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Job Design with Conflicting Tasks Reconsidered

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Abstract. A principal wants two sequential tasks to be performed by wealth-constrained agents. When the tasks are conflicting (i.e., when a first-stage success makes second-stage effort less effective), the principal’s profit-maximizing way to induce high efforts is to hire one agent to perform both tasks. In this case, the prospect to get a larger second-stage rent after a first-stage success motivates the agent to work hard in the first stage. In contrast, when the tasks are synergistic, the principal prefers to hire two different agents for the two tasks. These results are in contrast to previous studies that consider simultaneous tasks.

Keywords: moral hazard; limited liability; conflicting tasks; synergies

JEL Classification: D86

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1 Introduction

Decision-makers often have to choose between letting one agent be responsible for two tasks, or letting two different agents be responsible for one task each. For example, when an infrastructure is designed (first task) and subsequently built (second task), it has to be decided whether the same contractor or two different contractors should be in charge of the two tasks. For instance, in the recent case of two new Ohio River spans, the only method allowed under current Kentucky law is the traditional approach, which means that there are different contractors. Yet, also the alternative option of having one contractor in charge of both tasks is currently discussed, which would require action by the Kentucky General Assembly.¹ In the case of the Port of Miami Tunnel, a major construction project in Florida with an estimated cost of 1 billion U.S. dollars, it was decided to let the private contractor MAT Concessionaire LLC be in charge of both tasks (Miami Herald, April 17, 2010). Note that in both cases, the two tasks have to be performed sequentially.

Related problems may also arise when a new government is formed. There can be a single department responsible for different fields, or there can be separate departments in charge of the different fields. For instance, in the current Government of New South Wales led by Premier Barry O’Farrell, there now is a so-called “super-ministry” led by Andrew Stoner, who is both Minister for Trade and Investment and Minister for Regional Infrastructure and Services.² While the minister is responsible for both fields simultaneously, observe that there may also be regional infrastructure projects that have to be established first in order to facilitate subsequent trade, so the minister may also be in

¹See The Courier-Journal, October 6, 2011, under the headline “Bridges authority delays decision on how to build new Ohio River spans.”

²The Sydney Morning Herald (April 3, 2011), reported about the new government under the headline “New faces: O’Farrell launches super-ministries.”
charge of some successive tasks. Moreover, note that term limits in politics may rule out that the same decision maker is in charge of different issues that come up over a longer time span, which means that there are different agents in charge of these issues, while the same agent might be in charge of such consecutive tasks in the absence of term limits.\(^3\)

In contract theory, there is by now a large literature on multi-task principal-agent problems in the presence of moral hazard.\(^4\) Starting with Holmström and Milgrom (1991), many contributions in this literature are focused on the trade-off between insurance and incentives when agents are risk-averse. However, as has been pointed out by Dewatripont and Tirole (1999) and Bolton and Dewatripont (2005, Sections 6.2.2 and 6.4), interesting multitask problems may also arise when agents are risk-neutral but wealth-constrained.\(^5\) Traditional multi-task models were focused on the fact that one agent engaging in different activities may lead to higher (lower) effort costs when the tasks are substitutes (complements). In contrast, Bolton and Dewatripont (2005) assume that there are no cost advantages or disadvantages when an agent performs two tasks. Instead, they analyze the effects of direct conflicts between the tasks.

Tasks are said to be conflicting (synergistic) when effort exerted in one task may reduce (increase) the probability that the other task will be performed successfully. For instance, when an agent exerts effort and comes up with an innovative design, then this might either decrease or increase the proba-

3For an analysis of the behavior of U.S. governors facing term limits, see Besley and Case (1995).

4For reviews of the literature, see Dewatripont et al. (2000), Laffont and Martimort (2002, ch. 5), and Bolton and Dewatripont (2005, ch. 6).

5On moral hazard problems with risk-neutral but wealth-constrained agents, see also Innes (1990), Pitchford (1998), and Tirole (2001). These models are “efficiency wage” models in the contract-theoretic sense of Tirole (1999, p. 745) and Laffont and Martimort (2002, p. 174). See also Kragl and Schöttner (2011), who study whether a principal should hire one or two agents to perform simultaneous tasks in the presence of wage floors.

