Multiproduct search

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31 November 2011

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This Version: November 2011

Abstract

This paper presents a sequential search model where consumers look for several products among competitive multiproduct firms. In a multiproduct search market, both consumer behavior and firm behavior exhibit different features from the single-product case: a consumer often returns to previously visited firms before running out of options; and prices can decrease with search costs and increase with the number of firms. The framework is then extended in two directions. First, by introducing both single-product and multiproduct searchers, the model can explain the phenomenon of countercyclical pricing, i.e., prices of many retail products decline during peak-demand periods. Second, by allowing firms to use bundling strategies, the model sheds new light on how bundling affects market performance. In a search environment, bundling tends to reduce consumer search intensity, which can soften competition and reverse the usual welfare assessment of competitive bundling in a perfect information setting.

Keywords: consumer search, oligopoly, multiproduct pricing, countercyclical pricing, bundling

JEL classification: D11, D43, D83, L13

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1 Introduction

Consumers often look for several products during a given shopping process. For example, during ordinary grocery shopping they buy food, drinks and household products; in high street shopping they purchase clothes, shoes and other goods; in the Christmas season they often look for several presents. Sometimes a consumer seeks electronic combinations such as computer, printer and scanner; when furnishing a house they need several furniture items; when going on holiday or attending a conference they book both flights and hotels; and for new parents they look for many baby products. On the other side of the market, there are many multiproduct firms such as supermarkets, department stores, electronic retailers, and travel agencies which often supply most of the products a consumer is searching for in a particular shopping trip. Usually the shopping process also involves non-negligible search costs. Consumers need to reach the store, find out each product's price and how suitable they are, and then may decide to visit another store in pursuit of better deals. In effect, in many cases a consumer chooses to shop for several goods together to save on search costs.

Despite the ubiquity of multiproduct search and multiproduct firms,[1] the search literature has been largely concerned with single-product search markets. There are probably two reasons why multiproduct search is under-researched. First, as I will discuss in more detail later, a multiproduct search model is less tractable than a single-product one. Second, people may also be concerned regarding how useful a multiproduct search model will be. This paper develops a tractable model to study multiproduct search markets. I find that multiproduct consumer search actually has rich market implications, and the developed framework can be used to address several interesting economic issues. First, a multiproduct search market exhibits some qualitatively different properties compared to the single-product case. For example, in a multiproduct search market, prices can decline with search costs and rise with the number of firms. Second, the multiproduct search model can explain the phenomenon of countercyclical pricing, i.e., prices of many retail products fall during high-demand periods such as weekends and holidays. Third, the multiproduct search model provides an appropriate setting for studying bundling in search markets, and sheds new light on how bundling affects market performance.

The basic framework of this paper is a sequential search model in which consumers look for several products and care about both price and product suitability. Each firm supplies all relevant products, but each product is horizontally differentiated across firms. By incurring a search cost, a consumer can visit a firm and learn all product and price information. In particular, the cost of search is incurred jointly for all products, and the consumer does not need to buy all products from the same firm, i.e., they can mix and match after sampling at least two firms (if firms allow them to do so).

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[1] Multiproduct search is also relevant in the labor market, for example, when a couple, as a collective decision maker, is looking for jobs. See Guler, Guvenen, and Violante (2011) for a recent exploration on this topic.

In the basic model, I assume linear prices are used, i.e., firms set separate prices for each product. A distinctive feature of consumer behavior in multiproduct search is that a consumer may return to previously visited firms to buy some products before running out of options. While in a standard single-product sequential search model, a consumer never returns to earlier firms before having sampled all firms. As far as pricing is concerned, with multiproduct consumer search, if a firm lowers one product's price, this will induce more consumers who are visiting it to terminate search and buy some other products as well. That is, a reduction of one product's price also boosts the demand for the firm's other products. I term this the *joint search effect*. As a result, even independent products are priced like complements.

Due to the joint search effect, prices can decline with search costs in a multiproduct search market. When search costs increase, the standard effect is that consumers will become more reluctant to shop around, which will induce firms to raise their prices. However, in a multiproduct search market, higher search costs can also strengthen the joint search effect and make the products in each firm more like complements, which will induce firms to lower their prices. When the latter effect dominates prices will fall with search costs. A related observation is that prices can rise with the number of firms. This is because when there are more firms, it becomes more likely that a consumer will return to previous firms to buy some products when she stops searching. This weakens the joint search effect and so the complementary pricing problem.

Another prediction of our model is that firms set lower prices in the multiproduct search environment than in the single-product case. This is for two reasons: first, due to economies of scale in search, consumers on average sample more firms in the multiproduct search case than in the single-product search case, which tends to increase each product's own-price elasticity; second, multiproduct search causes the joint search effect, which gives rise to the complementary pricing problem and so increases products' cross-price elasticities. There is a substantial body of evidence that prices of many retail products drop during high-demand periods such as weekends and holidays.[2] Our model can offer a simple explanation for this phenomenon of countercyclical pricing. Suppose there are both single-product searchers and multiproduct searchers in the market, and suppose a higher proportion of consumers become multiproduct searchers during high-demand periods (e.g., many households conduct their weekly grocery shopping during weekends). Then the above result implies that prices will decline when demand surges.

The second part of this paper allows firms to engage in bundling (i.e., selling a package of goods in a particular price). Bundling is a widely observed multiproduct pricing strategy in the market. For example, many retailers offer a customer a discount or reward (e.g., free delivery) if she buys several products together from the same store. Bundling is usually explained as a price discrimination or entry deterrence device,[3] but in a search environment it has a new function: it can discourage consumers

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[2]See section 3.4 for related literature and other possible explanations for countercyclical pricing.

[3]See, for instance, Adams and Yellen (1976), and McAfee, McMillan, and Whinston (1989) for the view of price discrimination, and Whinston (1990) and Nalebuff (2004) for the view of entry deterrence.

from exploring rivals' deals. This is because bundling reduces the anticipated benefit from mixing-and-matching after visiting another firm. As such, firms may have a greater incentive to adopt the bundling strategy in a search market. Moreover, this search-discouraging effect works against the typical pro-competitive effect of competitive bundling in a perfect information scenario (see the related literature below).[4] When search costs are relatively high the new effect can be such that bundling benefits firms and harms consumers.[5]

Since the seminal work by Stigler (1961), there has been a vast literature on search, but most papers focus on single object search. There is a small branch of literature that investigates the optimal stopping rule in multiproduct search. In Burdett and Malueg (1981) and Carlson and McAfee (1984), consumers search for the lowest price of a basket of goods among a large number of stores. The former mainly deals with the case of free recall and the latter deals with the case of no recall. In both cases the optimal stopping rule possesses the reservation property.[6] Gatti (1999) considers a more general setting with free recall in which consumers search for prices to maximize an indirect utility function. He shows that the reservation property holds in multiproduct search if the indirect utility function is submodular in prices, i.e., if a better offer in one dimension (weakly) reduces the search incentive in the other dimension. (The often adopted additive setting is a special case of that.) This branch of literature has emphasized the similarity between single-product and multiproduct search in the sense that in both cases the stopping rule often features the static reservation property. However, I argue that despite this similarity, consumer search behavior still exhibits substantial differences between the two cases.

More importantly, the above works do not consider an active supply side, and the price (or surplus) distribution among firms is exogenously given. According to our knowledge, the only genuine equilibrium multiproduct search model is McAfee (1995).[7]

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[4]In different settings, Carbajo, de Meza and Seidmann (1990) and Chen (1997) argue that (asymmetric) bundling can create "vertical" product differentiation between firms, thereby softening price competition.

[5]The European Commission has recently branded all bundled financial products as anti-competitive and unfair. One of the main reasons is that the practice reduces consumer mobility. See the consultation document "On the Study of Tying and Other Potentially Unfair Commercial Practices in the Retail Financial Service Section", 2009.

[6]However, with free recall consumers purchase nothing until search is terminated, while with no recall consumers may buy some cheap goods first and then continue to search for the other goods.

[7]Lal and Matutes (1994) also present a multiproduct search model where each product is homogenous across firms and each consumer needs to pay a location-specific cost to reach firms and discover the price information. Their setting is subject to the Diamond paradox. That is, no consumers will participate in the market given that they expect each firm is charging the monopoly prices. Lal and Matutes argue that firms can avoid the market collapse by employing loss-leading strategy, i.e., by advertising (and committing to) low prices of some products to persuade consumers to visit the store. However, in equilibrium each consumer still only samples one firm. Shelegia (2009) studies a multiproduct version of Varian (1980) in which for some exogenous reasons one group of consumers visits only one store while the other visits two. The presence of heterogeneously informed consumers can be a

It studies multiproduct price dispersion by extending Burdett and Judd (1983) to the multiproduct case. Each product is homogenous across stores, and by incurring a search cost a consumer can learn price information from a random number of stores. In particular, some consumers only learn information from one store while others learn more. As a result, similar to the single-product case, firms adopt mixed pricing strategies, reflecting the trade-off between exploiting less informed consumers and competing for more informed consumers. However, multiproduct search generates multiple types of (symmetric) equilibria. In particular, there is a continuum of equilibria in which firms randomize prices on the reservation frontier such that one product's price decline must be associated with the rise of some other prices.[8] Although the model offers interesting insights, both the multiplicity of equilibria and the complication of equilibrium characterization restrict its applicability. Our paper develops an alternative multiproduct search framework with differentiated products where the symmetric equilibrium is unique. I do not aim to address price dispersion. Instead, I use the developed framework to address other important economic issues such as countercyclical pricing and bundling in search markets.

In terms of the modelling approach, our paper is built on the single-product search model with differentiated products. That framework was initiated by Weitzman (1979), and later developed and applied to a market context by Wolinsky (1986) and Anderson and Renault (1999). Compared to the homogeneous product search model, models with product differentiation often better reflect consumer behavior in markets that are typically characterized by nonstandardized products. Moreover, they avoid the well-known modelling difficulty suggested by Diamond (1971), who shows that with homogeneous products and positive search costs (no matter how small) all firms will charge a monopoly price and all consumers will stop searching at the first sampled firm. So rivalry between firms has no impact on price. In search models with product differentiation, there are some consumers who are ill-matched with their initial choice of supplier and then search further, so that the pro-competitive benefit of actual search is present.[9] Recently this framework has been adopted to study various economic issues such as prominence and non-random consumer search (Armstrong, Vickers, and Zhou, 2009), firms' incentive to use selling tactics such as exploding offers and buy-now

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consequence of rational search, but without an explicit search model the main insights from our paper are absent there. Rhodes (2010) proposes a multiproduct monopoly model in which each consumer knows her private valuations for all products but needs to incur a cost to reach the firm and learn prices. He shows that selling multiple products can solve the Diamond hold-up problem which would unravel the market in a single-product case with inelastic consumer demand.

[8]In the other type of equilibria, firms randomize prices over the acceptance set (*not* just on its border). They are, however, qualitatively similar to the single-product equilibrium in the sense that the marginal price distribution for each product is the same as in the single-product search case, and so is the profit from each product.

[9]In the homogeneous product scenario, the main approach to avoid the Diamond paradox is to introduce heterogeneously informed consumers. See, for example, Burdett and Judd (1983) and Stahl (1989), where price dispersion arises endogenously and so consumers have incentive to search.

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discounts (Armstrong and Zhou, 2010), how the decline of search costs affects product design (Bar-Isaac, Caruana, and Cuñat, 2009), and attention-grabbing advertising (Haan and Moraga-González, 2011). This paper extends the basic framework in this literature to the multiproduct case.

Last but not least, this paper contributes to the literature on competitive bundling. Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) have studied competitive pure bundling, and Matutes and Regibeau (1992), Anderson and Leruth (1993), Thanassoulis (2007), and Armstrong and Vickers (2010) have studied duopoly mixed bundling.[10] One important insight emerging from all these works is that bundling (whether pure or mixed) has a tendency to intensify price competition, and under the assumptions of unit demand and full market coverage (which are also retained in this paper) it typically reduces firm profits and boosts consumer welfare.[11] This paper is the first to study bundling in a search environment. Our findings indicate that assuming away information frictions (which usually do exist in consumer markets) may significantly distort the welfare assessment of bundling. In particular, when search costs are relatively high, bundling may actually benefit firms and harm consumers. As such, our work complements the existing literature.

The rest of the paper is organized as follows. Section 2 presents the basic model with linear pricing and analyzes consumer search behavior. Section 3 characterizes equilibrium linear prices in a duopoly and conducts comparative statics analysis, and an application to countercyclical pricing is then discussed. Section 4 studies bundling in a search market and examines its welfare impacts relative to linear pricing. Section 5 discusses the case with more firms and other extensions, and section 6 concludes. Omitted proofs and calculations are presented in the Appendix.

2   A Model of Multiproduct Search

There are a large number of consumers in the market, and the measure of them is normalized to one. Each consumer is looking for a number of products. For example, a consumer who is furnishing her house may need to buy several furniture items; a high-street shopper may be looking for both clothes and shoes. For simplicity, let us assume that each consumer needs two products 1 and 2, and they have unit demand for each product. There are $n \geq 2$ multiproduct firms in the market, each supplying both products at a constant marginal cost which is normalized to zero. Suppose each product is horizontally differentiated across firms. For example, different firms may supply different brands of furniture or clothes and shoes with different styles, and consumers often have idiosyncratic tastes. I model this scenario by extending the random utility model in Perloff and Salop (1985) to the two-product case. Specifically, a consumer's

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[10]Most of these studies adopt the two-dimensional Hotelling setting and assume that consumers are distributed uniformly on the square and have unit demand for each product. Armstrong and Vickers (2010) consider a fairly general setting with arbitrary distributions and elastic demand.

[11]The welfare impact of bundling in the monopoly case is ambiguous (e.g., Schmalensee, 1984).

valuations for the two products in each firm are randomly drawn from a common joint cumulative distribution function $F(u_1, u_2)$ defined on $[\underline{u}_1, \overline{u}_1] \times [\underline{u}_2, \overline{u}_2]$ which has a continuous density $f(u_1, u_2)$. The valuations are realized independently across firms and consumers (but a consumer may have correlated valuations for the two products in the same firm). For simplicity, I assume that the two products are neither complements nor substitutes, in the sense that a consumer obtains an additive utility $u_1 + u_2$ if product i has a match utility u_i, $i = 1, 2$. (See a discussion about intrinsic complements in section 5.5.) Let $F_i(u_i)$ and $H_i(u_i|u_j)$ denote the marginal and conditional distribution functions; $f_i(u_i)$ and $h_i(u_i|u_j)$ denote the marginal and conditional densities.

Following Perloff and Salop (1985), I assume that in equilibrium all consumers buy both products, i.e., the market is fully covered.[12] (This is the case, for example, when consumers have no outside options or when they have large basic valuations for each product on top of the above match utilities.) However, consumers do not need to purchase both products from the same firm. This possibility of multi-stop shopping is realistic and also important for our model. Otherwise, the multiproduct search model would degenerate to a single-product one with a composite product with match utility $u_1 + u_2$. In the basic model, firms must charge a separate price for each product. I refer to this case as "linear pricing" henceforth. (I will consider bundling in section 4.)

I introduce imperfect information and consumer search as Wolinsky (1986) and Anderson and Renault (1999) did in a single-product framework. Initially consumers are assumed to have imperfect information about the (actual) prices firms are charging and match utilities of all products.[13] But they can gather information through a sequential search process: by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out both prices (p_1, p_2) and both match utilities (u_1, u_2). At each firm (except the last one), the consumer faces the following options: stop searching and buy both products (maybe from firms visited earlier), or buy one product and keep searching for the other, or keep searching for both products. The cost of search is assumed to be the same no matter how many products a consumer is looking for, which reflects economies of scale in search. I also assume away other possible costs involved in sourcing supplies from more than one firm. Finally, following most of the literature on consumer search, I suppose that consumers have free recall, i.e., there are no extra costs in buying products from a previously visited store.

