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ABSTRACT

Corruption in the public sector erodes tax compliance and leads to higher tax evasion. Moreover, corrupt public officials abuse their public power to extort bribes from the private agents. In both types of interaction with the public sector, the private agents are bound to face uncertainty with respect to their disposable incomes. To analyse effects of this uncertainty, a stochastic dynamic growth model with the public sector is examined. It is shown that deterministic excessive red tape and corruption deteriorate the growth potential through income redistribution and public sector inefficiencies. Most importantly, it is demonstrated that the increase in corruption via higher uncertainty exerts adverse effects on capital accumulation, thus leading to lower growth rates.

Key words: Corruption, growth, public goods, tax evasion, uncertainty

JEL code: D92, D72, E20, E60, H26, H41,G11, O16, O41

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CORRUPTION, UNCERTAINTY AND GROWTH

1 INTRODUCTION

Corruption usually means a deviation from what is considered to be normal or required by regulations or law. In other words, corrupt public officials distort rules and regulations. They do it to create and capture rents for themselves.

I argue that since corruption is illegal the outcomes of the corrupt deals are associated with risk. The corrupt bureaucrats may try to create a system of bribery and graft that is well-defined and accepted (or at least believed to be unavoidable) by the society. If it was possible then the burden created by such corruption would not be different from the public sector burden imposed by means of taxation as all agents would be aware about the bribes and whom and when to pay. However, corruption is not something that bureaucrat involved in it would display ostentatiously. Corrupt transactions are clandestine and therefore, risky.

In the environment with corrupt bureaucracy the allocation of government permits and licenses is unpredictable. Therefore, the private firms' output depending on such permits and licenses are also subject to uncertainty.

The another important aspect is that in the environment with highly corrupt and predatory bureaucracy there is always a risk that a private agent can be framed and extorted bribes by the public officials.

For example, Polinsky and Shavell (2001) studies corruption in the imposition of sanctions for violations of law and shows that the outcomes for the private agents may vary quite significantly depending on the regulation and enforcement structure and attitudes to risk.

Put another way, corruption leads to increased uncertainty for the private agents, as their disposable income may vary depending on the interactions with the public officials. In this paper, I would like to investigate how this type of uncertainty affects economic growth.

There is a quite extensive body of literature on the effects of income volatility on saving and investment. The model introduced by Bewley (1977) pioneered studies on growth impact of the labour income risk. The Bewley-type models focus on the precautionary saving and wealth distribution caused by the labour income uncertainty (Aiyagari, 1994; Calvet, 2001; Huggett, 1997). These studies infer that the labour-income risk leads to lower interest rates and over-saving in the steady state.

The models developed by Angeletos (2005), Angeletos and Calvet (2005) investigate the impact of idiosyncratic risk in production and investment. These studies find that in the presence of the production uncertainty, risk aversion dominates over the precautionary saving behaviour of the agents.

The aforementioned Bewley type models are mainly built around the assumption that there is no aggregate uncertainty[1]. The analysis of the relationship between risk and growth at the aggregate level is also interesting. A group of related papers by Eaton (1981), Gertler and Grinols (1982), Grinols and Turnovsky (1993) (1998) among others address this question. They analyse how risk related to different aggregate variables affects growth. These models predict that volatility and the related risk adversely affect growth.

Interestingly, not all empirical studies confirm the negative association between volatility and growth. At least the earlier findings of Kormendi and Meguire (1985) and Grier and Tullock (1989) based on cross-country data show that higher output volatility is correlated with higher average growth rate. Analysing the US data, Tsai et al. (1997) find no evidence on the relationship between output volatility and growth after accounting for the volatility associated with terms of trade, government expenditure, and monetary environment.

Contrary to the foregoing results, Ramey and Ramey (1995) conclude based on their empirical analysis that a higher macroeconomic volatility is associated with lower long-term growth. Aizenman and Marion (1993) (1999) also argue that there is significant correlation between volatility and private investment.

[1]Although there are extensions of Bewley model in this direction too; for example Krusell and Smith (1998)

Denizer, Iyugun, and Owen (2000) state that corruption can be an important factor contributing to volatility. Campos (2001) argue that predictability of corruption is also important and plays significant role in determining its growth impact. The idea that the secrecy stemming out of the illegal nature of corruption imposes an additional burden on the economy has been indicated by Shleifer and Vishny (1993).

It is clear that secrecy of corrupt transactions adds to the uncertainty related to dealing with bureaucracy. This additional risk can contribute to the overall volatility in the economy, since due to corruption the volatility at the individual level increases.

The economic impact of corruption has been studied extensively (Acconcia and d'Amato, 2006; Barelli and Pessôa, Barelli and Pessôa; Barreto, 2000; Huntington, 1968; Leff, 1964; Lui, 1985, 1996; Mauro, 1995, 1998, 2004; Rivera-Baitiz, 2002; Rose-Ackerman, 1998, 1999, 2002, 2004). However, the literature on corruption has neglected the growth impact of the uncertainty created by corruption. The aim of the paper is to tackle this problem.

The idea that the institutional structure may cause uncertainty has been captured in the model by Lin and Yang (2001) and Eichhorn (2006). They investigate a stochastic growth model with the uncertainty caused by tax evasion. In this paper I essentially would like to extend their model by broadening the uncertainty related to institutions. To analyse the effect of corruption-caused uncertainty on the capital accumulation, I employ a stochastic dynamic growth model with a public sector. In the model, I suppose that the corrupt interactions between the private and public sectors create random shocks to private disposable income. In particular, the model of Lin and Yang (2001) is extended by assuming a production function with a public input and income shocks engendered by the public sector.

Lin and Yang (2001) find that the growth rate is always higher for the environment with tax evasion. Their finding hinges on the assumption that public goods do not affect both consumption and production, and the tax rates are not optimized. Eichhorn (2006) shows that the optimizing government chooses higher tax rates and achieves the same growth rates as if there were no tax evasion, or in other words, tax evasion is neutral to the rate of economic growth.

The uncertainty in my model stems out not only from tax evasion but also from the predatory behaviour of the corrupt public officials. In my model, corruption takes place both in tax collection and public good production.

The rationale for this assumption stems from the fact that corruption always involves the public sector, therefore, the effects of corruption should be considered as an integral part of the public sector- private sector interactions.

It is a standard approach to aggregate the interactions between the public and private sectors by means of two variables. Namely, the net taxes, T and the government spending G. Then it seems reasonable to analyse corruption as distortions created with regards to these two variables.

Based on this rationale we have the following distortions created by corruption: First, the corrupt tax inspectors conceal tax evasion for the bribes paid by detected tax evaders. Second, the corrupt public officials abuse the authority given to them by attaching excessive red tape to the public services they are supposed to provide.

It is supposed that excessive red tape is a set of unnecessary procedures that has no productive value for firms. The firms have to incur the burden of excessive red tape in order to obtain the essential public services.

The corrupt officials can decrease the excessive red tape for the bribes paid by the firms. As a result, the corrupt officials capture a part of firms' profits. This income redistribution from the firms to the corrupt officials effectively imposes an illegal tax on the firms.

The degree or extent of corruption depends on the quality of the institutions embodying the public sector. Contrary to Lin and Yang (2001), it is demonstrated that when the productive inputs provided by the public sector accounted for, tax evasion leads to suboptimal growth in the decentralised setting. The social planner can increase the tax rate to account for the loss in tax revenue from evasion and thus voids the impact of tax evasion on growth.

This result supports Eichhorn (2006), though he derived his result in a setting different from mine. Then it is shown that an increase in corruption via the associated income uncertainty leads to less capital accumulation. This results in further deterioration of growth.

The structure of the paper is as follows: first, the setup of the basic model is described, then the implications based on the optimal solution obtained for the model is analysed. Based on the results, the basic model is extended to the environment with corruption to analyse its growth effects.

2 THE BASIC MODEL

Let us start with the simple case. First, we want to examine the growth effect of tax evasion under the assumption that the government provides an essential productive input to private production as in Barro (1990). In this assumption my model is different from the model used in Lin and Yang (2001) and Eichhorn (2006), for they assume unproductive public services. In addition, we can assume that the probability of detection of a tax evader is endogenous and depends on the rate of tax evasion.

