International R&D Rivalry with a Shipping Firm

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Abstract

To examine the role of shipping firms in the international research and development (R&D) rivalry, we build a two-country (exporting and importing), two-firm (exporting and local) duopoly model with a shipping firm. The exporting firm competes with the local firm in the duopoly market of the local country but must pay a shipping fee to the shipping firm in order to sell its product in the local market. Similar to market competition, exporting and local firms engage in R&D competition. We compare two timing structures of the game: in one, the R&D stage is first, and in the other, the shipping firm is the leader. We show that when the R&D stage is first, there are ranges of parameter values such that the investment level of the exporting firm decreases as R&D becomes more efficient. When the shipping firm is the leader, we show that there are ranges of parameter values such that the profit of the local firm decreases as R&D becomes more efficient. Further, it is shown that consumers in the local country prefer the regime in which the shipping firm is the leader, whereas the government of the local country prefers the other regime.

JEL classification: F12; F13; L13

Keywords: International trade; R&D rivalry; Shipping firm

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1 Introduction

Strategic research and development (R&D) investment has been widely discussed in the literature on international trade. Following the seminal work of Spencer and Brander (1983), for example, Leahy and Neary (1996, 1999), Muniguurria and Singh (1997), Qiu and Tao (1998), Dewit and Leahy (2004, 2011), Kujal and Ruiz (2007, 2009), and Takauchi (2011) focus on the effects of R&D subsidy policy, the spillover effects of investment, trade liberalization effects on firms’ investment strategy and welfare, and the interactions between other trade policies and firms’ R&D incentives in oligopolistic markets.1

However, the literature ignores one important factor on international trade: the role of shipment.2 In fact, since producers and shippers are not necessarily the same, we must distinguish between producers and shippers for a realistic consideration of international competition with R&D.3 If a shipping fee is determined by a shipping firm, how is the international R&D rivalry between exporting and domestic firms affected? In other words, compared with a traditional result, does the nature of the output and the investment strategies of firms change?

To answer this question, in the present paper, we consider a model of international R&D rivalry with a shipping firm. There are two producers, exporting and local firms, and a non-producer, a shipping firm. Local and exporting firms compete à la Cournot in the market of the local country and engage in R&D activity, whereas the shipping firm carries the product produced by the exporting firm from the exporting country to the market of the local country. In this situation, we examine two different timing games: in the first, the R&D investment level is first chosen by the exporting and local firms; in the second, the shipping firm is the leader.

In cost-reducing R&D competition, it may be expected that if the efficiency of R&D improves (or a cost parameter of R&D decreases), firms have larger incentives to invest, the volume of R&D investment increases, and output and profit increase. Because the efficiency of R&D directly affects the volumes of investment and output, it has an important role. However, we found some paradoxical results when R&D is highly efficient. First, we show that when the R&D stage is first, the volume of R&D investment of the exporting firm is inverted-U shaped

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1There are other topics on international R&D rivalry that focus on firm heterogeneity. Assuming the R&D investment level of firm has no effect on the rival’s output choice and introducing a distribution function of cost, Long et al. (2011) examine trade liberalization effects with and without free entry in an oligopoly model. Their non-strategic R&D model is similar to the study of Haaland and Kind (2008), which assumes that each firm simultaneously chooses both R&D investment and output. Furthermore, although they do not consider international competition, there are recent works that focus on cost heterogeneity. Ishida et al. (2011) examine the relationship between firms’ competitive position and initial cost differences among firms.

2In international trade theory, there are studies that focus on transportation services. Francois and Wooton (2001) consider shipping industry in a general equilibrium setting. Andriamananjara (2004) examines the reduction of trade barriers in a two-way oligopoly model with the shipping industry. However, these studies do not consider firms’ R&D activity.

3For example, a famous shipping firm in Japan, Mitsui O.S.K. Lines, provides car carriers, container ships, and logistics from Japan overseas. See their web site: http://www.mol.co.jp/menu-e.html. Additionally, Mitsubishi Souko provides marine transportation by ship; this service is used by Mitsubishi Motors Company. See their web site: http://www.mitsubishi-logistics.co.jp/service/index.html (in Japanese).
for the efficiency of R&D: if R&D is highly efficient, the exporter’s investment level decreases as the efficiency of R&D improves. The key to this result is that the shipping fee decreases as the volume of the exporting firm’s investment decreases. For that reason, to reduce the shipping fee, the exporting firm commits to a smaller level of investment. Since this commitment also increases the rival’s investment level, the exporter’s investment level decreases when the rival has extremely larger incentives to invest.

Second, we show that when the shipping firm is the leader, the local firm’s profit is inverted-U shaped for the efficiency of R&D: if R&D investment is highly efficient, the profit of the local firm decreases as the efficiency of R&D improves. In this case, the local firm invests excessively given its much smaller increase in output. Since an increase in expenditure level in R&D dominates an increase in gains of investment, the local firm’s profit falls.

Third, we show that in the case that R&D investment level is first decided, the shipping fee is always higher than in the other case. This is because in the case that fee-setting is first decided, for fear of a disadvantage to the exporting firm, the shipping firm always offers a relatively low fee. As a result, in the case that R&D investment level is first decided, the profit of the exporting firm is smaller than in other case, but the local firm has a larger profit in this case.

Last, we demonstrate that consumers in the local country prefer the case that the shipping firm is the leader, whereas the government of the local country prefers the case that R&D investment is first decided. Commitment to a smaller investment level gives a larger benefit to the rival firm (the local firm), but this does not cover the losses in total outputs of two manufacturing firms. Therefore, in the case that the shipping firm is the leader, total output is relatively large, whereas the profit of local firm is relatively large in the other case.

The role of a shipping firm in an international oligopoly is inspired by the work of Lahiri and Ono (1999). They examine how a traditional argument on optimal tariff policy can be altered when the producer and seller differ. To an international oligopoly model, they introduce a middleman, called as “seller,” and demonstrate that the optimal policy is an import subsidy (i.e., a negative tariff rate) if the seller is the leader. Also they found that the sign of the optimal tariff rate depends on the shapes of production cost and market structures. Although their focus is on the tariff policy, in view of the presence of a middleman, our study is related to theirs, although R&D rivalry was not considered in their work.

Our study is also related to some works on labor unions with oligopolies (e.g., Calabuig and Gonzalez-Maestre, 2002; Haucap and Wey, 2004; and Manasakis and Petrakis, 2009). In our model, the shipping firm sets the fee in a manner similar to that employed in the above labor union model. Manasakis and Petrakis (2004) consider a cost-reducing R&D rivalry with a labor union and examine the formation of research joint ventures (RJVs) in duopoly firms.
However, our model is crucially different from the above labor union model. In the above studies, it is assumed that each firm has a union (i.e., a decentralized regime) or that all firms have a common union (i.e., a centralized regime). In our R&D competition model with a shipping firm, the only exporting firm must use the shipping firm and pays a fee. This point, that “one firm has a middleman but the other does not,” is basically different from the above unionized industry model.