Both consumers and firms are assumed to be risk neutral. I focus on symmetric equilibria in which firms set the same (linear) prices and consumers sample firms in

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[12]The assumption of full market coverage is often adopted for simplicity in oligopoly models. In this paper, neither the joint search effect nor the effect of bundling on consumer search incentive relies on this assumption, so the main insights can carry over even without this assumption (though the analysis will become more involved).

[13]In the markets (e.g., the grocery market) where consumers shop frequently, some consumers should be able to learn both price and product information if they do not vary over time. However, in reality both prices and product variants in many retailers change over time such that imperfect information might be a plausible presumption.

a random order (and without replacement).[14] I use the perfect Bayesian equilibrium concept. Firms set prices simultaneously, given their expectation of consumers' search behavior. Consumers search optimally, to maximize their expected surplus, given the match utility distribution and their rational beliefs about firms' pricing strategy. At each firm, even after observing off-equilibrium offers, consumers hold the equilibrium belief about the unsampled firms' prices.[15]

I have made several simplifying assumptions to make the model tractable.

Economies of scale in search. Our assumption that the cost of search is independent of the number of products a consumer is seeking is an approximation when the search cost is mainly for learning the existence of a seller or for reaching the store. In the other polar case where the cost of search is totally divisible among products (so no economies of scale in search at all), the multiproduct search problem degenerates to two separate single-product search problems. In reality, most situations are in between (e.g., a typical shopping process involves a fixed cost for reaching the store and also variable in-store search costs for finding and inspecting each product). Our simplification is both for analytical convenience and for highlighting the difference between multiproduct and single-product search.

Free recall. Free recall is often assumed in the consumer search literature. It could be appropriate, for instance, when a consumer can phone previously visited firms (e.g., furniture stores) to order the products she decides to buy, or when shopping online a consumer can leave the browsed websites open. Sometimes we also assume no recall at all (especially in the job search literature). In most consumer markets, however, there are usually positive returning costs but they are not so high that returning is totally banned. I choose to assume free recall both for tractability,[16] and for facilitating the comparison between our model and the corresponding single-product search model in Wolinsky (1986) and Anderson and Renault (1999) (both of which assume free recall). I will discuss how costly recall or no recall could affect our results in section 5.3.

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[14]As usual in search models, there exists an uninteresting equilibrium where consumers expect all firms to set very high prices which leave them with no surplus, consumers do not participate in the market at all, and so firms have no incentive to reduce their prices. I do not consider this equilibrium further. In addition, there may also exist asymmetric equlibria with active search, and I will discuss this issue in section 5.1.

[15]This off-equilibrium belief is reasonable because in our setting there are no correlated economic shocks (e.g., aggregate cost shocks) across firms and so their pricing decisions are independent to each other. In an alternative setting, say, with correlated cost shocks, a consumer who observes a low price in one firm may infer that other firms also have low costs and so anticipate that they are also charging low prices. See, for instance, Bénabou and Gertner (1993) for such a learning model in the single-product search scenario.

[16]According to my knowledge, Janssen and Parakhonyak (2010) is the only paper in the economics literature which studies the optimal stopping rule in the single-product search case with costly recall. They find that when there are more than two (but a finite number of) firms, the stopping rule is non-stationary and depends on the historical offers in an intricate way. The optimal stopping rule in multiproduct search with costly recall and an arbitrary number of firms is still an open question.

Two-stop shopping costs. Even if search costs and returning costs are absent, transacting with two firms may involve some other costs (e.g., the cost of paying two bills). But in many retail markets, this kind of two-stop shopping costs seem less important than search costs. I will discuss the difference between this market friction and search friction in section 5.4. (Two-stop shopping costs are also similar to the joint-purchase discount I will examine in the bundling part.)

2.1   The optimal stopping rule

I first derive the optimal stopping rule (which has been proved in Burdett and Malueg, 1981, or Gatti, 1999 in a price search scenario). The first observation is that given the indivisible search cost and free recall a consumer will never buy one product first and keep searching for the other. Hence, at any store (except the last one) the consumer faces only two options: stop searching and buy both products (one of which may be from a firm visited earlier), or keep searching for both.

Denote by

$$\zeta_i(x) \equiv \int_x^{\overline{u}_i} (u_i - x) dF_i(u_i) = \int_x^{\overline{u}_i} [1 - F_i(u_i)] du_i \tag{1}$$

the expected incremental benefit from sampling one more product i when the maximum utility of product i so far is x. (The second equality is from integration by parts.) Note that $\zeta_i(x)$ is decreasing and convex. Then the optimal stopping rule in a symmetric equilibrium is as follows.

Lemma 1 *Suppose prices are linear and symmetric across firms. Suppose the maximum match utility of product i observed so far is z_i and there are firms left unsampled. Then a consumer will stop searching if and only if*

$$\zeta_1(z_1) + \zeta_2(z_2) \leq s \ . \tag{2}$$

The left-hand side of (2) is the expected benefit from sampling one more firm given the pair of maximum utilities so far is (z_1, z_2), and the right-hand side is the search cost. This stopping rule seems "myopic" at the first glance, but it is indeed sequentially rational. It can be understood by backward induction. When in the penultimate firm, it is clear that (2) gives the optimal stopping rule because given (z_1, z_2) the expected benefit from sampling the last firm is $\mathbb{E}[\max(0, u_1 - z_1) + \max(0, u_2 - z_2)]$, which equals the left-hand side of (2). (Note that I did not assume u_1 and u_2 are independent of each other. The separability of the incremental benefit in (2) is because of the additivity of match utilities and the linearity of the expectation operator.) Now step back and consider the situation when the consumer is at the firm before that. If (2) is violated, then sampling one more firm is always desirable. By contrast, if (2) holds, then even if the consumer continues searching, she will stop at the next firm no matter what she will find there. So the benefit from keeping searching is the same as sampling one more firm. Expecting that, the consumer should actually cease her search now. (This stopping rule also carries over to the case with an infinite number of firms.)

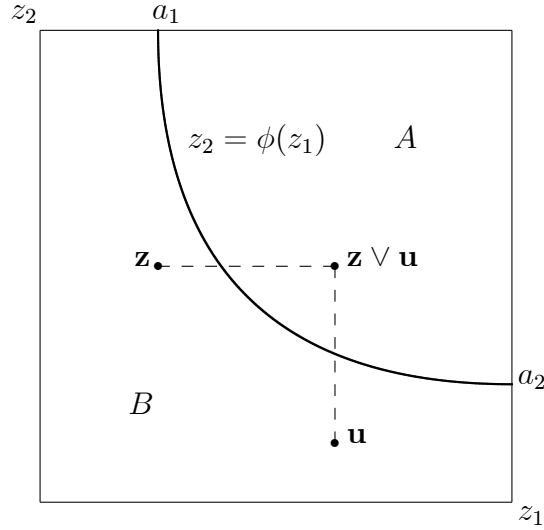Figure 1 below illustrates the optimal stopping rule.

Figure 1: The optimal stopping rule in multiproduct search with perfect recall

A is the set of (z_1, z_2) which satisfies (2) and let us refer to it as the *acceptance set*. Then a consumer will stop searching if and only if the maximum utility pair so far lies within A. Define the border of A as $z_2 = \phi(z_1)$, i.e., $(z_1, \phi(z_1))$ satisfies (2) with equality, and call it the *reservation frontier*. One can show that A is a convex set, and the reservation frontier is decreasing and convex.[17] Let B be the complement of A. Note that a_i on the graph is just the reservation utility level when the consumer is only searching for product i. It solves

$$\zeta_i(a_i) = s \ , \tag{3}$$

and satisfies $\phi(a_1) = \bar{u}_2$ and $\phi(\bar{u}_1) = a_2$. This is because when the maximum possible utility of one product has been achieved, the consumer will behave as if she is only searching for the other product.

It is worth mentioning that from (1) and (2), one can see that only the marginal distributions matter for the expected benefit of sampling one more firm. This implies that if the marginal distributions are fixed, the correlation of the two products' match utilities does not affect the reservation frontier.

Search behavior comparison. It is useful to compare consumer search behavior between single-product search and multiproduct search. The early literature has emphasized that in both cases (given additive utilities in the multiproduct case) the optimal stopping rule possesses the static reservation property. Despite this similarity, consumers' search behavior exhibits some differences between the two cases, which have not been discussed before.

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[17]From the equality of (2), we have

$$\phi'(z_1) = -\frac{1 - F_1(z_1)}{1 - F_2(\phi(z_1))} < 0 \ ,$$

and this derivative increases with z_1.

In single-product search with perfect recall, the stopping rule is characterized by a reservation utility a. When a consumer is already at some firm (except the last one), she will stop searching if and only if the current product has a utility greater than a. Previous offers are irrelevant because they must be worse than a (otherwise the consumer would not have come to this firm). As a result, a consumer never returns to previously visited firms until she finishes sampling all firms. In particular, if there are an infinite number of firms, the consumer actually never exercises the recall option.

However, in multiproduct search, a consumer's search decision may depend on *both* the current firm's offer $\mathbf{u}$ and the best offer so far $\mathbf{z}$. This can be seen from the example indicated in Figure 1, where the current offer $\mathbf{u}$ lies outside the acceptance set A but the consumer will stop searching because $\mathbf{z} \vee \mathbf{u} \in A$ (where $\vee$ denotes the "join" of two vectors). As a result, in multiproduct search (even with an infinite number of firms), although a consumer will buy at least one product at the firm where she stops searching, she may return to a previous firm and buy the other product even if there are firms left unsampled. In the above example, the consumer will go back to some previous firm to buy product 2.

These differences will complicate the demand analysis in multiproduct search. In particular, unlike the single-product search case, considering an infinite number of firms does not simplify the analysis (mainly because various types of returning consumers still exist). However, the complication can be avoided if there are only two firms. Moreover, as I will discuss in section 5.2, such a simplification does not lose the most important insights concerning firm pricing in a multiproduct search setting. Hence, in the following analysis, I mainly deal with the duopoly case. (A detailed analysis of the general case with more than two firms is provided in the online supplementary document at https://sites.google.com/site/jidongzhou77/research.)

3   Equilibrium Prices

3.1   The single-product benchmark

To facilitate comparison, I first report some results from the single-product search model (see Wolinsky, 1986 and Anderson and Renault, 1999 for an analysis with n firms). Suppose the product in question is product i, and the unit search cost is still s. Then the reservation utility level is a_i defined in (3), and it decreases with s. That is, in a symmetric equilibrium, a consumer will keep searching if and only if the maximum match utility so far is lower than a_i, and a higher search cost will make the consumer less willing to search on. In the following analysis, I will mainly focus on the case with a relatively small search cost:

$$s < \zeta_i(\underline{u}_i) \Leftrightarrow a_i > \underline{u}_i \text{ for both } i = 1, 2 \ . \tag{4}$$

(Remember that $\zeta_i(\underline{u}_i)$ is the expected benefit from sampling another product i when the current one has the lowest possible match value.) This condition ensures an active

search market even in the single-product case.

The symmetric equilibrium price p_i^0 is then determined by

$$\frac{1}{p_i^0} = \underbrace{f_i(a_i)[1 - F_i(a_i)] + 2\int_{\underline{u}_i}^{a_i} f_i(u)^2 du}_{\alpha_0} \ . \tag{5}$$

(Its intuition will be clear soon.) It follows that p_i^0 increases with the search cost s (or decreases with a_i) if

$$f_i(a_i)^2 + f_i'(a_i)[1 - F_i(a_i)] \geq 0 \ .$$

This condition is equivalent to an increasing hazard rate $f_i/(1 - F_i)$. Then we have the following result (Anderson and Renault, 1999 have shown this result for an arbitrary number of firms).

Proposition 1 *Suppose the consumer is only searching for product i and the search cost condition (4) holds. Then the equilibrium price defined in (5) increases with the search cost if the match utility has an increasing hazard rate $f_i/(1 - F_i)$.*

3.2 Equilibrium prices in multiproduct search

I now turn to the multiproduct search case. Let (p_1, p_2) be the symmetric equilibrium prices. For notational convenience, let (u_1, u_2) be the match utilities of firm 1, the firm in question, and (v_1, v_2) be the match utilities of firm 2, the rival firm. In equilibrium, for a consumer who samples firm 1 first, her reservation frontier $u_2 = \phi(u_1)$ is determined by

$$\zeta_1(u_1) + \zeta_2(\phi(u_1)) = s \ , \tag{6}$$

which simply says that the expected benefit of sampling firm 2 is equal to the search cost. Note that $\phi(u_1)$ is only defined for $u_1 \in [a_1, \overline{u}_1]$ (see Figure 2 below). For convenience, let us extend its domain to all possible values of u_1, but stipulate $\phi(u_1) > \overline{u}_2$ for $u_1 < a_1$.

Instead of writing down the demand functions and deriving the first-order conditions for the equilibrium prices directly, I use the following economically more illuminating method. Starting from an equilibrium, suppose firm 1 unilaterally decreases p_2 by a small ε. How will this adjustment affect firm 1's profits? Let us focus on the first-order effects. First of all, firm 1 suffers a loss from those consumers who only buy product 2 from it because they are now paying less. Since in equilibrium half of the consumers buy product 2 from firm 1 (remember the assumption of full market coverage), this loss is $\varepsilon/2$. Second, firm 1 gains from boosted demand: (i) For those consumers who visit firm 1 first, they will be more likely to stop searching since they hold equilibrium beliefs that the second firm is charging the equilibrium prices. Once they stop searching, they will buy *both* products from firm 1 immediately. (ii) For those consumers who eventually sample both firms, they will be more likely to buy product 2 from firm 1 due to the price reduction. In equilibrium, the loss and gain should be such that firm 1 has no incentive to deviate, which generates the first-order condition for p_2.

Now let us analyze in detail the two (first-order) gains from the proposed small price reduction. The first gain is from the effect of the price reduction on consumers' search decisions. How many consumers who sample firm 1 first will be induced to stop searching by the price reduction? (Note that the consumers who sample firm 2 first hold equilibrium beliefs and so their stopping decisions remain unchanged.) Denote by $\phi(u_1|\varepsilon)$ the new reservation frontier. Since reducing p_2 by ε is equivalent to increasing u_2 by ε, $\phi(u_1|\varepsilon)$ solves

$$\zeta_1(u_1) + \zeta_2(\phi(u_1|\varepsilon) + \varepsilon) = s \ ,$$

so $\phi(u_1|\varepsilon) = \phi(u_1) - \varepsilon$ according to the definition of $\phi(\cdot)$. That is, the reservation frontier moves downward everywhere by ε, and the stopping region A expands (i.e., more consumers buy immediately at firm 1) as illustrated in the figure below.
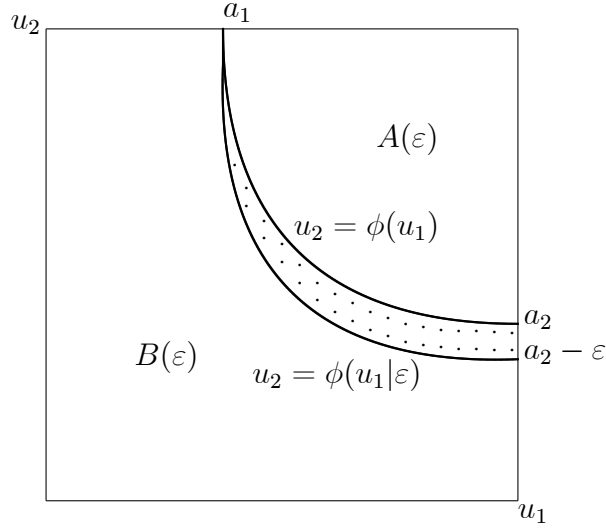

Figure 2: Price deviation and the stopping rule

For a small ε, the number of consumers who switch from keeping searching to buying immediately at firm 1 (i.e., the probability measure of the shaded area between $\phi(u_1)$ and $\phi(u_1|\varepsilon)$) is

$$\frac{\varepsilon}{2} \int_{a_1}^{\overline{u}_1} f(u, \phi(u)) du \ . \tag{7}$$

(Remember that half of the consumers sample firm 1 first. The integral term is the line integral along the reservation frontier in the u_1 dimension.)

