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Pinheiro, Roberto B. and Visschers, Ludo

University of Colorado at Boulder, Universidad Carlos III de Madrid

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Unemployment Risk and Wage Differentials*

ROBERTO PINHEIRO[†] AND LUDO VISSCHERS[‡]

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Abstract

Workers in less secure jobs are often paid less than identical-looking workers in more secure jobs. We show that this lack of compensating differentials for unemployment risk can arise in equilibrium when all workers are identical, and firms differ, but do so only in offered job security (the probability that the worker is not sent into unemployment). In a setting where workers search on and off the job, wages paid *increase* with job security for at least all firms in the risky tail of the distribution of firm-level unemployment risk. As a result, unemployment spells become persistent for low-wage and unemployed workers, a seeming pattern of ‘unemployment scarring’, that is created entirely by firm heterogeneity alone. Higher in the wage distribution, workers can take wage cuts to move to more stable employment.

Keywords: Layoff Rates, Unemployment risk, Wage Differentials, Unemployment Scarring

JEL Codes: J31, J63

1 Introduction

When a transition into unemployment inflicts a loss on the worker, a competitive labor market requires a higher risk of unemployment to be compensated by a higher wage. However, empirically, *at the firm level* the relation between job security - the probability of not becoming unemployed - and wages, seems to be positive or at the least not significantly different from zero.¹ For example, after controlling for worker and firm characteristics

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[†]University of Colorado, Roberto (dot) Pinheiro (at) colorado (dot) edu.

[‡]Department of Economics, U. Carlos III Madrid, lviissche (at) eco (dot) uc3m (dot) es

¹The literature testing for the relationship between job security *at the firm level* and wages is remarkably small. We know more about compensation for risk at different levels: for example, at the industry level the picture of compensating wage differentials is ambiguous, with some evidence in favor of it (Abowd and Aschenfelter 1981 e.g.). Moretti (2000) found evidence that seasonal work earns a premium versus year-around work. However, patterns at industry level are not necessarily informative about what occurs at the firm level. In this paper, we argue that search frictions, working between firms and workers within e.g. industries and occupations, can result in interesting non-linearities and interactions for the wage - unemployment risk relationship.

including firm profit levels and firm exit probabilities, Mayo and Murray (1991), and Arai and Heyman (2001) find a positive correlation between job security and wages.²

The lack of compensating wages is even more puzzling, once it is observed that transitions to unemployment also raise the prospect of shortened *employment* spells in the future. A job loss increases the probability of future job losses (e.g. Stevens 1997, Kletzer 1998). Such a reduction in subsequent employment durations is responsible for a significant part of the cost of a transition into unemployment (Eliason and Storrie 2006, Böheim and Taylor 2002, Arulampalam et al. 2000). For displaced workers of a given quality, commonly, new jobs come with lower wages, and *simultaneously* with a higher risk of renewed unemployment (Cappellari and Jenkins 2008, Uhlendorf 2006, Stewart 2007). These observations are in line with the absence of compensating wage differentials at the firm level.

In addition, job security is partially related to observable and unobservable characteristics of the firms offering such positions. For example, larger firms tend to offer more secure jobs (Morrisette 2004, Winter-Ebmer 1995, 2001). Escape from a sequence of low-pay and no-pay spells can occur when the same worker is lucky enough to land a high-wage job (Stewart 2007). In regression analyses of linked employer-employee data Holzer, Lane and Vilhuber (2004) and Andersson, Holzer and Lane (2005) find that this higher-wage, more stable, employment appears to be concentrated in a subset of 'good' firms, i.e. firms with a high firm fixed effect. These firms are typically also larger.

We propose an equilibrium theory that is consistent with (i) the lack of compensating wage differentials, (ii) a pattern of 'unemployment scarring' through both repeated unemployment spells and lower wages, and (iii) the suggested importance of firm heterogeneity in shaping these. In our model, we follow Burdett and Mortensen (1998), henceforth referred to as BM, and introduce search frictions and on-the-job search into an otherwise competitive setting. Our sole deviation from the BM setup, is that firms differ in the job security they provide, and not, for example, in the productivity of their workers. In this framework, even though workers ask for a risk premium to stay at a risky job, fully compensating wage differentials are not offered in the labor market equilibrium. Because of a shorter expected match duration, and a higher cost of offering the worker the same life-time utility value, firms that offer only low job security have no incentive to compete with more solid firms to keep workers for the long term. It then follows that workers move only from firms that offer risky jobs to firms that offer safe jobs. The model then naturally produces an aggregate hazard rate for transitions into unemployment declining with time, whereas in the BM model the rate is counterfactually constant.

The value of more job security is low in jobs where the lifetime expected utility is only slightly higher than the value of unemployment. As a result, in these jobs, not only do the expected values of employment increase in job security, but the actual wages themselves increase in job security – a strong failure of compensating differentials. This comes about because at the bottom of the wage distribution, even with different levels of job security, different types of firms are still in considerable wage competition with each other.³ We are able

²When firm profit levels or firm exit probabilities are not controlled for, this relationship becomes even stronger, as wages and firm failure probabilities are also negatively correlated, see for example Blanchflower (1991) and Carneiro and Portugal (2006)

³Hwang, Mortensen and Reed (1998) study a labor market with on-the-job search where firms pay wages and provide job amenities. In their model the valuation of the amenity is constant and given, in our model the valuation of job security depends endogenously on the firm's wage and the entire wage distribution. Job security also differs because it is strongly complementary with wages: the lowest wages will necessarily be at the riskiest firms, independently of, e.g., the firm distribution.

to show that the increasing relationship between job security and wages extends at least to the entire tail with firms offering the lowest job security.

On the other hand, higher up in the equilibrium wage distribution, workers value job security more, and could accept wage cuts to move to safer firms, and thus the model is consistent with wage cuts in job-to-job transitions, documented for example by Postel-Vinay and Robin (2002). We show that the extent to which this happens depends also on the degree of competition on the firms' side of the market: loosely, wage cuts can occur if, increasing with job security, there is a decreasing amount of firms with a similar extent of job security competing for the same workers.⁴

By themselves, the apparent lack of compensating differentials and unemployment scarring can also be explained by worker heterogeneity and learning about match quality as well. It is possible to distinguish our mechanism from those alternatives by looking at further implications of these theories. We discuss this in detail in the last section. These different implications could then naturally be used to measure *how much* worker, match, and firm heterogeneity each contribute to the risk of job loss. At this stage, it's worth reiterating that many of the empirical papers mentioned above have attempted to control for worker heterogeneity, and still found significant scarring effects, suggesting a significant role for alternative mechanisms.

In short, our model is an equilibrium model of the wage ladder, where, endogenously, the lowest rungs of the wage ladder are especially slippery. It can explain the lack of compensating differentials, 'genuine' unemployment scarring, and the correlation with firm identities and characteristics. The model also produces a negative correlation between firm size and unemployment risk that operates through the same channel as in the BM model. The literature has hitherto largely ignored the effects of heterogeneity in the firm-specific component of job security *by itself* (and not related to imminent plant closures) in shaping patterns of low future wages and repeated job loss for those currently unemployed.⁵ If *firm* heterogeneity is a leading factor behind these patterns, there are also clear policy implications. First, as it creates persistence in bad labor market outcomes, it would likely increase the inferred risk that a typical worker faces in the labor market. Then, incorporating that the first rungs of the wage ladder are more slippery has further implications for the consumption/savings tradeoff of workers who have just become unemployed, and, *ex ante*, for employed workers who face differing risks of becoming unemployed, with more dire consequences if they do fall off the job ladder, into unemployment.⁶ Secondly, it could mean that policies attempting to diminish the heterogeneity, both in workers' initial conditions, for example by investing in education, and the conditions later in life, to make up for perceived human capital loss while unemployed, might not be as effective as has sometimes been assumed. From a negative correlation between wages and job insecurity, and more importantly, a perceived joint cause

⁴In the most straightforward case, where a finite number of types exist, and hence, some firms do not face competition from very similar types on the upside and downside, wage cuts will occur, independently of parameter values, as shown in Pinheiro and Visschers 2009.

⁵Some have argued that heterogeneity on both sides of the market, with low-quality, unstable workers sorting into unstable firms, can explain the lack of compensating differentials (Evans and Leighton 1989, for example). These arguments seem to imply that worker heterogeneity is still a necessary condition to create patterns of wages increasing in job security. This paper argues, on the contrary, that firm heterogeneity alone can generate these patterns.

⁶See Lise (2011) for a model linking the climbs of the wage ladder through on-the-job search, and the drops from the wage ladder, to consumption and savings decisions. In his model, following BM, the risk of dropping from the ladder is constant across workers and jobs.

of both of these on the worker's side, it is tempting to conclude that increasing a worker's earnings capacity would also increase his or her job security in subsequent jobs. However, this gain in job security might not be realized when its cause lies not with the worker, but –without being affected by worker-centered policies– on the firm's side.

2 Model

A measure 1 of risk-neutral firms and a measure m of risk-neutral workers live forever, and discount the future at rate r . All firms produce an identical amount of output p per worker, but differ in the probability δ with which they send workers back into unemployment. We index firms by this probability, and will refer to a high- δ firm as a “risky firm”, and a low- δ firm as a “solid firm”. The distribution function of firm types is $H(\delta)$; this distribution can contain mass points. Apart from the differences in layoff risk δ , the setup further follows Burdett and Mortensen (1998). The labor market is subject to search frictions: unemployed workers receive a single job offer at rate λ_0 , employed workers at rate λ_1 . This offer can be accepted or rejected on the spot, no recall of a rejected offer is possible. An offer is a wage w to be paid by the firm as long as the match lasts; unemployed workers receive b each period they are without a job. Firms are committed to the posted wage, which has to be the same for every worker in the firm, and are able to hire everyone who accepts their offer. They maximize steady state profits.

2.1 Worker's Problem and Risk-Equivalent Wages

Consider that firms with layoff risk δ post according to a symmetric, possibly pure, strategy with cdf $\hat{F}(w|\delta)$. We can express the value functions of workers as follows: for unemployed workers

$$rV_0 = b + \lambda_0 \iint \max\{V(w, \delta) - V_0, 0\} d\hat{F}(w|\delta) dH(\delta). \quad (1)$$

Similarly, for employed workers

$$rV(w, \delta) = w + \lambda_1 \iint \max\{V(w', \delta') - V(w, \delta), 0\} d\hat{F}(w'|\delta') dH(\delta') + \delta(V_0 - V(w, \delta)) \quad (2)$$

The optimal policy is a reservation value policy, and by the monotonicity of value function (2) in w while keeping δ fixed, and vice versa, it follows that, given a current (w, δ) there is a reservation wage function $w' = w^r(\delta'; w, \delta)$ such that each (w', δ') yields a lifetime expected value equal to the value of employment at wage w in a firm with layoff probability δ . This function can be derived directly from the above equations, given that $V(w^r(\delta'; w, \delta), \delta') = V(w, \delta)$. When taking the wage at the most solid firm as the reference wage, we can define

$$w^*(w^s, \delta) \stackrel{\text{def}}{=} w^r(\delta, w^s, \underline{\delta}) = w^s + (\delta - \underline{\delta})(V(w^s, \underline{\delta}) - V_0), \quad (3)$$

which implies that the difference between a wage at a firm with unemployment risk δ and the equally preferred wage at the most solid firm is precisely the per-period expected loss due to decreased job security in the more risky firm. In case of this indifference, we refer to the latter as solid-firm equivalent wage. Below, to distinguish solid-firm equivalent wages from actual wages paid, we superscript the former, w^s , while star the latter, w^* , when necessary.

We can find the function that links wages at firms with unemployment risk δ with their solid-firm equivalent wage as the solution to a partial differential equation.

Lemma 1. *The reservation (equivalent) wage function $w^*(w^s, \delta)$ is the solution to the following partial differential equation*

$$\frac{\partial w^*(w^s, \delta)}{\partial w^s} = \frac{r + \delta + \lambda_1(1 - \int \hat{F}(w^*(w^s, \delta')|\delta')dH(\delta'))}{r + \underline{\delta} + \lambda_1(1 - \int \hat{F}(w^*(w^s, \delta')|\delta')dH(\delta'))} \quad (4)$$

$$\frac{\partial w^*(w^s, \delta)}{\partial \delta} = \frac{w^*(w^s, \delta) - w^s}{\delta - \underline{\delta}}, \quad (5)$$

with initial conditions for every δ ,

$$w^*(R_0, \delta) = R_0, \text{ where } R_0 \text{ solves } V(R_0, \underline{\delta}) = V_0. \quad (6)$$

We have relegated all proofs to the appendix. Now, with the wage equivalence function in hand, we can construct can map all wages into their solid-firm equivalent wage, and construct its cdf.