In the next section, we build a model and examine firms’ investment and output strategies. Section 3 compares outcomes and examines welfare implications. Section 4 concludes.

2 Model

We consider a two-country partial equilibrium trade model with a shipping firm. There are two countries, local and foreign. The local country has a product market, but the foreign country does not have. We call the foreign an “exporting” country. Three firms—an exporting firm, a local firm, and a shipping firm—exist in the world, and the exporting (resp. local) firm locates in the exporting (resp. local) country. Exporting and local firms are manufacturers of a product and compete à la Cournot in the local country’s market. Since the local firm is local, we assume that this firm supplies its product without a shipping fee. On the other hand, since the exporting firm is located overseas, it must use the shipping firm in order to carry its product to the local market across international borders. To transport the product, the exporting firm must pay a shipping fee, $f$, to the shipping firm.

Before market competition, exporting and local firms engage in cost-reducing R&D competition without spillover. The unit production cost is $c (> 0)$, and is the same between exporting and local firms. To reduce the initial unit cost, both manufacturing firms make cost-reducing investments. After investment, the unit cost becomes $c - x_i$, where $x_i$ is the investment level of manufacturing firm $i$. Variables associated with the two manufacturing firms are subscripted by $i = E, L$ (exporting and local). We assume that the R&D cost function is linear-quadratic: $(\gamma/2)(x_i)^2$, where $\gamma$ is the efficiency of R&D and a positive constant. That is, a smaller $\gamma$ corresponds to a higher efficiency in R&D.

A representative consumers’ utility function in the local country is $U = z + \alpha Q - (1/2)Q^2$.

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4Although Calabuig and Gonzalez-Maestre (2002) and Haucap and Wey (2004) consider a unionized industry, they do not examine R&D rivalry. In addition, there are other works on the unionized industry model. Lommerud et al. (2003) discuss a firm’s choice between exporting and foreign direct investment (FDI) with a labor union in a two-way oligopoly model. However, they do not consider R&D activity.

5Our model also differs from Manasakis and Petrakis (2009) on the following point. To examine cooperative R&D regimes (RJVs), they focus on the role of spillover effects of investment, but ignore the cost parameter of R&D. In contrast to Manasakis and Petrakis (2009), we focus on the role of the cost parameter of R&D, but do not consider the spillover effects of investment.

where $z$ is consumption of the numerare good and $Q$ is the total supply of both manufacturing firms: $Q = \sum_{i=E,L} q_i$, where $q_i$ is the supply of manufacturing firm $i$. Hence, the inverse market demand in local country is $p = a - Q$, where $p$ is the product price, $a$ is a positive constant, and $a > c$. Thus, the profit of each producing firm, $\Pi_i$, is given by

$$
\Pi_E \equiv [a - (q_E + q_L) - (c - x_E) - f] q_E - \frac{\gamma}{2} (x_E)^2, \quad (1)
$$

$$
\Pi_L \equiv [a - (q_E + q_L) - (c - x_L)] q_L - \frac{\gamma}{2} (x_L)^2. \quad (2)
$$

The shipping firm makes a take-it-or-leave-it offer to the exporting firm, and decides the fee, $f$. The profit of the shipping firm, $\pi_S$, is given by

$$
\pi_S \equiv (f - f_0) q_E, \quad (3)
$$

where $f_0$ is the minimum price of shipment. For simplicity, we normalize $f_0$ to zero.

To examine the strategic interaction between the shipping firm and the manufacturing firms, we consider two scenarios on the timing structures of the game. In the first, manufacturing firms’ R&D stage is first, and in the other, the shipping firm’s fee-setting stage is first (i.e., the shipping firm is the leader). In general, since investment decisions do not have the flexibility of other production decisions, it is plausible that the investment stage will take place in the first stage of the game. On the other hand, the shipping firm is a different player from manufacturing firms. For this reason, we must consider these two cases.

In Case 1, the following timing structure is considered. At stage 1, the exporting and local firms independently and simultaneously choose a volume of R&D investment. At stage 2, the shipping firm sets the shipping fee. At stage 3, the exporting and local firms compete à la Cournot in the local market.

In Case 2, the timing structure is as follows. At stage 1, the shipping firm sets shipping fee. At stage 2, the exporting and local firms independently and simultaneously choose a volume of R&D investment. At stage 3, the exporting and local firms compete à la Cournot in the local market.

The game has perfect information. Therefore, we use the subgame perfect Nash equilibrium as the equilibrium concept and solve the game using backward induction.

In the next subsection, we calculate the equilibrium outcomes in two timing structures and examine on firms strategies. We first consider Case 1.
2.1 Case 1: The R&D stage is first

**Third stage.** The exporting and local firms compete à la Cournot in the market of the local country. Using (1) and (2), the FOC for the profit maximization of each firm is

- Exporting firm: \( 0 = a - c - f - 2q_E - q_L + x_E, \)
- Local firm: \( 0 = a - c - q_E - 2q_L + x_L. \)

The above equations yield the following outputs in the third stage:

\[
q_E(x_E, x_L, f) = \frac{1}{3}(a - c - 2f + 2x_E - x_L); \quad q_L(x_E, x_L, f) = \frac{1}{3}(a - c + f - x_E + 2x_L). \tag{4}
\]

**Second stage.** In the second stage, the shipping firm chooses a level of the shipping fee \( f. \) From (3) and (4), the FOC is

\[
0 = -\frac{2}{3}f + \frac{1}{3}(a - c - 2f + 2x_E - x_L).
\]

Solving the above equation for \( f, \) we obtain the shipping fee in the second stage.

\[
f(x_E, x_L) = \frac{1}{4}(a - c + 2x_E - x_L). \tag{5}
\]

Thus, fee rises (falls) as the volume of R&D investment of the exporting firm increases (decreases).

