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Abstract

The Kuznets-Kaldor stylized facts are one of the most striking empirical regularities of the development process in industrialized countries: While massive factor reallocation across technologically distinct sectors takes place, the aggregate ratios of the economy are quite stable. This implies that cross-technology factor reallocation has a relatively weak impact on the aggregates, which is a puzzle from a theoretical point of view. We provide a model which can explain the Kuznets-Kaldor-puzzle by independent preferences and technologies. Furthermore, we show by empirical evidence that this explanation is in line with 55% of structural change in the USA between 1948 and 1987.

Keywords: *Kaldor facts, Kuznets facts, structural change, factor reallocation, sectors, balanced growth, unbalanced growth*

JEL Codes: O14, O41

1. Introduction

As shown by Kongsamut et al. (1997, 2001), the development process of industrialized countries over the last century satisfies two types of stylized facts: “Kuznets facts” and “Kaldor facts”.

Generally speaking, “Kuznets facts” state that strong structural change takes place during the development process.[1] Especially, in the early stages of economic development production factors are primarily reallocated from the agricultural sector to the industrial sector and in later stages of development factors are primarily reallocated from the manufacturing sector to the services sector. (It has also been shown that structural change takes place at more disaggregated level.)

On the other hand, “Kaldor facts” state that some key aggregate measures of the economy are quite stable during the development process; especially, the aggregate capital-to-output ratio and the aggregate income shares of capital and labour are quite stable, whereas the aggregate capital-to-labour ratio increases (at a fairly constant rate).[2] That is, the growth process seems to be “balanced” at the aggregate level.

As discussed by Kongsamut et al. (2001) and Acemoglu and Guerrieri (2008), the coexistence of Kuznets and Kaldor facts seems to be a puzzle, since strong factor-reallocations across sectors imply, in general, that Kaldor-facts are not satisfied (“unbalanced” growth of aggregates). Therefore, we name the empirically observable coexistence of Kuznets and Kaldor facts “Kuznets-Kaldor-puzzle”.

The literature which deals with the Kuznets-Kaldor-Puzzle (more or less explicitly) includes Kongsamut et al. (1997, 2001), Meckl (2002), Foellmi and Zweimueller (2008), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Boppart (2010). As discussed in detail by Stijepic (2011), we learn from this literature that

[1] Papers that provide empirical evidence for the massive labor reallocation across sectors during the growth process are e.g. Kuznets (1976), Maddison (1980), Kongsamut et al. (1997, 2001) and Ngai and Pissarides (2004). Kongsamut et al. (1997, 2001) formulate the following stylized facts of structural change for the last hundred years: 1.) the employment share of agriculture decreases during the growth process; 2.) the employment share of services increases during the growth process; 3.) the employment share of manufacturing is constant. Ngai and Pissarides (2007) note that the development of the manufacturing employment-share can be regarded as “hump-shaped” in the longer run.

[2] In detail, Kaldor’s stylized facts state that the growth rate of output per capita, the real rate of return on capital, the capital-to-output ratio and the income distribution (between labour and capital) are nearly constant in the long run; capital-to-labour ratio increases in the long run. It is widely accepted that these facts are an accurate shorthand description of the long run growth process (at the aggregate level) in industrialized countries. A discussion of these facts can be found in the paper by Kongsamut et al. (1997, 2001) and in the books by Maußner and Klump (1996) and Barro and Sala-i-Martin (2004).

the solution of the Kuznets-Kaldor-Puzzle in neoclassical growth frameworks requires, in general, the use of some knife-edge conditions. In fact, all papers used very severe restrictions to solve the Kuznets-Kaldor-Puzzle: all of them omitted some structural change determinants (which is the same as imposing some implicit knife-edge conditions) and/or imposed some explicit knife-edge parameter restrictions (like Kongsamut et al. (1997, 2001) and Meckl (2002)). Such (implicit and explicit) knife-edge conditions are severe restrictions, if their validity is not proven by empirical and/or theoretical reasoning. For an extensive discussion of these aspects see Stijepic (2011); for a discussion of structural change determinants see the next section.

We include all key structural change determinants into analysis and analyze whether the knife-edge conditions, which are required for the solution of the Kuznets-Kaldor-Puzzle, are empirically reasonable. Furthermore, we point to a possible theoretical micro-foundation of these knife-edge conditions.

The starting point of our analysis is the following fact: The key challenge to solving the Kuznets-Kaldor-Puzzle is already known since Baumol (1967): If production technology differs across sectors, the reallocation of factors across sectors causes unbalanced growth, i.e. Kaldor-facts are not satisfied.

Then, we approach as follows:

First, we show that Kaldor facts can be satisfied when factors are reallocated across technologically distinct sectors. In this sense our results postulate that structural change across technology can be "irrelevant" regarding the development of aggregate ratios. We name this type of structural change "neutral structural change". (Of course, the existence of neutral structural change requires some knife-edge conditions, which will be analyzed below, in our model.) Previously, Ngai and Pissarides (2007) have shown that neutral structural change can arise when all sectors have the same capital-intensity. However, Acemoglu and Guerrieri (2008) have shown that their results do not hold if capital-intensities differ across sectors, i.e. they show that in this case growth is, in general, unbalanced. In some sense, our result contradicts Acemoglu and Guerrieri (2008), since neutral structural change arises despite the fact that capital-intensities differ across sectors in our model. We are able to obtain our results, since, in contrast to Acemoglu and Guerrieri (2008), we assume a utility function that has non-unitary price elasticity of demand (i.e. each good has its own specific price elasticity) and since we assume that at least one of the three sectors uses two technologies. (As we will discuss in our essay, the latter assumption is consistent with empirical evidence, which postulates that, e.g., the services sector is quite technologically heterogeneous.) Furthermore, in contrast to Acemoglu and Guerrieri (2008), we model sectors which feature non-constant output-elasticities of inputs.

Second, we study the empirically observable patterns of structural change and analyze whether they were neutral or non-neutral. In this sense, we analyze implicitly whether the knife-edge conditions, which ensure the satisfaction of the Kuznets-Kaldor-facts in our model, are given in reality. We develop an index of neutrality of structural change and show with the data for the US between 1948 and 1987 that about 55% of structural change was neutral structural change. Hence, neutrality of structural change seems to be a relatively large explanatory variable regarding the Kuznets-Kaldor-puzzle. We show as well that this result applies to the most of the previous literature, implying that the previous literature can explain (maximally) 55% of structural change over the observation period.

Third, we show that low (no) correlation between preference parameters and technology parameters can explain the prevalence (existence) of neutral structural change in reality (our model).[3] We also suggest that the assumption of uncorrelated preferences and technologies may be theoretically reasonable in long run growth models. In this sense, the independency between preferences and technologies can be a theoretical foundation of the knife-edge conditions which are necessary for the solution of the Kuznets-Kaldor-Puzzle.

In the next section (section 2) we provide some evidence on sectoral structures which are observed in reality, in order to provide an empirical basis for our discussion and model assumptions. Then, in section 3, we provide a PBGP-model of structural change in order to show the existence of neutral structural change; we also generalize some of the model results in Proposition 4 of this section. Section 4 is dedicated to the empirical analysis, where among others we develop an index of neutrality of structural change and analyze the cross-capital-intensity structural change patterns in detail. In section 5 we discuss the assumption of low correlation between technology and preferences. Finally, in section 6 we provide some concluding remarks and hints for further research.

[3] It should be noted here that previously it has been mentioned by Foellmi and Zweimueller (2008) that some type of independency between technology and preferences may be useful for generating aggregate balanced growth. However, this topic has not been studied further by them.

2. Stylized facts of sectoral structures

2.1 Stylized facts regarding cross-sector-heterogeneity in production-technology

Empirical evidence implies the following stylized facts of sectoral production functions:

1. TFP-growth differs across sectors. Empirical evidence implies that TFP-growth-rates differ strongly across sectors. For example, Bernard and Jones (1996) (pp. 1221f.), who analyze sectoral TFP-growth in 14 OECD countries between 1970 and 1987, report that, e.g., the average TFP-growth rate in agriculture (3%) was more than three times as high as in services (0.8%). Similar results are obtained by Baumol et al. (1985), who report the TFP-growth-rates of US-sectors between 1947 and 1976.

2. Capital intensity differs across sectors. Empirical evidence implies that factor-shares in income differ strongly across sectors (hence, capital intensities differ strongly across sectors as well[4]). For example, Kongsamut, Rebelo and Xie (1997) provide evidence for the USA for the period 1959-1994. Their data implies that, for example, the labour income share was relatively high in manufacturing and construction (around 70%) in this period. At the same time, e.g. the labour income share in agriculture, finance, insurance and real estate was relatively low (around 20%). Similar results for the USA are obtained by Close and Shulenburger (1971) for the period 1948-1965 and by Acemoglu and Guerrieri (2008) for the period 1987-2004. Some new evidence for the USA (presented by Valentinyi and Herrendorf (2008)) supports these results as well. Gollin (2002) (p. 464) analyzes the data from 41 countries reported in the U.N. National Statistics. He confirms that factor income shares vary widely across sectors.

A model which analyzes structural change across *sectors* should be consistent with these "stylized" facts of *sectoral* production functions. This is especially important, since these stylized facts have an impact on structural change (and hence on aggregate balanced growth), as we will see now.

2.2 Structural change determinants

We name the attributes of preferences and technologies which cause structural change and determine its strength "structural change determinants". A detailed discussion of these determinants (especially empirical evidence) is provided by, e.g., Schettkat and Yocarini (2006). We provide here a different overview and some newer references, since they are important for the following discussion. There a four main determinants of structural change.

1. Non-homothetic preferences (inter-sectoral differences in income-elasticity of demand) – relevance for structural change analyzed empirically and theoretically by, e.g., Kongsamut et al. (1997, 2001).
2. Differences in TFP-growth across sectors – empirical relevance for structural change shown, e.g., by Baumol (1967); theoretical relevance for structural change shown by, e.g., Ngai and Pissarides (2007).
3. *Differences in capital intensities across sectors* – relevance for structural change analyzed empirically and theoretically by, e.g., Acemoglu and Guerrieri (2008).
4. *Shifts in intermediates production across sectors* – relevance for structural change analyzed empirically and theoretically by, e.g., Fixler and Siegel (1998).

These four determinants generate structural change and determine strength and direction of structural change. Since the aggregate economy is the weighted average of its sectors, the aggregate behaviour depends on the structural change patterns. Thus, all four structural change determinants influence the behaviour of the aggregate economy. Hence, only if we include all four structural change determinants into a model, we can adequately analyse why balanced growth of aggregates (Kaldor-facts) can coexist with structural change.

[4] If labour income shares (or: output elasticities of labour) differ across sectors, capital intensities differ across sectors as well, since optimal capital intensity is determined by factor prices and by output elasticities of capital and labour. We will see later that this is true within our model.

3. Model of neutral structural change

3.1 Model assumptions

3.1.1 Production

In the following we discuss a long-run multi-sector Ramsey-model. In fact, this model is the same as the one-sector Ramsey-model beside the fact that multiple consumption-goods-sectors are added to the model structure in neoclassical fashion. This sort of model is used in newer structural change literature, especially by Kongsamut et al. (1997, 2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).

We assume an economy where two technologies exist (the model could be modified such that it includes more technologies; the key results would be the same). The technologies differ by capital intensity (i.e. output elasticities of inputs differ across technologies) and by total factor productivity (TFP) growth. TFP-growth rates are constant and exogenously given. Goods $i = 1,...n$ are produced in the economy. Goods $i = 1,...m$ are produced by using technology 1 and goods $i = m+1,...n$ are produced by using technology 2 $(n > m)$. We assume that three inputs are used for production: capital (*K*), labour (*L*) and intermediates (*Z*). All capital, labour and intermediates are used in the production of goods $i = 1,...n$. The amount of available labour grows at constant rate (g_L). Since we want to model TFP-growth, we assume Hicks-neutral technological progress. It is well known that the existence of a balanced growth path in standard balanced growth frameworks requires the assumption of Cobb-Douglas production function(s) when technological progress is Hicks-neutral. (Later, we will see that the aggregate production function "inherits" the attributes of sectoral production functions along the PBGP, i.e. the aggregate production function is of type Cobb-Douglas.) These assumptions imply the following production functions:

(1) $Y_i = A(l_i L)^{\alpha} (k_i K)^{\beta} (z_i Z)^{\gamma}, \quad i = 1,...m$

where $\alpha + \beta + \gamma = 1; \quad \alpha, \beta, \gamma > 0; \quad \frac{\dot{A}}{A} = g_A = const.$

(2) $Y_i = B(l_i L)^{\chi} (k_i K)^{\nu} (z_i Z)^{\mu}, \quad i = m+1,...n$

where $\chi + \nu + \mu = 1; \quad \chi, \nu, \mu > 0; \quad \frac{\dot{B}}{B} = g_B = const.$

$$(3) \quad \sum_{i=1}^{n} l_i = 1; \qquad \sum_{i=1}^{n} k_i = 1; \qquad \sum_{i=1}^{n} z_i = 1$$

$$(4) \quad \frac{\dot{L}}{L} \equiv g_L = const.$$

where Y_i denotes the output of good *i*; l_i, k_i and z_i denote respectively the fraction of labour, capital and intermediates devoted to production of good *i;* K is the aggregate capital; L aggregate labour; Z aggregate intermediate index. Note that we omit here the time index. Furthermore, note that the index i denotes *not* sectors but a good or a group of similar goods. We will define sectors later.

Of course, it is not "realistic" that there are only two technologies and that some goods are produced by identical production functions. However, every model simplifies to some extent and it is only important that the simplification does not affect the meaningfulness of the results. Our assumption is only a "technical assumption", which is necessary to make our argumentation as simple as possible. Our key arguments (namely the existence of neutral structural change) could also be derived in a framework where each good is produced by a unique production function. (We show this fact in Proposition 4.) However, it would be much more difficult to formulate the independency assumptions (which are formulated in the next subsection). Instead of the simple restrictions, which we use in the next subsection, we would have to derive complex restrictions which would not be such transparent. Anyway, later our focus will be on the analysis of only three sectors (which are aggregates of the products i=1,…n); thus, two technologies are sufficient to generate technological heterogeneity between these three sectors. In this sense, we have introduced technological diversity into our framework in the simplest manner (by assuming that there are only two technologies).

It may be easier to accommodate with our assumption of only two technologies by imagining that an economist divides the whole set of products of an economy into two groups (a technologically progressive and a technologically backward) and estimates the average production function for the two groups. Such approaches are prominent in the literature: e.g. Baumol et al. (1985) and Acemoglu and Guerrieri

(2008) approach in similar way in the empirical parts of their argumentation. Furthermore, note that much of the new literature on the Kuznets-Kaldor-puzzle assume very similar sectoral production functions (e.g. Kongsamut et al. (2001) and Ngai and Pissarides (2007)) or assume even identical sectoral production functions (e.g. Foellmi and Zweimueller (2008)). Hence, our assumption of only two (completely distinct) technologies is an improvement in comparison to some previous literature. Note that the empirical study of our paper (section 4) uses the more general assumption, i.e. each good is produced by a unique production function.

We assume that all goods can be consumed and used as intermediates. Furthermore, we assume that only the good *m* can be used as capital. (Note that the model could be modified such that more than one good is used as capital e.g. in the manner of Ngai and Pissarides (2007).) This assumption implies:

(5) $Y_i = C_i + h_i, \quad \forall i \neq m$

(6) $Y_m = C_m + h_m + \dot{K} + \delta K$

where C_i denotes consumption of good *i*; δ denotes the constant depreciation rate of capital; h_i is the amount of good *i* which is used as intermediate input.

We assume that the intermediate-inputs-index *Z* is a Cobb-Douglas function of h_i's which is necessary for the existence of a PBGP (see Ngai and Pissarides 2007):

(7) $Z = \prod_{i=1}^{n} h_i^{\varepsilon_i}$

where $\varepsilon_i > 0, \forall i; \quad \sum_{i=1}^{n} \varepsilon_i = 1$

3.1.2 Utility function

We assume the following utility function, which is quite similar to the utility function used by Kongsamut et al. (1997, 2001):

(8) $U = \int_0^\infty u(C_1,...C_n)e^{-\rho t}dt, \quad \rho > 0$

where

(9) $u(C_1,...C_n) = \ln\left[\prod_{i=1}^{n}(C_i - \theta_i)^{\omega_i}\right]$

(10) $\sum_{i=1}^{m}\theta_i = 0$

(11) $\sum_{i=m+1}^{n}\theta_i = 0$

where U denotes the life-time utility of the representative household and ω_i, θ_i and ρ are constant parameters. In contrast to the model by Ngai and Pissarides (2007), the assumption of logarithmic utility function (equation (9)) is not necessary for our results, i.e. we could have assumed a constant intertemporal elasticity of substitution function of the consumption composite in equation (9).

We can see that this utility function is based on the Stone-Geary preferences. Without loss of generality we assume that θ_i s are not equal to zero and that they differ across goods i. The key reason for using this utility function is that it features non-unitary income-elasticity of demand and, especially, non-unitary price-elasticity of demand. That is, income elasticity of demand differs across goods and price-elasticity of demand differs across goods (as long as θ_i differ across goods). For example, the good i=4 has another price elasticity of demand than good i=7 (provided that $\theta_4 \neq \theta_7$). Due to this feature, we can determine price elasticity and income elasticity for groups of goods. For example, by setting the θ_i in a specific pattern we can determine that the (average) price elasticity of demand for goods i=7,…14 is larger than for goods i= 56,…79.

This is the key to our argumentation about preference and technology correlation later: By setting parameter restrictions (10) and (11) we determine that

1.) on average, the income elasticity of demand for technology-1-goods is not larger or smaller in comparison to the income elasticity of demand for technology-2-goods

2.) on average, the price-elasticity of demand for technology-1-goods is not larger or smaller in comparison to the price-elasticity of demand for technology-2-goods; i.e. elasticity of substitution between technology-1-goods and technology-2-goods is equal to one.

Hence, preferences and technologies are not correlated on average. This means for example, that demand for some of the goods which are produced by technology 1 can be price-inelastic and for some of the technology-1-goods price-elastic, while at the same time the demand for some goods which are produced by technology 2 can be price-elastic and for some of the technology-2-goods price inelastic. However, on average, the elasticity of substitution between technology-1-goods and technology-2-goods is equal to unity.

This restriction (equations (10) and (11)) reduces the generality of our model. Nevertheless, for our further argumentation it does not matter. It is simply a technical assumption in order to show in the simplest manner the existence of neutral structural change. That is, due to this assumption we can pursue our analysis along a PBGP, which is technically simple. Without this assumption, we would have to numerically solve the model and the distinction between neutral and non-neutral structural change would be quite difficult. Nevertheless, we will discuss the theoretical reasonability of this restriction later and we will show empirically that the largest part of structural change is in line with this restriction.

