Desert and inequity aversion in teams

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Abstract

Teams are becoming increasingly important in work settings. We develop a framework to study the strategic implications of a meritocratic notion of desert under which team members care about receiving what they feel they deserve. Team members find it painful to receive less than their perceived entitlement, while receiving more may induce pleasure or pain depending on whether preferences exhibit desert elation or desert guilt. Our notion of desert generalizes distributional concern models to situations in which effort choices affect the distribution perceived to be fair; in particular, desert nests inequity aversion over money net of effort costs as a special case. When identical teammates share team output equally, desert guilt generates a continuum of symmetric equilibria. Equilibrium effort can lie above or below the level in the absence of desert, so desert guilt generates behavior consistent with both positive and negative reciprocity and may underpin social norms of cooperation.

Keywords: Desert, Deservingness, Equity, Inequity aversion, Loss aversion, Reference-dependent preferences, Guilt, Reciprocity, Social norms, Team production

JEL Codes: D63, J33

1 Introduction

Teams have become increasingly important in work settings in recent years (Che and Yoo, 2001, document a number of examples). The growing popularity of teams is driven partly by the increasing complexity of work tasks in a knowledge-driven environment; this increasing complexity demands the input of many different complementary skills and makes monitoring and legally verifying the value of individual contributions more difficult, thus leading to a greater use of teams with some form of output sharing.

Equity may play an important role when agents work together in teams. In particular, a team member may feel dissatisfied if the payoff she receives from working as part of the team deviates from the payoff that she thinks she deserves. In this paper we develop a theoretical framework to study the strategic implications of desert considerations when agents work in teams. The increasing preponderance of teamwork makes understanding the incentives of team members more relevant than ever. A better understanding of incentives within teams which incorporates the implications of agents' desert concerns will provide a more solid foundation for future research to answer broader questions, such as: when will agents choose to join or form teams? when should employers use teams? how should employers design optimal team compensation? can employers help foster cooperation in teams by, e.g., providing information about the efforts of teammates or the efforts of others in similar teams? should profit-sharing partnerships be taxed differently from other types of companies?[1]

Our analysis complements that of Gill and Stone (2010), who study the implications of desert in competitive settings. We develop a meritocratic notion of desert or equity under which each team member compares her monetary payoff to the payoff that she feels she deserves, which in turn depends on how hard she has worked in relation to her teammates. When a team member receives less than she feels she deserves, she suffers a psychological cost which we call a *desert loss.* It is less clear whether she views getting more than she deserves as a good thing or a bad thing, so we allow for both. We say that she benefits from *desert elation* if she feels good about getting more than she feels she deserves. When, instead, the team member feels bad about getting more than she feels she deserves, we say that she suffers from *desert guilt.* Such feelings of guilt may be triggered by a desire to conform with a meritocratic social norm that the distribution of payoffs should reflect

[1] Our focus is on the impact of non-standard desert preferences on strategic behavior within teams under the simplest equal sharing rule, which for example is often used by partnerships. Holmstrom (1982) and Che and Yoo (2001) consider, in a static and dynamic context respectively, the choice of optimal sharing rule by a principal given standard preferences. We hope this paper will spur future research on how the multiplicity of equilibria that we find with desert preferences interacts with a principal's choice of sharing rule.

recipients' efforts.

Section 2 expounds our meritocratic notion of desert in more detail, relating it to the existing literature on equity and explaining how we formalize our notion using loss aversion around endogenous reference points. A significant body of empirical evidence from social psychology and experimental economics supports the idea that equity and desert are important when agents exert effort. However, the literature has not embedded desert in a formal framework suitable for studying its strategic implications.

Section 3 presents the formal model, which we apply in Section 4 to analyze the implications of desert for equilibrium effort choices in teams. When identical teammates share the team output equally, desert guilt generates a continuum of symmetric equilibria: some of these equilibria generate more effort than without desert; but, more surprisingly, other equilibria generate *less effort* than when desert considerations are absent. Desert guilt forges an endogenous complementarity between agents' efforts by giving the agents incentives to match the efforts of their teammates, and so generates behavior that is consistent with both positive and negative reciprocity. However, as outlined in Section 4.2, the mechanism which introduces reciprocity into our framework is different to that which drives intentions-based theories of reciprocal altruism. When desert guilt leads to cooperative behavior, the guilt can be thought of as underlying *social norms* of cooperation: desert guilt can make cooperation normatively appropriate for an agent, conditional on her teammates adhering to the norm of cooperative behavior. Thus desert guilt, which as noted above may itself be underpinned by a meritocratic social norm, can give rise to a specific norm of cooperation in our team setting. With desert elation, effort is always driven below the no-desert level as the team members feel no compunction about taking advantage of their teammates by slacking off. We also study the welfare implications of desert in Section 4.4.

The theoretical implications of desert in team settings in which effort creates a positive externality for teammates are very different to those when agents compete and so impose negative externalities on rivals. As noted above, Gill and Stone (2010) consider the implications of desert in a competitive environment, finding that when agents compete desert concerns push identical agents to *differentiate* their effort levels, with some agents working very hard and others slacking off substantially. Eisenkopf and Teyssier (2010) provide support for this prediction using evidence from a laboratory experiment. The fact that desert concerns imply such different predictions across these different settings is evidence of the scope and portability of our notion of desert. Desert concerns may influence behavior in any situation in which agents exert effort, and an agent's payoff depends on her own effort as well as on the efforts of some other agents that she interacts with.

Our notion of desert can be seen as a generalization of distributional concern models to situations in which effort choices affect the distribution that is perceived to be fair or equitable. Desert-concerned agents care not just about the distribution of monetary payoffs, but also on how the distribution came about. Indeed, our conception of desert is related to the inequity-aversion model of Fehr and Schmidt (1999), and one of the aims of our paper is to clarify this relationship. In Section 5, we show that in a team setting our model of desert nests Fehr and Schmidt (1999)-type inequity aversion over monetary payoffs *net of effort costs* as a special case. Inequity aversion over money net of effort costs implies one particular way to weight monetary payoffs in relation to effort exerted, while our notion of desert does not prescribe the exact form that this weighting should take.

Finally, in Section 6 we study a simple linearized example which allows the calculation of explicit analytical expressions for the range of possible equilibria. We hope that this example will prove useful in future applied theoretical and empirical work. Section 7 concludes.

2 Desert in teams

We start by outlining our general notion of desert and linking it to the existing literature. Suppose that a set of identical agents are members of a team: the agents exert costly effort to help produce some team output which is shared equally among the team members. Output sharing implies that each agent's effort confers a positive externality on her teammates. We capture a notion of *desert* or equity by supposing that each agent cares about how her monetary payoff compares to how much she feels she deserves, given by a reference point r_i which depends on how hard agent i has worked relative to her teammates. We also suppose that the agents share a common notion of desert, so they agree about the payoff each deserves. Our notion of desert is meritocratic: if an agent works harder than a teammate, she feels she deserves more than that teammate, while if she works less hard she feels she deserves less. Letting e_i represent agent $i's$ effort:

$$r_i \gtreqless r_j \Leftrightarrow e_i \gtreqless e_j. \tag{1}$$

Gill and Stone (2010) consider a similar notion of desert, but in a competitive context where efforts impose a negative externality on rivals and an agent's deserved reference point is given by her expected winnings. We suppose that desert-motivated agents feel hard done by when they receive less than what they feel they deserve, while feelings of elation or guilt are possible when they do better than they deserve. We operationalize our notion of desert by assuming that each agent is loss averse around her reference point, so losses relative to the deserved reference point are more painful than gains are pleasurable; indeed, doing better than is felt to be deserved may induce psychologically painful guilt rather than elation. Loss aversion captures the central stylized fact that has emerged from the empirical literature on reference-dependent preferences: losses relative to reference points loom larger than corresponding gains (see Rabin, 1998, and DellaVigna, 2009, for surveys, and the original paper by Kahneman and Tversky, 1979). In the terminology of Kőszegi and Rabin (2007), the reference points in our set-up are *choice-acclimating* and thus endogenous: the agents understand and anticipate how their effort choices influence their reference points. In a competitive tournament setting, Gill and Prowse (forthcoming) find experimental evidence of the importance of loss aversion around choice-acclimating reference points.[2]

