Estimation of the fiscal impact multiplier in reduced-form equations.

Stefan Belliveau

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in reduced-form equations
(Working paper)
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1. ABSTRACT.

An analysis of the multiplier for the US federal government's consumption expenditures is presented. A simple approach to identify the influence of the federal government's consumption expenditures on economic activity using reduced-form equations is clearly presented and examined using annual US data from 1929-2011. The conclusion from this analysis is that estimates from reduced-form equations can inform fiscal-policy decision making.

2. KEY WORDS: Business cycles, fiscal policy

3. JEL CODES: E32, E62
1. INTRODUCTION.

Recent economic development raises questions for policymakers about whether and how to respond. Economic theory has informed analysis that supports various fiscal-policy responses, including nonresponse by reason of no effect, stimulative fiscal response, and fiscal discipline as a response. Fiscal nonresponse is supported by economic theory which concludes that economic variables measuring such policy's action have no influence on a real economy. Stimulative fiscal response is supported by economic theory informed by Keynesian conclusions (e.g., the New Keynesian theory), where economic conditions may be augmented for a time by fiscal policy in such a way that fiscal policy as generally understood is a viable option. The response of fiscal discipline to poor economic conditions is informed by economic theory that hypothesizes that reducing government expenditures for a given government fiscal position will improve economic conditions. The structure of this paper's analysis is intended to show that analysis of reduced-form equations can inform a fiscal-policy decision-making process.

The structure of this working paper follows: this introduction followed by a brief exploration of the economic data, a presentation of reduced-form equations and their estimates, a summary, all tables and figures cited, a list of references, and a list of the data sources.

2. EXPLORATION OF THE ECONOMIC DATA.

Economic activity appears to rise and fall in a cyclical pattern. Reference to a business or economic cycle is common in both general news media and scholarly economic journals. Figure 1 shows measures and analysis of US economic activity for a long period. The solid black line depicts annual US real output on a logarithmic scale (left axis). This scale allows a linear view of an exponential process (i.e., economic growth). Presented over a long duration, it suggests an underlying process that is steady. The dashed line portraying the trend of real output on a logarithmic scale with
respect to time makes this explicit.

The shaded area in Figure 1 depicts deviation of real output from its (logarithmic) linear trend with respect to time in percent (right axis). It can be interpreted as a barometer of economic activity. This barometer of economic activity begins positive in 1929, sharply declines in the years 1930-3, rises greatly in the years 1941-4, again declines sharply in the years 1946-7, remains positive for all years 1950-81, and is sometimes negative for years 1982-2011. This is a reasonable measure of US economic activity for the period.

Comparisons showing how economic activity behaves while government spending fluctuates are presented to examine whether or not there is any relationship present. Table 1 lists the ten largest absolute changes to nominal federal consumption expenditures for the years 1929-2011 when measured as a percent of nominal output. Figure 2 compares economic activity with changes to federal government consumption expenditures in the years 1936-52, the range of years with the 10 largest absolute changes to nominal federal consumption expenditure when measured as a percent of nominal output.

Figure 2 shows that during the years with the largest changes to federal consumption expenditures as a percent of nominal output, there is a positive relationship between changes in real output and changes to federal consumption expenditures. During these years the mean of federal consumption expenditures as a percent of nominal output is 11.5, with large variation. This exploration of the economic data is suggestive, but insufficient of its own to inform policy decision making. It shows that at the end of the 1930s, after roughly a decade of poor economic performance, a large and (positive) exogenous shock to federal government purchases was coincident with improved economic conditions (see Figure 1). In narrowly defined economic terms this spending was largely wasteful (i.e., purchases of munitions and other war materials), yet economic conditions improved.

Restricting analysis to one country could limit its conclusions, as there are many factors that
undoubtedly influence the measure of real gross domestic product. Additional factors may include monetary policy and inflation. Therefore, this paper will include a brief exploration of international economic data to examine if the US is a special case among developed economies with respect to the relationship between government consumption expenditures and real output.

Countries that adopted the Euro as a common currency accept identical monetary policy and similar rates of inflation. This makes the countries of this currency union especially useful to explore for comparison purposes, because conclusions based on cross-section analysis of these countries’ economic data cannot be attributed to differences in monetary policy for the period of common currency. This is a control for differences in monetary policy.