Overall, our utility function allows for structural change caused by all structural change determinants: In general the goods have a price elasticity of demand which is different from one (as discussed above). Hence, changing relative prices can cause structural change in this model (see also Ngai and Pissarides 2007 on price elasticity and structural change). Intertemporal elasticity of substitution differs across goods i and is *not* equal to unity, despite of the fact that equation (9) is logarithmic. Equations (8)-(11) imply that the utility function is *non-homothetic across goods i,* i.e. income elasticity of demand differs across goods i (depending on the parameterization of the θ_i's).

3.1.3 Aggregates and sectors

We define aggregate output (*Y*), aggregate consumption expenditures (*E*) and aggregate intermediate inputs (*H*) in the standard-way:

$$(12)\quad Y \equiv \sum_{i=1}^{n} p_i Y_i \,; \qquad E \equiv \sum_{i=1}^{n} p_i C_i \,; \qquad H \equiv \sum_{i=1}^{n} p_i h_i$$

where p_i denotes the price of good *i*. We chose the good *m* as numéraire, hence:

(13) $p_m = 1$

Note that in reality the manufacturing sector is not the numéraire in the real GDP calculations. Hence, our definition of aggregate output Y is not the same as real GDP. However, the choice of numéraire is irrelevant when discussing ratios or shares (see e.g. Ngai and Pissarides (2004, 2007)), since the numéraire of the numerator and the denominator of a ratio offset each other. Therefore, we focus our discussion on the shares and ratios in our paper (e.g. aggregate capital-intensity, capital-to-output ratio, income-share of capital and labour), where the numéraire choice is irrelevant. Our results regarding the other Kaldor-facts, which are dealing with the development of the real-GDP-growth rate and the real interest rate, should be considered with caution. However, as discussed by Barro and Sala-i-Martin (2004), the constancy of the real interest rate (as a Kaldor fact) may anyway be questionable; see Ngai and Pissarides (2007). Furthermore, as shown by Ngai and Pissarides (2004, 2007) the real GDP as measured in reality behaves in similar way as real GDP in manufacturing terms. Therefore, to some extent our results may be relevant for real GDP as measured in reality.

Last but not least we have to define the sectors of our economy. Without loss of generality we assume here that there are three sectors which we name for reasons of convenience (according to the tree sector hypothesis): agriculture, manufacturing and services. Furthermore, without loss of generality we assume the following sector division

- agricultural sector includes goods $i = 1,...a;\ 1 < a < m$
- manufacturing sector includes goods $i = a+1,...s;\ \ m < s < n$
- services sector includes goods $i = s+1,...n$.

Hence, the agricultural sector uses only technology 1, the manufacturing sector uses technology 1 and 2 and the services sector uses only technology 2. Note that this whole division is not necessary for our argumentation, neither the naming of the sectors. We could also assume that the capital-producing manufacturing sector uses only one technology (and the services sector both technologies). We could even assume that there are more sectors (and more technologies). In all these cases our key results would be the same. Furthermore, note that the assumption that a sector uses both technologies is plausible. For example, the service sector includes services which feature high TFP-growth and/or high capital intensity, e.g. ICTbased services, as well as services which feature low TFP-growth and/or low capital intensity, e.g. some personal services like counselling and consulting (for discussion and empirical evidence see e.g. Baumol et al. 1985 and Blinder 2007). Similar examples can be found in the manufacturing sector (e.g. a traditional clock maker vs. a car producer). Furthermore, our sector-division implies that only the manufacturing sector produces capital. This is consistent with the empirical evidence which implies that most capital goods are produced by the manufacturing sector (see, e.g., Valentinyi and Herrendorf 2008).

According to our classification, we can define the outputs of the agricultural, services and manufacturing sector ($Y_{agr.}$, $Y_{man.}$ and $Y_{ser.}$) and the consumption expenditures on agriculture, manufacturing and services ($E_{agr.}$, $E_{man.}$ and $E_{ser.}$) as follows:

$$(14)\quad Y_{agr.} \equiv \sum_{i=1}^{a} p_i Y_i; \qquad Y_{man.} \equiv \sum_{i=a+1}^{s} p_i Y_i; \qquad Y_{ser.} \equiv \sum_{i=s+1}^{n} p_i Y_i$$

$$(15)\quad E_{agr.} \equiv \sum_{i=1}^{a} p_i C_i; \qquad E_{man.} \equiv \sum_{i=a+1}^{s} p_i C_i; \qquad E_{ser.} \equiv \sum_{i=s+1}^{n} p_i C_i$$

Furthermore, note that employment shares ($l_{agr.}$, $l_{man.}$ and $l_{ser.}$), capital shares ($k_{agr.}$, $k_{man.}$ and $k_{ser.}$) and intermediate shares ($z_{agr.}$, $z_{man.}$ and $z_{ser.}$) of sectors agriculture, manufacturing and services are given by:

$$\begin{aligned} & l_{agr.} \equiv \sum_{i=1}^{a} l_i;\; l_{man.} \equiv \sum_{i=a+1}^{s} l_i;\; l_{ser.} \equiv \sum_{i=s+1}^{n} l_i; \\ (16)\quad & k_{agr.} \equiv \sum_{i=1}^{a} k_i;\; k_{man.} \equiv \sum_{i=a+1}^{s} k_i;\; k_{ser.} \equiv \sum_{i=s+1}^{n} k_i; \\ & z_{agr.} \equiv \sum_{i=1}^{a} z_i;\; z_{man.} \equiv \sum_{i=a+1}^{s} z_i;\; z_{ser.} \equiv \sum_{i=s+1}^{n} z_i \end{aligned}$$

3.2 Model equilibrium

3.2.1 Optimality conditions

We have now specified the model completely. The intertemporal and intratemporal optimality conditions can be obtained by maximizing the utility function (equations (8)-(11)) subject to the equations (1)-(7) and (12)-(16) by using e.g. the

Hamiltonian. When there is free mobility of factors across goods and sectors these (first order) optimality conditions are given by:

(17) $p_i = \frac{\partial Y_m / \partial(l_m L)}{\partial Y_i / \partial(l_i L)} = \frac{\partial Y_m / \partial(k_m K)}{\partial Y_i / \partial(k_i K)} = \frac{\partial Y_m / \partial(z_m Z)}{\partial Y_i / \partial(z_i Z)} = \frac{\partial Y_m}{\partial(z_m Z)} \frac{\partial Z}{\partial h_i}, \quad \forall i$

(18) $p_i = \frac{\partial u(.) / \partial C_i}{\partial u(.) / \partial C_m}, \quad \forall i$

(19) $-\frac{\dot{u}_m}{u_m} = r - \delta - \rho$

where $u_m \equiv \partial u(.) / \partial C_m$ and $r \equiv \partial Y_m / \partial(k_m K)$ is the real interest rate (see APPENDIX A for proofs). We show in APPENDIX A that these are the sufficient conditions for an optimum (together with the transversality condition).

3.2.2 Development of aggregates in equilibrium

To be able to derive some theoretical arguments from the model, we have to insert equations (1) to (16) into optimality conditions (17) to (19) in order to transform the optimality conditions into some explicit functions of model-variables and model-parameters. To get an impression of how this is done, see the model by Stijepic and Wagner (2012) (see there APPENDIX A). Therefore, we present the following equations, which describe the optimal aggregate structure of the economy, without explicit proof:

(20) $Y = \dot{K} + \delta K + E + H$

(21) $\tilde{Y} = \left(\frac{k_m}{l_m}\right)^q GL^{1-q} K^q$

(22) $\frac{\dot{E}}{E} = \left(\frac{l_m}{k_m}\right)^{1-q} \beta GL^{1-q} K^{q-1} - \delta - \rho$

(23) $H = \gamma \tilde{Y} \left(c_1 + c_2 \frac{l_m}{k_m}\right)$

$$(24)\quad \frac{l_m}{k_m} = 1 - c_3 \frac{E}{\tilde{Y}} - c_4 \frac{H}{\tilde{Y}}$$

where

$$(25)\quad \tilde{Y} \equiv \frac{Y}{c_5 + c_6 \dfrac{l_m}{k_m}}$$

$$(27)\quad q \equiv \frac{\beta(1-\bar{\varepsilon}\mu) + \bar{\varepsilon}\gamma\nu}{1-\gamma(1-\bar{\varepsilon}) - \bar{\varepsilon}\mu} > 0$$

$$(28)\quad G \equiv A\left\{ A^{1-\bar{\varepsilon}} B^{\bar{\varepsilon}} \gamma \left[\frac{\chi}{\alpha}\left(\frac{\alpha\nu}{\chi\beta}\right)^{\nu}\left(\frac{\alpha\mu}{\chi\gamma}\right)^{\mu}\right]^{\bar{\varepsilon}} \prod_{i=1}^{n} \varepsilon_i^{\varepsilon_i} \right\}^{\frac{\gamma}{1-\gamma(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}}$$

$$(29)\quad \bar{\varepsilon} \equiv \sum_{i=m+1}^{n} \varepsilon_i$$

and

$$c_1 \equiv 1 - c_2, \qquad c_2 \equiv \frac{1-\dfrac{\alpha\mu}{\chi\gamma}}{1-\dfrac{\alpha\nu}{\chi\beta}}, \qquad c_3 \equiv \left(1-\frac{\alpha\nu}{\chi\beta}\right)\frac{\chi}{\alpha}\frac{\sum_{m+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}, \qquad c_4 \equiv \left(1-\frac{\alpha\nu}{\chi\beta}\right)\frac{\chi}{\alpha}\sum_{i=m+1}^{n}\varepsilon_i,$$

$$c_5 \equiv 1 - c_6 \text{ and } c_6 \equiv \frac{1-\dfrac{\alpha}{\chi}}{1-\dfrac{\alpha\nu}{\chi\beta}}.$$

Note that G grows at positive constant rate, q is positive and $\bar{\varepsilon} < 1$.[5]

Equations (20)-(28) look actually more complicated than they are. As we will see soon they are quite the same as in the standard one-sector Ramsey-Cass-Koopmans-model[6] or Solow-model. The key difference is that our equations feature the term l_m / k_m, which reflects the impact of cross-capital intensity structural change on the development of aggregates. However, before discussing

[5] The term within the {}-brackets in equation (28) grows at constant positive rate since $\bar{\varepsilon}$ is positive and smaller than one (see equation (29)). Furthermore, the exponent of the {}-brackets is positive as well, since $\gamma(1-\bar{\varepsilon}) + \mu\bar{\varepsilon} < 1$ (a weighted average of numbers that are smaller than one (γ and μ) is always smaller than one). As well, $q>0$, since $\gamma(1-\bar{\varepsilon}) + \mu\bar{\varepsilon} < 1$.

[6] For a discussion of the Ramsey-model see e.g. Barro, Sala-i-Martin (2004) pp. 85ff.

these facts we start with our definition of a dynamic-equilibrium growth path which is quite similar to the definition used by Ngai and Pissarides (2007).

Definition 1: *A partially balanced growth path (PBGP) is an equilibrium growth path where aggregates (Y, $\tilde{Y}$, K, E and H) grow at a constant rate.*

Note that this definition does require balanced growth for aggregate variables. However, it does not require balanced growth for sectoral variables (e.g. for sectoral outputs). Hence, it allows for structural change.

Lemma 1: *Equations (20) to (28) imply that there exists a unique PBGP, where aggregates (Y, $\tilde{Y}$, K, E and H) grow at constant rate g^* and where l_m / k_m is constant. The PBGP-growth rate is given by* $g^* = \frac{(1-\mu\bar{\varepsilon})g_A + \gamma\bar{\varepsilon}g_B}{(1-\mu\bar{\varepsilon})\alpha + \gamma\bar{\varepsilon}\chi} + g_L$.

Proof: See APPENDIX B.

Proposition 1: *a) A saddle-path, along which the economy converges to the PBGP, exists in the neighbourhood of the PBGP. b) If intermediates are omitted (i.e. if $\gamma = \mu = 0$), the PBGP is locally stable.*

Proof: See APPENDIX C.

Proposition 1 ensures that the economy will approach to the PBGP even if the initial capital level is not such that the economy starts on the PBGP.

Proposition 2: *The aggregate dynamics of the economy along the PBGP are represented by the following equations: $\hat{Y} = \dot{K} + \delta K + E$; $\hat{Y} = \tilde{G}L^{1-q}K^q$ and $\frac{\dot{E}}{E} = \lambda\frac{\hat{Y}}{K} - \delta - \rho$, where $\tilde{G}$ is an auxiliary variable growing at constant rate (a sort of "Hicks-neutral technological progress"), $\hat{Y}$ denotes aggregate output without intermediates production (i.e. Y-H) and λ is a constant auxiliary variable (see APPENDIX B for details of auxiliary variables).*

Proof: *See APPENDIX B.*

In fact Proposition 2 implies that the aggregate structure of our economy is quite the same as the structure of the standard Ramsey-Cass-Koopmans- or Solow-model (with Cobb-Douglas production function and logarithmic utility).

Now, the question arises, whether structural change takes place along the PBGP. We discuss this question in the following.

3.2.3 Development of sectors in equilibrium

The following equations, which describe the optimal sector structure of the economy (represented by employment shares), can be obtained by inserting equations (1) to (16) into optimality conditions (17) to (19):

(30a) $$l_{agr.} = \Lambda_{agr.} + \frac{1}{\tilde{Y}}\sum_{i=1}^{a}\theta_i$$

(30b) $$l_{man.} = \Lambda_{man.} + \frac{1}{\tilde{Y}}\sum_{i=a+1}^{m}\theta_i + \Gamma\sum_{i=m+1}^{s}\theta_i$$

(30c) $$l_{ser.} = \Lambda_{ser.} + \Gamma\sum_{i=s+1}^{n}\theta_i$$

where

(31a) $$\Lambda_{agr.} \equiv \frac{\sum_{i=1}^{a}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\sum_{i=1}^{a}\varepsilon_i$$

(31b) $$\Lambda_{man.} \equiv \frac{\sum_{i=a+1}^{m}\omega_i + \frac{\chi}{\alpha}\sum_{i=m+1}^{s}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\left(\sum_{i=a+1}^{m}\varepsilon_i + \frac{\chi}{\alpha}\sum_{i=m+1}^{s}\varepsilon_i\right) + \frac{\dot{K} + \delta K}{\tilde{Y}}$$

(31c) $$\Lambda_{ser.} \equiv \frac{\chi}{\alpha}\frac{\sum_{i=s+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\frac{\chi}{\alpha}\sum_{i=s+1}^{n}\varepsilon_i$$

(31d) $\Gamma \equiv \dfrac{1}{\left(\dfrac{\alpha\nu}{\chi\beta}\right)^{\nu}\left(\dfrac{\alpha\mu}{\chi\gamma}\right)^{\mu} B\left(\dfrac{G}{A}\right)^{\mu/\gamma} L\left(\dfrac{k_m K}{l_m L}\right)^{\nu+\frac{\mu\beta(1-\bar{\varepsilon})+\nu\bar{\varepsilon}}{1-(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}}}$

Again, to get an impression of how these equations can be derived, see the derivations in Stijepic and Wagner (2012) (especially APPENDIX A).

Note that $\Lambda_{agr.}$, $\Lambda_{man.}$, $\Lambda_{ser.}$ and Γ can be easily derived as functions of exogenous parameters along the PBGP.[7] However, we omit here the explicit proof, since it is trivial and irrelevant for further discussion (for a sketch of the proof see footnote 7).

Lemma 2: *Structural change takes place along the PBGP. That is, the employment shares of sectors agriculture ($l_{agr.}$), manufacturing ($l_{man.}$) and services ($l_{ser.}$) change over time along the PBGP.*

Proof: This Lemma is implied by equations (30) and (31). Note that $\Lambda_{agr.}$, $\Lambda_{man.}$, and $\Lambda_{ser.}$ are constant along the PBGP (due to Lemma 1); $\tilde{Y}$ grows at rate g^* along the PBGP (see Lemma 1). Γ decreases at constant rate along the PBGP. The latter fact comes from Lemma 1 and equation (28). Note that *G/A* grows at positive constant rate; see equation (28) and footnote 5. Furthermore, note that the exponent $\nu + \frac{\mu\beta(1-\bar{\varepsilon})+\nu\bar{\varepsilon}}{1-(1-\bar{\varepsilon})\gamma-\mu\bar{\varepsilon}}$ is positive, since $\gamma(1-\bar{\varepsilon})+\mu\bar{\varepsilon}<1$ as explained in footnote 5. ***Q.E.D.***

Now, the remaining exercise is to show that along the PBGP our model is indeed consistent with all the stylized facts mentioned in the introduction and section 2 of our paper.

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[7] In APPENDIX B (equation (B.17)) we have derived l_m / k_m as function of exogenous model parameters. This function can be used to derive $\tilde{Y}$ and Y as functions of exogenous model parameters by using equations (21) and (25). Then, when we have $\tilde{Y}$ and l_m / k_m as functions of exogenous model parameters, we can derive H as a function of exogenous model parameters by using equation (23). Finally, we can use Y and H to derive E as function of exogenous model parameters (via equation (20); note that the initial capital endowment K_0 is exogenously given; hence K can be calculated by using K_0 and the equilibrium growth rate of capital g^*, where g^* is a function of exogenous model parameters as shown in Lemma 1). When we have l_m / k_m, $\tilde{Y}$, K and E as functions of exogenous model parameters, we can derive $\Lambda_{agr.}$, $\Lambda_{man.}$, $\Lambda_{ser.}$ and Γ as functions of exogenous model parameters.

3.2.4 Consistency with stylized facts

Lemma 3: *The PBGP of our model satisfies the Kaldor facts regarding the development of the great ratios. That is, the aggregate capital intensity (K/L) is increasing; the aggregate capital-income-share (rK/Y or $rK/(Y-H)$), the aggregate labour-income-share (wL/Y or $wL/(Y-H)$) and the aggregegate capital-to-output ratio (K/Y or K/(Y-H)) are constant (where r is the real rate of return on capital and w is the real wage rate).*

Proof: The constancy of *K/Y* and *K/(Y-H)* as well as the increasing capital-intensity (*K/L*) are directly implied by Lemma 1. Since we assume perfect polypolisitic markets, the marginal productivity of capital (labour) in a sub-sector i is equal to the real rate of return on capital (real wage rate) for all i. This implies for example for $i = m$:

$$(32)\quad r = \frac{\partial Y_m}{\partial (k_m K)} = \beta \frac{l_m}{k_m} \frac{\tilde{Y}}{K}$$

$$(33)\quad w = \frac{\partial Y_m}{\partial (l_m L)} = \alpha \frac{\tilde{Y}}{L}$$

Hence, Lemma 1 and equations (32) and (33) imply that $\frac{rK}{Y}$, $\frac{rK}{Y-H}$, $\frac{wL}{Y}$ and $\frac{wL}{Y-H}$ are constant. ***Q.E.D.***

Note that there are two further Kaldor-facts: "growth rate of real GDP is constant" and the "real rate of return on capital is constant". As discussed in section 3.1, due to numéraire-choice we cannot say whether these two Kaldor-facts are satisfied approximately in our model. However, as mentioned before, the constancy of the real interest rate seems to be rather not a fact in reality. Furthermore, the results by Ngai and Pissarides (2004, 2007) imply that aggregate output expressed in manufacturing terms (as in our model) behaves in similar fashion as aggregate output measured in reality (by using some compound numéraire). Hence, our model could be consistent with a constant growth rate of aggregate output.

