# A coopetitive approach to financial markets stabilization and risk management

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**Abstract.** The aim of this paper is to propose a methodology to stabilize the financial markets by adopting Game Theory, in particular, the Complete Study of a Differentiable Game and the new mathematical model of Coopetitive Game, proposed recently in the literature by D. Carfì. Specifically, we will focus on two economic operators: a real economic subject and a financial institute (a bank, for example) with a big economic availability. For this purpose we will discuss about an interaction between the two above economic subjects: the Enterprise, our first player, and the Financial Institute, our second player. The only solution which allows both players to win something, and therefore the only one collectively desirable, is represented by an agreement between the two subjects: the Enterprise artificially causes an inconsistency between spot and future markets, and the Financial Institute, who was unable to make arbitrages alone, because of the introduction by the normative authority of a tax on economic transactions (that we propose to stabilize the financial market, in order to protect it from speculations), takes the opportunity to win the maximum possible collective (social) sum, which later will be divided with the Enterprise by contract. We propose hereunder two kinds of agreement: a fair transferable utility agreement on the an initial natural interaction and a same type of compromise on a quite extended coopetitive context.

**Keywords:** Financial Markets and Institutions; Financing Policy; Financial Risk; Financial Crises; Game Theory; Arbitrages; Coopetition

## 1 Introduction

The recent financial crisis has shown that, in order to stabilize markets, it is not enough to prohibit or to restrict short-selling, in fact:

- *big speculators can influence badly the market and take huge advantage from arbitrage opportunities, caused by themselves.*

In this paper, by the introduction of a tax on financial transactions, we propose a method in order to limit the speculations of medium and big financial operators and consequently a way to make more stable the financial market; our aim is attained without inhibiting the possibilities of profits. At this purpose we will present and study a natural and quite general normal-form game as a possible standard model of fair interaction between two financial operators, which gives to both players mutual economic advantages. Finally, we shall propose an even more advantageous coopetitive model and its possible compromise solution.

## 2 Methodologies

The normal-form game $G$, that we propose to model our financial interaction, requires a construction which takes place on 3 times, which we say time 0, time 1 and time 2.

At time 0 the Enterprise can choose if to buy futures contracts to hedge the market risk of the underlying commodity, which (the Enterprise knows) should be bought at time 1, in order to conduct its business activities.

The Financial Institute, on the other hand, acts with speculative purposes, on spot markets (buying or short-selling the goods at time 0) and future market (doing the action contrary to that one made on the spot market: if the Financial Institute short-sold goods on spot market, it purchases its on the futures market, and vice versa) of the same product, which is of interest for the Enterprise. The Financial Institute may so take advantage of the temporary misalignment of the spot and future prices that would be created as a result of a hedging strategy by the Enterprise. At the time 2 the Financial Institute will cash or pay the sum determined by its behavior in the futures market at time 1.

# 3 Financial preliminaries

Here we recall the financial concepts that we shall use in the present article.

- Any positive real number defines a ***purchasing strategy***, on the other hand a negative real number defines a ***selling strategy***.
- The ***spot market*** is the market where it is possible to buy and sell at current prices.
- ***Futures*** are contracts through which you undertake to exchange, at a predetermined price a certain quantity of the underlying commodity at the expiry of the contract.
- ***A hedging operation*** through futures by a trader consists in the purchase of future contracts in order to reduce exposure to specific risks on market variables (in this case on the price).
- A hedging operation is defined to be a ***perfect hedging operation*** when it completely eliminates the risk of the case.
- ***The future price*** is linked to the underlying spot price. Assuming that:
  1. the underlying commodity does not offer dividends;
  2. the underlying commodity hasn't storage costs and has not convenience yield to take physical possession of the goods rather than future contract.

  Then, the general relationship linking future price and spot price, with unique interest capitalization at the time $t$, is the following one:

  $$F_0 = S_0(1+i)^t.$$

  If not, the arbitrageurs would act on the market until future and spot prices return to levels indicated by the above relation.

# 4 The game and stabilizing proposal

## 4.1 The description of the game

We assume that our first player is an Enterprise that may choose if to buy futures contracts to hedge, in a way that we assume perfect, by an upwards change in the price of the underlying commodity that the Enterprise knows to buy at time 1, to the conduct of its business. Therefore, the Enterprise has the possibility to choose a strategy $x \in [0,1]$ which represents the percentage of the quantity of the underlying $M_1$ that the Enterprise itself will purchase through futures, depending on whether it intends:

1. to not hedge ($x = 0$),
2. to hedge partially ($0 < x < 1$),
3. to hedge totally ($x = 1$).

On the other hand, our second player is a Financial Institute operating on the spot market of the underlying asset that the Enterprise knows it should buy at time 1. The Financial Institute works in our game also on the futures market:

- taking advantage of possible gain opportunities - given by misalignment between spot prices and futures prices of the commodity;
- or accounting for the loss obtained, because it has to close the position of short sales opened on the spot market.

They are just these actions to determine the win or the loss of the Financial Institute.

