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# Home seekers in the Housing Market

Gaetano Lisi \*

## **Abstract**

*There are two types of home seekers in this housing market matching model: the homeless who search for a dwelling both in the rental market and in the homeownership market simultaneously; and the home seekers in the renter (tenant) state who want to buy a home and only search in the homeownership market. The search process leads to several types of matching and in turn this implies different prices of equilibrium. Furthermore, the search process connects the rental market with the homeownership market. Hence, this simple model is able to explain both the relationship between the rental price and the selling price and the price dispersion which exists in the housing market, relying only on the different states of agents in the search process.*

*Keywords:* rental market, homeownership market, housing price dispersion

*JEL Classification:* R21, R31, J63

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## 1. Introduction

Although recent, housing market studies that adopt search and matching models are not new in the economic literature (notably, Wheaton, 1990; Krainer, 2001; Albrecht *et al.*, 2007; Caplin and Leahy, 2008; Novy-Marx, 2009; Ngai and Tenreyro, 2009; Diaz and Jerez, 2009; Albrecht *et al.*, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011). Precisely, two goals are usually pursued: analysing the formation process of house price and its dynamics; explaining the behaviour of the housing market, in particular the price dispersion and the relationship between prices, time-on-the-market and sales.

All the papers in this literature formalise the important search and matching frictions which characterize the decentralized housing markets, but the key mechanism or insight behind the model is different: idiosyncratic shocks (Wheaton, 1990; Krainer, 2001; Albrecht *et al.*, 2007; Diaz and Jerez, 2009), preference shocks (Piazzesi and Schneider, 2009), mismatch between buyers and sellers (Caplin and Leahy, 2008), market tightness effect (Novy-Marx, 2009), thick-market effect on match-specific quality (Ngai and Tenreyro, 2009), demand shocks (Genesove and Han, 2010), and auction (Albrecht *et al.*, 2009).

The empirical “anomaly” known as ‘price dispersion’ is probably the most important distinctive feature of housing markets. It refers to the phenomenon of selling two houses with very similar attributes and in near locations at the same time but at very different prices. The literature has mainly responded to the price dispersion puzzle by introducing the heterogeneity of economic agents. In Leung and Zhang (2011), in fact, a necessary condition for explaining the housing price dispersion (as well as other basic facts) is the heterogeneity on the seller's and/or the buyer's side, which generates corresponding submarkets.

Nevertheless, price dispersion may arise from the very specific nature of the search process. In this model, there are in fact two types of home seekers: the homeless who search for a dwelling both in the rental market and in the homeownership market simultaneously; and the home seekers in the renter (tenant) state who want to buy a home and only search in the homeownership market. Hence, the search process leads to several types of matching; in turn, this implies different prices of equilibrium. Furthermore, the search process connects the rental market with the homeownership market. Indeed, as far as we are aware, the latter topic has been overlooked by housing market studies which adopt search and matching models. Indeed, papers in this literature omit from consideration

the rental housing market (Diaz and Jerez, 2009) or rely on the standard asset-market equilibrium condition (Ngai and Tenreyro, 2009),<sup>1</sup> thus assuming a rental market without frictions (Kashiwagi, 2011).<sup>2</sup>

Therefore, the main aim of this paper is to develop a search and matching model of the housing market which is able to explain both the price dispersion and the relationship between rental and selling prices, relying only on the different states of agents in the search process.

The rest of the paper is organised as follows: section 2 presents the housing market matching model; section 3 shows the existence of price dispersion, while section 4 describes the relationship between selling price and rental price; finally, section 5 closes the model and section 6 concludes the work.

## 2. The model

The housing market consists of the rental market and the homeownership market. In the homeownership market, the home-seeker who finds a dwelling and pays the selling price ( $p_s$ ) becomes the (new) owner of the house; whereas, this does not happen in the rental market, where the rental price ( $p_r$ ) only ensures the use of the house for a certain period of time. We distinguish these two markets by the subscript  $i = \{R, S\}$ , where  $R$  = rental market and  $S$  = homeownership market. A striking feature of the housing market is that today's buyers/home-seekers are potential tomorrow's sellers/landlords (Leung, Leong and Wong, 2006).