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bility that a building can be successfully constructed without exceeding the budget limits (see Hart, 2003). As another illustration, consider a principal who wants two goods to be sold. When the goods are imperfect substitutes (complements), then successfully selling one product will decrease (increase) the probability that also the second good will be sold.6

The punchline of Bolton and Dewatripont (2005) is that when simultaneous tasks are conflicting, then it is very difficult to motivate one agent to exert effort in both tasks, so that it is better to delegate the two tasks to two different agents. In contrast, when the tasks are not conflicting, only one agent should be in charge of both tasks, since then it is cheaper for the principal to incentivize one agent (a bonus must only be paid when both tasks are successful). The results of Bolton and Dewatripont (2005) are intuitively plausible and they were shown to also have bite in the laboratory in a recent study by Hoppe and Kusterer (2011).7

However, Bolton and Dewatripont (2005) consider only the case in which the two tasks are to be performed simultaneously. In contrast, in the present paper a variant of their model is studied in which two tasks have to be performed sequentially.8 It turns out that then Bolton and Dewatripont’s (2005)

6See Bolton and Dewatripont (2005, Section 6.2.2) and Hoppe and Kusterer (2011).

7Hoppe and Kusterer (2011) have conducted an experiment with 474 subjects. The agents were salespersons who could promote one or two products. When the products were substitutes, so that the tasks are conflicting in the sense of Bolton and Dewatripont (2005), high effort levels were observed significantly less often when there was one agent in charge of both tasks compared to the case of two agents. In the absence of conflict, the principal was better off when she hired just one agent, as predicted by Bolton and Dewatripont (2005).

8On agency problems with sequential tasks, see also Hirao (1993), Schmitz (2005), Khalil et al. (2006), Kräkel and Schöttner (2010, 2011), Müller (2011), and Ohlendorf and Schmitz (2012). Nieken and Schmitz (2012) provide experimental evidence. Yet, these contributions do not consider conflicting tasks in the sense of Bolton and Dewatripont (2005) which are the focus of the present paper.
results may be overturned. Surprisingly, if the tasks are in conflict, so that a success in the first task makes effort in the second task less effective, then the principal is better off when she hires only one agent in charge of both tasks. In contrast, if there are synergies between the tasks, so that a success in the first task makes effort in the second task more effective, then the principal prefers to hire two different agents for the two different tasks.

The intuitive explanation for the novel finding is as follows. In the presence of limited liability, the principal cannot make the agent pay a fine when there is no success. Hence, the only possibility to motivate an agent to exert unobservable effort is to offer him a bonus when there is a success, so that the agent enjoys a rent.9 In particular, when effort is not very effective in increasing the success probability, then the rent that the principal must promise the agent has to be large in order to give him an incentive to work hard.

Now consider a two-stage model. When exerting effort in the second stage becomes less effective, it becomes more difficult to motivate the agent in charge of the second stage to work, so that the principal has to increase the rent that she must leave to the agent when she wants to implement high effort. Hence, there is a new externality between the stages that is absent in a simultaneous framework. When the tasks are conflicting, an agent who is in charge in both stages now has an additional incentive to exert effort in the first stage, because by making second-stage effort less effective, he can increase the rent that he can enjoy in the second stage. In contrast, when there are synergies, it is better for the principal to hire two different agents, because a single agent would now be tempted to shirk in the first stage (and thus make second-stage effort less effective) in order to increase his second-stage rent.

The remainder of the paper is organized as follows. In the following section

9Laffont and Martimort (2002) use the term “limited liability rent” to distinguish the rent in moral hazard models with wealth constraints from the related concept of information rents that a principal has to leave to agents in adverse selection models.
the simplest model that allows for sequential conflicting tasks is introduced. Section 3 characterizes the principal’s optimal contract when she hires only one agent (scenario I). The case of two agents (scenario II) is analyzed in Section 4. The overall optimal contract is derived in section 5, where the principal’s profit in the two scenarios is compared. Finally, section 6 concludes.

2 The model

Consider a principal who wants two sequential tasks to be performed. The outcome of task $i \in \{1, 2\}$ is denoted by $q_i \in \{0, 1\}$. If task $i$ is a success ($q_i = 1$), the principal obtains a revenue $R$, otherwise her revenue in stage $i$ is zero. Two different scenarios are considered. In scenario I, the principal employs a single agent to perform both tasks, while in scenario II, she employs two different agents for the two different tasks. All parties are risk neutral. An agent has no wealth and his reservation utility is zero. Effort on task $i \in \{1, 2\}$ is denoted by $e_i \in \{0, 1\}$. An agent who exerts effort $e_i$ incurs a disutility of effort $\psi e_i$. The effort levels are not observable.