In the second stage, from (4) and (5), the outputs of exporting and local firms are

\[
q_E(x_E, x_L) = \frac{1}{6}(a - c + 2x_E - x_L); \quad q_L(x_E, x_L) = \frac{1}{12}[5(a - c) - 2x_E + 7x_L]. \tag{6}
\]

**First stage.** Hereafter, we use “∗” as the symbol of the subgame perfect Nash equilibrium in Case 1 of the game. In the first stage, the exporting and local firms choose their respective volumes of R&D investment. Using (1), (2), (5), and (6), FOCs in this R&D stage are

- Exporting firm: \( 0 = a - c + (2 - 9\gamma)x_E - x_L, \)
- Local firm: \( 0 = 35(a - c) - 14x_E + (49 - 72\gamma)x_L. \)

The above equations yield the equilibrium levels of R&D investment \( x^*_E \) and \( x^*_L \) (eq. (8), below).
The equilibrium output and profit of exporting and local firms are

\[ q^*_E = \frac{6(a - c)\gamma(6\gamma - 7)}{28 - 195\gamma + 216\gamma^2}, \quad q^*_L = \frac{6(a - c)\gamma(15\gamma - 4)}{28 - 195\gamma + 216\gamma^2}, \]  

\[ x^*_E = \frac{4(a - c)(6\gamma - 7)}{28 - 195\gamma + 216\gamma^2}, \quad x^*_L = \frac{7(a - c)(15\gamma - 4)}{28 - 195\gamma + 216\gamma^2}, \]  

\[ \Pi^*_E = \frac{4(a - c)^2(6\gamma - 7)^2\gamma(9\gamma - 2)}{(28 - 195\gamma + 216\gamma^2)^2}, \quad \Pi^*_L = \frac{(a - c)^2(15\gamma - 4)^2\gamma(72\gamma - 49)}{2(28 - 195\gamma + 216\gamma^2)^2}. \]  

The equilibrium shipping fee is

\[ f^* = \frac{9(a - c)\gamma(6\gamma - 7)}{28 - 195\gamma + 216\gamma^2}. \]  

From (3), (7), and (10), the equilibrium profit of the shipping firm is

\[ \pi^*_S = \frac{54(a - c)^2\gamma^2(6\gamma - 7)^2}{(28 - 195\gamma + 216\gamma^2)^2}. \]  

Throughout the analysis, we make the following assumption.

**Assumption 1** The efficiency of R&D is not too high, i.e., \( \gamma > 4/3 \simeq 1.3333 \).

To ensure positive quantities for all outcomes in the two cases of the game, we require the above assumption. Note that this is a stronger condition on \( \gamma \) than the condition required to satisfy the SOCs in both the R&D and fee-setting stages of the two timing structures.\(^7\) Thus, any other conditions are satisfied as long as Assumption 1 holds.

Eqs. (7)–(9) yield the following proposition.

**Proposition 1** In Case 1, (i) when R&D is highly efficient, the volume of R&D investment of the exporting firm decreases as the efficiency of R&D improves; otherwise it increases as the efficiency of R&D improves, i.e., \( \partial x^*_E / \partial \gamma > 0 \) if \( \gamma < \hat{\gamma} \equiv (1/12)(3\sqrt{7} + 14) \simeq 1.8281 \) but \( \partial x^*_E / \partial \gamma < 0 \) if \( \gamma > \hat{\gamma} \). However, the volume of R&D investment of the local firm increases as the efficiency of R&D improves. (ii) The output and profit of the exporting firm decrease, but those of the local firm increase as the efficiency of R&D improves.

**Proof.** See Appendix A.

In a usual cost-reducing R&D competition, the volume of investment increases as R&D improves (a decrease in the cost parameter of R&D).\(^8\) Thus, one might think that the result shown in Proposition 1 is in sharp contrast to the intuition. When R&D is highly efficient, why

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\(^7\)In case 1, the SOC for profit maximization in the R&D stage is \( \gamma > 2/9 \simeq 0.22222 \) for the exporting firm and \( \gamma > 49/72 \simeq 0.680556 \) for the local firm.

\(^8\)For example, see Motta (2004, Ch. 2).
dose the exporting firm reduce the volume of investment as the efficiency of R&D improves? The logic behind this result depends on two factors. The first is that the economy of scale works in R&D investment; the effect is stronger if R&D is highly efficient. The second is that to commit to a smaller level of investment reduces the shipping fee, which decreases as the exporting firm’s investment level decreases.

In investment, each firm will invest heavily if it believes it can sell heavily. Furthermore, since a smaller $\gamma$ corresponds to a higher R&D efficiency, investment incentives are larger. The exporting firm must pay the shipping fee, so it has a smaller market share than the local firm. That is, since the local firm has an advantage in market competition, the local firm invests actively if $\gamma$ is sufficiently small.

The second factor promotes the effects of the scale economy. Since an investment level is decided in the first stage of the game, the exporting firm can control the shipping fee. A reduction in $x_E$, from eq. (5), directly reduces the shipping fee and increases the rival’s investment level (i.e., strategic substitutes in the R&D stage). Additionally, from eq. (5), an increase in the rival’s investment level also reduces the fee. To reduce the shipping fee, the exporting firm always commits to a smaller level of investment; this behavior induces a larger level of investment by the rival. When the efficiency of R&D is sufficiently high, the effects of smaller investment extremely increase the rival’s investment and further reduces the investment level of the exporting firm. Therefore, if $\gamma$ becomes smaller than the critical level, $\gamma^*$, then the investment level of the exporting firm falls.

2.2 Case 2: The shipping firm is the leader

Note that in Case 2, the shipping fee is decided in the first stage of the game. Since the third stage (market stage) is the same as Case 1, we start the analysis from the second stage.
**Second stage.** In the second stage, each producing firm engages in R&D investment. Thus, each firm’s FOC is as follows:

Exporting firm: \[ 0 = \frac{1}{9} [4(a - c) - 8f - (9\gamma - 8)x_E - 4x_L], \]

Local firm: \[ 0 = \frac{1}{9} [4(a - c) + 4f - 4x_E - (9\gamma - 8)x_L]. \]

Solving the above FOCs for \( x_E \) and \( x_L \), we obtain the investment levels in the second stage.

\[
x_E(f) = \frac{4[(a - c)(3\gamma - 4) - 2(3\gamma - 2)f]}{(3\gamma - 4)(9\gamma - 4)}; \quad x_L(f) = \frac{4[(a - c)(3\gamma - 4) + 3\gamma f]}{(3\gamma - 4)(9\gamma - 4)}.
\] \( (12) \)

From (12), the outputs of both producing firms in the second stage are

\[
q_E(f) = \frac{3\gamma[(a - c)(3\gamma - 4) - 2(3\gamma - 2)f]}{(3\gamma - 4)(9\gamma - 4)}; \quad q_L(f) = \frac{3\gamma[(a - c)(3\gamma - 4) + 3\gamma f]}{(3\gamma - 4)(9\gamma - 4)}.
\] \( (13) \)

**First stage.** Hereafter, we use “\( \ast \ast \)” as the symbol of the subgame perfect Nash equilibrium in Case 2 of the game. The FOC for the shipping firm is

\[
0 = \frac{3\gamma[(a - c)(3\gamma - 4) - 4(3\gamma - 2)f]}{(3\gamma - 4)(9\gamma - 4)}.
\]