We adopt a standard matching framework *à la* Mortensen-Pissarides (see e.g. the textbook by Pissarides, 2000) with random search and prices determined by Nash bargaining. As regards the supply side, the expected values of posting a vacant house ( $V_i$ ) and of an occupied dwelling ( $D_i$ ) are the following:<sup>3</sup>

$$rV_R = -c_R + q(\vartheta_R) \cdot [D_R - V_R] \quad [1]$$

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<sup>1</sup> Assuming perfectly competitive housing markets, in equilibrium the risk-adjusted returns for homeowners and landlords should be equated across investments. This yields the usual user cost formula *à la* Poterba (1984) where the rental price covers the user cost of housing, which is equal to the house price multiplied by the user cost, i.e. the sum of the real after-tax interest rate, the combined depreciation and maintenance rate, and the expected future house price appreciation.

<sup>2</sup> Well-functioning rental markets can smooth out fluctuations in housing market liquidity (Krainer, 2001).

<sup>3</sup> Time is continuous and individuals are risk neutral, live infinitely and discount the future at the exogenous interest rate  $r > 0$ . As usual in matching-type models, the analysis is restricted to the stationary state.

$$rD_R = p_R + \delta \cdot [V_R - D_R] \quad [2]$$

$$rV_S = -c_S + q(\vartheta_S) \cdot [p_S - V_S] \quad [3]$$

where  $\vartheta_i$  is the housing market tightness (see later), with  $i = \{R, S\}$ ;  $c_i$  is the cost of posting vacancies;  $q(\vartheta_i)$  is the (instantaneous) probability of filling a vacant house, which depends on  $\vartheta_i$ ; and  $\delta$  is the lease destruction rate. Instead, in the homeownership market, if a contract is legally binding (as hypothesised) it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up. Hence, there is no destruction rate and the value of an occupied home is simple given by the selling price.

As regards the demand side, there are two types of home-seekers in this model: i) the homeless who search for a house in both markets simultaneously; ii) the home-seekers in the renter (tenant) state who want to buy a home and only search in the homeownership market. The value function of the homeless ( $H$ ) is the following:

$$\begin{aligned} rH &= -e_H - a + g(\vartheta_R) \cdot [T - H] + g(\vartheta_S) \cdot [x - p_S - H] \\ \Rightarrow H &= \frac{-e_H - a + g(\vartheta_R) \cdot T + g(\vartheta_S) \cdot [x - p_S]}{r + g(\vartheta_R) + g(\vartheta_S)} \end{aligned} \quad [4]$$

with  $\partial H/\partial T > 0$ ,  $\partial H/\partial p_S < 0$ , where  $T$  is the value of being a tenant;  $e_H$  is the effort (in monetary terms) made by the homeless to find and visit the largest possible number of houses;  $a$  is the cost of hotel accommodation;  $g(\vartheta_i)$  is the (instantaneous) probability of finding a vacant house, which depends on  $\vartheta_i$ , with  $i = \{R, S\}$ ; and  $x$  is the buyer's benefit (i.e. the value of the house). Instead,  $T$  is modelled as a staging post for searching in the homeownership market:

$$\begin{aligned} rT &= -e_T - p_R + g(\vartheta_S) \cdot [x - p_S - T] + \delta \cdot [H - T] \\ \Rightarrow T &= \frac{-e_T - p_R + g(\vartheta_S) \cdot [x - p_S] + \delta \cdot H}{r + g(\vartheta_S) + \delta} \end{aligned} \quad [5]$$

with  $\partial T/\partial H > 0$ ,  $\partial T/\partial p_S < 0$ ,  $\partial T/\partial p_R < 0$ , and  $e_H > e_T$ , since the homeless search in both markets.