Let us consider a closed economy with ex-ante identical infinitely-lived agents with zero population growth. Each agent has a measure of utility defined by a function on private consumption c. The utility function is given by

$$u(c) = \ln(c) \tag{1}$$

Assume that the agents maximize their expected utility

$$U(0) = E_0 \left\{ \int_0^\infty ln(c(t)) exp(-\rho t) dt \right\} \tag{2}$$

where ρ is the constant rate of time preference. Further, when it does not distort the underlying idea we omit the time argument.

Assume the production function has the following form:

$$y = Ak^a g^{1-a} \tag{3}$$

where y is output per capita, A is TFP coefficient, k is per worker capital, g is per worker public input. It is assumed that the production function is stationary within the planning horizon.

The government imposes an income tax at a flat rate τ. To increase their disposable income the agents evade taxes by underreporting their true income. We assume that the agent reports only $(1-e)y$ of his total income y in per capita terms. To combat tax evasion the government audits taxpayers randomly and depending on the evasion rate detects the evasion. The joint probability of being audited and detected is given by p. This probability is expressed as

$$p = \phi e \tag{4}$$

where $e < 1$ is the tax evasion rate, $\phi \in (0,1)$ is the institutional parameter that captures the effectiveness of the bureaucracy including tax administration. The detected evader pays back the due tax liability and some additional fine. This penalty is determined by a penalty rate θ=1+s, which includes the tax evaded and a surcharge for *s*. The tax paid by the agent is then either $T = \tau(1-e)y + \theta\tau ey$ with probability p, or $T = (1-e)y$ with probability $q = 1- p$. Consequently, the expected tax payment for the agent is expressed as

$$\overline{T}=\tau(1\text{-e})\text{y}+\text{p}\theta\tau\text{ey} \tag{5}$$

The random part of the agent's income can be described by the return on one unit of tax evaded: with probability p the return equals to $r = -s$, and with probability $q = 1- p$, equals $r = 1$. Then the expected return on a unit of tax evaded is found as

$$\overline{r} = p \cdot (-s) + (1-p) \cdot 1 = 1 - p(1+s) \tag{6}$$

By denoting θ=1+s we re-write (6) as

$$\overline{r} = 1 - p\theta \tag{7}$$

Then the agent's income is given by

$$y_d = (1-\tau)y + (1-p\theta)\tau ey + w_\tau \tag{8}$$

where the first two terms stand for the deterministic part of the income, $w_t = \sigma eyW$ is the stochastic part, σ is a constant, W is defined as a Wiener process. Later, we will discuss the rationale for this assumption.

By introducing a notation $\tau_e = \tau(1-\overline{r}e)$, the agent's income can be put as

$$y_d = (1-t_e)y + w_t \tag{9}$$

The variance of the return on tax evasion is given by the following [1] :

$$\sigma^2 = pq\theta^2\tau^2 \tag{10}$$

[1]The derivation is given in Appendix

It is assumed that $\bar{r}>0$ and thus risk-averse taxpayers evade taxes, as the expected return is positive. Since in our model all agents are identical *ex-ante*, all agents evade taxes as soon as the return to tax evasion is positive. Thus, the value of after-tax income is random and depends on being caught and penalized for tax evasion or being successful in the act of tax evasion.

Assuming that the probability of audit and detection is random for the taxpayer, we can conclude that this gamble of tax evasion has a binomial distribution. Then we can assume that the decision process of the taxpayers follows the Markov process. It is said that a stochastic process x_t has the Markov property if $\forall k \geq 2$ and *t*, the probability of outcome in period $t+1$ is conditional on the state in period t:

$$Pr(x_{t+1} \mid x_t, x_{t-1}, \dots, x_{t-k}) = Pr(x_{t+1} \mid x_t).$$

That is the future distribution of the states depends only on the current state.

This assumption is based on the following logic. Let us ask ourselves this question: If the tax evader is caught then should his future behaviour be affected and is the probability of the detection in the future for this particular taxpayer different? Let us try to elaborate on this.

Assume that the tax administration creates a "black-list" or prior tax-evasion records of the offenders and audits them with a higher probability in the next year, which means for those who has not been detected in the current year the probability of detection decreases even further, so that, they should increase their evasion. Thus, rational taxpayers should take account of this strategy employed by the tax administration and should not evade in the following year if caught in the current year. Therefore, the costly audit carried out by the tax administration would turn out in vain, and would be inefficient, as the other taxpayers not caught in the previous period would increase their evasion rate in the next period. It is evident, for the tax administration it is an efficient strategy to keep its audit random and with some probability (which depends on the cost of conducting an audit) equally faced by all taxpayers. That effectively means that the probability of audit and detection of this year should be independent from the similar events in other years.

Based on this logic, the administration path cannot be non-stationary. If we assume non-stationarity of the tax administration and taxpayer's behaviour that essentially means that we are assuming that if once caught for tax evasion the taxpayer's propensity to evasion decreases significantly forever. Then the government can in fact carry out tax audit of all taxpayers (as we assume a finite number of agents) over some finite number of years and thus significantly decrease tax evasion or eliminate it for good. Yet, the real life experience tells us that tax evasion is persistent even in the countries with very strong administrative machinery.

Based on the Markovian nature of the control variables it can be shown that the cumulative change of the agent's income $(y_t - y_0)$ follows a binomial distribution as we have only two outcomes. The random process, when the number of steps goes to infinity, converges to a Wiener process (also known as the Brownian motion)[1] . By ascertaining that the tax evasion process is the Wiener process we open a new avenue for analysis, as it allows us to employ a well-established stochastic dynamic optimization approach.

Lin and Yang (2001) first apply this methodology to capital accumulation problem with tax evasion and come up with a stochastic model of each individual's capital evolution. In my case I follow their approach; however, the first and second moments of the linear stochastic Itô differential equation are different as I am assuming a different functional form for the production process.

The households in their pursuit of utility maximization face a resource constraint in deciding what part of their income to consume and what part – to save. As the households are facing stochastic disposable incomes depending on the success of tax evasion, the amount of capital accumulation also follows a stochastic process. Based on our assumption that the random part of the disposable income of the agents follows the Wiener process, the amount of capital accumulation is given by the following linear stochastic Itô differential equation:

$$dk = [(1 - \tau + \overline{r}\tau e)y - c]dt + (\sigma e y)dW \qquad (11)$$

where σ is the standard deviation of the normalized process of random return on tax evasion, W is the Wiener process.

[1]As in Chang (2004); Dixit and Pindyck (1994)

We employ the dynamic programming method in dealing with the optimal control problems we face here. We digress here to describe the method following Dixit and Pindyck (1994)[1] .

2.1 Dynamic programming

We start with the description of theory of dynamic programming in an environment with uncertainty modelled as a discrete Markov process and then later show that the theory can be extended to continuous-time Wiener process.

Since we are interested in the decision of the households, we consider the household choosing its control variables and affecting its state variables. The current value of the state variable k_t is known, but all future values $k_{t+1}, k_{t+2}...$ are random variables. It is assumed that the values of the state variable follow the Markov process. At time t the household can choose its control variable value c_t.

The state and control variables at time t determine the household's utility $u_t(k_t, c_t)$. The k_t and c_t affect the probability distribution of the future states. The conditional cumulative probability distribution of the state in the next period is given by $\Phi_t(k_{t+1} \mid k_t, c_t)$.

The main idea employed in the dynamic programming is to split the decision sequence into two steps: the current period and all the future periods. Assume that the current time is t and the state is k_t Let us denote by $I_t(k_t)$ the expected present value of all of the households utility flows, when the household makes all decisions optimally from this point onwards.

When the household chooses the control variable c_t, it obtains current utility of $u_t(k_t, c_t)$. At the next period $(t+1)$, the state is determined by k_{t+1}. From that point in time onwards the sum of optimal utility flows are given by $I_{t+1}(k_{t+1})$. At the current period t the households treats it as an expected value $E_t[I_{t+1}(k_{t+1})]$.

[1]Dixit and Pindyck (1994), pp. 79-90

In the present-value terms, the overall utility is given by $u(k_t, c_t) + \frac{1}{1+\rho} E_t[I_{t+1}(k_{t+1})]$.

Then the households chooses c_t to maximize the lifetime utility, which is equal to $I_t(k_t)$. We write it as

$$I_t(k_t) = \max_c \left\{ u(k_t, c_t) + \frac{1}{1+\rho} E_t[I_{t+1}(k_{t+1})] \right\}. \tag{12}$$

The expression (12) in fact is Bellman's Principle of Optimality, which claims that if the policy is optimal then the remaining choices represent an optimal policy with respect to the problem starting at the state that resulted from the initial state due to whatever was the initial action. In other words, at any given period the agent needs to choose only the current control value optimally. Equation (12) is also called the Bellman equation.

