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**Abstract**

This paper considers a model of voluntary public good provision with two players and convex costs. I demonstrate that the provision of public good is higher in the sequential framework under fairly general conditions. This outcome shows that introducing convex costs may reverse under some condition the results of Varian ( 1994).

*Jel Codes: C72, D0, H40, H41*

*Keywords*: Public Goods, Contribution Games, Private Provision of Public Goods

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## 1 Introduction

The provision of public good has been investigated by two main approaches of the literature, one considers the case in which the public goods provided are continuous and the other one focused the analysis on the case in which the contributions of the agents are utilized to provide the discrete public goods.

There are many example of voluntary contribution environments: the provision of a public good by a work team, some members that voluntary contribute to provide a good for all the components of a community, the donors that give moneys for a fund raising, two or more countries that contribute in order to protect the environment, the nations belonging to a Federation that make a payment to manage the immigration ( Russo, Senatore, 2011). In all the previous situations the contribution can be made sequentially or simultaneously, in this paper I mainly analyze how the timing in the contribution can influence the total amount of public good provided. Sometimes in a work team one worker can decide to make an effort before the others or a nation can be first to commit

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in an agreement on an environmental or migratory problem. Certainly the total amount of public good provided depends from the timing of the contributions.

Starting from Warr ( 1982, 1983 ) many scholars have analyzed the economic aspects related to this kind of games. As Warr clearly states ( 1982 ), when the economic agents contribute voluntary to provide a public good that is motivated by utility interdependence and it can be observed that a non –Pareto optimal income arises due to the presence of the free rider problem. Afterwards there have been many other scientific contributions starting from Cornes and Sandler ( 1984 ) and Varian ( 1994 ) that describe why in case there are voluntary contributions the amount of public good is underprovided because of the free riding problem. Especially Varian ( 1994 ) examines a game where the agents decide to contribute sequentially to provide a public good. Basically, using this framework, he points up that the agent who moves the first can credibly commit his contribution, as a consequence he focalizes the analysis on the amount of public good provided in this situation, comparing this amount with the one provided in a situation where the agents contribute simultaneously. In order to analyzes the provision of public good he considers a simultaneous move game where contributors have a utility function that is quasi -linear in their private consumption and concave increasing function of public good provided. Varian gets two results, in case an agent likes the public good more than the other agent, she contributes the entire amount of public good and the other one free rides. When the two agents likes the good identically there is a whole range of equilibrium contribution and, at same time, a unique equilibrium amount of the public good. Moreover Varian considers the same situation when the players contribute to the public project sequentially in order to provide a public good. In his model he searches out other results: in case the agent who likes the good least is the first contributor his optimal choice is to contribute zero; in case the agent that likes the public good the most is the first contributor, either contributor may free ride. From the Varian's model come out two possible Stackelberg equilibria: the first one is also a Nash equilibria because the agent who likes the public good most contributes everything; the second one is when the agent who likes the good least contributes everything, but this not a Nash equilibrium considered that the agent who likes the public most is not a credible free rider in the simultaneous move game. As a consequence the best situation for each agent is to move first, given that there are only two possible outcomes and the player chooses the one that he prefers. Since Varian considers utility functions by which the sum of utilities is higher at the higher level of public good, the higher level of the public good may be provided if the agent who likes the good least move first. The central result of Varian's model is that, in case of private provision of public good, it is supplied a less quantity of this good if the agents move sequentially instead of simultaneously.

Bergstrom et al. (1986) show how a larger redistributions of wealth changes the set of contributors and consequently the equilibrium provision of public good. Buchholz, Konrad and Lommerud ( 1997 ) analyses the same model of Varian ( 1994 ) introducing the transfers in the Stackelberg game of private provision of a public good. In the Varian's Stackelberg equilibrium of private provision of public good it is considered the case in which there are equal contribution prices, Buchholz et al. don't remove this condition and examine the welfare effects of income redistribution. In particular they analyze the case in which there is a redistribution of income from follower to the leader before the Stackelberg contribution game, as a result they show how this kind of redistribution it is Pareto improving that is in contrast with the case of the redistribution in the Nash outcome. Furthermore, assuming that both contributors have identical contribution prices, it is clear that the increase of welfare is not related to a comparative advantage type specialization. Thus the income redistribution eliminate the incentive for the leader to commit herself to make only a small contribution. In addition they demonstrate that the follower has an incentive to make transfers to leader for a wide range of initial distributions of income, so the transfers received from the leader can induce this agent to become a standalone contributor in equilibrium.

Kerschbamer and Puppe ( 2001 ) also demonstrate that in the Varian's model of sequential contributions the private provision of a public good is not neutral if there is an income transfer from the follower to the leader, even if Varian shows that income transfer from the leader to the follower that do not exceed the leader's original contribution are neutral. Moreover they discuss the uniqueness of equilibrium in the sequential set - up. Vesterlund ( 2003 ) points out the importance of the informational value in a sequential fund-raising. In contrast with the theoretical predictions many times the fund-raisers choose to announce the contributions, this paper puts forward that an announcement strategy maybe optimal because it gives more information about the quality and the amount of public good provided.