Thus, solving the above FOC for \( f \), we obtain the equilibrium shipping fee \( f^{\ast \ast} \) in Case 2. \( ^{10} \)

\[
f^{\ast \ast} = \frac{(a - c)(3\gamma - 4)}{4(3\gamma - 2)}.
\] \( (14) \)

The equilibrium outcomes in Case 2 are

\[
q_E^{\ast \ast} = \frac{3(a - c)\gamma}{2(9\gamma - 4)}; \quad q_L^{\ast \ast} = \frac{3(a - c)\gamma(15\gamma - 8)}{4(3\gamma - 2)(9\gamma - 4)}, \quad x_E^{\ast \ast} = \frac{2(a - c)}{9\gamma - 4}; \quad x_L^{\ast \ast} = \frac{(a - c)(15\gamma - 8)}{(3\gamma - 2)(9\gamma - 4)}, \quad \text{and}
\]

\[
H_E^{\ast \ast} = \frac{(a - c)^2\gamma(9\gamma - 8)}{4(9\gamma - 4)^2}; \quad H_L^{\ast \ast} = \frac{(a - c)^2(15\gamma - 8)^2\gamma(9\gamma - 8)}{16(3\gamma - 2)^2(9\gamma - 4)^2}.
\] \( (16) \)

The equilibrium profit of the shipping firm is

\[
\pi_S^{\ast \ast} = \frac{3(a - c)^2\gamma(3\gamma - 4)}{8(3\gamma - 2)(9\gamma - 4)}.
\] \( (17) \)

\( ^{9} \)The SOC for both producing firms in R&D stage is \( \gamma > 8/9 \approx 0.888889. \)

\( ^{10} \)The SOC for the shipping firm is \( \partial^2 \pi_S/\partial f^2 = -12\gamma(3\gamma - 2)/(3\gamma - 4)(9\gamma - 4). \) Hence, the SOC holds as long as Assumption 1 holds.
Before the analysis on the outcomes of the two producing firms, let us compare the equilibrium shipping fees in the two cases. From (10), (11), (14), and (18), we establish the following proposition.

**Proposition 2** Regardless of the timing structures, (i) the shipping fee rises and becomes to $(1/4)(a-c)$ as the efficiency of R&I becomes worse; (ii) the profit of the shipping firm decreases as the efficiency of R&I improves.

**Proof.** (i) Differentiating (10) and (14) with respect to $\gamma$, we obtain

\[
\frac{\partial f^*}{\partial \gamma} = \frac{18(a-c)(-98+168\gamma+171\gamma^2)}{(28-195\gamma+216\gamma^2)^2}, \quad \frac{\partial f^{**}}{\partial \gamma} = \frac{3(a-c)}{2(3\gamma-2)^2} > 0.
\]

From the numerator of $\partial f^*/\partial \gamma$, $-98+168\gamma+171\gamma^2 \geq 0$ for all $\gamma \geq (7/57)(3\sqrt{6} - 4) \approx 0.411216$. Hence, $\partial f^*/\partial \gamma > 0$. Further, rearranging (10) and (14), we find that $\lim_{\gamma \to \infty} f^{*} = \lim_{\gamma \to \infty} (a-c)(54-63\gamma+216\gamma^2) = (1/4)(a-c) = \lim_{\gamma \to \infty} f^{**} = \lim_{\gamma \to \infty} \frac{(a-c)(3-(4/\gamma))}{4(3-2/\gamma)}$.

(ii) Differentiating (11) with respect to $\gamma$, we obtain

\[
\frac{\partial \pi^S}{\partial \gamma} = \frac{216(a-c)^2(6\gamma-7)(-98+168\gamma+171\gamma^2)}{(28-195\gamma+216\gamma^2)^3}, \quad \frac{\partial \pi^{**}}{\partial \gamma} = \frac{3(a-c)^2(-16+24\gamma+9\gamma^2)}{4(3\gamma-2)^2(9\gamma-4)^2}.
\]

From the above argument, $\partial \pi^S/\partial \gamma > 0$. From the numerator of $\partial \pi^{**}/\partial \gamma$, $-16+24\gamma+9\gamma^2 > 0$ for all $\gamma > (4/3)(\sqrt{2} - 1) \approx 0.552285$. Q.E.D.

In our model, exporting and shipping firms have a certain “cooperative relationship.” Since a shipping firm’s profit directly depends on the volume of output of the exporting firm, to ensure positive production for the exporting firm, the shipping firm reduces its fee when the exporting firm is extremely weak. Hence, the profit of shipping firm also goes down as the output of the exporting firm goes down. This logic holds in both cases.

In a cost-reducing R&I competition, investment is not performed if the cost of R&I increases indefinitely (i.e., $x_i \to 0$ as $\gamma \to \infty$). That the shipping fee has an upper limit corresponds to this fact.

From (15)–(17), we establish the following proposition.

**Proposition 3** In Case 2, (i) the output and volume of the R&I investment of local and exporting firms increases as the efficiency of R&I improves. (ii) When R&I is highly efficient, the profit of the local firm decreases as the efficiency of R&I improves; otherwise, it increases as the efficiency of R&I improves, i.e., $\partial \Pi_L^{**}/\partial \gamma > 0$ if $\gamma < \bar{\gamma} \simeq 1.74008$ but $\partial \Pi_L^{**}/\partial \gamma < 0$ if $\gamma > \bar{\gamma}$. However, the profit of the exporting firm increases as the efficiency of R&I improves.
Proof. (i) Differentiating (15) with respect to $\gamma$, we obtain

$$\frac{\partial q_i^*}{\partial \gamma} = \frac{-6(a-c)}{(9\gamma-4)^2} < 0, \quad \frac{\partial q_L^*}{\partial \gamma} = \frac{-3(a-c)(32-120\gamma+117\gamma^2)}{(9\gamma-4)^2}. $$

From the numerator of $\partial q_L^*/\partial \gamma$, $32-120\gamma+117\gamma^2 = 0$ has no real root, and $\gamma > 4/3$ has a positive value. Thus, $\partial q_L^*/\partial \gamma < 0$. Differentiating (16) with respect to $\gamma$, we obtain

$$\frac{\partial x_i^*}{\partial \gamma} = \frac{-18(a-c)}{(9\gamma-4)^2} < 0, \quad \frac{\partial x_L^*}{\partial \gamma} = \frac{-3(a-c)(40-144\gamma+135\gamma^2)}{(3\gamma-2)^2(9\gamma-4)^2}. $$

From the numerator of $\partial x_L^*/\partial \gamma$, $40-144\gamma+135\gamma^2 = 0$ has no real root, and $\gamma > 4/3$ has a positive value. Thus, $\partial x_L^*/\partial \gamma < 0$.