Figure 3 compares changes in real output with changes to nominal government final consumption expenditures for the years 2007-10.\(^1\) The selection of countries includes those in the economic and monetary union which adopted the Euro as common currency no later than 31 December 2006 (minus Cyprus and Malta) plus the US for comparison. Germany is plotted in yellow as a reference point, being the largest economy in the currency union. The US is plotted in blue for comparison. There is a positive relationship depicted between changes to nominal government final consumption expenditures and changes in real output. This analysis offers no support that the US is a special case with respect to the relationship between government consumption expenditures and real output.

3. REDUCED-FORM EQUATIONS.

Exploratory analysis of the economic data shows a positive relationship between changes to nominal government consumption expenditures and changes in real output, suggesting a viable role for fiscal policy. However, this cursory analysis is possibly deficient because there may exist excluded

1 Comparable data for 2011 were not available from the source at the time of this analysis. Data for Cyprus and Malta were also not available.
factors that are co-varying with changes to nominal government consumption expenditures and determining. Analysis in this section will address such potential deficiency in an attempt to reduce bias from excluded factors, as well as the potential issue of endogeneity.

The timing of large changes to nominal defense consumption expenditures, occurring when international relations break down (e.g., a war), is less likely than of total nominal federal consumption expenditures to be influenced by changes in other economic measures. Changes to nominal defense consumption expenditures will be used therefore to estimate the impact from changes to total nominal federal consumption expenditures in order to reduce bias from possible endogeneity. This approach has been used previously by Ramey.2

This paper's analysis will estimate the percent change in real output in terms of a change equal to 1% of nominal output in the economic measure analyzed.3 This is the definition of a multiplier for an economic measure henceforth herein. Equation (1) is used when estimating the short-run trade-off between output and inflation:

\[
y_t = \alpha + \beta \Delta x_t + \gamma y_{t-1} + \delta t ,
\]

where the log of real output is regressed on the change in nominal output, its own lag, and a time trend.4 This equation estimates real output as a function of change in nominal output, the assumption being that the effect not captured by real output is lost to inflation.

Taking instead the first difference of all terms and dropping the differenced time trend gives:

\[
\Delta y_t = \alpha + \beta \Delta x_t + \gamma \Delta y_{t-1} ,
\]

which has the form of a dynamic regression model. Analysis now continues by substituting relevant economic measures for change in nominal output (i.e., for the term \( \Delta x_t \) in equation (2)).

The first measures to substitute for change in nominal output are change to nominal defense

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2 Use of a reference to this work was made in another reference used.
3 The interval between observations is one year; the impact / short-run multiplier will be estimated.
4 See Ball et al. (1988).
consumption expenditures and change to nominal business investment:

\[ \Delta y_t = \alpha + \beta_1 \Delta (\text{NDC:Y})_t + \beta_2 \Delta (\text{I:Y})_t + \gamma \Delta y_{t-1}. \]

Change to nominal defense consumption expenditures is the proxy chosen for change to total nominal federal consumption expenditures, while change to nominal business investment is selected to reduce the potential for bias from excluded factors. Table 2 summarizes the OLS regression results for equation (3). Included in Table 2 are regression results for the years 1929-54 (these years include large changes to nominal defense consumption expenditures resulting from WWII and the Korean War). The estimated coefficients for defense consumption expenditures and business investment are similar. These coefficients are equivalent to the definition of multiplier stated previously and both estimates are significantly greater than one. This type of analysis could be used to inform fiscal-policy decision making.

Economic theory predicts that a fiscal multiplier will not remain constant through time. Some reasons are that government expenditure is subject to diminishing returns, that inflation will ensue per the well-known relationship expressed in the Phillips Curve, and that crowding out of private investment will occur. A static estimate of a fiscal multiplier with respect to time is therefore possibly deficient. Equation (4) allows for an estimate of a non-static fiscal impact multiplier:

\[ \Delta y_t = \alpha + \beta_1 \Delta \text{ndc}_t + \beta_2 \Delta i_t + \gamma \Delta y_{t-1}. \]

Both defense consumption expenditures and business investment as shares of nominal output are replaced in equation (4) by their logs.