In a more general form the Bellman equation is written as

$$I(k_t) = \max_c \left\{ u(k, c) + \frac{1}{1+\rho} E[I(k' \mid k, c)] \right\} \tag{13}$$

where instead of k_t and k_{t+1} we use *k* and *k*' as it could be any of possible states, and also the $I(k_t)$ is written without time index as the value function has the same form for all periods, by the virtue of the stationarity property.

The Bellman equation can be adjusted for the case when time is continuous. Assume that time changes in increments of Δt. Then the instantaneous utility is given by $u(k,c,t)$. The utility over the time of Δt is then $u(k,c,t)\ \Delta t$. Since ρ is the discount rate per unit of time, the total discounting over the time of Δt is given by $\frac{1}{1+\rho\Delta t}$. The Bellman equation then can be written as

$$I(k,\ t) = \max_c \ \left\{ u(k,c,t)\Delta t + \frac{1}{1+\rho\Delta t} \ \ E[I(k', t+\Delta t \mid k, c)] \right\}$$

Multiplying by $(1+\rho\Delta t)$ and rearranging yields

$$\begin{aligned}\rho\Delta t I(k,t) &= \max_c \left\{ u(k,c,t)\Delta t(1+\rho\Delta t) + E[I(k', t+\Delta t) - I(k,t)] \right\} \\ &= \max_c \left\{ u(k,c,t)\Delta t(1+\rho\Delta t) + E[\Delta I] \right\}\end{aligned}$$

Dividing this equation by Δt and taking the limit with regards to $\Delta t \to 0$, we get

$$\rho I(k,t) = \lim_{c} \left\{ u(k,c,t) + \frac{1}{dt} E[dI] \right\} \tag{14}$$

However, we need the Bellman equation that reflects the optimal decision process, when the state variable is defined by the Itô stochastic differential equation as in (11). Assume that the state variable, per capita capital stock *k*, follows a Wiener's process and given by

$$dk = a(k,t)dt + b(k,t)dW \tag{15}$$

Then the Itô's lemma states that for a function $I(k,t)$ which is twice differentiable in *k* and at least once in *t* the differential *dI* is given by

$$dI = \frac{\partial I}{\partial t}dt + \frac{\partial I}{\partial k}dk + \frac{1}{2}\frac{\partial^2 I}{\partial k^2}(dk)^2 \tag{16}$$

Taking into account (15) we write

$$dI = \left[\frac{\partial I}{\partial t} + a(k,t)\frac{\partial I}{\partial k} + \frac{1}{2}b^2(k,t)\frac{\partial^2 I}{\partial k^2} \right] dt + b(k,t)\frac{\partial I}{\partial x}dW \tag{17}$$

Applying the Itô's lemma to the value function we get

$$E\left[I(k+\Delta k, t+\Delta t) \,|\, k, c \right] = I(k,t) + \left[\frac{\partial I}{\partial t} + a(k,c,t)\frac{\partial I}{\partial k} + \frac{1}{2}b^2(k,c,t)\frac{\partial^2 I}{\partial k^2} \right]\Delta t + o(\Delta t)$$

where $o(\Delta t)$ stands for the terms that tend to zero faster than Δt. Using this result we can write (14) as

$$\rho I(k,t) = \max_{c} \left\{ u(k,c,t) + \frac{\partial I}{\partial t} + a(k,c,t)\frac{\partial I}{\partial k} + \frac{1}{2}b^2(k,c,t)\frac{\partial^2 I}{\partial k^2} \right\} \tag{18}$$

The specific form of $a(k,c,t)$ and $b(k,c,t)$ is found by comparing (15) and (11) or $a(k,c,t) = (1-\tau+\overline{r}\tau e)y - c$ and $b(k,c,t) = \sigma e y$. That yield the following state dynamics:

$$dk = [(1-\tau+\overline{r}\tau e)y - c]dt + (\sigma e y)dW \tag{19}$$

2.2 The household's optimization

An individual household maximizes its expected overall utility by choosing consumption level *c* and tax evasion rate *e* subject to the resource constraint.

$$max_{c,e} U = \int_0^\infty ln(c) exp(-\rho t) dt \tag{20}$$

$$s.t.\ dk = [(1 - \tau + \overline{r}\tau e)y - c]dt + (\sigma e y)dW \tag{21}$$

$$0 \le c \le (1 - \tau + \overline{r}\tau e)y, \quad 0 \le k, \quad k(0) = k_0 \tag{22}$$

$$0 \le e \le 1 \tag{23}$$

This problem is transformed to the Bellman equation as it is explained earlier:

$$\rho I(k,t) = \max_c \left\{ \ln(c(t)) + I'(k)\left[\left(1 - \tau + \overline{r}\tau e\right) y - c\right] + \frac{1}{2} I''(k)(\sigma e y)^2 \right\} \tag{24}$$

where $I(k) = max_{c,e} E_0 \left\{ \int_0^\infty ln(c) exp(-\rho t) dt \right\}$ s.t. (11), (21),(22), (23). In other words, $I(k)$ is the value function, E_0 is the conditional expectation operator for the given initial value of capital, $k(0) = k_0$.

The FOC of the Bellman equation (24) leads to

$$c(t) = \frac{1}{I'(k)} \tag{25}$$

$$e(t) = -\frac{I'(k)\overline{r}\tau}{I''(k)\sigma^2 y(t)} \tag{26}$$

Inserting back (26) and (25) into (24) we get

$$\begin{aligned} \rho I(k) &= \ln\left(1 / I'(k) \right) + I'(k)\left[\left(1 - \tau + \overline{r}\tau\left(-\frac{I'(k)\overline{r}\tau}{I''(k)\sigma^2 y(t)} \right)\right) y - \left(1 / I'(k) \right)\right] \\ &+ \frac{1}{2} I''(k)(\sigma\left(-\frac{I'(k)\overline{r}\tau}{I''(k)\sigma^2 y(t)} \right) y)^2 = \\ &= \ln\left(1 / I'(k) \right) - 1 + I'(k)(1 - \tau)A(g/k)^{1-\alpha} - \frac{[I'(k)]^2 (\overline{r}\tau)^2}{2 I''(k)\sigma^2} \end{aligned} \tag{27}$$

A general solution of this differential equation can be expressed in the following form, $I(k) = \frac{ln(k) + C}{\rho}$. A substitution for $I(k)$ in (27) leads to:

$$I(k) = \frac{1}{\rho}\left[\ln(\rho k) - 1 + \frac{(1-\tau)A(g/k)^{1-\alpha}}{\rho} + \frac{(\overline{r}\tau)^2}{2\rho\sigma^2} \right] \tag{28}$$

However, we know that due to tax evasion the public input g is given by

$$g = \tau(1-\overline{r}e)y = \varepsilon\tau y \tag{29}$$

Taking into account the production function given by (3), we write

$$g = \tau\varepsilon A k^{\alpha} g^{1-\alpha} \tag{30}$$

By rearranging (30), we obtain

$$\left(\frac{g}{k}\right)^{\alpha} = A\tau\varepsilon \tag{31}$$

By inserting (31) into (28), we get

$$I(k) = \frac{1}{\rho}\left[ln(\rho k) - 1 + \frac{(1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}}}{\rho} + \frac{(\overline{r}\tau)^2}{2\rho\sigma^2} \right] \tag{32}$$

Then we can find from (25) and (26) that,

$$c(t) = \rho k \tag{33}$$

$$e(t) = \frac{\overline{r}\tau}{\sigma^2 A\left(g/k\right)^{1-\alpha}} \tag{34}$$

Substituting (33) and (34) into (11) we obtain the equation of the capital evolution:

$$dk = ([1 - \tau + \frac{(\overline{r}\tau)^2}{\sigma^2 A\left(g/k\right)^{1-\alpha}}]Ak^{\alpha}g^{1-\alpha} - \rho k)dt + \frac{\overline{r}\tau k}{\sigma}dW \tag{35}$$

Dividing (35) by k we get

$$\frac{dk}{k} = ((1-\tau)A\left(g/k\right)^{1-\alpha} + \frac{(\overline{r}\tau)^2}{\sigma^2} - \rho)dt + \frac{\overline{r}\tau}{\sigma}dW \tag{36}$$

Then the per capita growth rate equals

$$\gamma_e = \frac{\dot{c}}{c} = E\left(\frac{dk}{k}\right) = (1-\tau)A\left(g/k\right)^{1-\alpha} + \frac{(\overline{r}\tau)^2}{\sigma^2} - \rho \tag{37}$$

By substituting for $\left(\frac{g}{k}\right)$ from (31) we get

$$\gamma_e = (1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} + \frac{(\overline{r}\tau)^2}{\sigma^2} - \rho \tag{38}$$

By analogy, we can show that in the absence of tax evasion the growth rate would be given by

$$\gamma_0 = (1-\tau)A(A\tau)^{\frac{1-\alpha}{\alpha}} - \rho \tag{39}$$

Comparing (38) and (39), we see that tax evasion does not increase the growth rate straightforwardly. There is a trade-off between the gain from tax evasion to disposable income and the loss because of the lower productivity due to lower public input. Therefore, the overall effect is ambiguous and depends on the on what effect is dominant.