Kempf and Graziosi ( 2010 ) focus their attention on the issue of leadership when two governments provide a public goods to their citizens with cross border externalities. Their analysis is strictly related to the Varian's model ( 1994 ) and extend the Varian's framework in two main directions: firstly they consider a sequential contribution game determining the leadership endogenously and secondly they don't consider the amounts of contribution as substitutes but consider all the possible situations. The conclusion of Varian's Model remains valid for the case in which the contribution are substitute and there is an endogenous leadership but fails in case of complementarity between the contributions.

More recently Bag and Roy ( 2011 ) study the influence of incomplete information on sequential and simultaneous contributions. In contrast with the case of complete information they show that the contributors having independent private valuations for the public good when contribute sequentially may provide a large amount of public good respect to the case in which they contribute simultaneously.

In this paper I consider a pure strategy contribution game in which two agents completely informed contribute simultaneously and sequentially in order to provide a continuous public good. For both the utility function is quasi - linear in their private consumption and concave increasing function of the public good provided. Furthermore I modify the Varian's framework ( 1994 ) introducing convex costs in the payoff functions of the agent. Once I have calculated the total amount of public good provided in equilibrium in simultaneous and sequential game I compare both cases showing that when the contributors has the same tastes the public good supplied in sequential framework is larger respect to the case in which the agents contribute simultaneously. This result not confirm Varian ( 1994 ). I repeat the same comparison introducing the possibility that the players have a kind of conflict in the preferences, in other words when one player likes more the public good the other one likes it less and viceversa. In this case the Varian's result is not confirmed as well. It is not the aim of this paper to develop an efficiency's analysis or determine normative issue, basically I want to point out under which condition the sequential contribution may determine a higher amount of the public good provided respect to the sequential scheme and which are the main economic reason of this result.

The paper proceeds as follows: In the Section 2 I describe the contribution game model. In the Section 3 I compare the total amount of public good provided in sequential and simultaneous game with agents having same preferences and convex cost. In Section 4 I develop the same analysis of Section 3 introducing an heterogeneity in the utility function of the contributors. The Section 5 concludes. The Section 6 and 7 are the Appendixes A and B containing the proofs.

## 2 The model

Two agents, $C$ and $L$ contribute respectively an amount of money $g_C$ and $g_L$ in order to provide a continuous public good $G = g_C + g_L$. Each of them has a wealth $w_i$ , with $i = C, L$, and distributes it between private consumption $x_i \geq 0$ and the contribution to a public good $g_i \geq 0$. As in Varian's model ( 1994 ) I assume that the agents' utility function is quasi - linear in their private consumption and a concave increasing function of $G$. If for any values of wealth $w_i$ it is always bigger than $\bar{g_i}$ that is the amount of public good that maximizes agent $i's$ utility, then it is possible to remove the wealth $w_i$ from the utility function. Unlike the Varian's model ( 1994 ) the cost of contribution is quadratic instead of linear and there is a parameter $\lambda \geqslant 1$.

The payoffs of each agent are:

$$U_C\left(g_C, g_L\right) = \left(g_C + g_L\right)^{\beta} - \frac{\lambda g_C^2}{2} \tag{1}$$

$$U_L\left(g_C, g_L\right) = \left(g_C + g_L\right)^{\beta} - \frac{\lambda g_L^2}{2} \tag{2}$$

with $0 < \beta < 1$

This is a non - cooperative game in pure strategies, the agents have perfect information and common knowledge.

# 3 Simultaneous v/s Sequential contribution with same preferences and convex costs

I start the analysis considering the situation in which the agents contribute simultaneously to provide a continuous public good, then I consider the case in which the contribution is made sequentially, at the end I compare the total amount of public good provided in both cases. The aim is to show in which scenario the total amount of public good provided is bigger.

## 3.1 Simultaneous Contributions with same preferences

Firstly I consider the case in which the utility functions are identical, then the two agents have the same tastes and the pay off functions can be rewritten in the following way:

For $C$ the pay off function is

$$U_C\left(g_C, g_L\right) = \left(g_C + g_L\right)^{\beta} - \frac{\lambda g_C^2}{2} \tag{3}$$

and for $L$ the pay off function is

$$U_L\left(g_C, g_L\right) = \left(g_C + g_L\right)^{\beta} - \frac{\lambda g_L^2}{2} \tag{4}$$

with $0 < \beta < 1$. Each agent, at simultaneous non -cooperative equilibrium, chooses the optimal amount of contribution that maximizes her pay off. The first order condition of agent $C$ is

$$\max_{g_C \geqslant 0} U_C\left(g_C, g_L\right), \quad g_L \ given \tag{5}$$

the first order condition of agent $L$ is

$$\max_{g_L \geqslant 0} U_L\left(g_C, g_L\right), \quad g_C \ given \tag{6}$$

The Nash equilibrium of this game $\left(g_C^{\mathrm{Si}\,m}, g_L^{\mathrm{Si}\,m}\right)$ is a pair that must satisfy simultaneously the conditions 5 and 6. There is a unique Nash Equilibrium in which the total sum of contribution is :

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{7}$$

the 7 is the total amount of public good provided in the simultaneous game with identical utility function.