A necessary condition for a non trivial equilibrium requires that:

$$(T - H) = \frac{(e_H - e_T) + a - p_R}{r + \delta + g(\vartheta_R) + g(\vartheta_S)} > 0$$

which is true if  $(e_H - e_T) + a > p_R$ , namely if the cost of being homeless is higher than the cost of being a tenant.

Market frictions in the rental and homeownership market are the following:

$$\vartheta_R = \frac{V_R}{h} \quad [6]$$

$$\vartheta_S = \frac{V_S}{h+t} \quad [7]$$

with  $q'(\vartheta_i) < 0$ , and  $g'(\vartheta_i) > 0$ ,  $\forall i$ , since  $v_i$  are the vacancies,  $t$  are the tenants, and  $h$  are the homeless.<sup>4</sup> The “zero profit” equilibrium condition (i.e.  $V_i = 0, \forall i$ ) normally used by matching models gives the market tensions of equilibrium (see Pissarides, 2000).<sup>5</sup> However, unlike the labour market matching model (which describes a negative relationship between market tightness and wage), in this case the free-entry condition yields a positive relationship between market tightness and price:

$$V_R = 0 \Rightarrow \frac{1}{q(\vartheta_R)} = \frac{p_R}{c_R \cdot (r + \delta)} \quad [8]$$

$$V_S = 0 \Rightarrow \frac{1}{q(\vartheta_S)} = \frac{p_S}{c_S} \quad [9]$$

This positive relationship is very intuitive: in fact, if the price increases, more vacancies will be on the market. We assume that market tensions are exogenous at the microeconomic level, in the sense that each individual takes  $\vartheta_R$  and  $\vartheta_S$  as given in the price bargaining.

### 3. Search and matching process, price bargaining and price dispersion

The generalised Nash bargaining solution, usually used for decentralised markets, allows the price to be obtained through the optimal subdivision of surplus deriving from a successful match. The surplus is defined as the sum of the seller/landlord’s and home-seeker’s value when the trade takes place, net of the respective external options (the value of continuing to search). Hence, a trade takes place between the parties at a price determined by Nash bargaining if the surplus is positive. Precisely, the price (both rental and selling) solves the following optimisation condition:

$$price = \operatorname{argmax} \left\{ (net\ gain\ of\ seller/landlord)^\gamma \cdot (net\ gain\ of\ homeseeker)^{1-\gamma} \right\} \quad [10]$$

where  $\gamma \in (0, 1)$  is the bargaining power of the seller/landlord.

<sup>4</sup> Standard technical assumptions are assumed:

$\lim_{\vartheta_i \rightarrow 0} q(\vartheta_i) = \lim_{\vartheta_i \rightarrow \infty} g(\vartheta_i) = \infty$ , and  $\lim_{\vartheta_i \rightarrow 0} g(\vartheta_i) = \lim_{\vartheta_i \rightarrow \infty} q(\vartheta_i) = 0$ ,  $\forall i$ .

<sup>5</sup> By definition, markets with frictions require positive and finite tightness, i.e.  $0 < \vartheta < \infty$ , since for  $\vartheta = 0$  the vacancies are always filled, whereas for  $\vartheta = \infty$  the home-seekers immediately find a vacant house.

Hence, the bargained price crucially depends on the surplus deriving from the matching. Precisely, in this model three kinds of matching can occur, thus leading to different surpluses:

- i. The homeless find a home in the rental market. This matching produces a rental price of equilibrium:  $p_R = \operatorname{argmax}\{(D_R - V_R)^\nu \cdot (T - H)^{1-\nu}\}$ ;
- ii. The homeless find a home in the homeownership market. This matching produces a selling price of equilibrium,  $p_S^A = \operatorname{argmax}\{(p_S - V_S)^\nu \cdot (x - p_S - H)^{1-\nu}\}$ ;
- iii. The home-seekers in the renter state find a home in the homeownership market. Hence, the selling price of equilibrium is  $p_S^B = \operatorname{argmax}\{(p_S - V_S)^\nu \cdot (x - p_S - T)^{1-\nu}\}$ .

