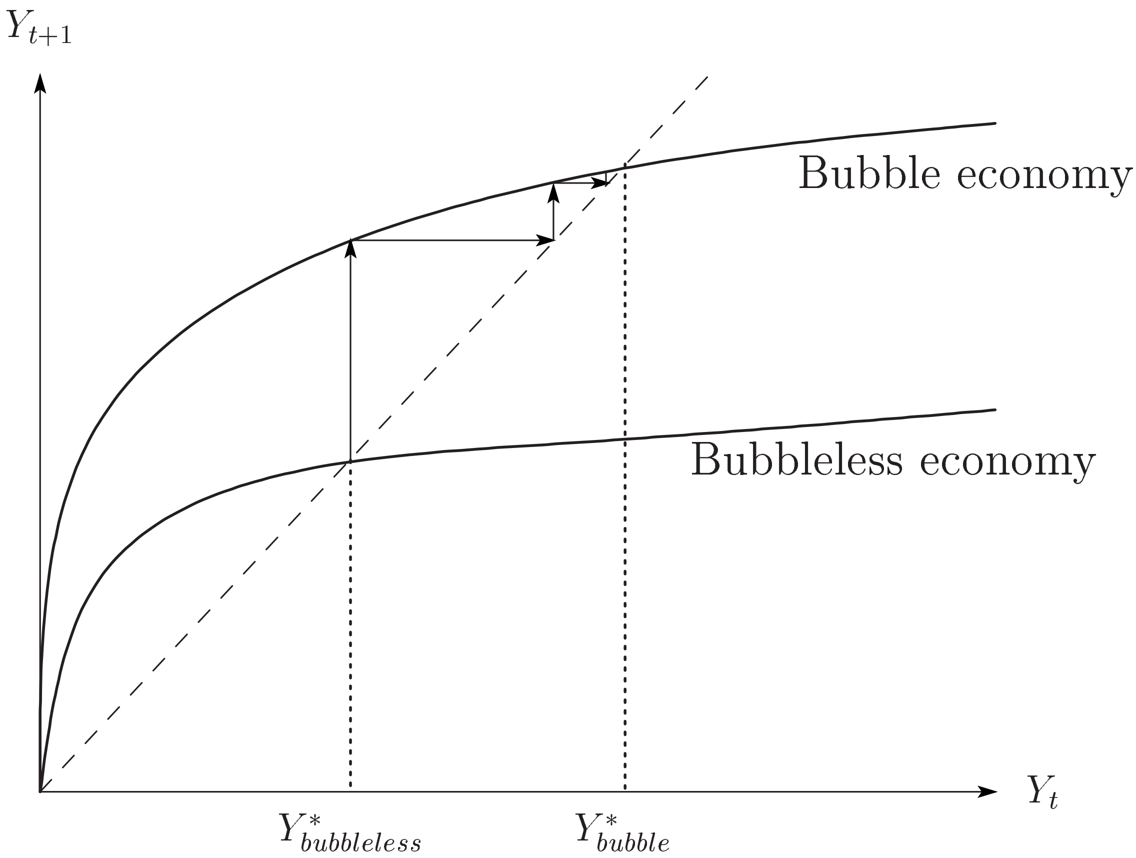

Figure 2: Bubbly Dynamics of Output

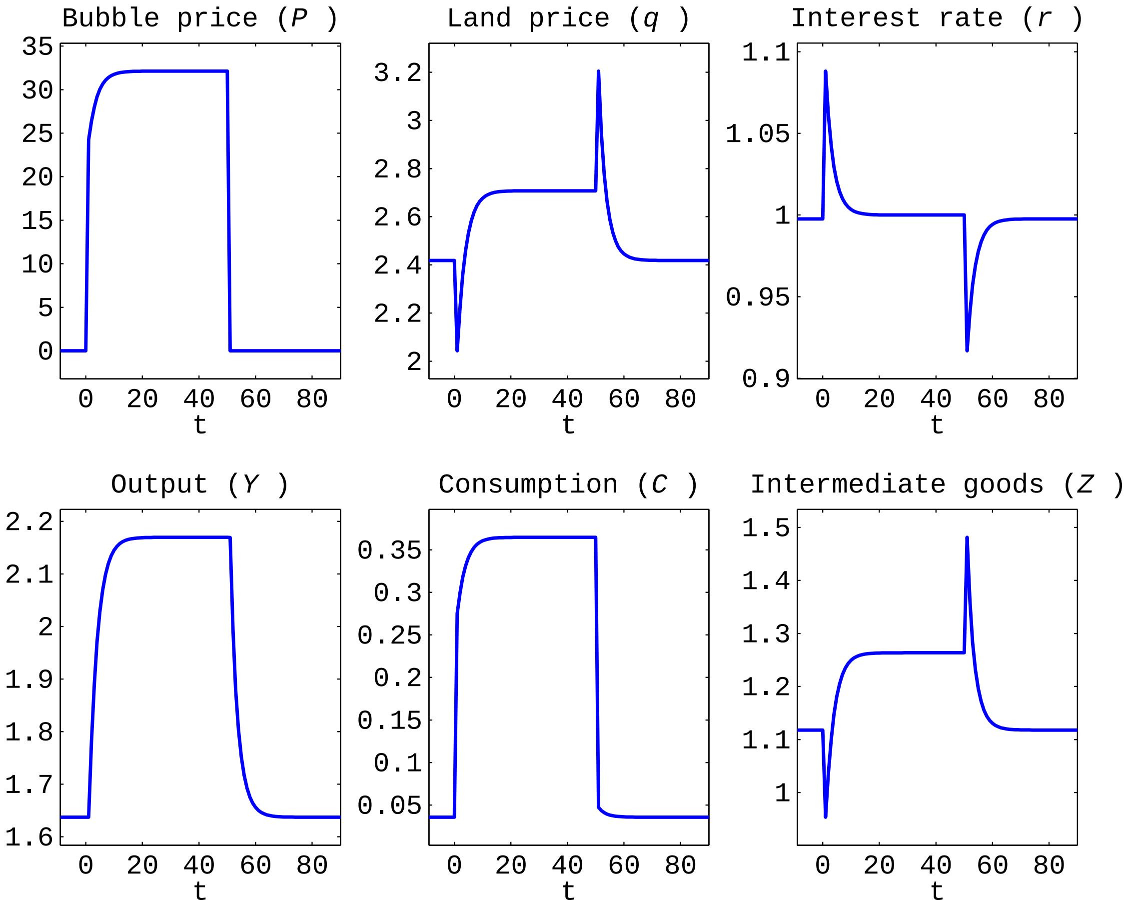

Figure 3: Effects of the Boom-Bust of the Bubble-Asset Price

