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Fairness and Fairness for Neighbors: The Difference between the Myerson Value and Component-Wise Egalitarian Solutions

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Abstract

We replace the axiom of fairness used in the characterization of the Myerson value (Myerson, 1977) by fairness for neighbors in order to characterize the component-wise egalitarian solution. When a link is broken, fairness states the two players incident to the link should be affected similarly while fairness for neighbors states that a player incident to the link and any of his other neighbors should be affected similarly. Fairness for neighbors is also used to characterize the component-wise egalitarian surplus solution and a two-step egalitarian solution. These results highlight that egalitarian and marginalistic allocation rules can be obtained by applying the same equal gain/loss property to different types of players.

Keywords: Myerson value, component-wise egalitarian solutions, fairness, fairness for neighbors, two-step value.
JEL Classification number: C71.

1 Introduction

Cooperative games with transferable utility describe situations in which any subset of the player set is able to form as a coalition and to earn the corresponding worth. However, in many situations the set of feasible coalitions is restricted by some hierarchical, technical or communicational structure. In this note we consider communication situations, which consist in a cooperative game and an undirected graph modeling the limited communication structure. The vertices in the graph represent the players and the edges represent the communication links between the players. One of the most famous allocation rules for communication situations is the Myerson value (Myerson, 1977), which is the Shapley value (Shapley, 1953) of a so-called Myerson restricted game. The Myerson value can be characterized by component efficiency and fairness. Component efficient means that the sum of payoffs in any component of the graph equals the worth of the component. Fairness means that the deletion of a communication link between two players hurts or benefits both players equally.

We keep component efficiency as an axiom and the equal gain/loss principle used in fairness. However, rather than requiring equal payoff variations between a player and the neighbor incident to the deleted link, we require equal payoff variations between this player and each of his other neighbors. So, the resulting axiom, which we call fairness for neighbors, relies on the same principle as fairness. The only difference is that the payoff variation involves those neighbors of the players incident to the deleted link that are neglected by the axiom of fairness. In other words, fairness points out the role of the neighbor with which a player is no longer linked while fairness neighbors points out the role of the neighbors with which a player continues to be linked. The rational behind this property can be understood as follows. When the removal of a link breaks a component into two parts, the players incident to that link do not communicate anymore. Nevertheless, both players still communicate with their other neighbors, and so are able to cooperate with the members of the corresponding component. Therefore, it makes sense to apply the equal gain/loss principle to pairs of players who continue to communicate in the resulting graph. It turns out that replacing fairness by fairness for neighbors and adding an equal treatment principle in two-player communication situations yields the characterization of the component-wise egalitarian solution, which distributes the worth of each component of the graph equally among its members.

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Replacing the equal treatment principle in two-player communication situations by the classical axiom of standardness characterizes the component-wise egalitarian surplus solution, which first assigns to every player its own worth, and then distributes the remaining surplus of his component equally among its members. Standardness requires the allocation rule to coincide with the standard solution in two-player components. Since the Myerson value coincides with component-wise egalitarian surplus solution in two-player communication situations, both allocations rules are comparable in the sense that they essentially differ with respect to pairs of players to whom the equal gain/loss principle applies when a communication link is severed. In a sense, the difference between an egalitarian rule and a marginalistic rule only results from the identity of the neighbors with which a player's payoff variation is evaluated.

A two-step procedure in which the component-wise egalitarian surplus solution is applied at the component level and then within each component is used to construct an efficient component-wise egalitarian surplus solution. This approach is inspired by the two-step Shapley value introduced by Kamijo (2009) for TU-games with a coalition structure. The resulting allocation rule satisfies fairness for neighbors and is characterized by replacing component efficiency in the characterization of the component-wise egalitarian surplus solution by efficiency, covariance and a natural axiom of symmetry for components.