**Claim 2** *For any values of $\beta$ and $\lambda$, the total amount of public good provided in the simultaneous game is* $\left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}}$ *( See Appendix A.1 )*

It is trivial to remark that in equilibrium the players split exactly the total amount of the contributions. When the players have the same preferences there is a unique Nash Equilibrium in which they maximize giving the same single amount of contribution.

## 3.2 Sequential Contributions with same preferences

Starting from the situation in which the players have the same preferences I analyze the situation in which they contribute sequentially, I consider again the payoff functions 3 and 4 . Suppose that the first mover is the contributor $C$. Solving the game by backward induction I define the first order condition for player $L$

$$\beta\left(g_C+g_L\right)^{\beta-1}=\lambda g_L. \tag{8}$$

Then I calculate the first order condition for agent $C$ given the optimal chosen strategy of agent $L$ in the first stage of the contribution game

$$\beta(g_C+g_L)^{\beta-1}\cdot\left(1+g_L'(g_C)\right)=\lambda g_C \tag{9}$$

Applying the Implicit Function Theorem to the First Order Condition of player $L$, I can rewrite the FOC of player $C$ in the following way( See Appendix A.2 )

$$\beta(g_C+g_L)^{\beta-1}\cdot\left[1-\frac{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}}{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}-\lambda}\right]=\lambda g_C \tag{10}$$

Solving the system with the two First Order Condition I get the following quadratic equation:

$$\frac{\beta}{\lambda}\left(\beta-1\right)\left(g_C+g_L\right)^{2\beta-4}-\left(g_C+g_L\right)^{\beta-2}\left(1+\beta\right)+\frac{\lambda}{\beta}=0 \tag{11}$$

**Claim 3** *For any values of $\beta$ and $\lambda$, the solutions of the quadratic equation 11 correspond to* $\left(g_C^{Seq}+g_L^{Seq}\right)^{\beta-2}$ *. Given that, the total amount of public good provided in the sequential game is* $\left(g_C^{Seq}+g_L^{Seq}\right)=\left(\frac{2\frac{\beta}{\lambda}(\beta-1)}{(1+\beta)\pm\sqrt[2]{1+2\beta+\beta^2-4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}\right)^{\frac{1}{2-\beta}}$ *( See Appendix A.2 )*

### 3.3 Comparison between the total amount of Public Good provided in simultaneous and sequential game

To compare the total amount of Public Good provided in the case when the players contribute simultaneously with the situation in which they interact sequentially, I investigate if the following inequality it is verified

$$\left(\frac{2\frac{\beta}{\lambda}(\beta-1)}{(1+\beta)-\sqrt[2]{1+2\beta+\beta^2-4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}\right)^{\frac{1}{2-\beta}} > \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{12}$$

As is clear the left hand side of the 12 is the total amount of public good provided in sequential game while the right hand side corresponds to the total amount of public good provided in simultaneous game. Furthermore I consider exclusively the positive values of the expression on the left hand of 12 because for negative values the public good it is not provided. (See Appendix B.1).

By solving 12 , it is straightforward to conclude that total contribution is higher in the sequential regime. This is summarized in the following proposition:

**Proposition 4** *In a two stages contribution game, with convex costs and a concave increasing utility function of public good, the total amount of public good provided is larger when the players contribute sequentially.*

**Proof.** See Appendix B.1 ■

The proposition states that the sequential game dominates the simultaneous game in terms of total contribution. In this case the players have same preferences and same cost functions. This result contradicts Varian (1994) on condition that the two contributors face convex costs.

## 4 Contribution game with heterogeneous preferences and convex costs

Let me introduce the case in which the two contributors have different tastes as previously described two agents, $C$ and $L$ contribute respectively an amount of money $g_C$ and $g_L$ in order to provide a continuous public good $G = g_C + g_L$. Each of them has a wealth $w_i$ , with $i = C, L$, and distributes it between private consumption $x_i \geq 0$ and the contribution to a public good $g_i \geq 0$. As in Varian's model ( 1994 ) I assume that the agents' utility function is quasi - linear in their private consumption and a concave increasing function of $G = g_C + g_L$. If for any values of the wealth $w_i$ it is always bigger than $\bar{g_i}$ that is the amount of public good that maximizes agent $i's$ utility, I can remove the wealth $w_i$ from the utility function. Unlike the Varian's model ( 1994 ) the cost of contribution is quadratic instead of linear and the utility function is affected from a parameter $\pi \in [0, 1]$.