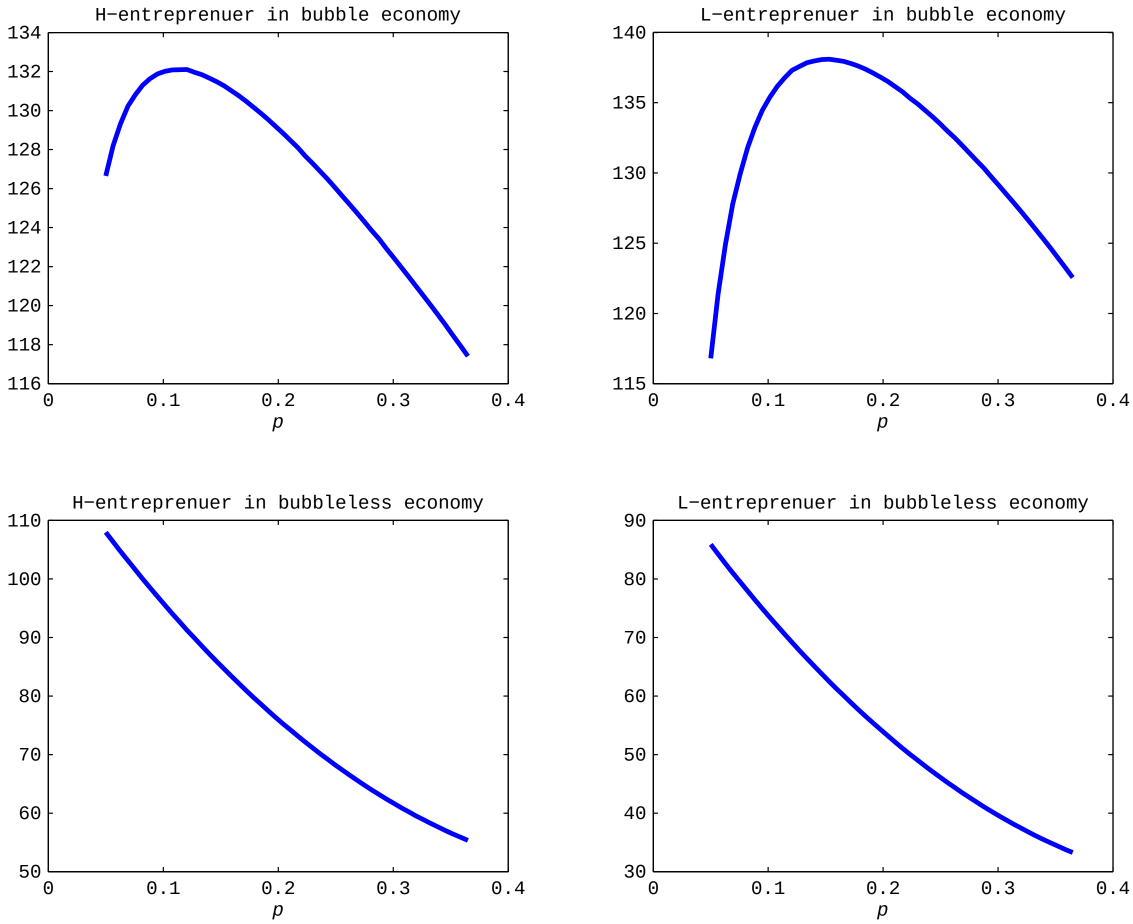

Figure 4: Ex-ante Welfare for Each Individual

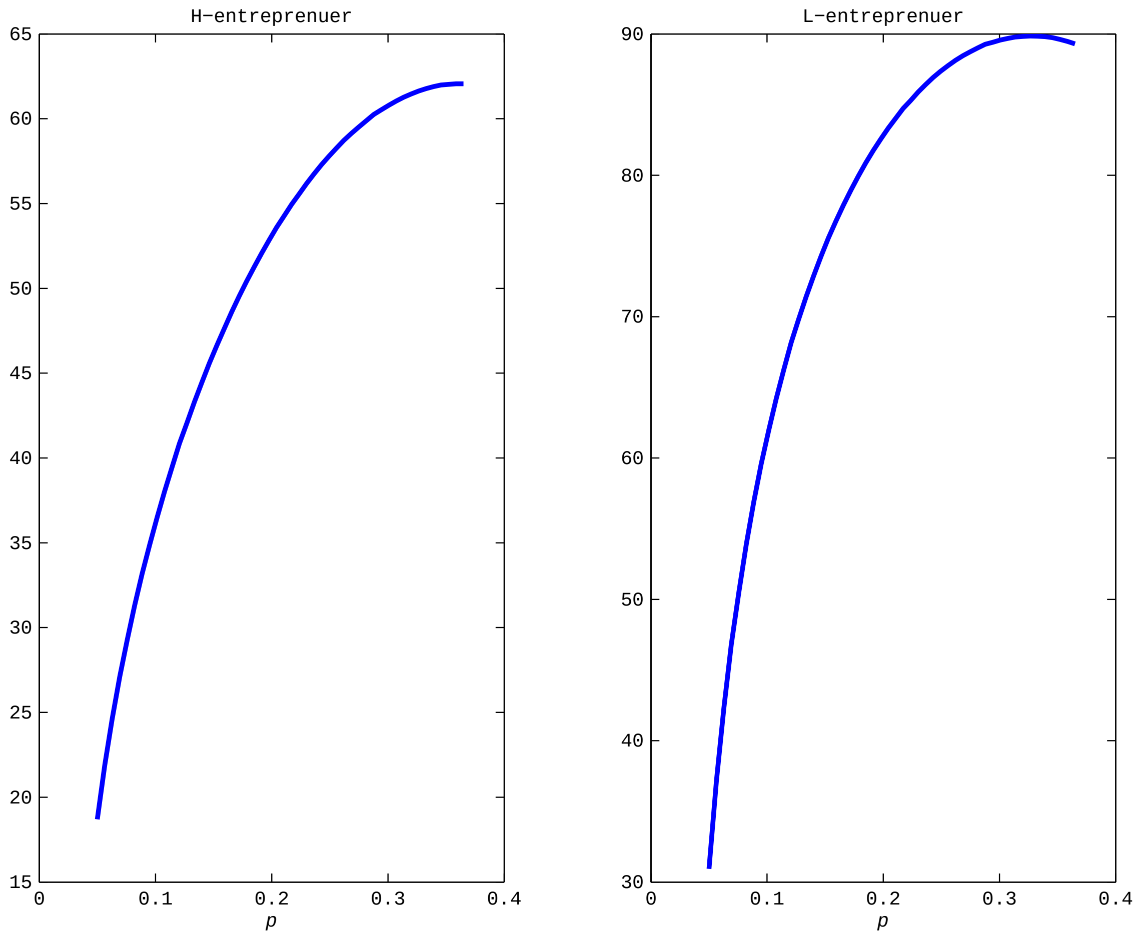

Figure 5: Difference of Ex-ante Welfare between Bubble and Bubbleless Economies

Appendices

A Proof of Proposition 1

In the steady-state bubble economy, from (23), u^σ is

$$u^\sigma = \frac{\alpha^H p\beta}{1-\beta+p\beta}. \tag{B1}$$

By substituting (B1) into (24), we can derive (26).