Lemma 4: *Along the PBGP the development of sectoral employment shares over time (equations (30)-(31)) can be monotonous (monotonously increasing,* *monotonously decreasing or constant) or non-monotonous ("hump-shaped" or "U-shaped"), depending on the parameterization of the model.*

Proof: This Lemma is implied by equations (30)-(31). In the proof of Lemma 2 we have shown that $\Lambda_{agr.}$, $\Lambda_{man.}$, and $\Lambda_{ser.}$ are constant along the PBGP, $\tilde{Y}$ grows at rate g^* along the PBGP (see Lemma 1) and Γ decreases at constant rate along the PBGP. Hence, since $1/\tilde{Y}$ and Γ grow at different rates, equation (30b) implies that the development of the manufacturing-employment-share over time ($l_{man.}$) can be non-monotonous, provided that $\sum_{i=a+1}^{m}\theta_i$ has not the same sign as $\sum_{i=m+1}^{s}\theta_i$. That is, it can be hump-shaped or U-shaped depending on the parameterization. Hence, the model can reproduce a "hump-shaped" development of the manufacturing-employment share over time, which has been emphasized by Ngai and Pissarides (2007) and Maddison (1980). Note that only sectors, which use at least two technologies, can feature non-monotonous development of their employment share over time. However, as discussed in section 3.1 the manufacturing sector (i.e. the capital producing sector) need not using two technologies, i.e. the model could be set up such that the agricultural sector or the services sector uses two technologies. Hence, in fact any of the sectors could feature non-monotonous dynamics of its employment-share over time. The proof that

- $l_{agr.}$ can be monotonously increasing, monotonously decreasing or constant,
- $l_{man.}$ can be monotonously increasing or monotonously decreasing, and
- $l_{ser.}$ can be monotonously increasing, monotonously decreasing or constant

is obvious when taking into account that $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$ can be negative, positive or equal to zero respectively. ***Q.E.D.***

Lemma 5: *Agriculture, manufacturing and services have different production functions in our model. Especially, the optimal capital-intensity differs across these sectors.*

Proof: Since we assumed that agriculture (services) uses only technology 1 (2) its production function is represented by technology 1 (2). Hence, we know that the technology (especially the TFP-growth-rate and the capital-intensity) differ across agriculture and services. Furthermore, manufacturing uses both technologies.

Hence, the average manufacturing technology is a mix of technology 1 and 2. Hence, the representative production function of the manufacturing sector is different in comparison to the services sector or the agricultural sector which each use only one technology. Nevertheless, since we have an emphasis on the cross-capital-intensity structural change, let us have a close look on the capital-intensity ($\frac{k_{agr.}K}{l_{agr.}L}$, $\frac{k_{man.}K}{l_{man.}L}$ and $\frac{k_{ser.}K}{l_{ser.}L}$), the output-elasticity of labor ($\lambda_{agr.}$, $\lambda_{man.}$ and $\lambda_{ser.}$) and the output-elasticity of capital ($\kappa_{agr.}$, $\kappa_{man.}$ and $\kappa_{ser.}$) in each sector:

$$(34)\quad \frac{k_{agr.}K}{l_{agr.}L} = \frac{k_m K}{l_m L} \neq \frac{k_{man.}K}{l_{man.}L} = \frac{k_m K}{l_m L}\left(1+\frac{\alpha\nu}{\chi\beta}\right) \neq \frac{k_{ser.}K}{l_{ser.}L} = \frac{k_m K}{l_m L}\frac{\alpha\nu}{\chi\beta}$$

$$(35)\quad \lambda_{agr.} = \frac{wl_{agr.}L}{Y_{agr.}} = \alpha \neq \lambda_{man.} = \frac{wl_{man.}L}{Y_{man.}} = \frac{\alpha l_{man.}}{\sum_{i=a+1}^{m} l_i + \frac{\alpha}{\chi}\sum_{i=m+1}^{s} l_i} \neq \lambda_{ser.} = \frac{wl_{ser.}L}{Y_{ser.}} = \chi$$

$$(36)\quad \kappa_{agr.} = \frac{rk_{agr.}K}{Y_{agr.}} = \beta \neq \kappa_{man.} = \frac{rk_{man.}K}{Y_{man.}} = \beta\frac{\sum_{i=a+1}^{m} l_i + \frac{\alpha\nu}{\chi\beta}\sum_{i=m+1}^{s} l_i}{\sum_{i=a+1}^{m} l_i + \frac{\alpha}{\chi}\sum_{i=m+1}^{s} l_i} \neq \kappa_{ser.} = \frac{rk_{ser.}K}{Y_{ser.}} = \nu$$

(Note that output-elasticity of factors is equal to the factor-income shares due to the assumption of perfect markets and perfect factor mobility in our model.) Overall, capital intensities and output-elasticities of inputs differ across sectors agriculture, manufacturing and services. ***Q.E.D.***

Lemma 6: *Along the PBGP the factor-reallocation across the agricultural, manufacturing and services sector is determined by cross-sector-TFP-growth-disparity, by cross sector capital-intensity-disparity and by non-homothetic preferences.*

Proof: As discussed above, the *TFP-growth rates* and the *capital-intensities* differ across the sectors agriculture, manufacturing and services; see also Lemma 5. Equations (30)-(31) (and equations (21) and (28)) imply that cross-sector-differences in TFP-growth-rates and cross-sector-differences in output-elasticities of inputs (which determine the capital-intensities) determine the strength of the factor reallocation between the sectors agriculture, manufacturing and services. Especially, they affect the sectoral employment shares ($l_{agr.}$, $l_{man.}$ and $l_{ser.}$) via the terms $\widetilde{Y}$ and Γ, which are among others functions of the parameters which determine the sectoral TFP-growth rates and sectoral capital intensities (see equations (21), (31d) and (28) and Lemma 5).

Furthermore, equations (8) to (11) imply that preferences are *non-homothetic* across sectors agriculture, manufacturing and services. A detailed proof is in APPENDIX D, where we show among others that the terms $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$ determine the pattern of non-homotheticity across sectors agriculture, manufacturing and services. Equations (30)-(31) imply that this non-homotheticity determines the strength and direction of structural change (via terms $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$). ***Q.E.D.***

Lemma 7: *Intersectoral outsourcing (i.e. shifts in intermediates production across sectors) takes place along the PBGP. That is, along the PBGP manufacturing-sector-producers shift more and more intermediates production to services-sector-producers (i.e.* h_i / h_j *changes), provided that services-sector-production becomes cheaper and cheaper (or less and less expensive) in comparison to manufacturing-sector-production (i.e. provided that relative prices change), and vice versa. Any direction of relative price changes (and hence any direction of intermediate-production shifts between the manufacturing and the services sector) can be generated along the PBGP, depending on the parameterization.*

Proof: See APPENDIX E.

Theorem 1: *The PBGP satisfies simultaneously the following stylized facts:*

- *Kaldor-facts regarding the development of the great ratios,*
- *Kuznets facts regarding structural change patterns,*
- *"stylized facts regarding cross-sector-heterogeneity in production-technology" (see section 2 as well), and*
- *empirical evidence on structural change determinants in industrialized countries (see section 2).*

Proof: The consistency of the PBGP with the *Kaldor facts* is implied by Lemma 3.

Note that empirical evidence on structural change between agriculture, manufacturing and services in industrial countries implies the following stylized facts for the development of the employment shares over the last century (see footnote 1):

- the agricultural sector featured a monotonously decreasing employment share,
- the services sector featured a monotonously increasing employment share, and
- the manufacturing sector featured a constant or "hump-shaped" employment share (depending on the length of the period considered).

In the proof of Lemma 4 we have shown that our model can reproduce these stylized facts regarding the development of the agricultural, manufacturing and services employment shares. Hence, the PBGP is consistent with the *Kuznets-facts*. The consistency of the PBGP with the *"stylized facts regarding cross-sector-heterogeneity in production-technology"* is shown in Lemma 5, where we show that production technology differs across agriculture, manufacturing and services in our model.

Finally the consistency of the PBGP with the *empirical evidence on structural-change-determinants in industrialized countries* is shown in Lemmas 6 and 7. ***Q.E.D.***

3.2.5 The relationship between structural change and aggregate-dynamics

Now we turn to the question about the relationship between structural change and aggregate growth, i.e. we ask how structural change affects aggregate growth, which is important for understanding the Kuznets-Kaldor-puzzle. In the following we will show that there are two types of structural change, which are distinguished according to their impact on the aggregate structure of the economy.

Definition 2: *The term "cross-capital-intensity structural change" stands for factor reallocation across sectors which differ by capital intensity.*

It can be shown that

$$(37)\quad \bar{l} \equiv \frac{l_m}{k_m}\frac{\beta_m}{\alpha_m} = \left(\frac{\kappa_{agr.}}{\lambda_{agr.}} l_{agr.} + \frac{\kappa_{man.}}{\lambda_{man.}} l_{man.} + \frac{\kappa_{ser.}}{\lambda_{ser.}} l_{ser.} \right)$$

where $\lambda_{agr.}$ ($\kappa_{agr.}$), $\lambda_{man.}$ ($\kappa_{man.}$) and $\lambda_{ser.}$ ($\kappa_{ser.}$) are respectively the income-share of labor (capital) in sectors agriculture, manufacturing and services. Equation (37) follows from the assumption of factor mobility across sectors and from the assumption of perfect markets.

Equation (37) and Lemma 1 imply that there are two sorts of cross-capital-intensity structural change:

(1) Cross-capital-intensity structural change where $\bar{l}$ is not constant. Lemma 1 implies that the economy is on a PBGP, only if l_m / k_m is constant; furthermore, equation (37) implies that the constancy of $\bar{l}$ is required for the constancy of l_m / k_m. Hence, as long as $\bar{l}$ is not constant, the economy is not on a PBGP and the Kaldor-facts are not satisfied (exactly). That is, the type of structural change which is associated with a change in $\bar{l}$ is not compatible with the Kaldor facts (unless structural change is very weak such that its impact via $\bar{l}$ is weak, which would imply that Kaldor facts are approximately satisfied).

(2) Cross-capital-intensity structural change which is compatible with a constant $\bar{l}$. If this sort of structural change exists in our model, an economy can be on a PBGP, even when cross-capital-intensity factor reallocation takes place, provided that this factor reallocation is such that $\bar{l}$ = const. (see also Lemma 1).

So we can postulate the following definition and theorem:

Definition 3: *"Neutral structural change" stands for cross-capital-intensity structural change which satisfies the following condition:*

$$(38)\ \bar{l} \equiv \left(\frac{\kappa_{agr.}}{\lambda_{agr.}} l_{agr.} + \frac{\kappa_{man.}}{\lambda_{man.}} l_{man.} + \frac{\kappa_{ser.}}{\lambda_{ser.}} l_{ser.} \right) = const.$$

Theorem 2: *The cross-capital-intensity structural change (between agriculture, manufacturing and services) along the PBGP is "neutral" in the sense of Definition 3.*

Proof: Note that we have shown in Lemma 5 that sectors agriculture, manufacturing and services differ by technology, and especially by capital intensity and by output-elasticities of inputs/income-shares of inputs. Lemma 2 implies that structural change takes place across these sectors. Equation (37), Definition 3 and

Lemma 1 (necessity of a constant l_m / k_m for a PBGP) imply the rest of the theorem. ***Q.E.D.***

Theorem 3*: Neutral structural change is an explanation for the Kuznets-Kaldor-Puzzle in our model.*

Proof: Remember that the Kuznets-Kaldor-puzzle was about the empirical question why (cross-capital-intensity) structural change is compatible with the stability of the great ratios (Kaldor facts). Theorem 2 implies that neutral-cross-capital-intensity structural change takes place along the PBGP, while Theorem 1 shows that the PBGP is consistent with the Kaldor facts. Thus, Kaldor-facts are satisfied, since cross-technology structural change needs not contradicting the Kaldor facts. In our model only "neutral structural change" (Definition 3) does not contradict Kaldor facts. Furthermore, Theorem 1 shows the generality of our proof: neutral cross-technology structural change is not only consistent with the Kaldor facts about the great ratios but also with the other stylized facts which are relevant for the analysis of the relationship between structural change and aggregates. Hence, Theorem 1 shows that we solved the Kuznets-Kaldor-puzzle under consideration of the most important structural change determinants and under assumption of sectoral cross-technology disparities observed in reality. ***Q.E.D.***

The convenient feature regarding latter two theorems is that we can use them to test our theory empirically: We can calculate $\bar{l}$, and then decompose which share of structural change does not change the value of $\bar{l}$ and which share of structural change changes the value of $\bar{l}$. In this way we can evaluate the quantitative significance of our model-explanation for the Kuznets-Kaldor-Puzzle, since our explanation focuses only on structural change which does not change $\bar{l}$ (due to Theorem 2).

However, before doing so we show two further interesting results

Proposition 3: *The output-elasticity of inputs ($\lambda_{man.}, \kappa_{man.}$) is not constant in the manufacturing sector along the PBGP, but changes according to the amount of inputs used in this sector.*

Proof: This is implied by equations (35) and (36). Note that any sector which uses two technologies has a non-constant output-elasticity of inputs in our model setting. ***Q.E.D.***

This result is interesting: in fact it implies that observed technology changes in sectors need not necessarily resulting form technological progress at sector level, but can also result from structural change. Of course this requires that sectors use several technologies, which seems to be a reasonable assumption. This fact could be of importance for further research, especially when analyzing endogenous technological progress at sector level. That is, Proposition 3 implies that such research will require considering technology change at sector level with caution, since some technology change may not result from technological progress at sector level but from structural change.

As discussed in section 3.1, we assume that there are only two technologies in our model, but that there is an arbitrary number of subsectors. Hence, some subsectors have to use identical technologies. As explained there, we use this assumption to explain the concept of "uncorrelated preferences and technologies" in a traceable way, which will be of interest later in this paper. However, the assumption of partly identical production functions is not necessary for the key results of the actual section: the following proposition shows that the key result of this section (namely for the existence of neutral structural change) can be derived even if all (sub-)sectors have completely different production functions.

Proposition 4: Generalization of our results: *In a framework where*

- *all sub-sectors (i) have sub-sector-specific production functions,*
- *sub-sectoral production functions are general neoclassical production functions with capital and labour as input-factors and labour-augmenting technological progress*

a necessary condition for neutrality of cross-capital-intensity structural change and for the satisfaction of Kaldor-facts is

(39) $\tilde{l} \equiv \sum_i \frac{l_i}{\lambda_i} = const.$

where λ_i is the output-elasticity of labour in subsector i which is equal to the labour-income share in sector i.

Proof: *See APPENDIX F.*

4. A measure of neutrality of cross-capital-intensity structural change

In the previous section, we have presented a model which explains the Kuznets-Kaldor-puzzle with a certain structural change pattern which we name "neutral structural change". In Theorem 2 and Proposition 4 we have shown that this structural change pattern must satisfy condition (38). Due to lack of data we cannot consider intermediates production explicitly. Therefore, we assume that capital and labour are the only inputs in the production function in this section. In this case condition (38) transforms into condition (39).

In Proposition 4 we have generalized the validity of condition (38) to a more general framework than that of section 3. Hence, the development of this condition is not only of interest for our model, but for all models which analyze PBGP's.

We can use condition (39) to asses to what extent neutral structural change takes place in reality.

For the calculations in this section we use the data for the U.S.A., which is available at the web-site of the U.S. Department of Commerce (Bureau of Economic Analysis). We use the U.S.-Gross-Domestic-Product-(GDP)-by-Industry-Data, which is based on the sector-definition from the "Standard Industrial Classification System" and which defines the following sectors:

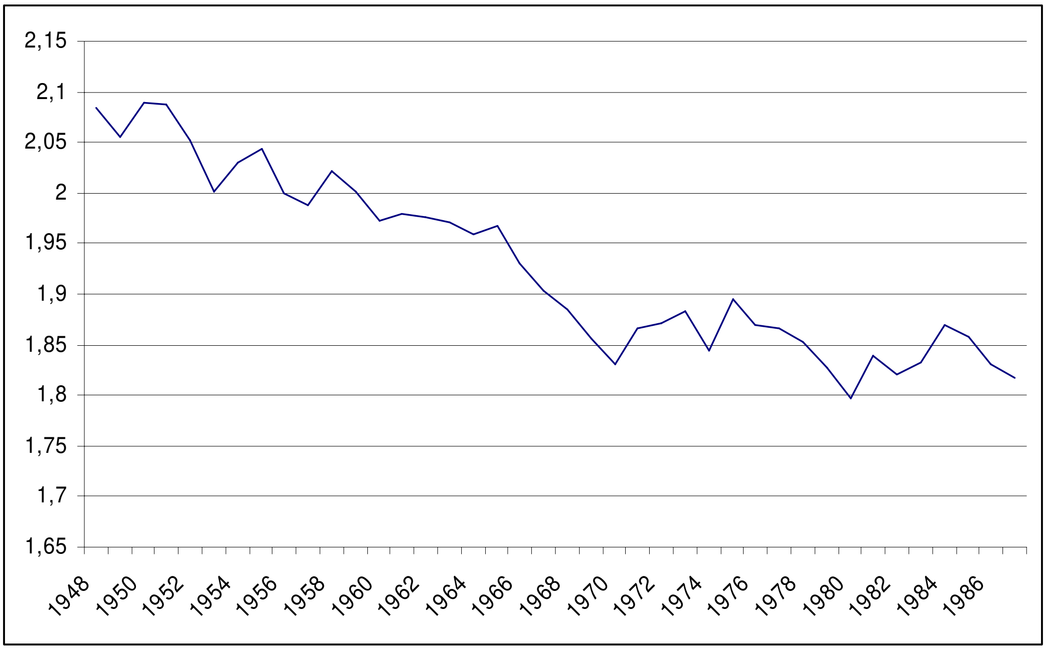
(1) Agriculture, forestry, and fishing
(2) Mining
(3) Construction
(4) Manufacturing
(5) Transportation and public utilities
(6) Wholesale trade
(7) Retail trade
(8) Finance, insurance, and real estate
(9) Services

Our calculations are based on the data for the period 1948-1987. Uniform data for longer time-periods is not available, since the "Standard Industrial Classification System" has been modified over time (hence, the sector definition after 1987 is not the same as the sector definition before 1987).

To calculate the sectoral labour income shares (λ_i) we divided "(Nominal) Compensation of Employees" by "(Nominal) Value Added by Industry" in each sector. The sectoral employment shares (l_i) are calculated by using the sectoral data on "Full-time Equivalent Employees". (This approach is similar to the one used by Acemogu and Guerrieri (2008)).