The parameter $\pi$ defines the situations in which the provision of public good $G$ can increase the utility of one agent diminishing the utility of the other one involved in the contribution game, in other word if one contributor prefers more public good the other one prefers less determining a conflict between the agent's utilities, then the payoffs of each agent are:

$$U_C(g_C, g_L) = \pi (g_C + g_L)^\beta - \frac{\lambda g_C^2}{2} \tag{13}$$

$$U_L(g_C, g_L) = (1-\pi)(g_C + g_L)^\beta - \frac{\lambda g_L^2}{2} \tag{14}$$

with $\beta < 1$ and $\pi \in [0, 1]$

Also in this case I analyze a non - cooperative game with pure strategies ,where the agents have perfect information and common knowledge.

# 5 Simultaneous v/s Sequential contribution with heterogenous preferences and convex costs

In this part of the paper I also compare the case in which the agents contribute simultaneously to provide the public good with the case in which they made a contribution sequentially, at the end I evaluate the total amount of public good provided in both cases. The aim is to show in which scenario the total amount of public good provided is bigger and if introducing the heterogeneity and conflict between the preferences the outcome it is different respect to the situation in which the agents have the same tastes.

## 5.1

## 5.2 Simultaneous Contributions with heterogeneous preferences

Now I consider the case in which the players contribute simultaneously, I consider the payoff functions 13 and 14 so there is a kind of heterogeneity in the utility functions and the costs are convex .

The first order condition of agent $C$ is

$$\max_{g_C \geqslant 0} U_C(g_C, g_L), \quad g_L \ given \tag{15}$$

the first order condition of agent $L$ is

$$\max_{g_L \geqslant 0} U_L\left(g_C, g_L\right), \quad g_C \textit{ given} \tag{16}$$

The Nash equilibrium of this game $\left(g_C^{\mathrm{Si}\,m}, g_L^{\mathrm{Si}\,m}\right)$ is a pair that must satisfy simultaneously the conditions 15 and 16. There is a unique Nash Equilibrium in which the total sum of contribution is :

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{17}$$

the 18 is the total amount of public good provided in the simultaneous game with identical utility function.

**Claim 5** *For any values of $\beta$ and $\lambda$, the total amount of public good provided in the simultaneous game is $\left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}}$ ( See Appendix A.3 )*

In addition the contribution for each agent are the following:

$$g_C^{\mathrm{Si}\,m} = \pi\left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{18}$$

$$g_L^{\mathrm{Si}\,m} = (1-\pi)\left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{19}$$

if $\pi = 0$ the contributions in equilibrium for each player are $g_C^{\mathrm{Si}\,m} = 0$ and $g_L^{\mathrm{Si}\,m} = \left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}}$ and if $\pi = 1$ the contributions in equilibrium are $g_C^{\mathrm{Si}\,m} = \left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}}$ and $g_L^{\mathrm{Si}\,m} = 0$.This result shows that the free riding problem occurs when there is total conflict between the two players, indeed in the simultaneous game when a contributor likes the public good too much gives all the sum in order to provide the total amount of public good while the other contributor free rides. Furthermore I can introduce the following proposition

**Proposition 6** *In a simultaneous contribution game with convex costs, a concave increasing utility function and heterogeneous preferences, the total amount of public good provided does not depend from the different tastes of the contributors .*

Clearly as the value of $\pi$ changes from 0 to 1 each player cover the amount of public good not provided from the other one in order to not change the level of public good that maximizes her outcome.

### 5.3 Sequential Contributions with heterogeneous preferences

In this paragraph I consider the case in which the agents play sequentially using the payoff functions 13 and 14. Suppose that the first mover is the contributor $C$. Solving the game by backward induction I define the first order condition for player $L$

$$(1-\pi)\,\beta\,(g_C+g_L)^{\beta-1}-\lambda g_L=0 \tag{20}$$

Then I calculate the first order condition for agent $C$ given the optimal chosen strategy of agent $L$ in the first stage of the contribution game

$$\pi\beta(g_C+g_L)^{\beta-1}\cdot(1+g_L'(g_C))-\lambda g_C=0 \tag{21}$$

Applying the Implicit Function Theorem to the First Order Condition of player $L$, I can rewrite the FOC of player $C$ in the following way( See Appendix A.2 )

$$\pi\beta(g_C+g_L)^{\beta-1}\cdot\left[1-\frac{(1-\pi)\,\beta\,(\beta-1)\,(g_C+g_L)^{\beta-2}}{(1-\pi)\,\beta\,(\beta-1)\,(g_C+g_L)^{\beta-2}-\lambda}\right]=\lambda g_C \tag{22}$$