As a first step, we focus on modeling the desert concerns of identical agents, who are thus of equal ability. If agents differed in their cost of effort functions, but team output continued to depend on effort, a meritocratic notion of desert would imply only that the reference point that an agent feels she deserves be increasing in her own effort, holding constant the efforts of her teammates. The specific requirement given by (1) would then correspond to a 'libertarian' (Cappelen et al., 2007) assumption about deservingness, in which desert depends only on individual output (here measured by effort) and is independent of inputs or ability. A 'liberal egalitarian' (Cappelen et al., 2007) assumption would instead allocate deservingness according to agents' inputs, as measured by the cost of exerted effort.

Although little theoretical work has been carried out to model desert concerns formally,[3] a significant body of literature supports the idea that people are motivated by a meritocratic notion of desert. Rabin (1998) writes that "desert will obviously be relevant in many situations - and the massive psychological literature on 'equity theory' shows that people feel that those who have put more effort into creating resources have more claim on those resources" (p. 18). Adams (1965) was the first modern proponent of equity theory: his work in social psychology led him to conclude that "when [a person] finds that his outcomes and inputs are not in balance in relation to those of others, feelings of inequity result" (p. 280) and that "there can be little doubt that inequity results

[2] Daido and Murooka (2011) show that when workers are loss averse around choice-acclimating expectations, to mitigate wage uncertainty a principal might choose to use a 'team' contract under which one worker's pay increases in the performance of another.

[3] An exception is Konow (2000), who considers only the optimal division of output by a dictator for given effort choices. Akerlof and Yellen (1990) invoke equity considerations to motivate an effort-supply function that is sensitive to the wage an agent receives relative to the wage she believes to be fair.

in dissatisfaction, in an unpleasant emotional state, be it anger or guilt" (p. 283). Using survey data, Konow (1996) distills an accountability principle according to which a person's entitlement varies in direct proportion to the value of his relevant discretionary variables, relative to others (p. 19). Experimental evidence also backs up the idea that people are sensitive to considerations of desert. For example, Konow (2000), Frohlich et al. (2004) and Cappelen et al. (2007) find evidence that when the amount to be distributed reflects agents' efforts, dictators tend to award a higher payoff to agents who have exerted more effort, while Abeler et al. (2010) show that when norms of equity are violated, so harder working agents are paid the same as those who exert less effort, the hard working agents start to withdraw effort even though it is in their interest to continue to work hard in the absence of social preferences. Finally, in an experimental setting with different productivities, Gantner et al. (2001) classify subjects according to their equity standards.

3 Formal model

We now imbed our notion of desert described above in Section 2 into a formal model.

3.1 Team production game

$N \geq 2$ identical agents simultaneously choose effort $e_i \geq 0$ at cost $C(e_i)$, with $C(0) = C'(0) = 0$, $C' > 0$ for $e_i > 0$, $C'' > 0$ and C' unbounded above. Team output Y depends on the sum of the agents' efforts, so $Y = f\left(\sum_{i=1}^{N} e_i\right)$, with $f(0) = 0$, $f' > 0$ and $f'' \leq 0$. The team output is distributed equally, so each agent receives a monetary payoff $y_i = Y/N$ which the agent values at $\phi(y_i)$ with $\phi(0) = 0$, $\phi' > 0$ and $\phi'' \leq 0$.[4] The agents can only produce as part of the team and not individually (perhaps because access to a crucial means of production is tied to the team).

In the absence of desert preferences, to be introduced shortly, each agent has a twice continuously differentiable utility function $\overline{U}_i(e_i, e_{-i}) = \phi(y_i) - C(e_i)$, which depends on the agent's own effort e_i and the vector of efforts chosen by the other team members e_{-i}. We call $\overline{U}_i$ agent *i's standard utility.*

3.2 Desert preferences

As explained in Section 2, we capture agents' desert concerns by supposing that each agent cares not only about her monetary payoff and effort cost, but also about how the monetary payoff y_i

[4] It is straightforward to extend our analysis to the case where each individual's output is partially or fully non-rival in consumption. Each team member then receives $y_i = \frac{aY}{N} = \frac{af}{N}$, where a ranges from 1 to N as we move from full rivalry to full non-rivalry, so we can simply replace f by af throughout.

compares to a reference point r_i that represents the payoff that the agent feels she deserves. We suppose that each agent's endogenous reference point is given by $r_i(e_i, e_{-i})$, which is continuously differentiable when $\sum_{i=1}^{N} e_i > 0$ and, letting $r_i' \equiv \partial r_i / \partial e_i$ and letting z_{-i} represent a common level of effort $e_j = z \ \ \forall j \neq i$, satisfies the following assumption:

Assumption 1 *If* $e_j = z > 0 \quad \forall j \neq i$ *then (i)* $e_i \gtreqless z \Rightarrow r_i(e_i, z_{-i}) \gtreqless \frac{f(e_i+(N-1)z))}{N}$ *and (ii)* $r_i'(z, z_{-i}) > \frac{f'(Nz)}{N}$.

Part (i) says that the identical agents adopt a meritocratic notion of desert such that if all agents put in a common level of effort z, each agent feels she deserves an equal N^{th} share of the resulting team output, while if agent i exerts more (less) effort than the common level of the other team members, she feels she deserves more (less) than the equal N^{th} share y_i that she receives. Part (i) follows from our general notion of desert (1) if we assume that $\sum_{i=1}^{N} r_i = Y$, so the amounts felt to be deserved sum to the team output available for distribution. Part (ii) says that, starting from a common effort level z, the payoff that agent i feels she deserves increases faster in her own effort e_i than does her actual payoff, which is equivalent to saying that the *share* of team output felt to be deserved is strictly increasing.[5] Part (ii) follows from part (i) in non-pathological cases.[6]

Each agent's utility U_i is assumed to take the following separable form:

$$U_i(e_i, e_{-i}) = \phi(y_i) + D(\phi(y_i) - \phi(r_i)) - C(e_i) = \overline{U}_i(e_i, e_{-i}) + D(\phi(y_i) - \phi(r_i)), \tag{2}$$

where *desert utility* $D(\phi(y_i) - \phi(r_i))$ represents the reference-dependent utility that the agent experiences from comparing her monetary payoff to her reference point. Desert utility depends on $\triangle_i \equiv \phi(y_i) - \phi(r_i)$, that is on the difference between the material utility derived from the agent's share of output and the material utility associated with receiving the deserved reference point. It is important to emphasize that in this formulation an agent's reference point is *choice-acclimating* and hence endogenous: as the agent changes her effort choice, her reference point adjusts, and the agent anticipates this when deciding how hard to work.