What is firm 1's net benefit from these marginal consumers? Realize that these marginal consumers now buy both products from firm 1 for sure, while before the price deviation they only bought each product from firm 1 with some probability less than one (i.e., when they search on but find worse products at firm 2). To be specific, consider a marginal consumer on the reservation frontier with match utilities $(u_1, \phi(u_1))$. If she chooses to sample firm 2, then she will find a worse product 1 at firm 2 (i.e., $v_1 < u_1$) with probability $F_1(u_1)$, in which case she will return to firm 1 and buy its product 1. Similarly, if she continues to sample firm 2, she will find a worse product 2 at firm

13

2 (i.e., $v_2 < \phi(u_1)$) with probability $F_2(\phi(u_1))$, in which case she will return to firm 1 and buy its product 2. Hence, the net benefit from inducing this marginal consumer from searching on is $p_1[1 - F_1(u)] + p_2[1 - F_2(\phi(u_1))]$. We then sum this benefit over all marginal consumers on the reservation frontier. By using (7), this total benefit is

$$\frac{\varepsilon}{2} \int_{a_1}^{\overline{u}_1} \{p_1[1 - F_1(u)] + p_2[1 - F_2(\phi(u_1))]\} f(u, \phi(u)) du .\tag{8}$$

The second gain is from those consumers who have sampled both firms because they will now buy product 2 from firm 1 more likely due to the price reduction. Consider first a consumer who visits firm 1 first and finds match utilities $(u_1, u_2) \in B(\varepsilon)$. She will then continue to visit firm 2, but will return to firm 1 and buy its product 2 if $v_2 < u_2 + \varepsilon$. The probability of that event is $F_2(u_2 + \varepsilon) \approx F_2(u_2) + \varepsilon f_2(u_2)$. So the small price adjustment increases the probability that this consumer buys product 2 from firm 1 by $\varepsilon f_2(u_2)$. Then the total increased probability from all such consumers is $\frac{\varepsilon}{2} \int_{B(\varepsilon)} f_2(u_2) dF(\mathbf{u}) \approx \frac{\varepsilon}{2} \int_B f_2(u_2) dF(\mathbf{u})$. (Since $B(\varepsilon)$ converges to B as $\varepsilon \to 0$, we can discard all higher order effects.) Similarly, one can show that the gain from those consumers who sample firm 2 first and then come to firm 1 is $\frac{\varepsilon}{2} \int_B f_2(v_2) dF(\mathbf{v})$. Adding these two benefits together gives us the second gain which is

$$p_2 \varepsilon \int_B f_2(u_2) dF(\mathbf{u}) .\tag{9}$$

In equilibrium, the (first-order) loss $\varepsilon/2$ from the small price reduction should be equal to the sum of the two (first-order) gains in (8) and (9). This yields the first-order condition for p_2:

$$1 = \underbrace{2p_2 \int_B f_2(u_2) dF(\mathbf{u}) + p_2 \int_{a_1}^{\overline{u}_1} [1 - F_2(\phi(u))] f(u, \phi(u)) du}_{\text{standard effect}}\tag{10}$$
$$+ \underbrace{p_1 \int_{a_1}^{\overline{u}_1} [1 - F_1(u)] f(u, \phi(u)) du}_{\text{joint search effect}} .$$

The first two terms on the right-hand side capture the standard effect of a product's price adjustment on its own demand: reducing p_2 increases demand for product 2. (This is similar to the right-hand side of (5) in the single-product search case.) The last term, however, captures a new feature of the multiproduct search model: when firm 1 reduces p_2, more consumers who sample it first will stop searching and buy *both* products, which increases the demand for its product 1 as well. This makes the two products supplied by the same firm like complements even if they are physically independent. This effect occurs because each consumer is searching for two products and the cost of search is incurred jointly for them, and so I refer to it as the *joint search effect* henceforth. Also notice that the size of the joint search effect (which determines the degree of "complementarity" between the two products in each firm) relies on the mass of marginal consumers on the reservation frontier, i.e., (7). It depends not only on

14

the density function f but also on the "length" of the reservation frontier as indicated in Figure 2. For example, in the uniform distribution case, when the search cost increases, the reservation frontier becomes longer such that the mass of marginal consumers rises and so the two products become more like complements. As we shall see below, this observation plays an important role in firms' pricing decisions.

Similarly, one can derive the first-order condition for p_1 as:

$$
\begin{aligned}
1 \;=\; & 2p_1 \int_B f_1(u_1)dF(\mathbf{u}) + p_1 \int_{a_2}^{\overline{u}_2} [1 - F_1(\phi^{-1}(u))]f(\phi^{-1}(u), u)du \qquad (11) \\
& + p_2 \int_{a_2}^{\overline{u}_2} [1 - F_2(u)]f(\phi^{-1}(u), u)du \ ,
\end{aligned}
$$

where ϕ^{-1} is the inverse function of ϕ. I summarize the results in the following lemma.[18]

Lemma 2 *Under the search cost condition (4), the first-order conditions for p_1 and p_2 to be the equilibrium prices are given in (10) and (11).*

Both (10) and (11) are linear equations in prices, and the system of them has a unique solution. Thus, the symmetric equilibrium, if it exists, will be unique.[19] Notice that if firms ignored the joint search effect, then the pricing problem would be actually separable between the two products. A special case is when $s = 0$ (so $a_i = \overline{u}_i$ and B equals the whole match utility domain). Then the effect of a price adjustment on consumer search behavior (i.e., (8)) disappears, and the first-order conditions simplify to

$$
\frac{1}{p_i} = 2 \int_{\underline{u}_i}^{\overline{u}_i} f_i(u)^2 du \ .
$$

In this case, the multiproduct model yields the same equilibrium prices as the single-product model.

---

[18]One can also derive the first-order conditions by calculating the demand functions directly. For example, when firm 1 unilaterally deviates to $(p_1 - \varepsilon_1, p_2 - \varepsilon_2)$, the demand for its product 1 is

$$
\frac{1}{2} \int_{\underline{u}_1}^{\overline{u}_1} [1 - H_2(\phi(u_1|\boldsymbol{\varepsilon})|u_1)(1 - F_1(u_1 + \varepsilon_1))] \, dF_1(u_1) + \frac{1}{2} \int_{\underline{u}_1}^{\overline{u}_1} H_2(\phi(v_1)|v_1)(1 - F_1(v_1 - \varepsilon_1))dF_1(v_1) \ ,
$$

where $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2)$, $\phi(u_1|\boldsymbol{\varepsilon}) = \phi(u_1 + \varepsilon_1) - \varepsilon_2$ is the reservation frontier associated with the deviation, and $H_i(\cdot|\cdot)$ is the conditional distribution function. Consumers who sample firm 1 first will buy its product 1 if they stop searching immediately or if they search on but find firm 2's product 1 is worse. Consumers who sample firm 2 first will purchase firm 1's product 1 if they come to firm 1 and find firm 1's product 1 is better. The deviation demand for product 2 is similar. However, this direct method will become less applicable in the case with more products, more firms, or mixed bundling.

[19]In my multiproduct search model, it is rather complicated to investigate the second-order condition in general. However, in the online supplementary document, I show that in the case with two symmetric products, each firm's profit function is locally concave around the prices defined in (10) and (11) under fairly general conditions. In the uniform and exponential examples (which use for illustration below), one can numerically verify that a firm's profit function is also globally concave such that the first-order conditions are sufficient for the equilibrium prices.

In the following analysis, I will often rely on the case of symmetric products. Slightly abusing the notation, let the one-variable functions $F(\cdot)$ and $f(\cdot)$ denote the common marginal distribution function and density function, respectively. Let a be the common reservation utility in each dimension. In particular, with symmetric products, we have $f(u_1, u_2) = f(u_2, u_1)$ and the reservation frontier satisfies $\phi(\cdot) = \phi^{-1}(\cdot)$, i.e., it is symmetric around the 45-degree line in the match utility space. If p is the equilibrium price of each product, then both (10) and (11) simplify to

$$\frac{1}{p} = \underbrace{2 \int_B f(u_i) f(u_i, u_j) d\mathbf{u} + \int_a^{\overline{u}} [1 - F(\phi(u))] f(u, \phi(u)) du}_{\text{standard effect: } \alpha} \qquad (12)$$

$$+ \underbrace{\int_a^{\overline{u}} [1 - F(u)] f(u, \phi(u)) du}_{\text{joint search effect: } \beta} .$$

Before proceeding to the comparative statics analysis, let us first study two examples.

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0, 1]$. Then $\zeta_i(x) = (1 - x)^2/2$. So $a = 1 - \sqrt{2s}$ and the search cost condition (4) requires $s \leq 1/2$. The reservation frontier satisfies

$$(1 - u)^2 + (1 - \phi(u))^2 = 2s ,$$

so the stopping region A is a quarter of a disk with a radius $\sqrt{2s}$. Then (12) implies[20]

$$p = \frac{1}{2 - (\pi/2 - 1)s} ,$$

where $\pi \approx 3.14$ is the mathematical constant.

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $\zeta_i(x) = e^{-x}$. So $e^{-a} = s$ and the search cost condition (4) requires $s \leq 1$. The reservation frontier satisfies

$$e^{-u} + e^{-\phi(u)} = s ,$$

so $\phi(u)$ is one branch of a hyperbola. Then (12) implies[21]

$$p = \frac{1}{1 + s^3/6} .$$

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[20]The standard effect is $\alpha = 2 - s\pi/2$: the first term in (12) is $2 \int_B d\mathbf{u}$, so it equals two times the area of region B, i.e., $2(1 - s\pi/2) = 2 - s\pi$; and the second term is $\int_a^1 [1 - \phi(u)] du$, which is the area of region A and so equals $s\pi/2$. The joint search effect is $\beta = \int_a^1 (1 - u) du = s$ according to the definition of a.

[21]One can check that the standard effect is $\alpha = 1$ and the joint search effect is $\beta = s^3/6$.

16

The prices in these two examples are depicted as the thick solid curves in Figure 3 below. The price increases with search costs in the uniform example, but it decreases with search costs in the exponential example. As we will see below, the result that prices can decline with search costs is not exceptional in our multiproduct search model.
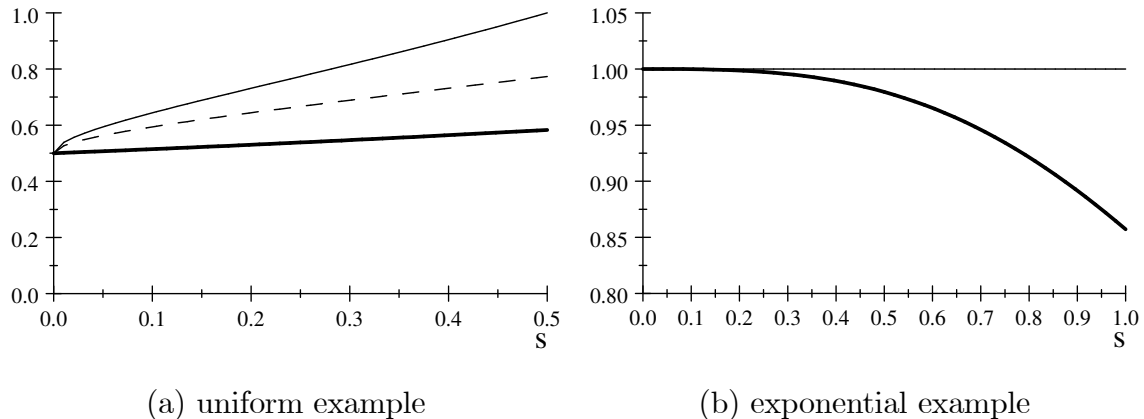

(a) uniform example (b) exponential example

Figure 3: Prices and search costs (symmetric products)

3.3 Search cost and price

This section investigates how search costs affect prices in our multiproduct search model. When search costs rise, there are two effects. First, consumers will become more reluctant to shop around, and so fewer of them will sample both firms (i.e., the region of B shrinks), which always induces firms to raise their prices. Second, when search costs rise, the mass of marginal consumers who distribute on the reservation frontier also changes. This is another determinant for prices. In particular, if the mass of marginal consumers increases with search costs (which occurs more likely in the multiproduct case than in single-product case because the reservation frontier often becomes "longer" as search costs rise in the permitted range of (4)), firms have an incentive to reduce their prices. This incentive is further strengthened in the multiproduct case due to the joint search effect (i.e., stopping a marginal consumer from searching on can boost demand for both products). The final prediction depends on which effect dominates.

Our first observation is that if the joint search effect were absent, then the marginal-consumer effect would be usually insufficient to outweigh the first. Denote by $\tilde{p}_i$, $i = 1, 2$, the hypothetical equilibrium prices if firms ignore the joint search effect (i.e., they solve (10) and (11) without the second line of each equation). (All omitted proofs can be found in Appendix A.)

Lemma 3 *Suppose the search cost condition (4) holds and*

$$\frac{h_i(u_i|u_j)}{1 - F_i(u_i)} \text{ increases with } u_i \text{ for any given } u_j. \tag{13}$$

Then $\tilde{p}_i$, $i = 1, 2$, weakly increase with the search cost.

17

That is, without the joint search effect, the relationship between prices and search costs would be actually similar to that in the single-product scenario. (Note that the condition (13) is just the standard increasing hazard rate condition if the two products have independent match utilities.) For instance, in the uniform example, we have $\tilde{p}_i = 1/(2 - \pi s/2)$ which increases with s; and in the exponential example which has a constant hazard rate, we have $\tilde{p}_i = 1$ which is independent of s (so p_i decreasing with s in this case is purely due to the joint search effect).

However, taking into account the joint search effect will qualitatively change the picture. I pursue this issue by considering two cases.

Symmetric products. Suppose first the two products are symmetric, and so the equilibrium price p is given in (12). Lemma 3 implies that the standard effect α indicated in (12) usually decreases with s. However, the joint search effect β can vary with s in either direction even under the regularity condition. If β also decreases with s, then the joint search effect will make the price increase with search costs even faster. Conversely, if β increases with s, then the joint search effect will mitigate or even overturn the usual relationship between price and search costs. As shown in the proof of next proposition,

$$\frac{d\beta}{ds} = f(a, \overline{u}) - \int_a^{\overline{u}} h'(u|\phi(u)) f(\phi(u)) du \ . \tag{14}$$

This derivative is positive, for example, when the (conditional) density function is weakly decreasing. This is true in both the uniform and exponential example.

As a result, the standard hazard rate condition is no longer enough to ensure that prices increase with search costs in our model. The following result gives a new condition.