The probability that the first task is a success is given by $\Pr\{q_1 = 1\} = \alpha + \rho e_1$. The probability that the second task is a success is given by $\Pr\{q_2 = 1\} = \alpha + \gamma q_1 e_2$. Throughout, we assume that the parameters $\alpha, \rho, \gamma_0, \gamma_1$ are strictly positive and $\alpha < 1 - \max\{\rho, \gamma_0, \gamma_1\}$, so that the expressions that describe probabilities lie between zero and one. Observe that even if the agent shirks, there is a success with probability $\alpha > 0$. Moreover, it may depend

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10 Notice that if the agents were not protected by limited liability, the principal could always attain the first-best solution by making an agent residual claimant; i.e. the principal would simply leave her revenue to the agent in exchange for a suitable up-front payment, so that the expected payoff of the agent would be zero.

11 Note that the first-best solution could always be attained if $\alpha$ were equal to zero, because then in case of a success the principal knew for sure that the agent has exerted high effort. The principal would then just reimburse the agent for his effort costs, so that the agent
on the outcome of the first stage \((q_1)\) how effective effort in the second stage is. Specifically, note that the two tasks are technologically independent if \(\gamma_1 = \gamma_0\). We say that the two tasks are conflicting if \(\gamma_1 < \gamma_0\). In this case, a success in the first stage makes effort in the second stage less effective (i.e., there is a negative externality). In contrast, we say that the tasks are synergistic if \(\gamma_1 > \gamma_0\). In this case, a success in the first stage makes effort in the second stage more effective (i.e., there is a positive externality).\(^{12}\)

Note that since the two agents are identical, in a first-best world (i.e., if effort were contractible) it would make no difference whether the principal hires one or two agents. Following Bolton and Dewatripont (2005), we assume throughout that the principal’s revenue \(R\) is sufficiently large so that she always wants to implement high effort. Hence, we can focus on the question in which of the two scenarios the principal’s agency costs are smaller. To induce an agent to exert effort, the principal can offer him a wage scheme \(w_{q_1,q_2} = w(q_1,q_2) \geq 0\) that is contingent on the outcomes of both tasks.

3 Scenario I: One agent

Suppose first that the principal has hired only one agent to perform both tasks. Since effort is unobservable, the principal must ensure that it is in the agent’s self-interest to choose high effort. Hence, the agent’s expected utility when he exerts high effort (incurring effort costs \(\psi\)) must be larger than his expected utility when he shirks.

\(^{12}\)For example, the first task might be to build an infrastructure and the second task might be to operate the infrastructure. As has been pointed out by Hart (2003), innovations in the first stage may either facilitate or hamper the operating efforts in the second stage.
Consider the second stage. The incentive compatibility constraints that ensure that the agent exerts high effort in the second stage are

$$(\alpha + \gamma_1)w_{11} + (1 - \alpha - \gamma_1)w_{10} - \psi \geq \alpha w_{11} + (1 - \alpha)w_{10}$$

for the case that the first stage was a success ($q_1 = 1$) and

$$(\alpha + \gamma_0)w_{01} + (1 - \alpha - \gamma_0)w_{00} - \psi \geq \alpha w_{01} + (1 - \alpha)w_{00}$$

for the case that the first stage was a failure ($q_1 = 0$).

Now consider the first stage. The agent is willing to exert high effort in the first stage if the incentive compatibility constraint

$$(\alpha + \rho)[(\alpha + \gamma_1)w_{11} + (1 - \alpha - \gamma_1)w_{10} - \psi] + (1 - \alpha - \rho)[(\alpha + \gamma_0)w_{01} + (1 - \alpha - \gamma_0)w_{00} - \psi] - \psi \geq \alpha[(\alpha + \gamma_1)w_{11} + (1 - \alpha - \gamma_1)w_{10} - \psi] + (1 - \alpha)[(\alpha + \gamma_0)w_{01} + (1 - \alpha - \gamma_0)w_{00} - \psi]$$

is satisfied.