Figure 1 depicts the development of $\tilde{l}$, calculated by these data:

Figure 1: Development of $\tilde{l}$ over time



We can see that $\tilde{l}$ is decreasing and not constant. Thus, structural change has not been neutral (in the sense of Definition 3). The question is, how small the decline of $\tilde{l}$ is. The decline in $\tilde{l}$ could have been much stronger or much weaker. If the decline was relatively small, we could postulate that $\tilde{l}$ is "approximately constant" from a theoretical point of view; hence, the model of neutral structural change would be relatively good in explaining the Kuznets-Kaldor puzzle. Hence, we have to develop an index which indicates how strong the decline is. In the following we develop such an index. This index is based on calculating the strongest possible decline in $\tilde{l}$ and then relating the actual decline to it.

Any actual $\tilde{l}$ can be expressed as a unique combination of neutral and "maximally non-neutral structural change". "Maximally non-neutral structural change" is the pattern of factor reallocation which causes the maximal decline in $\tilde{l}$ for a given amount of reallocated labour over a period. Hence, maximally non-neutral structural change is a diametric concept of neutral structural change: while neutral structural change is defined upon no change in $\tilde{l}$, maximally non-neutral structural change is defined upon maximal change in $\tilde{l}$. This allows us to create an index which shows us whether a given amount of reallocated labour has been reallocated rather in the neutral way or rather in the non-neutral way. According to this discussion the following relation must be true:

(40) $(\Delta\tilde{l})^{actual} = (1 - I_N)(\Delta\tilde{l})^{neutral} + I_N(\Delta\tilde{l})^{\max}$

where I_N is a weighting factor between neutral and maximally non-neutral structural change, i.e. it indicates whether structural change was rather neutral or rather non-neutral; if I_N =1, structural change is maximally non-neutral over the observation period; if I_N=0 structural change is neutral over the observation period. $(\Delta\tilde{l})^{actual}$ measures the change in $\tilde{l}$ which really took place between 1948 and 1987; $(\Delta\tilde{l})^{\max}$ measures the maximal change in $\tilde{l}$ which is (hypothetically) possible with the given amount of cross-sector factor reallocation between 1948 and 1987, i.e. $(\Delta\tilde{l})^{\max}$ stands for "maximally non-neutral structural change". $(\Delta\tilde{l})^{neutral}$ measures the change in $\tilde{l}$ which is caused by neutral structural change. Since per definition $(\Delta\tilde{l})^{neutral}$ is equal to zero, we can rearrange the condition from above as follows:

(41) $I_N \equiv \frac{(\Delta\tilde{l})^{actual}}{(\Delta\tilde{l})^{\max}}$

where $(\Delta\tilde{l})^{\max}$ and $(\Delta\tilde{l})^{actual}$ are defined as follows:

(42) $(\Delta\tilde{l})^{actual} \equiv \tilde{l}_{1987} - \tilde{l}_{1948} = \sum_i \frac{l_i^{1987}}{\lambda_i^{1987}} - \sum_i \frac{l_i^{1948}}{\lambda_i^{1948}}$

(43) $(\Delta\tilde{l})^{\max} \equiv \tilde{l}_{1987}^{\max} - \tilde{l}_{1948} = \sum_i \frac{l_i^{1987\max}}{\lambda_i^{1987}} - \sum_i \frac{l_i^{1948}}{\lambda_i^{1948}}$

where l_i^{1948}, l_i^{1987}, λ_i^{1948} and λ_i^{1987} denote respectively the employment share of sector i in 1948, the employment share of sector i in 1987, the labour-share of income in sector i in 1948 and the labour-share of income in sector i in1987. $l_i^{1987\max}$ stands for the hypothetical employment share of sector i which would result if the labour which has been reallocated between 1948 and 1987 were reallocated in such a manner that the maximal decrease in $\tilde{l}$ is accomplished between 1948 and 1987. That is, the $l_i^{1987\max}$'s stand for the hypothetical factor allocation in 1987 which yields the maximally non-neutral structural change between 1948 and 1987.

Last but not least, since our definition of $l_i^{1987\max}$ requires knowing how much labour has been reallocated between 1948 and 1987, we propose the following index of observable factor reallocation between 1948 and 1987:

$$\Delta l \equiv \frac{1}{2}\sum_i \left| l_i^{1987} - l_i^{1948} \right|$$

This measure indicates how much labour has been reallocated between 1948 and 1987. This measure is set up as follows: First, the change in the employment share in each sector is measured. The absolute values (modulus) of these changes are summed up (otherwise, without taking absolute values, the sum of the sectoral changes would always be equal to zero, since $\sum_i l_i = 1$ per definition). Since the change in the employment share in one sector has always a corresponding change in the employment shares of the other sectors (labour is reallocated across sectors), the sum of the absolute values of the changes must be divided by two to avoid double-counting.

It is possible that between 1948 and 1987 in some sectors the employment share increased first and decreased then. Hence, the pure difference $l_i^{1987} - l_i^{1948}$ would indicate less reallocation than actually took place. Our index of factor reallocation (Δl) neglects such non-monotonousity in sectoral employment shares. Hence, it underestimates the real amount of labour reallocated between 1948 and 1987. Therefore, our index I_N underestimates the neutrality of structural change: if more labour was reallocated during the period, the hypothetical maximal change in $\tilde{l}$ ($(\Delta\tilde{l})^{\max}$) would be larger; hence, I_N would be smaller, which would imply more neutrality. Overall, for these reasons, our index I_N indicates less neutrality than actually is.

Note that it is important that our measure of maximally non-neutral structural change ($(\Delta\tilde{l})^{\max}$) is based on the actual amount of reallocated labour (Δl). In this way we distinguish between *strength and direction* of structural change. *Strength* of structural change implies how much labour has been reallocated (e.g. as measured by Δl). The *direction* of structural change implies how the labour has been reallocated across technology. Neutrality of structural change is not related to strength but only to direction, since condition (39) can be satisfied by more or less strong structural change patterns. What counts for satisfying condition (39) is the *direction* of structural change. If there is no significant direction of structural change, (39) is satisfied. Therefore, when calculating the neutrality index it is important to be cautious about not defining $(\Delta\tilde{l})^{\max}$ such that it features stronger structural change than actual structural change is. Therefore, we calculate $(\Delta\tilde{l})^{\max}$ by using the *actual* amount of reallocated labor (Δl).

The data that we need for our calculations is given in the following table:

Table 1

Sector	$1/\lambda_i^{1948}$	$1/\lambda_i^{1987}$	l_i^{1948}	l_i^{1987}
(8)	5.248981966	3.997781119	0.039609477	0.077711379
(1)	6.874359747	3.921756596	0.05019623	0.019310549
(2)	2.62541713	3.240100098	0.024056398	0.008630482
(5)	1.632072868	2.20691581	0.099835263	0.063265508
(6)	1.937362752	1.72651328	0.062648384	0.070192118
(7)	1.988458748	1.649066345	0.141770435	0.191092947
(3)	1.495168451	1.505702087	0.056228499	0.059919498
(4)	1.505805486	1.447391372	0.376011435	0.229516495
(9)	1.681140684	1.444831355	0.149643878	0.280361023

Now, by using these data, we have to do the following steps to calculate I_N:

1.) Calculate the amount of reallocated labour between 1948 and (1987). This calculation yields: $\Delta l \approx$ 0.23.

2.) Calculate $\tilde{l}_{1987}^{\max}$. Note that there are always two ways of factor reallocation: the one which increases $\tilde{l}$ over time and the one which decreases $\tilde{l}$ over time. Figure 1 shows that factor reallocations over the observation period were *decreasing* $\tilde{l}$. Thus, for calculating $\tilde{l}_{1987}^{\max}$ we have to find the (hypothetical) factor allocation in 1987 which yields the strongest (hypothetical) *decrease* in $\tilde{l}$ over the observation period and, thus, the lowest $\tilde{l}_{1987}^{\max}$. According to our definition of $\tilde{l}_{1987}^{\max}$, we have to do the following steps:

a.) Find the sector which has the smallest $1/\lambda_i^{1987}$. This is actually sector (9).

b) Make a ranking of the *remaining* sectors according to their $1/\lambda_i^{1987}$. This ranking is given by (8)-(1)-(2)-(5)-(6)-(7)-(3)-(4), where sector (8) has the largest $1/\lambda_i^{1987}$ and sector (4) has the smallest $1/\lambda_i^{1987}$ in this ranking.

c) By using the ranking from b) reallocate the labour from the *sectors* which have the largest $1/\lambda_i^{1987}$ to sector (9). We first we use the whole amount of labour which has been employed in sector (8) in 1948, then the whole amount of labour which has been employed in sector (1) in 1948, and so on, stepping up in the ranking until we have hypothetically reallocated the whole $\Delta l \approx 0.23$. Hence, we obtain the following maximally non-neutral factor allocation for the year 1987

Table 2

Sector	$l_i^{1987\max}$
(8)	$= 0$
(1)	$= 0$
(2)	$= 0$
(5)	$= 0$
(6)	$= l_{(6)}^{1948} - (\Delta l - l_{(1)}^{1948} - l_{(2)}^{1948} - l_{(5)}^{1948} - l_{(8)}^{1948}) = 0.046969461$
(7)	$= l_{(7)}^{1948} = 0.141770435$
(3)	$= l_{(3)}^{1948} = 0.056228499$
(4)	$= l_{(4)}^{1948} = 0.376011435$
(9)	$= l_{(9)}^{1948} + \Delta l = 0.37902017$

3.) The rest of the calculations is quite simple: by inserting the data from Tables 1 and 2 into equations (41)-(43), we can obtain I_N.

Our calculations imply an index I_N = 0.45. This implies that actual structural change was slightly closer to its neutral extreme than to its non-neutral extreme. In other words, the actual structural change between 1948 and 1987 was by 55% neutral and by 45% maximally non-neutral.

In this sense, our model can explain 55% of the structural change between 1948 and 1987.

Note that our measure underestimates the neutrality of structural change. That is, in reality more than 55% of structural change can be regarded as neutral. There are two reasons: as discussed above, our measure assumes monotonousity of factor reallocation; furthermore, as will be discussed close to the end of next section, the period, which we used for analysis, is quite short and structural change is more neutral over very long periods of time.

5. On correlation between preferences and technologies

In section 3.1 we have assumed that preferences and technologies are uncorrelated. In detail, we have assumed that

- on average the income elasticity of demand is equal when comparing technology-1-goods and technology-2-goods
- on average the elasticity of substitution is equal to unity when comparing technology-1-goods and technology-2-goods.

In the following we will discuss the rationale for these assumptions. We focus here on the elasticity of substitution, but the corresponding arguments apply for the income-elasticity of demand.

Assuming that the elasticity of substitution between two goods is different from unity implies that the household has a certain preference for the one good over the other: Imagine that there are only two goods (good A and good B). If the relative price of the good A increases by one percent and the relative demand for this good decreases by less than one percent, good A is regarded as more important than good B by the household in the dynamic context. That is, the price change causes a weaker reaction than it would be if the two goods were regarded as equivalents. Only if two goods are regarded as equivalents, a one-percent-change in the relative price between these goods would yield a (minus) one-percent-change in the demand-relation between these goods (hence, elasticity of substitution between these goods being equal to one).

Now, the same argument could be applied to two groups of goods (group A and group B): if the household regards the two groups as equivalents, the average elasticity of substitution between the two groups is equal to unity. Otherwise, we would have to postulate that on average group A includes goods that are preferred over group B (or the other way around).

Now, imagine that the whole range of products in an economy is divided into two groups according to their production technology. Group A includes goods that are regarded as technologically progressive and group B includes goods that are produced by a backward technology. Furthermore, let us make the following assumptions:

(a) The household doesn't know anything about the production process, i.e. the household's preference depends only on the "objective taste" of the goods (but not on the knowledge that the good is produced at e.g. high-capital-intensity). "Objective taste" means the taste which depends only on the physical/chemical properties of the good or on the basic properties (i.e. actual quality) of the service, but not on the knowledge about the production process of the good or service. For example, if two goods are produced by different capital intensities, but if the two goods are basically the same (i.e. have the same physical and chemical properties), the objective taste of the two goods is the same. A further example is the following experiment: imagine that a live concert is recorded and then later replayed as a playback to a similar audience (while the original musicians pretend performing music). The labour-intensity of the original concert is higher in comparison to the playback concert, since pretending is easier (i.e. less labour-intense) in comparison to performing live music. The objective taste of the two concerts would be the same. (However, the "subjective taste" of the two concerts would differ, if the audience knew that the second concert is only a playback.)

(b) The "objective taste" of a good is on average not dependent on the technology which is used to produce it. That is, some very tasty goods are produced by progressive technology and some very tasty goods are produced by backward technology; as well, some less tasty goods are produced by progressive technology and some less tasty goods are produced by backward technology.

With these assumptions we would conclude that on average group-A-goods are not preferred over group-B-goods and group-B-goods are not preferred over group-A-goods. That is, the groups are regarded as equivalents; hence, on average the elasticity of substitution between these two groups will be *close to* one (according to the discussion above).

Now let us make a further assumption:

(c) We look only on the averages over very long periods of time and we assume that there are many technologies and goods.

Hence, from this perspective due to the law of large numbers the elasticity of substitution between the two groups is *equal to* unity.

In other words, if preferences and technologies are uncorrelated (i.e. if the taste does not depend on production technology), the household-behaviour will not display any preference for technology-level (group A or group B), provided that very long periods of time are considered and provided that there are many goods.

This is what we assumed in section 3.1: we assumed that there are two technologies and that there are many goods which are produced with these technologies and that the preference structure does not display any preference for a certain technology. This is what we did by assumptions (10) and (11). These assumptions ensure that on average the elasticity of substitution between technology-1-goods and technology-2-goods is equal to unity.

Now the question is whether the assumptions (a), (b) and (c) are suitable in long run growth models.

Assumption (c) seems not to be problematic, since long-run growth theory is anyway based on analyzing long-run-averages (e.g. the time preference rate is assumed to be constant in standard neoclassical growth models). Furthermore, since we look at very long run, any accidental correlations between technology and preferences, which may arise from a relatively low number of products, may as well offset each other over the period's average.

Assumption (b) is less problematic in comparison to assumption (a). In fact, technological progress during the last century has implicitly shown that the basic physical/chemical properties of a good are not necessarily dependent on its capital-intensity. In industrialized countries nearly all goods featured some technological progress which substituted labour by capital, while the basic physical properties of the goods remained the same basically. The most obvious example is agriculture. Food has for the most part the same basic physical properties today as earlier in the century, while the capital-intensity of agriculture increased significantly. Such developments are also apparent in manufacturing (e.g. regarding the increasing capital-intensity of car-production) and services (e.g. cash-teller-machines). Furthermore, today we can imagine for nearly every good or service a relatively realistic technology which could substitute the labour by capital, without changing the basic physical properties of the good. It is not plausible to assume that in the very long run technological progress is restricted to certain types of goods. In the last two decades many service-jobs which were regarded as labour-intensive were replaced by computer-machines and the substitutability of human by machines in services is increasing. Hence, when developing a long run theory of structural change, the dependency between technology and certain types of goods (and hence certain preferences) seems to be difficult to defend. Therefore, overall, the assumption that the "objective taste" of a good is independent of the capital-intensity of the production process seems to be acceptable to some degree, especially when assuming (c).

It is more difficult to evaluate *assumption (a)* a priori. Assumption (a) requires that the representative household behaves like he doesn't know about the actual capital intensity of a good, i.e. it is required that the household's demand reaction to a price and/or income change is based only on physical/chemical properties of a good. What we know from basic microeconomics (e.g. form the discussion about "Giffen-goods") is that the price elasticity (and income elasticity) depends on the basic physical/chemical properties of the good, i.e. whether the physical/chemical properties of a good are such that it is feasible to satisfy the basic needs of a household. (The price elasticity for such goods is low.) On the other hand, there is also a discussion about a "snob" effect, where some very labour-intensity services (like a full time servant) are used to signal the wealth of the household. Such services have a relatively high income-elasticity and price-elasticity. However, as well there are many high-capital-intensity-goods which have high price-elasticity of demand and high income-elasticity of demand, like very expensive cars. Hence, there is both: capital-intensive and labour-intensive goods which feature a relatively high price-elasticity and a relative high income-elasticity. Our model requires that *on average* (i.e. when looking at the average of all consumption goods) the income (price) elasticity of demand does not depend on the capital-intensity of a good.

Last not least, the increasing complexity of the products and of the production process, international outsourcing and increasing variety of products make it increasingly unlikely that the household has clear information about the capital-intensity of a large part of its consumption bundle.

All in all, the empirical evidence from the previous section implies that the assumption of no/low correlation between technology and preferences can explain a part of the Kuznets-Kaldor-puzzle. The fact that there is some correlation between technology and preferences results probably from the fact that assumption (a) has not been satisfied over the time-period of our sample. That is, probably high labour-intensity of a service has been regarded as an aspect of quality and/or luxury. Hence, high-labour-intensity services have probably had high income-elasticity of demand on average, which caused the correlation between technology and preferences in the past.

The fact that there has been some correlation between preferences and technologies in our sample does not necessarily imply that we can presume such correlation in future:

We analyzed only a 40 year period. This is a very short period to satisfy assumption (c) and to study growth theory empirically in general. Remember that Kaldor-facts (which we seek to explain in our paper) do not necessarily apply to such a short period. The probability is very high that over such a short period "accidental" correlation between technology and preferences arises, which does not persist over the long run. It seems that this was the case: The technological innovation between 1940 and 1980 allowed to a big part an increase in capital-intensity in non-service-sectors (such as manufacturing and agriculture). That is, the technological break-throughs were such that they were easy to implement in non-services sectors but they were hardly implementable in the services sector[8]. Hence, if services have high income-elasticity of demand, some correlation

[8] Of course, the term "services" means here rather personal services (i.e. services which require face-to-face contact, e.g. counselling) and rather not such services as transportation. The latter featured strong increases in capital-intensity. See for example Baumol et al. (1985) on discussion and empirical evidence about progressive and stagnant services.

between technology and preferences may have been arisen due to such biased technological progress. However, new sorts of technological break-through occurred after this period, especially in the information and communication technology. Such break-throughs have increased the capital-intensity in the services sector and have a high potential for increasing the capital-intensity of the services sector drastically (e.g. by progress in computers and robotics, which is implementable in services).

Hence, our empirical results probably over-estimate the long-run degree of correlation between preferences and technologies; the long-run correlation between preferences and technologies is probably very low or even inexistent. In this sense, our model of independent preferences and technologies predicts quite well the future structural change impacts on aggregates.

6. Concluding remarks

In this essay we have searched for a solution of the Kuznets-Kaldor-puzzle. In fact, the Kuznets-Kaldor-puzzle states that aggregate ratios behaved in a quite stable manner in industrialized countries, while at the same time massive factor reallocation took place across sectors which differ by technology (and especially by optimal capital-intensity).