Proposition 2 *Suppose the search cost condition (4) holds, and the two products are symmetric. Then the equilibrium price p defined in (12) increases with the search cost if and only if*

$$\int_a^{\overline{u}} \frac{f(\phi(u))}{1 - F(\phi(u))} \left\{ f(u)h(u|\phi(u)) + [2 - F(u) - F(\phi(u))]h'(u|\phi(u)) \right\} du > f(a, \overline{u}) \tag{15}$$

for all a. If the two products further have independent valuations, a sufficient condition for (15) is that the marginal density $f(u)$ is (weakly) increasing.

Condition (15), however, can be easily violated by some distributions having a decreasing or non-monotonic density (but still having an increasing hazard rate).[22] As well as the exponential case, other relatively simple examples include: the distribution with a decreasing density $f(u) = 2(1 - u)$ for $s \in [0, 1/3]$; and the logistic distribution $f(u) = e^u/(1 + e^u)^2$ for s less than about 1.

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[22]One may wonder, if $f(a, \overline{u})$ is bounded away from zero, whether the condition always fails to hold as $a \to \overline{u}$ (i.e., as $s \to 0$). This is not true because $\frac{f(\phi(u))}{1-F(\phi(u))}$ may converge to infinity at the same time. For example, in the uniform case, the left-hand side is equal to $\frac{\pi}{2} - 1 > 0$, independent of a.

On the other hand, if firms supply (and consumers need) more products, the joint search effect could have an even more pronounced impact such that prices fall with search costs more likely. I can extend the two-product model to the case with m products (see the details in Appendix A). In particular, in the uniform case, the equilibrium price p has a simple formula:

$$\frac{1}{p} = \underbrace{2 - \frac{V_m(\sqrt{2s})}{2^m}}_{\text{standard effect}} + \underbrace{\frac{(m-1)V_m(\sqrt{2s})}{2^{m-1}\pi}}_{\text{joint search effect}} \,, \tag{16}$$

where $s \in [0, 1/2]$ and $V_m(\sqrt{2s})$ is the volume of an m-dimensional sphere with a radius $\sqrt{2s}$.[23] One can check that p increases with s if and only if $m < 1 + \pi/2 \approx 2.6$. Then we have the following result.

Proposition 3 *Suppose the search cost condition (4) holds, and each firm supplies m symmetric products with independent valuations $u_i \sim U[0, 1]$. Then the equilibrium price p defined in (16) increases with s if $m \leq 2$ and decreases with s if $m \geq 3$.*

In this example, if the joint search effect were absent, the price would increase with the search cost for any m. But its presence makes the price decline with search costs whenever consumers are searching for more than two products.

Asymmetric products. Another force which could influence the relationship between prices and search costs is product asymmetry. Intuitively, when one product has a lower profit margin than the other, the joint search effect from adjusting its price is stronger (i.e., reducing its price can induce consumers to buy the more profitable product). Then this product's price may go down with the search cost. I confirm this possibility in a uniform example in which product 1 is a "small" item and has match utility uniformly distributed on $[0, 1]$, and product 2 is a "big" item and has match utility uniformly distributed on $[0, 4]$. Figure 4 below depicts how p_1 (in the left panel) and p_2 (in the right panel) vary with search costs. This example suggests that when the two products are asymmetric, search costs can affect their prices in different directions.

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[23]The volume formula for an m-dimensional sphere with a radius r is $V_m(r) = \frac{(r\sqrt{\pi})^m}{\Gamma(1+m/2)}$, where $\Gamma(\cdot)$ is the Gamma function. One can show that for any fixed r, $\lim_{m \to \infty} V_m(r) = 0$. Then as m goes to infinity, p will approach the perfect information price $1/2$. This is because for a fixed search cost, if each consumer is searching for a large number of products, they will almost surely sample both firms.
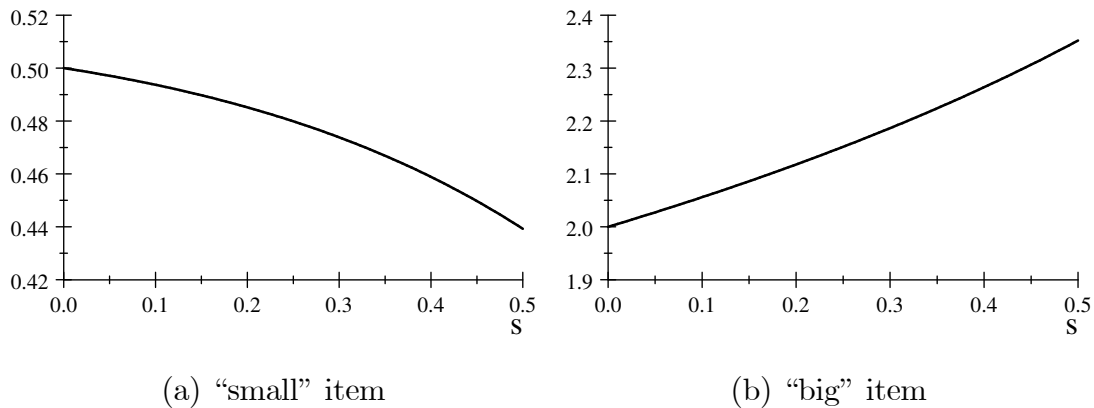
(a) "small" item   (b) "big" item

Figure 4: Prices and search costs (asymmetric products)

Discussion: larger search costs. The analysis so far has been restricted to relatively small search costs such that it is even worthwhile to search for one good alone. I now discuss the case with higher search costs beyond condition (4). (In some circumstances, a consumer conducts multiproduct search maybe just because it is not worthwhile to search for each good separately.) As we shall see later, this discussion will also be useful for understanding the results in the bundling case. For simplicity, let us focus on the case of symmetric products. Suppose the condition (4) is violated such that $s > \zeta_i(\underline{u})$ and $a < \underline{u}$. (But s cannot exceed $2\zeta_i(\underline{u})$ in order to ensure an active search market.) Then the reservation frontier is shown in Figure 5 below, where $c = \phi(\underline{u})$.
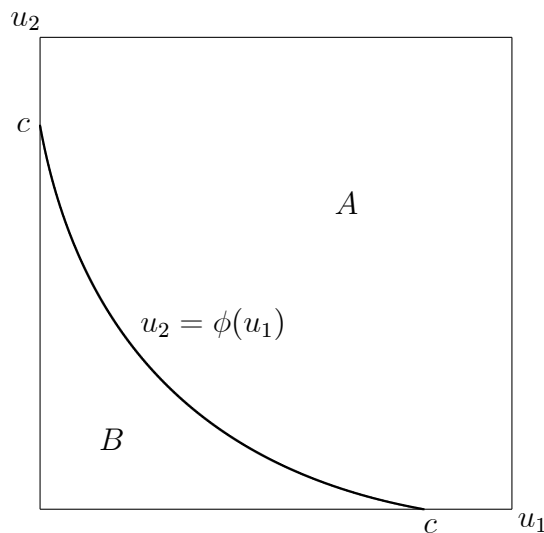

Figure 5: The optimal stopping rule for a large search cost

The key difference between this case and the case of small search costs is that now the frontier becomes "shorter" as search costs go up. This feature has a significant impact on how prices vary with search costs. For example, in the uniform case, a higher search cost now leads to fewer marginal consumers on the reservation frontier, which provides

firms with a greater incentive to raise prices. (In this case, the joint search effect strengthens the usual relationship between prices and search costs.)

In general, the following result suggests that prices often increase with search costs when they are beyond the condition (4). (The equilibrium price formula is given in the proof.)

Proposition 4 *Suppose the two products are symmetric and have independent match utilities, and search costs are relatively high such that $\zeta_i(\underline{u}) < s < 2\zeta_i(\underline{u})$. Then the equilibrium price p increases with search costs if each product's match utility has a monotonic density and an increasing hazard rate.*

For example, in the exponential example, we now have $p = 1/(4/3 - s^3/6)$ for $s \in (1, 2)$, which increases with s.

3.4   Price comparison with single-product search

As the end of this section, I compare the multiproduct search prices in section 3.2 with the single-product search prices in section 3.1, and discuss one possible empirical implication of the result.

Proposition 5 *Suppose the search cost condition (4) and the regularity condition (13) hold. Then $p_i \leq p_i^0$, $i = 1, 2$, i.e., each product's price is lower in multiproduct search than in single-product search.*

This result is intuitive. In our model, there are economies of scale in search, i.e., searching for two products is as costly as searching for only one, so more consumers are willing to sample both firms in multiproduct search, which intensifies the price competition. On top of that, the joint search effect gives rise to a complementary pricing problem and induces firms to further lower their prices. This result is illustrated in Figure 3 where the thin solid curves represent p_i^0 and the thick solid curves represent p_i (they coincide only when $s = 0$). For example, in the uniform case with $s = 0.1$, the multiproduct search price is 0.51, lower than the single-product search price 0.64 by 20%. I want to emphasize that even if economies of scale in search are weak (e.g., when multiproduct search is more costly than single-product search), the joint search effect can still induce substantial price reduction. For instance, in the uniform case, if single-product search is half as costly as two-product search (i.e., if its search cost is $s/2$), then the single-product search price becomes $1/(2 - \sqrt{s})$, depicted as the dashed curve in Figure 3(a). The multiproduct search price is still significantly lower than that. For example, when $s = 0.1$, the new single-product search price is 0.59, and the multiproduct search price is still lower than it by 13.5%.

If we extend the basic model by allowing for both single-product and multiproduct searchers in the market, the above result implies that market prices will decline when a higher proportion of consumers become multiproduct searchers, which is perhaps the case during high-demand periods such as weekends and holidays. For example,

many households conduct their weekly grocery shopping during weekends, and more people buy multiple gifts in Christmas season.[24] Thus, the multiproduct search model can provide a possible explanation for the phenomenon of *countercyclical pricing*, i.e., prices of many retail products fall during demand peaks such as holidays and weekends. (See relevant empirical evidence documented in Warner and Barsky (1995), MacDonald (2000), Chevalier, Kashyap, and Rossi (2003) and others. All these paper use data from multiproduct retailers such as supermarkets and department stores.)[25]

4   Bundling in Search Markets

Bundling is a widely used multiproduct pricing strategy. In practice, the most often adopted form is that alongside each separately priced product, a package of more than one product is sold at a discount relative to the components. For example, retailers such as electronic stores, travel agencies and online book shops often offer a customer a discount or reward (e.g., free delivery) if she buys more than one product from the same store.[26] This is termed *mixed bundling*. Another less often adopted form, termed *pure bundling*, is that the firm only sells a package of all its products, and no product is available for individual purchase.

Consumer search is clearly relevant in various circumstances where firms use bundling strategies, and could have a significant influence on firms' incentive to bundle and the welfare impacts of bundling. However, the existing literature on competitive bundling assumes perfect information in the consumer side and has not explored this issue. This section intends to fill this gap by allowing firms to adopt bundling strategies in the multiproduct search model presented in section 3. I continue to focus on the duopoly setting with two products. The rest of this section is organized as follows. I will first consider how bundling affects consumers' search incentive, which is the driving force behind the main result in this section. I will then show that starting from the linear pricing equilibrium, each firm does have an incentive to introduce bundling. After that,

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[24]Another possible justification is that the demand fluctuations may also arise endogenously: anticipating firms' pricing pattern, consumers may strategically accumulate their demand for various products and shop intensively during low-price periods, which in turn justifies firms' pricing strategies.

[25]Warner and Barsky (1995) have much earlier suggested such a possible explanation based on consumer search for countercyclical pricing, though they did not develop a formal search model. Their idea is wholly based on economies of scale in search, while my model suggests that even if economies of scale in search are weak, the joint search effect can still induce multiproduct firms to reduce their prices substantially. There are of course other existing explanations for countercyclical pricing. For example, it may be due to the dynamic interaction among competing retailers who are more likely to have a price war during demand booms (Rotemberg and Saloner, 1986). It may also be a consequence of a rise of demand elasticity caused by more price advertising during high-demand periods (Lal and Matutes, 1994).

[26]In a multiproduct environment, a two-part tariff—a fixed fee plus separate prices for each product— can also be regarded as a mixed bundling strategy. A typical example in the retail market is that the shipping fee is often independent of the number of products (e.g., furniture items) in the same order.

I will characterize bundling equilibrium and examine the welfare impacts of bundling relative to linear pricing.

4.1 Bundling and consumer search incentive

I first examine how bundling might affect consumers' search incentive. In the linear pricing case, given match utilities (u_1, u_2) at firm 1, the expected benefit from sampling firm 2 is

$$\mathbb{E}\left[\max\left(0, \sum_{i=1}^{2}(v_i - u_i), v_1 - u_1, v_2 - u_2\right)\right] . \tag{17}$$

(This merely rewrites the left-hand side of (6). The expectation operator is over (v_1, v_2).) If both products at firm 2 are a worse match, the consumer will return and buy at firm 1 and so the gain from the extra search will be zero; if both products at firm 2 are a better match, the consumer will buy at firm 2 and gain $\sum_{i=1}^{2}(v_i - u_i)$; if only product i at firm 2 is the better match, she will mix and match and the gain will be $v_i - u_i$.

Suppose now both firms adopt the mixed bundling strategy and charge the same prices. Let $\hat{p}_i$ denote the stand-alone price for product i and $\hat{P}$ denote the bundle price. In the meaningful case, the bundle should be cheaper than buying the two products separately (i.e., $\hat{P} \le \hat{p}_1 + \hat{p}_2$). (Otherwise, no consumers will opt for the package and we go back to the linear pricing case.) Let $\delta \equiv \hat{p}_1 + \hat{p}_2 - \hat{P}$ denote the *joint-purchase discount*. The bundle is also usually more expensive than each single product (i.e., $\hat{P} > \hat{p}_i$ for $i = 1, 2$). (If the bundle is cheaper than either single product, firms are in effect using the pure bundling strategy.) Then, conditional on (u_1, u_2), the expected benefit of sampling firm 2 becomes

$$\mathbb{E}\left[\max\left(0, \sum_{i=1}^{2}(v_i - u_i), v_1 - u_1 - \delta, v_2 - u_2 - \delta\right)\right] . \tag{18}$$

If the consumer buys both products from firm 2, the gain is the same as before (since the bundle price is the same across firms); but if she sources supplies from both firms, she must forgo the joint-purchase discount δ, which is the cost of mixing-and-matching. (Note that for consumers this tariff-intermediated cost plays the same role as an exogenous cost involved in dealing with multiple firms.) This benefit is clearly lower than (17), i.e., mixed bundling reduces a consumer's search incentive. (Pure bundling will lead to an even lower search incentive.[27]) In other words, when both firms bundle, consumers become more likely to stop at the first sampled firm. As I will demonstrate below, this may induce firms to compete less aggressively and reverse the usual welfare impacts of competitive bundling in a perfect information setting.

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[27] The exact search incentive with pure bundling depends on whether consumers can buy both bundles and mix and match. If this is permitted, the expected benefit of sampling firm 2 is $\mathbb{E}[\max(0, \sum_{i=1}^{2}(v_i - u_i), v_1 - u_1 - P, v_2 - u_2 - P)]$, since it now costs a bundle price P for a consumer to mix and match. If this is not permitted (e.g., if pure bundling introduces the compatibility problem), the expected benefit is $\mathbb{E}[\max(0, \sum_{i=1}^{2}(v_i - u_i))]$, since the consumer has totally lost the opportunity of mixing-and-matching.

4.2 Incentive to bundle

Before proceeding to the equilibrium analysis with bundling, I first investigate, starting from the linear pricing equilibrium, whether a firm has a unilateral incentive to bundle. I suppose that firms choose their bundling strategies and prices simultaneously, and both choices are unobservable to consumers until they reach the store. I will focus on the incentive to employ mixed bundling. (A similar result as below can be established for pure bundling if it is the only possible bundling strategy and if, once a firm bundles, consumers cannot mix and match.) First of all, given the rival's linear prices, introducing mixed bundling cannot make a firm worse off since it can at least set linear prices (by setting $\delta = 0$). What I will show below is that each firm has a strict incentive to choose $\delta > 0$.