We let desert utility $D(\triangle_i)$ be a continuous function with $D(\triangle_i) = L(\triangle_i)$ when $\triangle_i < 0$, $D(\triangle_i) = 0$ when $\triangle_i = 0$ and $D(\triangle_i) = G(\triangle_i)$ when $\triangle_i > 0$. Since $\phi' > 0$, $\triangle_i \gtreqless 0 \Leftrightarrow \phi(y_i) \gtreqless \phi(r_i) \Leftrightarrow y_i \gtreqless r_i$. Thus, $L(\triangle_i)$ represents the desert utility associated with situations in which

[5] The share felt to be deserved is given by $\frac{r_i}{f}$ and the derivative of this share with respect to e_i is given by $\frac{r_i' f - r_i f'}{f^2}$. At a common effort level $z > 0$, $\frac{r_i}{f} = \frac{1}{N}$, so this derivative is strictly positive if and only if (ii) holds.

[6] Part (i) implies that $r_i(e_i, z_{-i})$ crosses $\frac{f(e_i+(N-1)z)}{N}$ from below at $e_i = z$, so $r_i'(z, z_{-i}) \geq \frac{f'(Nz))}{N}$, with strict inequality in non-pathological cases.

$y_i < r_i$ where the agent receives less than she feels she deserves. In that case we say that the agent suffers a *desert loss,* and we assume that $L(\triangle_i) < 0$, so such losses are always unambiguously painful. $G(\triangle_i)$ represents the desert utility when $y_i > r_i$, so the agent receives more than she feels she deserves. When $G(\triangle_i) > 0$, we say the agent derives *desert elation* from $\triangle_i > 0$: she gains pleasure from doing better than deserved; when $G(\triangle_i) < 0$ we say the agent suffers *desert guilt* from $\triangle_i > 0$: doing better than is felt to be deserved induces a psychological cost which we call guilt.[7] Letting $G'(0) \equiv \lim_{\triangle_i \downarrow 0} G'(\triangle_i)$, we define local and global desert guilt and elation as follows:

Definition 1 *The agents exhibit local desert guilt if* $G'(0) < 0$. *They exhibit global desert guilt if* $G'(0) < 0$ *and* $G(\triangle_i) < 0$ *for all* $\triangle_i > 0$.

Definition 2 *The agents exhibit local desert elation if* $G'(0) > 0$. *They exhibit global desert elation if* $G'(0) > 0$ *and* $G(\triangle_i) > 0$ *for all* $\triangle_i > 0$.

As discussed in Section 2, we assume that each agent is loss averse around her choice-acclimating endogenous reference point. In particular, letting $L'(0) \equiv \lim_{\triangle_i \uparrow 0} L'(\triangle_i)$, we assume that desert utility $D(\triangle_i)$ is differentiable everywhere away from $\triangle_i = 0$, that $L'(0) > 0$, and that $L'(0) > G'(0)$. Thus, in the limit as the deviation from the reference point tends to zero, desert losses remain painful and desert losses are more painful than any desert elation is pleasurable: the agents are loss averse for small stakes. This corresponds to Assumption A4 in Kőszegi and Rabin's (2006) formal description of loss aversion, and implies a kink in utility at the reference point.[8] Models of loss aversion generally also assume loss aversion for large stakes, weak convexity in the loss domain and weak concavity in the gain domain (Assumptions A2 and A3 in Kőszegi and Rabin, 2006), but our results do not require such assumptions.

[7] We take the classical view of guilt as arising from "private feelings associated with a troubled conscience" (Smith et al., 2002, p. 138). However, nothing in the structure of our model precludes a role for other negative emotions linked to publicity such as shame which involves "an unpleasant emotional reaction by an individual to an actual or presumed negative judgment of himself by others" (Ausubel, 1955, p. 382). Note also that desert guilt differs from the guilt aversion of Battigalli and Dufwenberg (2007), who use dynamic psychological game theory to model guilt from reducing another agent's payoff below her expectations. In our model, an agent can feel guilt even when others' expectations are confirmed.

[8] Note that in Kőszegi and Rabin's (2006) formalization, $L'(0) > G'(0) > 0$, so that in the limit receiving less than the reference point is always painful but receiving more is always pleasurable. In our desert framework, we allow $G'(0) < 0$ to capture desert guilt.

3.3 Examples

To fix ideas, we now provide a few concrete examples of desert preferences which satisfy the assumptions outlined above. We start by presenting three plausible forms for the reference point. All three satisfy Assumption 1 and our general notion of desert (1). First, an agent could feel that she deserves a share of team output equal to her share of effort, so:

$$r_i = \frac{e_i}{\sum_{i=1}^N e_i} f\left(\sum_{i=1}^N e_i\right) \quad \text{when } \sum_{i=1}^N e_i > 0. \tag{3}$$

Second, the agent might feel that she deserves a share of team output equal to her share of the cost of effort, so:

$$r_i = \frac{C(e_i)}{\sum_{i=1}^N C(e_i)} f\left(\sum_{i=1}^N e_i\right) \quad \text{when } \sum_{i=1}^N e_i > 0 \text{ so } \sum_{i=1}^N C(e_i) > 0. \tag{4}$$

Finally, the agent could feel that she deserves an equal share of the team output that would have been produced had everybody worked as hard as she did, so:

$$r_i = \frac{f(Ne_i)}{N}. \tag{5}$$

Next, we present one simple piecewise-linear form for desert utility $D(\triangle_i)$ which satisfies our assumptions on desert utility outlined above. We present this example for illustrative purposes—piecewise linearity is not required for our equilibrium results in Section 4. With piecewise linearity, $D(\triangle_i) = L(\triangle_i) = l\triangle_i$ when $\triangle_i < 0$, $D(\triangle_i) = 0$ when $\triangle_i = 0$ and $D(\triangle_i) = G(\triangle_i) = g\triangle_i$ when $\triangle_i > 0$; thus l represents the slope of desert utility when the agent receives less than she feels she deserves and g represents the slope when the agent receives more. To ensure that an agent suffers a desert loss when she receives less than she feels she deserves, i.e., $L(\triangle_i) < 0$, we assume that $l > 0$. Desert guilt (Definition 1) implies that $g < 0$ while desert elation (Definition 2) implies that $g > 0$. Note that with piecewise linearity there is no longer a distinction between local and global desert guilt or between local and global desert elation, and our assumption of loss aversion boils down to assuming that $l > g$. Applications of loss aversion often use a piecewise-linear functional form (for some recent examples see Crawford and Meng, forthcoming, and Gill and Prowse, forthcoming); in particular, Gill and Stone's (2010) analysis of desert in competitive environments assumes piecewise linearity throughout.

3.4 Desert equilibrium

Taking the efforts of her teammates e_{-i} as given, each agent chooses her own effort e_i to maximize her utility U_i. Having exerted her chosen effort, each agent receives her monetary payoff and also observes the effort levels of her teammates. Observability implies that the agents do not have to try to infer other agents' efforts from the team output. When an agent's reference point r_i depends only on her own effort e_i and the sum of her teammates' efforts $\sum_{j \neq i} e_j$, as will be the case in the linear example of Section 6, the assumption of observability is superfluous as $\sum_{j \neq i} e_j$ can be inferred perfectly from the agent's monetary payoff y_i. We restrict attention to pure-strategy Nash equilibria, which we call *desert equilibria*.[9]

4 Equilibrium analysis

4.1 No desert

As a benchmark, we first solve for equilibrium play when the agents do not have desert preferences. The equilibrium is symmetric and interior. Defining social welfare as the sum of utilities, so welfare $W \equiv \sum_{i=1}^{N} U_i$, effort in the unique equilibrium is socially too low due to the positive externality inherent in the team production game.