This note therefore provides axiomatic characterizations of two egalitarian allocation rules for communication situations that are comparable with the Myerson value in the sense that they both incorporate a similar equal gain/loss principle. This research is related to the work of Slikker (2007) who provide comparable axiomatizations of the Myerson value and of the egalitarian solutions for cooperative games on networks. Our characterization of the component-wise egalitarian solution can be considered as closer to the characterization of the Myerson value since the equal gain/loss property, which is the corner stone of the axiom of fairness, is reused in the axiom of fairness for neighbors, even if it is applied to different pairs of players. The present article is also related to the work of van den Brink (2007) who shows that replacing null players by nullifying players in the characterization of the Shapley value characterizes the equal division rule for TU-games. A player is nullifying if its presence in a coalition generates zero worth.

The rest of the note is organized as follows. Section 2 provides preliminaries. The component-wise egalitarian solution and component-wise egalitarian surplus solution are characterized in sections 3 and 4 respectively. In section 5 we characterize the two-step component-wise egalitarian surplus solution. Section 6 concludes by a comparison table.

2 Preliminaries

Let $N = \{1, \dots, n\}$ be a finite set of players. A *cooperative game with transferable utility* on N, or simply TU-game, is a *characteristic function* $v : 2^N \longrightarrow \mathbb{R}$ such that $v(\emptyset) = 0$. For each $S \in 2^N$, $v(S)$ is the *worth* of *coalition* S and s its cardinality. A TU-game v is *zero-normalized* if $v(\{i\}) = 0$ for each $i \in N$. For any TU-game v, any real $a \in \mathbb{R}\backslash\{0\}$ and any $b \in \mathbb{R}^n$, the TU-game $(av + b)$ is such that, for each $S \in 2^N$, $(av + b)(S) = av(S) + \sum_{i \in S} b_i$.

A *communication graph* is a pair (N, L) where the nodes in N represent the players and edges in $L \subseteq \{\{i, j\} \subseteq N : i \neq j\}$ represent bilateral communication links between players. We shall use the short notation ij instead of $\{i, j\}$ for a link between i and j. For each player $i \in N$, $L_i = \{j \in N : ij \in L\}$ is the set of *neighbors* of i in (N, L). For each coalition S, $L(S) = \{ij \in L : i \in S, j \in S\}$ is the set of links between players in S. The graph $(S, L(S))$ is the *subgraph* of (N, L) induced by S. A sequence of $p \geq 1$ distinct players $(i_1, \dots, i_p)$ is a *path* in (N, L) if $i_q i_{q+1} \in L$ for $q = 1, \dots, p - 1$. A graph (N, L) is *connected* if there exists a path between any two players in N. A coalition S is connected in (N, L) if $(S, L(S))$ is a connected graph. A coalition C is a *component* of a graph (N, L) if the subgraph $(C, L(C))$ is connected and for each $i \in N\backslash C$, the subgraph $(C \cup \{i\}, L(C \cup \{i\}))$ is not connected. Let N/L and $S/L(S)$ be the sets of components of (N, L) and $(S, L(S))$ respectively.

A *communication situation* on N is a pair (v, L) such that v is a TU-game on N and (N, L) a communication graph. Denote by $\mathcal{C}_N$ the set of all communication situations on N. A *payoff vector* $x \in \mathbb{R}^n$ is an n-dimensional vector giving a payoff $x_i \in \mathbb{R}$ to each player $i \in N$. An *allocation rule* on $\mathcal{C}_N$ is a function f that assigns to each $(v, L) \in \mathcal{C}_N$ a payoff vector $f(v, L) \in \mathbb{R}^n$. Given a communication situation (v, L), the *graph-restricted game* v^L, introduced by Myerson (1977), is defined as:

$$\forall S \in 2^N, \quad v^L(S) = \sum_{T \in S/L(S)} v(T).$$

If a coalition is not connected, then its worth in v^L is given by the sum of the worths of its connected components. The Shapley value (Shapley, 1953) of v^L is known as the *Myerson value* of (v, L). More specifically, the Myerson value is the allocation rule μ on $\mathcal{C}_N$ defined as:

$$\forall (v, L) \in \mathcal{C}_N, \forall i \in N, \qquad \mu_i(v, L) = \sum_{S \in 2^N : i \in S} \frac{(n-s)!(s-1)!}{n!} \big(v^L(S) - v^L(S \backslash \{i\})\big).$$

Myerson (1977) characterizes μ by component efficiency and fairness. Component efficiency requires that an allocation rule assigns to any component C of L the total payoff $v(C)$. Fairness states that for every communication link in the graph the incident players lose or gain the same amount from breaking this link.