Suppose firm 2 sticks to the linear equilibrium prices (p_1, p_2). Consider the following deviation for firm 1: $\hat{p}_1 = p_1 + \varepsilon$, $\hat{p}_2 = p_2 + \varepsilon$, and $\hat{P} = p_1 + p_2$, where $\varepsilon > 0$. That is, firm 1 raises each stand-alone price by ε, but keeps the bundle price unchanged. I will examine the impact of such a deviation on firm 1's profit as ε approaches zero. First, the consumers who originally bought a single product from firm 1 now pay more, which of course brings firm 1 a benefit.

There are also two demand effects. First, for those consumers who sample firm 1 first, due to the joint-purchase discount, more of them will stop searching and buy both products immediately. More precisely, given (u_1, u_2), the expected benefit of sampling firm 2 now becomes $\mathbb{E}[\max(0, \sum_{i=1}^{2}(v_i - u_i), v_1 - u_1 - \varepsilon, v_2 - u_2 - \varepsilon)]$, since mixing and matching involves an extra outlay ε. This is clearly lower than the search incentive in the linear pricing case. (Note that in this model even *increasing* prices can reduce consumers' search incentive.[28]) Consumers who switch from keeping searching to buying immediately will make a positive contribution to firm 1's profit.

Second, for those consumers who eventually sample both firms, the introduced joint-purchase discount will make them buy from the same firm (but not necessarily firm 1) with a higher probability. Firm 1 gains from those consumers who switch from two-stop shopping to buying both products from it, but suffers from those who switch to buying both products from firm 2. However, I show in the proof of the following proposition that as $\varepsilon \approx 0$ the pros and cons just cancel out each other, such that this second demand effect has no first-order effect on firm 1's profit. Therefore, the proposed deviation is strictly profitable at least when ε is small.

Proposition 6 *Starting from the linear pricing equilibrium, each firm has a strict incentive to introduce mixed bundling.*

In other words, if bundling is permitted and is costless for firms to implement, then any symmetric equilibrium (if exists) must involve both firms using bundling strategy. Note that our argument works even if the search cost is zero (then the second

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[28]The increased stand-alone price is paid only when a consumer returns to firm 1 and buys a single product, so it generates a returning cost for consumers who want to mix and match.

effect disappears). That is, each firm has a strict incentive to bundle even in a perfect information scenario.[29] Costly search provides firms with an extra incentive to do so.

4.3 The welfare impacts of bundling

I now investigate the welfare impact of bundling relative to linear pricing in a search environment. The first observation is that total welfare—defined as the sum of industry profit and consumer surplus—must fall with bundling. With the assumption of full market coverage, consumer payment is a pure transfer and only the match efficiency (including search costs) matters. Bundling reduces efficiency because it not only results in insufficient consumer search (i.e., too few consumers search beyond the first sampled firm due to the joint-purchase discount) but also induces too many consumers who have sampled both firms to buy both products from the same firm than is efficient. This result holds no matter whether information frictions exist or not.

In the following, I focus on the impacts of bundling on industry profit and consumer surplus, which, however, depend on information frictions. To this end, I first need to characterize equilibrium prices when both firms bundle. However, the equilibrium analysis with mixed bundling and costly search is fairly intricate. By contrast, pure bundling is easier to analyze and can capture the key insight. Therefore, I will start with analyzing the pure bundling case.

4.3.1 Pure bundling

As a pricing strategy, pure bundling is less often observed than mixed bundling. But it may become relevant when implementing the mixed bundling strategy is rather costly for firms (e.g., when the number of products is large). In the following analysis, I assume that when both firms bundle, consumers buy only one of the two bundles, i.e., they will not buy both bundles to mix and match. This is the case, for instance, when pure bundling introduces the compatibility problem, or when the equilibrium bundle price is so high that it is not worthwhile to buy both bundles.[30] (Nalebuff, 2000, makes the same assumption in studying competitive pure bundling.)

Equilibrium prices with pure bundling. When both firms bundle, consumers face a single-product search problem: firm 1 offers a composite product with a match utility $U = u_1 + u_2$ and firm 2 offers another one with an independent match utility $V = v_1 + v_2$. Both U and V belong to $[\underline{U} = \underline{u}_1 + \underline{u}_2, \overline{U} = \overline{u}_1 + \overline{u}_2]$. Let $G(\cdot)$ and $g(\cdot)$ denote their common cdf and pdf, respectively. Denote by b the reservation utility level in this search

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[29]Armstrong and Vickers (2010) have shown a similar result in the Hotelling setting with perfect information.

[30]For example, in the uniform example below, when the search cost is relatively high, the bundle price is greater than 1. Then even for a consumer who values firm 1's products at $(1, 0)$ and firm 2's products at $(0, 1)$, it is not worthwhile to buy both bundles.

problem. It satisfies

$$\int_b^{\overline{U}} (U - b)dG(U) = s \ . \tag{19}$$

The left-hand side is the expected benefit from sampling the second bundle given the first one has a match utility b. Hence, in a symmetric equilibrium a consumer will visit the second firm if and only if the first bundle has a match utility below b. Since pure bundling reduces consumers' search incentive, the acceptance set expands, i.e., $b < u_1 + \phi(u_1)$ for $u_1 \in [a_1, \overline{u}_1]$. Figure 6 below illustrates this change in the consumer stopping rule, where the linear line is the reservation frontier in the pure bundling case and the new acceptance set is A plus the shaded area.

Let P be the equilibrium bundle price. Then, similar to (5), P is determined in

$$\frac{1}{P} = g(b)[1 - G(b)] + 2\int_{\underline{U}}^b g(U)^2 dU \ . \tag{20}$$

P increases with the search costs provided that U has an increasing hazard rate, which is true if each u_i has an increasing hazard rate and is independent from u_j (see, for instance, Miravete, 2002).
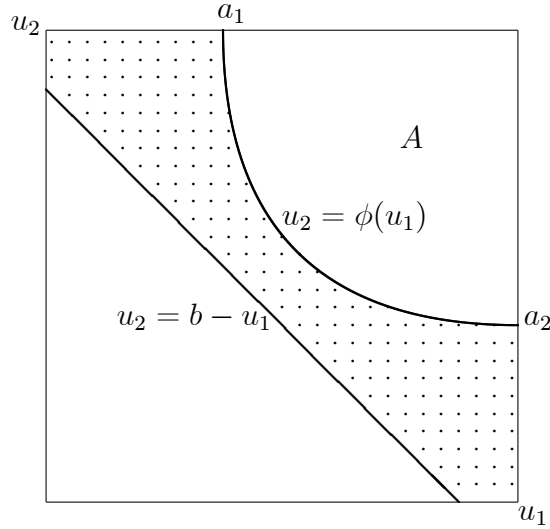

Figure 6: The optimal stopping rule—linear pricing *vs* pure bundling

Comparison with linear pricing. When information is perfect, Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) have shown in the two-dimensional Hotelling setting (with full market coverage) that pure bundling typically lowers price (and profit) and boosts consumer welfare. This is mainly because pure bundling makes a price reduction doubly profitable, thereby intensifying price competition (the so-called "Cournot effect").[31]

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[31]This intuition is, however, incomplete because bundling also affects the extent of product differentiation (see also Economides, 1989). For example, in our random utility setting, the bundle's match

The same argument applies in our setting when the search cost is zero.[32] Suppose the two products are symmetric. Then from (12) and (20) we can see that at $s = 0$ (so $a = \overline{u}$ and $b = \overline{U}$) pure bundling results in a lower bundle price ($P < 2p$) if and only if

$$\int_{\underline{u}}^{\overline{u}} f(u)^2 du < 2 \int_{\underline{U}}^{\overline{U}} g(U)^2 dU \ . \tag{21}$$

If the two products' valuations are independent, one can check that this condition holds for a variety of distributions such as uniform, normal and logistic. But it does not always hold. For instance, as we will see below, in the exponential case the *equality* of (21) holds.[33]

When search is costly, the pro-competitive effect of pure bundling still applies among the consumers who sample both firms. However, pure bundling weakens consumers' search incentive and so reduces the number of informed consumers in the first place, which has a tendency to soften price competition. The net effect hinges on the relative importance of these two forces. Intuitively, when the search cost is higher, there will be fewer fully informed consumers and the first effect will appear less important. Then pure bundling may lead to a higher bundle price. This intuition is confirmed in the following examples.[34]

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0,1]$. To facilitate the comparison with linear pricing, we keep the search cost condition $s \leq 1/2$. One can show that $G(U) = U^2/2$ and $g(U) = U$ if $U \in [0,1]$, and $G(U) = 1 - (2 - U)^2/2$ and $g(U) = 2 - U$ if $U \in [1,2]$. According to (19), the reservation utility b satisfies $(2 - b)^3/6 = s$ (so $b \geq 1$) if $s \in [0,1/6]$, and $1 - b + b^3/6 = s$ (so $b < 1$) if $s \in [1/6, 1/2]$. Then (20) implies

$$P = \begin{cases} \dfrac{1}{4/3 - s} & \text{if } s \in [0, 1/6) \\[2em] \dfrac{1}{b^3/6 + b} & \text{if } s \in [1/6, 1/2] \end{cases} .$$

One can check that P increases with s, but the speed is much faster when $s > 1/6$. (The upward sloping curve in Figure 7(a) below depicts how $P - 2p$ varies with search costs.) This is because in the range of $s \in [0, 1/6)$, $b > 1$ and so as s increases, the reservation frontier gets "longer" (i.e., there are more marginal

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utility has a greater variance than a single product, which usually softens price competition. Therefore, even with perfect information, whether pure bundling increases or decreases market price depends on a delicate interplay of these two effects. This accounts for why pure bundling does not always lead to lower prices even in the perfect information setting (see, for example, the exponential example below).

[32]With perfect information our random utility model can be converted into a two-dimensional Hotelling model.

[33]The opposite can also occur, for example, for a Weibull distribution $f(u) = ku^{k-1}e^{-u^k}$ with k less than but close to one.

[34]We can verify in both examples that (20) is also sufficient for the equilibrium price.

consumers), which mitigates firms' incentive to raise prices. By contrast, after s exceeds $1/6$, $b < 1$ and so the reservation frontier gets "shorter" as s increases, which strengthens firms' incentive to raise prices. In other words, when the reservation frontier is still getting longer in the linear pricing case, it already starts to get shorter in the bundling case. In particular, when the search cost exceeds roughly 0.26, the bundle price is higher in the pure bundling case than in the linear pricing case.

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $G(U) = 1 - (1 + U)e^{-U}$ and $g(U) = Ue^{-U}$. (Note that U has a strictly increasing hazard rate, though u_i has a constant one.) According to (19), the reservation utility b satisfies $(2 + b)e^{-b} = s$. Substituting G and g into (20) yields

$$P = \frac{2}{1 - e^{-2b}} \; ,$$

which increases with s and is *always* greater than the bundle price $2p$ in the linear pricing case (except $P = 2p$ at $s = 0$). (The upper curve in Figure 7(b) depicts how $P - 2p$ varies with search costs in this example.) With pure bundling, as s increases the reservation frontier always gets "shorter" in the exponential case, which explains why pure bundling reverses the relationship between price and search costs.

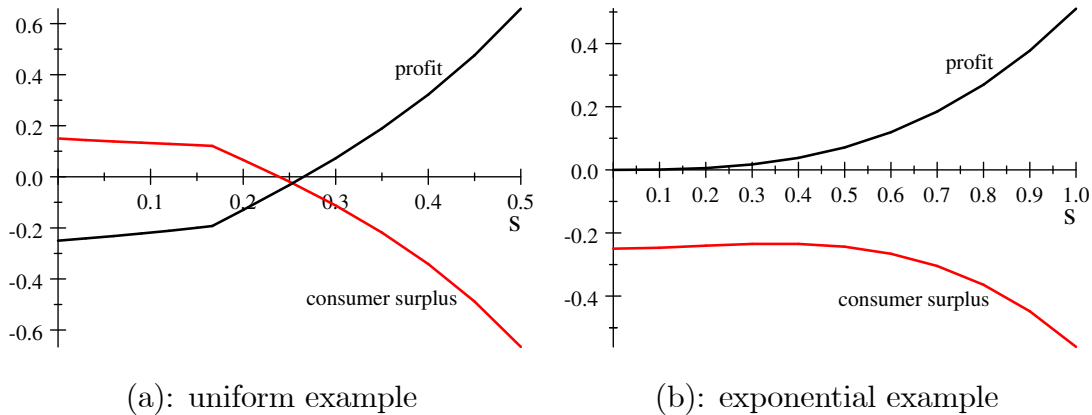(a): uniform example   (b): exponential example

Figure 7: The impacts of pure bundling

Let us turn to welfare impacts. First, each firm earns a higher profit whenever pure bundling leads to a higher bundle price (given the assumption of full market coverage). Hence, given that total welfare always falls with bundling, consumers must become worse off if the bundle price rises in the pure bundling case. But things are less clear when the bundle price falls because consumers also end up consuming less well matched goods. In the uniform example, as indicated by the downward sloping curve in Figure 7(a) which represents the impact of pure bundling on consumer surplus relative to linear pricing, pure bundling benefits consumers when the search costs are lower than about

0.24; while it harms consumers when the search costs exceed that threshold. In the exponential case, pure bundling always harms consumers since it (weakly) raises the bundle price for any search cost level. This is indicated by the lower curve in Figure 7(b).[35] (Calculating consumer surplus directly in our multiproduct search framework is complicated. I develop a more efficient indirect method in the Appendix B.)

In sum, in a search environment pure bundling can generate a significant competition-relaxing effect such that relative to linear pricing it can benefit firms and harm consumers, in contrast to the perfect information case.[36]

4.3.2 Mixed bundling

I aim to deliver a similar message as in the pure bundling case: since mixed bundling also reduces consumers' search incentive (and expands the stopping region), the reservation frontier starts to become shorter with search costs earlier than in the linear pricing case. As a result, when search costs are relatively high, prices with mixed bundling may increase with search costs much faster than in the linear pricing case. This in turn may lead to a positive impact of bundling on profits but a negative one on consumer surplus.

The details of characterizing the symmetric equilibrium prices $(\hat{p}_1, \hat{p}_2, \delta)$ are relegated to the online supplementary document. Unlike the linear pricing or pure bundling case, no analytical solution appears to be available even in the uniform or exponential example. Here I report the main numerical observations from the uniform example: (i) The joint-purchase discount δ decreases with search costs but it does not vary too much. When s increases from 0 to 0.5, δ decreases from 0.333 to about 0.294. (ii) Both the stand-alone prices and the bundle price increase with s. (iii) As depicted in Figure 8 below, relative to linear pricing, mixed bundling has a qualitatively similar impact on industry profit (the upward sloping curve) and consumer surplus (the downward sloping curve) as pure bundling. That is, when the search costs are relatively small, mixed-bundling harms firms but benefits consumers, while the opposite is true when the search costs are relatively high. However, since mixed-bundling is less able to deter consumers from searching than pure bundling, as we can see (by comparing Figures 7(a) and 8) higher search costs are needed to reverse the welfare impacts and the sizes

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[35]A more extreme example is when the two products are symmetric and have perfectly negatively correlated valuations. Then in the pure bundling case, the two bundles are in effect homogenous. With perfect information, we have the Bertrand competition and price will be equal to marginal cost, which is often better than linear pricing for consumers; while with costly search, we have the Diamond paradox in which all consumers stop at the first sampled firm (if the first search is costless) and the price will be the monopoly price (in our setting the consumer's willingness to pay), which is of course worse than linear pricing for consumers.