Proposition 1 *In the absence of desert, there is a unique and symmetric pure-strategy Nash equilibrium in which the agents exert strictly positive effort $\widetilde{e}$. Equilibrium effort is strictly lower than the socially optimal effort level e^w.*

Proof. See Appendix. ■

4.2 Desert guilt

When agents suffer from global desert guilt (see Definition 1), so the agents always dislike receiving more than they feel they deserve, desert generates a range of symmetric equilibria around the no-desert equilibrium $\widetilde{e}$ as Proposition 2 illustrates.

[9] Technically, our game is psychological (Geanakoplos et al., 1989) as agent $i's$ utility depends on her belief about the efforts of her teammates via the reference point. In particular, our game falls under Battigalli and Dufwenberg's (2009) framework of a dynamic psychological game as utility depends on terminal node (ex post) beliefs, so beliefs can update during the course of the game (in contrast to Geanakoplos et al., in which utilities only depend on initial beliefs). However, we have assumed that the agents observe each other's efforts ex post, so the actual efforts pin down these beliefs at the terminal nodes. Thus we do not need to introduce the apparatus of psychological games: we can write payoffs as a function of actions alone, given the actions determine the first-order beliefs. Even in the absence of observability, the set of pure-strategy equilibria would remain the same: the discussion in the third-from-last paragraph of Section 2 in Gill and Stone (2010) also applies here.

Proposition 2 *When the agents exhibit global desert guilt, a range of symmetric desert equilibria exists around the equilibrium level of effort in the absence of desert* $\widetilde{e}$, *with some equilibria strictly above and some strictly below* $\widetilde{e}$.

Proof. See Appendix. ■

The intuition is as follows. If an agent increases her work effort above the common effort level of her teammates, she raises the reference point that she feels she deserves above the equal share of the team output that she receives. Thus she suffers a desert loss as she receives less than she feels she deserves. If, instead, the agent reduces her work effort below that of her teammates, she suffers from desert guilt as her deserved reference point falls below an equal share of team output, and so she receives more than she feels she deserves. Thus, starting from a common effort level, desert considerations reduce both the incentive to increase and to decrease effort. When the common effort level is not too far from the no-desert equilibrium, these desert considerations dominate, thus generating a symmetric equilibrium. Essentially, desert forges an endogenous complementarity between agents' efforts by giving them incentives to match the efforts of their teammates in equilibrium.

Proposition 2 tells us that, perhaps unsurprisingly, desert guilt can make the agents work harder in equilibrium.[10] Thus desert guilt, which as noted in the Introduction may itself be underpinned by a desire to conform to a meritocratic social norm, can give rise to a specific social norm of cooperation in our team production setting: desert guilt can make cooperation normatively appropriate for an agent, conditional on her teammates adhering to the norm of cooperative behavior. Once a norm of cooperation starts to become established, desert guilt can help to ensure that the agents stick to the norm without the need for any external pressure or sanctions. In practice, however, we might expect desert guilt to interact with external enforcement mechanisms such as public shame and punishment in forming and underpinning social norms of cooperation.[11] The importance of social norms suggests that employers using teams might try to mold perceptions of norms by, for instance, providing team members with information about effort levels in other similar successful teams.

[10] Kandel and Lazear (1992) and Mohnen et al. (2008) study the effect of peer pressure on effort in a team production setting. Kandel and Lazear assume that peer pressure operates linearly, and so raises effort in the unique symmetric equilibrium. Mohnen et al. assume that the peer pressure function is convex, and so has no effect on equilibrium effort in a single-stage game.

[11] See Young (1998), Bicchieri (2006) and Krupka and Weber (2009) for further discussion and experimental evidence of the role of social norms and the importance of context and expectations in how norms drive behavior.

More unexpectedly, with desert guilt there is a whole range of possible symmetric equilibria, some of which involve *less* effort than in the absence of desert considerations. Our theory of desert thus endogenously generates behavior that is consistent with both positive and negative reciprocity, whereby agents reciprocate by matching the level of cooperation of their teammates.

A number of experiments provide evidence that a large proportion of subjects in contribution games exhibit conditionally cooperative behavior. In linear public good games, Fischbacher et al. (2001), Croson (2007), and Fischbacher and Gächter (2010) elicit contribution schedules, finding that a majority of subjects show a positive relationship between desired contribution levels and the group average, while Falk et al. (forthcoming) find that the same individual contributes more to a public good when he is a member of a group with higher average contributions. In these types of linear public good experiments, contributions are *monetary* and tend to fall over time towards the selfish Nash equilibrium. However, with *real effort* in a team production setting van Dijk et al. (2001) find no free-riding on average, with no tendency for the average amount of free-riding to increase over time. We would expect deservingness to be more salient when subjects exert real effort; thus in an environment with real effort desert-type considerations should be better able to sustain cooperative behavior over time. In non-linear public good experiments (with interior selfish Nash equilibria) average contributions sometimes fall below the selfish Nash equilibrium level, suggesting negative reciprocity (see Isaac and Walker, 1998, Cason et al., 2002, 2004, and the survey by Laury and Holt, 2008).

We stress, however, that the mechanism which introduces reciprocity into our framework is different to that which drives intentions-based theories of reciprocal altruism. Intentions-based models explain conditional cooperation by assuming that agents like to reciprocate kindness by helping those who are kind to them but hurting those who are mean to them (see for instance Rabin, 1993, Dufwenberg and Kirchsteiger, 2004, and Falk and Fischbacher, 2006). The perceived kindness of an action is determined by the perceived intention of the agent, which introduces hierarchies of beliefs into utility. In contrast, our theory of desert predicts reciprocal behavior without the need to introduce beliefs about motives or intentions of other agents.[12]

[12] Some authors impose reciprocity by simply assuming that certain actions induce a reciprocal response without any consideration of intentions, which of course can give rise to multiple equilibria. In various team production-type settings Sugden (1984), Rob and Zemsky (2002) and Huck et al. (2010) impose positive reciprocity, Sugden by assuming that at a minimum agents want to match the lowest of their rivals' efforts, Rob and Zemsky by assuming that the greater the degree of cooperation in the previous period, the more employees want to cooperate now, and Huck et al. by assuming that the strength of a preference for conforming to a social norm is increasing in the degree of conformity of the other players.

4.3 Desert elation

When agents exhibit local desert elation (see Definition 2), so the agents actively like receiving slightly more than they feel they deserve, Proposition 3 shows that effort in a symmetric equilibrium must decline compared to the case where the agents do not have desert preferences.

Proposition 3 *When agents exhibit local desert elation, effort in any symmetric desert equilibrium is strictly lower than the equilibrium level of effort in the absence of desert $\widetilde{e}$.*

Proof. See Appendix. ■

Local desert elation implies that, starting from a common level of effort, the local incentive to reduce effort is now higher than in the absence of desert considerations. Agents like receiving more than they feel they deserve, and by reducing effort an agent lowers her deserved reference point below the equal share of team output that she receives. Thus no common effort level at or above the no-desert equilibrium $\widetilde{e}$ can form a symmetric equilibrium, as the agents would want to slack off to enjoy some desert elation. The linear example in Section 6 illustrates that a range of symmetric equilibria may exist below $\widetilde{e}$. In this range, the common effort level is low enough that the desert elation from deviating to an even lower level of effort is outweighed by the reduction in standard utility $\overline{U}_i$.