Component efficiency. For each $(v, L) \in \mathcal{C}_N$ and each $C \in N/L$, it holds that

$$\sum_{i \in C} f_i(v, L) = v(C).$$

Fairness. For each $(v, L) \in \mathcal{C}_N$ and each $ij \in L$, it holds that

$$f_i(v, L) - f_i(v, L \backslash ij) = f_j(v, L) - f_j(v, L \backslash ij).$$

3 The component-wise egalitarian solution

The *component-wise egalitarian solution* is the allocation rule CE on $\mathcal{C}_N$ that distributes the worth of each component equally among its members in any communication situation, i.e

$$\forall (v, L) \in \mathcal{C}_N, \forall C \in N/L, \forall i \in C, \qquad CE_i(v, L) = \frac{v(C)}{c}.$$

The component-wise egalitarian solution satisfies component efficiency but not fairness. It does satisfy the following very similar equal gain/loss property.

Fairness for neighbors. For each $(v, L) \in \mathcal{C}_N$, each $ij \in L$, and each $k \in L_i \backslash \{j\}$, it holds that

$$f_i(v, L) - f_i\big(v, L \backslash ij\big) = f_k(v, L) - f_k\big(v, L \backslash ij\big).$$

When a link is removed from the set of communication links, fairness for neighbors requires that each player incident to this link enjoys the same payoff variation as any of his neighbors in the resulting graph. In other words, player i is linked in graph (N, L) with the players in L_i. When the link ij is cut, fairness compares the payoff variation between i and $j \in L_i$ but not between i and any $k \in L_i \backslash \{j\}$, while fairness for neighbors compares the payoff variation between i and each $k \in L_i \backslash \{j\}$ but not between i and $j \in L_i$. The component-wise egalitarian solution also satisfies the following equal treatment principle for two-player components of the form $\{i, j\}$.

Equal treatment for two-player components. For each $(v, L) \in \mathcal{C}_N$ and each $\{i, j\} \in N/L$, it holds that $f_i(v, L) = f_j(v, L)$.

Replacing fairness by fairness for neighbors and adding equal treatment for two-player components in the characterization of the Myerson value yields the component-wise egalitarian solution.

Proposition 1 *The component-wise egalitarian solution is the unique allocation rule on $\mathcal{C}_N$ that satisfies component efficiency, fairness for neighbors and equal treatment for two-player components.*

Proof. It is easy to check that the component-wise egalitarian solution satisfies the three properties. Conversely, consider an allocation rule f on $\mathcal{C}_N$ that satisfies component efficiency, fair treatment for neighbors and equal treatment for two-player components. We show that $f = CE$ on $\mathcal{C}_N$. The proof is by induction on the cardinality of L.

INITIAL STEP: for each $(v, L) \in \mathcal{C}_N$ such L is empty, it follows directly that f and CE coincide, since both rules satisfies component efficiency. Note also that for each $(v, L) \in \mathcal{C}_N$ such that L contains one link, f and CE coincide by component efficiency and equal treatment for two-player components.

INDUCTION HYPOTHESIS: let $l \in \{1, \ldots, n(n-1)/2\}$ and assume that CE and f coincide for each $(v, L) \in \mathcal{C}_N$ where L contains at most $l - 1$ links.

INDUCTION STEP: let (v, L) be such that L contains l links. Pick any component C in N/L. We have to show that $CE_i(v, L) = f_i(v, L)$ for each $i \in C$. If $L(C)$ contains at most one link, this equality follows directly from component efficiency and equal treatment for two-player components. So, assume that $L(C)$ contains at least two links.

Pick any link $ij \in L(C)$. There is $k \in C \backslash \{i, j\}$ such that $ki \in L(C)$ or $kj \in L(C)$. Without loss of generality, assume that $ki \in L(C)$. An application of fairness for neighbors yields:

$$f_i(v, L) - f_i(v, L \backslash ki) = f_j(v, L) - f_j(v, L \backslash ki).$$

Because i and j belong to the same component in $(N, L \backslash ki)$, the induction hypothesis yields $f_i(v, L \backslash ki) = f_j(v, L \backslash ki)$, and so $f_i(v, L) = f_j(v, L)$.