The principal’s problem is to find a wage scheme ($w_{00}, w_{10}, w_{01}, w_{11}$) in order to minimize her expected costs

$$(\alpha + \rho)[(\alpha + \gamma_1)w_{11} + (1 - \alpha - \gamma_1)w_{10}] + (1 - \alpha - \rho)[(\alpha + \gamma_0)w_{01} + (1 - \alpha - \gamma_0)w_{00}]$$

subject to the incentive compatibility constraints and the limited liability constraints $w_{q_1q_2} \geq 0$. Since the agent always has the possibility to choose low effort without incurring any costs, incentive compatibility and limited liability together imply that the agent’s participation constraint is always satisfied.

Note that the incentive compatibility constraints can be rewritten such that they read $\gamma_1(w_{11} - w_{10}) \geq \psi$ and $\gamma_0(w_{01} - w_{00}) \geq \psi$ in the second stage, and

$$\rho[(\alpha + \gamma_1)w_{11} + (1 - \alpha - \gamma_1)w_{10} - (\alpha + \gamma_0)w_{01} - (1 - \alpha - \gamma_0)w_{00}] \geq \psi$$

in the first stage. Thus, the following result can be proved.
Proposition 1 Consider the case in which the principal has delegated both tasks to one agent.

(i) If \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho \geq 0 \), it is optimal for the principal to offer the contract \( w_{00} = w_{10} = 0 \), \( w_{01} = \psi/\gamma_0 \), and \( w_{11} = \psi[\gamma_0 + \rho(\alpha + \gamma_0)]/[\rho \gamma_0 (\alpha + \gamma_1)] \). Then her expected costs are \( [(\alpha + \rho)/\rho + (\alpha + \gamma_0)/\gamma_0]\psi \).

(ii) If \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho < 0 \), the principal will offer the contract \( w_{00} = w_{10} = 0 \), \( w_{01} = \psi/\gamma_0 \), and \( w_{11} = \psi/\gamma_1 \). Then her expected costs are \( [(\alpha + \rho)(\alpha + \gamma_1)/\gamma_1 + (1 - \alpha - \rho)(\alpha + \gamma_0)/\gamma_0]\psi \).

Proof. See the Appendix.

Observe that it is optimal for the principal not to make a payment to the agent when the second stage was not successful, regardless of the outcome of the first stage \( (w_{00} = w_{10} = 0) \). Clearly, the principal does not want to reward the agent for a failure. However, a second-stage success is rewarded even if the first stage was a failure \( (w_{01} > 0) \). This is necessary in order to induce the agent to work hard in the second stage, even when he was not successful in the first stage (the second-stage incentive compatibility constraint conditional on a first-stage failure is always binding). With regard to the bonus \( w_{11} \) that is paid when both stages are successful, a case distinction has to be made. In case (i), the parameter constellation \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho \geq 0 \) is satisfied. This case always prevails if the tasks are synergistic \( (\gamma_1 > \gamma_0) \), and it also prevails if a conflict between the tasks is not too strong. It turns out that then the second-stage incentive compatibility constraint conditional on a first-stage success is not binding; i.e., the wage scheme that motivates the agent to work hard in the first stage is sufficient to also motivate him to work hard in the second stage after a first-stage success. If the conflict is very strong, \( \gamma_1 - \gamma_0 \) may be so negative that we are in case (ii). In this case, it is difficult to motivate the agent to work hard in the second stage following a first-stage success, so that the corresponding incentive compatibility constraint then is binding.
Finally, regarding the principal’s expected costs, observe that the principal must leave a rent to the agent (i.e., she must pay more to him than $2\psi$, which would be necessary to reimburse the effort costs), because a success might occur even if the agent is lazy.\(^\text{13}\)

4 Scenario II: Two agents

Suppose now that the principal has hired two different agents for the two different tasks. Let agent A be in charge of task 1, while agent B is responsible for task 2. Recall that agent A imposes an externality on agent B, since the effectiveness of agent B’s effort ($\gamma_{q_1}$) depends on whether ($q_1 = 1$) or not ($q_1 = 0$) agent A is successful in the first stage.