(ii) Differentiating (17) with respect to $\gamma$, we obtain

$$\frac{\partial \Pi_i^*}{\partial \gamma} = \frac{8(a-c)^2}{(9\gamma-4)^3} > 0, \quad \frac{\partial \Pi_L^*}{\partial \gamma} = \frac{-(a-c)^2(15\gamma-8)(-128+528\gamma-684\gamma^2+243\gamma^3)}{4(3\gamma-2)^3(9\gamma-4)^3}. $$

From the numerator of $\partial \Pi_L^*/\partial \gamma$, we find that $-128+528\gamma-684\gamma^2+243\gamma^3 \geq 0$ for $\gamma \geq \tilde{\gamma} \approx 1.74008$. Therefore, $\partial \Pi_L^*/\partial \gamma \leq 0$ for $\gamma \geq \tilde{\gamma}$ and $\partial \Pi_L^*/\partial \gamma > 0$ for $\gamma < \tilde{\gamma}$. Q.E.D

![Graph of $q_i^*$ and $q_L^*$](image-a)

**Panel (a): Graph of $x_i^*$ and $q_i^*$**

**Panel (b): Graph of $\Pi_L^*$**

**Figure 2: Inverted-U shaped equilibrium profit of local firm**

Panel (a) in Figure 2 shows that the local firm’s output and investment are decreasing with $\gamma$. In Case 2, the shipping fee is decided in the first stage, so the exporting firm does not directly control it. For this reason, the equilibrium investment and output levels of the two producing firms have an ordinal nature with respect to $\gamma$, i.e., $\partial q_i^*/\partial \gamma < 0$ and $\partial x_i^*/\partial \gamma < 0$, for $i = E, L$.

On the other hand, an interesting result occurs concerning the profit of local firm: it is
inverted-U shaped for \( \gamma \) (see Panel (b)). That is, if R&D is highly efficient, the local firm’s profit decreases as the efficiency of R&D improves.

Similar to Case 1 (Proposition 1), the local firm has a larger incentive to invest because the local firm has an advantage in market competition. From (15) and (16), we find the following relations in Case 2:

\[
\Delta x_q \equiv \frac{\partial x^*_L}{\partial \gamma} - \frac{\partial q^*_L}{\partial \gamma} = \frac{9(a - c)[16 + \gamma(51\gamma - 56)]}{2(3\gamma - 2)^2(9\gamma - 4)^2} > 0; \quad \frac{\partial \Delta x_q}{\partial \gamma} < 0.
\]

Thus, the difference between an increase in investment and an increase in output expands when R&D becomes more efficient.\(^{11}\)

Why does this result occur? As shown in Proposition 2, the shipping fee falls as the efficiency of R&D improves. For this reason, the cost difference between the exporting and local firms closes as the efficiency of R&D improves. The effect of a closing cost difference induces an increase in the output and profit of the exporting firm ((ii) of Proposition 3). Also, in Case 2, the shipping fee is decided in the first stage of the game, so the exporting firm cannot commit to a smaller level of investment to reduce the shipping fee. Thus, corresponding to an increase in output, the exporting firm increases its investment level if the efficiency of R&D improves. The local firm suffers a loss from such behavior by the exporting firm. This is because the local firm has an advantage in market competition from the beginning, but this advantage becomes smaller when the shipping fee falls. Therefore, the local firm’s output increases, but the increase in output is relatively smaller than the expenditure level of R&D. The effects of excessive investment expand as the efficiency of R&D improves (\( \partial \Delta x_q / \partial \gamma < 0 \)). Therefore, if \( \gamma \) becomes smaller than the critical level, \( \gamma_\text{c} \), an increase in the expenditure level of investment will dominate an increase in gains, and the profit of the local firm decreases.

3 Comparison among Outcomes/Welfare Implications

In this section, we compare welfares in Cases 1 and 2 and examine the relationship between each country’s welfare and the efficiency of R&D.

Before the welfare analysis, let us compare the outcomes in Cases 1 and 2. From eqs. (7)–(10) and (14)–(17), we establish the following proposition.

Proposition 4 Comparison between the outcomes in Case 1 (the R&D stage is first) and Case 2 (the shipping firm is the leader) yields

\(^{11}\)Formally, we obtain

\[
\frac{\partial \Delta x_q}{\partial \gamma} = -\frac{9(a - c)(-256 + 1296\gamma - 2268\gamma^2 + 1377\gamma^3)}{(3\gamma - 2)^4(9\gamma - 4)^4} < 0 \quad \text{for all } \gamma > 4/3.
\]
(i) $f^* > f^{**}$;

(ii) $\pi_S^* \geq \pi_S^{**}$ if $\gamma \leq \gamma_S$ and $\pi_S^* < \pi_S^{**}$ if $\gamma > \gamma_S$, where $\gamma_S \approx 1.4273$;

(iii) $q_E^{**} > q_E^*, \ q_L^* > q_L^{**}$, $x_E^* > x_E^*, \ x_L^* > x_L^{**}$, and the difference between $x_L^*$ and $x_L^{**}$ is maximized at $\gamma = \gamma' \approx 1.82817$; and

(iv) $\Pi_E^{**} > \Pi_E^*$ and $\Pi_L^* > \Pi_L^{**}$.

Proof. See Appendix C.

Figures 3 and 4 depict investment levels in the local firm (claim (iii)), shipping fees, and the difference in the profits of the shipping firm (claims (i) and (ii)). Claim (i) of Proposition 4 indicates that when the shipping fee can directly control the exporting firm’s investment strategy, for fear of a disadvantage to the exporting firm, the shipping firm offers a relatively lower fee. Thus, this is desirable for the exporting firm when the shipping firm is the leader, which corresponds to claim (iv), that is, $\Pi_E^{**} > \Pi_E^*$.

Intuitively, from claim (i) of Proposition 4, one might think that the shipping firm has a “second-mover advantage” in our model. However, in contrast to the result of the shipping fee, the profit of the shipping firm is larger in Case 1 than in Case 2 if R&D is highly efficient. Claim (ii) relies on claim (iii), that is, on the outputs ranking between the two cases: $q_E^{**} > q_E^*$. Since the exporting firm’s output is relatively small in Case 1 (although the shipping fee is relatively high in that case), the profit of shipping firm is possibly larger in Case 1 than in Case 2.

Claim (iii) of Proposition 4 has an interesting result. This is because world total investment, $x_E + x_L$, is larger in Case 2 than in Case 1. In particular, $x_L^{**} > x_L^*$ is important. This result corresponds to claim (ii) of Proposition 3, that is, the local firm invests excessively given its output.
Upper curve is $x_L^*$; Lower curve is $x_L^{**}$.

Panel (a): Graph of $x_L^*$ and $x_L^{**}$

Panel (b): Graph of $x_L^* - x_L^{**}$

Figure 4: Comparison $x_L^*$ and $x_L^{**}$

Claim (iv) is intuitive one. Since the profit of the producing firm depends on its output, claim (iv) corresponds to the result of claim (iii). The exporting and local firms’ desirable timing structures are opposite of one another.