Corollary 1: *Define $F(w^s) = \int \hat{F}(w^*(w^s, \delta')|\delta')dH(\delta')$. Then the left and right derivatives $F'_L(w^s), F'_R(w^s)$ exist a.e., and so do the second right/left derivatives of the equivalent wage function, with*

$$\frac{\partial^2 w^*}{\partial w^s \partial w^s} = \frac{\lambda_1(\delta - \underline{\delta})F'_L(w^s)}{(r + \lambda_1(1 - F(w^s)) + \underline{\delta})^2} \geq 0, \quad \frac{\partial^2 w^*}{\partial w^s \partial \delta} = \frac{1}{(r + \lambda_1(1 - F(w^s)) + \underline{\delta})} > 0, \quad (7)$$

The solution to the PDE allows us to compare wages, and construct a wage offer distribution in terms of solid-firm equivalent wages. Initial conditions (6) follow directly from (3), but are interesting in terms of economics as well: they state that at the reservation wage out of unemployment, the probability of becoming unemployed again is irrelevant.⁷ Job security is only valued above the reservation wage for the unemployed. Intuitively, if one is indifferent between being in state A or B, whether one transits from one to the other, and thus also how frequently, is irrelevant. Moreover, job security differs from e.g. standard differences in firms' productivities (as discussed in BM, or Bontemps et al. 1999) because value of job security is endogenous, and it also depends on the distribution of wages offered in equilibrium. To be indifferent at a wage above R_0 , more risky wage offers naturally should come with higher wages. However, the increase in wages required for risky firms is increasing more than proportionally with the increase in the wage offered by the solid firm, as wages get further away from the reservation wage. Put otherwise, there is an endogenous complementarity between job safety and wage levels, evidenced by the positive cross derivative in Corollary 1: safety becomes increasingly valuable at higher wages (and conversely, high wages are more valuable in safer jobs).

2.2 The Firm's Problem and Labor Market Equilibrium

The firm's profit can be split up in two components: (i) the wage it pays per worker, leaving an instantaneous per-worker profit flow $p - w^*(w^s, \delta)$, and (ii) the steady state amount of workers the firm has $l(w^s, \delta)$. Taking as given the firm's δ , there is a clear trade-off between the two components: higher wages mean less profit per

⁷See also Burdett and Mortensen 1980.

worker at a given moment – but a worker will stay longer at the firm, and moreover, more potential workers from other firms will accept offers from the firm, resulting in a larger firm size $l(w^s, \delta)$. Let $G(w^s, \delta)$ be the joint distribution of solid-firm equivalent wages and unemployment risks. Steady state dictates that the mass of workers with $w^{s'} \leq w^s$, and $\delta \leq \delta'$ is unchanging; with standard random matching (see Podczeck and Puzzello 2011), the dynamic evolution of the measure of workers can be expressed as⁸

$$\int_{\delta' \leq \delta, R_0 \leq w^{s'} \leq w^s} \left(\delta' + \lambda_1 \int_{\tilde{w}^s > w^s} dF(\tilde{w}^s, \tilde{\delta}) + \lambda_1 \int_{w^s \geq \tilde{w}^s > w^{s'}} dF(\tilde{w}^s, \tilde{\delta}) \right) dG(w^{s'}, \delta') (m - u) = \\ \int_{\delta' \leq \delta, R_0 \leq w^{s'} \leq w^s} \left(\lambda_0 u + \lambda_1 \int_{\tilde{\delta} > \delta, \tilde{w}^s < w^{s'}} dG(\tilde{w}^s, \tilde{\delta}) (m - u) \right) dF(w^{s'}, \delta') \quad (8)$$

where the LHS is the outflow consisting of (in order) the outflow to unemployment (δ'), to firms with a higher wage $\tilde{w}^s > w^s$, and to firms with different $\delta' > \delta$ that offer wages higher than the current wage, but weakly lower than w^s ; the inflow, on the RHS, comes from unemployment or from firms with $\delta' > \delta$, with lower equivalent wages. Steady state dictates that inflow equals outflow; using (8) we can derive the firm size. It follows that $G(w^s, \delta)$ is absolutely continuous with respect to $F(w^s, \delta)$: if a subset $A \in \mathbb{R}^2$ has probability $\int_A dF(w, \delta)$ equal to zero, then the LHS of (8), adapted to integrate only over the set A , equals zero; since $\delta > 0, \forall \delta$, it must be that $\int_A dG(w, \delta) = 0$ as well. Then, by the Radon-Nikodym theorem, a function $l(w^s, \delta)$ exists such that $(m - u)G(w^s, \delta) = \int_{w^s}^{w^s} \int_{\delta}^{\delta} l(\tilde{w}^s, \tilde{\delta}) dF(\tilde{w}^s, \tilde{\delta})$; Roughly, $l(w^s, \delta)$ corresponds to the measure of workers divided by the measure of firms, as both get very small, and we take this as the firm size.

Lemma 2. *The size of a firm posting a wage of which the equivalent wage is w^s , only depends on aggregate equivalent-wage distributions $F(w^s)$ and $G(w^s)$, and the firm's own δ ,*

$$l(w^s, \delta) = \frac{\lambda_0 u + \lambda_1 G^-(w^s)(m - u)}{\lambda_1 (1 - F^+(w^s)) + \delta}, \quad (9)$$

where $G^-(w^s) = \int_{w^{s'} < w^s} dG(w^{s'}, \delta')$, $F^+(w^s) = \int_{w^{s'} \leq w^s} dF(w^{s'}, \delta')$. (Note, these are integrated over the entire set of δ).

Likewise for unemployment,

$$\lambda_0 u \int_{w^s \geq R_0} dF(w^s, \delta) = \int \delta dG(w^s, \delta) \quad (10)$$

The size of a firm will be affected by both the (equivalent) wage and its own unemployment risk. This stands in contrast to BM and Bontemps et al., who allow many sources of heterogeneity on the firm and worker side, but keep the property that the firm size only depends on the wage. As a direct consequence of the dependence of firm size on (w^s, δ) , the distribution function of workers $G(w^s) = \int_{w \leq w^s} l(w, \delta) dF(w, \delta)$, depends on the distribution of $H(\delta)$ directly –it affects the inflows into unemployment–, as well as indirectly, through the equilibrium wage strategies for a given type. In particular, outflows of workers into unemployment are higher when high- δ firms dominate the lower part of the wage distribution, which makes the mass of employed workers who are willing to move to a firm with equivalent wage w^s smaller (assuming inflows from

⁸Note that, since mass can be concentrated at a single (w, δ) , we are explicit whether the boundaries are included.

unemployment are the same); this, in turn will affect wage strategies of firms. In equilibrium, we have to take these dependencies into account.

To return to the firm's optimization, a firm with layoff rate δ chooses w^s to maximize $(p - w^*(w^s, \delta))l(w^s, \delta)$. Combining lemma 1 and lemma 2, we can derive that wages will not be compensating fully for the employment risk

Proposition 1 (Ranking Property). *Suppose two firms with layoff risk δ_l, δ_h such that $\delta_l < \delta_h$ offer profit maximizing equivalent wages w_l^s and w_h^s . Then, we must have $w_l^s \geq w_h^s$.*

Proposition 1 is proved without reference to the shape of $H(\delta)$, and therefore holds also whether it is a discrete, continuous, or a mixture distribution. Intuitively, the gain of posting a higher equivalent wage is larger for more solid firms, because (i) the *increase* in the actual wage needed is lower, i.e. the marginal cost of an equivalent-wage increase is lower for the solid firm, and (ii) the increase in the steady state number of workers is higher, i.e. the marginal benefit of an equivalent-wage increase is higher. Each of the two forces by themselves would already yield the result of proposition 1. Overall, it means that safer firms have an advantage offering higher equivalent wages, and hence higher *values* of employment. However, we cannot make inferences yet about the *actual wages* posted; for this we have to study the equilibrium and its implications more deeply.

Definition 1. *The steady state equilibrium in this labor market consists of distributions $\hat{F}(w|\delta)$, $F(w^s|\delta)$, $G(w^s, \delta)$, $F(w^s), G(w^s)$; an unemployment rate u ; a reservation function $w^*(w^s, \delta)$ for employed workers, and a reservation wage R_0 for unemployed workers, such that*

1. *workers' utility maximization: acceptance decisions $w^*(w^s, \delta)$, R_0 are optimal, given $\hat{F}(w|\delta)$ and $H(\delta)$, and derived from (4)-(6)*
2. *Firms' profit maximization: given $F(w^s)$, $G(w^s)$, for each δ , $\exists \pi$ such that $\forall w^s \in \text{supp } F(w^s|\delta)$, it holds that $\pi = (p - w^*(w^s, \delta))l(w^s, \delta)$ and $\forall w^s \notin \text{supp } F(w^s|\delta)$, $\pi \geq (p - w^*(w^s, \delta))l(w^s, \delta)$*
3. *steady state distributions follow from individual decisions aggregated up: $F(w^s|\delta)$ is derived from $\hat{F}(w|\delta)$ and $H(\delta)$ using $w^*(w^s, \delta)$, while $G(w^s, \delta)$, and u follow from the steady state labor market flow accounting in (8)-(10), $F(w^s)$ and $G(w^s)$ follow from $\int \int_{w \leq w^s} d\hat{F}^s(w^s|\delta) dH(\delta)$, and $\int_{w \leq w^s} dG(w^s, \delta)$.*

Adapting the proofs in BM and Bontemps et al. to incorporate the heterogeneity in δ , we can show that $F(w^s)$ is a continuous, strictly increasing distribution function, and so is $G(w^s)$. The intuition for this, however, follows the aforementioned papers: mass points in the distribution of offered wages or intermediate intervals where no firms offer wages, allow discrete gains in firm size or profit per worker, while the costs of such deviation can be made arbitrarily small.

Proposition 2. *In equilibrium, we can derive the following about derived distribution $F(w^s)$: (i) The support of the distribution of equivalent wages w^s offered in equilibrium is a connected set, (ii) there are no mass points in $F(w^s)$, (iii) the lowest wage offered is R_0 , i.e. $F(R_0) = 0$. Properties (i)-(iii) likewise hold for $G(w^s)$ derived from $F(w^s)$ and (8).*

Combining proposition 1 and proposition 2, the conditional distribution function $\hat{F}^{-1}(\delta|w^s)$ has all probability mass concentrated at a unique δ . Conversely, if $H(\delta)$ has a continuous probability density, it also follows that each δ posts a unique equivalent wage. Neither implies that an *actual* wage w^* is offered by at most one δ -type of firm: overlaps in the actual wage distribution (with concomitant wage cuts in transitions) are possible, as we show below.

One of the strengths of the results is that, although now workers and firms are affected by two dimensions, wages and job security, and the valuation of the latter is endogenous, solving for the firm size distribution and subsequently the equilibrium wage distributions can be done as (almost) as easily and as explicitly as in BM. We turn to this now.

2.3 Equilibrium Firm Sizes

Above we have shown that equivalent wages are a sufficient statistics for workers' mobility decisions. Moreover, the ranking property tells us that in equilibrium the firm with rank z in the equivalent-wage distribution has the same unemployment risk as the firm that has rank $1 - z$ in firm-type distribution $H(\delta)$. As a result, we are able to solve for equilibrium firm sizes without reference to equilibrium wages paid, or equivalent wages, while incorporating that firms are heterogeneous in their unemployment risk; as detailed above, this affects their firm size even if the rank in the equivalent-wage distribution would be the same.

Formally, define $F(w^s) = z$; by proposition 2, we have the $F(w^s)$ is continuous and strictly increasing, $w^s(z) = F^{-1}(z)$ exists, and is unique. Also define $\delta(z)$ as the layoff risk associated with z^{th} firm, starting from the most *risky* firm. To deal with mass points in $H(\delta)$, define $\bar{H}(\delta)$ as the closed graph of $1 - H(\delta)$, then let $\delta(z) \stackrel{\text{def}}{=} \min\{\delta | \text{conv}(\bar{H}(\delta)) = z\}$.⁹ Similarly, define $G^z(z) = G(w^s(z))$. Moreover, using that $\hat{F}^{-1}(\delta|w^s)$ concentrates mass at a unique δ , and the absolute continuity of $F(w^s)$, which both follow from proposition 2, we have

$$\begin{aligned} G^z(z)(m-u) &= G(w^s)(m-u) = \int_{w^{s'} < w^s} l(w^{s'}, \delta) dF(w^{s'}, \delta) \\ &= \int_{R_0}^{w^s} \int_{\delta} \frac{\lambda_0 u + \lambda G(w^{s'})(m-u)}{\lambda_1(1-F(w^{s'})) + \delta} d\hat{F}^{-1}(\delta|w^{s'}) dF(w^{s'}) = \int_{R_0}^{w^s} \frac{\lambda_0 u + \lambda G(w^{s'})(m-u)}{\lambda_1(1-F(w^{s'})) + \delta(w^s)} dF(w^{s'}) \\ &= \int_0^z l(z') dz', \quad \text{where } l(z') = \frac{\lambda_0 u + \lambda G^z(z')(m-u)}{\lambda_1(1-z') + \delta(z')} \end{aligned} \tag{11}$$

Though the necessary substitutions above involve a surplus of notation, the intuition of the firm size is straightforward, and similar to the case of a finite number of firms: the steady state firm size is given by the ratio of the rate of worker inflows to the rate of outflows. From (11), we find that $G^z(z)$ is the solution to differential equation,

$$\frac{dG^z(z)}{dz} = \frac{\frac{\lambda_0 u}{m-u} + \lambda_1 G^z(z)}{\lambda(1-z) + \delta(z)}, \tag{12}$$

with initial condition $G^z(0) = 0$. Note that there is no reference to another equilibrium object in this formulation of the distribution but $G^z(z)$ itself. Solving for the distribution is now straightforward, and is done in the next lemma.