4.4 Welfare

Remember that we defined social welfare as the sum of utilities, so $W = \sum_{i=1}^{N} U_i$.[13] Part (i) of the following proposition tells us that desert guilt does not change how hard the agents should work in order to maximize welfare. Part (ii) tells us that desert guilt can raise welfare relative to the no-desert level if it partially or wholly overcomes the free-rider problem in the team production game by pushing the agents to match the higher effort levels of their teammates, but that desert guilt can also lower welfare if it lowers effort (see Proposition 2 which shows that desert guilt can either raise or lower efforts in equilibrium). The final part of the result tells us that, by lowering efforts, desert elation reduces welfare relative to the no-desert level.

[13] If desert concerns are understood as a moral judgment about what an agent feels she ought to do rather than as a component of her well-being, welfare should exclude desert utility. Of course this distinction has no effect on behavior, and since we focus on symmetric equilibria in which desert utility is zero in equilibrium, the distinction is not important for the results in this section.

Proposition 4

(i) If the agents exhibit global desert guilt, the socially optimal level of effort remains the same as in the absence of desert preferences.

(ii) Assuming the agents play a symmetric desert equilibrium, global desert guilt increases welfare if it raises effort towards the socially optimal level, but reduces welfare if it lowers effort below the level in the absence of desert preferences.

(iii) Assuming the agents play a symmetric desert equilibrium with strictly positive effort, local desert elation always reduces welfare.

Proof. See Appendix. ■

Desert guilt can raise welfare despite the negative psychological consequences of receiving more or less than is felt to be deserved. This is because when all the agents work equally hard, everybody feels that they deserve the equal share of team output that they receive, and so desert utility is zero. This raises the possibility that desert guilt evolved to coordinate behavior on welfare-improving effort choices. We note, however, that desert guilt can push the agents to work too hard if effort is raised above the socially optimal level, and it is even possible for desert guilt to push effort so high that welfare falls below the level in the no-desert benchmark.

5 Inequity aversion as a special case of desert

Our notion of desert can be seen as a generalization of distributional concern models to situations in which effort choices affect the distribution that is perceived to be fair or equitable. Desert-concerned agents care not just about the distribution of monetary payoffs, but also on how the distribution came about. Indeed, our notion of desert is related to the inequity-aversion models of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000), and one of the aims of this paper is to clarify this relationship in a team production setting. Inequity-averse agents, like desert-concerned agents, care about the distribution of resources in addition to their own material payoffs. Inequity aversion over monetary payoffs alone plays no role in our team production setting because all the agents receive the same equal share of the team output. Desert-concerned agents, on the other hand, care about the relationship between the distribution of monetary payoffs and the distribution of agents' efforts, and not just about the brute distribution of money. However, agents might be inequity averse over monetary payoffs *net of effort costs*, in which case they too will be concerned with the relationship between money and effort.[14]

In fact, it turns out that Fehr and Schmidt (1999) inequity aversion over money net of effort costs is a special case of our more general notion of desert: we show below that inequity aversion over money net of effort costs corresponds to a particular form of the reference point that is felt to be deserved, while our theory of desert leaves open the form of the functional relationship between efforts and deservingness. Also, for analytical tractability Fehr and Schmidt (1999) impose piecewise-linear loss aversion, while our theory allows desert utility to take on arbitrary non-linear shapes around the kink implied by loss aversion.

If we apply Fehr and Schmidt's (1999) model of inequity aversion (equation (1) at p. 822) to our team production game, and assume aversion to differences in monetary payoffs net of effort costs, then:

$$U_i\left(e_i, e_{-i}\right) = y_i - C(e_i) - \tfrac{\alpha}{N-1}\sum_{j\neq i}\max\left\{C(e_i) - C(e_j), 0\right\} - \tfrac{\beta}{N-1}\sum_{j\neq i}\max\left\{C(e_j) - C(e_i), 0\right\}, \quad (6)$$

where $\alpha \geq \beta$, $\alpha > 0$ and $\beta \in [0, 1)$.[15] Note that monetary payoffs drop out of the comparison terms because $y_i = y_j = f\left(\sum_{i=1}^{N} e_i\right)/N$ for all i, j pairs.

With just two agents, it is straightforward to see that (6) is a special case of our model of desert. The reference point that agent i feels she deserves is then given by:

$$r_i\left(e_i, e_{-i}\right) = \frac{f\left(\sum_{i=1}^{N} e_i\right)}{N} + C(e_i) - C(e_j), \quad (7)$$

which satisfies Assumption 1 and our general notion of desert (1). Money utility is linear, so $\phi\left(y_i\right) = y_i$. Desert utility $D\left(\triangle_i\right)$ takes the piecewise-linear form outlined in the second paragraph of Section 3.3, with $l = \alpha$, $g = -\beta$ and $\triangle_i = y_i - r_i = C(e_j) - C(e_i)$. The assumption of loss aversion, i.e., that $l > g$, corresponds to $\alpha > -\beta$ which always holds in Fehr and Schmidt (1999) given $\alpha > 0$ and $\beta \geq 0$. Thus we have the following result.

[14] A few papers analyze the consequences of inequity aversion over monetary payoffs net of effort costs. Demougin and Fluet (2003) evaluate the impact in tournaments, Kölle et al. (2011) study the impact on contributions to a public good (focusing on the role of initial wealth differences, they find that a continuum of equilibria may exist), and a burgeoning literature looks at the implications for contract design (Itoh, 2004, Demougin and Fluet, 2006, Demougin et al., 2006, Desiraju and Sappington, 2007, Rey Biel, 2008, Bartling and von Siemens, 2010, Bartling, forthcoming, von Siemens, forthcoming a, forthcoming, b).

[15] Fehr and Schmidt (1999) do not explicitly require that $\alpha > 0$, but when $\alpha = 0$, $\alpha = \beta = 0$, so the inequity terms disappear and their model collapses to the standard one.

Proposition 5 *When there are two agents, our model of desert nests inequity aversion over monetary payoffs net of effort costs as a special case.*

When the aversion to advantageous inequity is strict, i.e., $\beta > 0$, the agents exhibit global desert guilt (Definition 1) as $G'(0) = -\beta < 0$ and $G(\triangle_i) = -\beta \triangle_i < 0$ for all $\triangle_i > 0$. Thus Proposition 2 applies. Fehr and Schmidt (1999) do not allow agents to like advantageous inequity, i.e., they exclude the case where $\beta < 0$. Nonetheless, we also consider this case. When $\beta < 0$, the agents exhibit local desert elation (Definition 2) as $G'(0) = -\beta > 0$. Thus Proposition 3 applies, so long as we maintain the assumption of loss aversion so $\alpha > -\beta$.

When there are more than two agents, the Fehr and Schmidt (1999) model, outlined above in (6), involves a series of pairwise comparisons. To see the connection between inequity aversion over money net of effort costs and our model of desert, we therefore need to broaden our model of desert to also allow for pairwise desert comparisons.