[36]Nevertheless, the search-based anti-competitive effect of bundling is most pronounced when the number of goods a consumer is looking for is relatively small. For a given search cost, if a consumer is looking for a large number of goods, she will almost surely sample both firms and the situation will then be close to the perfect information case. In that case, as shown in a previous version of this paper, under a regularity condition (f is logconcave), pure bundling benefits consumers and harms firms.
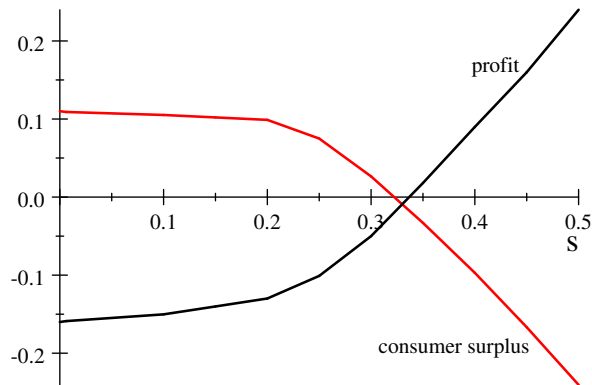
of impacts are also smaller.

Figure 8: The impact of mixed bundling (uniform example)

5   Discussions

5.1   Asymmetric equilibria

The analysis so far has been confined to symmetric equilibria. Here I discuss the possibility of two types of asymmetric equilibria in the duopoly model with linear pricing. In one possible case, one firm sells both products cheaper than its rival, and expecting that all consumers choose to visit it first. This type of equilibria could be sustained because consumers' non-random search order reveals information about their preferences—they visit the second firm only when they are unsatisfied with the first firm's products—and thereby the second firm indeed has an incentive to charge relatively high prices. However, an analytical investigation of this kind of equilibria is rather involved (mainly because the reservation frontier in equilibrium becomes now price dependent), and the existence of this kind of equilibria also relies on coordinated consumer expectation.

In the other possible case, firms may put different products on sales but consumers still search randomly. For example, in the case with two symmetric products, one firm charges price p_L for its product 1 and price $p_H > p_L$ for its product 2, and the other firm sets prices in the opposite way. However, as shown in the Appendix, this kind of equilibria cannot be sustained under a regularity condition.

5.2   More firms

Considering an arbitrary number of firms entails a more intricate analysis (see the online supplementary document for the details). But the main insights from the duopoly case can survive.

Search cost and price. In the linear pricing case, when a firm lowers one product's price, more consumers who are currently visiting it for the first time will stop searching, which boosts the demand for its both products. So the joint search effect is still present. However, a subtle difference emerges: for those consumers who stop searching at some

firm (except at the first one), they now do not necessarily buy both products from that firm. Instead, some of them may go back to a previous firm to buy one product. This tends to weaken the joint search effect, but does not eliminate it. For instance, in the exponential example with more firms prices can still decline with s (though not necessarily everywhere on $[0, 1]$).[37]

Bundling. Bundling still reduces the anticipated benefit from mixing-and-matching after sampling more firms, and so restrains consumers' search incentive. However, a new opposite force will come into play when $n \geq 3$—bundling now also restricts mixing-and-matching among previous offers and so lowers the maximum utility so far (except at the first firm), which can increase consumers' search incentive.[38] We can compare the expected search times and the bundle price between linear pricing and pure bundling in the uniform example. (Analyzing mixed bundling with more than two firms appears rather intractable.) I find that consumers search more intensively in the pure bundling case only if n is sufficiently large and s is sufficiently small. In particular, even if $n = \infty$, we need s to be lower than about 0.03. This suggests that the new force may be relatively weak most of the time. Consequently, pure bundling can still lead to a higher bundle price (and lower consumer surplus). For example, when $n = \infty$, this is true at least when s is greater than about 0.38.

The number of firms and price. The general model with n firms also allows us to examine how the number of firms affects market prices in a multiproduct search environment. In the single-product case, Anderson and Renault (1999) have shown that the equilibrium price decreases with n under the regularity condition. But this is no longer true in our multiproduct case. Although an analytical investigation is infeasible, numerical simulations suggest that prices can *increase* with n. For instance, in the uniform example with $s = 0.5$, the duopoly price is 0.583 while the price for $n = \infty$ is 0.602. (More examples are provided in the supplementary document.) The intuition is that when there are more firms, it becomes more likely that a consumer, when she stops searching, will return to previously visited firms to buy some products. This weakens the joint search effect and so the complementary pricing problem such that firms may raise their prices.

5.3   Costly recall

When recall is costly, the optimal stopping rule has a new feature: when one product is a good match and the other is a bad match, a consumer may buy the well matched product first (to avoid paying the returning cost) and then continue to search for the other. As a result, each firm will (endogenously) face both single-product searchers (who have bought one product from some previous firm) and multiproduct searchers.

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[37]When $n = \infty$, if prices tend to zero at $s \approx 0$, then they cannot decrease with s at least when s is small. However, the perfect-information prices for $n = \infty$ may not be equal to zero, for example, in the exponential case.

[38]For example, when the first firm offers $(0, 1)$ and the second firm offers $(1, 0)$, linear pricing obviously leads to higher maximum utilities so far than pure bundling.

The joint search effect survives, but the effect of bundling on consumer search can be different. For instance, in the polar case with no recall, since consumers cannot return to mix and match anyway, bundling does not reduce consumers' search incentive any more. However, in a more reasonable case where recall is costly but not totally banned, the search-discouraging effect of bundling, though reduced, will persist.

A complete analysis with costly recall is beyond the scope of this paper. In effect, when the returning cost is mild (such that returning consumers exist), the optimal stopping rule does not have a simple characterization even in the duopoly case.

5.4 Search costs *vs* shopping costs

Search costs usually mean the costs incurred to find and evaluate a new option. The literature sometimes also considers shopping costs. Literally, shopping costs should include all costs except payment involved in a shopping process, so search costs (if they exist) should be part of it. In a single-product case, these two terms are often used exchangeably, because if there are any shopping costs they are usually related with search activity. However, in a multiproduct case, even if information is perfect (e.g., when firms advertise both product and price information), there may still exist substantial shopping costs (e.g., the costs of conducting extra transactions) when the customer sources supplies from more than one firm (see, for instance, Klemperer, 1992, and Armstrong and Vickers, 2010). This type of shopping costs can cause a similar effect as our joint search effect, i.e., it renders two independent products in each firm complements and so has a tendency to intensify price competition. Nevertheless, there is an essential difference between search costs and this kind of shopping costs. Search costs always have their own anti-competition effect since they reduce consumers' incentive to shop around, while shopping costs in a perfect information setting are usually pro-competitive. In effect, shopping costs are similar to the joint-purchase discount in the mixed bundling scenario. If information is initially perfect, shopping costs tend to intensify competition and reduce market prices. While if information is initially imperfect and consumers need to conduct costly search, as we have learned from the bundling exercise shopping costs (which is similar to the joint-purchase discount) can work in the opposite way by reducing consumers' search incentive. Therefore, how (search unrelated) shopping costs affect competition may crucially depend on whether search costs are present or not in the same time.

5.5 Intrinsic complementary products

In reality the products a consumer is looking for in a particular shopping trip are rarely entirely independent as I assumed in the model. In many circumstances (e.g., when shop for clothes and shoes), they are more or less intrinsic complements in the sense that a higher valuation for one product increases the consumer's willingness to pay for the other (e.g., the utility function takes the form of $u_1 + u_2 + \lambda u_1 u_2$ with $\lambda > 0$). I assumed independent products mainly for tractability. (However, with the assumption

of full market coverage, we can actually regard the two products in the model as perfect complements, i.e., only consuming the two products together can generate utility.) If we consider intrinsic complements in the multiproduct search framework, then the reservation frontier may no longer be decreasing in the utility space.[39] This is because now finding a better matched product 1 may strictly *increase* a consumer's incentive to find a better matched product 2. This will greatly complicate calculation (e.g., when the utility function takes the form of $u_1 + u_2 + \lambda u_1 u_2$, I am unable to find an analytically tractable example). When there are more than two firms, considering intrinsic complements will render the optimal stopping rule non-stationary (see Gatti, 1999 for a related discussion), which will further complicate the analysis. However, the joint search effect and the effect of bundling on consumer search incentive should be still present, and so our main results may hold qualitatively.

Another point deserves mentioning is that intrinsic complementarity is different from the complementarity caused by the joint search effect. The latter means that reducing the price of a firm's one product will stop more consumers from searching on and so increase the demand for the *same* firm's other product as well. However, if information is perfect and the two products are intrinsic complements, then making a firm's one product cheaper will not influence the consumer decision of where to buy the other product. Hence, considering a perfect information setting with intrinsic complements cannot reproduce the main results in this paper.

6   Conclusion

This paper has explored a multiproduct search model and shown how consumers and firms may behave differently compared to a single-product search framework. In particular, the presence of the joint search effect may induce prices to decline with search costs and to rise with the number of firms. The developed framework has also been used to address other economic issues such as countercyclical pricing and bundling, and new insights emerged. For instance, I find that compared to the perfect information scenario, the welfare assessment of competitive bundling can be reversed in a search environment.

Our multiproduct search framework has other possible applications including the following.

Multiproduct vs single-product shops. In the market, large multiproduct sellers often coexist with smaller competitors (e.g., specialist shops). We can modify the basic model to investigate this kind of market structure. Consider a market with three asymmetric

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[39]For example, when the utility function takes the form of $u_1 + u_2 + \lambda u_1 u_2$, one can check that in the duopoly case the reservation frontier satisfies

$$\frac{1}{2}(1 - u_1)^2 + \frac{1}{2}(1 - u_2)^2 + \frac{\lambda}{4}\left[(u_1 - u_2)^2 + (1 - u_1 u_2)^2\right] = s \ ,$$

and it is not a monotonically decreasing curve in the (u_1, u_2) space if $\lambda > 0$.

firms: firm 1 supplies two products (say, clothes and shoes), while firms 2 and 3 are two single-product shops (say, firm 2 is a clothes shop and firm 3 is a shoe shop). Suppose the costs of reaching any firm are identical for all firms, and consumers visit firm 1 first (which can be rational in equilibrium). After visiting the multiproduct firm 1, a consumer will continue to visit firm 2 (firm 3) if and only if firm 1 offers unsatisfactory clothes (shoes). In this simple setting, changing the clothes price will no longer affect a consumer's decision whether to visit the shoe shop, so the joint search effect disappears and firms have two separate competitions for each product. However, other interesting insights will emerge. Given all consumers visit firm 1 first, their search order reveals information about their preferences: a consumer will visit a single-product shop only if she is unsatisfied with the product in the multiproduct shop. This gives the single-product shop extra monopoly power and induces it to charge a higher price.[40] Therefore, this variant can explain why multiproduct shops often set lower prices than their smaller competitors, without appealing to other exogenous reasons such as the multiproduct shop's economies of scale in operations or its advantage in bargaining with manufacturers.

Advertising and loss leaders. In the case of asymmetric products, we have found that firms have an incentive to sacrifice the profit from some small item to induce more consumers to buy the more profitable big item. This opens up the possibility of using the loss-leading strategy, but I have not found an example with some product being priced below its cost. Allowing for price advertising, however, may generate real loss leaders (see Lal and Matutes, 1994, for instance). Reducing a product's price (privately) can only stop some consumers who are already in the store from searching on, but advertising this price cut can increase the store traffic in the first place. This suggests that firms may compete intensely via advertised prices to attract consumers, and compensate the possibly resulted loss by charging high prices for unadvertised products (which can be sustained because of costly search). Compared to Lal and Matutes (1994), our richer setting may better predict which products will be sold as loss leaders. This remains another interesting future research topic.

Appendix A

Proof of Lemma 3: I only prove the result for $\tilde{p}_2$ (the proof for $\tilde{p}_1$ is similar). The price $\tilde{p}_2$, when both firms ignore the joint search effect, is given by

$$
\begin{aligned}
\frac{1}{\tilde{p}_2} &= 2 \int_B f_2(u_2) dF(\mathbf{u}) + \int_{a_1}^{\overline{u}_1} [1 - F_2(\phi(u_1))] f(u_1, \phi(u_1)) du_1 \qquad (22) \\
&= \int_{\underline{u}_1}^{\overline{u}_1} \left\{ 2 \int_{\underline{u}_2}^{\phi(u_1)} f_2(u_2) h_2(u_2 | u_1) du_2 + [1 - F_2(\phi(u_1))] h_2(\phi(u_1) | u_1) \right\} dF_1(u_1) .
\end{aligned}
$$

---

[40]See a similar logic in Armstrong, Vickers, and Zhou (2009) where a prominent firm which is always sampled first by consumers in a single-product search scenario charges a lower price than its non-prominent rivals.

(Note that for $u_1 < a_1$, $\phi(u_1)$ is independent of s and $1 - F_2(\phi(u_1)) = 0$.) The regularity condition (13) implies $f_2(x)h_2(x|u_1) + [1 - F_2(x)]h_2'(x|u_1) \geq 0$. Then the bracket term in (22) is an increasing function of $\phi(u_1)$. Moreover, $\phi(u_1)$ is the only element related with s and it decreases with s.[41] Therefore, (22) decreases with s, i.e., $\tilde{p}_2$ increases with s.

Proof of Proposition 2: In the case of symmetric products, from (12) we know the standard effect is

$$
\begin{aligned}
\alpha &= 2\int_B f(u_i)dF(\mathbf{u}) + \int_a^{\overline{u}}[1 - F(\phi(u))]f(u, \phi(u))du \\
&= \int_{\underline{u}}^{\overline{u}}\left\{2\int_{\underline{u}}^{\phi(u)} f(u_i)h(u_i|u)du_i + [1 - F(\phi(u))]h(\phi(u)|u)\right\}dF(u) .
\end{aligned}
$$

(Note that for $u < a$, $\phi(u)$ is independent of a and $1 - F(\phi(u)) = 0$.) Using the notation

$$
\lambda(x|u) \equiv f(x)h(x|u) + [1 - F(x)]h'(x|u) ,
\tag{23}
$$

we have

$$
\begin{aligned}
\frac{d\alpha}{ds} &= \int_a^{\overline{u}} \frac{d\phi(u)}{ds}\lambda(\phi(u)|u)dF(u) \\
&= \int_a^{\overline{u}} \frac{\phi'(u)}{1 - F(u)}\lambda(\phi(u)|u)dF(u) \\
&= -\int_a^{\overline{u}} \frac{f(\phi(x))}{1 - F(\phi(x))}\lambda(x|\phi(x))dx .
\end{aligned}
$$

The second step used

$$
\frac{d\phi(u)}{ds} = -\frac{1}{1 - F(\phi(u))}, \quad \phi'(u) = -\frac{1 - F(u)}{1 - F(\phi(u))} ,
\tag{24}
$$

which are both derived from the definition of $\phi(\cdot)$ in (6). The last step is from changing the integral variable from u to $x = \phi(u)$ and using the symmetry of $\phi(\cdot)$.