⁹Taking the minimum here is without loss of generality for our results, since alternative assumptions at points where the convex closure of $H(\delta)$ is an interval, would change δ only for a zero measure of firms.

Lemma 3. The cumulative density function $G^z(z)$, and equilibrium firm size $l(z)$ and measure of unemployed, are given by: $u = \frac{m \int_0^1 \delta(z)g(z)dz}{\lambda_0 + \int_0^1 \delta(z)g(z)dz} = (\lambda_0)^{-1} \int_0^1 \delta(z)l(z)dz$, and

$$G^z(z) = \frac{\lambda_0 u}{\lambda_1(m-u)} \left(e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z')+\delta(z')} dz'} - 1 \right), \quad (13)$$

$$l(z) = \frac{\lambda_0 u}{\lambda_1(1-z) + \delta(z)} e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z)+\delta(z')} dz'}. \quad (14)$$

The dependence of the size of the z th-ranked firm on the unemployment risk of all lower ranked firms is explicit in the integral term in the exponent. Solving firm size as a function of the firm rank does not only yield a clean expression for firm size, because it does not reference parameters or variables that do not affect firm size such as unemployment benefits b (which would turn up if we solved firm size as a function of wages, though changing b has no effect on firm sizes); it also allows for general probability distributions of firm types through $\delta(z)$. Concretely, this means that the formulation in lemma 2 can deal with discrete distributions as easily as it can deal continuous distributions, or any mixture of these. We expect that this approach can be applied more generally when a firm-specific factor leads to differences in firm sizes for the same rank in the workers' ranking of firms, as long as one can establish a ranking property along the lines of proposition 1.¹⁰

2.4 Equilibrium Wage Distributions

We can set up the maximization problem equivalently such that, given equilibrium equivalent-wages $w^s(z)$, no firm strictly prefers a *ranking* different than its own. Then, the problem becomes

$$\max_{z'} (p - w^s(w^s(z'), \delta(z))) l^d(z', \delta(z)), \quad (15)$$

where it follows from (9) and lemmas 2 and 3 that the firm size of a firm with δ , offering an equivalent wage $w^{s'}$ that is ranked at $z' = F(w^{s'})$, is

$$l^d(z', \delta) = \frac{\lambda_1(1-z') + \delta(z')}{\lambda_1(1-z') + \delta} l(z') = \frac{\lambda_0 u}{\lambda_1(1-z') + \delta} e^{\int_0^{z'} \frac{\lambda_1}{\lambda_1(1-\tilde{z})+\delta(\tilde{z})} d\tilde{z}}. \quad (16)$$

A firm can change the job-to-job separations by providing a higher value to the worker, but cannot change the unemployment risk. From (16), we see that $l^d(z', \delta(z))$ is differentiable in z' . The first order condition of (15)

¹⁰We can be more explicit in the case of a pure discrete and a purely continuous distribution. For a continuous probability density $h(\delta)$ with $H'(\delta) = h(\delta) > 0$, $\delta(z)$ is differentiable everywhere with $\delta'(z) > 0$, and with the appropriate change of variable , this results in

$$l(\delta) = \frac{\lambda_0 u}{\lambda_1 + \delta} e^{\int_{\delta}^{\bar{\delta}} -\left(\frac{2\lambda_1 h(\delta)+1}{\lambda_1(1-H(\delta))+\delta} \right) d\tilde{\delta}}.$$

In case of a discrete distribution $h(\delta_j)$, $j = 1, \dots, J$, with $\sum_j h(\delta_j) = 1$ and $\bar{\delta} = \delta_1 > \dots > \underline{\delta} = \delta_J$, lemma 3 tells us that the mass of workers in δ_i firms, $v(\delta_i)$ can be derived from (13), using $e^{\int_a^b \frac{\lambda_1}{\lambda_1(1-z)+\delta} dz} = \frac{\lambda_1(1-z_a)+\delta}{\lambda_1(1-z_b)+\delta}$. Suppose that $\sum_{i=1}^{j-1} h(\delta_i) < z < 1 - \sum_{i=j+1}^J h(\delta_i)$ for some j . Then from (13),

$$G(z) = \frac{\lambda_0 u}{\lambda_1(m-u)} \left(\frac{\lambda_1(1 - \sum_{i=j}^J h(\delta_i)) + \delta_j}{\lambda_1(1-z) + \delta_j} \prod_{i=1}^{j-1} \frac{\lambda_1(1 - \sum_{h=i}^J h(\delta_h)) + \delta_i}{\lambda_1(1 - \sum_{h=i+1}^J h(\delta_h)) + \delta_i} - 1 \right)$$

with respect to z' , evaluated at the equilibrium choice, $z = z'$ is

$$(p - w^*(w^s(z), \delta(z))) \frac{\partial l^d(z, \delta)}{\partial z} \Big|_{\delta=\delta(z)} - \left(\frac{\partial w^*(w^s(z), \delta(z))}{\partial w^s(z)} \frac{dw^s(z)}{dz} \right) l^d(z, \delta(z)) = 0 \quad (17)$$

This can be rewritten as

$$\frac{\frac{\partial w^*(w^s(z), \delta(z))}{\partial w^s(z)} \frac{dw^s(z)}{dz}}{(p - w^*(w^s(z), \delta(z)))} = \frac{\frac{\partial l^d(z', \delta)}{\partial z'} \Big|_{\delta=\delta(z')}}{l(z, \delta(z))} \quad (18)$$

Safer firms gain more, relatively and absolutely, when they improve their position in the firm ranking: $\frac{\partial^2 \ln l(z', \delta)}{\partial z' \partial \delta} = -\frac{2\lambda_1}{(\lambda_1(1-z')+\delta)^2}$. Then, *ceteris paribus*, safer firms have a higher term on the RHS of (18) and thus will compete more heavily; on the LHS of (18), this force pushes equivalent wages $w^s(z)$ further upwards with z .

The change of wage actually paid with the firm rank is

$$\frac{dw^*(w^s(z), \delta(z))}{dz} = \frac{\partial w^*(w^s(z), d(z))}{\partial w^s(z)} \frac{dw^s(z)}{dz} + \frac{\partial w^*(w^s(z), d(z))}{\partial \delta(z)} \delta'(z). \quad (19)$$

where we used that the function $\delta(z)$ is differentiable a.e. and everywhere right-differentiable; with abuse of notation $\delta'(z)$ is the associated right-derivative. Then, we can decompose this wage change into two parts: the competition component discussed above in (18), $\frac{\partial w^*(w^s(z), \delta(z))}{\partial w^s(z)} \frac{dw^s(z)}{dz}$, and the effect through the composition of firms on the labor market; with $\delta'(z)$ derived from $H(\delta)$, capturing how fast job security increases with firm rank. Substituting (18) into the last expression, yields

$$\frac{dw^*(z)}{dz} = (p - w^*(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)(V(w^*(z)), \delta(z) - V_0). \quad (20)$$

While the firm's optimization pins down $\frac{dw^s(z)}{dz}$, increased job security of the higher ranked firm itself could deliver part of the increased w^s (for a given wage paid). If workers value job security a lot, i.e. $V(w^*(z), \delta(z)) - V_0$ is high, or whenever, the firm's job security is much higher, i.e. $\delta'(z) < 0$ and large in absolute value, this force can be strong.

The strength of these forces varies with the rank of the firm in the firm-level job-security distribution. For firms high in the the distribution, the value of employment is significantly different from the value of unemployment; this means that the term $V(w^*(z), \delta(z)) - V_0$ is relatively large. However, simultaneously, for these safer firms, the gains of holding on to their workers are larger, and hence these firms will compete more fiercely, as argued below equation (18). It is therefore not a foregone conclusion whether wages paid will rise or fall with firm-level job security, nor is it immediate that a greater valuation of job security on the worker side will indeed lead to wage cuts, as it simultaneously also raises the value of retaining a worker to the firm. One can see that $\delta'(z)$ can potentially play an important role here, scaling $V(w^*(z), \delta(z)) - V_0$, and therefore the strength of the forces. In the next section, we study the occurrence or absence of wage cuts in exchange for job security: both can occur but depend in part on the distribution of firm types, which is an important determinant of the extent of competition among firms. The workers' marginal rate of substitution between wages and job security (derived in section 2.1) by itself is only half of the story; firms' imperfect competition is the other half.

Let us finish this section by putting all pieces together: we can find $(w^*(z), w^s(z))$ as the solution a system of *two* differential equations, one from using (17) which tells us $\frac{\partial w^s(z)}{\partial z}$, and (20) combined with (3),

which tells us $\frac{\partial w^*(z)}{\partial z}$, both as functions of parameters, distributions and wages $w^s(z), w^*(z)$. The solution $\{w^s(z), w^*(z)\}$ fully characterizes the equilibrium. Moreover, we are able to establish the existence and uniqueness of this equilibrium.

Theorem 1 (Existence, Uniqueness, Characterization). *Consider functions $\{w^*(z), w^s(z)\}$, and $R_0 \in \mathbb{R}$, such that $w^*(z), w^s(z)$ are a solution to the system of two ODEs, with, for all z at which $\delta(z)$ is continuous,*

$$\frac{dw^s(z)}{dz} = (p - w^*(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{r + \underline{\delta} + \lambda_1(1-z)}{r + \delta(z) + \lambda_1(1-z)} \quad (21)$$

$$\frac{dw^*(z)}{dz} = (p - w^*(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \frac{\delta'(z)}{(\delta(z) - \underline{\delta})} (w^*(z) - w^s(z)), \quad (22)$$

and a jump discontinuity at every \check{z} such that $\lim_{z \uparrow \check{z}} \delta(z) > \delta(\check{z})$, such that $w^*(z)$ will jump according to

$$w^*(\check{z}) = \frac{\delta(\check{z}) - \underline{\delta}}{\lim_{z \uparrow \check{z}} \delta(z) - \underline{\delta}} \left(\lim_{z \uparrow \check{z}} w^*(z) - w^s(\check{z}) \right) + w^s(\check{z}), \quad (23)$$

under initial conditions $w^s(0) = w^*(z) = R_0$, where R_0 additionally satisfies

$$R_0 = b + (\lambda_0 - \lambda_1) \int_0^1 \frac{1-z}{r + \lambda_1(1-z) + \underline{\delta}} \frac{dw^s(z)}{dz} dz \quad (24)$$

Denote the inverse of $w^s(z)$ at a given ω^s by $(w^s)^{-1}(\omega^s)$. The distribution functions $F(\omega^s) = (w^s)^{-1}(\omega^s)$, $G(\omega^s) = G^z((w^s)^{-1}(\omega^s))$, reservation function $w^*(w^s, \delta)$, and $u, \hat{F}(w|\delta), G(w^s, \delta), F(w^s|\delta)$, all constructed from $\{w^s(z), w^*(z), R_0\}$ are the functions associated with the steady state equilibrium in the environment; if $0 \leq \lambda_1 \leq \lambda_0$, this steady state is unique.

In this setting it is necessary to follow a path different from BM and Bontemps et al. towards characterizing the equilibrium wage distribution: neither wages or values (which, in our setting, maps one-to-one to equivalent wages) alone are sufficient to characterize the equilibrium. How powerful competition is driving up the values offered to workers depends on wages through the instantaneous profit flows $p - w^*$, and it depends on the job security of the firm in question. On the other hand, how wages comove with job security depends on the valuation of job security, which consist of the job value (or equivalent wages) lost when losing a job, and how likely this transition is. Crucially, not all elements of equilibrium depend on wages and equivalent wages: firm size only depends on the rank of the firm and its associated job security. Exploiting this, we are able to solve for equilibrium firm sizes first, and then simultaneously find the wages and equivalent wage distributions that have to arise with these firm size.