Summary of our results

For the first time in the literature, we have shown that a PBGP can exist even when factors are reallocated across sectors which differ by capital intensity. We name the cross-capital-intensity structural change which is compatible with a PBGP "neutral structural change".

To test the actual neutrality of structural change we developed an index of neutrality. In fact, our measure of neutrality indicates the weighting between two measures $\left(\Delta\tilde{l}\right)^{neutral}$ and $\left(\Delta\tilde{l}\right)^{\max}$. $\left(\Delta\tilde{l}\right)^{neutral}$ measures the hypothetical change in $\tilde{l}$ which would result if the empirically observed amount of reallocated labour (Δl) was reallocated in the neutral way. $\left(\Delta\tilde{l}\right)^{\max}$ measures the hypothetical change in $\tilde{l}$ which would result if Δl was reallocated in the maximally non-neutral way. Hence, the weighting between these two measures implies how much labour has been reallocated in the neutral way and how much labour has been reallocated in the non-neutral way between 1948 and 1987. This index implies that 55% of structural change can be regarded as neutral. We provided also some theoretical/verbal arguments which imply that over the (very) long run significantly more than 55% of structural change is neutral (see section 5).

We also made a first step towards a micro-foundation of neutrality of structural change by showing that neutral structural change can arise if preferences and technologies are uncorrelated. Therefore, our neutrality index could also be interpreted as an index of correlation between technology and preferences. In this sense, our empirical findings imply that the correlation between preferences and technologies is rather low. Exactly speaking, the actual correlation was closer to the extreme of "no correlation" than to the extreme of "maximal correlation".

Discussion

Our empirical findings are valid for all the literature which analyses structural change along PBGP's (and where capital is included into analysis): We have shown in Proposition 4 that every PBGP which satisfies the Kaldor-facts (exactly) must feature neutral structural change. Hence, we can say that the papers by Kongsamut et al. (2001), Ngai and Pissarides (2007) and Foellmi and Zweimueller (2008) are compatible with 55% of structural change observed over the period 1948-1987.

The latter papers – especially the one by Kongsamut et al. (2001) – have been criticized by Buera and Kaboski (2009) for not being able to reproduce some structural change dynamics. This criticism is primarily based on the Stone-Geary type preference structure which is used in this model. Note that, although our empirical results seem to be in line with Buera and Kaboski's (2009)-results at first look, they are rather not comparable, since we analyse something completely different. Our empirical study is a test of the empirical validity of the partially balanced growth concept and of the independency assumption regarding preferences and technologies. For example, in contrast to Buera and Kaboski (2009), we do not assume any preference structure in our empirical test. Our results imply that, although independency between preferences and technologies has a large explanatory value regarding the Kuznets-Kaldor-puzzle (55% of factor-reallocations "offset" each other), the partially balanced growth concept is not very useful for predicting "shorter run" structural change – over periods of 40(!) years or so –, since it covers "only" 55% of structural change over such "short run" periods. Thus, to some extent our results imply that the new ways of structural change modelling which are suggested by Buera and Kaboski (2009) will not be good predictors of structural change over "shorter run", if they are implemented into partially balanced growth frameworks.

Overall, our explanation for the Kuznets-Kaldor-puzzle is the following: There is a certain degree of independency between technologies and preferences. As discussed in the previous section, over the very long run such independency comes from the assumption that the household's consumption decisions are based on the physical and chemical properties of the goods, but not on the capital-intensity (i.e. households are not interested in the production process of the consumption goods but only on the "taste" of the goods). If preferences and technologies are uncorrelated (or independent), structural change patterns which satisfy all the empirical observations associated with the Kuznets-Kaldor-puzzle can arise (especially factors are reallocated across sectors which differ by capital intensity). We show that this explanation is compatible with 55% of structural change.

The remaining task is to find a theoretical explanation for the fact that the remaining 45% of structural change must be compatible with the Kaldor facts as well (if we take the Kuznets-Kaldor-facts seriously). We suggest three explanations:

Our preferred explanation of this fact is the following: We suggest that over the longer run (100 years or so) those 45% vanish, i.e. preferences and technologies are completely independent. In this sense, the 45% of structural change which seem to represent dependency between preferences and technologies result from "short run" effects, e.g. unbalanced capital accumulation (which shifts production factors to the manufacturing sector); remember that over shorter run (40 years) Kaldor-facts are not satisfied anyway. Currently, we are working on a method to eliminate the effects of unbalanced capital expansion (and of the resulting shift towards manufacturing) in our empirical study.

Another interesting explanation may be that these 45% are quantitatively small; hence, their aggregate impact is relatively low (in comparison to the other aggregate-growth determinants, e.g. technological progress) at least at the level of stylized facts. In fact, this is implied by the paper by Acemogly and Guerrieri (2008).

The third explanation: The aggregate effect of these 45% of structural change may be offset by the aggregate effects of other growth determinants, e.g. some sort of "economy-wide technological progress" may have accelerated between 1948 and 1987 which would have offset the (negative) impacts of non-neutral structural change. Further research could analyze this question in more detail.

Furthermore, it seems interesting to search for other micro-foundations of neutral structural change: we explained the parameter restrictions which are necessary for the existence of neutral structural change by uncorrelated preferences and technologies; however, there are certainly other micro-foundations which can explain these parameter restrictions.

Our model features exogenous technology. Especially, the fact that technologies differ across sectors ("technology-bias") is exogenous in our model. Such assumptions are often criticized today. In a separate paper (Stijepic and Wagner 2011) we present an extension of our model where the technology-bias is endogenous. This extension has interesting results, e.g. it implies that technology-bias does not vanish. However, this extension does not change any of the key results which we have summarized in this section.

Note that we could try to assess the degree of correlation between preferences and technologies in an alternative way: First we would have to estimate the price elasticity of demand, the income elasticity of demand and the production functions for all sectors and then we would have to try to somehow figure out the degree of correlation between the estimated preference and technology parameters. This approach would be problematic for two reasons:

(1) Estimation of preference parameters (and especially of income elasticity of demand) is very difficult, since there are problems in measuring the changes in quality of goods and services. Hence, it is difficult to isolate whether demand for a good increased due to relatively high income-elasticity of demand or due to an increase in quality of the service; see e.g. Ngai and Pissarides (2007).

(2) Even if we could measure the preference and technology parameters exactly there would be a problem in defining a measure of correlation between preferences and technologies, since we have actually two sorts of preference parameters (income elasticity of demand and price elasticity of demand). Hence, if we have

two economies (A and B), which are identical except for their correlation between income elasticity and technology and between price elasticity and technology, it would be difficult to say in which economy the correlation between preferences and technologies is lower: For example, if the correlation between *income elasticity* and technology is slightly lower in country A in comparison to country B and if the correlation between *price elasticity* and technology is slightly lower in country B in comparison to country A, we could not say whether preferences and technologies are more or less correlated in country A in comparison to country B. Our approach omits this problem by focusing on the factor reallocation across technology which, as modelled in our paper, reflects the degree of correlation between preferences and technologies.

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APPENDIX A

There are two approaches to solve our model, which are known from the literature on the Ramsey-Cass-Koopmans model: (1) We can assume that there is a social planer who maximizes the welfare of the representative household ("benevolent dictator"); or (2) we can assume that there are many marginalistic households and entrepreneurs who maximize their life-time utility and profits in perfect markets. Both ways of solution lead to the same first order optimality conditions. We explain approach (1) in short and focus on the approach (2).

APPROACH (1):

Necessary (first order) conditions for an optimum

The benevolent dictator maximizes the utility function of the representative household (equations (8)-(11)) subject to the equations (1)-(7) and (12)-(16).

The Hamiltonian for this control problem is given by:

$$HAM = u(C_1, C_2, \ldots C_n) + \psi_H (Y_m - \delta K - C_m - h_m)$$

where ψ_H is the co-state variable.

The variables of this Hamiltonian are determined as follows:

C_i are given by $C_i = Y_i - h_i$, $\forall i \neq m$ (cf. (5)),

Y_i are given by (1) and (2),

Z is given by (7)

k_m is given by $k_m = 1 - \sum_{i \neq m} k_i$ (cf. (3)),

l_m is given by $l_m = 1 - \sum_{i \neq m} l_i$ (cf. (3)),

z_m is given by $z_m = 1 - \sum_{i \neq m} z_i$ (cf. (3))

Control variables are:

$l_1, l_2, \ldots l_{m-1}, l_{m+1}, \ldots l_n$,

$k_1, k_2, \ldots k_{m-1}, k_{m+1}, \ldots k_n$,

$z_1, z_2, \ldots z_{m-1}, z_{m+1}, \ldots z_n$,

$h_1, \ldots h_n$,

C_m

K is state variable.

The first order optimality condition can be derived by

- setting the first derivatives of the Hamiltonian with respect to the *control variables* equal to zero
- setting the first derivative of the Hamiltonian with respect to the *state variable* equal to $\rho\psi_H - \dot{\psi}_H$.

Then after some algebra, the first order optimality conditions (17)-(19) can be obtained. Q.E.D. We omit this derivation, since it is trivial.

Proof that sufficient (second order) conditions are satisfied

Note that the proof that the first order conditions are sufficient for an optimum is quite difficult in this APPROACH (1). Especially the proof of concavity in **Step 1** becomes quite "impossible" as we will see. Therefore, the following proof of sufficiency of the optimality conditions may be regarded as incomplete. As we will see, in APPROACH (2) this problem does not arise.

To prove the sufficiency of these necessary conditions we use the Arrow-Kurz-criterion, which can be applied in three steps. Note that in the following we omitted intermediates production, i.e. $\gamma = \mu = 0$, for simplicity. (Analogous results can be obtained with intermediate production.)

Step 1: Maximize the Hamiltonian with respect to the control variables for given state variable, co-state variable and time.

In fact, this implies $\frac{\partial HAM}{\partial C_m} \overset{!}{=} 0$ and (17). The latter together with (1), (2), (3) and (12) implies (21), (24) and (25).

From (9) and (17) we obtain

(A.1) $C_i = \frac{\omega_i}{\omega_m}\frac{C_m - \theta_m}{p_i} + \theta_i \quad \forall i$

Inserting this equation into (12) yields

(A.2) $E = \frac{C_m - \theta_m}{\omega_m}$

Inserting (A.1) and (A.2) into (9) yields after some algebra:

(A.3) $u(.) = \ln E - \overline{\omega}_n \ln p_n + \overline{\omega}$

where $\overline{\omega}_n \equiv \sum_{i=m+1}^{n} \omega_i$ and $\overline{\varpi} \equiv \sum_i \omega_i \ln(\omega_i)$

and where we have obtained from (1), (2) and (17)

$$\text{(A.4)}\quad p_n \equiv \left(\frac{\alpha}{\chi}\right)^{\chi} \left(\frac{\beta}{\nu}\right)^{\nu} B^{-1} A^{\chi/\alpha} \left(\frac{Kk_m}{Ll_m}\right)^{\beta-\nu}$$

Equations (9) and (A.2) and condition $\frac{\partial HAM}{\partial C_m} \overset{!}{=} 0$ imply

$$\text{(A.5)}\quad \psi_H = \frac{1}{E}$$

Now, note that we have just derived the first order conditions for a maximum. These conditions are ***sufficient*** only if the Hamiltonian is jointly concave in the control variables for given state variable, co-state variable and time. This requires determining the signs of the first minors of the Hessian determinant of the Hamiltonian (with respect to the control variables for given state variable, co-state variable and time); see e.g. Chiang 1984, p.336. Since we have an arbitrary (and large) number of state variables this becomes impossible (at least for us), due to the difficulties in calculating determinants. (Note that sometimes these difficulties do not arise if the Hessian is a diagonal matrix. However, in our model it is not.) Therefore, **Step 1** may be regarded as incomplete. We have not researched for a solution of this problem, since, as mentioned above, the model can be solved by using APPROACH (2). The proofs of sufficiency in APPROACH (2) are feasible for us.

Step 2: Insert the optimality conditions from Step 1 into the Hamiltonian, in order to obtain $\tilde{HAM} = \tilde{HAM}(\psi_H, K, time)$!

Inserting (A.3) and (A.5) into the Hamiltonian yields:

$$\text{(A.6)}\quad \tilde{HAM}(\psi_H, K, t) = \ln\left(\frac{1}{\psi_H}\right) - \overline{\omega}_n \ln p_n + \overline{\varpi} + \psi_H\left(Y - \frac{1}{\psi_H} - \delta K\right)$$

where p_n can be derived as (implicit) function of ψ_H by using equations (A.4), (24), (21) and (A.5) (remember that in equation (24) H=0, due to (23) and $\gamma = \mu = 0$) and where Y can be derived as (implicit) function of ψ_H by using equations (25), (21), (24) and (A.5) (again, remember that in equation (24) H=0, due to (23) and $\gamma = \mu = 0$).

Step 3: Show that $\tilde{HAM} = \tilde{HAM}(\psi_H, K, t)$ is concave in K for given ψ_H and time, by showing that $\frac{\partial^2 \tilde{HAM}(\psi_H, K, t)}{(\partial K)^2} < 0$.

This step is quite lengthy and includes calculating implicit derivatives, but straight forward. After some algebra it can be shown that

$$\text{(A.7)} \quad \frac{\partial^2 \tilde{HAM}(\psi_H, K, t)}{(\partial K)^2} = \frac{-\overline{\omega}_n(\chi - \alpha)}{K^2\left(\frac{\beta k_m}{l_m} + \alpha\right)^2\left(\frac{k_m}{l_m} - 1\right)}(1+\beta) < 0$$

This relation is true, since equation (24) implies that $\frac{k_m}{l_m} - 1 > (<) 0$ if $\chi - \alpha > (<) 0$ (remember that in equation (24) H=0, due to (23) and $\gamma = \mu = 0$).

Since we have accomplished all three steps, the Arrow-Kurz-criterion implies that conditions (17)-(19) are sufficient for an optimum (together with the transversality condition). Q.E.D. (Remember, however, that there are some difficulties in **Step 1**, as explained there.)

APPROACH (2)

As mentioned above in this section we assume that there exist many marginalistic and identical households and producers. (Of course the producers are identical within a sector, while they differ across sectors.) The assumption of marginalistic agents implies that all agents consider the prices and factor prices as exogenous; i.e. all agents are "price-takers". The prices, factor prices and quantities are determined by laws of (aggregate) demand and (aggregate) supply on the corresponding markets (where market clearing is assumed).

This interpretation of the Ramsey-Cass-Koopmans model is suggested by Cass (1965) and it is well known in the literature (see any book on growth economics, e.g. Barro and Sala-i-Martin (2004), pp.86ff).

Remember, however, that although APPROACH (1) and APPROACH (2) interpret our model in different ways, both approaches yield the same first order optimality conditions (and results in general).

For simplicity we omit intermediates production in this section, i.e. we set $\gamma = \mu = 0$. (Analogous results can be obtained with intermediate production.)

Producers

Since we have assumed that each sector is polypolistic and since there is perfect mobility of factors across sectors, we know that the value of marginal factor-productivity in each sector must be equal to the (economy-wide) factor-price, i.e.

(A.8) $p_i \frac{\partial Y_i}{\partial (l_i L)} = w, \quad \forall i$

(A.9) $p_i \frac{\partial Y_i}{\partial (k_i K)} = r, \quad \forall i$

where w is the real wage rate and r is the real rate of return on capital; see also, e.g., Kongsamut et al. (2001). These conditions can be obtained by maximizing the sector-profit function $\{p_i Y_i - w l_i L - r k_i K\}$ with respect to factor inputs $l_i L$ and $k_i K$, while sector demand, sector-price and factor-prices are exogenous. (That is, the sector behaves like a price-taker; the reason for this fact is that all entrepreneurs of the sector are price-takers. This fact could be proved by modelling explicitly each sector as consisting of identical marginalistic profit-maximizing producers; then conditions (A.8) and (A.9) could be obtained by calculating the first-order conditions for profit-maximization of each individual producer and by aggregating over all producers of a sector.)

We know that the wage rate and the rental rate of capital are equal across sectors due to the following fact: differences in factor-prices across sectors are eliminated instantly by cross-sector factor-migration due to the assumption of perfect cross-sector factor-mobility.

Equations (A.8) and (A.9) imply $p_i \frac{\partial Y_i}{\partial (l_i L)} = p_j \frac{\partial Y_j}{\partial (l_j L)}, \quad \forall i, j$ and $p_i \frac{\partial Y_i}{\partial (k_i K)} = p_j \frac{\partial Y_j}{\partial (k_j K)}, \quad \forall i, j$. This in turn implies for $j = m$ due to (13):

(A.10) $p_i = \frac{\partial Y_m / \partial(l_m L)}{\partial Y_i / \partial(l_i L)} = \frac{\partial Y_m / \partial(k_m K)}{\partial Y_i / \partial(k_i K)}, \quad \forall i$

which is part of optimality condition (17) (Q.E.D.).

Inserting (1) and (2) into (A.10) yields

(A.11) $p_i = 1, \quad i = 1,...m$

(A.12) $p_i = \left(\frac{\alpha}{\chi}\right)^{\chi} \left(\frac{\beta}{\nu}\right)^{\nu} B^{-1} A^{\chi/\alpha} \left(\frac{Kk_m}{Ll_m}\right)^{\beta-\nu} \equiv p_n, \quad i = m+1,...n$

Households

In this section the index ι denotes the corresponding variable of the individual household. For example, while E stands for consumption expenditures of the whole economy, E^{ι} stands for consumption expenditures of the household ι. We assume that there is an arbitrary and large number of households ($\iota = 1,...x$), sufficiently large to constitute marginalistic behaviour of households. Hence, it follows from equations (8)-(11) that each household has the following utility

(A.13) $U^{\iota} = \int_0^{\infty} u(C_1^{\iota},...C_n^{\iota}) e^{-\rho t} dt, \quad \forall \iota, \qquad \rho > 0$

where

(A.14) $u(C_1^{\iota},...C_n^{\iota}) = \ln\left[\prod_{i=1}^{n} (C_i^{\iota} - \theta_i^{\iota})^{\omega_i}\right], \quad \forall \iota$

(A.15) $\sum_{i=1}^{m} \theta_i^{\iota} = 0$

(A.16) $\sum_{i=m+1}^{n} \theta_i^{\iota} = 0$

Furthermore, each household has the following dynamic constraint:

(A.17) $\dot{W}^{\iota} = w\overline{L} + (r - \delta)W^{\iota} - E^{\iota}, \quad \forall \iota$

where W^{ι} is the wealth/assets of household ι, E^{ι} are consumption expenditures of household ι and $\overline{L}$ is the (exogenous) labour-supply of household ι. The latter implies that each household supplies the same amount of labour at the market.

The dynamic constraint (A.17) is standard (compare for example Barro and Sala-i-Martin (2004), p.88). It implies that the wealth of the household increases by labour-income and by (net-) interest-rate-payments and decreases by consumption expenditures.

Note that we assume that the labour supply of each household is exogenously determined.