Let $q(e_i, e_j, e_{-ij})$ represent how much more or less agent i fells that she deserves relative to agent j, where e_{-ij} represents the vector of effort choices of all the other agents. In order to satisfy our general notion of desert (1), which says that agent i feels she deserves more (less) than agent j if and only if she works harder (less hard), we impose that:

$$q(e_i, e_j, e_{-ij}) \gtreqless 0 \Leftrightarrow e_i \gtreqless e_j. \tag{8}$$

Letting $\triangle_{ij} \equiv -q(e_i, e_j, e_{-ij})$, the desert utility component of total utility U_i now takes the following form:

$$\frac{1}{N-1} \sum_{j \neq i} D(\triangle_{ij}), \tag{9}$$

where the properties of $D(\triangle_{ij})$ match those previously imposed on $D(\triangle_i)$ in Section 3.2. Thus a given pairwise comparison induces a desert loss when agent i feels she deserves more than agent j so $q(e_i, e_j, e_{-ij}) > 0$ and hence $\triangle_{ij} < 0$, and induces desert elation or guilt when i feels she deserves less than j so $q(e_i, e_j, e_{-ij}) < 0$ and hence $\triangle_{ij} > 0$. This generalization nests our earlier model of desert when we set $q(e_i, e_j, e_{-ij}) = -(\phi(y_i) - \phi(r_i))$, so $\triangle_{ij} = \triangle_i = \phi(y_i) - \phi(r_i)$ for all i, j pairs, and hence desert utility $\left(\sum_{j \neq i} D(\triangle_{ij})\right) / (N-1) = D(\triangle_i) = D(\phi(y_i) - \phi(r_i))$.[16] The generalization also nests inequity aversion over money net of effort costs, given by (6), when $\phi(y_i) = y_i$, $q(e_i, e_j, e_{-ij}) = C(e_i) - C(e_j)$ so $\triangle_{ij} = C(e_j) - C(e_i)$, and $D(\triangle_{ij})$ takes the same

[16] Of course, $q(e_i, e_j, e_{-ij})$ can then no longer be interpreted directly in terms of relative desert as we no longer have any notion of pairwise comparisons, and so we should not seek to impose (8).

piecewise-linear form as $D(\triangle_i)$ in the two-agent case above with $l = \alpha,\ \ g = -\beta$, and $l > g$ as $\alpha > -\beta$. Thus we get the following result.

Proposition 6 *When there are more than two agents, a generalization of our model of desert to allow for pairwise desert comparisons continues to nest inequity aversion over monetary payoffs net of effort costs as a special case.*

Our model of desert focuses on the case where agents are identical. As discussed in Section 2, if the agents differed in their ability as measured by their cost of effort functions, then requirement (1) would correspond to a 'libertarian' assumption about deservingness in which desert depends only on individual outputs and is independent of ability. The deserved reference point implied by inequity aversion over money net of effort costs (7) would instead correspond to a 'liberal egalitarianism' assumption (Cappelen et al., 2007) about deservingness, where agents who incur higher effort costs are more deserving. The deserved reference point based on effort cost shares given by (4) would provide another example of liberal egalitarianism.

6 Linear example

In this section we linearize our model in order to work with an analytically tractable example. We undertake this exercise for a number of reasons. First, the analysis clarifies the more abstract results above in an applied setting. Second, we can say more in the example: in particular we can rule out asymmetric desert equilibria and we find a range of equilibria with desert elation. Third, the example allows us to get a feel for how the range of equilibria varies with the parameters of the model. Finally, we hope that the example will prove useful in future applied theoretical and empirical work.

We linearize money utility, so $\phi(y_i) = y_i$, and we linearize team output as a function of efforts, so $f\left(\sum_{i=1}^{N} e_i\right) = \sum_{i=1}^{N} e_i$. We assume that the cost of effort function is quadratic, i.e., $C(e_i) = \left(ce_i^2\right)/2$ with $c > 0$, so marginal cost is linearized. We further assume that the deserved reference point takes the form given by (5), which gives $r_i = e_i$ due to the linearity of the team output function f (note that (3) would give also give $r_i = e_i$). Finally, we assume that desert utility $D(\triangle_i)$ takes the piecewise-linear form described in the second paragraph of Section 3.3, so $l > 0$ represents the slope of desert utility when the agent receives less than she feels she deserves and g represents the slope when the agent receives more. Our assumption of loss aversion implies that $l > g$. As noted in Section 3.3, under piecewise-linearity there is no distinction between local and global desert guilt or between local and global desert elation, so we will simply refer to desert guilt when $g < 0$ and desert elation when $g > 0$. Under these conditions, we get the following result.

Proposition 7

(i) In the absence of desert, there is a unique and symmetric pure-strategy Nash equilibrium in which the agents exert effort $\widetilde{e} = \frac{1}{cN} > 0$, *which is strictly lower than the socially efficient level* $e^w = \frac{1}{c}$.

(ii) With desert, any effort $e^* \in \left[\frac{1}{cN} - l\left(\frac{N-1}{cN}\right),\ \max\left\{\frac{1}{cN} - g\left(\frac{N-1}{cN}\right), 0\right\}\right] \cap \mathbb{R}_+$ *forms a symmetric desert equilibrium. There are no other desert equilibria.*

(iii) Desert guilt ($g < 0$) *gives a range of symmetric desert equilibria around the equilibrium level of effort in the absence of desert* $\widetilde{e}$. *The top of the range tends to the socially efficient level of effort* e^w *as* g *tends to* -1.

(iv) Desert elation ($g > 0$) *implies that equilibrium effort is always strictly lower than the level in the absence of desert* $\widetilde{e}$, *with a range of symmetric desert equilibria when* $g < \frac{1}{N-1}$ *and a unique symmetric desert equilibrium at zero effort when* $g \geq \frac{1}{N-1}$.

Proof. See Appendix. ■

Part (i) corresponds to Proposition 1 for the more general model. Part (ii) gives us the range of symmetric equilibria with desert as an explicit function of the parameters of the linear example. Part (ii) also tells us that in this linear example there can be no asymmetric equilibria - we were not able to rule these out in our more general model. Part (iii) confirms Proposition 2, but further tells us that as g tends to -1, so desert utility with guilt tends to be as steep as money utility, we approach social efficiency if the agents coordinate on the highest effort equilibrium. Part (iv) confirms Proposition 3, but also tells us that a range of equilibria exists when the desert elation is not too strong.

We now look at how the range of equilibria given by part (ii) changes with the strength of desert preferences. When desert elation becomes weaker or desert guilt becomes stronger (i.e., g falls), the highest feasible equilibrium level of effort goes up. When desert losses become more strongly felt (i.e., l rises), the lowest feasible equilibrium level of effort goes down. With desert guilt ($g < 0$), the difference between the highest feasible equilibrium level of effort and the no-desert level as a proportion of the no-desert level $\widetilde{e} = (cN)^{-1}$ is given by $-g\,(N-1) > 0$: this proportion is increasing in the strength of desert guilt and in the number of agents N.

7 Conclusion

In this paper we developed a theoretical framework to study the strategic implications of desert considerations when agents work in teams. Our notion of desert can be seen as a generalization of distributional concern models to situations in which effort choices affect the distribution that is perceived to be fair or equitable. We focused on the strategic implications of desert for the team members themselves. However, we hope that our framework and insights will provide a useful building block for future research drawing out the wider implications of desert concerns, e.g., for employers deciding whether to use teams in the workplace and designing optimal team incentive schemes, for policy-makers deciding how to tax partnerships and team-based bonuses, and for workers themselves deciding whether or not to join teams. We also hope that our model will spur testing to determine whether agents who interact in teams behave as if desert concerns matter to them. Finally, we hope that researchers will use our framework to analyze the equilibrium implications of desert in broader settings where, for instance, teammates interact repeatedly, or simultaneously cooperate in teams but compete for promotions.