The joint search effect is $\beta = \int_a^{\overline{u}}[1 - F(u)]f(u, \phi(u))du$, and so

$$
\begin{aligned}
\frac{d\beta}{ds} &= f(a, \overline{u}) - \int_a^{\overline{u}} \frac{d\phi(u)}{ds}[1 - F(u)]h'(\phi(u)|u)f(u)du \\
&= f(a, \overline{u}) - \int_a^{\overline{u}} [-\phi'(u)]h'(\phi(u)|u)f(u)du \\
&= f(a, \overline{u}) - \int_a^{\overline{u}} h'(x|\phi(x))f(\phi(x))dx .
\end{aligned}
$$

---

[41] From the definition of $\phi(\cdot)$ in (6), we have $\frac{d\phi(u_1)}{ds} = -\frac{1}{1 - F_2(\phi(u_1))} < 0$ for $u_1 > a_1$, i.e., the reservation frontier moves downward as the search cost rises; and $\phi(u_1) > \overline{u}_2$ for $u_1 < a_1$ and is independent of s.

The first step used $\frac{da}{ds} = -1/[1 - F(a)]$, the second step used (24), and the last step is again from changing the integral variable from u to $x = \phi(u)$. Therefore, $p = 1/(\alpha + \beta)$ increases with s if and only if $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$ or the condition (15) in the main text holds.

Now suppose the two products have independent valuations and the marginal density satisfies $f'(u) \geq 0$. Then

$$
\begin{aligned}
-\frac{d\alpha}{ds} &= \int_a^{\overline{u}} \frac{f(\phi(x))}{1 - F(\phi(x))} \{f(x)^2 + [1 - F(x)]f'(x)\}dx \\
&\geq \int_a^{\overline{u}} \frac{f(\phi(x))}{1 - F(\phi(x))} f(x)^2 dx \\
&\geq \frac{f(a)}{1 - F(a)} \int_a^{\overline{u}} f(x)^2 dx \\
&\geq \frac{f(a)^2}{1 - F(a)} \int_a^{\overline{u}} f(x)dx = f(a)^2 \; ,
\end{aligned}
$$

and

$$
\begin{aligned}
-\frac{d\beta}{ds} &= \int_a^{\overline{u}} f'(x)f(\phi(x))dx - f(a)f(\overline{u}) \\
&\geq f(a)[f(\overline{u}) - f(a)] - f(a)f(\overline{u}) = -f(a)^2 \; .
\end{aligned}
$$

Therefore, $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$, i.e., p increases with s.

Proof of Proposition 3: I first derive the first-order conditions for the linear pricing case with m products. Let $\mathbf{u}_{-i} \equiv (u_j)_{j \neq i} \in \mathbb{R}^{m-1}$. In a symmetric equilibrium, without loss of generality the reservation frontier can be defined as $u_m = \phi(\mathbf{u}_{-m})$, where $\phi(\mathbf{u}_{-m})$ satisfies

$$
\sum_{i=1}^{m-1} \zeta_i(u_i) + \zeta_m(\phi(\mathbf{u}_{-m})) = s \; .
$$

As in the two-product case, let A denote the acceptance set and B denote its complement. Suppose firm 2 sticks to the equilibrium prices, and firm 1 lowers p_m by a small ε. Following the same logic as in the two-product case, we have the first-order condition for p_m:

$$
1 = \underbrace{2p_m \int_B f_m(u_m)dF(\mathbf{u}) + p_m \int_{A_{-m}} [1 - F_m(\phi(\mathbf{u}_{-m}))]f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m}))d\mathbf{u}_{-m}}_{\text{standard effect}}
$$
$$
+ \underbrace{\sum_{i=1}^{m-1} p_i \int_{A_{-m}} [1 - F_i(u_i)]f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m}))d\mathbf{u}_{-m}}_{\text{joint search effect}} \; . \quad (25)
$$

This can be understood as follows. First of all, the price reduction leads to a loss $\varepsilon/2$ since the half consumers who buy product m from firm 1 now pay less. The gain from this price reduction consists of three parts. (i) The consumers who sample both firms

will buy product m from firm 1 more likely, and this benefit is $\varepsilon/2$ times the first term in (25). (ii) Some consumers who sample firm 1 first will switch from searching on to buying at firm 1 immediately. More precisely, the reservation frontier moves downward by ε along the dimension of u_m. Denote by A_{-m} the projection of A on an $(m-1)$-dimensional hyperplane with a fixed u_m. Then the measure of these marginal consumers is

$$\frac{\varepsilon}{2} \int_{A_{-m}} f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m} .$$

For a marginal consumer with $(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m}))$, she would come back to buy product m with a probability $F_m(\phi(\mathbf{u}_{-m}))$ even if she searched on. So the net benefit from the increased demand for product m is $\varepsilon/2$ times the second term in (25). (iii) Similarly, the net benefit from the increased demand for all other products is $\varepsilon/2$ times the third term in (25), which is the joint search effect.

Now consider the uniform case with m symmetric products and independent valuations. Then the first integral in (25) measures the volume of solid B, and so it equals one minus the volume of solid A. Since A is $1/2^m$ of an m-dimensional sphere with a radius $\sqrt{2s}$, we get

$$1 - \frac{V_m(\sqrt{2s})}{2^m} .$$

(See the expression for $V_m(\cdot)$ in footnote 23.) The second integral equals

$$\int_{A_{-m}} [1 - \phi(\mathbf{u}_{-m})] d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^m} ,$$

since it just measures the volume of A. Finally, the third integral equals

$$\int_{A_{-m}} (1 - u_1) d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^{m-1}\pi} . \tag{26}$$

(This equality has no straightforward geometric interpretation. See its proof below.) Then (16) in the main text follows.

Proof of (26): For $m = 2$, $A_{-m} = [a, 1]$ and (26) is easy to be verified. Now consider $m \geq 3$. Let $A_{-1,m}(u_1)$ be a "slice" of A_{-m} at u_1. Then we have

$$\int_{A_{-m}} (1 - u_1) d\mathbf{u}_{-m} = \int_a^1 (1 - u_1) \left(\int_{A_{-1,m}(u_1)} d\mathbf{u}_{-1,m} \right) du_1 .$$

Since $A_{-1,m}(u_1)$ is $1/2^{m-2}$ of an $(m-2)$-dimensional sphere with a radius $r = \sqrt{2s - (1 - u_1)^2}$, the internal integral term equals

$$\frac{V_{m-2}(r)}{2^{m-2}} = \frac{\pi^{(m-2)/2} \cdot r^{m-2}}{2^{m-2}\Gamma(m/2)} ,$$

where $\Gamma(\cdot)$ is the Gamma function. Hence,

$$
\begin{aligned}
\int_{A_{-m}} (1-u_1)d\mathbf{u}_{-m} &= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \int_a^1 (1-u_1)\left(\sqrt{2s-(1-u_1)^2}\right)^{m-2} du_1 \\
&= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \frac{\left(\sqrt{2s}\right)^m}{m} \\
&= \frac{V_m(\sqrt{2s})}{2^{m-1}\pi}.
\end{aligned}
$$

The second step used $a = 1 - \sqrt{2s}$ and the fact that the integrand is the derivative of $\frac{1}{m}(\sqrt{2s-(1-u_1)^2})^m$ with respect to u_1. The last step used the expression for $V_m(\cdot)$ and the fact $x\Gamma(x) = \Gamma(x+1)$.

Proof of Proposition 4: In the case with two symmetric products, if the search costs satisfy $\zeta_i(\underline{u}) < s < 2\zeta_i(\underline{u})$, the equilibrium price p is given by

$$
\frac{1}{p} = \underbrace{2\int_B f(u_i)dF(\mathbf{u}) + \int_{\underline{u}}^c [1-F(\phi(u))]f(u,\phi(u))du}_{\text{standard effect: } \alpha} + \underbrace{\int_{\underline{u}}^c [1-F(u)]f(u,\phi(u))du}_{\text{joint search effect: } \beta},
$$

where $c = \phi(\underline{u})$. This is the same as (12), except the domain of $\phi(\cdot)$ is now different (see Figure 5). Following the same logic as in the proof of Proposition 2, one can verify that

$$
\frac{d\alpha}{ds} = -\frac{f(c,\underline{u})}{1-F(c)} - \int_{\underline{u}}^c \frac{f(\phi(u))}{1-F(\phi(u))}\lambda(u|\phi(u))du
$$

and

$$
\frac{d\beta}{ds} = -f(c,\underline{u}) - \int_{\underline{u}}^c h'(u|\phi(u))f(\phi(u))du,
$$

where $\lambda(\cdot)$ is defined in (23).

I aim to show $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$ under the proposed conditions. Independent valuations and increasing hazard rate imply $\lambda(u|\phi(u)) \geq 0$. So it suffices to show

$$
f(c)f(\underline{u})\frac{2-F(c)}{1-F(c)} + \int_{\underline{u}}^c f'(u)f(\phi(u))du \geq 0. \tag{27}
$$

If $f' \geq 0$, (27) is obviously true. Now suppose $f' < 0$. Since $f' \geq -\frac{f^2}{1-F}$ (which is implied by the increasing hazard rate condition), the second term in (27) is greater than

$$
\begin{aligned}
-\int_{\underline{u}}^c \frac{f(u)^2}{1-F(u)}f(\phi(u))du &\geq -\frac{f(c)}{1-F(c)}\int_{\underline{u}}^c f(u)f(\phi(u))du \\
&\geq -\frac{f(c)f(\underline{u})}{1-F(c)}\int_{\underline{u}}^c f(u)du \\
&= -\frac{f(c)f(\underline{u})}{1-F(c)}F(c).
\end{aligned}
$$

38

The first inequality used increasing hazard rate, and the second one used $f' < 0$. Hence, we only need $[2 - F(c)] - F(c) \geq 0$, which is always true.

Proof of Proposition 5: Recall $\tilde{p}_i$ is product i's price if both firms ignore the joint search effect, and $p_i \leq \tilde{p}_i$. So it suffices to show $\tilde{p}_i \leq p_i^0$. Let us consider product 2 (the proof for product 1 is similar). The price $\tilde{p}_2$ is defined in (22). We have known that under the regularity condition (13), the bracket term in (22) is an increasing function of $\phi(u_1)$. Since $\phi(u_1) \geq a_2$, it is greater than

$$2 \int_{\underline{u}_2}^{a_2} f_2(u_2) h_2(u_2|u_1) du_2 + [1 - F_2(a_2)] h_2(a_2|u_1) \ .$$

Realizing $\int_{\underline{u}_1}^{\overline{u}_1} h_2(x|u_1) dF_1(u_1) = f_2(x)$, we obtain

$$\frac{1}{\tilde{p}_2} \geq \int_{\underline{u}_1}^{\overline{u}_1} \left\{ 2 \int_{\underline{u}_2}^{a_2} f_2(u_2) h_2(u_2|u_1) du_2 + [1 - F_2(a_2)] h_2(a_2|u_1) \right\} dF_1(u_1)$$

$$= 2 \int_{\underline{u}_2}^{a_2} f_2(u_2)^2 du_2 + [1 - F_2(a_2)] f_2(a_2) = \frac{1}{p_2^0} \ .$$

Proof of Proposition 6: Following the argument in the main text, I only need to show that the second demand effect has no first-order impact on firm 1's profit. Given that firm 2 sticks to the linear prices (p_1, p_2) and firm 1 deviates to $\hat{p}_i = p_i + \varepsilon$ and $\hat{P} = p_1 + p_2$, a consumer's demand pattern after sampling both firms (conditional on the match utilities at the first sampled firm) is depicted in Figure 10 below, where the dashed lines indicate the boundaries in the linear pricing equilibrium and the solid lines indicate the boundaries after firm 1 deviates. It is clear that more consumers now buy both products from the same firm. For a consumer who samples firm 1 first and then visits firm 2 (so she must have $\mathbf{u} \in B(\varepsilon)$ which converges to B as $\varepsilon \to 0$), the shaded areas in Figure A1(a) represent the probability (conditional on (u_1, u_2)) that she switches from two-stop shopping to buying from the same firm. The pros and cons of such a change for firm 1 are also indicated in the figure. For example, firm 1 gains p_2 from a consumer who originally only bought product 1 from firm 1 but now buys both products from it, indicated by "$+p_2$" in the left shaded strip. A similar change occurs to a consumer who samples firm 2 first and then visits firm 1 (so she must have $\mathbf{v} \in B$). This is depicted in Figure A1(b). Due to the symmetry of firms (i.e., for every $\mathbf{u} \in B$ in Figure A1(a), there is a corresponding $\mathbf{v} \in B$ in Figure A1(b)) and the random search order, one can see that all effects in these two figures just cancel out each other.
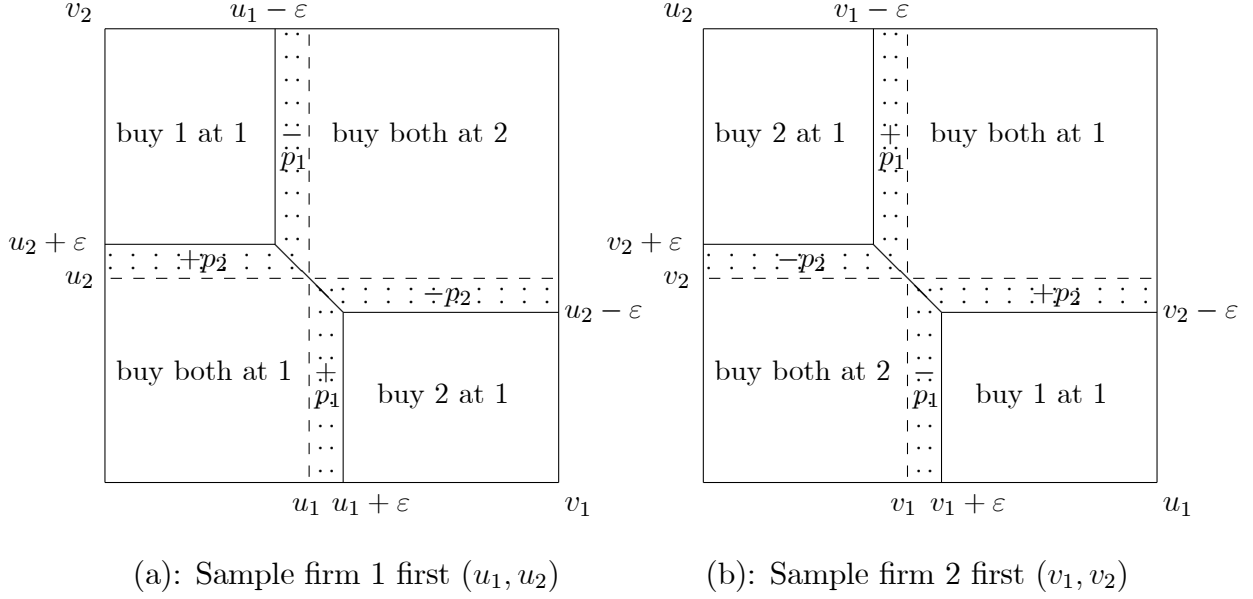
(a): Sample firm 1 first (u_1, u_2) (b): Sample firm 2 first (v_1, v_2)

Figure A1: Demand pattern after sampling both firms

About asymmetric equilibria in the duopoly model: This part aims to show that in the duopoly setting with two symmetric products and independent valuations, under the regularity condition that the marginal density f is logconcave, there do *not* exist asymmetric equilibria in which one firm, say, firm 1, charges (p_L, p_H) and firm 2 charges (p_H, p_L) with $p_L < p_H$ and consumers search in a random order. To this end, I will argue that starting from a hypothetical asymmetric equilibrium as above, it is a profitable deviation for firm 1 to set prices $(p_L + \varepsilon, p_H - \varepsilon)$ with a small $\varepsilon > 0$, given that firm 2 sticks to the equilibrium strategy and consumers hold the equilibrium belief. For notational convenience, let $\Delta \equiv p_H - p_L > 0$.