The incentive compatibility constraint ensuring that agent A chooses high effort in the first stage (given that agent B will be induced to exert high effort in the second stage) reads

\[
(\alpha + \rho)\left[ (\alpha + \gamma_1)w_{11}^A + (1 - \alpha - \gamma_1)w_{10}^A \right] \\
+ (1 - \alpha - \rho)\left[ (\alpha + \gamma_0)w_{01}^A + (1 - \alpha - \gamma_0)w_{00}^A \right] - \psi \\
\geq \alpha\left[ (\alpha + \gamma_1)w_{11}^A + (1 - \alpha - \gamma_1)w_{10}^A \right] \\
+ (1 - \alpha)\left[ (\alpha + \gamma_0)w_{01}^A + (1 - \alpha - \gamma_0)w_{00}^A \right].
\]

The incentive compatibility constraints that ensure that agent B chooses high effort in the second stage are

\[
(\alpha + \gamma_1)w_{11}^B + (1 - \alpha - \gamma_1)w_{10}^B - \psi \geq \alpha w_{11}^B + (1 - \alpha)w_{10}^B
\]

for the case that the first stage was a success and

\[
(\alpha + \gamma_0)w_{01}^B + (1 - \alpha - \gamma_0)w_{00}^B - \psi \geq \alpha w_{01}^B + (1 - \alpha)w_{00}^B
\]

\(^{13}\)To see this in case (i), recall that $\alpha > 0$. With regard to case (ii), note that the condition $\gamma_0\gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho < 0$ can be used to show that $(\alpha + \rho)(\alpha + \gamma_1)/(\gamma_1 + (1 - \alpha - \rho)(\alpha + \gamma_0)/\gamma_0 > 2$ must hold.
for the case that the first stage was a failure.

The principal designs wage schemes \((w_{00}^A, w_{10}^A, w_{01}^A, w_{11}^A)\) and \((w_{00}^B, w_{10}^B, w_{01}^B, w_{11}^B)\) in order to minimize her expected costs

\[
(\alpha + \rho)[(\alpha + \gamma_1)(w_{11}^A + w_{11}^B) + (1 - \alpha - \gamma_1)(w_{10}^A + w_{10}^B)]
\]

\[
+(1 - \alpha - \rho)[(\alpha + \gamma_0)(w_{01}^A + w_{01}^B) + (1 - \alpha - \gamma_0)(w_{00}^A + w_{00}^B)]
\]

subject to the incentive compatibility constraints and the limited liability constraints \(w_{q_1q_2}^A \geq 0\) and \(w_{q_1q_2}^B \geq 0\). Note that these constraints again imply that the participation constraints are satisfied.

It is easy to see that agent A’s incentive compatibility constraint can be simplified to

\[
\rho[(\alpha + \gamma_1)w_{11}^A + (1 - \alpha - \gamma_1)w_{10}^A - (\alpha + \gamma_0)w_{01}^A - (1 - \alpha - \gamma_0)w_{00}^A] \geq \psi.
\]

Moreover, agent B’s incentive compatibility constraints can be rewritten as \(\gamma_1(w_{11}^B - w_{10}^B) \geq \psi\) and \(\gamma_0(w_{01}^B - w_{00}^B) \geq \psi\). Thus, the following result must hold.

**Proposition 2** Consider the case in which the principal has hired two different agents to work on the two different tasks. It is optimal for the principal to offer the contracts \(w_{11}^A = w_{10}^A = \psi/\rho, w_{01}^A = w_{00}^A = 0\) and \(w_{11}^B = \psi/\gamma_1, w_{10}^B = \psi/\gamma_0, w_{01}^B = w_{00}^B = 0\). Then the principal’s expected costs are \((\alpha + \rho)[\psi/\rho + (\alpha + \gamma_1)\psi/\gamma_1] + (1 - \alpha - \rho)[(\alpha + \gamma_0)\psi/\gamma_0]\).

**Proof.** See the Appendix.

Observe that agent A is rewarded whenever the first stage is successful \((w_{11}^A = w_{10}^A > 0)\) and agent B is rewarded whenever the second stage is successful \((w_{11}^B > 0, w_{01}^B > 0)\), while the other wages are zero. All incentive compatibility constraints are binding. Note that the reward that agent B gets after a first-stage success \((w_{11}^B = \psi/\gamma_1)\) is larger than the reward he gets after a first-stage failure \((w_{01}^B = \psi/\gamma_0)\) whenever the tasks are conflicting \((\gamma_1 < \gamma_0)\), and vice versa if the tasks are synergistic.
Furthermore, with regard to the principal’s expected costs, observe again that the principal has to leave rents to the agents, since a success might occur even if an agent shirks.\textsuperscript{14}

5 One agent or two agents?

We can now compare the principal’s expected costs in the two scenarios in order to determine when the principal is better off hiring one agent or two agents. Propositions 1 and 2 imply that the principal’s expected costs in scenario I are smaller than in scenario II whenever $\gamma_1$ is smaller than $\gamma_0$. Our main result can thus be stated as follows.