The Financial Institute can therefore choose a strategy $y \in [-1,1]$ which represents the percentage of the quantity of the underlying $M_2$ that it can buy (in algebraic sense) with its financial resources, depending on whether it intends:

1. to purchase the underlying on the spot market ($y > 0$);
2. to short sell the underlying on the spot market ($y < 0$);
3. to not intervene on the market of the underlying ($y = 0$).

Now we illustrate graphically the bi-strategy space $E \times F$ of the game:

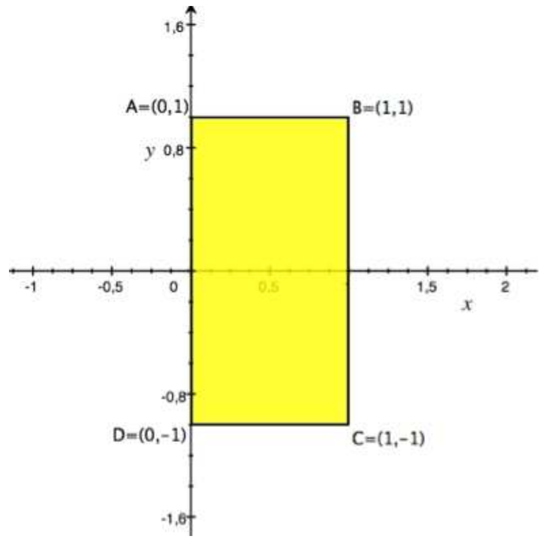


**Fig. 1.** The bi-strategy space of the game

### 4.2 The payoff function of the Enterprise

The payoff function of the Enterprise, that is the function which represents quantitative loss or win of the Enterprise, referred to time 1, will be given by the win (or loss) obtained on goods not covered. The win relating with the not covered goods will be given by the quantity of the uncovered goods $(1-x)M_1$, multiplied by the difference $F_0 - S_1(y)$, between the future price at time 0 (the term $F_0$) - which the Enterprise should pay, if it decides to hedge - and the spot price $S_1(y)$ at time 1, when the Enterprise actually will buy the goods which it did not hedge.

In mathematical language, the payoff function of the Enterprise is given by

$$f_1(x,y) = M_1(1-x)(F_0 - S_1(y)), \tag{1}$$

for every bi-strategy $(x,y)$ in $E \times F$, where:

- $M_1$ is the amount of goods that the Enterprise should buy at time 1;
- $(1-x)$ is the percentage of the underlying asset that the Enterprise will buy on the spot market at time 1 without any coverage (and therefore exposed to the fluctuations of the spot price of the goods);
- $F_0$ is the future price at time 0. It represents the price established at time 0 that the Enterprise will have to pay at time 1 to buy the goods. By definition, assuming the absence of dividends, known income, storage costs and convenience yield to keep possession the underlying, the future price after $(t-0)$ time units is given by

$$F_0 = S_0(1+i)^t, \tag{2}$$

where $(1+i)^t$ is the capitalization factor with rate $i$ at time $t$. By $i$ we mean the risk-free interest rate charged by banks on deposits of other banks, the so-called "LIBOR" rate. $S_0$ is, on the other hand, the spot price of the underlying asset at time 0. $S_0$ is a constant because it does not influence our strategies $x$ and $y$.

– $S_1(y)$ is the spot price of the underlying at time 1, after that the Financial Institute has played its strategy $y$. It is given by

$$S_1(y) = S_0(1+i) + ny(1+i) \tag{3}$$

where $n$ is the marginal coefficient representing the effect of the trategy $y$ on the price $S_1$. The price function $S_1$ depends on $y$ because, if the Financial Institute intervenes in the spot market by a strategy $y$ not equal to 0, then the price $S_1(y)$ changes because any demand change has an effect on the asset price. We are assuming the dependence $n \mapsto ny$ in $S_1(y)$ as linear by assumption. The value $S_0$ and the value $ny$ should be capitalized, because they should be "transferred" from time 0 to time 1.

***The payoff function of the Enterprise.*** Therefore, remembering the Eq. (3), that is

$$S_1(y) = (S_0 + ny)(1+i),$$

and the Eq. (2), that is

$$F_0 = S_0(1+i),$$

and replacing them in the Eq. (1), that is

$$f_1(x,y) = M_1((1-x)(F_0 - S_1(y)),$$

we have:

$$f_1(x,y) = M_1((1-x)[S_0(1+i) - (S_0+ny)(1+i)].$$

After the appropriate simplifications, here is represented the payoff function of the Enterprise:

$$f_1(x,y) = M_1(1-x)(-ny(1+i)). \tag{4}$$

From now the value $n(1+i)$ will be called $\nu_1$ for simplicity of calculation.

### 4.3 The payoff function of the Financial Institute

The payoff function of the Financial Institute, that is the function representing the algebraic gain of the Financial Institute at time 1, is the multiplication of the quantity of goods bought on the spot market, that is $yM_2$, by the difference between the future price $F_1(x,y)$ (it is a price established at time 1 but cashed at time 2) transferred to time 1, that is

$$F_1(x,y)(1+i)^{-1},$$

and the purchase price of goods at time 0, say $S_0$, capitalized at time 1 (in other words we are accounting for all balances at time 1).