Appendix

Proof of Proposition 1. For any vector of efforts e_{-i} for the other team members:

$$\overline{U}_i' \equiv \frac{\partial \overline{U}_i}{\partial e_i} = \phi'\left(\frac{f}{N}\right)\frac{f'}{N} - C'; \tag{10}$$

$$\overline{U}_i'' \equiv \frac{\partial^2 \overline{U}_i}{\partial e_i^2} = \phi''\left(\frac{f}{N}\right)\left(\frac{f'}{N}\right)^2 + \phi'\left(\frac{f}{N}\right)\frac{f''}{N} - C'' < 0. \tag{11}$$

Thus for any e_{-i}, $\overline{U}_i$ is strictly concave; and furthermore $\overline{U}_i' < 0$ for high enough e_i given C' is unbounded above. Therefore a strict best response e_i^* exists and is unique for any e_{-i}.

No asymmetric equilibrium can exist. Suppose one did. The agent(s) with the highest effort must have $\overline{U}_i' = 0$. Any agent with a strictly lower effort will share the same $\phi'\left(\frac{f}{N}\right)\frac{f'}{N}$ and have a strictly lower C' and so will have a strict incentive to increase effort.

Differentiating $\overline{U}_i'$ w.r.t. a common effort level z:

$$\frac{\partial \overline{U}_i'}{\partial z} = \phi''\left(\frac{f(Nz)}{N}\right)\left(\frac{f'(Nz)}{N}\right)^2 N + \phi'\left(\frac{f(Nz)}{N}\right)\frac{f''(Nz)}{N} N - C''(z) < 0. \tag{12}$$

Thus a unique symmetric equilibrium $\widetilde{e} > 0$ exists where $\widetilde{e} = C'^{-1}\left(\phi'\left(\frac{f(N\widetilde{e})}{N}\right)\frac{f'(N\widetilde{e})}{N}\right)$ as $\overline{U}_i' > 0$

for $z = 0$ while $\overline{U}_i' < 0$ for high enough z given C' is unbounded above.

Consider now the socially optimal vector of efforts. Because $C'' > 0$ and Y depends on $\sum_{i=1}^{N} e_i$, all the agents' efforts must be the same at a social optimum. At a common effort level z:

$$\frac{\partial W}{\partial z} = N\left(\phi'\left(\frac{f(Nz)}{N}\right)\frac{f'(Nz)}{N}N - C'(z)\right); \tag{13}$$

$$\frac{\partial^2 W}{\partial z^2} = N\left(\phi''\left(\frac{f(Nz)}{N}\right)\left(\frac{f'(Nz)}{N}\right)^2 N^2 + \phi'\left(\frac{f(Nz)}{N}\right)\frac{f''(Nz)}{N}N^2 - C''(z)\right) < 0. \tag{14}$$

Any social optimum must have $e^w > \widetilde{e}$ as at $z = \widetilde{e}$, $\frac{\partial W}{\partial z} > 0$. A unique optimum must exist where $e^w = C'^{-1}\left(\phi'\left(\frac{f(Ne^w)}{N}\right)\frac{f'(Ne^w)}{N}N\right)$, as W is strictly concave in z and $\frac{\partial W}{\partial z} < 0$ for high enough z given C' is unbounded above. ■

Proof of Proposition 2. Consider agent i's incentive to deviate from a common effort level $z > 0$. We show that for any z sufficiently close to the no-desert equilibrium effort $\widetilde{e} > 0$, the reduction in $i's$ desert utility $D(\triangle_i)$ arising from such a deviation outweighs any gain in standard utility $\overline{U}_i$, so $e_i = z$ is a best response to z_{-i}, that is to the vector of others' efforts e_{-i} in which $e_j = z \ \ \forall j \neq i$.

From Assumption 1(i) and $\phi' > 0$, $\triangle_i \gtreqless 0 \Leftrightarrow y_i \gtreqless r_i \Leftrightarrow e_i \lesseqgtr z$. Thus $e_i = z \Rightarrow D(\triangle_i) = D(0) = 0$, while a deviation upward to $e_i > z \Rightarrow D(\triangle_i) < 0$ as $L(\triangle_i) < 0$ for $\triangle_i < 0$, and a deviation downward to $e_i < z \Rightarrow D(\triangle_i) < 0$ as $G(\triangle_i) < 0$ for $\triangle_i > 0$ by the assumption of global desert guilt. Thus any deviation strictly reduces D, so deviations must increase $\overline{U}_i$ sufficiently to compensate.

(a) First consider $z \in (0, \widetilde{e}]$. From the proof of Proposition 1, $\overline{U}_i'' < 0$ with $\overline{U}_i' < 0$ for e_i sufficiently high, and $\overline{U}_i'(z, z_{-i}) \geq 0$ given $z \leq \widetilde{e}$. Thus downward deviations strictly reduce $\overline{U}_i$, while the strict concavity of $\overline{U}_i$ in e_i and its continuity in z ensures that large enough upward deviations must always reduce $\overline{U}_i$, i.e., $\exists \widehat{e} > \widetilde{e}$ such that $\forall z \leq \widetilde{e},\ \overline{U}_i(e_i, z_{-i}) \geq \overline{U}_i(z, z_{-i}) \Rightarrow e_i \in [z, \widehat{e}]$. As all deviations strictly reduce D, we can therefore restrict attention to deviations with $e_i \in (z, \widehat{e}]$.

Now take a given $z \in (0, \widetilde{e}]$. The gain in $\overline{U}_i$ from deviating to a specific $e_i \in (z, \widehat{e}]$ is bounded above by $\overline{U}_i'(z, z_{-i})(e_i - z)$ given $\overline{U}_i'' < 0$. The desert loss from a deviation to the specific $e_i \in (z, \widehat{e}]$, $D(\triangle_i) < 0$, is bounded above by $\left(\sup_{e_i \in (z, \widehat{e}]} \frac{D(\triangle_i)}{e_i - z}\right)(e_i - z)$. If $\frac{D(\triangle_i)}{e_i - z}$ has a maximal value $m(z)$ over $e_i \in (z, \widehat{e}]$, then $\sup_{e_i \in (z, \widehat{e}]} \frac{D(\triangle_i)}{e_i - z} = m(z) < 0$. If not, $\sup_{e_i \in (z, \widehat{e}]} \frac{D(\triangle_i)}{e_i - z} = \lim_{e_i \downarrow z} \frac{D(\triangle_i)}{e_i - z}$. Applying

L'Hôpital's Rule, and using Assumption 1 and $L'(0) > 0$:

$$\lim_{e_i \downarrow z} \frac{D(\triangle_i)}{e_i - z} = \lim_{e_i \downarrow z} D'\left(\phi\left(\frac{f}{N}\right) - \phi(r_i)\right)\left(\phi'\left(\frac{f}{N}\right)\frac{f'}{N} - \phi'(r_i)\, r_i'\right) \tag{15}$$

$$= L'(0)\left(\phi'\left(\frac{f(Nz)}{N}\right)\left(\frac{f'(Nz)}{N} - r_i'(z, z_{-i})\right)\right) < 0. \tag{16}$$