In line with (12), consumption expenditures of a household are given by

(A.18) $E^{\iota} = \sum_{i} p_i C_i^{\iota}, \quad \forall \iota$

Each household maximizes its life-time-utility (A.13)-(A.16) subject to its dynamic constraint (A.17). Since this optimization problem is time-separable (due to the assumption of separable time-preference and marginalistic household), it can be divided into two steps; see also, e.g., Foellmi and Zweimüller (2008), p.1320f:

1.) Intratemporal (static) optimization: For a given level of consumption-budget (E^{ι}), the household optimizes the allocation of consumption-budget across goods.

2.) Intertemporal (dynamic) optimization: The household determines the optimal allocation of consumption-budget across time.

Intratemporal optimization:

The household maximizes its instantaneous utility (A.14)-(A.16) subject to the constraint (A.18), where it regards the consumption-budget (E^{ι}) and prices (p_i) as exogenous. (Remember that the household is price-taker.) The corresponding Lagrange-function is given by

$$LG = \ln\left[\prod_{i=1}^{n} (C_i^{\iota} - \theta_i^{\iota})^{\omega_i}\right] - \psi_L\left[E^{\iota} - \sum_i p_i C_i^{\iota}\right], \quad \forall \iota$$

where ψ_L is the LaGrange-multiplier (shadow-price).

The first order necessary optimality conditions are given by

(A.19) $\dfrac{\omega_i}{C_i^{\iota} - \theta_i^{\iota}} - \psi_L p_i = 0, \quad \forall i, \iota$

These conditions are also sufficient for an optimum (maximum), since the target function is concave and the restriction linear. (The non-negativity constraints are studied in the phase diagram in APPENDIX C.)

From (A.19) and (13), we have

(A.20) $C_i^\iota = \frac{\omega_i}{\omega_m}\frac{C_m^\iota - \theta_m^\iota}{p_i} + \theta_i^\iota, \quad \forall i, \iota$

Inserting (A.20) into (A.18) yields

(A.21) $E^\iota = \frac{C_m^\iota - \theta_m}{\omega_m}, \quad \forall \iota$

Intertemporal optimization

Inserting (A.11), (A.12), (A.20) and (A.21) into (A.14)-(A.16) yields after some algebra:

(A.22) $u(.) = \ln E^\iota - \overline{\omega}_n \ln p_n + \overline{\omega}, \quad \forall \iota$

where $\overline{\omega}_n \equiv \sum_{i=m+1}^{n} \omega_i$ and $\overline{\omega} \equiv \sum_i \omega_i \ln(\omega_i)$

Now, we have determined the instantaneous utility as function of consumption-budget (and prices). (Remember that the household is price-taker, i.e. prices are exogenous from the household's point of view.) Inserting (A.22) into (A.13) yields

(A.23) $U^\iota = \int_0^\infty \left(\ln E^\iota - \overline{\omega}_n \ln p_n + \overline{\omega}\right) e^{-\rho t} dt, \quad \forall \iota$

Thus, the intertemporal optimization problem is to optimize (A.23) subject to the dynamic constraint (A.17). This is a typical optimal control problem. The Hamiltonian for this problem is as follows:

(A.24) $HAM = \ln E^\iota - \overline{\omega}_n \ln p_n + \overline{\omega} + \psi_H \left[w\overline{L} + (r - \delta)W^\iota - E^\iota \right], \quad \forall \iota$

where ψ_H is the co-state variable. E^ι is control-variable and W^ι is state variable. The prices (p_n) and factor prices (w and $r - \delta$) are regarded by the household as exogenous (since the household is marginalistic and thus price-taker.) Remember that $\overline{L}$ is exogenous. It may be confusing that p_n is time varying (while being regarded as exogenous in the optimal control problem of the household). However, this fact does not prevent us from using the Hamiltonian, since the Hamiltonian function allows, in general, that time enters the target function explicitly (i.e. via exogenous "parameters"); see e.g. Gandolfo (1996), p.375, on a general formulation of the control-problems which are solvable by using the Hamiltonian.

The first order optimality conditions are given by $\frac{\partial HAM}{\partial E^{\iota}} \overset{!}{=} 0$ and $-\frac{\partial HAM}{\partial W^{\iota}} \overset{!}{=} \dot{\psi}_H + \rho\psi_H$. These conditions imply (after some algebra) that

(A.25) $\frac{\dot{E}^{\iota}}{E^{\iota}} = r - \delta - \rho, \ \ \forall \iota$

Note that this first order condition is also a sufficient condition for an optimum. This can be immediately concluded from the Hamiltonian. Equation (A.25) has the same concavity features as the Hamiltonian of the standard one-sector Ramsey-Cass-Koopmans model. Especially, the target function is concave in the control-variable and the restriction (i.e. the term within the squared brackets) is linear in the control and the state variable. **Therefore, we know that the Hamiltonian is concave; therefore, the optimality conditions are sufficient. Q.E.D.**

Relationship between individual variables and economy-wide aggregates

Aggregate consumption expenditures are given by

(A.26) $E = \sum_{\iota} E^{\iota} = \sum_{i} p_i C_i^{\iota}$

where the following relation holds:

(A.27) $C_i = \sum_{\iota} C_i^{\iota}, \quad \forall i$

There is no unemployment, i.e.

(A.28) $L = \sum_{\iota} \overline{L}$

Last not least, since the wealth/assets can only be invested in production-capital (K), the following relation must be true

(A.29) $K = \sum_{\iota} W^{\iota}$

(see also, e.g. Barro and Sala-i-Martin (2004), p.97). That is, all assets are invested in capital (capital-market-clearing).

Furthermore, the "subsistence needs" of the whole economy are simply equal to the sum of the subsistence needs of its individuals, i.e.

(A.30) $\theta_i = \sum_{\iota} \theta_i^{\iota}, \quad \forall i$

Equation (A.20), (A.27) and (A.30) imply

(A.31) $C_i = \frac{\omega_i}{\omega_m} \frac{C_m - \theta_m}{p_i} + \theta_i \ , \quad \forall i$

This equation corresponds to equation (18). Q.E.D. Exactly speaking, inserting (9) into (18) yields (A.31).

(A.25) and (A.26) imply

(A.32) $\frac{\dot{E}}{E} = r - \delta - \rho$

This equation corresponds to equation (19). Q.E.D. Exactly speaking, (19) can be transformed into (A.32) by using (1), (2), (8)-(12), (17) and (18).

APPENDIX B

Equations (20) to (29) are relevant for aggregate analysis. Now let us search, like in the "normal" Ramsey model, for a growth path where E and K grow at constant rate, i.e.

(B.1) $$\frac{\dot{E}}{E} \overset{!}{=} g_E$$

(B.2) $$\frac{\dot{K}}{K} \overset{!}{=} g_K$$

Equations (B.1) and (22) imply that

(B.3) $$\frac{\dot{k}_m}{k_m} + \frac{\dot{K}}{K} - \frac{\dot{l}_m}{l_m} - \frac{\dot{L}}{L} = const.$$

Requirement (B.3) and equation (21) imply that

(B.4) $$\frac{\dot{\tilde{Y}}}{\tilde{Y}} = const.$$

(B.2) and (B.3) imply

(B.5) $$\frac{\left(k_m \dot{/} l_m\right)}{k_m / l_m} = const.$$

(B.1), (B.2) and (B.4) imply

(B.6) $$\frac{\left(E \dot{/} \tilde{Y}\right)}{E / \tilde{Y}} = const. \text{ and } \frac{\left(K \dot{/} \tilde{Y}\right)}{K / \tilde{Y}} = const.$$

Equations (B.2), (20) and (25) imply

(B.7) $$c_5 + c_6 \frac{l_m}{k_m} = \frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}} + (g_K + \delta) \frac{K}{\tilde{Y}}$$

Solving equation (24) for $\frac{H}{\tilde{Y}}$ and inserting it into equation (B.7) yields after some algebra:

(B.8) $$c_5 - \frac{1}{c_4} = \left(1 - \frac{c_3}{c_4}\right) \frac{E}{\tilde{Y}} - \left(c_6 + \frac{1}{c_4}\right) \frac{l_m}{k_m} + (g_K + \delta) \frac{K}{\tilde{Y}}$$

Remember that c_3, c_4, c_5, c_6, g_K and δ are constants. Furthermore, note that (B.5) and (B.6) imply that $\frac{E}{\tilde{Y}}$, $\frac{l_m}{k_m}$ and $\frac{K}{\tilde{Y}}$ grow at constant rate. Hence, equation (B.8) can be satisfied at any point of time only if $\frac{E}{\tilde{Y}}$, $\frac{l_m}{k_m}$ and $\frac{K}{\tilde{Y}}$ are constant (i.e. they grow at the constant rate zero), i.e.

(B.9) $$\frac{E}{\tilde{Y}} = const., \quad \frac{l_m}{k_m} = const., \quad \frac{K}{\tilde{Y}} = const.$$

Equations (B.9), (23), (25) imply

(B.10) $$\frac{\dot{E}}{E} = \frac{\dot{\tilde{Y}}}{\tilde{Y}} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{H}}{H} = const.$$

Q.E.D.

Let g^* denote the constant growth rate from equation (B.10). Hence, (B.9), (B.10), (21) and (26) imply

(B.11) $$g^* = \frac{\frac{\dot{G}}{G}}{1-q} + g_L$$

Inserting equations (27) and (28) into equation (B.11) yields after some algebra:

(B.12) $$g^* = \frac{(1-\mu\bar{\varepsilon})g_A + \gamma\bar{\varepsilon}g_B}{(1-\mu\bar{\varepsilon})\alpha + \gamma\bar{\varepsilon}\chi} + g_L$$

Q.E.D.

Note that in all the calculations from above we searched for an equilibrium growth path where E and K grow at constant rate. As a result we obtained that H grows at constant rate along this growth path. Hence, we can treat H like exogenous technological progress along this growth path. Let $\hat{Y} \equiv Y - H$. In this case equation (20) can be written as follows:

(B.13) $$\hat{Y} = \dot{K} + \delta K + E$$

Q.E.D.

Inserting equations (21), (23) and (25) into $\hat{Y} \equiv Y - H$ yields:

(B.14) $$\hat{Y} = \tilde{G}L^{1-q}K^q$$

where $\tilde{G} \equiv G\left(\frac{k_m}{l_m}\right)^q \left(c_5 - \gamma c_1 + (c_6 - \gamma c_2)\frac{l_m}{k_m}\right)$ grows at constant positive rate due to (B.9) ($\tilde{G}$ grows at constant positive rate). ***Q.E.D.***

Inserting equation (B.14) into equation (22) yields:

$$\text{(B.15)} \qquad \frac{\dot{E}}{E} = \lambda \frac{\hat{Y}}{K} - \delta - \rho$$

where $\lambda \equiv \beta \frac{l_m}{k_m}\left(c_5 - \gamma c_1 + (c_6 - \gamma c_2)\frac{l_m}{k_m}\right)$ is constant due to (B.9). ***Q.E.D.***

Equations (B.14) and (B.15) include the term l_m / k_m. This term is constant along the equilibrium growth path and can be derived as function of model parameters by setting equation (22) equal to g^* and solving afterwards for l_m / k_m:

$$\text{(B.16)} \qquad \frac{k_m}{l_m} = \left(\frac{g^* + \delta + \rho}{\beta}\right)^{\frac{1}{q-1}} G^{\frac{1}{1-q}} \frac{L}{K}$$

Note that the term $G^{\frac{1}{1-q}} L$ is a function of exogenous parameters and grows at rate g^* (see equation (B.11) for g^*). K grows at rate g^* along the equilibrium growth path as well (see Lemma 1). Hence, the term $G^{\frac{1}{1-q}} \frac{L}{K}$ is constant along the equilibrium growth path; thus, we can rewrite equation (B.16) in terms of initial values of exogenous parameters (the index zero denotes the initial value of the corresponding variable):

$$\text{(B.17)} \qquad \frac{k_m}{l_m} = \left(\frac{g^* + \delta + \rho}{\beta}\right)^{\frac{1}{q-1}} (G_0)^{\frac{1}{1-q}} \frac{L_0}{K_0}$$

where q, G_0 and g^* are given by equations (27), (28) and (B.12). ***Q.E.D.***

We have shown now that along an equilibrium growth path where E and K grow at constant rate H grows at constant rate as well and k_m / l_m is constant. When this fact is taken into account, the economy in aggregates is represented by equations (B.13)-(B.15). These equations are similar to the Ramsey-model regarding all relevant features; hence, they imply that this equilibrium growth path exists and is unique. ***Q.E.D.***

APPENDIX C

First, we show by using linear approximation that the saddle-path-feature of the PBGP is given (Proposition 1a). Then we prove local stability by using a phase diagram (Proposition 1b).

Existence of a saddle-path (Proposition 1a)

First we rearrange the aggregate equation system (20)-(29) as follows:

(C.1) $$\dot{\hat{K}} = \left(\frac{l_m}{k_m}\right)^{-q} (\alpha + \beta \frac{l_m}{k_m}) \hat{K}^q - \hat{E} - (\delta + g_L + \frac{g_G}{1-q}) \hat{K}$$

(C.2) $$\frac{\dot{\hat{E}}}{\hat{E}} = \beta \left(\frac{l_m}{k_m}\right)^{1-q} \hat{K}^{q-1} - \delta - \rho - g_L - \frac{g_G}{1-q}$$

(C.3) $$\frac{l_m}{k_m} = \frac{1 - \mu\bar{\varepsilon} + \gamma\bar{\varepsilon}\frac{\nu}{\beta} - \frac{\chi\beta - \alpha\nu}{\alpha\beta}\overline{\omega}_n \frac{\hat{E}}{\hat{K}^q \left(\frac{l_m}{k_m}\right)^{-q}}}{1 - \mu\bar{\varepsilon} + \gamma\bar{\varepsilon}\frac{\chi}{\alpha}}$$

where aggregate variables are expressed in "labor-efficiency units", i.e. they are divided by $LG^{\frac{1}{1-q}}$; hence $\hat{K} \equiv \frac{K}{LG^{\frac{1}{1-q}}}$ and $\hat{E} \equiv \frac{E}{LG^{\frac{1}{1-q}}}$. Furthermore, g_G is the growth rate of G given by (28) and $\overline{\omega}_n \equiv \sum_{i=m+1}^{n} \omega_i$.

These equations imply that $\hat{K}$, $\hat{E}$ and l_m / k_m have the following values along the PBGP

(C.4) $$\hat{K}^* = \sigma^{\frac{1}{1-q}} \left(\frac{l_m}{k_m}\right)^*$$

(C.5) $$\hat{E}^* = \alpha\sigma^{\frac{q}{1-q}} + \rho\sigma^{\frac{1}{1-q}} \left(\frac{l_m}{k_m}\right)^*$$

(C.6) $$\left(\frac{l_m}{k_m}\right)^* = \frac{\alpha}{\beta} \frac{\beta + (\nu\gamma - \mu\beta)\bar{\varepsilon} - (\chi\beta - \alpha\nu)\overline{\omega}_n}{\alpha + (\chi\gamma - \mu\alpha)\bar{\varepsilon} + (\chi\beta - \alpha\nu)\overline{\omega}_n \frac{\rho}{\beta}\sigma}$$

where $\sigma \equiv \dfrac{\beta}{\delta + \rho + g_L + \dfrac{g_G}{1-q}}$

where an asterisk denotes the PBGP-value of the corresponding variable.

The proof of local saddle-path-stability of the PBGP is analogous to the proof by Acemoglu and Guerrieri (2008) (see there for details and see also Acemoglu (2009), pp. 269-273, 926).

First, we have to show that the determinant of the Jacobian of the differential equation system (C.1)-(C.2) (where l_m / k_m is given by equation (C.3)) is different from zero when evaluated at the PBGP (i.e. for $\hat{K}^*$, $\hat{E}^*$, $\left(\dfrac{l_m}{k_m}\right)^*$ from equations (C.4)-(C.6)). This implies that this differential equation system is hyperbolic and can be linearly approximated around $\hat{K}^*$, $\hat{E}^*$, $\left(\dfrac{l_m}{k_m}\right)^*$ (Grobman-Hartman-Theorem; see as well Acemoglu (2009), p. 926, and Acemoglu and Guerrieri (2008)). The determinant of the Jacobian is given by:

$$\text{(C.7)} \qquad |J| = \begin{vmatrix} \dfrac{\partial \dot{\hat{K}}}{\partial \hat{K}} & \dfrac{\partial \dot{\hat{K}}}{\partial \hat{E}} \\ \dfrac{\partial \dot{\hat{E}}}{\partial \hat{K}} & \dfrac{\partial \dot{\hat{E}}}{\partial \hat{E}} \end{vmatrix} = \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} - \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} \frac{\partial \dot{\hat{K}}}{\partial \hat{E}}$$

The derivatives of equations (C.1)-(C.2) are given by:

(C.8)

$$\frac{\partial \dot{\hat{K}}}{\partial \hat{K}} = q\hat{K}^{q-1}\left(\alpha\left(\frac{l_m}{k_m}\right)^{-q} + \beta\left(\frac{l_m}{k_m}\right)^{1-q}\right)$$

$$+ \hat{K}^{q}\left(-q\alpha\left(\frac{l_m}{k_m}\right)^{-q-1} + (1-q)\beta\left(\frac{l_m}{k_m}\right)^{-q}\right)\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{K}} - \left(\delta + g_L + \frac{g_G}{1-q}\right)$$

$$\frac{\partial \dot{\hat{K}}}{\partial \hat{E}} = \hat{K}^{q}\left(-q\alpha\left(\frac{l_m}{k_m}\right)^{-q-1} + (1-q)\beta\left(\frac{l_m}{k_m}\right)^{-q}\right)\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{E}} - 1$$

$$\frac{\partial \dot{\hat{E}}}{\partial \hat{K}} = \beta\hat{E}\left((q-1)\hat{K}^{q-2}\left(\frac{l_m}{k_m}\right)^{1-q} + (1-q)\hat{K}^{q-1}\left(\frac{l_m}{k_m}\right)^{-q}\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{K}}\right)$$

$$\frac{\partial \dot{\hat{E}}}{\partial \hat{E}} = \left(\beta\left(\frac{l_m}{k_m}\right)^{1-q}\hat{K}^{q-1} - \delta - \rho - g_L - \frac{g_G}{1-q}\right)$$

$$+ \beta\hat{E}\hat{K}^{q-1}(1-q)\left(\frac{l_m}{k_m}\right)^{-q}\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{E}}$$

where the derivatives of equation (C.3) are given by

(C.9)

$$\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{E}} = -\frac{\frac{\chi\beta - \alpha\nu}{\alpha\beta}\overline{\omega}_n\frac{\left(\frac{l_m}{k_m}\right)^{q}}{\hat{K}^{q}}}{\left(1+\overline{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)\left(1+\left(1+\overline{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)^{-1}\frac{\chi\beta-\alpha\nu}{\alpha\beta}\overline{\omega}_n q\frac{\hat{E}}{\hat{K}^{q}\left(\frac{l_m}{k_m}\right)^{1-q}}\right)}$$

$$\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial \hat{K}} = \frac{\frac{\chi\beta - \alpha\nu}{\alpha\beta}\overline{\omega}_n q\frac{\hat{E}}{\hat{K}^{q+1}\left(\frac{l_m}{k_m}\right)^{-q}}}{\left(1+\overline{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)\left(1+\left(1+\overline{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)^{-1}\frac{\chi\beta-\alpha\nu}{\alpha\beta}\overline{\omega}_n q\frac{\hat{E}}{\hat{K}^{q}\left(\frac{l_m}{k_m}\right)^{1-q}}\right)}$$

Inserting the derivatives (C.8) and (C.9) into (C.7) and inserting the PBGP-values from equations (C.4)-(C.6) yields after some algebra the following value of the determinant of the Jacobian evaluated at the PBGP:

$$\text{(C.10)} \qquad |J|^* = \frac{-(1-q)(\chi\beta-\alpha\nu)\overline{\omega}_n \dfrac{\hat{E}^*}{\hat{K}^*}\left[\rho + \dfrac{\overline{\alpha}}{(\chi\beta-\alpha\nu)\overline{\omega}_n}\dfrac{\beta}{\sigma}\right]}{\overline{\alpha} + \dfrac{\chi\beta-\alpha\nu}{\beta}\overline{\omega}_n q \dfrac{\hat{E}^*}{(\hat{K}^*)^q \left[\left(\dfrac{l_m}{k_m}\right)^*\right]^{1-q}}}$$

where $\overline{\alpha} \equiv \alpha + \overline{\varepsilon}(\chi\gamma - \alpha\mu) > 0$ and q is given by equation (27).