**Stabilizing strategy of normative authority.** We therefore propose that - in order to avoid speculations on spot and future markets by the Financial Institute, which in this model is the only one able to determine the spot price (and consequently also the future price) of the underlying commodity - the normative authority imposes to the Financial Institute the payment of a tax on the sale of the futures. So the Financial Institute can't take advantage of price swings caused by itself. This tax will be fairly equal to the incidence of the strategy of the Financial Institute on the spot price, so the price effectively cashed or paid for the futures by the Financial Institute is

$$F_1(x,y)(1+i)^{-1} - ny(1+i),$$

where $ny(1+i)$ is the tax paid by the Financial Institute, referred to time 1.

**Observation**. We note that if the Financial Institute wins, it will act on the future market at time 2 to cash the win, but also in case of loss it must necessarily act in the future market and account for its loss because at time 2 (in the future market) it should close the short-sale position opened on the spot market.

In mathematical terms, the payoff function of the Financial Institute is:

$$f_2(x, y) = yM_2[F_1(x, y)(1 + i)^{-1} - ny(1 + i) - S_0(1 + i)], \tag{5}$$

where:

- $y$ is the percentage of goods that the Financial Institute purchases or sells on the spot market of the underlying;
- $M_2$ is the amount of goods that the Financial Institute can buy or sell on the spot market according to its disposable income;
- $S_0$ is the price at which the Financial Institute bought the goods. $S_0$ is a constant because our strategies $x$ and $y$ does not have impact on it.
- $ny(1 + i)$ is the normative tax on the price of the futures paid at time 1. We are assuming the tax is equal to the incidence of the strategy y of the Financial Institute on the price $S_1$.
- $F_1(x, y)$ is the price of the future market (established) at time 1, after the Enterprise has played its strategy $x$.
  The price $F_1(x, y)$ is given by
  $$F_1(x, y) = S_1(1 + i) + mx(1 + i), \tag{6}$$
  where $(1 + i)$ is the factor of capitalization of interests. By $i$ we mean risk-free interest rate charged by banks on deposits of other banks, the so-called LIBOR rate. With $m$ we intend the marginal coefficient that measures the impact of $x$ on $F_1(x, y)$. The price $F_1(x, y)$ depends on $x$ because, if the Enterprise buys futures with a strategy $x \neq 0$, the price $F_1$ changes because an increase of future demand influences the future price. The value $S_1$ should be capitalized because it follows the relationship between futures and spot prices expressed in Eq. 3.1. The value $mx$ is also capitalized because the strategy $x$ is played at time 0 but has effect on the future price at time 1.
- $(1+i)^{-1}$ is the discount factor. $F_1(x, y)$ must be actualized at time 1 because the money for the sale of futures will be cashed in a time 2.

***The payoff function of the Financial Institute.*** Recalling the Eq. (6), that is

$$F_1(x, y) = S_1(y)(1 + i) + mx(1 + i),$$

and replacing it in the Eq. (5), that is

$$f_2(x, y) = yM_2[F_1(x, y)(1 + i)^{-1} - ny(1 + i) - S_0(1 + i)],$$

we have, after the appropriate simplifications, the payoff function of the Financial Institute:

$$f_2 : E \times F \to \mathbb{R} : f_2(x, y) = yM_2mx. \tag{7}$$

Summarizing, the payoff function of our game is the following:

$$f : E \times F \to \mathbb{R}^2 : f(x, y) = (-\nu_1 yM_1(1 - x), yM_2mx). \tag{8}$$

# 5 Study of the game

## 5.1 Nash equilibria

If the two players want to think only for themselves, they would choose the strategy that makes maximum their win regardless of the other player's strategy. In this case we talk about multifunction of best reply. It means to maximize for each player its payoff function considering every possible strategy of the other player. In mathematical language the multifunction of best reply of the Enterprise is:

$$B_1 : F \to E : y \mapsto \max_{f_1(\cdot,y)} E$$

(i.e. the strategies in $E$ of the Enterprise which maximize the section $f_1(\cdot, y)$).

On the other hand the multifunction of best reply of the Financial Institute is:

$$B_2 : E \to F : x \mapsto \max_{f_2(x,\cdot)} F$$

(i.e. the strategies in $F$ of the Financial Institute which maximize the section $f_2(x,\cdot)$).

Remembering that $M_1 = 1$, $\nu_1 = 1/2$, $M_2 = 2$ and $m = 1/2$ are always positive numbers (strictly greater than 0), and the Eq. (4), that is $f_1(x,y) = M_1[-\nu_1 y(1-x)]$, we have

$$\partial_1 f_1(x,y) = M_1\nu_1 y,$$

this derivative is positive iff $M_1\nu_1 y > 0$, and so:

$$B_1(y) = \{1\} \iff y > 0$$

$$B_1(y) = E \iff y = 0$$

$$B_1(y) = \{0\} \iff y < 0$$

Remembering also the Eq. (7), that is $f_2(x,y) = M_2 mxy$, we have

$$\partial_2 f_2(x,y) = M_2 mx$$

$$M_2 mx > 0$$

and so:

$$B_2(x) = \{1\} \iff x > 0$$

$$B_2(x) = F \iff x = 0.$$

Representing in red the graph of $B_1$, and in blue that one of $B_2$ we have:

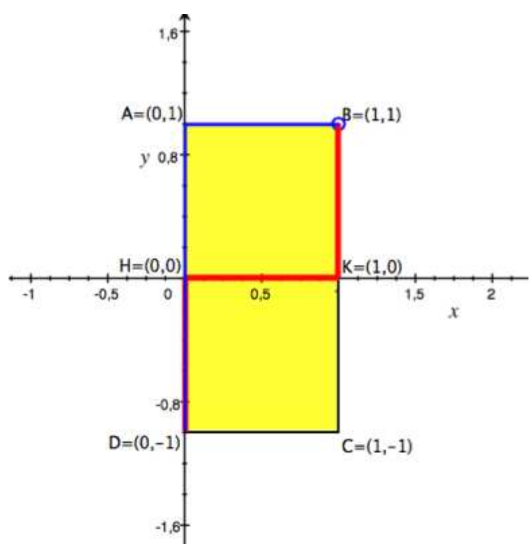


**Fig. 2.** Nash equiliria

The set of Nash equilibria, that is the intersection of the two best reply graphs, is

$$\mathrm{Eq}(\mathrm{B}_1, \mathrm{B}_2) = (1,1) \cup [\mathrm{H}, \mathrm{D}].$$

The Nash equilibria can be considered quite good, because they are on the weak maximal Pareto boundary. It is clear that if the two players pursue as aim the profit, and decide to choose their selfish strategy to obtain the maximum possible win, they will arrive on the weak maximal boundary. The selfishness, in this case, pays well. This purely mechanical examination, however, leaves us unsatisfied. The Enterprise has two Nash possible alternatives: not to hedge, playing

$x = 0$, or to hedge totally, playing $x = 1$. Playing $x = 0$ it could both to win or lose, depending on the strategy played by the Financial Institute; opting instead for $x = 1$, the Enterprise guarantee to himself to leave the game without any loss and without any win.

Analyzing the strategies of the Financial Institute relevant for Nash, we see that if the Enterprise adopts a strategy $x$ different from 0, the Financial Institute plays the strategy $y = 1$ winning something, or else if the Enterprise plays $x = 0$ the Financial Institute can play all its strategy set $F$, indiscriminately, without obtaining any win or loss. These considerations lead us to believe that the Financial Institute will play $y = 1$, in order to try to win at least "something", because if the Enterprise plays $x = 0$, its strategy $y$ does not affect its win. The Enterprise, which knows the situation of the Financial Institute that very likely chooses the strategy $y = 1$, will hedge playing $x = 1$.

So, despite the Nash equilibria are infinite, it is likely that the two players arrive in $B = (1, 1)$, which is part of the proper maximal Pareto boundary. Nash is a viable, feasible and satisfactory solution, at least for one of two players, presumably the Financial Institute.

### 5.2 Cooperative solutions

The best way for two players to get both a win is to find a cooperative solution. One way would be to divide the maximum collective profit, determined by the maximum of the collective gain functional $g$, defined by

$$g(X, Y) = X + Y$$

on the payoffs space of the game $G$, i.e the profit $W = \max_{f(E\times F)} g$.

The maximum collective profit $W$ is attained (with evidence) at the point $B'$, which is the only bi-win belonging to the straight line $g^{-1}(1)$ with equation $X + Y = 1$ and to the payoff space $f(E \times F)$.

So the Enterprise and the Financial Institute play $x = 1$ and $y = 1$, in order to arrive at the payoff $B'$ and then they split the obtained bi-win $B'$ by means of a contract.

Practically: the Enterprise buys futures to create artificially a misalignment between future value and spot prices, misalignment that is exploited by the Financial Institute, which get the maximum win $W = 1$.

For a possible quantitative division of this win $W = 1$, between the Financial Institute and the Enterprise, we use the transferable utility solution, applying to the transferable utility Pareto boundary of the payoff space the non-standard Kalai-Smorodinsky solution (non-standard because we do not consider the whole game, but only the maximal Pareto boundary).

We proceed finding the supremum of our maximal Pareto boundary, which is

$$\sup \partial^* f(E \times F) =: \alpha = (1/2, 1);$$

and join it with the infimum of our maximal Pareto boundary, which is given by

$$\inf \partial^* f(E \times F) = (0, 0).$$

We note that the infimum of our maximal Pareto boundary is equal to $v^\sharp = (0, 0)$.

The coordinates of the intersection point $P$, between the straight line of maximum collective win (i.e. $X+Y = 1$) and the straight line joininig the supremum of the maximal Pareto boundary with the infimum (i.e., the line $Y = 2X$) give us the desirable division of the maximum collective win $W = 1$ between the two players.

We can see the following figure in order to make us more aware of the situation:

In order to find the coordinates of the point P is enough to put in a system of equations $X + Y = 1$ and $Y = 2X$. Substituting the Y in the first equation we have $X + 2X = 1$ and therefore $X = 1/3$. Substituting now the X in the second equation, we have $Y = 2/3$.