2.3 Welfare optimization

Assume that government chooses tax rate so that the social welfare is maximized. Since, the economy populated with the identical individuals, this problem reduces to the maximization of the utility of the representative agent. It can be shown that for the current setup the welfare maximization is equivalent to maximization of the growth rate of the individual's consumption. This result is formulated as the following corollary. The proof is given in Appendix 5.2

Corollary 1. *For the government of the economy with identical individuals maximizing log utility function and producing with CRTS technology, welfare maximization is equivalent to growth maximization.*

Based on this rationale we consider growth maximization by government by setting the optimal tax rate for the individuals. Let us start with the Cobb-Douglas case given by (3). By substituting for $\sigma^2 = e\phi(1-e\phi)\theta^2\tau^2$, $\overline{r} = 1 - e\phi\theta$, and $p = \phi e$ we state the growth maximization problem as:

$$\max_{\tau} \gamma_e = (1-\tau)A(A\tau)^{\frac{1-\alpha}{\alpha}}(1-(1-e\phi\theta)e)^{\frac{1-\alpha}{\alpha}} + \frac{(1-e\phi\theta)^2}{e\phi(1-e\phi)\theta^2} - \rho \tag{40}$$

Since the growth rate here is a result of the optimal reaction of the taxpayer to the given tax system, tax evasion, e, is not a variable but is a specific value and thus independent of tax rate. The FOC of (40) gives us

$$\begin{aligned} \frac{d\gamma_e}{d\tau} &= -A^{\frac{1}{\alpha}}(1-(1-e\phi\theta)e)^{\frac{1-\alpha}{\alpha}}(\tau)^{\frac{1-\alpha}{\alpha}} \\ &+(1-\tau)A^{\frac{1}{\alpha}}\frac{1-\alpha}{\alpha}(1-(1-e\phi\theta)e)^{\frac{1-\alpha}{\alpha}}(\tau)^{\frac{1-\alpha}{\alpha}} - 1 = 0 \end{aligned} \tag{41}$$

Solving this yields

$$\tau^* = 1 - \alpha \tag{42}$$

This result for the optimal tax rate is the same as we get from the optimization for the non-evasion case

$$\max_c \gamma_0 = (1-\tau)A(A\tau)^{\frac{1-\alpha}{\alpha}} - \rho \Rightarrow \tau_0^* = 1 - \alpha \ ,$$

which is known as the Barro's optimal tax rate.

In order to draw general conclusions we need to consider the problem in a more general setting. For this purpose, we investigate the optimal tax problem for an economy with a more general form of the production function. In presentation, we follow Barro's (1990) specification.

2.3.1 CRTS production case

Assume that the technology is given by a function with constant returns to scale (CRTS), $y = F(k, g)$. Then, using the constant returns to scale property, we can write

$$y = F(k, g) = k\varphi\ (g/k) \tag{43}$$

The government expenditure is given by

$$g = \ \tau y = \ \tau k\varphi\ (g/k) \tag{44}$$

It is known that the growth rate for such economies is given by $\gamma = \frac{1}{\upsilon}[r - \rho]$, where $\rho > 0$ is a constant rate of time preference, $\frac{1}{\upsilon} > 0$ is a constant intertemporal elasticity of substitution. However, the equilibrium interest rate should equal the private return to capital $r = (1-\tau)\frac{\partial y}{\partial k}$. For the production function given by (43) this return is then written as

$$(1-\tau)\frac{\partial y}{\partial k} = (1-\tau)\varphi(g/k)[1-(g/y)\varphi'] \tag{45}$$

Therefore, the growth rate in this environment can be stated as

$$\gamma = \frac{1}{\upsilon}\left[(1-\tau)\varphi(1-\eta) - \rho\right], \tag{46}$$

where $\eta = \tau\varphi'$ is the elasticity of y with respect to g for given k. The steady-state growth implies that if the tax rate is constant, then g / y, g / k and η are constant.

The optimization procedure leads to the condition:

$$\begin{aligned}
\frac{d\gamma}{d\tau} &= \frac{d}{d\tau}\left\{\frac{1}{\upsilon}\left[(1-\tau)\varphi(1-\eta)-\rho\right]\right\} \\
&= \frac{1}{\upsilon}(1-\eta)\left[-\varphi+(1-\tau)\frac{d}{d\tau}(\varphi)\right] \\
&= \frac{1}{\upsilon}(1-\eta)\left[-\varphi+\frac{(1-\tau)\varphi'\varphi}{1-\eta}\right] \\
&= \frac{1}{\upsilon}\left[-\varphi(1-\eta)+(1-\tau)\varphi'\varphi\right] \\
&= \frac{1}{\upsilon}\left[\varphi(\varphi'-1)\right] = 0
\end{aligned} \tag{47}$$

$\frac{d}{d\tau}(\varphi)$ is found by making use of the fact that $g / k = \tau\varphi(g / k)$:

$$\frac{d}{d\tau}(\varphi) = \varphi'\left(\varphi + \tau\frac{d}{d\tau}(\varphi)\right) \tag{48}$$

By simplifying, we get

$$\begin{aligned}
&\Rightarrow \frac{d}{d\tau}(\varphi)(1-\tau\varphi') = \varphi'\varphi \\
&\Rightarrow \frac{d}{d\tau}(\varphi) = \frac{\varphi'\varphi}{(1-\tau\varphi')} = \frac{\varphi'\varphi}{(1-\eta)}
\end{aligned} \tag{49}$$

We infer from the result in (49) that the growth maximization is attained when $\varphi' = 1$.

Let us repeat this optimization procedure for the case with tax evasion. Assume that the equilibrium tax evasion is determined by $e^{*} = (1-\varepsilon)$, where ε is the equilibrium income-reporting rate. Then we re-write the growth rate as

$$\gamma_e = \frac{1}{\upsilon}\left[(1-\varepsilon\tau)\varphi(1-\varepsilon\eta)-\rho\right] \tag{50}$$

Here $g = \varepsilon\tau y$ and $\eta = \tau\varphi'$.

The FOC of (50) with respect to τ is found as

$$
\begin{aligned}
\frac{d\gamma_e}{d\tau} &= \frac{d}{d\tau}\left\{\frac{1}{\upsilon}\left[(1-\varepsilon\tau)\varphi(1-\varepsilon\eta)-\rho\right]\right\} \\
&= \frac{1}{\upsilon}(1-\varepsilon\eta)\left[-\varepsilon\varphi+(1-\varepsilon\tau)\frac{d}{d\tau}(\varphi)\right] \\
&= \frac{1}{\upsilon}(1-\varepsilon\eta)\left[-\varepsilon\varphi+\frac{(1-\varepsilon\tau)\varepsilon\varphi'\varphi}{1-\varepsilon\eta}\right] \\
&= \frac{1}{\upsilon}\left[-\varphi\varepsilon(1-\varepsilon\eta)+(1-\varepsilon\tau)\varepsilon\varphi'\varphi\right] \\
&= \frac{1}{\upsilon}\left[\varphi\varepsilon(\varphi'-1)\right]=0
\end{aligned} \tag{51}
$$

Again, we obtain an optimality condition given by $\varphi'=1$ as in the case with no-evasion. Hence, we conclude that the optimality condition for the public input is not affected by tax evasion, and formulate the following proposition:

Proposition 1: *Given that all tax revenue is used to produce public input to private production and the efficiency of the public sector is independent of tax evasion, and the production function exhibits CRTS, tax evasion has no effect on the optimality condition for the public input.*

Implication of this proposition is that in order to achieve the optimal level of the public input to production, the government chooses tax rate in a way that it takes into account the rate of tax evasion, and thus targets the effective tax rate. The government's task then becomes to equate the effective tax rate to the optimal tax rate, as it stems from the optimality condition we obtained earlier.