Therefore, the existence of price dispersion can be straightforwardly shown. In fact, in the homeownership market the net gain of home-seekers is different and this produces two different surpluses. Eventually, from equation [10] two different selling prices ( $p_S^A$  and  $p_S^B$ ) are obtained. It follows that the origin of price dispersion is due to the specific nature of the search and matching process. Indeed, this result holds true even in the presence of an identical bargaining power, identical search costs and also when the same house is considered.

#### 4. The relation between selling price and rental price

As regards the selling prices, i.e. the matching (ii) and (iii) in the homeownership market, solving the optimisation conditions yields (recall that in equilibrium  $V_i = 0$ ,  $\forall i$ ):

$$(x - p_S^A - H) = \frac{1-\nu}{\nu} \cdot p_S^A \Rightarrow p_S^A = \nu \cdot (x - H)$$

$$(x - p_S^B - T) = \frac{1-\nu}{\nu} \cdot p_S^B \Rightarrow p_S^B = \nu \cdot (x - T)$$

Given the properties of equations [4] and [5], both  $p_S^A$  and  $p_S^B$  depend positively on  $p_R$ , yet remaining different since  $T \neq H$ . In fact, an increase in the rental price reduces both  $T$  (directly) and  $H$  (indirectly through  $T$ ). Therefore, without loss of generality, we can express this relationship in a broader form as follows:<sup>6</sup>

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<sup>6</sup> Alternatively, one could see  $p_S$  as a function of the two selling prices ( $p_S^A, p_S^B$ ) and set up a system of four equations in four unknowns ( $p_S, p_S^A, p_S^B, p_R$ ). However, this solution would add complexity but no further insight.

$$p_S = p_S(p_R) \quad [11]$$

with  $\partial p_S / \partial p_R > 0$ . Furthermore, if the rental price tends to zero, no one will have convenience to buy a house and the value of being a tenant will be at the maximum. As a result, the selling price will also tend to zero, since it cannot be negative or null.

Instead, as regards the matching (i) in the rental market, we obtain:

$$(T - H) = [(1 - \gamma) / \gamma] \cdot (D_R - V_R)$$

$$\Rightarrow (T - H) = \frac{1 - \gamma}{\gamma} \cdot \frac{p_R + c_R}{r + \delta + q(\vartheta_R)} \Rightarrow \frac{\gamma \cdot (r + \delta + q(\vartheta_R))}{1 - \gamma} \cdot (T - H) - c_R = p_R$$

We know that an increase in selling price reduces both  $T$  and  $H$ , since both home-seekers search in the homeownership market. Nevertheless, as long as the renter state is an appealing perspective, i.e. as long as  $g(\vartheta_R) > \delta$ , the decrease in  $T$  is stronger than the decrease in  $H$ . Indeed, buying a home is the only future perspective for a tenant. Hence, in this case we obtain a negative relationship between rental price and selling price:

$$p_R = p_R(p_S) \quad [12]$$

with  $\partial p_R / \partial p_S < 0$ . Therefore, the relationship between selling and rental prices can be represented in the diagram with axes  $[p_S, p_R]$ , where only a steady-state equilibrium exists in the housing market with positive prices (see Figure 1a).