Thus $P = (1/3, 2/3)$ suggests as solution that the Enterprise receives $1/3$ by contract by the Financial Institute, while at the Financial Institute remains the win $2/3$.

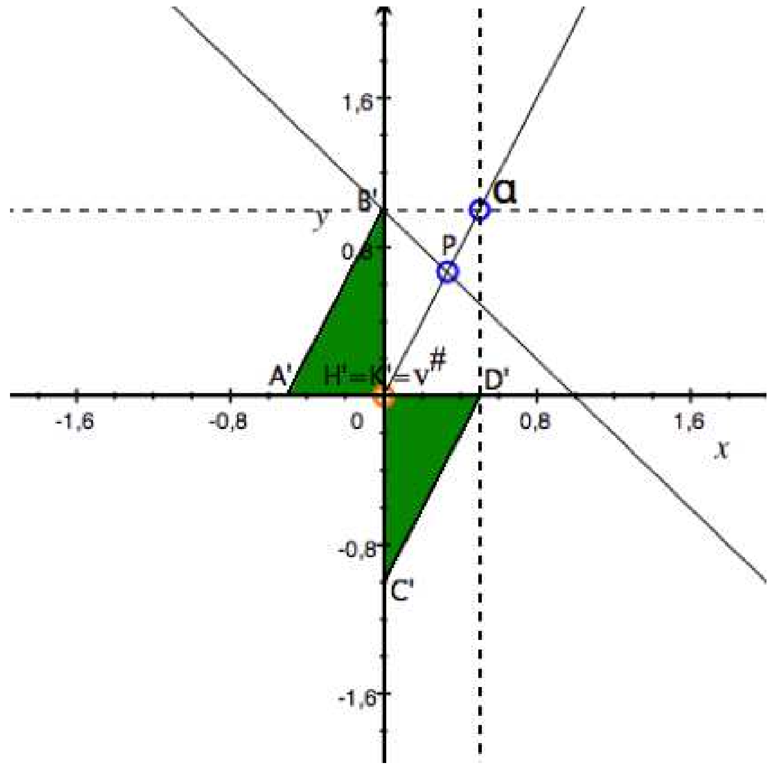


**Fig. 3.** Transferable utility solution: cooperative solution

# 6 Coopetitive approach.

## 6.1 The shared strategy

Now we pass to a coopetitive approach of our game G.

***Interpretation.*** We have two players, the Enterprise and the Financial Institute, each of them has a strategy set in which to choose his strategy; moreover, the two players can cooperatively choose a strategy z in a third set C. The two players will choose their cooperative strategy z to maximize (in some sense) the gain function f.

The strategy $z \in [0, A]$ is a shared strategy, which consists in the possibility for the Enterprise to use money borrowed by the Financial Institute from the European Central Bank with a very low interest rate (hypothesis highly plausible given the recent anti-crisis measures adopted by the ECB), rate which by convention we assume equal to 0. The two players want the loan so that the Enterprise can create an even higher misalignment between spot and futures price, misalignment which will be exploited by the Financial Institute. In this way, both players can get a greater win than that one obtained without a shared strategy z.

The two players can then choose a shared strategy depending on they want:

1. to not use the money of the ECB ($z = 0$)
2. to use a part of the money of the ECB so that the Enterprise purchases futures ($0 < z < A$)
3. to use totally the money of the ECB so that the Enterprise purchases futures ($z = A$)

## 6.2 The payoff function of the Enterprise

In practice, in the Eq. (4), that is

$$f_1(x, y) = M_1(1 - x)(-ny(1 + i)),$$

we must add the action of the Enterprise to buy futures contracts and after sell them. The win of the Enterprise is given by the quantity of futures brought $zF_0^{-1}$ multiplied by the difference

$$F_1(1 + i)^{-1} - F_0$$

between the future price at time 1 when it sell the futures and the future price at time 0 when it buys the futures.

**Remark.** Similarly to what happened to the Financial Institute, also the Enterprise will have to pay on the sale of the futures contracts a tax equal to its impact on the price of the futures, in order to avoid speculative acts, created by itself. In mathematical terms we have:

$$f_1(x,y,z) = -\nu_1 y M_1(1-x) + zF_0^{-1}(F_1(x,y,z)(1+i)^{-1} - m(x+zA^{-1}) - F_0) \tag{9}$$

where

- $zF_0^{-1}$ is the quantity of futures purchased. It is the ratio between the money z taken on loan from the ECB and $F_0$, the futures price at time 0.
- $m(x+zA^{-1})$ is the normative tax paid by the Enterprise on the sale of futures, referred to the time 1. In keeping with the size of x, also $zA^{-1}$ is a percentage. In fact $zA^{-1}$ is given by the quantity of future purchased with the strategy z (i.e. $zF_0^{-1}$) multiplied by $F_0A^{-1}$, that is the total maximum quantity of futures that the Enterprise can buy with the strategy z.
  **Remark.** From now on the value A will be equal to 1 for semplicity of calculation.
- $F_1(x,y,z)$ is the price of the future market (established) at time 1, after the Enterprise has played its strategies x and z. The price $F_1(x,y,z)$ is given by