We have proven that if $ij \in L(C)$ then $f_i(v, L) = f_j(v, L)$. It immediately follows (by induction on path lengths) that if i and j belong to a same component C, then $f_i(v, L) = f_j(v, L)$. Therefore, component efficiency implies $f_i(v, L) = CE_i(v, L)$ for each $i \in C$. This implies that $f = CE$ on $\mathcal{C}_N$. ■

4 The component-wise egalitarian surplus solution

The *component-wise egalitarian surplus solution* is the allocation rule CES on $\mathcal{C}_N$ that first assigns to each player $i \in N$ his stand-alone worth and then distributes, for each component $C \in N/L$, the remainder of $v(C)$ equally among all players in C, i.e.

$$\forall (v, L) \in \mathcal{C}_N, \forall C \in N/L, \forall i \in C, \qquad CES_i(v, L) = v(\{i\}) + \frac{v(C) - \sum_{j \in C} v(\{j\})}{c}.$$

Although the component-wise egalitarian surplus solution violates equal treatment for two-player components, it satisfies the very natural following axiom of standardness, which requires that the allocation rule coincides with the standard solution for two-player components of the form $\{i, j\}$.

Standardness For each $(v, L) \in \mathcal{C}_N$, each $\{i, j\} \in N/L$ and each $k \in \{i, j\}$, it holds that

$$f_k(v, L) = v(\{k\}) + \frac{v(\{i, j\}) - v(\{i\}) - v(\{j\})}{2}.$$

The axiom of standardness is used, among others, by Hart and Mas-Colell (1989) in order to characterize the Shapley value for TU-games. It turns out that replacing equal treatment for two-player components by standardness in the characterization of the component-wise egalitarian solution yields the component-wise egalitarian surplus solution. In a sense, the difference between the component-wise egalitarian solution and the component-wise egalitarian surplus solution can be explained only by differences in two-player components.

Proposition 2 *The component-wise egalitarian surplus solution is the unique allocation rule on $\mathcal{C}_N$ that satisfies component efficiency, fairness for neighbors and standardness.*

Proof. Obviously, the component-wise egalitarian surplus solution satisfies the three properties. The proof of uniqueness is done by induction on $|L|$ and is similar to the one given in Proposition 1. The initial steps in the induction follow from component efficiency if $|L| = 0$ and by component efficiency and standardness if $|L| = 1$. In the induction step, fairness for neighbors yields the following system: for each $C \in N/L$, each $i, j \in C$,

$$f_i(v, L) - f_j(v, L) = v(\{i\}) - v(\{j\}), \qquad \text{and} \qquad \sum_{k \in C} f_k(v, L) = v(C).$$

Let us show that $CES(v, L)$ is the unique solution of this system. Pick any $C \in N/L$ and any $i \in C$. Summing over all $j \in C$ we obtain:

$$cf_i(v, L) = \sum_{j \in C} f_j(v, L) - \sum_{j \in C} v(\{j\}) + cv(\{i\}).$$

Using component efficiency, this yields $f_i(v, L) = CES_i(v, L)$ as desired. ■

It is also possible to provide an alternative characterization of the component-wise egalitarian surplus solution by using the axiom of equal treatment for two-player components only on zero-normalized communication situations and adding the following well-known property of covariance.

Covariance. For each $(v, L) \in \mathcal{C}_N$ and each $(av + b, L) \in \mathcal{C}_N$, it holds that $f(av + b, L) = af(v, L) + b$.

Proposition 3 *The component-wise egalitarian surplus solution is the unique allocation rule on $\mathcal{C}_N$ that satisfies component efficiency, fairness for neighbors, equal treatment for two-player components on zero-normalized communication situations, and covariance.*

Proof. One can easily check that the component-wise egalitarian surplus solution satisfies the four properties. The proof of uniqueness for the class of zero-normalized communication situations is identical to the proof for the equal component division rule given in Proposition 1. Finally, uniqueness for the other communication situations follows from covariance. ■

Note that the Myerson value satisfies standardness. Hence, one can also use fairness for two-player components instead of standardness in order to highlight the difference between the Myerson value and the component-wise egalitarian surplus solution.