It is perhaps insightful to compare the case with heterogeneous unemployment risk to the standard case in BM and Bontemps et al. without this heterogeneity ($\delta(z) = \tilde{\delta} \forall z$). In the absence of heterogeneity in $\delta(z)$, the two equations (21) and (22) become identical to each other: the differential equation $w'(z) = (p - w(z))(2\lambda_1/(\lambda_1(1-z) + \delta))$ with initial condition $w(0) = R_0$ has solution

$$\frac{p - w(z)}{p - R_0} = \left(\frac{\lambda_1(1-z) + \delta}{\lambda_1 + \delta} \right)^2 \implies F(w) = \frac{\delta + \lambda_1}{\lambda_1} \left(1 - \left(\frac{p - w}{p - R_0} \right)^{0.5} \right), \quad (25)$$

which, on the LHS, is precisely the wage distribution in BM.¹¹

Note that one way of showing existence and uniqueness in Burdett-Mortensen model would combine the left expression in (25) with (24), to show that $T(R_0)$ is linear in R_0 , while $T(p) = b$, and $T(r) < r$, for r small enough. The proof of theorem 1 relies on the same method: we can rescale the differential equation (22) by $p - w^*$, and show that the term $\frac{w^*(z) - w^s(z)}{p - w^*(z)}$ is independent of R_0 , and therefore just a function of parameters, unemployment risk distribution $H(\delta)$ and firm rank z . Then, one can show that a term $A(z)$ exists and again depends only on parameters, $H(\delta)$, and firm rank, such that $p - w^*(z) = (p - R_0)A(z)$, which establishes uniqueness, and the continuity needed for the existence proof.

3 Wages and Transition Hazards

In the previous section, we derived equations which characterized how wages, worker's values, and firm qualities are linked in equilibrium. In this section, we look more concretely at the labor market outcomes implied by the characterization.

First, since safer jobs are more attractive jobs, workers in safe jobs are much less likely to separate from these jobs, whether to unemployment or to another job. This implies the following (where we have, once again, relegated all proofs to the appendix),

Result 1. *The transition rate into unemployment as a function of tenure is decreasing in tenure.*

To condense language, we will refer to this particular transition rate as the unemployment hazard. Thus, the standard BM model, augmented with firm heterogeneity in unemployment risk, is able to reproduce an unemployment hazard that in the aggregate declines with tenure (as well as with time spent in employment), as it does in the data.¹²

Next, we consider the relationship between the unemployment risk a worker faces, and the wage he receives. If wages are increasing in the job security that the firm offers, there is in some sense a *strong failure* of compensating wage differentials: not only do riskier firms offer lower employment values (established in proposition 1), but in fact they offer values so much lower that in addition to a higher unemployment risk they actually pay lower wages. For those jobs at the bottom of the wage distribution, we can derive the following, without restrictions on parameters or the firm distribution.

Result 2. *The lowest wage, R_0 , is paid by the firm with the highest unemployment risk. There exists a nontrivial interval of wages $[R_0, \hat{w}]$ where job security increases with wages*

Under typical conditions, spelled out next, this interval can span a large part of the wage distribution, while on the other hand, wage cuts can also occur higher up in the wage distribution. For analytic simplicity and to be consistent with steady state profit maximization, we let $r \rightarrow 0$, and consider mainly the case of a distribution of firm unemployment risk with a differentiable pdf $h(\delta)$.

¹¹Exploiting the ranking property inherent in BM-type models allows one to incorporate more heterogeneity in standard models. Here, we deal with firm heterogeneity that cannot be incorporated straightforwardly in the standard model, where e.g. there is a simple, unique one-dimensional mapping between wages and worker's values. Moscarini and Postel-Vinay (2010) exploit a similar ranking property to deal with time-varying aggregate productivity, an otherwise notoriously difficult problem.

¹²See e.g. Menzio et al. 2012.

Result 3. *In equilibrium the relation between wages and job security depends on the firm distribution of unemployment risk in the following way:*

1. If $h'(\delta) \leq 0$, wages increase with job security (i.e. $\frac{dw^*}{dz} > 0$ at \tilde{z} such that $\tilde{z} = 1 - H(\delta)$.)
2. Wage cuts for increased job security will occur if

$$\frac{h(\delta)}{\delta + \lambda(1 - H(\delta))} < \int_{\delta}^{\tilde{\delta}} \frac{\frac{\delta}{\tilde{\delta}} + \lambda_1(1 - H(\tilde{\delta}))}{(\frac{\delta}{\tilde{\delta}} + \lambda(1 - H(\tilde{\delta})))^3} h(\tilde{\delta}) d\tilde{\delta} \quad (26)$$

As an example where this can arise, consider densities $h(\delta)$ that have a thin left-tail with sufficient kurtosis, where $\frac{h(\delta)}{\delta}$ rises sufficiently fast in the left tail. In particular, for any distribution with $\frac{h(\delta)}{\delta} < \int_{\delta}^{\tilde{\delta}} \frac{\frac{\delta}{\tilde{\delta}} + \lambda_1(1 - H(\tilde{\delta}))}{(\frac{\delta}{\tilde{\delta}} + \lambda(1 - H(\tilde{\delta})))^3} h(\tilde{\delta}) d\tilde{\delta}$, wage cuts will occur for λ_1 small enough. In case of a discrete distribution, the existence of wage cuts will follow directly from (23)¹³; however, intuitively, this relates closely to the case where $h(\delta) = 0$ on an interval, which also leads to wage cuts according to (26).

In figure 1, we have drawn wages as a function of underlying unemployment risk as an example, for a particular set of parameters.¹⁴ Note that the firm distribution with almost completely decreasing density does not generate any wage cuts, but for the other two distributions, with the clear left tails, wage cuts occur when moving to the safest firms. Since climbing up the ladder occurs in increasingly smaller steps, and the steady state mass of workers is distributed heavily towards the safest firms, this means that a significant amount of workers will be taking wage cuts. For example, in a job with an unemployment risk near or below 2%, any subsequent job-to-job move will come with a wage cut, in case of the dashed distribution (which is a log-normal with standard deviation 0.03). On the other end, the lowest wages come with significantly higher unemployment risk. At these wages, who are taken in relatively large proportion by the unemployed, we see a complete absence of compensating wage differentials.

If the distribution of firm types is uniform, the (increasing) competition between firms of similar types drives up the wages, even though workers value job security increasingly. More generally, this result implies that there cannot be any wage cuts where the density is falling in δ . Concretely, for any unimodal distribution, wages will be increasing with job security at least until the modal firm. Wage cuts can occur in the left tail when safer firms become increasingly rare; those firms do not face as much direct competition from similar firms, and as a result can post wages that keep the worker closer to their indifference with respect to the job security of lower ranked firms. In general, the complementarity between wages and job security works both on the firm's and worker's sides, and can push actual wages either up or down, depending on the distribution. We think that this is a nice illustration of the value of studying unemployment risk and wage setting in a full-fledged labor market equilibrium setting, as the willingness to take wage cuts for safety for workers could be offset by the increased competition by firms for now longer valuable workers. Finally, note that the presence of wage cuts to transition to a more secure job means that, controlling for wages, the transition probability (to other firms, to unemployment, and therefore also the general separation hazard) decreases in tenure.

The degree to which firms are in competition with each other is linked to parameter λ_1 : an increase in this parameter makes it easier for higher ranked firms to poach workers from lower-ranked firms, and thus raises

¹³We discussed this type of wage cuts extensively in a previous version of the paper.

¹⁴Note that the proof of theorem 1 establishes that the shape of the function that links wages to unemployment risk does not depend on R_0 , b , λ_0 .

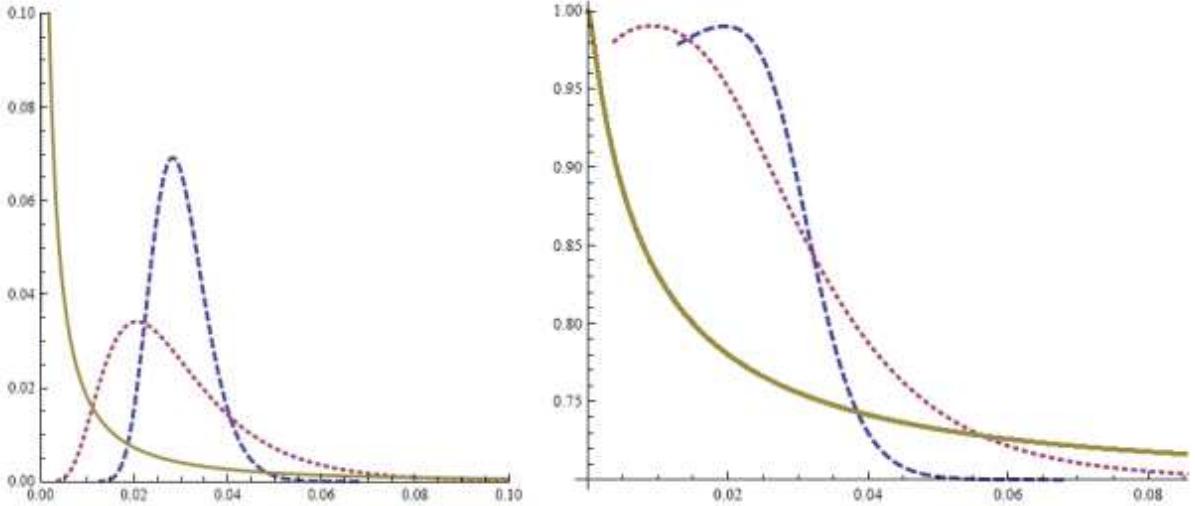


Figure 1: Left panel: log-normal distributions of δ , with standard deviation 0.03 (dashed), 0.2 (dotted), 2.0 (solid). Right panel: wages as a function of unemployment risk ($\lambda_1 = 0.23, r = 0.0025, R_0 = 0.7, p = 1$)

firm competition. One could think that an increase in competition among firms will lead firms to offer less dispersed employment values in equilibrium and hence trace out more closely the workers' marginal rate of substitution between job security and wages. This turns out not to occur; instead we can find a lower bound on $\tilde{\lambda}_1$ for a given distribution $H(\delta)$ (with $h'(\delta)/(h(\delta))^2$ bounded from above), such that above this $\tilde{\lambda}_1$, for any λ_0 and b , wages will be increasing in job security throughout.

Result 4. *If $\lambda_1 > h'(\delta)/(h(\delta))^2$, wages will be increasing in job security at δ ($dw^*(z)/dz > 0$ at $z = 1 - H(\delta)$), for any λ_0, b . If $h'(\delta)/(h(\delta))^2$ is bounded from above, there exists $\tilde{\lambda}_1$ such that for all $\lambda > \tilde{\lambda}_1$ wages are increasing in job security for all δ , for any b, λ_0 .*

Thus, as the labor market gets more competitive, the scope for wage cuts disappears. In this result, we keep b and λ_0 constant¹⁵; while it becomes progressively easier for employed workers to move from job to job, unemployed workers keep leaving unemployment at the same rate. This keeps the cost of losing one's job bounded away from zero, even as λ_1 becomes very large. (In the limit: $V_1(\underline{\delta}) - V_0 = \frac{p-b}{r+\lambda+\underline{\delta}}$.) Thus when λ_1 becomes large enough, the increased competition between firms will drive up wages with job security throughout the *entire* wage distribution, even though workers keep experiencing a loss of lifetime utility when becoming unemployed. Increased competition among firms does not lead to the payment of compensating wage differentials, it does, quite surprisingly, lead to the opposite, as it strengthens the motive of the low- δ firms to compete with similar firms. To prove this, we heavily rely on the ranking property, which holds for every λ_1 , and thus the firm ranking is preserved throughout any limit taking with respect to λ_1 (and λ_0). Thus, we can calculate firm sizes easily as a function of the rank of the firm as we approach the limit without having to recalculate the wage distribution. In turn, firm profit maximizing decisions are then still easily characterized, following theorem 1, even as we move towards the limit, $\lambda_1 \rightarrow \infty$.

¹⁵Result 4 is actually stronger, it says that this bound on λ_1 will hold, entirely independent of λ_0 and b .

We can also study the case where we let search frictions for both unemployed and employed workers disappear in the limit.

Result 5. Let $\lambda_0 > \lambda_1$, $\lambda_1 \rightarrow \infty$, $\lambda_0 \rightarrow \infty$, while keeping $\frac{\lambda_1}{\lambda_0} = \alpha < 1$ constant. Then $w^*(z) \rightarrow p$ for all z .