$$F_1(x,y,z) = ((S_0+ny)(1+i)^2 + m(x+zA^{-1}))(1+i), \tag{10}$$

  where $(1+i)$ is the factor of capitalization of interests. By i we mean risk-free interest rate charged by banks on deposits of other banks, the so-called LIBOR rate. With m we intend the marginal coefficient that measures the impact of x and $zA^{-1}$ on $F_1(x,y,z)$. $F_1(x,y,z)$ depends on x and z because, if the Enterprise buys futures with a strategy x not equal to 0 or z not equal to 0, the price $F_1$ changes because an increase of future demand influences the future price.
- $(1+i)^{-1}$ is the discount factor. $F_1(x,y,z)$ must be actualized at time t = 1 because the money for the sale of futures will be cashed in a time t = 2.
- $F_0$ is the future price at time 0. It represents the price established at time 0 that will have to pay at time 1 to buy the goods. It is given by

$$F_0 = S_0(1+i)^t,$$

  where $(1+i)^t$ is the capitalization factor with rate i at time t. By i we mean risk-free interest rate charged by banks on deposits of other banks, the so-called "LIBOR" rate. $S_0$ is, on the other hand, the spot price of the underlying asset at time 0. $S_0$ is a constant because it does not influence our strategies x, y and z.

***The payoff function of the Enterprise.*** Remembering the Eq. (10), that is

$$F_1(x,y,z) = (S_0+ny)(1+i)^2 + m(x+zA^{-1})(1+i),$$

and the Eq. (2), that is

$$F_0 = S_0(1+i),$$

and replacing in the Eq. (9), that is

$$f_1(x,y,z) = -\nu_1 y M_1(1-x) + zF_0^{-1}(F_1(x,y,z)(1+i)^{-1} - m(x+zA^{-1}) - F_0),$$

setting $u := 1+i$, we have:

$$f_1(x,y,z) = -\nu_1 y M_1(1-x) + zF_0^{-1}(((S_0+ny)u^2 + m(x+zA^{-1})u)u^{-1} - m(x+zA^{-1}) - S_0u).$$

After calculations, we have

$$f_1(x,y,z) = -\nu_1 y M_1(1-x) + zF_0^{-1}\nu_1 y. \tag{11}$$

From now on, the value $F_0$ will be equal to 1, for simplicity of calculation.

### 6.3 The payoff function of the Financial Institute

***The payoff function of the Financial Institute.*** In the payoff function of the second player, we have to add the strategy z multiplied by $A^{-1}$ (that, as we remember, is equal to 1) to the strategy x played by the Enterprise. In mathematical terms, remembering the Eq. (7), that is

$$f_2(x, y) = yM_2mx,$$

we have

$$f_2(x, y, z) = yM_2m(x + z).$$

After calculations, we obtain

$$f_2(x, y, z) = yM_2mx + yM_2mz \tag{12}$$

Summarizing, we have

$$f(x, y, z) = (-\nu_1 yM_1(1 - x), yM_2mx) + yz(\nu_1, M_2m). \tag{13}$$

### 6.4 The coopetitive translating vectors

We note immediately as the result is the same payoff function of the original game

$$f(x, y) = (-\nu_1 yM_1(1 - x), yM_2mx),$$

translated by the vector

$$v(y) := zy(\nu_1, M_2m).$$

Recalling that $y \in [-1, 1]$ and $z \in [0, 1]$, we see that the vector belongs to the vector range

$$yz(\nu_1, M_2m) \in [-1, 1](\nu_1, M_2m).$$

We can note a significant result: our old payoff space was partitioned into two relevant parts. In fact, recalling the Eq. (8.7), that is

$$f(x, y, z) = (-\nu_1 yM_1(1 - x), yM_2mx) + yz(\nu_1, M_2m),$$

and recalling that $z \in [0, 1]$, it is clear that with a shared strategy $z > 0$ the part of the game where $y$ is greater than 0 is translated upwards, while the part of the game where $y$ is less than 0 is translated downwards. Because our coopetitive game makes sense if and only if the two players will agree and collaborate to maximize their wins with the strategies $x$ and $y$ greater than 0 (it would be paradoxical to choose a strategy $z > 0$ to increase the loss), our bi-strategic space is reduced to the square $[0, 1]^2$. We now show that the shared strategy that maximizes the wins when $y \geq 0$ is always $z = 1$. In other words we want show that: $f(x, y, z) \leq f(x, y, 1)$, for every $y \geq 0$ and every $x$ in $E$.

Remembering the Eq. (13), that is

$$f(x, y, z) = (-\nu_1 yM_1(1 - x), yM_2mx) + yz(\nu_1, M_2m),$$

we have

$$(-\nu_1 yM_1(1 - x), yM_2mx) + yz(\nu_1, M_2m) \leq (-\nu_1 yM_1(1 - x), yM_2mx) + y(\nu_1, M_2m).$$

After calculations, we have

$$yz(\nu_1, M_2m) \leq y(\nu_1, M_2m)$$

and therefore $yz \leq y$, which is indeed verified only for any $y \geq 0$.