In both cases, we have the optimality condition $\varphi'_n = \varphi'_e = 1$, where subscripts *n* an *e* stand for no-tax evasion and tax evasion cases respectively. Denoting by g_n and g_e the amount of public input in the environment with no-evasion and with evasion correspondingly, we write down the equality of optimality conditions $\varphi'_n\left(\frac{g_n}{k}\right)=\varphi'_e\left(\frac{g_e}{k}\right)$. This leads to the equality of the variables $g_n = g_e$, for the given amount of k on the optimal growth path. However, under the assumption of the balanced government budget, we can substitute the public output with the tax revenue and write

$$
\tau_n y = \varepsilon\tau_e y \tag{52}
$$

where τ_n is the tax rate in no evasion environment, τ_e is the tax rate for the environment with evasion. Then we can find the optimal tax rate for the environment with tax evasion.

$$\tau_e^* = \frac{\tau_n^*}{\varepsilon} \tag{53}$$

As $\varepsilon < 1$ in the presence of tax evasion, the optimal tax rate for the environment with tax evasion must be adjusted upwards in order to equalize the effective tax rate with the optimal tax rate for the environment without tax evasion. The result is formulated in the following corollary of Proposition 1.

Corollary 2: *Welfare maximizing government sets the optimal tax rate in order to account for tax evasion, so that the effective tax rate is equal to the optimal tax rate for the no tax evasion case.*

This result is valid under the assumption that tax evasion does not cause any cost except the uncertainty it creates.

2.4 Decentralised equilibrium

Based on Proposition 1, we can also show that tax evasion is not growth enhancing in a decentralised setting. Suppose the government ignores tax evasion and sets the tax rate equal to the optimal rate for the no-evasion environment, then the resulting growth path would be suboptimal. Note that in the presence of tax evasion the effective tax rate is found as $\tau_e = \tau(1 - \overline{r}e)$ (as in(9)). For the sake of simplicity of exposition, we denote the equilibrium income-reporting rate as

$$\varepsilon = (1 - \overline{r}e) \tag{54}$$

and write $\tau_e = \tau e$. Next, we note that the disposable income in the presence of tax evasion given by $y_{de} = (1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} + \frac{(\overline{r}\tau)^2}{\sigma^2}$ can also be expressed as

$$\gamma_e = (1-\tau\varepsilon)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} - \rho \tag{55}$$

As both expressions of the growth rate are equivalent, we write the following identity:

$$(1-\tau\varepsilon)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} - \rho \equiv (1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} + \frac{(\overline{r}\tau)^2}{\sigma^2} - \rho \tag{56}$$

From (56) we infer that growth effect of tax evasion is two-prong: namely, there is a positive impact, $\Lambda = \frac{(\overline{r}\tau)^2}{\sigma^2}$, and a negative impact through lower public input provision. Some manipulation of (56) leads us to

$$\Lambda = A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}\varepsilon^{\frac{1-\alpha}{\alpha}}\tau(1-\varepsilon) \tag{57}$$

The loss is found as a difference of the first terms of (56) and (57)

$$\begin{aligned} Z &= (1-\tau)A(A\tau)^{\frac{1-\alpha}{\alpha}} - (1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} \\ &= (1-\tau)A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}(1-\varepsilon^{\frac{1-\alpha}{\alpha}}) \end{aligned} \tag{58}$$

Now we can state the overall growth effect of the tax evasion as a sum of these two effects

$$\begin{aligned} \Delta\gamma &= \Lambda - Z = A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}\varepsilon^{\frac{1-\alpha}{\alpha}}\tau(1-\varepsilon) - (1-\tau)A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}(1-\varepsilon^{\frac{1-\alpha}{\alpha}}) \\ &= A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}[\varepsilon^{\frac{1-\alpha}{\alpha}}\tau(1-\varepsilon) - (1-\tau)(1-\varepsilon^{\frac{1-\alpha}{\alpha}})] \\ &= A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}[\varepsilon^{\frac{1-\alpha}{\alpha}}(1-\tau\varepsilon) - (1-\tau)] \end{aligned} \tag{59}$$

With all other things being equal, only tax evasion can make the growth rate different for the economy with the possibility of tax evasion. Given this condition what is the tax evasion choice that maximizes the growth effect? This gives rise to the following:

$$\max_{c} \Delta\gamma = A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}[\varepsilon^{\frac{1-\alpha}{\alpha}}(1-\tau\varepsilon)-(1-\tau)] \tag{60}$$

The FOC of (60) leads to

$$\frac{d\Delta\gamma}{d\varepsilon} = A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}[\frac{1-\alpha}{\alpha}\varepsilon^{\frac{1-\alpha}{\alpha}-1} - \tau\varepsilon^{\frac{1-\alpha}{\alpha}}(\frac{1-\alpha}{\alpha}+1)] = 0 \tag{61}$$

A solution of which by the virtue of the fact that $\tau^* = 1-\alpha$ yields

$$\varepsilon^* = 1 \tag{62}$$

This result tells us that for the given specification of the economy, when the tax rate is set to the optimal level that is when $\tau^* = 1-\alpha$, the socially optimal tax evasion rate must be zero. This is not surprising, as any deviation from the optimal government size would lessen the growth rate.

We can see that if the tax rate were higher than the optimal rate then some positive evasion would be growth enhancing. On the other hand, if the tax rate were less than optimal than tax evasion would only harm growth. However, the private agent takes the amount of the public services as given, hence does not take into account the adverse effect of his tax evasion on the public good production. Since the equilibrium tax evasion is non-zero, the decentralized outcome should be suboptimal.

For the CRTS case, instead of maximizing the difference in the growth rates, as we did for the Cobb-Douglas case, we just carry out a growth maximization exercise with regards to tax evasion (which is the income reporting rate $\varepsilon = 1 - \overline{re}$ technically). The optimization leads us to the FOC:

$$\frac{\partial \gamma_e}{\partial \varepsilon} = \frac{1}{\upsilon}\left[-\tau\varphi(1-\varepsilon\eta) + (1-\varepsilon\tau)\frac{d}{de}[\varphi(1-\varepsilon\eta)]\right] = 0 \tag{63}$$

Noting that,

$$\frac{d}{d\varepsilon}[\varphi(1-\varepsilon\eta)] = -\varphi\eta + (1-\varepsilon\eta)\frac{d\varphi}{d\varepsilon}. \tag{64}$$

and taking into account that $g/k = \varepsilon\tau\varphi(g/k)$, we obtain $\frac{d\varphi}{d\varepsilon} = \varphi'(\tau\varphi + \varepsilon\tau\frac{d\varphi}{d\varepsilon})$.

This leads to $\frac{d}{d\varepsilon}[\varphi(1-\varepsilon\eta)] = -\varphi\eta + \frac{(1-\varepsilon\eta)}{(1-\varepsilon\tau)}\varphi$, then substituting it into (63) we get

$$\begin{aligned} \frac{\partial \gamma_e}{\partial \varepsilon} &= \frac{1}{\upsilon}\left[-\tau\varphi(1-\varepsilon\eta) + (1-\varepsilon\tau)[-\varphi\eta + \frac{(1-\varepsilon\eta)}{(1-\varepsilon\tau)}\varphi]\right] \\ &= \frac{1}{\upsilon}[-\tau\varphi(1-\varepsilon\eta) - (1-\varepsilon\tau)\varphi\eta + (1-\varepsilon\eta)\varphi]] \\ &= \frac{\varphi}{\upsilon}[(1-\tau)(1-\varepsilon\eta) - (1-\varepsilon\tau)\eta] = 0 \end{aligned} \tag{65}$$

We recall that for the optimal tax case $\varphi' = 1$ holds. This means $\eta = \tau\varphi' = \tau$. So, we transform (65) to

$$\frac{\partial \gamma_e}{\partial \varepsilon} = \frac{\varphi}{\upsilon}[(1-\tau)(1-\varepsilon\eta) - (1-\varepsilon\tau)\eta] = \frac{\varphi}{\upsilon}(1-\varepsilon\tau)(1-2\tau) = 0$$

The solution satisfying the FOC is $\varepsilon > 1$ as the tax rate is always $\tau < 1$. However, $\varepsilon > 1$ is not possible by definition, thus we have a corner solution here with optimal income reporting rate $\varepsilon = 1$. This is equivalent to saying that tax evasion rate is zero. Therefore, when the tax rate is set to the optimal level the socially optimal tax evasion rate is zero.