If we let the frictions for the unemployed disappear, we converge to the same limit as in the standard BM model without heterogeneity in δ , which equals competitive outcome $w(z) = p \forall z$, again without any compensation for unemployment risk. To see this, note that for reservation wage out of unemployment, R_0 , the following holds

$$\frac{R_0 - b}{p - R_0} = (\alpha - 1) \int_0^1 \frac{\lambda_1(1-z)}{\underline{\delta} + \lambda(1-z)} \frac{\underline{\delta} + \lambda_1(1-z)}{\delta(z) + \lambda_1(1-z)} \frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \frac{p - w^*(z)}{p - R_0} dz \quad (27)$$

$$\geq (\alpha - 1) \int_0^1 \frac{2\lambda_1^2(1-z)}{(\bar{\delta} + \lambda(1-z))^2} dz = -2 \frac{\lambda_1}{\lambda_1 + \bar{\delta}} + 2 \log \frac{\bar{\delta} + \lambda_1}{\bar{\delta}} \quad (28)$$

As we let λ_1 go to infinity in (27), it follows that $R_0 \rightarrow p$, as the RHS goes to infinity. Since $p > w^*(z) \geq R_0$, it follows that all wages go to p . Since the bound in result 4 is uniform in λ_0 , we also know that for λ_1 large enough, wages will become increasing in job security for all δ in the process.

As we approach the competitive limit no compensating wages are paid, and all firms are still active. In the case where search frictions also disappear in the limit for the unemployed, job security will cease to be a payoff relevant dimension for workers. This is intuitive because, apart from the loss of ‘search capital’, there is no additional cost to unemployment.¹⁶ Decreasing λ_1, λ_0 means that search frictions become more important, which implies that job security becomes more important, and competition between firms becomes more limited, which raises the potential for wage cuts. Thus somewhat ironically, wage cuts, which seem to relate closely to the notion of compensating wages paid in competitive settings, are in the environment we study actually associated with a low degree of competition among firms. Though ironic, the result is intuitive: a low λ_1 means that climbing up the job ladder is a slow process in which gains are lost when becoming unemployed; therefore, at a lower λ_1 , workers will value job security more, *ceteris paribus*. Likewise, a lower λ_1 lowers the competition among firms, i.e. the relative gains of being higher in the wage ranking are lower when λ_1 is low, hence higher ranked firms will not increase the values (equivalent wages) that they offer workers as much. This, however, does not mean that result 4 immediately follows from the intuition: we need to use, explicitly, the equilibrium characterization, because the lower values offered by the firms due to the lower competition reduce the valuation of job security, potentially more than offsetting the direct (*ceteris paribus*) effect of the decrease in λ_1 on the workers’ valuation of job security. However, result 4 implies that this is not the case, and with a lower λ_1 more cases can occur where firms with higher job security promise a lower equivalent wage increase than is delivered by their increased job security alone, and as a result will offer lower wages, thus leading to wage cuts in equilibrium.

¹⁶If a transition into unemployment comes with an explicit cost instead or in addition to a search cost, then in the limiting economy only the low turnover firms would survive.

4 Discussion

Above we have showed that heterogeneity on the firm side alone can explain, in frictional labor markets, the absence of compensating wages for unemployment risk and the persistence of low wages and unemployment spells for workers. The same observations are typically explained by (additional) heterogeneity elsewhere: perhaps most prominently, persistent differences in worker qualities, or alternatively, differences in the match qualities which need to be to some extent unknown at the start of the match. These different dimensions of heterogeneity have further empirical implications that can be used to gauge their importance. Also, concretely, they imply differences in the risks workers face in the labor market. From this follow different implications for welfare in general, and policy effectiveness in particular. While these other dimensions are doubtlessly important, we argue below that a potentially large role is left open for heterogeneity in the firm-level component of job security, and we discuss how one can distinguish the latter from other heterogeneity.

Persistent heterogeneity in worker quality In this line of explanation, we don't see compensating wages for job security for seemingly identical workers because the workers in low-paid, insecure jobs are in fact different from those in better-paid, secure jobs. Workers' low (unobservable) ability, however, does not in itself imply that their job durations have to be short: it is necessary to have, additionally, informational frictions or another dimension of heterogeneity. For example, the correlation between wages and job security could occur because low-ability workers are also unstable workers: they *prefer* not to stay with the same employer for long (Salop and Salop 1976). Alternatively, their skills are less job specific, making them more mobile (Neal 1998), or they are repeatedly screened out during a lower-wage probationary period (Wang and Weiss 1998). Finally, to incorporate the correlation between firm characteristics with wages and job security, one can introduce firm heterogeneity such that low-ability workers sort into the relevant subset of firms. Employment in these firms is insecure either because of aforementioned explanations, or because these firms are themselves unstable (as e.g. proposed in Evans and Leighton 1989).¹⁷

If differences in persistent worker quality are indeed behind the aforementioned patterns on the labor market, every piece of a worker's labor market history will be informative about his underlying quality, including the unobservable component. On the contrary, if behind these patterns is heterogeneity non-specific to the worker, in firm or match quality, the labor market history is only relevant to the extent that it is correlated with the current firm or match quality. In this extreme case, unemployed workers –who are not in a match with a firm– are all alike, and the labor market history before the unemployment spell, or even the previous duration of the current unemployment spell, becomes irrelevant. At the same time, since being unemployed can imply that subsequent employment occurs in a lower quality match or firm, being unemployed can still predict worse future labor market outcomes, compared to those who remained employed. Suggestive of the importance of firm or match heterogeneity rather than worker heterogeneity is the irrelevance of previous history is the ex-

¹⁷To study this setting theoretically, one could, for example, extend the model of Albrecht and Vroman (2002) with on-the-job search and increased match-breakup rates for the low productivity firms. Since this model would incorporate both firm and worker heterogeneity, it would be a good tool to measure the contribution of firm and worker heterogeneity, and their interaction. Kaas and Carrillo-Tudela (2011) show that in a setting with on-the-job search, and persistent worker heterogeneity, initially unobservable for the firm, heterogeneity in the wage strategies can give rise to low wage employment and repeated unemployment spells of low ability workers.

perience of workers who lost their job in a mass layoff, and can expect lower wages and a higher likelihood of repeated spells of unemployment, even when their previous labor market history was spotless (see, e.g. Stevens 1997)¹⁸ Likewise, we could test whether workers with ‘bad’ labor market histories can make these irrelevant by finding employment in good matches or good firms. Stewart (2007) argues that this is indeed an important feature of the labor market, while using linked employer-employee data Holzer, Lane and Vilhuber (2005) argue that good employment that allows for an escape from low earnings, is concentrated in ‘good’ firms.

Thus, there are ways to econometrically identify whether diminished labor market outcomes after an unemployment spell are “due to a causal effect of being unemployed (or working) or a manifestation of a stable trait?” (Heckman 2001). Further, it is also possible (though not necessarily easy) to distinguish the causal effect of unemployment per se, from the effect of unemployment duration, which could pick up skill depreciation or a worsening perception of unobserved quality. Controlling for observed and unobserved heterogeneity among workers along these lines, Arulampalam et. al. 2000 find that unemployment status per se generate a higher probability of future unemployment spells. (They find that it explains 40% of the persistence in employment for mature men). Böheim and Taylor (2002) find that it is unemployment incidence, rather than duration that has the major impact on future labor market outcomes. Recently, attention has also turned to identifying how future labor market outcomes are affected by being in a low-paid job – while similarly attempting to control for selection on unobserved quality into low-paid jobs. Although data limitations often allow only a coarse categorization of jobs as low- or high-paid, Cappellari and Jenkins (2008), and Uhlendorf (2006) find that being in a low-paid job itself increases the probability of becoming unemployed relative to the same person being in a high-paid job, thus showing once a worker is in unemployment, there is a risk of a low-pay/no-pay cycle. All this is suggestive that there is a role to play for heterogeneity on the firm side, as argued in this paper.

Match Heterogeneity When match quality cannot be observed prior to forming the match, but is instead slowly learned as time in the job increases, the unemployment hazard that results is typically initially increasing, then decreasing. Also, wages and job security will be positively correlated, reflecting the weeding out of the bad matches, and as such is consistent with the lack of compensating differentials. However, in this theory, in its most basic form, there is no role for firms. After controlling for a worker’s tenure, there should be no correlation of the unemployment hazard and firm characteristics. However, firm characteristics typically do correlate significantly with the unemployment hazard after controlling for tenure (e.g. Winter-Ebmer 2001). Learning is linked to the variation of the unemployment hazard with tenure, *ceteris paribus*, while our paper concerns itself with the variation in average unemployment risk across firms. It would be very interesting to decompose the variation in unemployment hazard into these across-tenure and across-firm variation.¹⁹

¹⁸Plant closing arguably reduces the selection problem among workers, once it sends all workers, good or bad, to unemployment. Often it is argued that still, there is some selection issues as the workers that stay around in the firm until the final moments might not be an unbiased sample. However, such problem can be address by including all workers who left in the years before the closure, as well as by constructing the right control group, as e.g. in Eliason and Storrie 2006, where wage and repeat unemployment effects are still found. See also Hijzen, Upward and Wright (2008).

¹⁹It would be even better if we could use matched employer-employee data and decompose the variation into worker-specific, firm-specific and tenure-specific variation. Arai and Heyman (2001) using Swedish matched employer-employee data, is the only

Looking at wages, we are further able to distinguish between *ex ante* known firm-specific job security and learning. For learning theory, maintaining the standard assumption that wages are tied to expected productivity (see Moscarini 2005 for a discussion), the separation hazard goes up with tenure when one controls for wages, while in our theory it decreases with tenure. Behind this lies a different impact of uncertainty: in the case of learning, among two matches with the same wage and thus same expected productivity, the most uncertain match is preferred. The reason is that the option to move out of the match insures the worker against bad realizations, diminishing the impact of the increased downside risk, while the increase in the upside risk is more fully enjoyed by the worker. In the empirically relevant setting with firm-specific job security, an increase in uncertainty is a decrease in job security, which lowers the value of a match for a given wage.

Focussing on wage cuts, we can likewise distinguish between the two theories. In the theory of learning about match quality, workers can move to a new match with a wage cut when the new match has a lower expected match quality but at the same time sufficiently more dispersion of possible outcomes, and hence more upside potential. Then, conditional on match survival, one would observe higher wage growth after a wage cut. This motive also arises elsewhere, in a different set of models where employed workers meet other firms occasionally, but upon such a meeting a bidding war for the worker in question is triggered (Postel-Vinay and Robin 2002, e.g.). In this setting, a worker is willing to take a lower initial wages in firms that will improve his wage prospects for the future: either because the current firm will pay more later to retain the worker, when it has to compete for the worker with another firm, or because the worker is able to secure a higher starting value when it is poached by another firm. In this case and in the case of the learning model, wage cuts are an investment in future wage growth. In our model, a wage cut is an investment in job security, and as such will not show up in eventually increased wages, but in terms of increased job durations.

Connolly and Gottschalk (2008) investigate the empirical importance of this motive for wage cuts and find that for males only 20% of the transitions with immediate wage cuts can be rationalized by future wage growth, thus suggesting that other factors, including job security, could be a significant driving force for wage cuts. Likewise, in Postel-Vinay and Robin (2002), the option-value motive can explain a part of the extent of downward wage mobility, but it also leaves open a significant part for other explanations.²⁰

Heterogeneity and risk in the labor market The amount of uncertainty a worker faces depends on the relevant dimensions of heterogeneity that underlie different outcomes. In the case of worker heterogeneity, uncertainty has resolved mostly before the labor market: workers have their own job ladders, some more slippery than others. In the case of match heterogeneity, every transition involves a temporary increase in uncertainty when match quality is initially unknown. This means that (in the absence of firm-specific policies that reduce this uncertainty before the match is formed), transitions *at any point* in the wage ladder carry risk – though this risk might take the form of wage risk or unemployment risk, depending how good the match

paper we know that does this; they find that tenure, worker fixed effects and firm fixed effects all turn up as significant correlates of unemployment risk.

²⁰The wages generated by the simulations from the estimated model in Postel-Vinay and Robin (2002) imply a constant or lower wage (i.e. wage cut) after the first job-to-job transition in 24%-38% of the cases, and a wage cut of five or more percent in 28.5%-42.9% of the cases. In the data on which the model is estimated, this percentage varies from 32.1%-54.5%, respectively 4.4%-22.9% for a five or more percent wage cut. These ranges are the values among the seven occupational categories on which the model is estimated separately.

looked from an *ex ante* perspective (allowing for *ex ante* signals). Every step on the job ladder is, in some sense, taken blindly, and the worker might end up higher or lower than expected. A worker who, after learning the match quality, stays put, is not subjecting himself to this uncertainty anymore. On the contrary, in this paper, job transitions progressively decrease the risk of job loss, and the higher the worker is on the job ladder, the more stable he stands. A fall, however, results in a new start at the bottom of the wage ladder. Moreover, contrary to the case of learning, there is no added uncertainty associated with taking a step, but rather a gain of stability.