We can show also that $f(x, y, z) \geq f(x, y, 0)$, for every $y \geq 0$ and every x in E.

Remembering the Eq. (13), that is

$$f(x, y, z) = (-\nu_1 yM_1(1 - x), yM_2mx) + yz(\nu_1, M_2m),$$

we have

$$(-\nu_1 y M_1(1-x), y M_2 m x) + y z \nu_1, M_2 m) \geq (-\nu_1 y M_1(1-x), y M_2 m x).$$

After calculations, we have

$$yz(\nu_1, M_2 m) \geq 0$$

and therefore $yz \geq 0$, which is indeed verified only for any $y \geq 0$.

Because with $x \in [0,1]$ and $y \in [0,1]$ we have

$$f(x,y,0) \leq f(x,y,z) \leq f(x,y,1),$$

we arrive to a very important result: we know that all possible combinations of the bi-strategic space $[0,1] \times [0,1]$ are included in the payoff space of the transformations of $f(x,y,0)$ and $f(x,y,1)$.

So, transforming our bi-strategic space $[0,1]^2$ on the payoff space with $z=0$ (in dark green) and $z=1$ (in light green), we have the whole payoff space of the our coopetitive game and obtain the following figure:

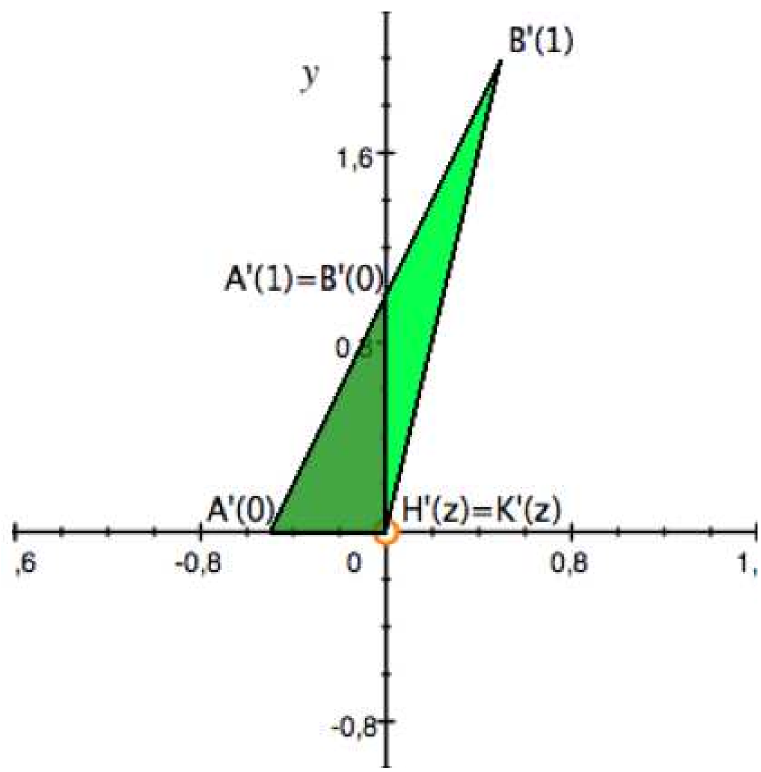


**Fig. 4.** The payoff space of the coopetitive game $f([0,1]^3)$

If the Enterprise and the Financial Institute play the strategies $x=1$, $y=1$ - respectively - and the shared strategy $z=1$, they arrive at the point $B'(1)$, that is the maximum of the coopetitive game $G$, so the Enterprise wins 1/2 (amount greater than 1/3 obtained in the co-operative phase) while the Financial Institute wins even 2 (an amount much greater than 2/3, value obtained in the first cooperative phase).

### 6.5 Kalai-smorodisky solution.

The point $B'(1)$ is the maximum of the game. But the Enterprise could be not satisfied by the win 1/2, value that is much more little than the win 2 of the Financial Institute. In addition playing the shared strategy $z=1$, the Enterprise increases slightly the win obtained in the non-coopetitive game, while the Financial Institute even wins more than double. For this reason, precisely to avoid that the envy of the Enterprise can affect the game, the Financial Institute might be willing to cede part of its win by contract to the Enterprise in order to make more balanced the distribution of money. One way would be to divide the maximum collective profit, determined by the maximum of the function of collective gain

$$g(X, Y) = X + Y$$

on the payoffs space of the game G, i.e the profit $W = \max_S g$.

The maximum collective profit is represented with evidence by the point $B'(1)$, which is the only bi-win belonging to the straight line $X + Y = 5/2$ and to the payoff space.

So the Enterprise and the Financial Institute play $x = 1$, $y = 1$ and $z = 1$ in order to arrive at the payoff $B'(1)$ and then split the wins obtained by contract.

Practically: the Enterprise buys futures to create artificially (also thanks to the money borrowed from the European Central Bank) a very big misalignment between future and spot prices, misalignment that is exploited by the Financial Institute, which get the maximum win $W = 5/2$.

For a possible quantitative division of this win $W = 5/2$ between the Financial Institute and the Enterprise, we use the transferable utility solution applying the Kalai Smorodinsky.