Again, the atomistic private agent does not take into account his adverse effect of his tax evasion on production and maximizes his private gain from tax evasion, which leads to positive tax evasion. That is to say that the private agent's problem is:

$$\max_e \Lambda = (1 - e\phi\theta)\tau ek\varphi \tag{66}$$

where we make use of $\Lambda = \overline{r}\tau ey = \overline{r}\tau ek\varphi = (1 - e\phi\theta)\tau ek\varphi$. Then the equilibrium value for the tax evasion rate is positive and given by

$$e^* = \frac{1}{2\phi\theta} < 1 \tag{67}$$

Therefore, providing that the tax rate is set without accounting for tax evasion, the decentralized equilibrium is socially suboptimal in the environment with more general CRTS technologies. By tacking account of tax evasion the government at best achieves the same growth rate as in the absence of tax evasion.

3 THE ECONOMY WITH CORRUPTION

Since tax evasion may take place even in the absence of corruption, we need to incorporate corruption into the model specified above. The act of corruption happens when public officials attempt to capture rents by abusing the power entrusted to them from the government or the private agents. Corruption can take place *ex ante* or *ex post*. In the *ex ante* case, the corrupt bureaucracy creates a situation, when the public good is sold at a price higher than its marginal cost. For that, the corrupt bureaucrat either decreases supply of the public good or bundles it with excessive red tape. In the either case, the cost of public good is higher for the private agent. In case of shortage of the public good, the private agent pays higher price for the good as it is described by Shleifer and Vishny (1993).

The other situations described by Barreto (2000) involves red tape in public good provision. Note, that in Barreto (2000) red tape generally is not a product of corruption, but rather a feature of any bureaucracy. Guriev (2004) also analyses red tape and corruption, and shows that when the bureaucracy is corrupt the level of red tape is above the social optimum. In general, red tape is a type of the public service that produces useful information about the private agents. I assume that red tape is just an unproductive hurdle created by corrupt officials. It is assumed that all other useful properties of the public sector including the informative red tape are embodied in the public goods they provide. In other words, my red tape is only the excessive red tape induced by the corrupt officials to coerce the private agents to pay bribes.

The *ex post* corruption happens only after the interaction between the private and public agents. It usually involves the situation when the public agent obtains some information about the private agent's failure in law or regulation abidance. Then in order to avoid the penalty for the infringement the private agent is willing to pay bribes to the public agent. A corrupt public agent chooses bribes and conceals the infringement. Thus, a corrupt deal occurs only if it is beneficial for the both agents.

Based on the discussion above we specify the following forms of corruption:

1. corruption in tax collection, which is manifested by concealment of the fact of tax evasion in case of detection by a corrupt tax inspector;
2. corruption in public good production, which is manifested by use of public position to create excessive red tape (transfer services) to extort income from the private individuals.

Since our modelling approach is to incorporate different types of corruption into the basic model we considered earlier, we need to discuss how each such an addition will complicate the dynamics of the model. We start with a simple case of corruption, when the tax inspectors assigned to audit the taxpayers engage in corruption and with some probability conceals the tax evasion for the bribe paid by the tax evader. Then we extend the model to include corruption in the public good provision.

3.1 Corruption in tax collection

We assume the same tax system as in the basic model described above. In other words, the probability of detection of the tax evader is given by $p = \phi e$ and when detected the taxpayer should pay fine equal to $\theta \tau e y,$ where $\theta > 1$.

An individual taxpayer treats the tax rate, tax audit probability, and penalty rate as given. We are also introducing corruptibility of the tax inspectors conducting the tax audits. We assume that a tax inspector can be corruptible with probability p_1 .

The extent of corruptibility depends on the quality of the institutions or specifically in their effectiveness in controlling corruption. Therefore, we can again write that the probability of the tax inspector being corrupt is a function of the quality of the public institutions:

$$p_1 = p_1(\phi)$$

Due to corruption the penalty rate becomes random as when audited and detected a taxpayer may pay bribe instead of tax penalty. In other words, θ in (8) should be adjusted to the following:

$$\theta_1 = \begin{cases} \theta - \text{with probability } q_1 = 1 - p_1 \\ b - \text{with probability } p_1 \end{cases} \tag{68}$$

Where $b < \theta$ is the bribe rate, so tax evasion costs the bribe paid instead of penalty, if the inspector is corrupt.

In general the bribe rate depends on the bargaining power of the involved private agents, which again depends on the institutional arrangements. The less the risk for the tax inspector to be caught and punished the more bargaining power he wields.

The expected value of the random penalty rate then is given by

$$E[\theta_1] = \overline{\theta} = p_1 b + q_1 \theta \tag{69}$$

Since $0 \leq p_1 < 1$, the expected penalty rate is lower when the tax inspectors are corrupt.

Given the context, for an individual taxpayer being audited and getting a corrupt deal is random. Thus, disposable income after taxes and audit is also a random variable given by,

$$y_d = \begin{cases} (1-\tau)y + (1-\overline{\theta})\tau ey, \text{with probability } p \\ (1-\tau)y + \tau ey, \text{with probability } 1-p \end{cases} \tag{70}$$

Then the random return on one unit of evaded tax is determined as $r_c = 1$ with probability (1-*p*) and $r_c = -(\overline{\theta} - 1)$ with probability *p*. The expected return to one unit of evaded tax is given by $\overline{r_c} = 1 - e\phi\overline{\theta}$. We notice that if the decrease in the effective penalty rate is dominates the increase in tax evasion rate, or $\overline{r_c} \geq \overline{r}$, the expected return on evaded tax with corruption is higher than without corruption. In any case, we conclude that tax evasion increases with corruption in taxation, as the effective penalty rate is strictly lower.

The variance of the return on tax evasion is found as in the case for the tax evasion without corruption:

$$\sigma_c^2 = pq\overline{\theta}^2\tau^2 \tag{71}$$

With corruption in the tax collection the level of the public good is defined as

$$g_c = \varepsilon\tau y \tag{72}$$

With corruption in taxation, $\varepsilon < 1 - \overline{r}e$, as a part of the amount paid by the taxpayer is the bribes paid to the tax inpectors.

Since $\overline{r_c} \geq \overline{r}$ and the tax evasion rate is greater, we conclude from (72) that with corruption the amount of public good produced is less than in the environment without corruption.

For the setting with corruption in taxation, the private disposable income is expressed by

$$y_d = (1 - \tau + \tau\overline{r_c}e)y + w_\tau \tag{73}$$

We notice that except the rate of return to tax evasion $\overline{r_c} \geq \overline{r}$ and its variance σ_c^2, the disposable income is found in similar way to the case without corruption. Using the expression obtained for disposable income, we can state the transition equation of this dynamic system:

$$dk = \left([1 - \tau + \frac{(\overline{r_c}\tau)^2}{\sigma_c{}^2 A\left(g_c/k\right)^{1-\alpha}}]Ak^\alpha g_c{}^{1-\alpha} - \rho k \right) dt + \frac{\overline{r_c}\tau k}{\sigma_c} dW \tag{74}$$

Analogously with the basic model described earlier, we can find the growth rate and the control functions:

$$\gamma_c = (1-\tau)A(A\tau\varepsilon)^{\frac{1-\alpha}{\alpha}} + \frac{(\overline{r}_c\tau)^2}{\sigma_c^{\,2}} - \rho \tag{75}$$

$$e = \frac{\overline{r}_c\tau}{\sigma_c^{\,2}A\left(\frac{g_c}{k}\right)^{1-\alpha}} \tag{76}$$

$$c = \rho k \tag{77}$$

Examining (75), (76), and (77) we infer that the corruption in tax collection only increases tax evasion, but does not change the structure of the model. Consequently, the dynamics of the model does not change with the introduction of corruption in tax collection. Therefore, we conclude that **Proposition 1** holds for the environment with corruption in tax collection.