This equation can be transformed further by using equations (27) and (C.4)-(C.6):

$$\text{(C.11)} \qquad |J|^* = \frac{-\dfrac{\hat{E}^*}{\hat{K}^*}\dfrac{\alpha}{\sigma}\left[\overline{\beta} - (\chi\beta-\alpha\nu)\overline{\omega}_n\right]}{\overline{\alpha}\left(\dfrac{l_m}{k_m}\right)^* + \dfrac{\alpha}{\beta}\dfrac{\overline{\beta}^2}{\overline{\alpha}}}$$

where $\overline{\beta} \equiv \beta + \overline{\varepsilon}(\nu\gamma - \beta\mu) > 0$. Note that $\dfrac{\hat{E}^*}{\hat{K}^*}$ and $\left(\dfrac{l_m}{k_m}\right)^*$ are positive and are given by equations (C.4)-(C.6). Furthermore, note that following relations, which are useful for deriving equation (C.11), are true: $\overline{\alpha} + \overline{\beta} = 1 - \gamma + (\gamma - \mu)\overline{\varepsilon}$, $q = \dfrac{\overline{\beta}}{\overline{\alpha} + \overline{\beta}}$ (from (27)) and $\left(\dfrac{l_m}{k_m}\right)^* = \dfrac{\alpha}{\beta}\dfrac{\overline{\beta} - (\chi\beta-\alpha\nu)\overline{\omega}_n}{\overline{\alpha} + (\chi\beta-\alpha\nu)\overline{\omega}_n \dfrac{\rho}{\beta}\sigma}$ (from (C.6)).

We can see that the determinant evaluated at PBGP is different form zero. Hence, the PBGP is hyperbolic. Furthermore, equations (C.10) and (C.11) imply that $|J|^* < 0$. (Equation (C.10) implies that $|J|^* < 0$, if $\chi\beta - \alpha\nu > 0$; equation (C.11) implies that $|J|^* < 0$, if $\chi\beta - \alpha\nu < 0$ as well.)

Our differential equation system consists of two differential equations ((C.1) and (C.2)) and of two variables ($\hat{E}$ and $\hat{K}$), where we have one state and one control-variable. Hence, saddle-path-stability of the PBGP requires that there exist one negative (and one positive) eigenvalue of the differential equation system when evaluated at PBGP (see also Acemoglu and Guerrieri (2008) and Acemoglu (2009), pp. 269-273). Since $|J|^* < 0$ we can be sure that this is the case. ($|J|^* < 0$ can exist only if one eigenvalue is positive and the other eigenvalue is negative. If both eigenvalues were negative or if both eigenvalues were positive, the determinant $|J|^*$ would be positive.) Therefore, the PBGP is locally saddle-path-stable, i.e. Proposition 1a is proved. ***Q.E.D.***

Local stability (Proposition 1b)

In the following, we omit intermediates for simplicity, i.e. we set $\gamma = \mu = 0$. Furthermore, we study here only the case where output-elasticity of capital in investment goods industries (i=m) is relatively low in comparison to the output-elasticity of capital in the consumption goods industries ($\forall i \neq m$), i.e. we assume $\chi < \alpha$. This is consistent with the empirical evidence presented and discussed in Valentinyi and Herrendorf (2008) (see there especially p.826). Note, however, that the qualitative stability results for the other case (i.e. $\chi > \alpha$) are the same.

To show the stability-features of the PBGP, the three-dimensional system (C.1)-(C.3) has to be transformed into a two dimensional system, in order to allow us using a phase-diagram. By defining the variable $\kappa \equiv \frac{\hat{K}k_m}{l_m}$, the system (C.1)-(C.3) can be reformulated as follows (after some algebra):

(C.12) $$\frac{\dot{\hat{E}}}{\hat{E}} = \beta\kappa^{\beta-1} - \left(\delta + \rho + g_L + \frac{g_G}{1-\beta}\right)$$

(C.13) $$\frac{\dot{\kappa}}{\kappa} = \frac{\kappa^{\beta-1} - (\delta + g_L + \frac{g_G}{1-\beta}) - \frac{\hat{E}}{\kappa}\left(1 - \frac{\alpha-\chi}{\alpha\beta}\overline{\omega}_n \rho \kappa^{1-\beta}\right)}{1 + \frac{\alpha-\chi}{\beta}\overline{\omega}_n \frac{\hat{E}}{\kappa^{\beta}}}$$

where $\overline{\omega}_n \equiv \sum_{i=m+1}^{n} \omega_i$

We can focus attention on showing that the stationary point of this differential equation system is stable: The discussion in APPENDIX B implies that κ and $\hat{E}$ are jointly in steady state only if $\hat{K}$, $\hat{E}$ and k_m / l_m are jointly in steady state and that $\hat{K}$, $\hat{E}$ and k_m / l_m are jointly in steady state only if κ and $\hat{E}$ are jointly in steady state. Therefore, the proof of stability of the stationary point of system (C.12)-(C.13) implies stability of the stationary point of system (C.1)-(C.3). Hence, in the following we will prove stability of the stationary point of system (C.12)-(C.13).

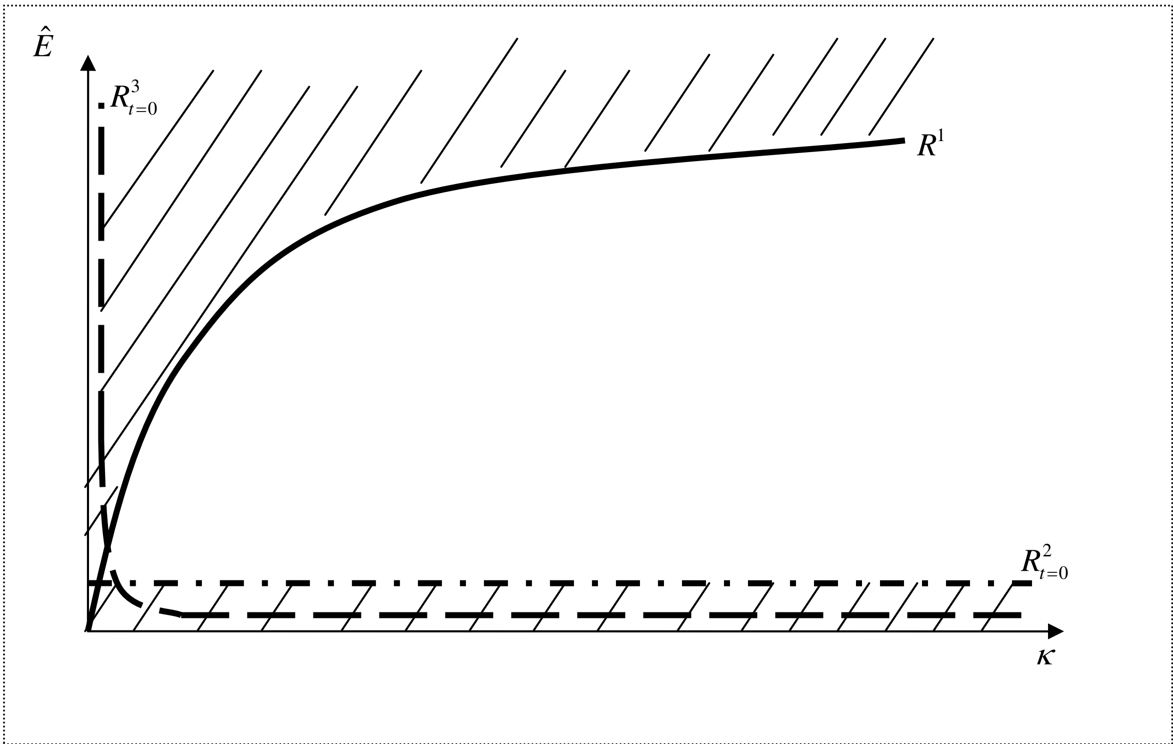
It follows from equations (C.12) and (C.13) that the steady-state-loci of the two variables are given by

(C.12a) $$\frac{\dot{\hat{E}}}{\hat{E}} = 0: \ \kappa^* = \left(\frac{\beta}{\delta + \rho + g_L + \frac{g_G}{1-\beta}} \right)^{\frac{1}{1-\beta}}$$

(C.13a) $$\frac{\dot{\kappa}}{\kappa} = 0: \quad \hat{E}_{\dot{\kappa}=0} = \frac{\kappa^{\beta-1} - (\delta + g_L + \frac{g_G}{1-\beta})}{1 - \rho \frac{\alpha - \chi}{\alpha\beta} \overline{\omega}_n \kappa^{1-\beta}} \kappa$$

Now, we could depict the differential equation system (C.12)-(C.13) in the phase space ($\hat{E},\kappa$). Before doing so, we show that not the whole phase space ($\hat{E},\kappa$) is economically meaningful. The economically meaningful phase-space is restricted by three curves (R^1, R_t^2, R_t^3), as shown in the following figure and as derived below:

Figure C.1: Relevant space of the phase diagram



Only the space below the R^1-line is economically meaningful, since the employment-share of at least one sub-sector i is negative in the space above the R^1-line. This can be seen from the following fact:

It follows from equations (1), (2), (3) and (17) after some algebra that

(C.14) $$\frac{l_m}{k_m} = 1 - \left(\frac{\chi\beta - \alpha\nu}{\chi\beta}\right)\sum_{i=m+1}^{n} l_i$$

Note that $\chi\beta - \alpha\nu = \chi - \alpha$ when $\gamma = \mu = 0$.

Since, l_i cannot be negative (hence , $0 \leq \sum_{i=m+1}^{n} l_i \leq 1$) this equation implies that

(C.15) $$\frac{l_m}{k_m} < \frac{\alpha\nu}{\chi\beta}$$

Inserting equation (24) into this relation yields

(C.16) $$R^1 : \hat{E} < \frac{\alpha}{\chi}\frac{1}{\omega_n}\kappa^{\beta}$$

(remember that in equation (24) H=0, due to (23) and $\gamma = \mu = 0$).

Hence, the space above R^1 is not feasible. When the economy reaches a point on R^1, no labour is used in sub-sectors i=1,…m. If we impose Inada-conditions on the production functions, as usual, this means that the output of sub-sectors i=1,…m is equal to zero, which means that the consumption of these sectors is equal to zero. Our utility function implies that life-time utility is infinitely negative in this case. Hence, the household prefers not to be at the R^1-curve. Note that actually the R^1-curve is only an outer limit: Since we have existence-minima in our utility function, the utility function becomes infinitely negative when the consumption of one of these goods falls below its subsistence level. Hence, even when the consumption of all goods is positive, it may be the case that the utility function is infinitely negative due to violation of some existence minima. Therefore, the actual constraint is somewhere below the R^1-curve. However, this fact does not change the qualitative results of the stability analysis.

Now we turn to the R_t^2 and R_t^3-curves. We have to take account of the non-negativity-constraints on consumption ($C_i > 0 \ \forall i$), since our Stone-Geary-type utility function can give rise to negative consumption. By using equations (A.1), (A.2) (A.11) and (A.12) from APPENDIX A and equations (27) and (28) the non-negativity-constraints ($C_i > 0 \ \forall i$) can be transformed as follows (remember that we assume here $\gamma = \mu = 0$):

(C.17) $$\hat{E} > \frac{-\theta_i}{\omega_i} \frac{1}{LA^{\frac{1}{\alpha}}} \qquad i = 1,...m$$

(C.18) $$\hat{E} > \frac{-\theta_i}{\omega_i} \left(\frac{\alpha \nu}{\chi \beta} \right)^{\chi} \frac{\beta}{\nu} \frac{1}{LBA^{\frac{2\nu - \beta}{\alpha}}} \frac{1}{\kappa^{\nu - \beta}} \qquad i = m+1,...n$$

This set of constraints implies that at any point of time only two constraints are binding, namely those with respectively the largest $\frac{-\theta_i}{\omega_i}$. Hence, the set (C.17), (C.18) can be reduced to the following set:

(C.19) $$R_t^2 : \ \hat{E} > \frac{-\theta_j}{\omega_j} \frac{1}{LA^{\frac{1}{\alpha}}}$$

where $\frac{-\theta_j}{\omega_j} > \frac{-\theta_i}{\omega_i} \quad i = 1,...m$

and $1 \leq j \leq m$.

(C.20) $$R_t^3 : \ \hat{E} > \frac{-\theta_x}{\omega_x} \left(\frac{\alpha \nu}{\chi \beta} \right)^{\chi} \frac{\beta}{\nu} \frac{1}{LBA^{\frac{2\nu - \beta}{\alpha}}} \frac{1}{\kappa^{\nu - \beta}}$$

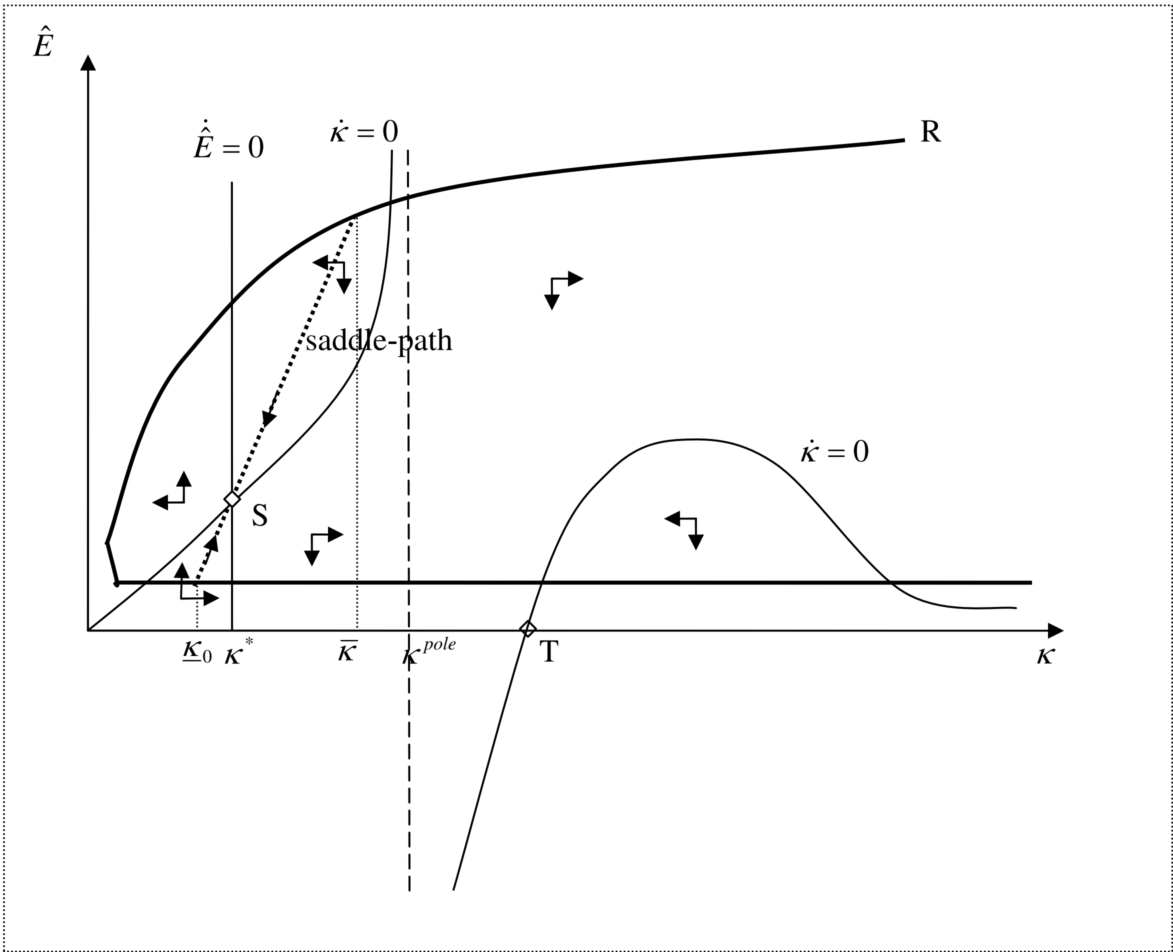
where $\frac{-\theta_x}{\omega_x} > \frac{-\theta_i}{\omega_i} \quad i = m+1,...n$

and $m+1 \leq x \leq n$

These constraints are time-dependent. It depends upon the parameter setting whether R_t^2 or whether R_t^3 is binding at a point of time. In Figure C.1 we have depicted examples for these constraints for the initial state of the system. Only the space above the constraints is economically meaningful, since below the constraints the consumption of at least one good is negative. Last not least, note that equations (C.19)/(C.20) imply that the R_t^2-curve and the R_t^3-curve converge to the axes of the phase-diagram as time approaches infinity.

Now, we depict the differential equation system (C.12)-(C.13) in the phase space ($\hat{E}, \kappa$).

Figure C.2: The differential equation system (C.12)-(C.13) in the phase-space for $\frac{\alpha\beta}{\overline{\omega}_n\rho(\alpha-\chi)} < \frac{1}{\delta + g_L + \frac{g_G}{1-\beta}}$



Note that we have depicted here only the relevant (or: binding) parts of the restriction-set of Figure C.1 as a bold line R.

As we can see, the $\dot{\kappa} = 0$-locus has a pole at $\kappa^{pole} = \left(\frac{\alpha\beta}{\overline{\omega}_n\rho(\alpha-\chi)}\right)^{\frac{1}{1-\beta}}$.