Next, we consider welfare and consumer surplus in the local country. The social welfare of the local country is given by $W_L \equiv CS + II_L$, where consumer surplus is defined by $CS = (1/2)(q_E + q_L)^2$. Thus, welfare and consumer surplus in the two cases are

$$CS^* = \frac{18(a - c)^2\gamma^2(21\gamma - 11)^2}{(28 - 195\gamma + 216\gamma^2)^2}, \quad (19)$$

$$CS^{**} = \frac{81(a - c)^2\gamma^2(7\gamma - 4)^2}{32(3\gamma - 2)^2(9\gamma - 4)^2}, \quad (20)$$

$$W_L^* = \frac{(a - c)^2\gamma(-784 + 11388\gamma - 36297\gamma^2 + 32076\gamma^3)}{2(28 - 195\gamma + 216\gamma^2)^2}, \quad (21)$$

$$W_L^{**} = \frac{(a - c)^2\gamma(-1024 + 6288\gamma - 12456\gamma^2 + 8019\gamma^3)}{32(3\gamma - 2)^2(9\gamma - 4)^2}. \quad (22)$$

Consumer surplus and welfare in two cases increase as the efficiency of R&D improves: $\partial CS/\partial \gamma < 0$ and $\partial W_L/\partial \gamma < 0$ (see Appendix D).

Comparing (19) with (20), we obtain

$$CS^* - CS^{**} = -\frac{9(a - c)^2\gamma^2(368 - 1056\gamma + 729\gamma^2)(-1040 + 6912\gamma - 14103\gamma^2 + 9072\gamma^3)}{32(3\gamma - 2)^2(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^2}. \quad (23)$$

Subsequently, comparing (21) with (22), we obtain

$$W_L^* - W_L^{**} = \frac{3(a - c)^2\gamma^2A}{32(3\gamma - 2)^2(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^2}, \quad (24)$$
where \( A \equiv 523520 - 3860736\gamma + 10300032\gamma^2 - 11625120\gamma^3 + 4221639\gamma^4 + 629856\gamma^5 \).

From (23) and (24), we establish the following proposition.

**Proposition 5** While consumers in the local country prefer the regime in which the shipping firm is the leader, the government of the local country prefers the regime in which the R&D stage is first.

**Proof.** From the right hand side of (23), 
\[ 368 - 1056\gamma + 729\gamma^2 > 0 \text{ for all } \gamma > (4/243)(\sqrt{73}+44) \simeq 0.864922; \]
\[ -1040 + 6912\gamma - 14103\gamma^2 + 9072\gamma^3 > 0 \text{ for all } \gamma > 0.700737. \]
Subsequently from the right hand side of (24), by numerical calculation, \( H > 0 \) for all \( \gamma > 0.695157 \). Therefore, \( CS^{**} > CS^{*} \) and \( W^{**}_L > W^{*}_L \) from Assumption 1. Q.E.D.

Proposition 5 implies that either the profit of the local firm or consumer surplus has the larger effect on the welfare of local country. As shown in (iv) of Proposition 4, the local firm’s profit is larger in Case 1 than in Case 2. However, from Proposition 5, consumer surplus (i.e., volume of total output) is larger in Case 2 than in Case 1. Since the national welfare of local country is the sum of the local firm’s profit and consumer surplus, we find that a gain in the profit of the local firm in Case 1 dominates a loss in consumer surplus in Case 2. In Case 1, to reduce the shipping fee, the exporting firm commits to a smaller level of investment (Proposition 1). In Proposition 5, we can also verify that this commitment effect of the exporting firm gives a larger benefit to the local firm.

Finally, we consider the social welfare of the exporting country. Hereafter, we assume that the shipping firm is located in the exporting country. Then, the social welfare of the exporting country, \( W_E \), is sum of the profits of the shipping and exporting firms, \( \Pi_E + \pi_S \). The social welfare is given by

\[
W^{**}_E = \frac{(a - c)^2(80 - 228\gamma + 135\gamma^2)}{8(9\gamma - 4)^2(3\gamma - 2)}.
\]

In contrast to the welfare of the local country, the welfare of the exporting country decreases as the efficiency of R&D improves: \( \partial W_E / \partial \gamma > 0 \) (see Appendix D).

Eq. (25) yields the following result.

**Corollary 1** Suppose that the shipping firm is located in the exporting country. Then, (i) the social welfare of the exporting country is larger in Case 2 than in Case 1. (ii) The difference between the social welfare of the exporting country in Cases 1 and 2 is maximized at \( \gamma = \bar{\gamma} \simeq 1.63951 \).

**Proof.** See Appendix C.

As shown in Proposition 4, the profit of the exporting firm is always larger in Case 2 than in Case 1. On the other hand, if R&D is highly efficient, the profit of the shipping firm is larger in
Case 1 than in Case 2. At first glance, welfare appears larger in Case 1 than in Case 2. However, the welfare level in Case 2 dominates that in Case 1. Whereas the profit of the shipping firm becomes smaller as the efficiency of R&D improves, the profit of the exporting firm in Case 2 increases as the efficiency of R&D improves. Therefore, the volume of the shipping firm’s profit where R&D is sufficiently efficient is too small to reverse the ranking in the exporting firm’s profit in Cases 1 and 2.

4 Conclusion

This paper builds a model of a two-country (exporting and local countries), two-firm (exporting and local firms) international R&D rivalry with a shipping firm. Although R&D rivalry has been broadly discussed in the literature of trade theory, the role of the shipping firm on industrial competition has received little attention (e.g., Spencer and Brander, 1983; Leahy and Neary, 1996, 1999; Muniagurria and Singh, 1997; Qiu and Tao, 1998; Dewit and Leahy, 2004, 2011; Kujal and Ruiz, 2007, 2009; Takauchi, 2011). To examine the interaction between the endogenously decided shipping fee and the producing firm’s strategies, we consider different two timing structures of the model. In one, the R&D investment level is first decided; in the other, the shipping firm is the leader.

We have shown first that the exporting firm’s investment level is inverted-U shaped for the efficiency of R&D when the R&D investment level is first decided in the game. That is, there are ranges of parameter values such that the investment level goes down as the efficiency of R&D improves. This is because the shipping fee decreases as the exporting firm’s investment level decreases, and thus the exporting firm commits to a smaller investment level to reduce the shipping fee. This behavior leads to an increase in the investment level of the rival firm. As a result, the exporting firm decreases its investment level when the rival firm has stronger incentives to invest.