All of the above three dimensions of heterogeneity imply that workers differ in their expectations of income and in the extent of uncertainty about this income, but each dimension has very different implications for endogenous responses of workers, such as the degree of self-insurance. Workers might select a different amount or type of human capital to avoid ending up with characteristics that lead to insecure, low-wage employment. In the case of match uncertainty, risk-averse workers might save more before undertaking a risky job change, or stay with a current match to avoid being subject to additional uncertainty, something becoming more important when e.g. close to a borrowing constraint.²¹ When jobs out of unemployment are likely at more insecure firms, workers can improve their income security by *moving* to a more stable firm (as opposed to staying in a match). Foreseeing that loss of employment would raise the risk of repeat unemployment spells, employed workers might self-insure more, but as the unemployment probability at the top of the wage ladder is low while the climb back is hard, they might, *ex post*, still suffer a larger loss of utility upon becoming unemployed than e.g. when the unemployment risk is more evenly spread across firms.

These considerations also have implications for the public policies to allow further insurance against adverse income and employment shocks. If employment security is in part determined at the firm level, then worker training might not be as effective as thought in mitigating a seeming ‘unemployment scar’, as trained workers might be hired by the same firms at the bottom of the firm distribution. If firm-specific unemployment risk and wages are negatively correlated, workers have more dispersed lifetime discounted utilities than those based on wages combined with the same, average, unemployment risk counterfactually assigned to all matches. Then, unemployment benefits could conceivably be more beneficial than in standard settings, because the expected discounted lifetime income loss upon becoming unemployed is larger when subsequent jobs are more likely to end in unemployment.²²

All in all, firm-level heterogeneity in unemployment risk can provide an explanation for worker-flow and wage patterns that were the focus of this paper, but moreover, it has implications that differ from theories based on match or worker heterogeneity. On one hand, this means that one can distinguish between these dimensions of heterogeneity and measure their importance, and on the other hand, that care has to be taken that policies are optimal given the measured importance of each dimension.²³

²¹These different responses imply that one could use savings and consumption data to further distinguish between the different dimensions of uncertainty.

²²Moreover, the moral hazard implications of unemployment benefits might be different when separations into unemployment are driven in part by firm characteristics (and firms’ choices), rather than workers’ choices, though this also necessitates an appropriate contracting friction that drives a wedge between the two.

²³Another example where allowing firm-heterogeneity, in addition to match heterogeneity, can lead to different conclusions, is Kahn (2008). She finds that those hired in recessions have lower job durations, but that this is driven by the selection of firms that hire in recessions, rather than an on average worse quality of the matches formed in a recession.

Conclusion

In this paper, we have presented a model with homogeneous workers and search frictions in which, in equilibrium, wages do not compensate for differences in unemployment risk. Therefore, workers move, whenever they have the chance, from risky companies to more stable firms, which then are also larger. We show that wages increase with job security for the lowest wages; this pattern can extend over a significant part of the wage distribution. While safer firms can offer lower wages while still attracting more workers, the increased job duration makes a worker more valuable to the firm, and hence raises their incentive to prevent worker mobility to other firms, which puts an upwards force on wages. The second force dominates at low wages, but higher in the wage distribution, depending on the distribution of the heterogeneous firms, the first force can dominate – thus leading to wage cuts.

The model also generates an unemployment hazard rate that is declining with tenure, as in the data, while in the standard Burdett and Mortensen (1998) model it is counterfactually constant. We thus show that unemployment scarring in terms of wages *and* risk of repeated job losses, arises in equilibrium, resulting neither from a decline in (perceived) productivity of workers when they become unemployed, nor as a manifestation of a selection effect on workers, but because of heterogeneity on the firm side.

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APPENDIX

A Proofs

Proof of lemma 1 The initial conditions (6) follow directly from putting $V(w, \delta) - V_0 = 0$ in (3). For (4), note first that it can be established straightforwardly that $V(w, \delta)$ is continuous, increasing and bounded in w , and therefore differentiable a.e., and therefore it follows that the same properties also hold for $w^*(w^s, \delta)$ by (3). The value of employment at $(w, \underline{\delta})$ satisfies

$$(r + \underline{\delta})V(w^s, \underline{\delta}) = w^s + \underline{\delta}V_0 + \lambda_1 \int \int_{w^*(w^s, \delta)} V(w', \delta'^s, \underline{\delta}) d\hat{F}(w'|\delta') dH(\delta'). \quad (29)$$

$$\implies \frac{\partial V(w^s, \underline{\delta})}{\partial w^s} = \frac{1}{r + \underline{\delta} + \lambda_1 \int \int_{w^*(w^s, \delta)}^{\bar{w}} d\hat{F}(w'|\delta') dH(\delta')}, \text{ a.e.} \quad (30)$$

where we have used that the derivative with respect to lower bound of the inner integral, $w^*(w^s, \delta)$, equals zero for every δ , since it is evaluated where $V(w^*(w^s, \delta), \delta) = V(w^s, \underline{\delta})$. Using (3) and (29), the latter both evaluated at $V(w, \delta)$ and $V_0 = V(R_0, \delta)$, we have

$$\begin{aligned} w^*(w^s, \delta) &= w^s + (\delta - \underline{\delta}) \left((r + \underline{\delta})^{-1} \left(w^s - R_0 - \lambda_1 \left(\int \int_{R_0}^{w^*(w^s, \delta')} V(w', \delta') d\hat{F}(w'|\delta') dH(\delta') \right. \right. \right. \\ &\quad \left. \left. \left. + \int \int_{w^*(w^s, \delta')}^{\bar{w}} V(w^s, \underline{\delta}) d\hat{F}(w'|\delta') dH(\delta') - \int \int_{R_0}^{\bar{w}} V_0 d\hat{F}(w'|\delta') dH(\delta') \right) \right) \right) \end{aligned} \quad (31)$$

$$\begin{aligned} \implies \frac{\partial w^*(w^s, \delta)}{\partial w^s} &= \frac{r + \delta}{r + \underline{\delta}} - \frac{(\delta - \underline{\delta})}{r + \underline{\delta}} \lambda_1 \int \int_{w^*(w^s, \delta')} d\hat{F}(w'|\delta') dH(\delta') \frac{\partial V(w^s, \underline{\delta})}{\partial w^s} \\ &= \frac{r + \delta + \lambda_1 \int \int_{w^*(w^s, \delta')} d\hat{F}(w'|\delta') dH(\delta')}{r + \underline{\delta} + \lambda_1 \int \int_{w^*(w^s, \delta')} d\hat{F}(w'|\delta') dH(\delta')}, \text{ a.e.} \end{aligned} \quad (32)$$

and (4) now follows. For (5) note that $\frac{\partial w^*(w^s, \delta)}{\partial \delta} = V(w^s, \underline{\delta}) - V_0 = \frac{w^*(w^s, \delta) - w^s}{\delta - \underline{\delta}}$. The second derivatives follow from straightforward differentiation.

Proof of lemma 2 In this proof, we show that the appropriate ratio of limits of a sequence of sets agrees with (9) almost everywhere (with respect to $F(w, \delta)$). (We do not have to worry about a set of measure zero for overall outcomes: anything that happens on a set of measure zero won't affect the profit maximization of other agents, or is otherwise relevant for outcomes in equilibrium. Hence, if we can establish $l(w^s, \delta)$ a.e. we are done.) First, we can define

$$\begin{aligned} I(\delta, w^s) &\stackrel{\text{def}}{=} \int_{\delta' \leq \delta, R_0 \leq w^{s'} \leq w^s} \left(\delta' + \lambda_1 \int_{\tilde{w}^s > w^s} dF(\tilde{w}^s, \tilde{\delta}) + \lambda_1 \int_{w^s \geq \tilde{w}^s > w^{s'}} dF(\tilde{w}^s, \tilde{\delta}) \right) dG(w^{s'}, \delta')(m - u) \\ &\quad - \int_{\delta' \leq \delta, R_0 \leq w^{s'} \leq w^s} \left(\lambda_0 u + \lambda_1 \int_{\tilde{\delta} > \delta, \tilde{w}^s < w^{s'}} dG(\tilde{w}^s, \tilde{\delta})(m - u) \right) dF(w^{s'}, \delta') \end{aligned} \quad (33)$$

Then, for $\delta'' > \delta'$ and $w^{s''} > w^{s'}$, we have $I(\delta'', w^{s''}) - I(\delta', w^{s''}) - I(\delta'', w^{s'}) + I(\delta', w^{s'}) = 0$. After some tedious algebra, in which we drop the terms (i.e. flows) that cancel each other out, add up the remaining flows where possible, but split the integral such that in one set the upper bound is not included and in the other set the wage to integrate over is a singleton $\{w^{s''}\}$; this results in

$$\begin{aligned} & \int_{w^{s'} < w < w^{s''}}_{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{w}^s > w^{s''}} dF(\tilde{w}^s, \tilde{\delta}) + \lambda_1 \int_{w < \tilde{w} \leq w^{s''}} dF(\tilde{w}, \tilde{\delta}) \right) dG(w, \delta) \\ & + \int_{w=w^{s''}}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{w}^s > w^{s''}} dF(\tilde{w}^s, \tilde{\delta}) \right) dG(w, \delta) \\ & = \int_{w^{s'} < w < w^{s''}}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{w}^s \leq w^{s'}} dG(\tilde{w}^s, \tilde{\delta}) + \lambda_1 \int_{w^{s'} \leq \tilde{w} < w} dG(\tilde{w}, \tilde{\delta}) \right) dF(w, \delta) \\ & + \int_{w=w^{s''}}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{w}^s \leq w^{s'}} dG(\tilde{w}^s, \tilde{\delta}) \right) dF(w, \delta) \end{aligned} \quad (34)$$

Now, we can take the limit as $\delta' \rightarrow \delta''$ and $w^{s'} \rightarrow w^{s''}$. There are two cases: (i) $\int_{w=w^{s''}, \text{all } \delta} dF(w, \delta) = 0$, and (ii) $\int_{w=w^{s''}, \text{all } \delta} dF(w, \delta) > 0$. In case (i), the terms on the second and fourth line equal zero, while the rightmost terms in the integral on the first and third line are equal in value to an integral that has a strict upper or lower on wages, i.e. $\lambda_1 \int_{\substack{\delta' < \delta \leq \delta'' \\ w < \tilde{w} \leq w^{s''}}} dF(\tilde{w}, \tilde{\delta}) = \lambda_1 \int_{\substack{\delta' < \delta \leq \delta'' \\ w < \tilde{w} < w^{s''}}} dF(\tilde{w}, \tilde{\delta}) \stackrel{\text{def}}{=} T(w, \delta')$. Moreover, T is continuous at $(\delta'', w^{s''})$ with $T(\delta'', w^{s''}) = 0$, so we have

$$(\delta + \lambda_1(1 - F(w^{s''})) \frac{\lim_{\substack{\delta' \rightarrow \delta'' \\ w^{s'} \rightarrow w^{s''}}} \int_{\substack{\delta' < \delta \leq \delta'' \\ w^{s'} < w \leq w^{s''}}} dG(w, \delta)}{\lim_{\substack{\delta' \rightarrow \delta'' \\ w^{s'} \rightarrow w^{s''}}} \int_{\substack{\delta' < \delta \leq \delta'' \\ w^{s'} < w \leq w^{s''}}} dF(w, \delta)} = (\delta + \lambda_1(1 - F(w^{s''}))l(w^{s''}, \delta) = \lambda_0 u + \lambda G(w^{s''}),$$

using that $\frac{dG(w^{s''}, \delta)}{dF(w^{s''}, \delta)} = l(w^{s''}, \delta)$, and that $G(w^{s''}), F(w^{s''})$ are continuous at $w^{s''}$. Rearranging yields (9).