The OLS regression estimates are found in Table 3. The Breusch-Godfrey Lagrange-multiplier statistic (through order 5) for the regression estimation is not significant at the 10% level given the standard null hypothesis for the years 1929-2011. The Breusch-Pagan Lagrange-multiplier statistic for the regression estimation is significant at the 5% level for the years 1929-2011; estimates of robust standard error with respect to heteroscedasticity are included.
for the years 1929-54. Given the respective levels of defense consumption expenditures and business investment during the years 1929-2011, these estimates are similar with respect to the relative size of implied impact to those from Table 2. Figure 4 shows the fiscal impact multipliers that result from the relevant estimate of equation (4) compared with the level of nominal federal consumption expenditures for the years 1947-2011.

Informed by economic theory, an inverse relationship between the fiscal impact multiplier and nominal federal consumption expenditures is depicted now. Analysis supports that for most ranges of federal government expenditures observed, fiscal policy’s estimated potential impact on total output is greater than one; for a few observed levels its estimated potential impact is less than one but greater than zero; and for no observed levels is its estimated potential impact zero (although theoretically it is possible at the zero asymptote). This particular analysis could be used to inform a fiscal-policy decision-making process.

Concurrent action by monetary authorities could have an effect on the potential impact from a policy where government expenditures are deliberately shocked. For example, if a monetary authority purchases the debt issued to pay for a fiscal response, some economic theory suggests that this would be additionally stimulative.⁷ This effect will be referred to as monetization, and equation (5) includes measures to estimate this effect:

\[
\Delta y_t = \alpha + \beta_1 \Delta h_t + \beta_2 \Delta h_{t-1} + \gamma_1 \Delta ndc_t + \gamma_2 \Delta ndc_{t-1} + \delta_1 \Delta MM_t + \delta_2 \Delta MM_{t-1},
\]

where “h” is the log of high-powered money and “MM” is the M1 multiplier. This equation is a variant of the St. Louis Equation.⁸ Table 4 shows estimates for both OLS and generalized least squares (GLS (functionally equivalent to a model with form ARMAX)) regressions for the years 1929-2011.⁹ ¹⁰

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⁷ Financial intermediaries and holders of deposits are assumed to co-influence this effect on the impact.
⁸ See Andersen and Jordan (1968).
⁹ The Breusch-Godfrey Lagrange-multiplier statistic for the OLS regression estimation is significant at the 5% level given the standard null hypothesis. The Breusch-Pagan Lagrange-multiplier statistic for the OLS regression estimation is not significant at the 10% level given the standard null hypothesis.
¹⁰ The mean estimate is deliberately omitted.
Equation (5) does not include nominal business investment as a regressor, the measure selected for inclusion to minimize bias from excluded factors. Equation (6) includes this measure:

\[
(6) \Delta y_t = \alpha + \beta_1 \Delta h_t + \beta_2 \Delta h_{t-1} + \gamma_1 \Delta ndc_t + \gamma_2 \Delta ndc_{t-1} + \delta_1 \Delta MM_t + \delta_2 \Delta MM_{t-1} + \\
\zeta_1 \Delta i_t + \zeta_2 \Delta i_{t-1}.
\]

Table 5 shows estimates for both OLS and GLS regressions for the years 1929-2011.\textsuperscript{11} \textsuperscript{12} \textsuperscript{13} This analysis supports the hypothesis that there is a joint outcome between fiscal policy and the effect of debt monetization.

4. SUMMARY.

An exploration of the economic data demonstrated a positive relationship between economic activity and government consumption expenditures. This relationship was shown in the US and among developed economies with a common currency. Cross-section analysis did not support that the US was a special case with respect to the relationship between government consumption expenditures and economic activity. The narrative attributing economic recovery from the Great Depression to a large exogenous shock to government expenditures is congruous with the economic data. It was noted that mere correlation between one economic measure and economic activity is insufficient to inform policy decision making.

Caution was taken in the analysis in order to reduce bias and increase efficiencies of the regression estimates. A proxy for government consumption expenditures that is less likely to be influenced by other economic factors was incorporated into the analysis in order to reduce the potential for bias from endogeneity. A measure of nominal business investment was included in the analysis

\textsuperscript{11} Both the Breusch-Godfrey and Breusch-Pagan Lagrange-multiplier statistics for the OLS regression estimation are significant at the 5% level given the standard null hypotheses. Estimates of robust standard error with respect to heteroscedasticity and autocorrelation are included with the OLS estimates.
\textsuperscript{12} The mean estimate is deliberately omitted.
\textsuperscript{13} Equation (6) and its corresponding regression estimates were not included in the previous version of this working paper.
specifically as a means to minimize bias from excluded factors. Given known issues of autocorrelation and heteroscedasticity with econometric analysis of time-series data, techniques were used to identify potential issues and to mitigate their effects.