Solving the system with the two First Order Condition I get the following quadratic equation:

$$\frac{(1-\pi)^2\,\beta\,(\beta-1)\,(g_C+g_L)^{2\beta-4}}{\lambda}-(g_C+g_L)^{\beta-2}\,(\beta-\pi\,(\beta-1))+\frac{\lambda}{\beta}=0 \tag{23}$$

**Claim 7** *For any values of $\beta$ and $\lambda$, the solutions of the quadratic equation 24 correspond to $\left(g_C^{Seq}+g_L^{Seq}\right)^{\beta-2}$. Given that, the total amount of public good provided in the sequential game is $\left(g_C^{Seq}+g_L^{Seq}\right)=\left(\frac{2(1-\pi)^2\frac{\beta}{\lambda}(\beta-1)}{\beta-\pi(\beta-1)\pm\sqrt[2]{(\beta-\pi(\beta-1))^2-4(1-\pi)^2\beta(\beta-1)}}\right)^{\frac{1}{2-\beta}}$ ( See Appendix A.3 )*

### 5.4 Comparison between the total amount of Public Good provided in simultaneous and sequential game with convex costs and heterogeneous utility functions

The comparison between the sequential and the simultaneous contribution game can be carry out solving the following inequality

$$\left(\frac{2(1-\pi)^2 \frac{\beta}{\lambda}(\beta-1)}{\beta-\pi(\beta-1) \pm \sqrt[2]{(\beta-\pi(\beta-1))^2-4(1-\pi)^2\beta(\beta-1)}}\right)^{\frac{1}{2-\beta}} > \left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{24}$$

As is clear the left hand side of the 24 is the total amount of public good provided in sequential game while the right hand side corresponds to the total amount of public good provided in simultaneous game. As in the case of same preferences I consider exclusively the positive values of the expression on the left hand of 24 because for negative values the public good it is not provided. (See Appendix B.2).

By comparing the equilibrium solutions in two scenarios, it is straightforward to conclude that total contribution is higher in the sequential regime.

This is summarized in the following proposition:

**Proposition 8** *In a two stages contribution game, with convex cost and a concave increasing and heterogeneous utility functions, the total amount of public good provided it is always larger when the players contribute sequentially.*

**Proof.** See Appendix B.2 ■

The proposition simply states that the sequential game dominates the simultaneous game in terms of total contribution. Also in case there is a conflict between the players on the total amount of the public good to provide the result not confirms Varian (1994), this implies that the existence of convex costs determines an increase of public good provided when the players contribute sequentially .

# 6 Conclusions

The model presented in this paper has several implications for public goods provision in the presence of convex costs. In contrast to Varian ( 1994 ), in presence of convex and identical preferences the public good supplied is larger in sequential framework. This result shows that the introduction of timing in some circumstances can assure an high level of public good provided even in presence of the free rider problem. Furthermore even in case the contributors have different taste and conflict in the preferences, more precisely when the increase in the utility of one agent automatically decreases the utility of the other agent the Varian's result is not confirmed. It is possible to conclude that with convex costs in order to provide more public good the sequential framework in the contributions is preferable.

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# 7 Appendix A

## 7.1 Appendix A.1: Total Amount of Public Good Provided in Simultaneous Game with convex costs

$$U_C(g_C, g_L) = (g_C + g_L)^{\beta} - \frac{\lambda g_C^2}{2} \qquad \text{(Payoff of Player } C\text{)}$$

$$U_L(g_C, g_L) = (g_C + g_L)^{\beta} - \frac{\lambda g_L^2}{2} \qquad \text{(Payoff of Player } L\text{)}$$

FOC for player $g_C$, $\frac{\partial U_C}{\partial g_C} = 0$, gives:

$$\beta (g_C + g_L)^{\beta-1} = \lambda g_C. \qquad (\text{FOC}_C)$$

SOC clearly satisfied.
FOC for player $g_L$, $\frac{\partial U_L}{\partial g_L} = 0$, gives:

$$\beta (g_C + g_L)^{\beta-1} = \lambda g_L. \qquad (\text{FOC}_L)$$

SOC clearly satisfied.

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}}$$

The total contribution in the simultaneous game equilibrium is:

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \qquad \text{(TOTAL CONTRIBUTION)}$$

## 7.2 Appendix A.2 Total Amount of Public Good Provided in Sequential Game with convex costs

$$U_C(g_C, g_L) = (g_C + g_L)^{\beta} - \frac{\lambda g_C^2}{2} \qquad \text{(Payoff of Player } C\text{)}$$

$$U_L(g_C, g_L) = (g_C + g_L)^{\beta} - \frac{\lambda g_L^2}{2} \qquad \text{(Payoff of Player } L\text{)}$$