For case (ii), we can first take the limit $w^{s'} \rightarrow w^{s''}$ on both sides of the equation. The terms on the first and third line go to zero. If the second and fourth line are zero as well, we are dealing a set $\{(w^s, \delta) | w^s = w^{s''}, \delta \in (\delta', \delta'']\}$ that is of measure zero in F , which wlog for the aggregate patterns, we can ignore. Suppose therefore that $B(\delta', \delta'') \stackrel{\text{def}}{=} \{(w^s, \delta) | w^s = w^{s''}, \delta \in (\delta', \delta'']\}$ is of positive measure. Then in the limit as $w^{s'} \rightarrow w^{s''}$ (34) reduces to

$$\int_{w=w^{s''}}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{w}^s > w^{s''}} dF(\tilde{w}^s, \tilde{\delta}) \right) dG(w, \delta) = \int_{w=w^{s''}}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{w}^s < w^{s''}} dG(\tilde{w}^s, \tilde{\delta}) \right) dF(w, \delta) \quad (35)$$

Consider now the limit as $\delta' \rightarrow \delta''$ while $B(\delta', \delta'')$ stays of positive measure (if it becomes of zero measure, we can ignore it, wlog). The terms between brackets inside the integrals stay constant, and hence can be taken outside the integrals. Dividing both sides by $\int_{\substack{\delta' < \delta \leq \delta'' \\ w=w^{s''}}} dF(w, \delta)$, and taking the limit wrt δ , we have

$$(\delta + \lambda_1(1 - F^+(w^{s''})) \frac{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta \leq \delta'' \\ w^{s'}=w^{s''}}} dG(w, \delta)}{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta \leq \delta'' \\ w^{s'}=w^{s''}}} dF(w, \delta)} = (\delta + \lambda_1(1 - F^+(w^{s''}))l(w^{s''}, \delta) = \lambda_0 u + \lambda G^-(w^{s''}),$$

where $(1 - F^+(w^{s''})) = \int_{\tilde{w}^s > w^{s''}} dF(\tilde{w}^s, \tilde{\delta})$ and $G^-(w^s) = \int_{\tilde{w}^s < w^{s''}} dG(\tilde{w}^s, \tilde{\delta})$. \square

Proof of proposition 1 Suppose there exists equivalent wages w_B, w_A , such that $w_B > w_A$, and w_B is offered by the more risky firm while w_A by the more solid firm. Then it must be that

$$(p - w^*(w_B, \delta_h))l(w_B, \delta_h) \geq (p - w^*(w_A, \delta_l))l(w_A, \delta_h). \quad (36)$$

Now, note the following, by (9) $\frac{l(w^s, \delta_l)}{l(w^s, \delta_h)} = \frac{\delta_h + \lambda_1(1 - F(w^s))}{\delta_l + \lambda_1(1 - F(w^s))} > 1$ and both right and left derivatives of this ratio with respect to w^s are $\frac{\lambda_1 F'_i(w^s)(\delta_h - \delta_l)}{(\delta_l + \lambda_1(1 - F(w^s)))^2} \geq 0$ with $i = \{L, R\}$. Therefore, this ratio is larger than 1 and increasing, and in particular

$$\frac{l(w_B, \delta_l)}{l(w_A, \delta_l)} \geq \frac{l(w_B, \delta_h)}{l(w_A, \delta_h)}. \quad (37)$$

To study the instantaneous profit per worker, notice

$$\frac{d \left(\frac{p - w^*(w^s, \delta_l)}{p - w^*(w^s, \delta_h)} \right)}{dw^s} = \frac{-\frac{dw^*(w^s, \delta_l)}{dw^s}(p - w^*(w^s, \delta_h)) + \frac{dw^*(w^s, \delta_h)}{dw^s}(p - w^*(w^s, \delta_l))}{(p - w^*(w^s, \delta_h))^2} > 0$$

since $p - w^*(w^s, \delta_l) > p - w^*(w^s, \delta_h)$ for $w^s > R_0$, and $\frac{dw^*(w^s, \delta)}{dw^s}$ is strictly increasing in δ for all w^s , all by lemma 1. But this implies

$$\frac{p - w^*(w_B, \delta_l)}{p - w^*(w_A, \delta_l)} > \frac{p - w^*(w_B, \delta_h)}{p - w^*(w_A, \delta_h)}. \quad (38)$$

Now, putting (37) and (38) together, it follows that (36) implies

$$(p - w^*(w_B, \delta_l))l(w_B, \delta_l) > (p - w^*(w_A, \delta_l))l(w_A, \delta_l),$$

which contradicts that w_A was the profit maximizing choice of the solid firm. \square

Proof of proposition 2. First, the same argument that established that $G(w^s, \delta)$ is absolutely continuous with respect to $F(w^s, \delta)$ can be made to establish that $F(w^s, \delta)$ is absolutely continuous with respect to $G(w^s, \delta)$, which then necessarily implies that each property (i)-(iii) applies to $F(w^s)$, if and only if it applies to $G(w)$.

To establish property (ii), towards a contradiction, consider a w^s at which a strictly positive mass of wages is offered, with δ_1 as the infimum of those types offering this equivalent wage. The expected profit gain for this type when offering a wage $w^s + \varepsilon$ is greater than

$$\frac{d\pi}{d\varepsilon} = - \left(\frac{dw^*(w^s, \delta)}{dw^s} \varepsilon + \xi(\varepsilon) \right) l(w^s, \delta_1) + (p - w^*(w^s, \delta)) \frac{\lambda_1 G^-(w^s + \varepsilon) - \lambda_1 G^-(w^s)}{\lambda_1(1 - F^+(w^s)) + \delta_1}, \quad (39)$$

where $\xi(\varepsilon)$ is a term with $\frac{\xi(\varepsilon)}{\varepsilon} \rightarrow 0$ as $\varepsilon \rightarrow 0$. Note that a mass point at w^s implies that, there exists a $\eta > 0$, such that for all $\varepsilon > 0$, $G^-(w^s + \varepsilon) - G^-(w^s) \geq \frac{2\eta(\lambda_1 + 1)}{\lambda_1} > 0$. Then, as $\varepsilon \rightarrow 0$, $\frac{d\pi}{d\varepsilon} > \eta$. Moreover, since $\frac{dw^*(w^s, \delta)}{dw^s}$ is continuous by (4), this also holds for a $\delta_2 > \delta_1$ close enough to δ_1 , in particular δ_2 is also close enough to δ_1 to offer equivalent wage w^s to workers. This δ_2 then has a profitable deviation, which is the contradiction we were looking for. This establishes the absence of mass points, and hence the continuity of $F(w^s)$ and $G(w^s)$.

Next, consider the case where the support of $F(w^s)$ is not connected. Let \underline{w}^s be the minimum equivalent wage in the support of $F(w^s)$, and \bar{w}^s the corresponding maximum wage. Then, there exist $\underline{w}^s < w_1^s < w_2^s < \bar{w}^s$ such that $F(w_1^s) = F(w_2^s)$, and $G(w_1^s) = G(w_2^s)$ by the continuity of $F(w^s)$, $G(w^s)$. Consider a firm

posting at w_2^s , this firm can keep the same firm size but make strictly more profit when deviating to w^s . If $w^s > R_0$, the same argument applies: the firm offering w^s can deviate to R_0 , which does not affect his firm size, but strictly raises the profit per worker. \square

Proof of theorem 1 There are three steps in this proof. First, one can show that the equilibrium objects $F(w^s), G(w^s), w^*(w, \delta), \hat{F}(w|\delta), R_0$ constructed from $w^*(z), w^s(z)$ satisfy workers' and firms' optimization. This is straightforward, with this and the derivation in the paper, we have established that a steady state equilibrium corresponds to $\{w^s(z), w^*(z), R_0\}$ and vice versa.²⁴

Pseudoconcavity of the firm's problem Secondly, we have to check that the first-order conditions indeed pick the maximum in the firm's problem, at any point where $\delta(z)$ is continuous. We can verify that the problem is pseudo-concave, using (4) and (17) by showing the expression below is negative when (18) holds.

$$\frac{d}{dz'} \left((p - w^*(w^s(z'), \delta(z))) \frac{\partial l^d(z', \delta(z))}{\partial z'} - \left(\frac{\partial w^*(w^s(z'), \delta(z))}{\partial w^s(z')} \frac{dw^s(z')}{dz'} \right) l^d(z', \delta(z)) \right) \quad (40)$$

Evaluated at a point where the first order condition is equal to zero ($z = z'$), this has the same sign as

$$\frac{\partial}{\partial z'} \left((p - w^*(w^s(z'), \delta(z))) \frac{2\lambda_1}{\lambda_1(1-z') + \delta(z)} \right) - \frac{\partial}{\partial z'} \left(\frac{\partial w^*(w^s(z'), \delta(z))}{\partial w^s(z')} \frac{\partial w^s(z')}{\partial z'} \right) \quad (41)$$

At a $z = z'$ the first term equals $-\frac{4\lambda_1^2}{(\lambda_1(1-z)+\delta(z))^2} (p - w^*(w^s(z), \delta(z))) + \frac{2\lambda_1^2}{(\lambda_1(1-z)+\delta(z))^2} (p - w^*(w^s(z), \delta(z)))$ which reduces to $-\frac{2\lambda_1^2}{(\lambda_1(1-z)+\delta(z))^2} (p - w^*(w^s(z), \delta(z)))$. The second term equals $\frac{\partial^2 w^*(w^s, \delta)}{\partial w^s \partial w^s} \left(\frac{dw^s}{dz} \right)^2 + \frac{\partial w^*}{\partial w^s} \frac{d^2 w^s}{dz dz}$. Focussing on the last term, differentiating (21) wrt z , we find that it equals

$$-\frac{dw^*(z)}{dz} \frac{2\lambda_1}{\lambda_1(1-z) + \delta} + (p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta)^2} + (p - w^*) \frac{2\lambda_1}{\lambda_1(1-z) + \delta} \frac{\lambda_1(\delta - \delta)}{(r + \delta + \lambda(1-z))^2} \frac{dw^*(w^s, \delta)}{dw^s} \quad (42)$$

By (22), $\frac{dw^*(z)}{dz} < (p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta)}$, and therefore in absolute value the first two terms of the expression above jointly are smaller than $(p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta)^2}$. The last term in (42) is negative, and is equal $-\frac{\partial^2 w^*(w^s, \delta)}{\partial w^s \partial w^s} \left(\frac{dw^s}{dz} \right)^2$. Substituting the above in (40), and canceling out terms yields that $\frac{d}{dz'} (\cdot) < 0$.

Existence and uniqueness of the fixed point R_0 . Finally, we have to show existence and uniqueness of the reservation wage $R_0 = w^s(0) = w^*(0)$ satisfying (21), (22), (24). From this, the existence and uniqueness of the steady state equilibrium then follows. Index $\frac{dw^s(z; R_0)}{dz}, \frac{dw^*(z; R_0)}{dz}$ by their initial conditions R_0 ; then (24) is the solution to the following fixed point

$$R_0 = T(R_0), \text{ where } T(R_0) = b + (\lambda_0 - \lambda_1) \int_0^1 \frac{1-z}{r + \lambda_1(1-z) + \underline{\delta}} \frac{dw^s(z; R_0)}{dz} dz \quad (43)$$

Note that $\frac{dw^s(z)}{dz}$ depends implicitly on the reservation only through $(p - w^*(z))$.

²⁴For completeness, this in appendix B.

Manipulating (21) and (22), we can find

$$\frac{d(p - w^*(z))}{dz} = -(p - w^*(z)) \left(\frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \frac{\delta'(z)}{\delta(z) - \underline{\delta}} x(z) \right) \quad (44)$$

$$\frac{dx(z)}{dz} = \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{\delta(z) - \underline{\delta}}{r + \lambda_1(1-z) + \delta(z)} + \left(\frac{\delta'(z)}{\delta(z) - \underline{\delta}} + \frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \right) x(z) + \frac{\delta'(z)}{\delta(z) - \underline{\delta}} x(z)^2, \quad (45)$$

where $x(z) = \frac{w^*(z) - w^s(z)}{p - w^*(z)}$. Note that $p - w^*(0) = p - R_0$, and $x(0) = 0$. Note that $\frac{d(p-w^*(z))}{dR_0}$, by standard FODE theory, is continuous in R_0 . Consider first the interval $[0, \tilde{z}]$ on which $\delta(z)$ is continuous. On this interval, $x(z)$ does not depend on R_0 . We can rewrite (44) to get

$$\frac{\frac{d(p-w^*(z))}{dz}}{(p - w^*(z))} = - \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right); \quad (46)$$

Integrating over z yields

$$p - w^*(z) = e^{- \int_0^z \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right) dz} (p - R_0), \quad (47)$$

where the exponential term does not depend on R_0 . It follows immediately that

$$\frac{d(p - w^*(z))}{dR_0} = -e^{- \int_0^z \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right) dz} < 0.$$

To generalize this to general distributions $H(\delta)$, consider next a point where $\delta(z)$ is discontinuous: this a point where $\delta(z)$ drops discretely. We want to show that the properties of $\frac{d(p-w^*(z))}{dR_0}$, $\frac{dx(z)}{dR_0}$ are preserved. Consider first $x(z)$, from (23),

$$(p - w^*(\tilde{z}))x(\tilde{z}) = \frac{\delta(\tilde{z}) - \underline{\delta}}{\lim_{z \uparrow \tilde{z}} \delta(z) - \underline{\delta}} \left(\lim_{z \uparrow \tilde{z}} (p - w^*(z))x(z) \right). \quad (48)$$

Alternatively, the limit $\lim_{z \uparrow \tilde{z}} y(z) = y_L(z)$ for any variable y , and $\frac{\delta(z) - \underline{\delta}}{\delta_L(z) - \underline{\delta}} = \Delta$ we can rewrite (23) as

$$w^* = \Delta(w_L^*(z) - w^s(z)) - w^s(z) \quad (49)$$

$$\implies (p - w^*(z)) = (p - w_L^*(z))(1 + (1 - \Delta)x_L(z)). \quad (50)$$

From the previous results then, given $\frac{d(p-w_L^*(z))}{dR} < 0$, $\frac{dx_L(z)}{dR} = 0$ combined with (50), it follows that $\frac{d(p-w^*(z))}{dR} < 0$; and thus, $\frac{d(p-w^*(z))}{dR}$ is preserved when $\delta(z)$ drops discretely.