Proposition 3 (i) If the two tasks are conflicting ($\gamma_1 < \gamma_0$), then the principal prefers to hire one agent who is in charge of both tasks.

(ii) If the two tasks are synergistic ($\gamma_1 > \gamma_0$), then the principal prefers to hire two different agents for the two different tasks.

(iii) If the two tasks are independent ($\gamma_1 = \gamma_0$), then the principal is indifferent between hiring one or two agents.

Proof. See the Appendix.

The discussion following Proposition 2 has shown that when the tasks are conflicting, then the wage that must be paid in order to induce high effort in the second stage following a first-stage success is larger than the wage that must be paid following a first-stage failure. The reason is that with conflicting tasks, a first-stage success makes second-stage effort less effective, which means that in the second stage the agent must get a larger wage to be motivated to exert high effort. Hence, the principal can benefit from letting the same agent be in charge of both stages. The agent will then have an additional incentive

\textsuperscript{14}Notice that if $\alpha$ were equal to zero, the principal’s expected costs would again be equal to the agents’ total effort costs $2\psi$. 

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to work hard in the first stage, because of the prospect to earn a larger rent in the second stage.

When the tasks are synergistic, the opposite holds. A success in the first stage makes effort in the second stage more effective, so that the principal must then pay only a small rent to induce high second-stage effort. When the same agent is in charge of both stages, the prospect to earn a higher rent after a first-stage failure demotivates effort in the first stage. In this case, it is thus better for the principal to hire two different agents for the two different tasks.

Hence, Proposition 3 is driven by the fact that the second-stage rents depend on the first-stage results. This explains why the sequential nature of the tasks is crucial.

Of course, our results depend on the assumptions that we have made. Recall that in the absence of wealth-constraints (or if $\alpha$ were equal to zero), the principal would always implement the first-best solution without leaving any rent to an agent. Hence, in this case she would be indifferent between hiring one or two agents. Note also that the conclusions reached depend on the assumption that the principal’s return $R$ is sufficiently large so that she always wants to implement high effort. This assumption, which was also made in Bolton and Dewatripont (2005), is most plausible if $\rho, \gamma_0$, and $\gamma_1$ are not too close to zero.\(^{15}\)

Finally, it should be pointed out that in case of conflicting tasks, it is the success of the first stage which reduces the success probability in the second stage. Alternatively, one could conceive a model in which it is the first-stage effort that reduces the second-stage success probability. In this case, the effects

\(^{15}\)For instance, if $\gamma_1$ were close to zero and $R$ were small, then the principal might not want to induce high effort following a first-stage success. In this case, the benefit of hiring one agent when the tasks are conflicting that was highlighted in the model (i.e., the fact that a large second-stage rent can be earned when the first stage was successful), would no longer prevail.
highlighted in the present paper would not occur. The assumption that the outcome (and not the effort) of the first stage is decisive for the effectiveness of second-stage effort seems to be plausible in many applications. For instance, in an infrastructure project the success probability in the building stage depends on whether or not in the prior stage an innovative design was developed (and not on how hard the agent tried to come up with an innovation). Similarly, when an agent was successful in selling a product in the first stage, then this outcome (and not the agent’s efforts to sell the product) may reduce the probability of also selling a close substitute.

6 Concluding remarks

We have shown that when agents are risk-neutral but wealth-constrained and a principal wants to induce high efforts in two sequential tasks, then for incentive reasons she may be better off hiring one agent if the tasks are in conflict, while she may prefer to hire two different agents if there are synergies between the tasks. The reason is that when the tasks are conflicting, then a success in the first stage reduces the effectiveness of effort in the second stage, so that in order to induce high second-stage effort, the agent must get a larger second-stage rent. The prospect to get this larger rent provides a new incentive to exert effort in the first stage, provided that the same agent is in charge of both stages.

Our somewhat surprising results are entirely due to the sequential nature of the tasks and they are thus in sharp contrast to the findings of Bolton and Dewatripont (2005), who consider a framework where tasks are to be performed simultaneously. In their framework, two different agents should be hired to perform two conflicting tasks.