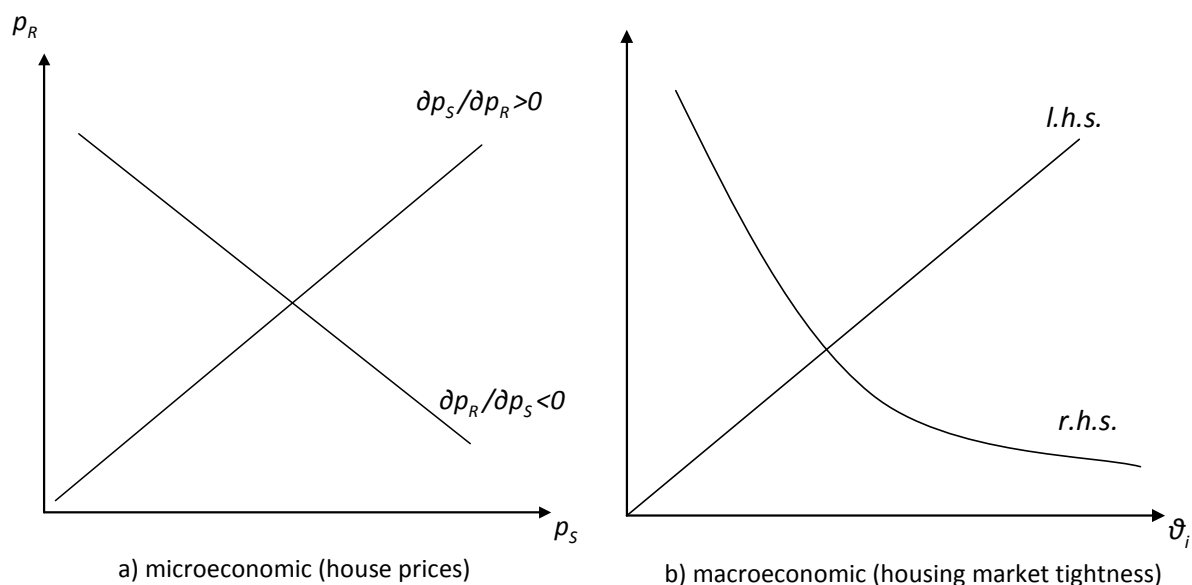


Figure 1. Equilibrium

Eventually, given  $p_R^*$  and  $p_S^*$ , we get a unique value of tightness for each market ( $\vartheta_R^*$  and  $\vartheta_S^*$ ) at the macroeconomic level. This testable proposition is made possible by a



downward sloping price function (in fact, *ceteris paribus*,  $\partial p_r / \partial \vartheta_r < 0$  and  $\partial p_s / \partial \vartheta_s < 0$ ), which forms the right hand side (*r.h.s.*) of the free-entry conditions (see equations [8]-[9] and *Figure 1b*).

## 5. Closing the model with the homelessness equation

In order to close the model, we normalise the home-seekers in the housing market to the unit:  $1 = h + t$ . In fact, the home-seekers in the renter state and the homeless who become homeowners exit the market (alternatively, one can assume that they become sellers or landlords).

The evolution of homelessness in the course of time ( $\dot{h}$ ) is thus the following:

$$\dot{h} = \delta \cdot (1 - h) - h \cdot [g(\vartheta_r) + g(\vartheta_s)] \quad [13]$$

$\delta \cdot (1 - h)$  represents homelessness inflows, i.e. existing leases cancelled at rate  $\delta$ , whereas  $h \cdot [g(\vartheta_r) + g(\vartheta_s)]$  describes the homelessness outflows, i.e. the homeless that find a home (as renter or as homeowner). Furthermore, the homelessness equation is independent of the transition rate which connects the renter (tenant) state to the homeowner state.

Finally, in steady state equilibrium, where homelessness is constant over time ( $\dot{h} = 0$ ), it follows that:

$$h = \frac{\delta}{\delta + g(\vartheta_r) + g(\vartheta_s)} \quad [14]$$

which has very intuitive properties:  $\partial h / \partial \delta > 0$ ,  $\partial h / \partial g(\vartheta_r) < 0$ , and  $\partial h / \partial g(\vartheta_s) < 0$ .

## 6. Conclusions

The literature has mainly responded to the price dispersion puzzle by introducing the heterogeneity of economic agents. Furthermore, the link between rental and homeownership markets has been overlooked by housing market studies which adopt search and matching models. This paper develops a search and matching model of the housing market which is able to explain both the price dispersion and the relationship between rental and selling prices, relying only on the different states of agents in the search and matching process.

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