FOC for player $L$, $\frac{\partial U_L}{\partial g_L} = 0$

$$\beta (g_C + g_L)^{\beta-1} - \lambda g_L = 0$$

$$\beta\left(g_C+g_L\right)^{\beta-1}=\lambda g_L \tag{FOC$_L$}$$

substituting in $U_C$

$$U_C=\left(g_C+g_L(g_C)\right)^{\beta}-\frac{\lambda g_C^2}{2}$$

$$\frac{\partial U_C}{\partial g_C}=\beta(g_C+g_L)^{\beta-1}\cdot\left(1+g_L'(g_C)\right)-\lambda g_C$$

Applying the IFT to the FOC of player $C$, I can rewrite the FOC of player $C$ in the following way

$$\frac{\partial U_C}{\partial g_C}=\beta(g_C+g_L)^{\beta-1}\cdot\left[1-\frac{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}}{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}-\lambda}\right]-\lambda g_C$$

FOC for player $C, \frac{\partial U_C}{\partial g_C}=0$

$$\beta(g_C+g_L)^{\beta-1}\cdot\left[1-\frac{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}}{\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}-\lambda}\right]=\lambda g_C \tag{FOC$_C$}$$

In that way the players maximize sequentially and I can build a system in order to get the value of $g_C+g_L$ the two FOCs

$$\begin{cases} \beta\left(g_C+g_L\right)^{\beta-1}=\lambda g_L \\ \beta(g_C+g_L)^{\beta-1}\cdot\left[1-\frac{\beta(\beta-1)(g_C+g_L)^{\beta-2}}{\beta(\beta-1)(g_C+g_L)^{\beta-2}-\lambda}\right]=\lambda g_C \end{cases}$$

Solving the system I get:

$$\beta^2\left(\beta-1\right)\left(g_C+g_L\right)^{2\beta-4}-\lambda\beta\left(g_C+g_L\right)^{\beta-2}-\lambda\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}+\lambda\pi\beta\left(\beta-1\right)\left(g_C+g_L\right)^{\beta-2}+\lambda^2=0$$

that is nothing else that a quadratic equation where is possible to calculate the value of $\left(g_C+g_L\right)^{\beta-2}$, so the two solutions are

$$\left(g_C+g_L\right)^{\beta-2}=\frac{(1+\beta)\pm\sqrt{1+2\beta+\beta^2-4\frac{\beta}{\lambda}\left(\beta-1\right)\frac{\lambda}{\beta}}}{2\frac{\beta}{\lambda}\left(\beta-1\right)}$$

from these solutions I have the possibility to calculate the total amount of public good provided in sequential game.

$$g_C^{Seq}+g_L^{Seq}=\left(\frac{(1+\beta)\pm\sqrt{1+2\beta+\beta^2-4\frac{\beta}{\lambda}\left(\beta-1\right)\frac{\lambda}{\beta}}}{2\frac{\beta}{\lambda}\left(\beta-1\right)}\right)^{\frac{1}{2-\beta}}$$

## 7.3

## 7.4 Appendix A.3 Total Amount of Public Good Provided in Simultaneous Game with convex costs and heterogeneity in the preferences

The payoff functions of the two contributors are the following

$$U_c = \pi \left(g_C + g_L\right)^{\beta} - \frac{1}{2}\lambda g_C^2 \qquad \text{(Payoff of Player } C\text{)}$$

$$U_c = (1-\pi)\left(g_C + g_L\right)^{\beta} - \frac{1}{2}\lambda g_L^2 \qquad \text{(Payoff of Player } L\text{)}$$

where $0 < \beta < 1$ and $\pi \in [0, 1]$.

FOC for player $C$, $\frac{\partial U_c}{\partial c} = 0$, gives:

$$\pi\beta \left(g_C + g_L\right)^{\beta-1} = \lambda g_C. \qquad (\text{FOC}_L)$$

SOC clearly satisfied.
FOC for player $L$, $\frac{\partial U_l}{\partial l} = 0$, gives:

$$(1-\pi)\,\beta \left(g_C + g_L\right)^{\beta-1} = \lambda g_L. \qquad (\text{FOC}_C)$$

SOC clearly satisfied.
Solving the system with the two FOCs I get the total contribution in the simultaneous game:

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \qquad \text{(TOTAL CONTRIBUTION)}$$

which does not depend on $\pi$.

## 7.5 Appendix A.4 Total Amount of Public Good Provided

## in Sequential Game with convex costs and heterogeneity in the preferences

The payoff functions of the two contributors are the following

$$U_c = \pi \left(g_C + g_L\right)^{\beta} - \frac{1}{2}\lambda g_C^2 \qquad \text{(Payoff of Player } C\text{)}$$

$$U_c = (1-\pi)\left(g_C + g_L\right)^{\beta} - \frac{1}{2}\lambda g_L^2 \qquad \text{(Payoff of Player } L\text{)}$$

where $0 < \beta < 1$ and $\pi \in [0,1]$.