We proceed finding the inferior extremum of our game, which is

$$\inf \partial^* f(E \times F \times C) = (-1/2, 0)$$

and join it with the superior extremum according to Kalai-Smorodinsky method, which is given by

$$\sup \partial^* f(E \times F \times C) = (5/2, 3).$$

The coordinates of the point of intersection $P'$ between the straight line of maximum collective win (i.e. $X + Y = 2.5$) and the straight line which joins the supremum with the infimum (i.e. $Y = X + 1/2$) give us the desirable division of the maximum collective win $W = 2.5$ between the two players.

We can see the following figure in order to make us more aware of the situation:

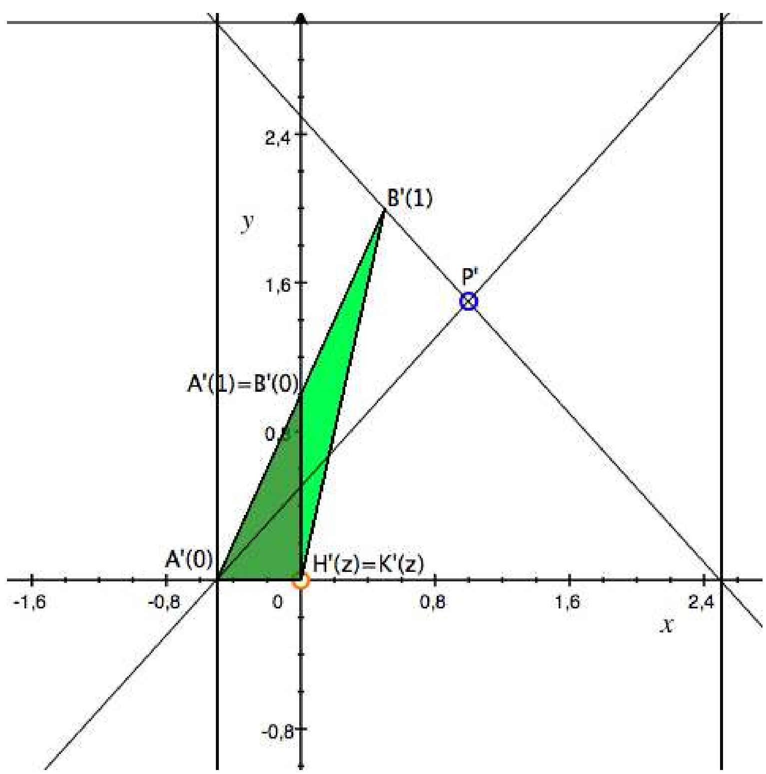


**Fig. 5.** Transferable utility solution in the coopetitive game: cooperative solution.

In order to find the coordinates of the point $P'$ is enough to put in a system of equations $X+Y=2.5$ and $Y=X+1/2$. Substituting the $Y$ in the first equation we have $X+X+1/2=2.5$ and therefore $X=1$. Substituting now the $X$ in the second equation, we have $Y=3/2$.

Thus $P'=(1,3/2)$ suggests as solution that the Enterprise receives 1 (the triple than the win obtained in the cooperative phase of the non-coopetitive game) by contract by the Financial Institute, while at the Financial Institute remains the win 3/2 (more than double than the win obtained in the cooperative phase of the non-coopetitive game).

## 7 Conclusions

The game just studied suggests a possible regulatory model that provides the stabilization of the financial market through the introduction of a tax on financial transactions. In fact, in this way it could be possible to avoid speculation by which our modern economy is constantly affected, and the Financial Institute could equally wins without burdening on the entire financial system with its unilateral manipulation of the asset price that it trades.

**Non-coopetitive game.** The only optimal solution is the cooperative one exposed in the section , otherwise the game appears like a sort of "your death, my life", as often happens in the economic competition, which leaves no escape if either player decides to work alone, without a mutual collaboration. In fact, all non-cooperative solutions lead dramatically to mediocre results for at least one of the parties. Now it is possible to provide an interesting key in order to understand the conclusions which we reached using the transferable utility solution. Since the point $B=(1,1)$ is also the most likely Nash equilibrium, the number 1/3 (that the Financial Institute pays by contract to the Enterprise) can be seen as the fair price paid by the Financial Institute to be sure that the Enterprise chooses the strategy $x=1$, so they arrive effectively to more likely Nash equilibrium $B=(1,1)$, which is also the optimal solution for the Financial Institute.

**Coopetitive game.** We can see that the game becomes much easier to solve in a satisfactory manner for both players. Both the Enterprise and the Financial Institute reduce their chances of losing than the non-coopetitive game, and even they can easily reach to the maximum of the game: so the Enterprise wins 1/2 and the Financial Institute wins 2. If they instead take the tranfer utility solution with the Kalai-Smorodisky method, the Enterprise triples the payout obtained in non-coopetitive game (1 instead of 1/3), while the Financial Institute wins more of twice than before (3/2 instead of 2/3). We have moved from an initial competitive situation that was not so profitable to a coopetitive highly profitable situation both for the Enterprise and for the Financial Institute.

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