B Proof of Proposition 2

In order to prove Proposition 2, we first characterize an equilibrium of the bubbleless economy. When there is no bubble, the interest rate, user cost, output, and wealth evolve over time as, respectively,

$$r'_t = \frac{\alpha^L}{u'^\sigma_t}, \tag{A1}$$

$$q_t = u_t + \frac{u^\sigma_t}{\alpha^L} q_{t+1}(1-\gamma) \tag{A2}$$

$$Y'_{t+1} = \frac{\alpha^H}{\sigma}(u'_t)^{1-\sigma} K'^H_t + \frac{\alpha^L}{\sigma}(u'_t)^{1-\sigma} K'^L_t, \tag{A3}$$

$$A'_{t+1} = \frac{\alpha^H}{u'^\sigma_t}\beta p A'_t + \frac{\alpha^L}{u'^\sigma_t}\beta(1-p)A'_t. \tag{A4}$$

In the bubbleless economy, the interest rate equals the rate of return of L-projects, so even L-entrepreneurs end up producing in equilibrium, which means that $K'^L \geq 0$ and $Z'^L \geq 0$.

Given an initial condition, Y_0, and $P_t = 0$ for all $t \geq 0$, the perfect foresight equilibrium path of the bubbleless economy is described by sequences $\{q_t, A_t, r_t, u_t, Y_t\}_{t=o}^{\infty}$, satisfying (10), (22), and (A1)-(A4).

In the steady-state, the user cost is

$$u'^\sigma = \alpha^H \beta p + \alpha^L \beta(1-p). \tag{A5}$$

Condition (26) is equivalent to

$$u^\sigma \geq u'^\sigma. \tag{A6}$$

This means that as long as bubbles can exist, the user cost in the steady-state bubble economy is greater than that in the steady-state bubbleless economy.

Since $u^\sigma \geq u'^\sigma$ and $K^H \geq K'^H$, from (20) and (A3),

$$Y \geq Y'. \tag{A7}$$

Moreover, $u^\sigma \geq u'^\sigma$ and $K^H \geq K'^H$ mean

$$\frac{uK^H}{\sigma} \geq \frac{u'K'^H}{\sigma}. \tag{A8}$$

Hence,

$$Z^H \geq Z'^H, \tag{A9}$$

$$A \geq A', \tag{A10}$$

because in equilibrium, $uK^H/\sigma = Z^H/1-\sigma = \beta pA$ and $u'K'^H/\sigma = Z'^H/1-\sigma = \beta pA'$ hold.

We also know that aggregate consumption is a fraction $1-\beta$ of the aggregate wealth. Hence,

$$C \geq C'. \tag{A11}$$

Aggregate TFP can be defined as

$$\mathrm{TFP} \equiv \frac{Y}{\left(\frac{K}{\sigma}\right)^\sigma \left(\frac{Z}{1-\sigma}\right)^{1-\sigma}}, \quad \mathrm{TFP}' \equiv \frac{Y'}{\left(\frac{K'}{\sigma}\right)^\sigma \left(\frac{Z'}{1-\sigma}\right)^{1-\sigma}}, \tag{A12}$$

where $Z = Z^H + Z^L$ and $Z' = Z'^H + Z'^L$. Hence,

$$\mathrm{TFP} = \alpha^H \geq \mathrm{TFP}' \equiv \frac{\alpha^H Z'^H - \alpha^L Z'^L}{Z'}. \tag{A13}$$

C Proof of Proposition 3

By substituting (18), (22), and (23) into (21), we obtain

$$\phi_t = \sigma p + \frac{\delta - \phi_t}{\delta - \phi_{t+1}}\phi_{t+1}(1-\gamma), \tag{C1}$$

where $\phi_t \equiv q_t/\beta A_t$.

Given a state variable Y_t, there is a unique price path $\{q_t, P_t\}_{t=0}^{\infty}$ where ϕ_t becomes constant over time and satisfies

$$\phi = \sigma p/\gamma. \tag{C2}$$

Once ϕ_t becomes constant, then we can derive (28)-(30) from (22) and (C2). Using (18), (20) can be rewritten as

$$Y_{t+1} = \frac{\alpha^H}{\sigma}\left(\sigma\beta p A_t\right)^{1-\sigma}. \tag{C3}$$

(C3) can be rearranged as (27) using (28), (29), and the definition of A_t. Hence, the economy converges to the steadystate according to (27), and (28)-(30). The initial values of $\{q_0, P_0\}$ are determined so that the economy gets on the saddle path.