Proposition 4 *The component-wise egalitarian surplus solution is the unique allocation rule on $\mathcal{C}_N$ that satisfies component efficiency, fairness for neighbors, and fairness for two-player components.*

5 A two-step component-wise egalitarian surplus solution

In this section, we examine a two-step allocation rule in the spirit of the two-step Shapley value for TU-games with a coalition structure studied by Kamijo (2009) and Calvo and Guttiérrez (2010). Instead of applying twice the Shapley value, we apply twice the equal surplus division principle that defines CES, at the component level and within each component. This procedure constructs an efficient variation of the component-wise egalitarian surplus solution. Efficient allocation rules for communication situations are also studied by van den Brink, Khmelnitskaya and van der Laan (2011).

For a communication situation (v, L), a first application of the equal surplus division principle of CES gives to each component $C \in N/L$ the total payoff

$$v(C) + \frac{v(N) - \sum_{T \in N/L} v(T)}{|N/L|} = v(C) + \frac{v(N) - v^L(N)}{|N/L|}.$$

Denote by $r^L(C)$ this quantity. Next, for each component $C \in N/L$, a second application of the equal surplus division principle of CES assigns to each player $i \in C$ the payoff

$$v(\{i\}) + \frac{r^L(C) - \sum_{j \in C} v(\{j\})}{c}.$$

By definition of $r^L(C)$, the latter payoff is equivalent to

$$CES_i(v,L) + \frac{v(N) - v^L(N)}{c \times |N/L|}. \tag{1}$$

The *two-step component-wise egalitarian surplus solution* is the allocation rule $ECES$ on $\mathcal{C}_N$ such that

$$\forall (v,L) \in \mathcal{C}_N, \forall C \in N/L, \forall i \in C, \qquad ECES_i(v,L) = CES_i(v,L) + \frac{v(N) - v^L(N)}{c \times |N/L|}.$$

The two-step component-wise egalitarian surplus solution satisfies fairness for neighbors. It turns out that replacing component efficiency by the classical axiom of efficiency and another natural axiom of component symmetry in the characterization of the component-wise egalitarian surplus solution yields the two-step component-wise egalitarian surplus solution.

Efficiency. For each $(v,L) \in \mathcal{C}_N$, it holds that $\sum_{i\in N} f_i(v,L) = v(N)$.

Component symmetry. For each $(v,L) \in \mathcal{C}_N$, if $v(C) = v(R)$ for some pair $\{C,R\} \subseteq N/L$, it holds that $\sum_{i\in C} f_i(v,L) = \sum_{i\in R} f_i(v,L)$.

Component symmetry means that two components of the communication graph should get an equal total payoff if they have the same worth. Observe that component efficiency implies component symmetry while component symmetry does not imply component efficiency. In a sense, component symmetry can be considered as a rather limited departure from component efficiency.

Proposition 5 *The two-step component-wise egalitarian surplus solution is the unique allocation rule on $\mathcal{C}_N$ that satisfies efficiency, fairness for neighbors, fairness for two-player components, covariance and component symmetry.*

Proof. It is easy to check that $ECES$ satisfies the five axioms. Next, consider any allocation rule f on $\mathcal{C}_N$ that satisfies the five axioms. The proof that $f = ECES$ is by induction on the cardinality of L.
INITIAL STEP: for each $(v,L) \in \mathcal{C}_N$ such L is empty, consider the zero-normalization v^0 of v, i.e. for each $S \in 2^N$, $v^0(S) = v(S) - \sum_{i\in S} v(\{i\})$. In (v^0,L), each component $\{i\}$, $i \in N$, has a zero worth. By component symmetry, $f_i(v^0,L) = f_j(v^0,L)$ for each $i,j \in N$. By efficiency, we get $f_i(v^0,L) = v^0(N)/n$ for each $i \in N$. By covariance, $f_i(v,L) = f_i(v^0,L) + v(\{i\})$ for each $i \in N$. Since $v^0(N) = v(N) - v^L(N)$, we get $f(v,L) = ECES(v,L)$ for $L = \emptyset$.