Second, we have shown that the local firm’s profit is inverted-U shaped for the efficiency of R&D when the shipping firm is the leader. That is, there are ranges of parameter values such that profit decreases as the efficiency of R&D improves. In this case, the local firm invests excessively given its output. Since the loss in investment expenditure dominates the investment gain, the profit of the local firm falls when R&D is highly efficient.

Third, we have shown that in the case that the R&D investment level is first decided, the shipping fee is always higher than in the other case. For fear of a disadvantage to the exporting firm, the shipping firm offers a relatively lower fee if it can control investment level of the exporting firm. For this reason, in the case that the R&D stage is first, the profit of the exporting firm is smaller than in the other case, but local firm has a larger profit.

Last, we have demonstrated that consumers in the local country prefer the regime in which
the shipping firm is the leader, whereas the government of local country prefers the regime in which R&D stage is first. To commit to a smaller investment level gives a larger benefit to the rival firm; this effect does not cover the losses in total output. Therefore, total output is relatively large when the shipping firm is the leader, whereas the profit of the local firm is relatively large when R&D stage is first.

In the cost-reducing R&D rivalry, the efficiency of R&D plays an important role. If the cost of R&D improves, investment incentives and output increase. This undoubtedly increases profit of firms. However, our findings highlight some paradoxical results that occur if R&D is highly efficient. Since all these findings depend on the behavior of the shipping firm, we believe that our model developed herein offers new insight on international R&D competition.

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Appendix

A. Proof of Proposition 1

(i) Differentiating (8) with respect to $\gamma$, we obtain

$$\frac{\partial x^*_E}{\partial \gamma} = -\frac{36(a-c)(133-336\gamma+144\gamma^2)}{(28-195\gamma+216\gamma^2)^2}, \quad \frac{\partial x^*_L}{\partial \gamma} = -\frac{504(a-c)(5-24\gamma+45\gamma^2)}{(28-195\gamma+216\gamma^2)^2}.$$  

From the numerator of $\frac{\partial x^*_E}{\partial \gamma}$, $133-336\gamma+144\gamma^2 \geq 0$ for $\gamma \geq (1/12)(3\sqrt{7} + 14) \approx 1.8281$ and $\gamma \leq (1/12)(-3\sqrt{7} + 14) \approx 0.505229$. This yields $\frac{\partial x^*_E}{\partial \gamma} \geq 0$ if $\gamma \leq (1/12)(3\sqrt{7} + 14)$, but $\frac{\partial x^*_E}{\partial \gamma} < 0$ if $\gamma > (1/12)(3\sqrt{7} + 14)$. Subsequently, from the numerator of $\frac{\partial x^*_L}{\partial \gamma}$, we find that $5-24\gamma+45\gamma^2 > 0$ for all $\gamma$. Thus, $\frac{\partial x^*_L}{\partial \gamma} < 0$.

(ii) Differentiating (7) with respect to $\gamma$, we obtain

$$\frac{\partial q^*_E}{\partial \gamma} = \frac{12(a-c)(-98+168\gamma+171\gamma^2)}{(28-195\gamma+216\gamma^2)^2}, \quad \frac{\partial q^*_L}{\partial \gamma} = -\frac{6(a-c)(112-840\gamma+2061\gamma^2)}{(28-195\gamma+216\gamma^2)^2}.$$  

From the numerator of $\frac{\partial q^*_E}{\partial \gamma}$, $-98+168\gamma+171\gamma^2 > 0$ for $\gamma > (7/57)(3\sqrt{6} - 4) \approx 0.411216$. Thus, $\frac{\partial q^*_E}{\partial \gamma} > 0$. Subsequently, from the numerator of $\frac{\partial q^*_L}{\partial \gamma}$, we find that $112-840\gamma+2061\gamma^2 = 0$ has no real root and a positive value. Thus, $\frac{\partial q^*_L}{\partial \gamma} < 0$.  

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Differentiating (9) with respect to \( \gamma \), we obtain
\[
\frac{\partial \Pi^*_{E}}{\partial \gamma} = 8(a - c)^2(6\gamma - 7)(196 - 903\gamma - 342\gamma^2 + 4374\gamma^3) \\
\frac{\partial \Pi^*_{L}}{\partial \gamma} = -\frac{(a - c)^2(15\gamma - 4)(-5488 + 39648\gamma - 137277\gamma^2 + 138024\gamma^3)}{2(28 - 195\gamma + 216\gamma^2)^3}.
\]

From numerical calculation, we find that \( 196 - 903\gamma - 342\gamma^2 + 4374\gamma^3 \geq 0 \) for all \( \gamma \geq \gamma_2 \simeq -0.50553 \). Thus, \( \partial \Pi^*_{E}/\partial \gamma > 0 \). Subsequently, from numerical calculation, we find that \(-5488 + 39648\gamma - 137277\gamma^2 + 138024\gamma^3 > 0 \) for all \( \gamma > \gamma_3 \simeq 0.64468 \). Thus, \( \partial \Pi^*_{L}/\partial \gamma < 0 \). Q.E.D.

\textbf{B. Proof of Proposition 4}

(i) From (10) and (14), we obtain
\[
f^* - f^{**} = \frac{(a - c)(261\gamma^2 - 360\gamma + 112)}{4(3\gamma - 2)(216\gamma^2 - 195\gamma + 28)}.
\]
Since \( 261\gamma^2 - 360\gamma + 112 > 0 \) for all \( \gamma > (4/87)(\sqrt{22} + 15) \simeq 0.905306, f^* > f^{**} \).

(ii) From (11) and (18), we obtain
\[
\pi^*_E - \pi^{**}_E = \frac{-3(a - c)\gamma B}{8(3\gamma - 2)(9\gamma - 4)(28 - 195\gamma^2 + 216\gamma^2)^2},
\]
where \( B \equiv -3136 - 10416\gamma + 75204\gamma^2 - 107541\gamma^3 + 42768\gamma^4 \). By numerical calculation, \( B \leq 0 \) for \( \gamma \leq \gamma_S \simeq 1.4273 \) but \( B > 0 \) for \( \gamma > \gamma_S \simeq 1.4273 \).

(iii) From (7) and (15), we obtain
\[
q^*_E - q^{**}_E = \frac{-9(a - c)\gamma(51\gamma - 28)}{2(9\gamma - 4)(28 - 195\gamma + 216\gamma^2)}, \quad q^*_L - q^{**}_L = \frac{3(a - c)\gamma(-32 - 60\gamma + 189\gamma^2)}{4(3\gamma - 2)(9\gamma - 4)(28 - 195\gamma + 216\gamma^2)}.
\]
Thus, \( q^*_E > q^{**}_E \). Since \(-32 - 60\gamma + 189\gamma^2 > 0 \) for all \( \gamma > (2/63)(\sqrt{193} + 5) \simeq 0.59976, q^*_L > q^{**}_L \).