The use of reduced-form equations and measures of monetary aggregates in econometric analysis is not popular currently. Instead, simulations (e.g., dynamic stochastic general equilibrium models) and policy rules incorporating interest rates are preferred. This working paper's analysis broadens the current literature by applying concepts that are no longer widely applied, yet at one time had more influence. If the approaches currently favored require reevaluation, then reduced-form equations can serve in part as a guide.

Analysis from reduced-form equations can inform a decision-making process for considering fiscal-policy response. Table 6 summarizes estimates of the fiscal impact multiplier from this working paper's analysis for the years 1947-2011. A recent work summarizes estimates of the one-year fiscal multiplier from various structural policy models; the range for the average of estimates in various scenarios given monetary accommodation is 1.2 to 1.55.

14 Averages of coefficient estimates were used.
15 See Coenen et al. (2012).
5. TABLES AND FIGURES.

(A) TABLES 1-6.

**TABLE 1:** Largest Absolute Changes to Federal Consumption Expenditures (as % of Y)

<table>
<thead>
<tr>
<th>RANK</th>
<th>YEAR</th>
<th>ABS($\Delta$(FCE:Y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1946</td>
<td>15.2</td>
</tr>
<tr>
<td>2.</td>
<td>1942</td>
<td>9.3</td>
</tr>
<tr>
<td>3.</td>
<td>1943</td>
<td>7.6</td>
</tr>
<tr>
<td>4.</td>
<td>1947</td>
<td>3.6</td>
</tr>
<tr>
<td>5.</td>
<td>1944</td>
<td>3.4</td>
</tr>
<tr>
<td>6.</td>
<td>1951</td>
<td>2.6</td>
</tr>
<tr>
<td>7.</td>
<td>1941</td>
<td>2.6</td>
</tr>
<tr>
<td>8.</td>
<td>1952</td>
<td>2.2</td>
</tr>
<tr>
<td>9.</td>
<td>1936</td>
<td>2.1</td>
</tr>
<tr>
<td>10.</td>
<td>1950</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**TABLE 2:** $\Delta y = \alpha + \beta_1 \Delta(NDC:Y) + \beta_2 \Delta(I:Y) + \gamma \Delta y_{t-1}$

<table>
<thead>
<tr>
<th>REGRESSOR</th>
<th>COEFF.</th>
<th>OLS.SE</th>
<th>H-R.SE</th>
<th>REGRESSOR</th>
<th>COEFF.</th>
<th>OLS.SE</th>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
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<td>0.00</td>
<td>0.00</td>
<td>Intercept</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta(NDC:Y)$</td>
<td>1.60</td>
<td>0.16</td>
<td>0.14</td>
<td>$\Delta(NDC:Y)$</td>
<td>1.60</td>
<td>0.28</td>
</tr>
<tr>
<td>$\Delta(I:Y)$</td>
<td>1.64</td>
<td>0.19</td>
<td>0.23</td>
<td>$\Delta(I:Y)$</td>
<td>1.58</td>
<td>0.35</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.17</td>
<td>0.07</td>
<td>0.09</td>
<td>$\Delta y_{t-1}$</td>
<td>0.19</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**TABLE 3:** $\Delta y = \alpha + \beta_1 \Delta ndc + \beta_2 \Delta i + \gamma \Delta y_{t-1}$

<table>
<thead>
<tr>
<th>REGRESSOR</th>
<th>COEFF.</th>
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<th>COEFF.</th>
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<td>0.00</td>
<td>Intercept</td>
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<tr>
<td>$\Delta ndc$</td>
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<td>0.01</td>
<td>0.02</td>
<td>$\Delta ndc$</td>
<td>0.12</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>0.06</td>
<td>0.01</td>
<td>0.01</td>
<td>$\Delta i$</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.19</td>
<td>0.07</td>
<td>0.09</td>
<td>$\Delta y_{t-1}$</td>
<td>0.20</td>
<td>0.12</td>
</tr>
</tbody>
</table>
### TABLE 4: $\Delta y = \alpha + \beta_1 \Delta h_{t} + \beta_2 \Delta h_{t-1} + \gamma_1 \Delta ndc_{t} + \gamma_2 \Delta ndc_{t-1} + \delta_1 \Delta MM_{t} + \delta_2 \Delta MM_{t-1}$