Several avenues for future research seem to be promising. The model was kept as simple as possible to highlight the effects of moral hazard in a clear
way. In future work, the model could be extended to also cover adverse selection aspects, where agents have private information about their types.\textsuperscript{16} The interaction of limited liability rents and information rents can be complicated (see Laffont and Martimort, 2002), but might lead to interesting new insights. Moreover, since the model is very simple, it might be useful as a building block in more applied work. For instance, starting with Hart (2003) and Bennett and Iossa (2006), several authors have recently pointed out that an important characteristic of so-called public-private partnerships is that the two stages of building and subsequently managing a public facility are delegated to one agent (a consortium), while under traditional procurement the two sequential tasks of building and managing are delegated to two different contractors. While the relevance of positive and negative externalities between the stages is also a common theme in this applied literature,\textsuperscript{17} the effects of conflicting tasks in a moral hazard framework as analyzed in the present paper have not yet been considered there. Integrating these kinds of externalities might lead to interesting novel insights that so far have escaped the literature on public-private partnerships. In particular, one could try to open the black box of contracting within the consortium and thus investigate whether there are differences between delegating a task to two different agents or to one agent who then subcontracts with another agent.\textsuperscript{18}

\textsuperscript{16}Models analyzing task assignment and job design from an adverse selection perspective include Riordan and Sappington (1987), Dana (1993), Gilbert and Riordan (1995), and Lewis and Sappington (1997).

\textsuperscript{17}See also Martimort and Pouyet (2008), Chen and Chiu (2010, 2011), Hoppe and Schmitz (2010), De Brux and Desrieux (2011), and Iossa and Martimort (2011).

\textsuperscript{18}See also Hoppe et al. (2012), who study subcontracting in an experiment on public-private partnerships. It turns out that there may be subtle differences between subcontracting within a consortium and direct contracting with the principal, because reciprocal behavior tends to occur with regard to the party with whom a contractor deals directly. These aspects have so far not been integrated in the formal literature on public-private partnerships.
Appendix

Proof of Proposition 1.

Note first that \( w_{00} = 0 \) must hold in the solution to the principal’s problem.\(^{19}\) Hence, the incentive compatibility constraint for the second stage after a first-stage failure now reads \( \gamma_0 w_{01} \geq \psi \). Note that in the optimum this constraint must be binding, \( w_{01} = \psi / \gamma_0 \). The first-stage incentive compatibility constraint can thus be rewritten as

\[
\rho [w_{10} + (\alpha + \gamma_1)(w_{11} - w_{10}) - (\alpha + \gamma_0)\psi / \gamma_0] \geq \psi.
\]

(i) Ignore for a moment the second-stage incentive compatibility constraint conditional on a first-stage success, \( \gamma_1 (w_{11} - w_{10}) \geq \psi \). Then the binding first-stage incentive compatibility constraint implies

\[
w_{11} = w_{10} + \frac{\psi / \rho + (\alpha + \gamma_0)\psi / \gamma_0 - w_{10}}{\alpha + \gamma_1}.
\]

The omitted constraint \( \gamma_1 (w_{11} - w_{10}) \geq \psi \) is thus satisfied whenever \( w_{10} \leq \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho \gamma_0 \gamma_1 / \rho \gamma_0 (\alpha + \gamma_1) \). Hence, we have found the solution in the case \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho \geq 0 \). Note that the principal has some freedom in choosing \( w_{11} \) and \( w_{10} \) when \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho > 0 \), since there are multiple combinations of these two wages leading to the (uniquely determined) minimal expected costs \([ (\alpha + \rho) / \rho + (\alpha + \gamma_0) / \gamma_0 ] \psi \). Specifically, the principal can always set \( w_{10} = 0 \) and \( w_{11} = \psi [\gamma_0 + \rho (\alpha + \gamma_0)] / [\rho \gamma_0 (\alpha + \gamma_1)] \), as stated in the proposition.

(ii) Next consider the case \( \gamma_0 \gamma_1 + (\gamma_1 - \gamma_0)\alpha \rho < 0 \), so that the constraint \( \gamma_1 (w_{11} - w_{10}) \geq \psi \) must be binding. Hence, \( w_{11} = \psi / \gamma_1 + w_{10} \). The first-stage incentive compatibility constraint is then satisfied whenever \( w_{10} \geq \psi / \rho - (\alpha + \gamma_1) (\psi / \gamma_1) + (\alpha + \gamma_0) \psi / \gamma_0 \). The right-hand side of this constraint is negative, partnerships.