Equation (50) tells us that $\frac{d \ln(p-w^*(z))}{dR} = \frac{d \ln(p-w_L^*(z))}{dR} + \frac{d \ln(1+(1-\Delta)x_L(z))}{dR}$. Rewriting (48) similarly, using (50), we find

$$x(z) = \frac{\Delta x_L(z)}{1 + (1 - \Delta)x_L(z)}, \quad (51)$$

from which it follows that $\frac{dx(z)}{dR} = 0$ whenever $\frac{dx_L(z)}{dR} = 0$. Thus, the irresponsiveness of $\frac{dx(z)}{dR}$ is also preserved whenever $\delta(z)$ drops discretely. Let $Z = \{\zeta_i\}$ be the countable set of ranks at which $\delta(z)$ drops

discretely; define additionally $\zeta_0 = 0$. Then, letting $\bar{\zeta}(z) = \sup\{\zeta \in Z | \zeta < z\}$

$$p - w^*(z) = (p - R_0) \left(\prod_{\{i | \zeta_i \in Z, \zeta_i < z\}} e^{-\int_{\zeta_{i-1}}^{\zeta_i} \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} (1 + (1 - \Delta(\zeta_i))x_L(\zeta_i)) \right) e^{-\int_{\bar{\zeta}(z)}^z \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'}. \quad (52)$$

From this it follows that $\frac{d(p-w^*(z))}{dR_0} = -S(z) < 0$, where $S(z)$ is the entire term postmultiplying $(p - R_0)$ on the RHS of (52). It is negative and does not vary with R_0 .²⁵

To consider existence, note that $T(p) = b$, and for $\lambda_0 \geq \lambda_1 \geq 0$ $T(b) \geq b$, while $T(R_0)$ is continuous in R_0 , as $\frac{dw^s(z; R_0)}{dz}$ is continuous in R_0 , since, from the above, $(p - w^*(z))$ is continuous in R_0 . For $0 < \lambda_0 < \lambda_1$, because $T(R_0) = b + (\lambda_0 - \lambda_1) \int (V(w; \delta) - V_0) dF(w; \delta)$ and $\int (V(w; \delta) - V_0) dF(w; \delta)$ is bounded by $\frac{p-b}{r}$, and hence $T(R_0)$ is bounded from below by $b - \lambda_1 \frac{p-b}{r}$, as we let R_0 go negatively enough, eventually $T(R_0) > R_0$.

To consider uniqueness, notice that the slope of $T(R_0)$ does not depends on R_0 . Hence, there can only be at most one intersection with R_0 . (Since $T(p) = b < p$, we can rule out the case that $T(R_0)$ has slope 1, and has a continuum of equilibrium). \square .

Proof of Result 1 The inflow into employment $\lambda_0 u + \lambda G(z) = l(z)(\delta(z) + \lambda(1-z))$, the probability that an inflow at time t survives until $t + \tau$ is $e^{(\delta(z) + \lambda_1(1-z))\tau}$; thus the number of workers in the z^{th} firm who have been around τ periods $t_{eu}(z, \tau) \stackrel{def}{=} l(z)(\delta(z) + \lambda(1-z)e^{-(\delta(z) + \lambda(1-z))\tau})$. Then, the derivative of the empirical hazard rate with respect to tenure is

$$\frac{d \ln \left(\frac{\int_0^1 \delta(z) t_{eu}(z, \tau) dz}{\int_0^1 t_{eu}(z, \tau) dz} \right)}{d\tau} = - \int_0^1 \frac{(\delta(z) - \delta^{ave}) t_{eu}(z, \tau) (\delta(z) + \lambda_1(1-z)) dz}{\int \delta^{ave} t_{eu}(z', \tau) dz'} < 0 \quad (53)$$

The derivative $dt_{eu}(z, \tau)/d\tau = -t_{eu}(z, \tau)(\delta(z) + \lambda_1(1-z))$. Define $\delta^{ave} \int t_{eu}(z', \tau) dz' = \int \delta(z') t_{eu}(z', \tau) dz'$. Then $\int_0^1 (\delta(z) - \delta^{ave}) t_{eu}(z, \tau) dz$ equals zero; since $\delta(z) - \delta^{ave}$ and $(\delta + \lambda_1(1-z))$ are both decreasing, the latter one strictly, the integral term in (53) is positive, establishing the result.

Proof of Result 2 By proposition 1, locally, for z close 0, we have $w^*(z)$ strictly increasing. Then there exists $\check{z} > 0$ such that $w^*(z)$ is strictly increasing for all $0 < z < \check{z}$, and by (21) so is $w^s(\check{z})$. Now, suppose that there exists a sequence $\{z_n\}$ with $z_n < \check{z}$, such that $w^*(z_n) \rightarrow R_0$. By (3), then also $w^s(z_n) \rightarrow R_0$. Then, there exists an n , such that $w^s(z_n) < w^s(\check{z})$, contradicting the ranking property.

²⁵A similar result holds true in the standard Burdett and Mortensen model, where

$$p - w(z) = (p - R_0) \left(\frac{\lambda_1(1-z) + \delta}{\lambda_1 + \delta} \right)^2,$$

however, here we have take care of the heterogeneity in δ , and the resulting influence on the wages (with wage cuts etc.). Notice that if we set $\delta'(z) = 0$ and hence $\Delta(z) = 0$ and $x(z) = 0 \forall z$, the Burdett and Mortensen result in fact follows. We find it encouraging that a similar nice property can be derived in our more complicated setting.

Proof of Result 3 Note that since $dw^*(0)/dz > 0$, at the the \tilde{z} from which onwards an interval of wage cuts occurs, both $dw^*(\tilde{z})/dz = 0$ and $d^2w^*(\tilde{z})/(dz dz) < 0$, i.e. $dw^*(z)/dz$ cuts 0 from above, at \tilde{z} . The second derivative at \tilde{z} equals

$$\begin{aligned} \frac{d^2w^*}{dz dz} &= -\frac{dw^*}{dz} \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + (p - w^*(z)) \frac{2\lambda_1(\lambda_1 - \delta'(z))}{(\lambda_1(1-z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \\ &\quad + \delta'(z) \frac{1}{r + \underline{\delta} + \lambda_1(1-z)} \frac{dw^s}{dz}. \end{aligned} \quad (54)$$

Note that after substituting in $dw^s(z)/dz$ from (21), the terms with $\delta'(z)$ cancel out. Evaluated at a point where $dw^*/dz = 0$, and letting $r \rightarrow 0$, this turns into

$$\left. \frac{d^2w^*}{dz dz} \right|_{\frac{dw^*}{dz}=0} = (p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \quad (55)$$

This can be smaller than zero only if $\delta''(z) < 0$, which in turn occurs if and only if $h'(\delta) > 0$, since $\delta'(z) = -1/h(\delta)$ and $\delta''(z) = \frac{h'(\delta)}{h(\delta)^2} \delta'(z)$.

The second point follows from (22) being negative. Substituting in $w^s(z)$ into (22), a change of the integrating variable ($dz = -h(\delta)d\delta$), this can be written equivalently as

$$(p - w^*(1 - H(\delta))) \frac{2\lambda_1}{\delta + \lambda_1(1 - H(\delta))} < \frac{1}{h'(\delta)} \int_{\delta}^{\tilde{\delta}} \frac{2\lambda_1(\underline{\delta} + \lambda_1(1 - H(\tilde{\delta})))}{(\tilde{\delta} + \lambda_1(1 - H(\tilde{\delta})))^3} (p - w^*(1 - H(\tilde{\delta}))) h(\tilde{\delta}) d\tilde{\delta}$$

Since $p - w^*(\delta(1 - H(\tilde{\delta}))) > p - w^*(\delta)$ for $\tilde{\delta} > \delta$, the above equation is negative whenever (26) holds.

Proof of Result 4 We can show this by establishing that $\frac{d^2w^*(\tilde{z})}{dwdw} < 0$, and $\frac{dw^*(\tilde{z})}{dz} = 0$ cannot occur for $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$. Note that the existence of wage cuts implies a z such that $dw^*(z)/dz = 0$, $\frac{d^2w^*(z)}{dz dz} \leq 0$. This implies that at that z , from (22) and (55),

$$(p - w^*(z)) \frac{2(\lambda_1^n)^2}{\delta(z) + \lambda_1(1 - z)} < -\delta''(z) \int_0^z \frac{1}{\delta(z') + \lambda_1(1 - z')} \frac{dw^s(z')}{dz'} dz \quad (56)$$

$$(p - w^*(z)) \frac{2\lambda_1}{\delta(z) + \lambda_1(1 - z)} = -\delta'(z) \int_0^z \frac{1}{\delta(z') + \lambda_1(1 - z')} \frac{dw^s(z')}{dz'} dz \quad (57)$$

Dividing the RHS of (56) by the RHS of (56), and similarly for the LHS, this yields

$$\lambda_1 < \delta''(z)/\delta'(z) = \frac{h'(\delta)}{(h(\delta))^2}$$

as a necessary condition. Hence if $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$, we do not satisfy the necessary condition, and therefore can rule out $dw^*(z)/dz < 0$ at $z = 1 - H(\delta)$.

Proof of Result 5 is in the main text

B Additional Material

proof of proposition 1, verification In the paper, we derived $\{w^s(z), w^*(z), R_0\}$ satisfying (21), (22), (24) from the equilibrium conditions. Here, we outline the reverse argument: any set $\{w^s(z), w^*(z), R_0\}$ satisfying (21), (22) and (24) corresponds to a steady state equilibrium. Since we have established that the tuple $\{w^s(z), w^*(z), R_0\}$ satisfying the aforementioned conditions exists and is unique, we have also established existence and uniqueness of the steady state equilibrium.

The rest of proof consists of the rather straightforward construction of equilibrium. We construct (i) the aggregate conditions, (ii) the worker's optimization, (iii) firm's optimization. First invert $\omega = w^s(z)$, to construct $F(\omega) = (w^s)^{-1}(\omega)$, $G(w^s) = G^z(w^s)^{-1}(\omega)$ at $w^s = w^s(z)$, and $\tilde{w}^*(w^s, \delta)$ for those wages that are actually posted on the equilibrium path from $\tilde{w}^*(\tilde{w}^s, \tilde{\delta}) = \{w^*(z) | \tilde{w}^s = w^s(z), \tilde{\delta} = \delta(z) \text{ for some } z\}$. Then we can construct $\hat{F}(w|\delta)$ from $F(w^s), \delta(z), w^*(w^s, \delta)$. $F(w^s, \delta), G(w^s, \delta)$ can be constructed straightforwardly from $F(w^s), \delta(z)$. Now we have to show that these distribution are consistent with optimal decision making, (ii)-(iii). Given $F(w^s), \tilde{w}^*(w^s, \delta)$, where for off-equilibrium (w^s, δ) we use the same partial derivatives as in lemma 1, to derive the entire function $w^*(w^s, \delta)$. For worker optimization and firm optimization, consider that firms choose w^s in the profit maximization, which leads to the following first order condition

$$(p - w^*(w^s(z), \delta(z))) \frac{\partial l(w^s, \delta)}{\partial w^s} - \left(\frac{\partial w^*(w^s, \delta)}{\partial w^s} \right) l(w^s, \delta) = 0, \quad (58)$$

which can be rewritten as

$$\frac{\frac{\partial w^*(w^s, \delta)}{\partial w^s}}{p - w^*(w^s(z), \delta(z))} - \frac{\frac{\partial l(w^s, \delta)}{\partial w^s}}{l(w^s, \delta)} = 0, \iff \frac{\frac{\partial w^*(w^s, \delta)}{\partial w^s}}{p - w^*(w^s(z), \delta(z))} = \frac{2\lambda_1 F'(w^s)}{\delta + \lambda(1 - F(w^s))} \quad (59)$$

Using $F'(w^s) = 1/w^{s\prime}(z)$, we can verify that (17) and (19) combined with (21) and (22) imply (4)-(5) and the equivalence of (58) and (17), thus establishing both $\tilde{w}^*(w^s, \delta) = w^*(w^s, \delta)$ and the optimality of $z = F^{-1}(w^s)$. Finally, the equivalence of R_0 can be established by a simple change of variable in the integral (24).