Consider the above hypothetical asymmetric equilibrium. Denote by $\phi_\Delta(u_1)$ the reservation frontier for those consumers who sample firm 1 first. It satisfies

$$\zeta(u_1 + \Delta) + \zeta(\phi_\Delta(u_1) - \Delta) = s.$$

(Charging a price lower/higher than its rival by Δ is equivalent to supplying a product with match utility higher/lower than its rival by Δ.) Then

$$\phi_\Delta(u_1) = \phi(u_1 + \Delta) + \Delta. \tag{28}$$

Relative to the reservation frontier $\phi(\cdot)$ in the symmetric equilibrium with $\Delta = 0$, the new reservation frontier $\phi_\Delta(\cdot)$ has shifted leftward by Δ and then upward by Δ. Similarly, for those consumers who sample firm 2 first, their stopping rule can be characterized by $\phi_\Delta(v_2)$ which solves $\zeta(\phi_\Delta(v_2) - \Delta) + \zeta(v_2 + \Delta) = s$. The following graphs illustrate the reservation frontiers (the thick lines) in the hypothetical equilibrium, relative to those (the feint lines) in the symmetric equilibrium.
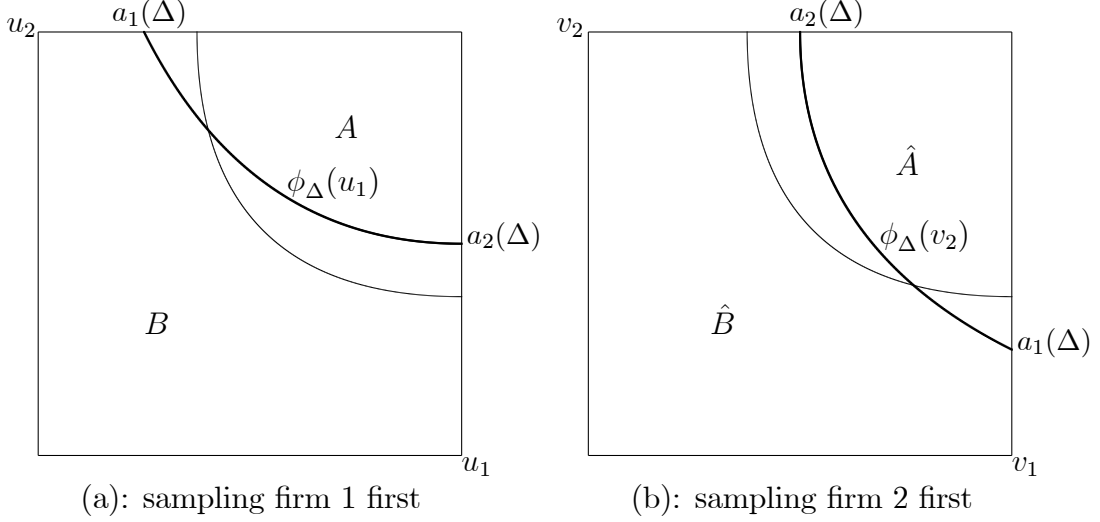
(a): sampling firm 1 first   (b): sampling firm 2 first

Figure A2: Asymmetric pricing and the stopping rule

It is ready to see $a_1(\Delta) = \phi(\overline{u} - \Delta) - \Delta$ (if $\overline{u} = \infty$, $a_1(\Delta) = a - \Delta$) and $a_2(\Delta) = a + \Delta$. Notice that $\phi_\Delta(\cdot)$ is no longer symmetric around the 45-degree line, and if $\overline{u} < \infty$, it also has a flat segment on the interval $[\overline{u} - \Delta, \overline{u}]$ (which has not been precisely indicated in the above graphs). In the following, for notational simplicity, I will restrict the discussion to the case with $\overline{u} = \infty$.

Now I start to derive the first-order effects of the small deviation $(p_L + \varepsilon, p_H - \varepsilon)$ on firm 1's profits. Denote by Q_i the number of consumers who buy product i from firm 1 in the hypothetical equilibrium.

(i) The first-order effects of lowering firm 1's p_H by ε (but keeping p_L unchanged). I follow the same logic as that used in the main text of the paper to derive the first-order conditions for the symmetric equilibrium. The direct loss from this deviation is εQ_2. But it also leads to two gains. First, those consumers who sample firm 1 first will stop searching more likely (the reservation frontier shifts downward by ε), which generates a gain

$$\frac{\varepsilon}{2} p_H \int_{a-\Delta}^{\infty} [1 - F(\phi_\Delta(u_1) - \Delta)] f(u_1, \phi_\Delta(u_1)) du_1 + \frac{\varepsilon}{2} p_L \int_{a-\Delta}^{\infty} [1 - F(u_1 + \Delta)] f(u_1, \phi_\Delta(u_1)) du_1.$$

Recalling $\phi_\Delta(u_1) = \phi(u_1 + \Delta) + \Delta$ from (28) and changing the integral variable from u_1 to $u = u_1 + \Delta$, we can rewrite the above expression as

$$\frac{\varepsilon}{2} p_H \int_a^{\infty} [1 - F(\phi)] f(u - \Delta, \phi + \Delta) du + \frac{\varepsilon}{2} p_L \int_a^{\infty} [1 - F(u)] f(u - \Delta, \phi + \Delta) du. \quad (29)$$

(The dependent variable in $\phi(u)$ has been suppressed.) Second, for those consumers who sample both firms, they will buy product 2 from firm 1 more likely, which generates a gain

$$\frac{\varepsilon}{2} p_H \left(\int_B f(u_2 - \Delta) dF(\mathbf{u}) + \int_{\hat{B}} f(v_2 + \Delta) dF(\mathbf{v}) \right). \quad (30)$$

(Here $B(\hat{B})$ is the non-stopping region for those consumers who sample firm 1(2) first as illustrated in Figure A2.)

(ii) The first-order effects of raising firm 1's p_L by ε (but keeping p_H unchanged). The direct gain from this deviation is εQ_1. But it also causes two losses. First, those consumers who visit firm 1 first will continue to search more likely (the reservation frontier shifts rightward by ε), which leads to a loss

$$\frac{\varepsilon}{2}p_H\int_{a+\Delta}^{\infty}[1-F(u_2-\Delta)]f(\phi_\Delta^{-1}(u_2),u_2)du_2+\frac{\varepsilon}{2}p_L\int_{a+\Delta}^{\infty}[1-F(\phi_\Delta^{-1}(u_2)+\Delta)]f(\phi_\Delta^{-1}(u_2),u_2)du_2.$$

By changing the integral variable from u_2 to $u=\phi_\Delta^{-1}(u_2)+\Delta$ (and so $u_2=\phi_\Delta(u-\Delta)=\phi(u)+\Delta$), we can rewrite it as

$$\frac{\varepsilon}{2}p_H\int_a^{\infty}[1-F(\phi)]f(u-\Delta,\phi+\Delta)(-\phi')du+\frac{\varepsilon}{2}p_L\int_a^{\infty}[1-F(u)]f(u-\Delta,\phi+\Delta)(-\phi')du. \tag{31}$$

Second, for those consumers who sample both firms, they will buy product 1 from firm 1 less likely, which leads to a loss

$$\frac{\varepsilon}{2}p_L\left(\int_B f(u_1+\Delta)dF(\mathbf{u})+\int_{\hat{B}}f(v_1-\Delta)dF(\mathbf{v})\right) \tag{32}$$

I claim the following result, which completes the argument.

Claim 1 *Suppose the two products have independent valuations, i.e., $f(u_1,u_2)=f(u_1)f(u_2)$, and the marginal density f is logconcave. Then the sum of all gains from the deviation, i.e., $\varepsilon Q_1+(29)+(30)$, is greater than the sum of all losses, i.e., $\varepsilon Q_2+(31)+(32)$.*

Proof. First, $Q_1>Q_2$ since product 1 is cheaper but product 2 is more expensive at firm 1 than at firm 2. So the gain εQ_1 from raising p_L by ε is greater than the loss εQ_2 from lowering p_H by ε. Second, the symmetry of the setting implies

$$\int_B f(u_2-\Delta)dF(\mathbf{u})=\int_{\hat{B}}f(v_1-\Delta)dF(\mathbf{v});\quad\int_B f(u_1+\Delta)dF(\mathbf{u})=\int_{\hat{B}}f(v_2+\Delta)dF(\mathbf{v}).$$

Thus, the gain in (30) is greater than the loss in (32).

Finally, I show that the gain in (29) is also greater than the loss in (31) if the two products have independent valuations and the marginal density f is logconcave. Notice that

$$\int_a^{\infty}[1-F(\phi)]f(u-\Delta,\phi+\Delta)(-\phi')du=\int_a^{\infty}[1-F(u)]f(u-\Delta,\phi+\Delta)du.$$

(Recall $-\phi'=\frac{1-F(u)}{1-F(\phi)}$.) Then it suffices to show that

$$p_H\int_a^{\infty}[F(u)-F(\phi)]f(u-\Delta,\phi+\Delta)du$$

$$\geq\ p_L\int_a^{\infty}[1-F(u)]f(u-\Delta,\phi+\Delta)(-\phi'-1)du$$

$$=\ p_L\int_a^{\infty}[F(\phi)-F(u)]f(u-\Delta,\phi+\Delta)(-\phi')du$$

$$=\ p_L\int_a^{\infty}[F(u)-F(\phi)]f(\phi-\Delta,u+\Delta)du. \tag{33}$$

(The last step is from changing the integral variable.)

I first argue that $\int_a^\infty [F(u) - F(\phi)]f(u - \Delta)f(\phi + \Delta)du > 0$ if f is logconcave. Let $\hat{u}$ solve $u = \phi(u)$. Then the left-hand side equals

$$
\int_a^{\hat{u}} [F(u) - F(\phi)]f(u - \Delta)f(\phi + \Delta)du + \int_{\hat{u}}^\infty [F(u) - F(\phi)]f(u - \Delta)f(\phi + \Delta)du
$$

$$
= \int_{\hat{u}}^\infty [F(\phi) - F(u)]f(\phi - \Delta)f(u + \Delta)(-\phi')du + \int_{\hat{u}}^\infty [F(u) - F(\phi)]f(u - \Delta)f(\phi + \Delta)du
$$

$$
> \int_{\hat{u}}^\infty [F(\phi) - F(u)]f(\phi - \Delta)f(u + \Delta)du + \int_{\hat{u}}^\infty [F(u) - F(\phi)]f(u - \Delta)f(\phi + \Delta)du
$$

$$
= \int_{\hat{u}}^\infty [F(u) - F(\phi)][f(u - \Delta)f(\phi + \Delta) - f(\phi - \Delta)f(u + \Delta)]du. \tag{34}
$$

(The first equality is from changing the integral variable. The inequality is because $\phi < u$ and $-\phi' \in (0, 1)$ for $u \in (\hat{u}, \infty)$.) If f is logconcave, then we have

$$
\ln f(\phi + \Delta) - \ln f(\phi - \Delta) \geq \ln f(u + \Delta) - \ln f(u - \Delta)
$$

given $\phi < u$ and $\Delta > 0$, which implies that (34) is positive.

Then to have (33), it remains to prove

$$
\int_a^\infty [F(u) - F(\phi)][f(u - \Delta)f(\phi + \Delta) - f(\phi - \Delta)f(u + \Delta)]du \geq 0.
$$

This can be done by dividing the integral interval into $[a, \hat{u}]$ and $[\hat{u}, \infty]$ and then applying the same logic as in showing (34) to be positive. ∎

Appendix B: Calculating Consumer Surplus

In our search model (especially in the case of linear pricing or mixed bundling), it is complicated to calculate consumer surplus directly. Here I develop a more efficient indirect method (which also carries over to the case with more than two firms).

For any given symmetric price vector $\mathbf{p}$ (which can a linear pricing, pure bundling, or mixed bundling scheme) and search cost s, consumer surplus is

$$
v(s|\mathbf{p}) = \sup_{\sigma \in \Sigma} [U(\sigma|\mathbf{p}) - s \cdot t(\sigma)] \ , \tag{35}
$$

where Σ is the (well-defined) set of all possible stopping rules, $U(\sigma|\mathbf{p})$ is the expected match utility minus payment if the consumer chooses a particular stopping rule σ, and $t(\sigma)$ is the expected search times. Let $\sigma(s|\mathbf{p})$ be the optimal stopping rule associated with $\mathbf{p}$ and s. Since the objective function in (35) is linear in s, $v(s|\mathbf{p})$ is convex in s and so is differentiable almost everywhere. Then the envelope theorem implies that

$$
v'(s|\mathbf{p}) = -t(\sigma(s|\mathbf{p})) \equiv -\hat{t}(s|\mathbf{p}) \ .
$$

If $\mathbf{p}$ is an equilibrium price vector, then $\hat{t}(s|\mathbf{p})$ is just the corresponding equilibrium number of searches. (In the duopoly case, it equals two minus the measure of the stopping region.) We can then decompose consumer surplus into two parts:

$$v(s|\mathbf{p}) = v(0|\mathbf{p}) - \int_0^s \hat{t}(x|\mathbf{p})dx , \qquad (36)$$

where the first term captures the surplus when the information is perfect (but given prices $\mathbf{p}$), and the second term reflects the inefficiency caused by imperfect information and costly search.

We can apply the general formula (36) to any case discussed in this paper. For example, in the linear pricing case with two firms, $v(0|\mathbf{p}) = \sum_{i=1}^2 (\mathbb{E}[\max(u_i, v_i)] - p_i)$, and the optimal stopping rule is independent of p and so $\hat{t}(x) = 2 - A(x)$, where $A(x)$ is the measure of the acceptance set when the search cost is x. In the pure bundling case, $v(0|\mathbf{p}) = \mathbb{E}[\max(U, V)] - P$ and $\hat{t}(x) = 1 + G(b(x))$. However, the implementation in the mixed bundling case is slightly more complicated. First, the joint-purchase discount δ affects the optimal stopping rule and so the equilibrium number of searches. Second, how to calculate $v(0|\mathbf{p})$ is now not so straightforward. Realize that in the mixed bundling case with $\mathbf{p} = (\hat{p}_1, \hat{p}_2, \delta)$, we have

$$v(0|\mathbf{p}) = \underbrace{\mathbb{E}\left[\max(u_1 + u_2 + \delta, v_1 + v_2 + \delta, u_1 + v_2, v_1 + u_2)\right]}_{w(\delta)} - (\hat{p}_1 + \hat{p}_2) .$$

The expectation is taken over all random variables u_i and v_i. Although $w(0) = \sum_{i=1}^2 \mathbb{E}[\max(u_i, v_i)]$ is straightforward to calculate, $w(\delta)$ for $\delta > 0$ is not. In the following, I explain how to derive a formula for $w(\delta)$. Realize that

$$w'(\delta) = 2\bar{Q}_{12}(\delta) = 1 - 2\bar{Q}_1(\delta) ,$$

where $2\bar{Q}_{12}(\delta)$ is the probability that a consumer buys both products from the same firm in the perfect information case (i.e., the probability that either of the first two terms in $w(\delta)$ dominates), and $2\bar{Q}_1(\delta)$ is the probability of a consumer mixing and matching (i.e., the probability that the third or fourth term in $w(\delta)$ dominates). The first equality is because when the joint-purchase discount δ is increased by a small ε, a consumer will benefit ε when she buys both products from the same firm, which occurs with a probability $2\bar{Q}_{12}(\delta)$. (Of course, the change of δ also affects which term in $w(\delta)$ dominates, but that effect on $w(\delta)$ is of second order when ε is small.) Thus, we obtain

$$w(\delta) = w(0) + \int_0^\delta [1 - 2\bar{Q}_1(z)]dz .$$

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