<table>
<thead>
<tr>
<th>REGRESSOR</th>
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<th>REGRESSOR</th>
<th>COEFF.</th>
<th>SE</th>
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<td>Intercept</td>
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<td>$\Delta h_{t}$</td>
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<td>$\Delta h_{t-1}$</td>
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<tr>
<td>$\Delta ndc_{t}$</td>
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<td>0.02</td>
<td>$\Delta ndc_{t}$</td>
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<td>0.02</td>
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<td>$\Delta ndc_{t-1}$</td>
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<td>0.02</td>
<td>$\Delta ndc_{t-1}$</td>
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<td>0.02</td>
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<tr>
<td>$\Delta MM_{t}$</td>
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<td>0.11</td>
<td>$\Delta MM_{t}$</td>
<td>0.68</td>
<td>0.11</td>
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<td>$\Delta MM_{t-1}$</td>
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<td>0.10</td>
</tr>
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</table>

$\sum \Delta h = \beta$ 0.20
$\sum \Delta ndc = \gamma$ 0.07
$\sum \Delta MM = \delta$ 0.43
$R^2$ 0.63

### TABLE 5: $\Delta y = \alpha + \beta_1 \Delta h_{t} + \beta_2 \Delta h_{t-1} + \gamma_1 \Delta ndc_{t} + \gamma_2 \Delta ndc_{t-1} + \delta_1 \Delta MM_{t} + \delta_2 \Delta MM_{t-1} + \zeta_1 \Delta i_{t} + \zeta_2 \Delta i_{t-1}$

<table>
<thead>
<tr>
<th>REGRESSOR</th>
<th>COEFF.</th>
<th>SE</th>
<th>R.SE</th>
<th>REGRESSOR</th>
<th>COEFF.</th>
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<td>0.00</td>
<td>Intercept</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta h_{t}$</td>
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<td>0.09</td>
<td>$\Delta h_{t}$</td>
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<td>$\Delta h_{t-1}$</td>
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<tr>
<td>$\Delta ndc_{t}$</td>
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<td>0.02</td>
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<td>$\Delta ndc_{t-1}$</td>
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<tr>
<td>$\Delta MM_{t}$</td>
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<td>0.07</td>
<td>$\Delta MM_{t-1}$</td>
<td>-0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>$\Delta i_{t}$</td>
<td>0.05</td>
<td>0.01</td>
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<td>$\Delta i_{t}$</td>
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<td>0.01</td>
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<tr>
<td>$\Delta i_{t-1}$</td>
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<td>0.01</td>
<td>0.02</td>
<td>$\Delta i_{t-1}$</td>
<td>0.00</td>
<td>0.01</td>
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</table>

$\sum \Delta h = \beta$ 0.06
$\sum \Delta ndc = \gamma$ 0.11
$\sum \Delta MM = \delta$ 0.24
$\sum \Delta i = \zeta$ 0.05
$R^2$ 0.73
TABLE 6:
Average of Estimates of Fiscal Multiplier (1947-2011)

<table>
<thead>
<tr>
<th>Static</th>
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<tr>
<td>Variable</td>
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</table>

(B) FIGURES 1-4.

FIGURE 1: Annual US Real Gross Domestic Product (1929-2011)
FIGURE 2: Annual US Δ Real GDP by Δ Federal Consumption Expenditures (1936-1952)

FIGURE 3: Δ Annual Real GDP by Δ Government Final Consumption Expenditures (2007-10)
FIGURE 4: US Federal Government Fiscal Multiplier
(1947-2011)

Nominal Federal Consumption Expenditures (% of Y)

Estimate of Fiscal Multiplier (One-year)

Nominal Federal Consumption Expenditures (% of Y)
REFERENCES USED.


Romer, Christina D. “What do we know about the effects of fiscal policy? Separating evidence from ideology.” Hamilton College, NY: Published remarks (7 November 2011).


* Use of a reference to this work was made in another reference used.
DATA SOURCES AND (ONE) IMPUTATION.

1. Federal Reserve. Monetary measure Adjusted Monetary Base (St. Louis), aggregated through averaging.
2. Federal Reserve. Monetary measure M1 from 1959 to 2011, aggregated through averaging. **
4. OECD. Data for the cross-section analysis.

** An imputation of the measure M1 for the years 1959-61 was performed in order to reduce variance resulting from different values reported by two sources.