Suppose now that $(v,L) \in \mathcal{C}_N$ is such that $|L| = 1$ and let $L = \{ij\}$. For each $C \in N/L$ and each $k \in C$, let $b_k = -v(C)/c$. From v construct the game v^0 such that for each $S \in 2^N$, $v^0(S) = v(S) - \sum_{k\in S} b_k$. In a sense v^0 consists in zero-normalizing the components of (N,L). Note that for each $C \in N/L$, we have $v^0(C) = 0$. Component symmetry and efficiency yield that $\sum_{k\in C} f_k(v^0,L) = v^0(N)/|N/L|$ for each $C \in N/L$. By covariance we get $\sum_{k\in C} f_k(v,L) = v^0(N)/|N/L| + v(C)$. So the payoff of each player $k \in N\backslash\{i,j\}$ is equal to $ECES_k(v,L)$ since $\{k\}$ is a component. Regarding component $\{i,j\}$, fairness for two-player components implies

$$f_i(v,L) - f_i(v,L\backslash ij) = f_j(v,L) - f_j(v,L\backslash ij) \Longleftrightarrow f_i(v,L) - v(\{i\}) = f_j(v,L) - v(\{j\}).$$

Together with $f_i(v,L) + f_j(v,L) = v^0(N)/|N/L| + v(\{i,j\})$, this yields a system of two equations. The unique solution of this system is precisely the pair $(ECES_i(v,L), ECES_j(v,L))$ as desired.
INDUCTION HYPOTHESIS: let $l \in \{1,\ldots,n(n-1)/2\}$ and assume that $f = ECES$ for each $(v,L) \in \mathcal{C}_N$ where L contains at most $l-1$ links.
INDUCTION STEP: let (v,L) be such that L contains l links. As before construct the game v^0 such that for each $S \in 2^N$, $v^0(S) = v(S) - \sum_{k\in S} b_k$ where for each $C \in N/L$ and each $k \in C$, $b_k = v(C)/c$. Once again, for each component $C \in N/L$, we have $v^0(C) = 0$. By component symmetry, efficiency and covariance, we obtain the cumulated payoff $\sum_{i\in C} f_i(v,L) = v^0(N)/|N/L| + v(C)$ for each $C \in N/L$. The rest of

the proof is similar to the one given in Proposition 1. Indeed, from fairness for neighbors, we obtain the following system: for each $C \in N/L$ and each $i, j \in C$,

$$f_i(v,L) - f_j(v,L) = v(\{i\}) - v(\{j\}), \qquad \text{and} \qquad \sum_{k \in C} f_k(v,L) = v^0(N)/|N/L| + v(C).$$

Summing on all $i \in C$, using the cumulated payoffs of C and the fact that $v^0(N) = v(N) - v^L(N)$, we can see that $ECES(v,L)$ is the unique solution of this system. ■

6 Conclusion

To conclude, we summarize our results in the following table, in which a "+" means that the allocation rule satisfies the axiom, in which "−" has the converse meaning and in which the "⊕" symbols indicate the characterizing sets of axioms in Myerson (1977) and in the present article. The superscripts in the CES column stand for the number of the associated Proposition.

	μ	CE	CES	$ECES$
Efficiency	−	−	−	⊕
Component efficiency	⊕	⊕	$\oplus^{2,3,4}$	−
Fairness	⊕	−	−	−
Fairness for two-player components	+	−	$\oplus^{4}$	⊕
Fairness for neighbors	−	⊕	$\oplus^{2,3,4}$	⊕
Equal treatment for two-player components	−	⊕	−	−
Equal treatment for two-player components on zero-normalized c.s.	+	+	$\oplus^{3}$	+
Standardness	+	−	$\oplus^{2}$	−
Covariance	+	−	$\oplus^{3}$	⊕
Component symmetry	+	+	+	⊕

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