Thus, as a sufficient condition, deviation to any $e_i \in (z, \widehat{e}]$ strictly reduces U_i if $\overline{U}_i'(z, z_{-i}) < -\sup_{e_i \in (z,\widehat{e}]} \frac{D(\triangle_i)}{e_i - z}$. From the proof of Proposition 1, $\overline{U}_i'(\widetilde{e}, \widetilde{e}_{-i}) = 0$, and therefore by continuity $\lim_{z \uparrow \widetilde{e}} \overline{U}_i'(z, z_{-i}) = 0$. By continuity, $\lim_{z\uparrow \widetilde{e}} \left(-\sup_{e_i \in (z,\widehat{e}]} \frac{D(\triangle_i)}{e_i - z}\right) = -\sup_{e_i \in (\widetilde{e},\widehat{e}]} \frac{D(\triangle_i)}{e_i - \widetilde{e}}$. In turn, we can see that $-\sup_{e_i \in (\widetilde{e},\widehat{e}]} \frac{D(\triangle_i)}{e_i - \widetilde{e}} > 0$, as the sup either equals $m(\widetilde{e}) < 0$ if such a maximal value exists, or $L'(0)\left(\phi'\left(\frac{f(N\widetilde{e})}{N}\right)\left(\frac{f'(N\widetilde{e})}{N} - r_i'(\widetilde{e}, \widetilde{e}_{-i})\right)\right) < 0$. Thus for z sufficiently close to $\widetilde{e}$, and for $z = \widetilde{e}$, all deviations strictly reduce U_i so we have a (strict) equilibrium.

(b) When considering $z > \widetilde{e}$, a similar argument to (a) holds, inverting the directions of deviations. $\overline{U}_i'(z, z_{-i}) < 0$ given $z > \widetilde{e}$. As $\overline{U}_i'' < 0$, we need only consider downward deviations, which in this case have a natural bound at 0. The sufficient no-deviation condition in this case is $-\overline{U}_i'(z, z_{-i}) < -\sup_{e_i \in [0,z)} \frac{D(\triangle_i)}{z - e_i}$, and $\lim_{e_i \uparrow z} \frac{D(\triangle_i)}{z - e_i} = -G'(0)\,\phi'\left(\frac{f(Nz)}{N}\right)\left(\frac{f'(Nz)}{N} - r_i'(z, z_{-i})\right) < 0$ using Assumption 1 and $G'(0) < 0$ from global desert guilt. ■

Proof of Proposition 3. Suppose we have a common effort level $z \geq \widetilde{e}$. From the proof of Proposition 1, $\overline{U}_i'(z, z_{-i}) \leq 0$ given $z \geq \widetilde{e}$. Applying L'Hôpital's Rule and using Assumption 1 together with $G'(0) > 0$ from local desert elation:

$$\lim_{e_i \uparrow z} \frac{D(\triangle_i) - 0}{e_i - z} = \lim_{e_i \uparrow z} D'\left(\phi\left(\frac{f}{N}\right) - \phi(r_i)\right)\left(\phi'\left(\frac{f}{N}\right)\frac{f'}{N} - \phi'(r_i)\, r_i'\right) \tag{17}$$

$$= G'(0)\left(\phi'\left(\frac{f(Nz)}{N}\right)\left(\frac{f'(Nz)}{N} - r_i'(z, z_{-i})\right)\right) < 0. \tag{18}$$

Thus local downward deviations strictly increase U_i, so we cannot have an equilibrium. ■

Proof of Proposition 4. Note first that if the agents exhibit global desert guilt (Definition 1), desert utility $D(\triangle_i) \leq 0$ at any vector of efforts. Note second that when the agents all exert a common effort level $z > 0$, $D(\triangle_i) = 0$: from Assumption 1 each agent's deserved reference point matches the equal share of team output that she receives, so $\triangle_i = 0$. Part (i) then follows as when all agents exert the socially optimal level of effort in the absence of desert, given by $e^w > 0$ from Proposition 1, the sum of standard utilities $\sum_{i=1}^N \overline{U}_i$ is maximized and $D(\triangle_i) = 0$, so $W = \sum_{i=1}^N U_i$ is maximized also.

Next note that, from (14), welfare in the absence of desert is strictly concave in a common effort level z around e^w. When considering $z > 0$ in a symmetric desert equilibrium, $D(\triangle_i) = 0$ from above, so welfare with desert preference matches welfare without desert. With global desert guilt and $z = 0$, $D(\triangle_i) \leq 0$, so welfare with desert is weakly lower than without desert. Thus parts (ii) and (iii) follow immediately from this concavity (remembering from Proposition 3 that local desert elation always reduces effort in a symmetric equilibrium). ■

Proof of Proposition 7.

(i) Follows immediately from Proposition 1 and the expressions for $\widetilde{e}$ and e^w in its proof.

(ii) Let $s_{-i} \equiv \frac{\sum_{j \neq i} e_j}{N-1}$. If agent i sets $e_i > s_{-i}$, then $y_i = \frac{e_i+(N-1)s_{-i}}{N} < e_i = r_i$ so the agent suffers a desert loss and $U_i = \frac{e_i+(N-1)s_{-i}}{N} + l\left(\frac{e_i+(N-1)s_{-i}}{N} - e_i\right) - \frac{ce_i^2}{2}$. If $e_i < s_{-i}$, then $y_i > e_i = r_i$ and the agent feels desert elation or guilt with $U_i = \frac{e_i+(N-1)s_{-i}}{N} + g\left(\frac{e_i+(N-1)s_{-i}}{N} - e_i\right) - \frac{ce_i^2}{2}$. If $e_i = s_{-i}$, $y_i = e_i = r_i$ so $U_i = \frac{e_i+(N-1)s_{-i}}{N} - \frac{ce_i^2}{2}$.

At $e_i = s_{-i}$, the right-hand side derivative $\left(\frac{\partial U_i}{\partial e_i}\right)^+ = \frac{1}{N} - l\left(\frac{N-1}{N}\right) - ce_i \leq 0 \Leftrightarrow \frac{1}{cN} - l\left(\frac{N-1}{cN}\right) \leq e_i$. At $e_i = s_{-i}$, the left-hand side derivative $\left(\frac{\partial U_i}{\partial e_i}\right)^- = \frac{1}{N} - g\left(\frac{N-1}{N}\right) - ce_i \geq 0 \Leftrightarrow \frac{1}{cN} - g\left(\frac{N-1}{cN}\right) \geq e_i$. Furthermore, U_i is everywhere strictly concave. Remembering that $e_i \geq 0$ and that $l > g$ from our assumption of loss aversion, it follows that any $e^* \in \left[\frac{1}{cN} - l\left(\frac{N-1}{cN}\right),\ \max\left\{\frac{1}{cN} - g\left(\frac{N-1}{cN}\right), 0\right\}\right] \cap \mathbb{R}_+$ forms a symmetric desert equilibrium and that there can be no other symmetric desert equilibria.

Suppose now that an asymmetric desert equilibrium exists. Let $e_i^{\max}$ represent the highest equilibrium effort. Then $e_i^{\max} > s_{-i}$ and $e_i^{\max} > 0$, so $e_i^{\max} = \frac{1}{cN} - l\left(\frac{N-1}{cN}\right) > 0$ given U_i is everywhere strictly concave. Similarly, $e_i^{\min} < s_{-i}$, so $e_i^{\min} = \max\left\{\frac{1}{cN} - g\left(\frac{N-1}{cN}\right), 0\right\}$. But this gives a contradiction given $l > g$ and $e_i^{\max} > e_i^{\min}$, so there can be no asymmetric desert equilibria.

(iii) Follows immediately from parts (i) and (ii) given $l > 0$ and $g < 0$.

(iv) Follows immediately from parts (i) and (ii) given $l > g > 0$. ■

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