Subsequently, from (8) and (16), we obtain
\[
x^*_E - x^{**}_E = \frac{-2(a - c)(-28 - 21\gamma + 108\gamma^2)}{(9\gamma - 4)(28 - 195\gamma + 216\gamma^2)}.
\]
Since \(-28 - 21\gamma + 108\gamma^2 > 0 \) for all \( \gamma > (1/72)(\sqrt{1393} + 7) \simeq 0.615596, x^*_E > x^{**}_E \).

Subsequently, we obtain
\[
x^*_L - x^{**}_L = -\frac{3(a - c)\gamma(100 - 249\gamma + 135\gamma^2)}{(3\gamma - 2)(9\gamma - 4)(28 - 195\gamma + 216\gamma^2)}.
\]
Since \(100 - 249\gamma + 135\gamma^2 > 0\) for all \(\gamma > (1/90)(\sqrt{889} + 83) \simeq 1.25351\), \(x_L^* > x^*_L\).

Subsequently, we have

\[
\frac{\partial(x^*_L - x^*_L^*)}{\partial \gamma} = \frac{3(a - c)C}{(3\gamma - 2)^2(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^2},
\]

where \(C \equiv -22400 + 111552\gamma + 145080\gamma^2 - 1701000\gamma^3 + 3549015\gamma^4 - 2904336\gamma^5 + 787320\gamma^6\).

By numerical calculation, \(C = 0\) for \(\gamma = \gamma \simeq 1.82817\).

(iv) From (9) and (17), we obtain

\[
\Pi^*_E - \Pi^*_E^* = \frac{-3(a - c)^2\gamma D}{4(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^2},
\]

where \(D \equiv 6272 - 58128\gamma + 180432\gamma^2 - 233685\gamma^3 + 104976\gamma^4\). By numerical calculation, \(D > 0\) for all \(\gamma > 1.00598\). Thus, \(\Pi^*_E > \Pi^*_E^*\). Subsequently, we have

\[
\Pi^*_L - \Pi^*_L^* = \frac{3(a - c)^2\gamma^2 F}{16(3\gamma - 2)^2(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^3},
\]

where \(F \equiv -312320 + 3532416\gamma - 14720688\gamma^2 + 29092608\gamma^3 - 27680859\gamma^4 + 10235160\gamma^5\). By numerical calculation, \(F > 0\) for all \(\gamma > 0.569386\). Thus, \(\Pi^*_L > \Pi^*_L^*\). Q.E.D.

C. Proof of Corollary 1

(i) From (25), the difference between \(W^*_E\) and \(W^*_E^*\) yields

\[
W^*_E - W^*_E^* = \frac{-3(a - c)^2\gamma G}{8(3\gamma - 2)(9\gamma - 4)^2(28 - 195\gamma + 216\gamma^2)^2},
\]

where \(G \equiv -12544 + 283584\gamma - 1465056\gamma^2 + 3124332\gamma^3 - 2960955\gamma^4 + 1014768\gamma^5\). By numerical calculation, \(G > 0\) for all \(\gamma > 1.193\). Thus, \(W^*_E > W^*_E^*\).

(ii) Differentiating \(W^*_E - W^*_E^*\) with respect to \(\gamma\), we obtain

\[
\frac{\partial(W^*_E - W^*_E^*)}{\partial \gamma} = \frac{3(a - c)^2 H}{4(3\gamma - 2)^2(9\gamma - 4)^3(28 - 195\gamma + 216\gamma^2)^3},
\]

where \(H \equiv 1404928 - 50577408\gamma + 402506496\gamma^2 - 1160452224\gamma^3 + 77639472\gamma^4 + 7514537832\gamma^5 - 19546529013\gamma^6 + 23059815480\gamma^7 - 13281773472\gamma^8 + 2959063488\gamma^9\). By numerical calculation, we find that \(H = 0\) has one real root that satisfies Assumption 1, that is, \(H = 0\) if and only if \(\gamma = \bar{\gamma} \simeq 1.63951\). Q.E.D.
D. The Effects of $\gamma$ on Welfare

In view of welfare the intuitive results hold on the cost parameter of R&D. From (19)–(22), we obtain the following.

$$\frac{\partial CS^*}{\partial \gamma} = -\frac{36(a-c)^2\gamma(21\gamma - 11)(308 - 1176\gamma + 1719\gamma^2)}{(28 - 195\gamma + 216\gamma^2)^3}.$$ 

Since $308 - 1176\gamma + 1719\gamma^2 = 0$ has no real root and positive values, $\partial CS^*/\partial \gamma < 0$.

$$\frac{\partial CS^{**}}{\partial \gamma} = -\frac{81(a-c)^2\gamma(7\gamma - 4)[16 + \gamma(51\gamma - 56)]}{8(3\gamma - 2)^3(9\gamma - 4)^3} < 0,$$

$$\frac{\partial W^*_L}{\partial \gamma} = -\frac{(a-c)^2(21952 - 484848\gamma + 5750811\gamma^2 + 4669488\gamma^4)}{2(28 - 195\gamma + 216\gamma^2)^3}.$$ 

By numerical calculation, since $21952 - 484848\gamma + 5750811\gamma^2 + 4669488\gamma^4 > 0$ for all $\gamma > 0.573553$, $\partial W^*_L/\partial \gamma < 0$.

$$\frac{\partial W^{**}_L}{\partial \gamma} = -\frac{(a-c)^2(2048 - 17472\gamma + 54000\gamma^2 - 72684\gamma^3 + 36207\gamma^4)}{8(3\gamma - 2)^3(9\gamma - 4)^3}.$$ 

By numerical calculation, since $2048 - 17472\gamma + 54000\gamma^2 - 72684\gamma^3 + 36207\gamma^4 > 0$ for all $\gamma > 0.617807$, $\partial W^{**}_L/\partial \gamma < 0$.

$$\frac{\partial W^*_E}{\partial \gamma} = \frac{8(a-c)^2(6\gamma - 7)(196 - 3549\gamma + 4194\gamma^2 + 8991\gamma^3)}{(28 - 195\gamma + 216\gamma^2)^3},$$

$$\frac{\partial W^{**}_E}{\partial \gamma} = \frac{(a-c)^2(320 - 1104\gamma + 828\gamma^2 + 243\gamma^3)}{4(3\gamma - 2)^2(9\gamma - 4)^3},$$

where $196 - 3549\gamma + 4194\gamma^2 + 8991\gamma^3 > 0$ for all $\gamma > 0.394337$ and $320 - 1104\gamma + 828\gamma^2 + 243\gamma^3 > 0$ for all $\gamma > -4.48568$. Thus, $\partial W^*_E/\partial \gamma > 0$ and $\partial W^{**}_E/\partial \gamma > 0$.

References