The phase diagram implies that there must be a saddle-path along which the system converges to the stationary point S (where S is actually the PBGP). The length of the saddle-path is restricted by the restrictions of the meaningful space R^1, R_t^2, R_t^3 (bold line). In other words, only if the initial κ (κ_0) is somewhere between $\underline{\kappa}_0$ and $\overline{\kappa}$[9], the economy can be on the saddle-path. Therefore, the system can be only

[9] Note that $\overline{\kappa}$ must be somewhat smaller than depicted in this diagram, since, as discussed above, R^1-curve is only an „outer limit".

locally saddle-path stable. Now, we have to show that the system will be on the saddle-path if $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$. Furthermore, we have to discuss what happens if κ_0 is not within this range.

Every trajectory, which starts *above the saddle-path or left from* $\underline{\kappa}_0$, reaches the R^1-curve in finite time. As discussed above, the life-time utility becomes infinitely negative if the household reaches the R^1-curve. These arguments imply that the representative household will never choose to start above the saddle path if $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$, since all the trajectories above the saddle-path lead to a state where life-time-utility is infinitely negative.

Furthermore, all initial points which are situated *below the saddle-path or right from* $\overline{\kappa}$ converge to the point T. If the system reaches one of the constraints (R_t^2, R_t^3) during this convergence process, it moves along the binding constraint towards T. However, the transversality condition is violated in T. Therefore, T is not an equilibrium. To see that the transversality condition is violated in T consider the following facts: The transversality condition is given by $\lim_{t\to\infty} \psi K e^{-\rho t} > 0$, where ψ is the costate variable in the Hamiltonian function (shadow-price of capital; see also APPENDIX A). By using the equations from APPENDIX A this transversality condition can be reformulated such that we obtain: $\lim_{t\to\infty} \beta\kappa^{\beta-1} - \delta - g_L - \frac{g_G}{1-\beta} > 0$,

which is equivalent to: $\lim_{t\to\infty} \kappa < \left(\frac{\beta}{\delta + g_L + \frac{g_G}{1-\beta}} \right)^{\frac{1}{1-\beta}}$. However, equation (C.13a)

implies that in point T in Figure C.2 $\kappa = \left(\frac{1}{\delta + g_L + \frac{g_G}{1-\beta}} \right)^{\frac{1}{1-\beta}}$. Hence, the

transversality condition is violated if the system converges to point T.

Overall, we know that, if $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$, the household always decides to be on the saddle-path. Hence, we know that for $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$ the economy converges to the PBGP. In this sense, **the PBGP is locally stable (within the range $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$).**

If the initial capital is to small ($\kappa_0 < \underline{\kappa}_0$), the economy converges to a state where some existence minima are not satisfied (curve R^1) and thus utility becomes infinitely negative. This may be interpreted as a **development trap**. For example, Malthusian theories imply that in this case some part of the population dies, which would yield an increase in per-capita-capital (and hence an increase in κ_0).

On the other hand, **if initial capital-level is too large** ($\kappa_0 > \bar{\kappa}$**),** all trajectories violate the transversality condition. Therefore, in this case, **the representative household must waste a part of its initial capital** to come into the feasible area ($\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$). This case may be interpreted as **"development trap of the rich"**.

Furthermore, note that there are always some happenings which reduce the capital stock, e.g. wars (like the Second World War) or natural catastrophes. These happenings could shift the economy into feasible space ($\underline{\kappa}_0 < \kappa_0 < \bar{\kappa}$). These thoughts could be analyzed further in order to develop a theory that the Second World War is the reason for the fact that many economies satisfy the Kaldor-facts today.

The alternative is to assume that the transversality condition needs not to hold necessarily. In this case the point T would be an equilibrium. All economies which start at $\kappa_0 > \bar{\kappa}$ would converge to this equilibrium. However, we have no idea of how we could omit the transversality condition. We know that the transversality condition implies that the value of capital is not allowed to be negative at the household's death (at infinity). In the actual model, there seems to be no adequate theory of allowing for the violation of the transversality condition.

Note that equation (C.3) and definition $\kappa \equiv \frac{\hat{K}k_m}{l_m}$ **imply that** κ **is a strictly monotonously increasing function of** K **in the relevant space of the phase diagram (ceteris paribus).** Hence, the κ-ranges which determine the ranges of the "development trap", the local stability and the "development trap of the rich" can be directly transformed into K-ranges. That is, if initial capital (K_0) is relatively low, the economy is in the "development trap"; if K_0 is relatively high, the economy is in the "development trap of the rich"; if K_0 is somewhere in-between the economy is in the space of local stability.

Furthermore, note that Figure C.2 depicts the phase diagram for parameter constellations, which satisfy the condition $\frac{\alpha\beta}{\overline{\omega}_n \rho(\alpha-\chi)} < \frac{1}{\delta + g_L + \frac{g_G}{1-\beta}}$. For parameter constellations, which satisfy the condition $\frac{\alpha\beta}{\overline{\omega}_n \rho(\alpha-\chi)} > \frac{1}{\delta + g_L + \frac{g_G}{1-\beta}}$, the discussion and the qualitative results are nearly the same. The only difference is that the $\dot{\kappa} = 0$-locus is hump-shaped (concave) for $\kappa < \kappa^{pole}$. However, all the qualitative results remain the same (local stability of PBGP for some range $\underline{\kappa}_0 < \kappa_0 < \overline{\kappa}$ and "infeasibility" for $\kappa_0 < \underline{\kappa}_0$ and $\kappa_0 > \overline{\kappa}$). **Q.E.D.**

APPENDIX D

It follows from the optimality condition (18) that

(D.1) $$C_i = \frac{\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{p_i} + \theta_i \qquad \forall i$$

For the sake of simplicity we consider only the non-homotheticity between the services sector and the conglomerate of the agriculture and manufacturing sector. Inserting equation (D.1) into equations (15) yields (remember equation (10)):

(D.2) $$E_{agr.+man.} = d_1 E + d_2$$

(D.3) $$E_{ser.} = d_3 E + d_4$$

where $E_{agr.+man} = E_{agr.} + E_{ser.}$ $d_1 \equiv \frac{\sum_{i=1}^{s}\omega_i}{\sum_{i=1}^{n}\omega_i}$, $d_2 \equiv p\sum_{i=m+1}^{s}\theta_i$, $d_3 \equiv \frac{\sum_{i=s+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}$ and $d_4 \equiv p\sum_{i=s+1}^{n}\theta_i$. Note that p is given by $p = \frac{\partial Y_m / \partial(l_m L)}{\partial Y_n / \partial(l_n L)}$ and stands for the relative price of sub-sectors $i = m+1,...n$.

If preferences are non-homothetic across sectors consumption expenditures on agriculture and manufacturing ($E_{agr.+man.}$) do not grow at the same rate as consumption expenditures on services ($E_{ser.}$), when treating relative prices as constants. Hence, we have to show that $E_{agr.+man.}$ and $E_{ser.}$ do not grow at the same rate when treating d_1-d_4 as constants. It follows from equations (D.2) and (D.3) that, when treating d_1-d_4 as constants, the following equations are true

(D.4) $$\frac{\dot{E}_{agr.+man.}}{E_{agr.+man.}} = \frac{\dot{E}}{E}\frac{1}{1+\frac{d_2}{d_1}E}$$

(D.5) $$\frac{\dot{E}_{ser.}}{E_{ser.}} = \frac{\dot{E}}{E}\frac{1}{1+\frac{d_4}{d_3}E}$$

which shows that $E_{agr.+man.}$ and $E_{ser.}$ do not grow at the same rate when treating d_1-d_4 as constants, i.e., preferences are non-homothetic between the services sector and the conglomerate of the agriculture sector. In the same way it can be shown that preferences are non-homothetic between the manufacturing sector and the agriculture sector. ***Q.E.D.***

APPENDIX E

The optimality condition (17) implies after some algebra that

(E.1) $$h_i = \varepsilon_i \frac{H}{p_i}, \quad \forall i$$

Hence,

(E.2) $$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j}\frac{p_j}{p_i} \quad \text{for } i = a+1,\ldots s \quad \text{and} \quad j = s+1,\ldots n$$

In equation (E.2) i stands for the manufacturing sector and j for the services sector. Let us now take a look at an arbitrary producer of the manufacturing sector, e.g. the producer i = 3, where a+1<3 < s. we rewrite equation (E.2) as follows to show the viewpoint of "producer 3":

(E.3) $$\frac{h_3}{h_j} = \frac{\varepsilon_3}{\varepsilon_j}\frac{p_j}{p_3} \quad \text{for } j = s+1,\ldots n$$

From the view point of "producer 3" equation (E.3) determines the ratio between the input of own intermediates (i.e. the amount of intermediates which is produced by "producer 3" and used by "producer 3") and input of services-sector-produced intermediates (i.e. the amount of intermediates which is produced by "producer j" from the services sector and used by "producer 3"). (Remember that h_3 and h_j enter the production function of "producer 3" via equations (1) and (7).) Hence, for example, a decrease in $\frac{h_3}{h_j}$ means that "producer 3" increases the input of producer-j-intermediates relatively more strongly than the input of own intermediates, i.e. "producer 3" substitutes own intermediate inputs by external intermediate inputs, i.e. "producer 3" outsources additional intermediates production to producer j. Therefore, we can conclude from equation (E.3) that "producer 3" outsources more and more to "producer j" (i.e. $\frac{h_3}{h_j}$ decreases), provided that $\frac{\dot{p}_j}{p_j} - \frac{\dot{p}_3}{p_3} < 0$ (i.e. provided that the price for the good j in terms of the good 3 $(\frac{p_j}{p_3})$ decreases; or in other words: provided that the output of "producer j" becomes cheaper and cheaper (or less and less expensive) in comparison to the output of "producer 3").

From this discussion and from equation (E.2) we can conclude the following: manufacturing-sector-producers ($i = a+1,...s$) shift more and more intermediates production to services-sector-producers ($j = s+1,...n$), i.e. $\frac{h_i}{h_j}$ decreases, provided that services-sector-production becomes cheaper and cheaper (or less and less expensive) in comparison to manufacturing-production, i.e. provided that $\frac{\dot{p}_j}{p_j} - \frac{\dot{p}_i}{p_i} < 0$, and vice versa. ***Q.E.D.***

Note that relative prices are determined by exogenous parameters. Hence, which producers outsource and whether outsourcing from manufacturing to services increases (or the other way around) depends on the parameterization of the model. In general both cases are possible. By using optimality condition (17) the relative prices can be calculated, so that we can reformulate equation (E.2) after some algebra as follows

(E.4) $$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j}\frac{\alpha}{\chi}\frac{A}{B}\frac{1}{\vartheta}\left(\frac{k_m}{l_m}\frac{K}{L}\right)^{\beta-\nu+\varpi(\gamma-\mu)} D^{\gamma-\mu} \quad \text{for } i = a+1,...m \quad \text{and}$$

$j = s+1,...n$

(E.5) $$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j} \quad \text{for } i = m+1,...s \quad \text{and} \quad j = s+1,...n$$

where $\vartheta \equiv \left(\frac{\alpha}{\chi}\frac{\nu}{\beta}\right)^{\nu}\left(\frac{\alpha}{\chi}\frac{\mu}{\gamma}\right)^{\mu}$, $\varpi \equiv \frac{\beta-(\beta-\nu)\bar{\varepsilon}}{1+(\gamma-\mu)\bar{\varepsilon}-\gamma}$ and

$$D \equiv \left\{\gamma A\left[\frac{\chi}{\alpha}\frac{B}{A}\left(\frac{\alpha\nu}{\chi\beta}\right)^{\nu}\left(\frac{\alpha\mu}{\chi\gamma}\right)^{\mu}\right]^{\bar{\varepsilon}}\prod_{i=1}^{n}\varepsilon_i^{\varepsilon_i}\right\}^{\frac{1}{1-\gamma(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}}.$$

From equation (E.5) we can see that some of the manufacturing sector producers (i.e. producers $i = m+1,...s$) do not change their outsourcing behavior (i.e. these producers keep their ratio of external to own intermediates production ($\frac{h_i}{h_j}$) constant). Equation (E.4) implies that the rest of the manufacturing sector producers (i.e. the producers $i = a+1,...m$) change their outsourcing behavior. Calculating the growth rate of equation (E.4) yields (remember Lemma 1):

(E.6) $$\frac{(h_i \dot{/} h_j)}{h_i / h_j} = g_A - g_B + (\beta - \nu + \varpi(\gamma - \mu))g^* + (\gamma - \mu)\frac{\dot{D}}{D} \text{ for } i = a+1,...m$$

and $j = s+1,...n$

ϖ, g^* and $\dot{D}/D$ are positive. We omit here a detailed discussion of $\frac{(h_i \dot{/} h_j)}{h_i / h_j}$, since it is less relevant for our purposes. The only important thing is that $\frac{(h_i \dot{/} h_j)}{h_i / h_j}$ can be positive (e.g. if $g_A - g_B > 0$, $\beta - \nu > 0$ and $\gamma - \mu > 0$) or negative (e.g. if $g_A - g_B < 0$, $\beta - \nu < 0$ and $\gamma - \mu < 0$) depending on the parameterization of the model. Hence, the intermediates-production may be shifted from manufacturing to services or the other way around, depending on the parameterization of the model. ***Q.E.D.***

APPENDIX F

It is well known that balanced growth requires either labor-augmenting technological progress (or production function(s) of type Cobb-Douglas.) Furthermore, a standard assumption in macroeconomic models is that the production function has constant returns to scale. (Later, we will see that the aggregate production function has the same structure as the sectoral production functions.) Since we want to reassess the standard growth theory we do not depart from these assumptions. Therefore, we assume now that sectoral production functions are given by:

(1)' $Y_i = B_i l_i L f_i(\Omega_i) \quad \forall i = 1,...n$

where

(26)' $$\Omega_i \equiv \frac{k_i K}{l_i L B_i} \quad \forall i = 1,...n$$

B_i stands for the level of sector-specific and labor augmenting technological progress; $f_i(\Omega_i)$ is a sector-specific function of Ω_i; it is the intensive form of a "standard" constant returns to scale function, where in this appendix Ω_i denotes the capital-to-labor ratio in efficiency units in sector i.

The sectoral growth rates of labor-augmenting technological progress (g_i) are constant, i.e. $\dot{B}_i / B_i = g_i \quad \forall i$. The following equations remain the same as in the previous discussion:

(3)' $$\sum_i k_i = 1$$

(3)'' $$\sum_i l_i = 1$$

(12)' $$Y \equiv \sum_i p_i Y_i$$

We still assume that sector m is numéraire (m<n) (although we do not make here any assumptions about which sector produces capital). Hence, equation (13) holds.

When labor and capital are mobile across sectors and markets are polypolistic the following efficiency conditions must be true:

(17)' $$\frac{p_i}{p_j} = \frac{\partial Y_j / \partial(k_j K)}{\partial Y_i / \partial(k_i K)} = \frac{\partial Y_j / \partial(l_j L)}{\partial Y_i / \partial(l_i L)} \quad \forall i, j$$

(32)' $$r + \delta = p_i \partial Y_i / \partial(k_i K) \quad \forall i$$

Note that we do not make here any assumption about the household behavior. The assumptions above are sufficient to derive Proposition 4.

The capital share of income in sector i (or: the elasticity of capital with respect to output in sector i) is given by:

(F.1) $$\kappa_i(\Omega_i) \equiv \frac{(\partial Y_i / \partial k_i K) k_i K}{Y_i} = \Omega_i \frac{f_i'(\Omega_i)}{f_i(\Omega_i)}$$

where $f_i'(\Omega_i) \equiv \frac{\partial f_i(\Omega_i)}{\partial \Omega_i}$.

By inserting equations (1)', (26)' and (13) into equation (32)' we obtain:

(F.2) $$r + \delta = f_m'(\kappa_m)$$

Inserting first equations (1)' and (26)' into equation (17)' and then inserting equation (F.1) into this term yields:

(F.3) $$\frac{k_i}{l_i} \frac{1-\kappa_i(\Omega_i)}{\kappa_i(\Omega_i)} = \frac{k_m}{l_m} \frac{1-\kappa_m(\Omega_m)}{\kappa_m(\Omega_m)} \quad \forall i$$

Solving this term for k_i and inserting it into equation (3)' yields (remember that $\frac{\kappa_i}{1-\kappa_i} = \frac{1}{1-\kappa_i} - 1$ and $\sum_i l_i = 1$):

(F.4) $$\frac{k_m}{l_m} \frac{1-\kappa_m(\Omega_m)}{\kappa_m(\Omega_m)} = \left(\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} - 1 \right)^{-1}$$

Equations (13) and (17)' imply:

(F.5) $$p_i = \frac{B_m \left(f_m(\Omega_m) - \Omega_m f_m'(\Omega_m) \right)}{B_i \left(f_i(\Omega_i) - \Omega_i f_i'(\Omega_i) \right)} \quad \forall i$$

Inserting equations (1)', (F.1) and (F.5) into equation (12)' yields:

(F.6) $$Y = B_m L f_m(\Omega_m)(1-\kappa_m(\Omega_m)) \sum_i \frac{l_i}{1-\kappa_i(\kappa_i)}$$

Inserting equation (F.4) into equation (F.6) yields equation

(F.7) $$\frac{Y}{K} = \frac{f_m(\Omega_m)}{\Omega_m} \left(\kappa_m(\Omega_m) + \left[1-\kappa_m(\Omega_t^m) \right] \frac{k_m}{l_m} \right)$$

Definition F.1: *A PBGP is a growth path where* $\frac{Y}{K}$ *and* $\frac{rK}{Y}$ *are constant.*

Definition F.1 is consistent with Definition 1 (and with the Kaldor facts). In fact both definitions yield the same equilibrium growth path (but Definition 1 is stronger than necessary). However, now we use Definition F.1 in order to demonstrate that the necessary condition for the PBGP is independent of the numéraire. (Remember that, since $\frac{Y}{K}$ and $\frac{rK}{Y}$ are ratios, they are always the same irrespective of the choice of the numérarire.)

Lemma F.1: *A necessary condition for the existence of a PBGP (according to Definition F.1) is* $l_m / k_m = const.$ *or equivalently* $\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} = const.$.

Proof: Definition F.1 requires that $\frac{Y}{K}$ and $\frac{rK}{Y}$ are constant; hence r must be constant; hence Ω_m must be constant (due to equation (F.2)). Due to equation (F.7), $\Omega_m = const.$ and $\frac{Y}{K} = const.$ require $l_m / k_m = const.$. $l_m / k_m = const.$ and $\Omega_m = const.$ require $\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} = const.$ (due to equation (F.4)). Note that $1-\kappa_i(\Omega_i) = \lambda_i$, since we assume that there are only two production factors capital and labor. (λ_i stands for the output-elasticity of labor in sector i or equivalently for the labor-income share in sector i.) ***Q.E.D.***