The agent $C$ contributes first so we solve the game by using the backward induction:

FOC for player $L$, $\frac{\partial U_l}{\partial g_L} = 0$, gives:

$$(1-\pi)\,\beta\left(g_C + g_L\right)^{\beta-1} = \lambda g_L. \qquad (\text{FOC}_L)$$

SOC clearly satisfied.

substituting in $U_C$

$$U_C = \pi\left(g_C + g_L(g_C)\right)^{\beta} - \frac{\lambda g_C^2}{2}$$

FOC for player $C$,$\frac{\partial U_C}{\partial g_C} = 0$

$$\frac{\partial U_C}{\partial g_C} = \pi\beta(g_C + g_L)^{\beta-1}\cdot(1 + g_L'(g_C)) - \lambda g_C \qquad (\text{FOC}_C)$$

Applying the IFT to the FOC of player $C$, I calculate calculate $\frac{dg_L}{dg_C}$ from the FOC of $L$ I can rewrite the FOC of player $C$ in the following way

$$\frac{\partial U_C}{\partial g_C} = \pi\beta(g_C + g_L)^{\beta-1}\cdot\left[1 - \frac{(1-\pi)\,\beta\,(\beta-1)\left(g_C + g_L\right)^{\beta-2}}{(1-\pi)\,\beta\,(\beta-1)\left(g_C + g_L\right)^{\beta-2} - \lambda}\right] - \lambda g_C$$

now I can build a system in order to get the value of $g_C + g_L$ the two FOCs

$$\begin{cases} (1-\pi)\,\beta\left(g_C + g_L\right)^{\beta-1} = \lambda g_L \\ \pi\beta(g_C + g_L)^{\beta-1}\cdot\left[1 - \frac{(1-\pi)\beta(\beta-1)(g_C+g_L)^{\beta-2}}{(1-\pi)\beta(\beta-1)(g_C+g_L)^{\beta-2}-\lambda}\right] = \lambda g_C \end{cases}$$

Solving the system I get::

$$(1-\pi)^2\beta^2(\beta-1)\left(g_C + g_L\right)^{2\beta-4} - \lambda\beta\left(g_C + g_L\right)^{\beta-2} -$$

$$-\lambda\beta(\beta-1)\left(g_C + g_L\right)^{\beta-2} + \lambda\pi\beta(\beta-1)\left(g_C + g_L\right)^{\beta-2} + \lambda^2 = 0$$

that expression is nothing else that a quadratic equation from which it is possible to calculate the value of $\left(g_C + g_L\right)^{\beta-2}$, so the two solutions are:

$$(g_C + g_L)^{\beta-2} = \frac{(\beta - \pi(\beta-1)) \pm \sqrt[2]{(\beta - \pi(\beta-1))^2 - 4\frac{\lambda}{\beta}\cdot(1-\pi)^2\frac{\beta}{\lambda}(\beta-1)}}{2(1-\pi)^2\frac{\beta}{\lambda}(\beta-1)}$$

from these solutions I have the possibility to calculate the total amount of public good provided in sequential game.

$$g_C^{Seq} + g_L^{Seq} = \left(\frac{2(1-\pi)^2\frac{\beta}{\lambda}(\beta-1)}{(\beta - \pi(\beta-1)) \pm \sqrt[2]{(\beta - \pi(\beta-1))^2 - 4\frac{\lambda}{\beta}\cdot(1-\pi)^2\frac{\beta}{\lambda}(\beta-1)}}\right)^{\frac{1}{2-\beta}}$$

# 8 Appendix B

## 8.1 Appendix B.1: Proof of Proposition 2

I want to prove that $\left(g_C^{Seq} + g_L^{Seq}\right) > \left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right)$ the total amount of public good provided is higher in sequential game.

The total amount of public good provided in the simultaneous scenario is

$$\left(g_C^{\mathrm{Si}\,m} + g_L^{\mathrm{Si}\,m}\right) = \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \quad (b.1.1)$$

the total amount of public good provided in the sequential scenario corresponds to the positive solutions of the following expression:

$$\left(g_C^{Seq} + g_L^{Seq}\right) = \left(\frac{2\frac{\beta}{\lambda}(\beta-1)}{(1+\beta) \pm \sqrt[2]{1 + 2\beta + \beta^2 - 4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}\right)^{\frac{1}{2-\beta}} \quad (b.1.2)$$

Comparing the two values

$$\left(\frac{2\frac{\beta}{\lambda}(\beta-1)}{(1+\beta) \pm \sqrt[2]{1 + 2\beta + \beta^2 - 4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}\right)^{\frac{1}{2-\beta}} > \left(\frac{2\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \quad (b.1.3)$$

I have to consider only positive values of public good provided, so I find which expression of the right hand side of $b.1.3$ ensures positive values.

The numerator of the right hand side of $b.1.3$ is negative so I have to use an expression of denominator that is negative in order to consider only the case in which the total amount of public good provided is positive

$$(1+\beta)-\sqrt{1+2\beta+\beta^2-4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}<0 \qquad (b.1.4)$$

then I get:

$$4-4\beta>0$$

so the $b.1.4$ it is verified.