\(^{19}\)To see this, assume that in the solution \( w_{00} > 0 \) would hold. Then the principal’s expected profit could be increased by reducing \( w_{00} \) without violating any constraints, contradicting the optimality of \( w_{00} > 0 \).
since $\gamma_0 \gamma_1 + (\gamma_1 - \gamma_0) \alpha \rho < 0$. Thus, the condition is always satisfied when the principal sets $w_{10}$ as small as possible, $w_{10} = 0$. Therefore, if $\gamma_0 \gamma_1 + (\gamma_1 - \gamma_0) \alpha \rho < 0$, the principal sets $w_{11} = \psi / \gamma_1$ and her expected costs are given by $[\psi / \gamma_1 + (1 - \alpha - \rho) \alpha \rho / \gamma_0]$. 

Proof of Proposition 2.
The incentive compatibility constraints of agent $B$ are given by $\gamma_1 (w_{11}^B - w_{10}^B) \geq \psi$ and $\gamma_0 (w_{01}^B - w_{00}^B) \geq \psi$. Hence, the principal will set $w_{00}^B = w_{10}^B = 0$, so that the binding constraints imply $w_{11}^B = \psi / \gamma_1$ and $w_{01}^B = \psi / \gamma_0$. 

With regard to agent $A$, the principal has to set $w_{00}^A = w_{01}^A = 0$ in order to minimize her expected costs. The principal has some freedom in designing the wages $w_{11}^A$ and $w_{10}^A$. All combinations of $w_{11}^A$ and $w_{10}^A$ that satisfy agent $A$'s binding incentive compatibility constraint $(\alpha + \gamma_1) w_{11}^A + (1 - \alpha - \gamma_1) w_{10}^A = \psi / \rho$ minimize the principal’s expected costs. Specifically, it seems to make sense not to condition agent A’s wages on the outcome of the second stage, $w_{11}^A = w_{10}^A = \psi / \rho$. In any case, the principal’s expected costs are uniquely determined; they are given by $(\alpha + \rho) [\psi / \rho + (\alpha + \gamma_1) \psi / \gamma_1] + (1 - \alpha - \rho) [(\alpha + \gamma_0) \psi / \gamma_0]$. 

Proof of Proposition 3.
Consider first the case $\gamma_0 \gamma_1 + (\gamma_1 - \gamma_0) \alpha \rho \geq 0$. Inspection of Propositions 1 and 2 immediately reveals that the principal prefers to hire only one agent in charge of both tasks whenever

$$[(\alpha + \rho) / \rho + (\alpha + \gamma_0) / \gamma_0] \psi$$

$$\leq (\alpha + \rho) [\psi / \rho + (\alpha + \gamma_1) \psi / \gamma_1] + (1 - \alpha - \rho) [(\alpha + \gamma_0) \psi / \gamma_0],$$

which is equivalent to $(\alpha + \gamma_0) \gamma_1 \leq (\alpha + \rho)(\alpha + \gamma_1) \gamma_0 + (1 - \alpha - \rho)(\alpha + \gamma_0) \gamma_1$ and which can be further simplified to $\gamma_1 \leq \gamma_0$. Hence, the principal prefers to hire one agent (two agents) whenever the two tasks are conflicting (synergistic).

Next, consider the case $\gamma_0 \gamma_1 + (\gamma_1 - \gamma_0) \alpha \rho < 0$. Note that this case can occur only if the tasks are conflicting ($\gamma_1 < \gamma_0$). In this case, it follows from
Propositions 1 and 2 that the principal prefers to hire only one agent in charge of both tasks whenever

\[
[(\alpha + \rho)(\alpha + \gamma_1)/\gamma_1 + (1 - \alpha - \rho)(\alpha + \gamma_0)/\gamma_0] \psi \\
\leq (\alpha + \rho)[\psi/\rho + (\alpha + \gamma_1)\psi/\gamma_1] + (1 - \alpha - \rho)[(\alpha + \gamma_0)\psi/\gamma_0].
\]

This condition can be rewritten as \(0 \leq (\alpha + \rho)/\rho\), which is always satisfied. Hence, the proposition follows immediately. ■
References