3.2 Excessive red tape and deterministic income extortion

It is logical to start the dynamic analysis of corruption in public good provision with deterministic case, then incorporate the uncertainty this type of corruption may give rise. As we discussed earlier, corruption in public good provision occurs through creation of red tape, which is used to extract rents from the private agents. Specifically, we suppose that the public officials create excessive red tape and extort a part of private after-tax income.

Effectively, the tax revenue accrued to the government is used to produce of the public input into the private production and excessive red tape, which enables the corrupt officials to coerce the private agents to pay bribes. One may argue that the cost of red tape creation is negligible so should be ignored. So we assume that red tape does not cost anything to produce. Also we should introduce a concept of technology for the public good production. After all public goods are also produced by the human beings and thus the way they organize this production process should determine their output.

Since the public sector is embodied in different public institutions, the quality or organizational level of these institutions should play a crucial role in determining the productivity or the efficiency of the public sector. We assumed in (4) that the quality of the public institutions can be measured by index $\phi \in (0,1)$. The values of this index in the public good provision context can be interpreted as follows: As all public institutions are established for some purpose, then an institutions with ϕ=1 implies that this institution serves its purpose fully.

In order to make the setup a bit more general, we can assume that the productivity of the public sector depends of the average quality index ϕ and given by

$$\psi = \psi(\phi)$$

Then depending on the quality (or efficiency) of the public sector the same amount of tax revenue can be transformed to varying amount of public goods. This condition is formulated as the budget balance given by

$$g = \psi\varepsilon\tau y \quad (78)$$

where g is the productive public input, ψ is the productivity coefficient, while $\varepsilon\tau y = T$.

It is also intuitive that the capability of the bureaucracy to create excessive red tape depends on the quality of the institutions of the government. In countries with weak public institutions, the bureaucrats may have plenty of opportunities to be engaged in the rent seeking through excessive red tape, as the possibility of detection and punishment is low.

At the same time, the larger public sector size also should be associated with greater red tape. The reason for that is that the more interactions between the government and the private sector the more chances for the bureaucrats to extort bribes from the private agents. In general, the increase in extortions should not be proportional to the increase in the government size. Nevertheless, for the simplicity we are assuming this linear dependence as an approximation.

Suppose, as a result, the public official by accepting bribes captures a rent equal to

$$R = R(\phi, g, b) \quad (79)$$

Put another way, the rent captured by the bureaucracy is related to the rate bribes in taxation, the size of the public sector and its quality. It is reasonable to assume that an increase in the size of the government should lead to a proportional increase in rents captured by the bureaucracy. The quality of the public institutions should be embodied in the efficiency of the extortive behaviour of the bureaucrats. The bribe rates in all public sectors should be comparable, thus the bribes paid in taxation should affect bribes paid in other public sector activities.

Let us assume that this efficiency parameter is given by

$$\chi = \chi(\phi)$$

The higher values of ϕ imply lower efficiency of the bureaucrats in getting bribes.

Under this rationale we then assume the following explicit functional form for the rents of the corrupt bureaucracy:

$$R = \chi\psi\tilde{\varepsilon}\tau y \tag{80}$$

where $\tilde{\varepsilon} = 1 - \overline{r}e$. This is the effective quotient for the tax burden, which includes the bribes paid by the taxpayers.

In other words, we are saying that the rents captured through extortions are proportional to the gross burden of the public sector including the bribes taken at the taxation stage. Note, only if there is no bribes paid in taxation the gross public burden equals to the tax revenue collected by the government. With corruption there is always a wedge between the burden borne by the taxpayer and the revenue collected by the government.

Equivalently, due to corrupt extortions the taxpayer departs with a part of after-tax income equal to *R*. That leaves him with the disposable income given by

$$y_d = (1-\tau)y + \tau\overline{r}ey + w_\tau - R$$

So we see that if we account for predation of the corrupt officials then after-tax income is lower than with tax evasion only. Recalling the expression (80) for *R* we can write:

$$y_d = (1-\tau)y + \tau\overline{r}ey + w_\tau - \chi\psi\varepsilon\tau y \tag{81}$$

Taking into account that $\tilde{\varepsilon} = 1 - \overline{r}e$ and denoting

$$\tilde{\tau} = \tau(1+\chi\psi) \tag{82}$$

we re-write (81) as

$$y_d = (1-\tilde{\tau} + \tilde{\tau}\overline{r}e)y + w_\tau \tag{83}$$

The transition equation by analogy with the previous case is given by:

$$dk = ([1 - \tau + \frac{(\overline{r}\tilde{\tau})^2}{\sigma^2 A\left(g/k\right)^{1-\alpha}}]Ak^{\alpha}g^{1-\alpha} - \rho k)dt + \frac{\overline{r}\tilde{\tau}k}{\sigma}dW \tag{84}$$

This result indicates that the optimization problem has not changed mathematically as the transition equation is the same as in the basic model specified earlier. So that, when the private agents are subject to deterministic income extortions through excessive red tape, the income uncertainty still is generated by tax evasion only. Therefore, the dynamic characteristic of the model does not change. Then the policy functions are determined analogously to the tax evasion case.

$$c(t) = \rho k \tag{85}$$

$$e(t) = \frac{\overline{r}\tilde{\tau}k}{\sigma^2 y} \tag{86}$$

As the tax evasion process has not changed, the expected returns to tax evasion and its volatility has not changed too. However, $\tilde{\tau} = \tau(1+\chi\psi) > \tau$ then from (86) we conclude that when the public sector is predatory the optimal tax evasion rate increases.

The growth rate for this environment is obtained in a similar fashion to the tax evasion case and given by

$$\gamma_c = (1-\tilde{\tau})A\left(\psi\varepsilon\tau A\right)^{\frac{1-\alpha}{\alpha}} + \frac{[\overline{r}\tilde{\tau}]^2}{\sigma^2} - \rho \tag{87}$$

Recall that $g = \psi\varepsilon\tau Ak^{\alpha}g^{1-\alpha}$, which yields us $\left(\frac{g}{k}\right)^{\alpha} = \psi\varepsilon\tau A$. Substituting it in (87), we obtain

$$\gamma_c = (1-\tilde{\tau})A\left(\psi\varepsilon\tau A\right)^{\frac{1-\alpha}{\alpha}} + \frac{[\overline{r}\tilde{\tau}]^2}{\sigma^2} - \rho \tag{88}$$

The growth rate in such an environment is lower than if there was tax evasion only. Firstly, the burden imposed by the public sector is heavier due to red tape. Secondly, less productive public input is offered due to greater tax evasion and the waste caused by red tape.

In a more general specification, we write the production function as

$$y = F(k, g)$$

If we adjust for the distortions caused by tax evasion and corruption, engendered red tape we obtain the following expression for the growth rate:

$$\gamma_c = \frac{1}{\upsilon}[(1-\tau)F_k(k,\tilde{g}) + \frac{[\overline{r}\tilde{\tau}]^2}{\sigma^2} - \rho] \tag{89}$$

where $\tilde{g}=\psi\varepsilon\tau y$ is the government input in the corrupt environment, $F_k(k,\tilde{g})$ is the marginal product of capital.

We know that for a well-behaved production function $y = F(k,\tilde{g})$, the growth optimization yields a unique value for the optimal tax rate. The optimal tax rate then matched with the only optimal public input value for the given technology. The public input satisfies the condition $\tilde{g} = \psi\varepsilon g_0 < g_0$ as red tape and tax evasion strictly decrease the public input to private production. When $\tilde{\tau} < \tau$, the growth rate given by (89) is suboptimal, as both the tax rate and the public input are inefficient in this case. If the effective burden is greater than the statutory burden $\tilde{\tau}>\tau$, then private disposable income is lower. Therefore, the growth rate in such a corrupt environment is again strictly lower than in the clean environment. Consequently, in either outcome with corruption the growth rate is suboptimal or

$$\gamma > \gamma_c \tag{90}$$

where γ is the optimal growth rate in the clean environment, γ_c is the growth rate for the corrupt environment. The finding is summarised as follows:

Proposition 2. *In the environment with tax evasion, misuse of public funds, and extortion of private income by corrupt public officials, private disposable income is lower due to:*

- *lower productivity caused by lower public input,*
- *higher public burden caused by extortions.*

The lower disposable income leads to lower capital accumulation and that in turn - to lower growth.