Then I evaluate the following inequality in order to prove if the $\left(g_C^{Seq}+g_L^{Seq}\right)>\left(g_C^{\mathrm{Si}\,m}+g_L^{\mathrm{Si}\,m}\right)$

$$\frac{2\frac{\beta}{\lambda}(\beta-1)}{(1+\beta)-\sqrt[2]{1+2\beta+\beta^2-4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}>\frac{2\beta}{\lambda} \qquad (b.1.5)$$

dividing both side by $2\frac{\beta}{\lambda}(\beta-1)$ I get

$$\frac{1}{(1+\beta)-\sqrt[2]{1+2\beta+\beta^2-4\frac{\beta}{\lambda}(\beta-1)\frac{\lambda}{\beta}}}<\frac{1}{\beta-1} \qquad (b.1.6)$$

$$+\beta^2-2\beta+1>0 \qquad (b.1.7)$$

the $b.1.7$ it is verified for every values of $\beta$ except for $\beta=1$, remind that in the model $0<\beta<1$ so $b.1.5$ is verified this prove that the total amount of public good provided is higher in sequential game.

### 8.2

### 8.3 Appendix B.2: Proof of Proposition 3

I want to prove that $\left(g_C^{Seq}+g_L^{Seq}\right)>\left(g_C^{\mathrm{Si}\,m}+g_L^{\mathrm{Si}\,m}\right)$ the total amount of public good provided is higher in sequential game.

The total amount of public good provided in the simultaneous scenario is

$$\left(g_C^{\mathrm{Si}\,m}+g_L^{\mathrm{Si}\,m}\right)=\left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \qquad (b.2.1)$$

the total amount of public good provided in the sequential scenario corresponds to the positive solutions of the following expression:

$$\left(g_C^{Seq}+g_L^{Seq}\right)=\left(\frac{2\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}{\left(\beta-\pi\left(\beta-1\right)\right)\pm\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\frac{\lambda}{\beta}\cdot\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}}\right)^{\frac{1}{2-\beta}}$$

Comparing the two values

$$\left(\frac{2\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}{\left(\beta-\pi\left(\beta-1\right)\right)\pm\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\frac{\lambda}{\beta}\cdot\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}}\right)^{\frac{1}{2-\beta}}>\left(\frac{\beta}{\lambda}\right)^{\frac{1}{2-\beta}} \tag{b.2.2}$$

I

I have to consider only positive values of public good provided, so I find which expression of the right hand side of $b.2.2$ ensures positive values.

The numerator of the right hand side of $b.2.3$ is negative so I have to use an expression of denominator that is negative in order to consider only the case in which the total amount of public good provided is positive

$$\beta-\pi\left(\beta-1\right)-\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\left(1-\pi\right)^2\left(\beta-1\right)}<0 \tag{b.2.3}$$

$$-4\left(1-\pi\right)^2\left(\beta-1\right)>0$$

the $b.2.3$ it is verified so I evaluate the following inequality in order to prove if the $\left(g_C^{Seq}+g_L^{Seq}\right)>\left(g_C^{\mathrm{Si}\,m}+g_L^{\mathrm{Si}\,m}\right)$ is

$$\frac{2\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}{\left(\beta-\pi\left(\beta-1\right)\right)-\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\frac{\lambda}{\beta}\cdot\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}}>\frac{\beta}{\lambda} \tag{b.2.4}$$

dividing both side by $2\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)$ I get

$$\frac{1}{\left(\beta-\pi\left(\beta-1\right)\right)-\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\frac{\lambda}{\beta}\cdot\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}}<\frac{1}{2\left(1-\pi\right)^2\left(\beta-1\right)} \tag{b.2.5}$$

both sides are negative so I have to show that

$$\left(\beta-\pi\left(\beta-1\right)\right)-\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\frac{\lambda}{\beta}\cdot\left(1-\pi\right)^2\frac{\beta}{\lambda}\left(\beta-1\right)}<2\left(1-\pi\right)^2\left(\beta-1\right) \tag{b.2.7}$$

$$\sqrt[2]{\left(\beta-\pi\left(\beta-1\right)\right)^2-4\cdot\left(1-\pi\right)^2\left(\beta-1\right)}>-2\left(1-\pi\right)^2\left(\beta-1\right)+\beta-\pi\left(\beta-1\right)$$

dividing by $(1-\pi)^2$and $(\beta-1)$ that is negative

$$-4<4\left(1-\pi\right)^2\left(\beta-1\right)-4\beta+4\pi\left(\beta-1\right)$$

that is

$$0<1-\beta \qquad (b.2.8)$$

$b.2.8$ it is verified because $0<\beta<1$ so $b.2.5$ is verified and this prove that the total amount of public good provided is higher in sequential game.