3.3 Excessive red tape and stochastic income extortion

It is more realistic to assume that the extortion takes place with some probability. This setting also allows us to see how an increase in income uncertainty affects growth. With this additional assumption, disposable income is found as interaction of two stochastic processes:

1. tax evasion,
2. and income extortion by corrupt officials.

Suppose, the corrupt public official by accepting bribes captures a rent equal to $\tilde{R} = R + w_R$. Here the first term is the deterministic part of the extorted rents and the second term is its stochastic part. Then we adjust the expression for disposable income given in (83) to

$$y_d = (1 - \tilde{\tau} + \tilde{\tau}\overline{r}e)y + w_\tau + w_R. \tag{91}$$

The only difference form the previous case is that now we have a second stochastic term w_R in the equation. From the tax evasion dynamics, we know that $w_\tau = (\sigma ey)W$. However, we do not know the form of the second stochastic term. In general, both stochastic process can be autonomous and may have different parameters. However, that makes our algebra unwieldy. It is reasonable to assume that the income uncertainty from both tax evasion and excessive red tape is correlated, as both depend on the quality of the same underlying institutions. In other words, we assume that $w_R = (\sigma_R ey)W$. On this grounds we aggregate the stochastic processes and write

$$w = w_\tau + w_R = [(\sigma + \sigma_R)ey]W = (\tilde{\sigma}ey)W \tag{92}$$

Then we the policy functions and the transition functions are determined similar to the deterministic income extortions case:

$$c(t) = \rho k \tag{93}$$

$$e(t) = \frac{\overline{r}\tilde{\tau}k}{\tilde{\sigma}^2 y} \tag{94}$$

$$dk = \left[(1-\tilde{\tau})Ak^\alpha g^{1-\alpha} + \frac{[\overline{r}\tilde{\tau}]^2 k}{\tilde{\sigma}^2} - \rho k\right]dt - \frac{\overline{r}\tilde{\tau}k}{\tilde{\sigma}}dW \tag{95}$$

The per capita growth rate is given by:

$$\gamma_{cs} = (1-\tilde{\tau})A\big((1-\zeta)\varepsilon\tau A\big)^{\frac{1-\alpha}{\alpha}} + \frac{[\overline{r}\tilde{\tau}]^2}{\tilde{\sigma}^2} - \rho \tag{96}$$

As the uncertainty is higher in the stochastic extortion case than in the deterministic extortion case, the expected return to tax evasion decreases. In other words, $\tilde{\sigma} > \sigma$, implies the following important result:

$$\frac{[\overline{r}\tilde{\tau}]^2}{\tilde{\sigma}^2} < \frac{[\overline{r}\tilde{\tau}]^2}{\sigma^2} \tag{97}$$

Therefore, in the economy with stochastic predation by bureaucracy and tax evasion, growth is lower than in the economy with stochastic tax evasion only. That is

$$\gamma_c > \gamma_{cs} \tag{98}$$

where γ_c is the optimal growth rate for the case of deterministic extortions, γ_{cs} is the growth rate in the presence of stochastic extortions. Thus we conclude that the higher corruption increases uncertainty of disposable income. The higher uncertainty then decreases capital accumulation, thus retarding the growth potential of the economy. This result is formulated as the following proposition:

Proposition 3. *An increase in corruption increases the uncertainty of disposable income, which in turn decreases capital accumulation.*

More generally, we can state that the adverse economic effect of corruption is not limited to the income redistribution and inefficiencies in the public sector, but also corruption affects the saving and investment decisions by increasing the uncertainty related to capital incomes.

The ambiguous conclusion about the growth effects of corruption one may get based on the deterministic model is now tilted towards the conclusion that corruption is not good for growth. Even if the heavy burden of the public sector is lowered by corruption, the uncertainty created by corrupt bureaucracy can counterbalance the gains.

4 CONCLUSIONS

An analysis of a simple stochastic model that captures the uncertainty in disposable income caused by tax evasion demonstrates that when the tax rate is set to the optimal level, the extent of tax evasion should be zero for the socially optimal equilibrium. In the decentralized outcome, the equilibrium degree of tax evasion is greater than zero, which results in suboptimal amount of public input into private production. With the lower public input the private productivity decreases, which leads to lower a private level of income. Thus, the tax evasion even in the absence of corruption exerts a negative effect on growth.

The extensions to the basic model to incorporate corruption lead to the conclusions that corruption increases uncertainty faced by the private agents. Due to corruption, the expected burden of the public sector becomes heavier than the optimal burden in the absence of corruption. Therefore, it decreases the expected returns on private capital, which entails low capital accumulation and growth.

By incorporating an additional uncertainty stemming from predation of the corrupt bureaucracy, we show that as a result the overall uncertainty increases with the increase in the extent of corruption. The solution of the model for this higher uncertainty setting yields a lower steady-state growth rate. Therefore, we conclude that corruption adversely affects growth by imposing a higher degree of uncertainty with respect to disposable incomes.

The adverse economic effect of corruption is not limited to income redistribution and inefficiencies in the public sector, but also corruption affects the saving and investment decisions by increasing uncertainty related to capital incomes.

5 APPENDIX

5.1 Derivation of the variance

We denote the return on tax evasion by x. The variance of the return on tax evasion is then determined by

$$var(x) = E[(x)^2] - (E[x])^2 \tag{99}$$

where E is the expectation operator. The first term is determined as $E[(x)^2] = p(-s\tau)^2 + (1-p)\tau^2 = \tau^2[1-p(1-s^2)]$. Then the variance of this random variable is given by

$$\begin{aligned} \mathrm{var}(x) &= E[(x)^2] - (E[x])^2 = \tau^2[1-p(1-s^2)] - [(1-p(1+s))\tau]^2 \\ &= \tau^2[1-p(1-s^2)] - [1-2p(1+s)+p^2(1+s)^2] \\ &= \tau^2[p-p^2](1+s)^2 = p(1-p)(\theta\tau)^2 \end{aligned} \tag{100}$$

By denoting the variance of the return on tax evasion by σ^2 and obtain the following:

$$\sigma^2 = pq\theta^2\tau^2 \tag{101}$$

5.2 Welfare optimization

Assume that government chooses tax rate so that the social welfare is maximized. Since, the economy populated with the identical individuals, this problem reduces to the maximization of the utility of the representative agent,

$$\max_{\tau} U(\tau) = \int_0^\infty u(c)exp(-\rho t)dt \tag{102}$$

$$\text{s.t. } \gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \left[\left(1 - \tau + \bar{r}\tau e\right) {}^{y}\!/\!{}_{k} - \rho \right],\ k(0) = k_0$$

Therefore, in time t capital per capita is given by

$$k(t) = k_0 exp(gt) \tag{103}$$

Noting that $c(t) = \rho k(t)$ from (33) (which is valid for CRTS functions) and substituting for $k(t)$ we write the representative individuals utility function in the following form:

$$u(c) = ln(\rho k_0 exp(\gamma t)). \tag{104}$$

Then the optimization problem becomes

$$\max_{\tau} \tilde{U}(\tau) = \int_0^\infty \left[\ln(\rho k_0 \exp(\gamma t)) \right] \exp(-\rho t)dt \tag{105}$$

This is simplified further as

$$\tilde{U} = \gamma \int_0^\infty t \cdot exp(-\rho t)dt + ln(\rho k_0) \int_0^\infty exp(-\rho t)dt \tag{106}$$

We note that the second term of (106) is not a function of tax rate. Therefore, this and other constant terms can be ignored. In other words, $\max_{\tau} \tilde{U}(\tau)$ is equivalent to $\max_{\tau} \hat{U} = \gamma \int_0^\infty texp(-\rho t)dt$. This integration is solved as

$$\int_0^\infty te^{-\rho t}dt = -\frac{t}{\rho}e^{-\rho t} \Big|_0^\infty + \frac{1}{\rho}\int_0^\infty e^{-\rho t}dt = -\frac{1}{\rho^2}e^{-\rho t} \Big|_0^\infty = \frac{1}{\rho^2}$$

Then the first term of (106) becomes

$$\hat{U} = \frac{\gamma(\tau)}{\rho^2} \tag{107}$$

It is evident that the welfare maximization problem given by (105) is equivalent to maximization of the objective function given by (107). In other words, the welfare maximization is equivalent to maximization of the growth rate of the individual's consumption. We summarise the results as Corollary 1.

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